Calibrating a regional PMP model of agricultural supply under multiple constraints: a set of matryoshka doll conditions

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1 Introduction

Positive mathematical programming (PMP) has been widely used for policy analysis, before and after its formalisation by Howitt (1995b). The methodology allows the exact calibration of agricultural production models against observed economic behavior—typically a single observation or average of observations on the allocation of inputs across activities—without the use of artificial flexibility constraints, while requiring minimal data (Howitt, 1995b; Heckelei and Wolff, 2003). PMP is often preferred to linear programming as it avoids overspecialisation and yields smooth responses to policy changes. Existing agricultural policy models that rely on PMP principles include, among others, the US Regional Environment and Agriculture Programming (REAP, formerly USMP) model (Johansson et al., 2007), the European Common Agricultural Policy Regionalised Impact (CAPRI) modelling system (capri-model.org), the Canadian Regionalized Agricultural Model (CRAM) (Horner et al., 1992), the Dutch Regionalised Agricultural Model (DRAM) (Helming, 2005), or the California StateWide Agricultural Production (SWAP) model (swap.ucdavis.edu). PMP models can accommodate various types of constraints such as resource limitations (e.g., land, water), nutrient balance constraints, and policy constraints (e.g., set-aside). They can also easily accommodate exogenous information on yields and soil processes from biophysical models such as DAYCENT (De Gryze et al., 2010), a critical advantage for the *ex ante* evaluation of agro-environmental policies. In this respect, calibrated PMP models represent a valuable tool for the analysis of climate change-related policies, such as the inclusion of agriculture in greenhouse gas emission trading schemes (Pérez Dominguez et al., 2009).

The "standard" PMP calibration procedure as outlined in Howitt (1995b) or Howitt (1995a) did not control for the model's implied supply response. In fact, the first-order calibration problem is typically underdetermined, in the sense that infinitely many sets of model parameters have the ability to exactly reproduce the observed cropping pattern, each resulting in a different supply response pattern. Over time, the literature has advocated the use of exogenous information, mostly related to costs and supply elasticities, to mitigate the underdeterminacy issue; a good survey is to be found in Heckelei and Britz (2005). More recent literature has argued in favor of incorporating available exogenous information on supply elasticities into PMP models (Heckelei and Wolff, 2003; Heckelei and Britz, 2005; Mérel and Bucaram, 2010; Mérel et al., 2011a), and the present article builds upon this strand. While some earlier models have resorted to *myopic* calibration—implicitly ignoring the change in the shadow price of the linear constraints—the focus here is on *exact* calibration. The second-order calibration problem consists of choosing a—hopefully unique—set of model parameters such that the implied own-price supply elasticities for each activity coincide with a set of exogenous values. Because exogenous information on supply elasticities typically comes from econometric studies that implicitly reflect the limitations faced by farmers, in particular the land constraint, it is important to account for the change in the shadow values of constraints in the calibration phase. Otherwise, as pointed out by Buysse et al. (2007), the model would reflect the constraints twice: once in the econometric values used for myopically calibrating model parameters, and once in the explicit constraint set. An attendant implication of this remark is that *only* those constraints that are reflected in the econometric estimates available to the analyst should be incorporated into the calibration phase. Other constraints should be ignored for calibration purposes, and reintroduced *ex post* for policy analysis. Hence, the number of constraints to be considered for calibration against supply elasticities need not be very large.[1]

Even with a few constraints, calibration against elasticity priors is not a trivial exercise. Because exact calibration takes full account of the change in the shadow prices of constraints, no crop can be calibrated independently of the others, making the calibration problem more difficult than with myopic calibration. The first hurdle was to derive a closed-form expression for the model's implied elasticities: this issue was resolved by Heckelei (2002) for the simple Leontief-quadratic model; Mérel and Bucaram (2010) provide a technique to derive model elasticities in all other cases, and here we build upon their findings.

The second hurdle arises from the form of these implied model elasticities: they are typically non-linear in the calibrating parameters. One issue germane to the exact calibration problem is thus the delineation of the set of exogenous elasticities that, conditional on the model specification and the observed reference allocation, can be reproduced by an appropriate choice of model parameters. As noted by Mérel et al. (2011a), though a single observation does not, in principle, provide information on the second-order properties of the objective function, it does put restrictions on the set of supply elasticities that can be reproduced—in a way, it *set identifies* the reproducible elasticity vector. Mérel et al. (2011a) delineate the reproducible set of supply elasticities for the fixed-proportions model with land constraint. Our purpose here is to extend their results to the more realistic case of multiple constraints.

