Measuring flexibility of multi-output firms: a primal and a dual measure

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1 INTRODUCTION

Enterprises in all sectors of the economy including agriculture are faced with changing economic, legal, and political conditions. The competitiveness of a firm and even the long term survival depend not only on its efficiency or productivity, but also on its ability to respond to these changes with adjustments in their production programs. However, output adjustment is often associated with an increase in average costs of production at firm level. A flexible and adaptable production technology is required to meet this challenge. In this context, flexibility can be considered as a crucial factor of competitive advantage.

The aim of this paper is to develop a primal and a dual flexibility measure for multi-product firms, which can be obtained by estimating both cost and input distance functions. Our measure is formulated for both short and long run functions and can be decomposed into three effects, which appears useful in investigating possible sources of flexibility of a firm. In our empirical implementation, we show how this decomposition can be used for flexibility analysis drawing on individual data of Polish farms.

Researchers have been interested in firms' flexibility since the topic was introduced in literature by STIGLER (1939). He defined flexibility as those attributes of cost curves that determine how responsive output decisions are to demand fluctuations. He discussed flexibility in terms of the relative convexity (the second derivative) of the average cost curve. Thus, the flatter the average curve the greater the flexibility. Therefore, in line with Stigler, we consider flexibility as an extent of average cost changes in response to output variations. Since firms are not likely to be single-output-producers and in most cases and especially in agriculture produce more than one output, we use a flexibility measure for the multi-output case which was recently proposed by CREMIEUX ET AL. (2004) in their comparison study of hospital services. Based on a procedure similar to that of CHAVAS AND KIM (2010) we decompose flexibility into three components. In doing so, we distinguish between cost response associated with changing the production level of individual products (scale effect), cost response associated with the product-line diversification (scope effect or complementarity effect), and cost response associated with the growth of the marginal costs (concavity effect). The proposed measure allows analyzing both the interdependence between scope, scale and concavity effects as well as their contribution to the overall flexibility of the firm. Further we distinguish between a short-run and long-run flexibility measure. In the short run, the flexibility index is based on the elasticities of the variable cost function conditional on a given level of fixed factors, while in the long run all inputs are variable. Using a dual production technology we present two alternative indices given as elasticities of the cost and the dual input distance function, both for the short and the long run. To obtain the dual flexibility measure we make use of the dual scale and scope measures proposed in economic literature (FÄRE ET AL. (1986), HAJARGASHT ET AL. (2008)). The advantage of the dual flexibility measure is that flexibility can be derived when an econometric estimation of the cost function is not possible due to data availability or other problems which we discuss further.

The structure of this paper is as follows. Next section introduces and discusses the flexibility measure based on the average cost function. This measure is formulated for the single-output and multi-product case. The economic interpretation of the flexibility measure and its components is discussed in section 3. Using relationships between variable and long run total cost functions we derive a long run flexibility measure in section 4. In section 5 we develop a dual flexibility measure for both the short and the long run, which can be empirically estimated using elasticities of the short run input distance function. Empirical implementation and econometric issues are discussed in section 6. In section 7 we introduce the data set used for our study, discuss estimated results and present conclusions.

2 DEFINITION AND MEASURE OF FLEXIBLITY

Following STIGLER (1939), we consider flexibility as an attribute of production technology to accommodate output variations at lower costs. According to Stigler's definition, flexibility varies inversely with the curvature of the average cost curve. Assuming U-shaped average cost curves this means that the steeper average costs rise around their minimum point, the less flexible is the production technology of the firm. In contrast to more flexible technologies with flatter average cost curves, such a technology will lead to larger changes in the average costs caused by changes in output levels. Because the curvature of a curve is measured by the second derivative, the firm is considered to be more flexible the smaller the second derivative of its average cost function with respect to the output. Thus, the measure for the single-output case can be formally expressed as follows:

$$Flex = \frac{\partial (C/y)}{\partial y} = \frac{C_{yy}}{y} + \frac{2C}{y^3}(1-\varepsilon_y^C)\,, \qquad (1)$$

where C is a cost function, satisfying the usual homogeneity, monotonicity and curvature properties (see CHAMBERS, 1997). Further, C_{yy} is the second order derivative of the cost function with respect to the output y, and $\varepsilon^C{}_y$ is the cost elasticity with respect to the output[1].

The existing flexibility literature is mainly focused on the *single-output case*, which is insufficient when production is highly diversified. For this reason, the definition and measure of flexibility have to be extended for the *multi-product case*. One possibility is provided by CREMIEUX ET AL. (2005). Their measure is based on the concept of ray average cost by BAUMOL ET AL. (1998). Ray average cost is characterized by the cost change induced by proportional changes in all outputs along a particular output ray. These proportional changes in the output set are determined by the positive scalar t.[2] In analogy to the single-output case, flexibility is measured by the second derivative of the ray average cost with respect to the scalar t:

$$Flex = y' C_{yy} y + 2C \cdot \left(1 - \sum_{j=1}^{J} \varepsilon_{y_j}^C\right), \qquad (2a)$$

where C_{yy} is the Hessian matrix of second order derivatives of the cost function with respect to output y_j and $\varepsilon^C{}_{yj}$ is the partial cost elasticity with respect to output y_j.

