

# Estimating the Technical Optimal Scale of Production in Danish Agriculture

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# Estimating the Technical Optimal Scale of Production in Danish Agriculture

By Svend Rasmussen

## *Abstract*

This paper uses representative farm account data for 1985-2007 to estimate stochastic production frontiers in the form of input distance functions for Danish crop, dairy and pig farms. The objective is to study and compare scale economies for the three farm types. The estimated technical efficiency is relatively constant over time for all three farm types, but the elasticity of scale differs. Although the size of all farm types has increased considerably during the last 20 years, more than 95 % of the crop farms and 85 % of the dairy and pig farms are still below the estimated technical optimal scale of production. The results support the hypothesis that the restrictions concerning the amalgamation of farms and the purchase of farm land have seriously prevented Danish farmers, and especially cash crop farmers, from taking full advantage of scale economies.

**Keywords:** Scale economies, agriculture, SPF, input distance function, technical optimal scale, elasticity of scale

## 1 Introduction

The farm structure in many countries is regulated by government intervention. While the intervention may fulfil certain political objectives, they also limit the farmers' ability to adjust farm size to changes in economic and technological conditions. Agricultural economists have argued that this comes at a cost, because exploitation of scale economies is essential for productivity changes. Rules and regulations that prevent farmers from reaching the efficient scale of operation will therefore limit productivity changes, and thereby the competitiveness of the agricultural sector.

In Denmark the farm structure has historically been regulated by the Agricultural Act. The objective has been to protect the family farm, and to prevent farm land from being concentrated in the hands of a few large landowners. The Agricultural Act has been adjusted over the years, but basically it limits the amount of land a farmer is allowed to hold, whilst it also regulates ownership structure, the amalgamation of farms, and restricts the number of livestock allowed per hectare of land.

The objective of this paper is to study scale economies in Danish agriculture. The hypothesis is that even though the size of farms has increased significantly over the last 30-40 years, Danish average full time farms have been, and still are, (considerably) below the optimal scale of production.

The study of scale economies in agriculture and related subjects has been on the research agenda for many years (Chavas, 2001). The empirical results have been mixed, but concerning agriculture in the developed countries, the empirical evidence suggests that the average cost function has a typical L shape, indicating increasing returns to scale up to a certain farm size, and then a fairly wide range of farms sizes with constant returns to scale (Chavas, 2001, p.268). Kisløv and Peterson (1996) found no clear evidence of scale economies in U.S. agriculture. However, later papers applying more precise models show other results. Morrison Paul *et al.* (2004) found that U.S. corn-belt family farms are scale inefficient and face significant scale economies. Alvarez and Arias (2003) estimated an average cost function to analyse Spanish dairy farmers and found that limited managerial ability can be an important source of diseconomies of size. Mosheim and Lovell (2009) used a cost function approach to study U.S. dairy farms and found significant scale economies.

They found no region of decreasing returns to scale. Mosheim and Lovell also stress that the more precise model used supports a conclusion that returns to scale are larger at all levels of output than previously believed (*ibid* p. 793).

In the following we will measure the (long run) elasticity of scale in Danish agriculture based on representative farm account data covering the period 1985-2007. Like Morrison Paul *et al.* (2004), we use the input distance function to estimate scale elasticity. Although the scale elasticity derived from the input distance function is a purely technical concept (an attribute of the production function), it can be given an economic interpretation as mentioned in Färe *et al.* (1986). They show that assuming fixed input prices "...an equiproportionate change in all outputs has exactly the same proportionate effect on minimised cost as it has on the value of the input distance function, apart from a change in sign." (*ibid* p. 178). In short, this means that the elasticity of scale and the elasticity of size coincide at cost minimising points.

Under standard conditions (convex technology set, perfect competition on the input and output markets (free entry and exit) and profit maximising agents), the optimal size of the farm is when the elasticity of size is equal to one. The problem is, however, that although an elasticity of size equal to one implies that the elasticity of scale is also equal to one, the opposite is not necessarily the case. Therefore, one cannot base conclusions concerning the optimal farm size on estimated elasticity of scale unless further assumptions are made. However, returns to scale is an important characteristic of the production function in the sense that it is a measure of productivity. With increasing returns to scale, the productivity and the profit increases by increasing the scale of production until the point where the returns to scale is equal to one.

The main results are that the majority of Danish full-time farms operate below their technical optimal scale, and that cash crop farms especially have a size which is considerably below the technical optimal scale of production.

The rest of the article is structured as follows. Section 2 provides a short review of the relationship between the two concepts, returns to scale and returns to size. Section 3 describes the methodology of using the input distance function to estimate the elasticity of scale, and the technical optimal scale of production. In section 4, the data are described whilst the empirical results are presented and discussed in section 5. Finally, a conclusion is reached in section 6.