Many existing PMP models, including the USDA REAP model, the European Commission's CAPRI model and the California SWAP model, introduce the nonlinearity in the objective function through quadratic cost adjustment terms, while the production function displays constant returns to scale. In contrast, standard microeconomic theory implies that well-specified production functions should display decreasing returns to scale for a solution function to the profit maximisation problem to exist, while cost terms should be expressed as linear functions of input use. Mérel et al. (2011a) suggest that the generalised CES model and its fixed-proportions variant, in addition to satisfying these requirements, are more flexible than their quadratic counterparts, in the sense that the set of elasticities they can reproduce is larger than—*contains*—that reproducible by quadratic models. Therefore, here we focus on the fixed-proportions model with decreasing returns to scale and linear cost. We leave the treatment of models with input substitution, such as the generalised CES model, to further work.

The article is organised as follows. In the following section, we derive the elasticity calibration system for the fixed-proportions model subject to any number of constraints. Section 3 contains the main contribution of the article: the calibration criterion for the two-constraint case. Because of space limitations, we relegate the derivation of the calibration criterion for the three-constraint model to our website.[2] Section 4 provides an empirical application. Section 5 concludes.

The calibration criteria we derive consist of a set of necessary and sufficient conditions, the

[1] Kanellopoulos et al. (2010) argue that PMP models typically include a small number of constraints.

[2] See home page.

number of which is directly related to the number of binding constraints. This suggests a tradeoff between the range of elasticity priors compatible with the reference allocation and the number of binding constraints in the model. In addition, calibration conditions are nested across models, in the sense that calibration conditions for the 3-constraint model include calibration conditions for a virtual model with two constraints, which in turn include calibration conditions for a virtual model with one constraint; hence the "matryoshka doll" conditions. The nesting structure of the calibration criteria across models implies that criteria can easily be derived for models with a larger number of constraints.

2 The calibration system

Throughout the article, the letter I denotes the number of non-zero activities in the base year, L denotes the number of inputs, and K the number of binding linear constraints, with $K < N$. For convenience, we also adopt the notation $I \equiv \{1, ..., I\}$. The constraints can reflect resource and/or policy constraints and typically include a land constraint. The input l allocated to activity i is denoted x_{il}, with the first input denoting acreage. Total output in activity i is denoted q_i. The price of crop i is p_i, and the price of input l, assumed to be the same for all activities, is denoted c_l.[3] Outputs and inputs in the reference allocation are denoted with bars. It is assumed that stage 1 of PMP, or any alternative technique deemed appropriate to recover the shadow prices of constraints, has been conducted. The resulting vector of shadow values is denoted $\bar{\boldsymbol{\lambda}} = (\bar{\lambda}_1, \ldots, \bar{\lambda}_K)$.

As in Mérel et al. (2011a), the production function for activity i is assumed to exhibit decreasing returns to scale, and here we assume that inputs contribute to each activity in fixed proportions, that is

$$q_i = \alpha_i \left[\min \left(x_{i1}, \frac{x_{i2}}{\mu_{i2}}, ..., \frac{x_{iL}}{\mu_{iL}} \right) \right]^{\delta_i}$$

for positive input coefficients $\mu_{il} = \frac{\bar{x}_{il}}{\bar{x}_{i1}}$ and technological parameters $\alpha_i > 0$ and $\delta_i \in (0, 1)$. For notational convenience, we write $x_i \equiv x_{i1}$, and denote $\mathbf{x}$ the $I \times 1$ vector of acreages.

Given fixed proportions, the per-acre cost is $C_i = \sum_{l \in L} c_l \mu_{il}$. We assume that the constraints are linear in the input levels x_{il}, which ensures, given fixed proportions, that the constraint set can be written $\mathbf{Ax} = \mathbf{v}$, where $\mathbf{A} = (a_{k,i})$ is the matrix of constraint coefficients and $\mathbf{v}$ a $K \times 1$ vector of real numbers.[4]

The optimisation program can then be written as

$$\max_{\mathbf{x} \geq 0} \quad \sum_i p_i \alpha_i x_i^{\delta_i} - (C_i + \lambda_{2i})\, x_i \quad \text{subject to} \quad \mathbf{Ax} = \mathbf{v} \quad [\boldsymbol{\lambda}]. \tag{1}$$

[3] This assumption is not crucial to the model.