Smaller values of the second derivative of the average cost function with respect to output correspond to flatter average cost curves. Thus, lower values of Flex imply more flexible technologies.

3 ECONOMIC INTERPRETATION

The multi-product flexibility index (2) measures the ease with which a firm can respond to a change in demand for one or more of its outputs. The more technology allows it to reallocate inputs in order to reduce unit costs by changes in the production program, the more the firm benefits from flexibility. Using the Flex-Index we can distinguish between different sources of these flexibility benefits. As follows from equation (2a), the degree of flexibility depends on the first and second order derivative of the initial cost function with respect to different

[1] Cost elasticity is defined as the percentage change in costs caused by a 1percent increase of output: $\varepsilon_y^C = \frac{\partial C}{\partial y} \cdot \frac{y}{C}$.

[2] The change in the outputs has not to be proportional for all products. Indeed, the positive scalar t could be replaced by the function reflecting the reaction of a certain output to relative price changes on the product market.

outputs. For the purpose of a more detailed analysis of the sources of flexibility we divide the matrix of second order derivatives of the cost function with respect to outputs into two matrices: $C_{yy} = C_{yy}^{D} + C_{yy}^{-D}$ where C_{yy}^{D} is the diagonal matrix containing only the diagonal elements of C_{yy} and, correspondingly, matrix C_{yy}^{-D} containing off diagonal elements of C_{yy} and zero values on the main diagonal.

$$Flex = y' C_{yy}^{-D} y + y' C_{yy}^{D} y + 2C \cdot \left(1 - \sum_{j=1}^{J} \varepsilon_{y_j}^{C}\right). \tag{2b}$$

Thus, the multiproduct flexibility measure in (2b) can be decomposed into three additive terms: scope (or complementarity) effect, concavity effect and scale effect. Considering the role of scope and scale economies, as well as concavity properties of the production technology, this decomposition provides useful insights on possible sources of flexibility for the multiproduct firm.

1. *Scope effect*

The first component on the right hand side of the measure (2b) reflects the scope effect (or complementarity effect). It can be obtained through the multiplication of the off diagonal elements of the Hessian matrix by the corresponding output vectors.

$$F\_scope = y' C_{yy}^{-D} y. \tag{3}$$

The scope effect measures cost savings which can be achieved by using economies of scope which result in lower unit costs due to diversification. The term economies of scope refers to cost reductions through the production of a variety of products rather than specializing in the production of single output. Positive economies of scope may arise from the sharing or joint utilization of input resources by diversified production technologies and lead to reductions in unit costs. On the other hand, if the joint production of two outputs is more costly than independent production by two independent firms, there exist diseconomies of scope. According to BAUMOL ET AL (1998), economies of scope depend on the signs of the second order derivatives of the multiproduct cost function with respect to outputs. Negative values denote weak cost complementarities among two outputs, which is a sufficient condition for the existence of economies of scope. Indeed, complementarity in outputs implies a reduction of the marginal cost of a particular output when the production of another output is increased.[3] Thus, the scope effect indicates how the utilization of economies of scope contributes to the ability of a firm to meet changes in demand fluctuations through the adjustment of its output levels. Lower values of *F_scope* correspond to flatter average cost curves and imply more flexible production technologies.

2. *Concavity effect*

The second term can be interpreted as concavity effect. It considers diagonal elements of the Hessian matrix, which can take on negative as well as positive values.

$$F\_concave = y' C_{yy}^{D} y. \tag{4}$$

[3] Examples of cost complementarity are the allocation of land, labor and management resources across farm activities and enterprises within a given period, so that these inputs can be utilized more efficiently. Further example is the production technology where products form one enterprise can be used as inputs to another. The use of manure as an organic fertilizer in crop production or the utilization of crop residues and products in animal feeding are the examples from agriculture.

The concavity effect is based on the curvature concept since the sign of $F\_concave$ depends on the concavity or convexity property of the cost function in y.[4] Decreasing marginal costs with respect to output j generate a negative sign for $F\_concave$, leading to a concave cost function in y_j, that contributes to higher flexibility, and vice versa. Therefore, the lower the growth rate in the cost increase, the more flexible the production technology.