## 2 Scale economies and optimal farm size

There are alternative ways of measuring scale economies. In a multi-input, one-output production framework, *elasticity of scale*, and the corresponding *returns to scale*, are well-defined and intuitive concepts (Chambers 1988, p. 24): A production function exhibits constant returns to scale (CRS) if changing all inputs by a positive proportional factor has the effect of increasing output by that factor (elasticity of scale equal to one). If output increases more than that factor, the production function exhibits increasing returns to scale (IRS) (elasticity of scale greater than one), and if output increases less than that factor, the production function exhibits decreasing returns to scale (DRS) (elasticity of scale less than one). In a multiple-input, multiple-output framework, the elasticity of the cost function, with respect to outputs, has been a useful concept in measuring scale economies. Färe *et al.* (1986) have shown that the elasticity of the input distance function provides exactly the same measure of elasticity of scale as the cost function. In the following, the measurement of the elasticity of scale will be based on the input distance function, whilst the intuitive interpretation of the elasticity of scale is therefore that it is a measure of the proportionate effect on the minimised cost of an equiproportionate change in all outputs (Färe *et al.*, 1986).

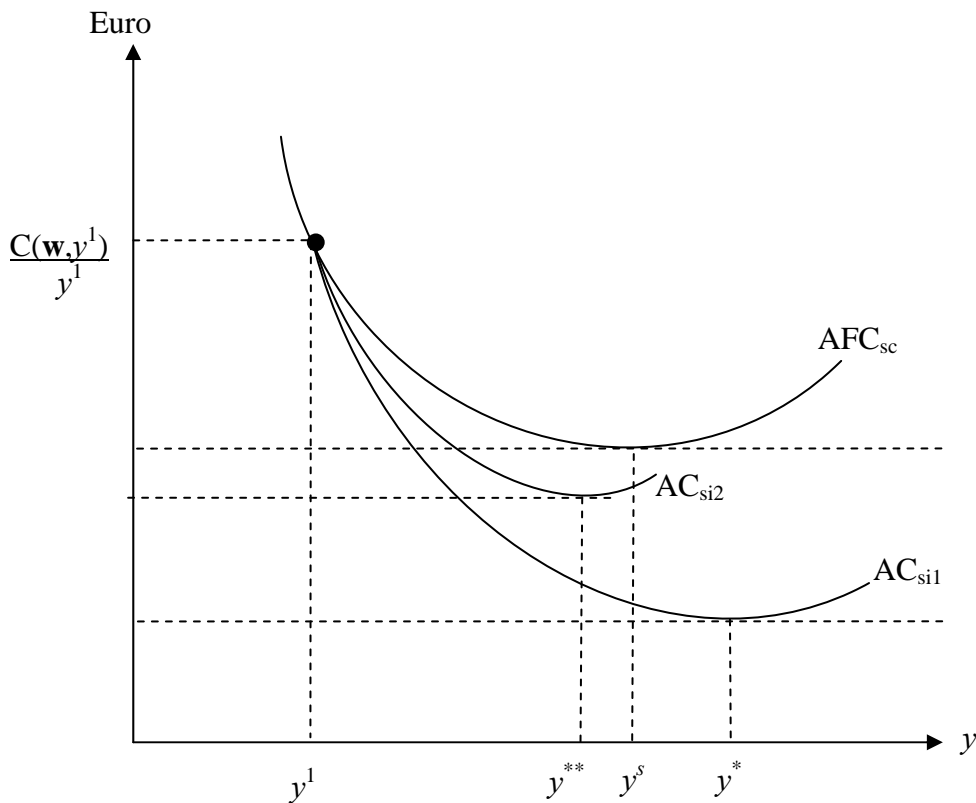


Figure 1. Economies of scale and economies of size

The concept, returns to scale, is a pure technical concept and the elasticity of scale therefore does not necessarily provide a measure of the optimal firm size, which in a competitive industry is the size with the lowest long run average costs. The formal concept describing changes in costs when the production changes, is *returns to size*, which is a measure of changes in the long run average costs when output changes. Increasing returns to size (*economies of size*) means that the long run average costs decreases as the size (the production) of the firm increases, and decreasing returns to size (*diseconomies of size*) means that the long run average costs increases as the size (the production) of the firm increases (Chambers, 1988, p. 70). The optimal farm size is the size at which the long run average costs are at minimum (constant returns to size).<sup>1</sup>

If input/output observations are cost-minimising points then the two concepts, returns to scale and returns to size, coincide, and the optimal farm size may be identified as the input/output combination where there is constant returns to scale (elasticity of scale equal to one). However, the opposite is not the case. Figure 1 illustrates this.