[4] This assumption somewhat restricts the nature of the constraints that can be imposed. For instance, an output quota cannot be easily modeled in this framework, because it would require $q_i \leq Q_i$ for some quota level Q_i, an expression that is nonlinear in x_i due to decreasing returns to scale.

The set of exogenous supply elasticities is denoted $\bar{\boldsymbol{\eta}} = (\bar{\eta}_1, \ldots, \bar{\eta}_I)$.[5]

It can be shown that the calibration problem is recursive, so that the model can be calibrated in two steps. First, the analyst chooses the vector of returns to scale parameters $\boldsymbol{\delta} = (\delta_1, ..., \delta_I)$ that replicates the set of prior elasticities $\bar{\boldsymbol{\eta}}$ at the reference allocation. Second, the analyst chooses the parameters $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_I)$ and the calibrating parameters $\boldsymbol{\lambda}_2 = (\lambda_{21}, \ldots, \lambda_{2I})$ to replicate the reference allocation $(\bar{\mathbf{q}}, \bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})$.[6] Hence, the calibrating parameters $\boldsymbol{\delta}$ are independent from the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}_2$.

By applying the implicit function theorem, one can derive the set of calibrating equations for the supply elasticities (evaluated at the reference allocation) as[7]

$$\bar{\boldsymbol{\eta}} = \text{VecDiag}\left[\mathbf{D}(\mathbf{I}_I - \mathbf{A}^T(\mathbf{A}\boldsymbol{\Delta}\mathbf{A}^T)^{-1}\mathbf{A}\boldsymbol{\Delta})\right] \tag{2}$$

where the operator VecDiag creates an I-vector out of the I diagonal elements of an $I \times I$ matrix, $\boldsymbol{\Delta}$ is the $I \times I$ diagonal matrix with typical element $\frac{b_i}{\delta_i(1-\delta_i)}$ with $b_i \equiv \frac{\bar{x}_i^2}{p_i \bar{q}_i}$, $\mathbf{D}$ is the $I \times I$ diagonal matrix with typical element $\frac{\delta_i}{1-\delta_i}$ and $\mathbf{I}_I$ is the $I \times I$ identity matrix.

Using Lemma 1 and Lemma 2 in Mérel and Bucaram (2010), we can rewrite the calibration system (2) as

$$\forall i \in I \quad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i}\left[1 - \frac{\sum\limits_{\{j_1,\ldots,j_{I-K}\} \in \mathcal{P}_{I-K}(I_{-i})} \det(\mathbf{A}_{-j_1,\ldots,j_{I-K}} \boldsymbol{\Delta}_{-j_1,\ldots,j_{I-K}} \mathbf{A}^T_{-j_1,\ldots,j_{I-K}})}{\sum\limits_{\{j_1,\ldots,j_{I-K}\} \in \mathcal{P}_{I-K}(I)} \det(\mathbf{A}_{-j_1,\ldots,j_{I-K}} \boldsymbol{\Delta}_{-j_1,\ldots,j_{I-K}} \mathbf{A}^T_{-j_1,\ldots,j_{I-K}})}\right] \tag{3}$$

where I_{-i} denotes the set $\{1, \ldots, i-1, i+1, \ldots, I\}$, $\mathcal{P}_{I-K}(I)$ denotes the set of $(I-K)$-combinations (without repetition) of elements of set I, $\mathbf{A}_{-j_1,\ldots,j_{I-K}}$ denotes the $K \times K$ matrix obtained from deleting the columns $j_1, \ldots, j_{I-K}$ from matrix $\mathbf{A}$, and similarly for $\boldsymbol{\Delta}_{-j_1,\ldots,j_{I-K}}$. Note that since these matrices are square, all determinants in (3) can be written as products of determinants.