3. Scale effect

Finally, the third component corresponds to the concept of economies of scale associated with the relationship between total (variable) cost and output changes, hereinafter referred to as scale effect:

$$F\_Scale = 2C \cdot \left(1 - \sum_{j=1}^{J} \varepsilon_{y_j}^{C}\right). \tag{5}$$

Scale elasticity measures the proportional increase in cost resulting from a proportional increase in the level of output. According to some definitions in the literature (cf. BROWN ET AL. (1979), CHRISTENSEN AND GREENE (1976)), scale elasticity may be expressed as unity minus the sum of cost elasticities with respect to outputs, which results in positive values for economies of scale and negative values for diseconomies of scale:

$$Scale = 1 - \sum_{j=1}^{J} \varepsilon_{y_j}^{C} . \tag{6}$$

Thereby, the scale effect in (5) measures the cost responsibility associated with intensification of production (increase in the output level of individual products) due to economies of scale. Firms which show higher economies of scale in the sum of all outputs possess steeper ray average cost curves, associated with inflexible production technology.

4 LONG-RUN FLEXIBILITY MEASURE

Further one can distinguish between *short-run* and the *long-run* flexibility measures. In the short run, some of the inputs are considered as quasi-fixed factors. Thus, the short-run flexibility measure is based on the variable cost function, derived from the cost minimization problem for the given values of quasi-fixed factors:

$$VC^{s}(w, y, k) = \min_{x>0} \left\{ w \cdot x \middle| x \in V(y, k) \right\}$$

with $\boldsymbol{x}$ – vector of variable inputs, $\boldsymbol{y}$ – vector of outputs, $\boldsymbol{w}$ – vector of input prices, $\boldsymbol{k}$ – vector of quasi-fixed factors.

For the long run analysis, we derive the flexibility measure from the long-run total cost function which corresponds to minimizing long-run total costs considering fixed costs as follows:

$$\min_{k} TC^{l}(y, w, k) = VC^{s}(w, y, k) + r\,'k \qquad \text{with } r \text{ - vector of the quasi-fixed factor prices.}$$

In the long run equilibrium, a firm minimizes its long-run total cost by choosing the optimal value for the quasi-fixed factor $\boldsymbol{k}$ where market prices of the quasi-fixed factors equal their shadow prices, which are defined as the derivative of the variable cost functions with respect to the quasi-fixed factor $r_m = -\partial VC^s/\partial k_m$. Thus, in terms of elasticities the long-run total cost function can be rewritten as:

[4] Färe and Lehmijoki (1987) argue, that necessary and sufficient condition for cost function to be quasi-convex in output is the homotheticity of the input correspondence.

$$TC^{l} = VC^{s}\left(1-\sum_{m}\varepsilon_{\kappa_m}^{VC}\right) \text{ with } \varepsilon_{\kappa_m}^{VC} = \frac{\partial VC^{s}}{\partial k_m}\frac{k_m}{VC^{s}} = \frac{\partial \ln VC^{s}}{\partial \ln k_m}. \tag{7}$$

Based on this relationship we can derive both a flexibility measure and its decomposed components – concavity, scope and scale effect in the long run.
Inserting the optimal solution of the cost minimization problem – optimal quasi-fixed factor demand functions *k*(y,w,r)* - into the long-run cost function and applying the envelope theorem by deriving *TC* with respect to *y* will yield the following relationship[5]:

$$TC_{y}^{l} = VC_{y}^{s} + VC_{k}^{s}{}' k_{y} + r' k_{y}, \tag{8}$$

where k_y is the matrix of partial derivatives of the fixed factors with respect to different outputs. In the long-run equilibrium holds: $r = -VC^{s}_{k}$, thus the expression in (8) leads to:

$$TC_{y}^{l} = VC_{y}^{s}. \tag{8a}$$

After differentiating both sides with respect to *y,* rearranging and considering the envelope theorem, we obtain the following expression for the matrix of the second order derivatives of the long-run cost function with respect to *y* in terms of the variable cost function[6]:

$$TC_{yy}^{l} = VC_{yy}^{s} - VC_{yk}^{s}\left(VC_{kk}^{s}\right)^{-1}VC_{ky}^{s}. \tag{9}$$

Using (9) we can now calculate the second order derivative of the long-run total cost function using second order own and cross-partial derivatives of the short-run variable cost function with respect to *y* and *k*. Similar to the short-run case, the diagonal elements of the matrix TC^{l}_{yy} can be used for investigating long-run convexity effects, while the elements offside the main diagonal indicate the existence of economies or diseconomies of scope on the long-run.

Further, we can derive a long-run measure for economies of scale utilizing the following relationship (7)[7]:

$$Scale^{l} = 1-\sum_{i}\varepsilon_{y_i}^{TC} = 1-\sum_{i}\varepsilon_{y_i}^{VC}\left(1-\sum_{m}\varepsilon_{k_m}^{VC}\right)^{-1}. \tag{10}$$

Long-run scale effect then can be expressed as:

$$F\_Scale^{l} = 2TC^{l}\left(1-\sum_{i}\varepsilon_{y_i}^{TC}\right) = 2VC^{s}\left(1-\sum_{i}\varepsilon_{y_i}^{VC}-\sum_{m}\varepsilon_{k_m}^{VC}\right). \tag{11}$$

And finally, using (9) and (11), we derive the *long-run flexibility measure* using elasticities and derivatives of the short-run variable cost function as follows:

$$Flex^{l} = y' TC_{yy}^{-D} y + y' TC_{yy}^{D} y + 2VC^{s}\left(1-\sum_{i}\varepsilon_{y_i}^{VC}-\sum_{m}\varepsilon_{k_m}^{VC}\right), \tag{12}$$

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[5] Hereinafter we use the following notation: VC_z is the vector of first-order partial derivatives and VC_{zz} is the matrix of second-order or cross-term derivatives of the short-run variable cost function *VC(w,y,k)* with respect to *z = (k,y)*.