The elasticity calibration system (3) is nonlinear in the unknown parameters $\boldsymbol{\delta}$. It does not depend on $\boldsymbol{\alpha}$ or $\boldsymbol{\lambda}_2$, meaning that calibration against supply elasticities can be conducted separately from calibration against the reference allocation. This is not to say that the calibrated parameters $\boldsymbol{\delta}$ will be independent from the reference allocation: in fact, the quantities $b_i = \frac{\bar{x}_i^2}{p_i \bar{q}_i}$, which represent the ratio of observed acreage to per acre gross revenue, appear in the matrices $\boldsymbol{\Delta}_{-j_1,\ldots,j_{I-K}}$.

Because it is nonlinear, system (3) typically has more than one solution for the unrestricted set of parameters $\boldsymbol{\delta}$. (It is equivalent to a polynomial system in the parameters δ_i.) However, we show in the following section that at most one solution lies within the restricted set $(0,1)^I$, hereafter referred to as the *acceptable range*. The solution to (3) that lies in the acceptable range is called the

[5] Throughout the article, the notation $(x_1, ..., x_I)$ (with commas separating elements) always denotes a column vector.

[6] The calibrating parameter $\boldsymbol{\lambda}_2$ is typical of PMP models and is necessary to replicate the reference allocation. It is heuristically justified as a unobserved cost component.

[7] See home page for the derivation.

acceptable solution. We derive the necessary and sufficient conditions under which an acceptable solution exists. That is, we delineate the set of supply elasticities $\bar{\boldsymbol{\eta}}$ that are "compatible" with the reference allocation, in the sense that at this reference allocation, the model can exactly reproduce the supply elasticities. We explicitly do so for the case $K = 2$, and refer the reader to the online material for the case $K = 3$.

Our main results take the form of two propositions and one corollary. The first proposition is entirely proven analytically in the working paper by Mérel et al. (2011a). For the other proposition and the corollary, we provide analytical proofs whenever possible, and numerical evidence based on an extremely large number of simulations otherwise.[8] The reason why analytical proofs are not always available is that most of the calibration conditions involve mathematical objects that are defined as implicit functions of the data. When these objects cannot be expressed explicitly as functions of the data, it becomes extremely difficult—if not impossible—to show the results analytically. The fact that the calibration conditions involve implicit functions of the data does not affect their practical usefulness, as these functions can be solved by any nonlinear solver.

3 The calibration conditions

3.1 The case $K = 0$

When there are no constraints, the elasticity calibration system is degenerate and has the form

$$\forall i \in I \quad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i}. \tag{4}$$

We will denote this system $\mathcal{S}^0(I, \bar{\boldsymbol{\eta}})$. The first argument denotes the size of the system, and the remaining vector the parameters that enter the system. $\mathcal{S}^0(I, \bar{\boldsymbol{\eta}})$ is trivially solved by choosing, for all $i \in I$, $\delta_i = \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$, a value that automatically lies within the acceptable range. There are no restrictions on the set of elasticities that can be reproduced, and the solution is unique. Therefore, as long as $I \geq 1$ and $\bar{\boldsymbol{\eta}} >> 0$, the system $\mathcal{S}^0(I, \bar{\boldsymbol{\eta}})$ always has a unique acceptable solution.

3.2 The case $K = 1$

With one constraint written $\sum_{i \in I} a_{1,i} x_i = v_1$, the calibration system (3) specialises to

$$\forall i \in I \quad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i} \left[1 - \frac{\frac{a_{1,i}^2 b_i}{\delta_i (1-\delta_i)}}{\sum_{j \in I} \frac{a_{1,j}^2 b_j}{\delta_j (1-\delta_j)}} \right].$$

[8]Due to page limitations, we relegate all analytical proofs and numerical simulation results to our website, home page. All simulations were run in MATLAB.

Defining $B_i \equiv a_{1,i}^2 b_i$, this system can be rewritten as

$$\forall i \in I \quad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i}\left[1 - \frac{\frac{B_i}{\delta_i(1-\delta_i)}}{\sum\limits_{j\in I}\frac{B_j}{\delta_j(1-\delta_j)}}\right]. \tag{5}$$

Since system (5) depends on the parameters $(B_i)_{i\in I}$ in addition to the vector of elasticities $\bar{\boldsymbol{\eta}}$, we denote it $\mathcal{S}^1\left(I, (B_i)_{i\in I}, \bar{\boldsymbol{\eta}}\right)$. (The superscript "1" indicates that there is one constraint.)

For $i \in I$, we denote $\bar{\boldsymbol{\eta}}_{-i} \equiv (\bar{\eta}_1, \ldots, \bar{\eta}_{i-1}, \bar{\eta}_{i+1}, \ldots, \bar{\eta}_I)$. Throughout the article, the notation $\mathbf{d}_{-i} \equiv (d_1, \ldots, d_{i-1}, d_{i+1}, \ldots)$ denotes an $(I-1)$-vector for which the components d_j are indexed in increasing order but skipping the component d_i. Given the set I and the family of coefficients $(B_i)_{i\in I}$, for each element $i \in I$ we introduce the following function of the vector $\mathbf{d}_{-i} \in (0,1)^{I-1}$:

$$\mathcal{R}_i^1\left(\mathbf{d}_{-i}; I, (B_i)_{i\in I}\right) = \frac{\sum\limits_{j\in I_{-i}}\frac{B_j}{d_j(1-d_j)}}{B_i}.$$

Proposition 1 *(Mérel et al., 2011a). Suppose that $I \geq 2$. For $i \in I$, denote $\hat{\boldsymbol{\delta}}_{-i}$ the unique acceptable solution to the system $\mathcal{S}^0(I-1, \bar{\boldsymbol{\eta}}_{-i})$. The calibration system $\mathcal{S}^1\left(I, (B_i)_{i\in I}, \bar{\boldsymbol{\eta}}\right)$ has an acceptable solution if and only if, for all $i \in I$*

$$\bar{\eta}_i < \mathcal{R}_i^1\left(\hat{\boldsymbol{\delta}}_{-i}; I, (B_i)_{i\in I}\right).$$

When this condition, to be denoted $\mathcal{C}_i^1\left(I, (B_i)_{i\in I}, \bar{\boldsymbol{\eta}}\right)$, is satisfied for all $i \in I$, the acceptable solution $\boldsymbol{\delta}$ is unique and satisfies $\delta_i \geq \frac{\bar{\eta}_i}{1+\bar{\eta}_i}$ for all $i \in I$.

Corollary 1 *The condition $\mathcal{C}_i^1\left(I, (B_i)_{i\in I}, \bar{\boldsymbol{\eta}}\right)$ is either satisfied for all $i \in I$ (which is equivalent to saying that $\mathcal{S}^1\left(I, (B_i)_{i\in I}, \bar{\boldsymbol{\eta}}\right)$ has a unique acceptable solution) or violated by at most one $i \in I$.*[9]

3.3 The case $K = 2$

When $K = 2$, the calibration system (3) specialises to

$$\forall i \in I \quad \bar{\eta}_i = \frac{\delta_i}{1-\delta_i}\left[1 - \frac{\sum\limits_{j\in I_{-i}}\frac{B_{ij}}{\delta_i(1-\delta_i)\delta_j(1-\delta_j)}}{\sum\limits_{\{j,k\}\in\mathcal{P}_2(I)}\frac{B_{jk}}{\delta_j(1-\delta_j)\delta_k(1-\delta_k)}}\right] \tag{6}$$

where $B_{ij} = b_i b_j \det(\mathbf{A}_{i,j})^2$, with $\mathbf{A}_{i,j}$ denoting the 2×2 matrix obtained by keeping the columns of matrix $\mathbf{A}$ with indices i and j. (Note that the order of i and j in the submatrix $\mathbf{A}_{i,j}$ does not matter here since the determinant is squared. We have $B_{ii} = 0$ and $B_{ij} = B_{ji}$.)

[9] The analytical proof of Corollary 1 is trivial given that $\hat{\boldsymbol{\delta}}_{-i} = \left(\frac{\bar{\eta}_1}{1+\bar{\eta}_1}, \ldots, \frac{\bar{\eta}_{i-1}}{1+\bar{\eta}_{i-1}}, \frac{\bar{\eta}_{i+1}}{1+\bar{\eta}_{i+1}}, \ldots, \frac{\bar{\eta}_I}{1+\bar{\eta}_I}\right)$.

Since system (6) depends on the family of parameters $(B_{ij})_{(i,j)\in I^2}$ and the elasticities $\bar{\boldsymbol{\eta}}$, we denote it $\mathcal{S}^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$, where the superscript "2" indicates a calibration system for a 2-constraint model.