[6] Differentiating $r = -VC_k^{s}$ with respect to *y* leads to: $0 = -VC_{ky}^{s} - VC_{kk}^{s}k_{y}$, from which we can now derive $k_y = -\left(VC_{kk}^{s}\right)^{-1}VC_{ky}^{s}$. Differentiating $TC_{y}^{l} = VC_{y}^{s}$ with respect to *y* leads to: $TC_{yy}^{l} = VC_{yy}^{s} + VC_{yk}^{s}k_{y}$. After replacing of k_y we get the expression in (9).

[7] For the discussion of the measure of long run scale elasticity based on the variable cost function see Braetigam and Daughety (1983).

with second derivatives of the long-run cost function TC^{l}_{yy} derived from the variable cost function given by (9).

5 DUAL MEASURE OF FLEXIBILITY BASED ON THE DISTANCE FUNCTION

For an empirical analysis of flexibility the measures in (2) and (11) may be directly derived from the elasticities of the econometrically estimated short run multi-output-cost function. However, the estimation of the cost function may be problematic in some instances. A first problem might arise from data availability as the input price data required for estimating cost functions are not always available. Second, due to the unpriced nature of many inputs in family farm agriculture (for example family work, owned land) it may be difficult to estimate overall cost. Finally, even if one is able to find an approximation for some input price data, a third kind of problems can occur if estimated parameters of the cost function are not consistent with the theoretical assumptions due to inappropriate calculation of price data (for example, when quasi-concavity in input prices is not fulfilled). In order to avoid these problems, we propose a dual flexibility measure which can be derived by estimating parameters of the input distance function. Both cost and input distance function are valid representations of a multiple production technology. Though the input distance function is less restricted compared to the cost function, it does not require any behavioral assumptions and only input and output quantity data are needed to estimate the parameters of this function.

Using duality theory we define the short-run input distance function dual to the cost function as follows:

$$D_i^s(x,y) = \inf_x \{wx : VC(y,w,k) \geq 1\}$$

The standard properties of the input distance function are: (i) decreasing in each output level, (ii) increasing in each input level, (iii) homogeneous of degree one and (iv) concave in all inputs (SHEPHARD (1970), FÄRE AND PRIMONT (1995)).

Applying functional relationships between the cost and input distance function we can obtain the dual flexibility measure. In doing so we make use of dual measures of economies of scale and scope as proposed in the economic literature. Following FÄRE ET AL. (1986), the multi-output measure of economies of scale computed from the input distance function and formulated in terms of cost elasticities can be written as:[8]

$$Scale = 1 - \sum_j \varepsilon^{VC}_{y_j} = 1 + y' D_y \cdot D^{-1}. \tag{13}$$

According TO HAJARGASHT ET AL. (2008) we derive the matrix of the second order derivatives of the short-run variable cost function with respect to the output vector in terms of the derivatives of the input distance function as follows:[9]

$$VC^s{}_{yy} = VC^s \cdot \left\lfloor D_y D_y' - D_{yy} + D_{yx}\left(D_{xx} + D_x D_x'\right)^{-1} D_{xy} \right\rfloor. \tag{14}$$

Note, that the elements on the main diagonal of the VC^s_{yy} matrix are part of the concavity effect while the elements outside the main diagonal are considered by measuring the scope effect.

Further, using the solution of the first order condition of the multi-product cost minimization problem with production constraints expressed by the input distance function and the optimal

[8] The equality of the primal and dual measures of economies of scale holds by convex input sets, see Färe et al.

[9] Hereinafter we use the following notation: D_z is the vector of first derivatives and D_{zz} is the matrix of second-order derivatives of the input distance function $D_i^s(x,y)$ with respect to $z = (x,y)$.

value of the Lagrangian multiplier $\lambda^*(y,w)$ which is equal to the cost function $C(y,w)$, we derive following relationship between the cost and the distance function[10]:

$$VC^s = D_{x_i}{}^s / w_i \,. \qquad (15)$$

With w_i - price of one of the variable inputs *i* and D_{x_i} - first derivative of the input distance function with respect to a particular input *i*.