Given the set I and the family of coefficients $(B_{ij})_{(i,j)\in I^2}$, for each element $i\in I$ we introduce the following function of the vector $\mathbf{d}_{-i}\in(0,1)^{I-1}$:

$$\mathcal{R}_i^2\left(\mathbf{d}_{-i};I,(B_{ij})_{(i,j)\in I^2}\right)=\frac{\sum\limits_{\{j,k\}\in\mathcal{P}_2(I_{-i})}\frac{B_{jk}}{d_j(1-d_j)d_k(1-d_k)}}{\sum\limits_{j\in I_{-i}}\frac{B_{ij}}{d_j(1-d_j)}}.$$

For $(i,j)\in I^2$, $i<j$, we denote $\bar{\boldsymbol{\eta}}_{-i,j}\equiv(\bar{\eta}_1,\ldots,\bar{\eta}_{i-1},\bar{\eta}_{i+1},\ldots,\bar{\eta}_{j-1},\bar{\eta}_{j+1},\ldots,\bar{\eta}_I)$. We also denote by $\mathbf{d}_{-i,j}$ an $(I-2)$-vector for which the components d_k are indexed in increasing order but skipping the components d_i and d_j.

We now derive the main proposition of this article, which establishes the conditions under which system $\mathcal{S}^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$ has a (unique) acceptable solution.

Proposition 2 *Suppose that $I\geq 3$. For $\{i,j\}\in\mathcal{P}_2(I)$, denote $\hat{\boldsymbol{\delta}}_{-i,j}$ the unique acceptable solution to $\mathcal{S}^0(I-2,\bar{\boldsymbol{\eta}}_{-i,j})$. The calibration system $\mathcal{S}^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$ has an acceptable solution if and only if, for all $i\in I$*

(i) if $\mathcal{C}_j^1\left(I-1,(B_{ij})_{j\in I_{-i}},\bar{\boldsymbol{\eta}}_{-i}\right)$ is violated for some (necessarily unique, see Corollary 1) $j\in I_{-i}$, then $\bar{\eta}_i<\mathcal{R}_i^1\left(\hat{\boldsymbol{\delta}}_{-i,j};I-1,(B_{ij})_{i\in I_{-j}}\right)$

(ii) if $\mathcal{S}^1\left(I-1,(B_{ij})_{j\in I_{-i}},\bar{\boldsymbol{\eta}}_{-i}\right)$ has a (necessarily unique, see Proposition 1) acceptable solution, denoted $\hat{\hat{\boldsymbol{\delta}}}_{-i}$, then $\bar{\eta}_i<\mathcal{R}_i^2\left(\hat{\hat{\boldsymbol{\delta}}}_{-i};I,(B_{ij})_{(i,j)\in I^2}\right)$.

When this condition, to be denoted $\mathcal{C}_i^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$, is satisfied for all $i\in I$, the acceptable solution $\boldsymbol{\delta}$ is unique and satisfies $\delta_i\geq\frac{\bar{\eta}_i}{1+\bar{\eta}_i}$ for all $i\in I$.[10]

3.4 Interpretation

Let us first try to visualise the calibration region. For the case $I=2$ and $K=1$, Figure 1 depicts the calibration region when $B_1=B_2$. The shape of the calibration region is that delineated by the outer portions of two cones. Calibration is feasible below the bottoms of the cones (both elasticities are small enough), close enough to one of the axes (one elasticity is small enough, the other being large) and in the tunnel delineated by the two cones (both elasticities can be large but one cannot be too large compared to the other). The infeasible region may thus be characterised by the fact that one of the elasticities is neither small enough, nor large enough, compared to the other one. When $B_1\neq B_2$ the calibration region becomes dissymmetric but is still delineated by the outer portions of two cones.

[10] We proved analytically the sufficiency of conditions $\mathcal{C}_i^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$, $i=1,\ldots,I$. Using MATLAB, we showed numerically the necessity of conditions $\mathcal{C}_i^2\left(I,(B_{ij})_{(i,j)\in I^2},\bar{\boldsymbol{\eta}}\right)$, as well as the uniqueness result.

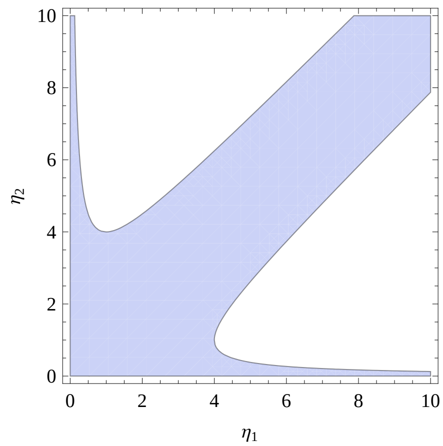
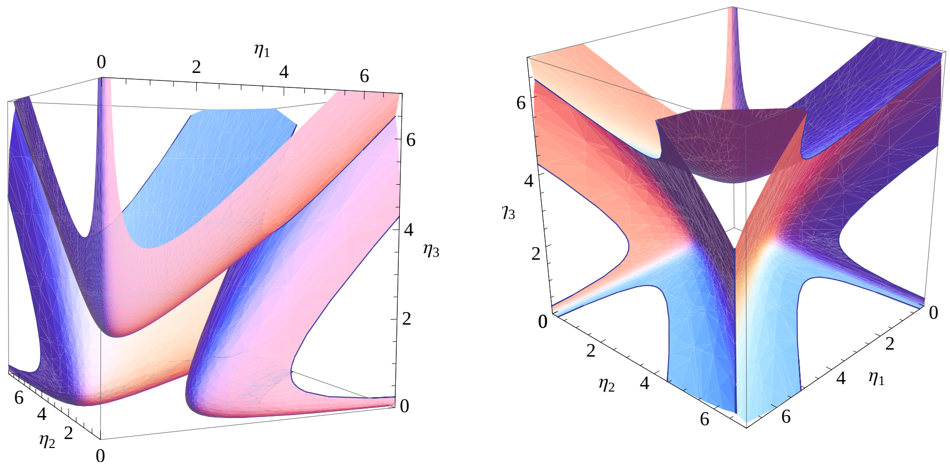


Figure 1: Calibration region for $I = 2$ and $K = 1$

Now consider the case $I = 3$ and $K = 2$, with parameters $B_{12} = 329, B_{13} = 337, B_{23} = 353$. (These parameters were generated from an initial matrix $\mathbf{A}$ and a set of values for b_1, b_2 and b_3.) Figure 2 depicts the feasible region determined by the calibration criterion of Proposition 2. A unique solution to the calibration system exists when the elasticities lie below the cones in the bottom right corner of panel (a), close enough to one of the axes, or in the tunnel delineated by the three cones in panel (b). This calibration region is the natural 3-dimensional extension of the one for $I = 2$ crops and $K = 1$ constraints depicted in Figure 1.

The existence of restrictions on the set of reproducible elasticities, conditional on the reference allocation, as well as the uniqueness property—when calibration is feasible,—have fundamental implications for the calibration of programming models of agricultural supply. Unlike first-order calibration against the reference allocation, which, as shown for instance in Howitt (1995b), is *always* feasible (a desirable property) and can be achieved in *many* ways (an undesirable property), second-order calibration against exogenous supply responses is *only sometimes* feasible, yet when it is it can only be achieved in *one* way.

Besides the obviously useful uniqueness result, being able, *ex ante*, to determine whether a given set of elasticity priors lies in the calibration region is far from being a luxury. Imagine for instance that the analyst wishes to replicate the set of supply elasticities $\bar{\boldsymbol{\eta}}$, and finds out that it lies outside of the calibration region. Being able to delineate the calibration region will allow the analyst to (i) identify precisely the source of the infeasibility and (ii) modify the prior in the least costly way in order to fall back into the calibration region. Here, "least costly" should be understood in two ways: first, the analyst will know where to search, as opposed to proceeding by trial and error until he finds a feasible elasticity vector. Second, and perhaps more importantly, the analyst will not "overshoot": he will be able to reach the calibration region while *minimising the departure* from the elasticity prior. This could be done, for instance, by solving a simple generalised maximum entropy problem. In the next section we present a concrete example.



(a) (b)

Figure 2: Calibration region for $I = 3$ and $K = 2$

4 Empirical implementation

Consider the simple three-crop, two-constraint model of California agriculture used in Howitt (1995a).[11] The three crops are cotton (C), wheat (W) and rice (R) and the four inputs are land (1), water (2), capital (3) and chemical inputs (4). In Howitt (1995a), California faces two binding resource constraints: land and water. The elasticity prior is $\bar{\boldsymbol{\eta}} = (\bar{\eta}_C, \bar{\eta}_W, \bar{\eta}_R) = (0.47, 0.40, 0.80)$. (These elasticities are the ones used in the California SWAP model.) The parameters B_{ij} are easily constructed using the information provided in Howitt (1995a).