After replacing the correspondent parts of the formula (2a) with (13), (14) and (15) we derive the dual measure of short run flexibility of a multi-product firm based on the parameters of the input distance function:

$$Flex^s = \frac{D_{x_i}}{w_i} \cdot y' \cdot \left[D_y D_y' - D_{yy} + D_{yx}\left(D_{xx} + D_x D_x'\right)^{-1} D_{xy} \right] \cdot y + \frac{2D_{x_i}}{w_i}\left(1 + y' D_y \cdot D^{-1}\right). \qquad (16)$$

For the *long run flexibility analysis* we use dual relationships between the shadow price of the quasi-fixed factor based and the short run input distance function:[11]

$$\sum_m \varepsilon_{k_m}^{VC} = k' VC_k^s \left(VC^s\right)^{-1} = -k' D_k D^{-1} \,. \qquad (17)$$

The dual long-run measure of economies of scale is derived by substituting relationships from (13) and (17) into formula (10):

$$Scale^l = 1 + y' D_y D^{-1} \left(1 + k' D_k D^{-1}\right)^{-1} . \qquad (18)$$

The scale effect on the long run can be expressed in terms of distance function as:

$$F\_Scale^l = 2\frac{D_{x_i}}{w_i}\left(1 + y' D_y D^{-1} + k' D_k D^{-1}\right). \qquad (19)$$

In order to obtain the Hessian matrix of the long run total cost function from the derivatives of the short-run distance function, we have to replace the parts of formula (9) with their corresponding dual vectors and matrices. In analogy to the derivation of (14), we can derive the corresponding matrices VC_{yk}, VC_{kk} and VC_{ky} of the long-run measure based on the derivative matrices of the short-run input distance function as follows:

$$VC^s{}_{ky} = VC^s (D_k D_y' - D_{ky} + D_{kx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xy}) \,. \qquad (20)$$

$$VC^s{}_{kk} = VC^s (D_k D_k' - D_{kk} + D_{kx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xk}) \,. \qquad (21)$$

$$VC^s{}_{yk} = VC^s (D_y D_k' - D_{yk} + D_{yx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xk}) \,. \qquad (22)$$

After substituting relationships (20) - (22) and (14) into formula (9) we can now derive dual long-run measures for scope and concavity effects:

$$TC_{yy}^l = \frac{D_{x_i}}{w_i}\left[\begin{array}{l} D_y \cdot D_y' - D_{yy} + D_{yx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xy} - (D_y D_k' - D_{yk} + D_{yx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xk}) \times \\ \left[(D_k D_k' - D_{kk} + D_{kx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xk})\right]^{-1} \times (D_k D_y' - D_{ky} + D_{kx}\left[D_{xx} + D_x D_x'\right]^{-1} D_{xy}) \end{array}\right]$$

[10] For derivative properties and dual relationships between cost and distance functions see Färe and Primont (1995), p.51ff.

[11] This relationship is based on the envelope theorem applied to the cost minimization problem $C_k^s = -C^s(w,y,k) \cdot D_k(x^*(w,y,k),y,k)$, where both sides were multiplied by k'/C^s which leads to $k' C_k^s / C^s = -k' D_k / D$ and can be expressed in elasticity terms as follows: $\partial \ln C_k^s / \partial \ln k_m = -\partial \ln D_k^s / \partial \ln k_m$.

6 ECONOMETRIC SPECIFICATION

For the econometric estimation of the short-run multiproduct input distance function we specify a translog functional form. We differentiate between various technologies by adding dummy variables which capture specialization in different production processes, namely crop production, grazing livestock or granivores. Thus, a parametric specification of the short-run multiproduct input distance function can be expressed as:

$$
\begin{aligned}
\ln D_{If} &= \beta_0 + \beta_d D\_Sp + \beta_t t + \beta_{tt} t^2 + \sum_{j=1}^{J} \beta_j \ln y_{jf} + \sum_{i=1}^{I} \beta_i \ln x_{if} + \sum_{m=1}^{M} \beta_m \ln k_{mf} + \\
&+ \sum_{j=1}^{J} \beta_{dj} \ln y_{jf} \cdot D\_Sp + \sum_{i=1}^{I} \beta_{di} \ln x_{if} \cdot D\_Sp + \sum_{m=1}^{M} \beta_{dm} \ln k_{mf} \cdot D\_Sp \\
&+ \sum_{j=1}^{J} \beta_{jt} \ln y_{jf} \cdot t + \sum_{i=2}^{I} \beta_{it} \ln x_{if} \cdot t + \sum_{m=1}^{M} \beta_{mt} \ln k_{mf} \cdot t + \\
&+ 0.5 \sum_{j=1}^{J} \sum_{p=1}^{P} \beta_{jp} \ln y_{jf} \ln y_{pf} + 0.5 \sum_{i=1}^{I} \sum_{s=1}^{S} \beta_{is} \ln x_{if} \ln x_{sf} + 0.5 \sum_{m=1}^{M} \sum_{n=1}^{N} \beta_{mn} \ln k_{mf} \ln k_{nf} \\
&+ \sum_{i=1}^{I} \sum_{j=1}^{J} \beta_{ij} \ln x_{if} \ln y_{jf} + \sum_{m=1}^{M} \sum_{j=1}^{J} \beta_{mj} \ln k_{mf} \ln y_{jf} + \sum_{i=1}^{I} \sum_{m=1}^{M} \beta_{im} \ln x_{if} \ln k_{mf}
\end{aligned}
$$

where y_{jf} represents the quantity of the *j*th output, x_{if} is the quantity of the *i*th variable input, k_{mf} is the quantity of the *m*th quasi-fixed factor, and $f = 1, \ldots, F$ denotes an agriculture firm. The time dummy t accounts for technological change and D_Sp are the dummies accounting for specialization in crop production, grazing livestock or granivores.[12]