Using the calibration criterion of Proposition 2, we find that part (ii) of condition $\mathcal{C}_i^2\left(3, (B_{ij}), \bar{\boldsymbol{\eta}}\right)$ is violated for rice, because $\bar{\eta}_R > \mathcal{R}_R^2\left(\hat{\hat{\boldsymbol{\delta}}}_{-R}; 3, (B_{ij})\right) = 0.54$. Therefore, given the reference allocation, it is not possible to exactly calibrate the model's implied supply elasticities against the prior $\bar{\boldsymbol{\eta}}$. Figure 3 shows that the elasticity prior (represented by the red dot) indeed lies outside the calibration region.

Solving a simple generalised maximum entropy (GME) program with equidistant support points for each crop, such that η_i^{GME} is restricted to lie in an interval $[\bar{\eta}_i - \gamma, \bar{\eta}_i + \gamma]$, we obtain the elasticity vector $\boldsymbol{\eta}^{GME} = (0.43, 0.42, 0.55)$ that now falls in the calibration region. Because the support intervals of all elasticities have the same width and are centered on the prior, the GME solution in this case yields the feasible elasticity vector that lies closest to the prior $\bar{\boldsymbol{\eta}}$ in the sense of the Euclidian distance. If the analyst had more confidence in one of the elasticities, departure from that particular elasticity could be reduced by narrowing its support interval. Figure 3 shows

[11] The program written in GAMS is available upon request.

the GME projection of the prior onto the calibration region (the green dot). The associated unique parameter vector can then be found by solving system (6) numerically, with $\bar{\boldsymbol{\eta}}$ replaced by $\boldsymbol{\eta}^{GME}$.

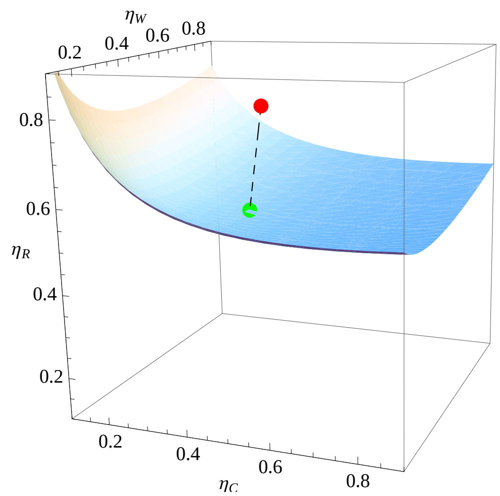


Figure 3: GME projection of the prior onto the calibration region

5 Conclusion

Recent literature has advocated the use of exogenous information on supply responses to calibrate agricultural production models. However, unlike first-order calibration against a reference allocation, second-order calibration is not always feasible. In this paper, we extended the results of Mérel et al. (2011a) to fixed-proportions models with decreasing returns to scale and linear cost subject to multiple constraints. We first generalised the elasticity calibration system to models with any number of constraints $K \geq 1$. Then, we explicitly derived the calibration criterion under which a unique acceptable solution to the calibration system exists in the case $K = 2$. (The calibration criterion for $K = 3$ is available at home page.)

As illustrated in the previous section, the availability of a calibration criterion allows for a systematic rationalisation of the second-order calibration of agricultural supply models.

Another potentially important application of these criteria is to permit the disaggregation of econometric estimates of supply elasticities down to the regional level. Often, econometric estimates of supply responses are only available at an aggregate level, typically state- or nation-wide. Yet, there is growing interest in modelling *regional* production patterns, in particular to better account for local environmental conditions through the coupling of programming models with biophysical models and soil process models (Godard et al., 2008; Mérel et al., 2011b). Wherever a second-order calibration criterion is violated, the analyst will need to depart from the initial, statewide prior in order to fall into the calibration region. Because the calibration criterion involves region-specific parameters (the "B" parameters), it is different for each region and will thus lead to different elasticity vectors in each region where it is violated. The existence of restrictions on the set of reproducible supply elasticities can thus be exploited to generate regional variation in agricultural supply responses.

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