Following the standard practice in the stochastic frontier literature we set $0 = \ln D_{ft} + v_{ft} - u_{ft}$, where the random disturbance term v_{ft} is assumed to be normally distributed with a zero mean, and u_{ft} as one-sided random variable according to a half normal distribution. The term u_{ft} is usually interpreted as technical inefficiency. Input homogeneity is imposed by normalizing the distance function with one of the variable inputs x_0 so that the regression model takes the form: $\ln x_{0ft} = \ln D^*_{ft} + v_{ft} + y_{ft}$ with D^* - short run distance function with variable inputs normalized by x_0.[13] Parameters of the stochastic input distance function were estimated using the maximum likelihood procedure by running the Limdep 9.0.

7 DATA AND EMPIRICAL RESULTS

In the empirical application we utilized a data set including eight years of observations, from 1994 to 2001, on 580 Polish agricultural farms; the total number of observations was 4,640. The data set was provided by the Polish Institute of Agricultural and Food Economics - National Research Institute (IERiGZ-PIB). Variables contain both farm-specific accountancy information and socio-demographic characteristics.

Variables used for the estimation of the input distance function include following variables. Outputs are total output values of crops and crop products (y_1), output from grazing including milk production, cattle, sheep and goats (y_2), and granivores (y_3) including pigs, poultry and

[12] Firms that produce more than 50% of the overall production in one of the three output groups - crop production (*D_sp=1*), grazing livestock (*D_sp=2*)or granivores (*D_sp=3*) - considered as specialized. Remaining firms are considered as not specialized or diversified (*D_sp=0*).

[13] Some authors (Krumbhakar et al (2008)) argue that normalized distance functions could lead to endogeneity problems, since one of the arbitrary chosen input variables is considered as exogenous while all other inputs are assumed to be endogenous. However, Coelli (2000) proves that under typically accepted behavioral assumptions OLS yields consistent functional estimates for a Translog functional form, and, thus, the endogeneity problem is less important than expected.

other animal production. These indicators were estimated taking into account sales, home consumption and stock changes. Since the individual figures for outputs were in current values, the variables were deflated by the corresponding price indices provided by the Statistical Office in Poland (GUS var. issues, a, b). As variable inputs we used the implicit quantity index for specific inputs of crop (x_1) and animal production (x_2) respectively, and other variable costs (x_3). The first two variables were obtained by deflating components of the variable costs going into crop or animal production by their corresponding price indexes calculated in 1994 price levels. Other variable costs were deflated by the national price index for fuel, oils and technical lubricants since these categories of costs result in 80% of other variable inputs. This variable was used as a normalizing variable, i.e. all input variables were divided by other variable costs. The vector of quasi-fixed factors contains the following variables: labour (k_1), land (k_2), and capital (k_3). Labor was given by agricultural working units for both family and hired labor. Land input was approximated by the sum of arable land and grassland in use. Capital input was approximated by the sum of expenditure on capital services and depreciation of building, machinery and equipments, deflated by the price index of agricultural investment. Definitions and descriptive statistics of the data used in our econometric model are summarized Table A 1 in Appendix.

We normalized all variables by their geometric means so that the coefficients of the first-order effects can be interpreted as the corresponding elasticities at the point of approximation. Results of the maximum likelihood estimation of the stochastic short-run input distance function model are presented in Table A 1 in Appendix.

The monotonicity requirements for inputs and outputs were fulfilled for about 85% of the observations. The distance function is quasi-concave as (n-1) eigenvalues of the Hessian Matrix were negative while one eigenvalue was positive for all observations. This was valid for 97% of the observations. In order to avoid a wrong interpretation, we excluded all improper values that were inconsistent with the theoretical properties of the distance function, leading to remaining 4056 observations. Further we excluded all extreme values for flexibility indicators. After the elimination we ended up with a total of 3895 observations.
Using estimated parameters of the *short-run* input distance function, we calculated indicators for scale and scope economies as well as flexibility values for each firm according to the suggested dual measures in (2) – (6). Figures for descriptive statistics of the flexibility components are represented by different types of farms in Table 1.

Table 1: Short-run flexibility

Type of farming	Overall flexibility		Scope effect		Convexity effect		Scale effect	
	Mean/(Std.Dev.)		Mean/(Std.Dev.)		Mean/(Std.Dev.)		Mean/(Std.Dev.)	
Mixed farms	**0.632**	(3.729)	**1.132**	(2.062)	-3.845	(2.927)	3.344	(3.009)
Specialist crops	1.743	(2.747)	1.332	(1.249)	*-2.492*	(1.970)	**2.904**	(3.305)
Specialist grazing livestock	2.428	(3.903)	1.596	(3.742)	-3.348	(3.381)	4.181	(4.162)
Specialist granivores	*2.821*	(4.499)	*2.460*	(2.115)	**-5.163**	(3.337)	*5.525*	(4.288)
Total	1.438		1.382	(2.347)	-3.511	(2.940)	3.568	(3.518)

Number of observations: 3895

According to these figures, the average value of overall flexibility was lowest for *mixed farms* which indicates that this type of farming is more flexible compared to other types of farming (0.632 compared to total average of 1.438). Being more diversified, farms of this category gain some advantages of scope economies as evident from the lowest value of the scope effect. The main source of flexibility for farms specializing in *crop production* comes from the scale effect. The relative low level of the average convexity effect on the other hand reduces the overall flexibility of the crop specializing farms. Although crop production farms possess less concave cost functions and thus cannot benefit from the significant decline in the growth rate of marginal cost, they perform at production levels closer to constant economies of scale than farms of other categories. Generally, farms specializing in *livestock production* (grazing, granivores) are less flexible in the short run (average overall flexibility is 2.428 and 2.821 respectively). In particular this concerns specialists in granivores which benefit less than other farms from scope and scale economies. In fact these farms have the highest scope effect values, and thus are affected by diseconomies of scope. Farms specializing in the production of pigs, poultry and other granivores show economies of scope only in 3 from 327 cases. In all other cases there are no complementarities with grazing or crop production.

This high share of diseconomies of scope is characteristically not only for livestock farms. In 93% of observations marginal costs of the considered outputs raise when the production of another output is increased, which indicates diseconomies of scale. This result is likely to be overestimated due to aggregation of the input and output variables. Using these variables we can only observe the cost complementarities between crop and grazing livestock production, crop and granivores and grazing production and granivores considering aggregated inputs of crop and animal production and other inputs. We would probably get much more observations with positive economies of scope by using more disaggregated data, thus considering for example complementarities between different crops that utilize the same input. However, more disaggregated variables would increase the number of parameters of the production function and make the estimation of a stochastic frontier model impossible.

Table 2: Long-run flexibility

Type of farming	Overall flexibility		Scope effect		Convexity effect		Scale effect	
	Mean/(Std.Dev.)		Mean/(Std.Dev.)		Mean/(Std.Dev.)		Mean/(Std.Dev.)	
Mixed farms	-3.130	(8.106)	**0.267**	(4.480)	**-2.825**	(3.925)	-0.573	(0.822)
Specialist crops	*-0.311*	(5.235)	1.169	(2.984)	*-1.457*	(2.671)	*-0.023*	(0.576)
Specialist grazing livestock	-1.203	(8.693)	*1.506*	(5.719)	-2.126	(3.732)	-0.584	(0.925)
Specialist granivores	**-4.006**	(12.014)	1.496	(6.928)	**-2.836**	(5.213)	**-2.667**	(2.030)
Total	-2.112	(8.117)	0.835	(4.704)	-2.340	(3.781)	-0.607	(1.163)

Number of observations: 3895

Further we also calculated the corresponding indicators in the *long run* where quasi-fixed factors were assumed to be utilized at their optimal long run equilibrium levels. Average values and standard deviations of the long run indicators are presented in Table 2. Generally, farms are more flexible on the long run. This also holds for all components of flexibility. Following the Samuelson's Le Chatelier Principle (SAMUELSON, 1947), according to which a firm's response to market changes is the bigger the fewer inputs are held fixed, this is not a surprising result. In the long run, farms were more able to use cost complementarities between outputs than in the short run. This can be explained by the utilization of common production factors such as capital, land and labor, which are considered to be variable in the long-run. In more than a one-third of the cases we could observe negative values for the scope effect, indicating positive effects of diversification compared to only 7% of observations with economies of scope in the short-run. As in the short-run, *mixed farms* benefit more from scope economies than specialized farms which showed the lowest average scope effect of 0.267. More than 50% of these farms possess positive economies of scope. Specializing granivores have the lowest negative value of overall flexibility (-4.006), resulting from the lowest value of the scale effect (-2.667). Almost all of these farms operate in the area of diseconomies of scale, which positively influences flexibility. Having less convex cost functions, specialists in crop production benefit neither from the scale economies nor from the decline in the growth rate of marginal cost, which makes them less flexible on the long run compared to farms from other categories.

8 CONCUSSION

In this paper we provide a flexibility measure for multi-product firms which can be obtained by estimation both cost and input distance functions, formulated on the short and on the long run. We decompose the primal and the dual flexibility measure in order to distinguish between three effects, which affect firm's flexibility in different ways: scope (or complementarity) effect, concavity effect and scale effect. In our empirical application we analyze the role of scope and scale economies, as well as concavity properties of the production technology on the flexibility of different types of farming using data on Polish farms. Mixed farms, being more diversified, are more flexible on the short run due to gains form economies of scope, while farms specializing in the production of pigs, poultry and other granivores are more flexible on the long run due to scale and convexity effects.

Our flexibility analysis has some limitations because it is formulated using a static neoclassical theory and does not consider dynamic effects. All flexibility measures are derived from cost or distance functions which assume static optimization and do not consider intertemporal interdependencies. Further studies considering such dynamic effects are needed.

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APPENDIX

Table A 1: Descriptive Statistics by farm type

	Variable	Definition	Total*	Mixed farms	Specialist crops	Specialist grazing livestock	Specialist granivores
Outputs:							
y_1	Crop	Total output crops & crop production in Zloty	12450.2 *(12281.4)*	9835.0 *(8041.8)*	19556.1 *(17545.5)*	8658.6 *(6953.6)*	13642.7 *(12360.8)*
y_2	Grazing	Total output grazing livestock (milk products, cattle, sheep etc.) in Zl	9344.9 *(11751.1)*	8208.4 *(6944.8)*	5764.8 *(6380.9)*	19348.3 *(20649.4)*	4841.6 *(5128.1)*
y_3	Granivores	Total output granivores (pigs, poultry and other granivores) in Zl	8302.8 *(13354.8)*	7954.0 *(7748.4)*	5388.0 *(7450.8)*	2574.4 *(3815.7)*	31575.7 *(30519.9)*
Quasi-fixed Faxtors:							
k1	Land	Total arable land and grassland in use in ha	14.2 *(11.8)*	12.6 *(9.3)*	16.1 *(15.1)*	15.1 *(11.4)*	15.1 *(12.5)*
k2	Labour	Total labour input in annual work unit (AWU)	3996.6 *(1728.2)*	4016.6 *(1599.9)*	3858.0 *(1816.9)*	3991.7 *(1684.1)*	4317.7 *(2145.9)*
k3	Capital	Depreciation of farm assets plus expenditure on services in Zloty	3956.0 *(2512.2)*	3491.7 *(2085.8)*	4478.9 *(2766.5)*	4068.0 *(2796.8)*	4709.8 *(2702.8)*
Variable Inputs:							
x1	Input crop	Specific costs of crop production in Zloty	2683.2 *(3071.2)*	2084.2 *(2054.8)*	4077.5 *(4510.8)*	2072.3 *(1950.6)*	19986.8 *(2942.1)*
x2	Input animal	Specific costs of animal production in Zloty	8924.3 *(10216.0)*	8303.1 *(6135.6)*	6384.2 *(5427.2)*	8986.9 *(14056.9)*	3455.6 *(18956.6)*
x3	Other var. inputs	Other variable costs in Zloty	2602.0 *(2129.8)*	2255.7 *(1728.8)*	2983.3 *(2460.9)*	2563.9 *(2140.1)*	3106.6 *(2577.5)*

*Standard deviations are given in parenthesis

Table A 2: Estimated parameters of the stochastic short run input distance function

First order effects and dummies:

Variable	Coeff.	Time	Specialisation-dummies for		
			Crops	Grazing	Granivores
Time	0.026***	-0.005***			
Crops	-0.439***	0.001	.24E-4***	.95E-4***	.60E-4***
Grazing	-0.104***	-0.001	-.64E-4***	-.86E-4***	.44E-4*
Granivors	-0.200***	0.001	-.54E-4***	.12E-3***	.41E-4***
Input Cops	0.250***	-0.004*	-0.023	-0.019	0.062**
Input Anim	0.586***	-0.001	-0.116***	0.017	-0.069***
Labour	-0.123***	-0.001	-0.086***	0.044*	-0.045*
Land	-0.050***	0.006	0.134***	0.021	-0.011
Capital	-0.113***	-0.009**	-0.009	-0.041	-0.038
Constant	0.553***		0.167***	-0.113***	-0.134***

Second order effects:

Variable	Crops	Grazing	Granivors	Input Cops	Input Anim	Labour	Land	Capital
Crops	-0.068***	-0.003***	-0.005**	0.020	0.011***	0.089***	-0.051**	0.116***
Grazing		-0.006***	0.007***	0.004***	-0.003***	-0.002**	0.000	0.000
Granivors			-0.009***	0.011***	-0.006***	0.003	0.007***	0.000
Input Cops				0.094***	-0.021***	-0.131***	-0.012	0.025
Input Anim					0.026***	-0.001	-0.003	0.011***
Labour						-0.107***	-0.003	0.015
Land							0.119***	-0.097***
Capital								-0.112***

Number of observations: 4634 Lambda: 0.80533
Log likelihood value: 1326.43 Sigma: 0.21005
Note: ***, **, * =Significance at 1%, 5%, 10% level