Assume that an observed input-output combination  $(\mathbf{x}^1, y^1)$  of an input vector  $\mathbf{x}$  and an output  $y$  is a cost minimizing point, which means the cost of producing  $y^1$  using  $\mathbf{x}^1$  is  $C(\mathbf{w}, y^1)$  where  $\mathbf{w}$  is a vector of input prices and  $C$  is the cost function. Assume further that there is increasing returns to scale at  $(\mathbf{x}^1, y^1)$ , and that therefore it is beneficial to increase the production. Consider the following two alternative ways of increasing the production: 1) Adjustment of all inputs by a positive proportional factor  $t > 1$ , i.e. increase of the production along a scale line through  $\mathbf{x}^1$ . 2) Adjust-

<sup>1</sup> Notice that others use different definitions. Thus, according to Chavas (2001), "Returns to scale reflects the relationship between average production cost and firm size" (p. 267).

ment of the inputs in a cost minimising way, i.e. increase of the production along the expansion path. In the first case, average factor cost is  $AFC_{sc} = \mathbf{w}t\mathbf{x}^1 / y$  with a minimum at  $y^s$  as illustrated in Figure 4. In the second case average cost is  $AC_{si} = C(\mathbf{w}, y) / y$  illustrated by two alternative average cost functions  $AC_{si1}$  and  $AC_{si2}$ . The minimum of this average cost function may either be to the right ( $y^*$ ) or to the left ( $y^{**}$ ) of  $y^s$ , or it may even be the same as  $y^s$ , which is the *technical optimal scale of production*.

While the technical optimal scale of production does not necessarily correspond to the optimal firm size, the profit increases when the scale of production is increased along the scale line until the technical optimal scale of production,  $y^s$  is reached. Thus, comparison of the actual scale of production and the technical optimal scale of production illustrates the economic potential of merging firms having the actual scale of production into firms having the technical optimal scale of production.

### 3 Methodology

The methodology is similar to the methodology used by Morrison Paul *et al.* (2004) and Rasmussen (2010) i.e. derivation of the elasticity of scale from an input distance function. The input distance-function first introduced by Shephard (1970), describes how much an input vector may be proportionally contracted with the output vector held fixed. It is non-decreasing, linearly homogenous and concave in input  $x$ , and non-increasing and quasi-concave in output  $y$  (Färe and Primont, 1995). As Morrison Paul *et al.*, we use a translog form, which has the advantage of being a flexible functional form. The model has the form:

$$\begin{aligned} \ln D^t(\mathbf{x}_t, \mathbf{y}_t) = & \beta_0 + \sum_{n=1}^N \beta_n \ln x_{nt} + \frac{1}{2} \sum_{n=1}^N \sum_{k=1}^N \beta_{nk} \ln x_{nt} \ln x_{kt} + \sum_{m=1}^M \alpha_m \ln y_{mt} \\ & + \frac{1}{2} \sum_{m=1}^M \sum_{l=1}^M \alpha_{ml} \ln y_{mt} \ln y_{lt} + \sum_{m=1}^M \sum_{n=1}^N \gamma_{mn} \ln y_{mt} \ln x_{nt} + \sum_{n=1}^N \beta_{nt} t \ln x_{nt} \\ & + \sum_{m=1}^M \gamma_{mt} t \ln y_{mt} + \sum_{s=2}^T \tau_s C_s \end{aligned} \quad (1)$$

According to Färe, Grosskopf and Lovell (1986), it is possible from the input distance function to estimate a local measure of elasticity of scale as:<sup>2</sup>

$$\epsilon^t(\mathbf{x}, \mathbf{y}) = - \left[ \sum_{m=1}^M \frac{\partial \ln D^t(\mathbf{x}, \mathbf{y})}{\partial \ln y_m} \right]^{-1} \quad (2)$$

where  $D^t$  is the input distance function in period  $t$ , and  $y_m$  ( $m=1 \dots M$ ) is output  $m$ . The technical optimal scale of production is defined as the scale of production at which the elasticity of scale  $\epsilon^t(\mathbf{x}, \mathbf{y})$  (EOS) is equal to one. To identify how far the actual scale of production ( $\mathbf{x}^0, \mathbf{y}^0$ ) is from the technical optimal scale of production assuming unchanged technical efficiency, we can determine the values of  $\theta > 0$  and  $\psi > 0$  such that:

$$\epsilon^t(\theta \mathbf{x}^0, \psi \mathbf{y}^0) = 1 \quad (3)$$

and

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<sup>2</sup> Notice that Morrison Paul *et al.* (2004) use the reciprocal of this term as a measure of elasticity.

$$\ln D(\theta \mathbf{x}^0, \psi \mathbf{y}^0) = \ln D(\mathbf{x}^0, \mathbf{y}^0) \quad (4)$$

Using equations (1) and (2), and following a procedure similar to the one used by Balk (2001, s. 167), it can be shown that condition (3) is equivalent to:

$$\sum_{m=1}^M \alpha_m + \sum_{m=1}^M \sum_{l=1}^M \alpha_{ml} \ln(\psi y_l^0) + \sum_{m=1}^M \sum_{n=1}^N \gamma_{mn} \ln(\theta x_n^0) = -1 \quad (5)$$

Using the restriction  $\sum_n \gamma_{mn} = 0$  ( $m=1, \dots, M$ ), the solution to (5) with respect to  $\ln \psi$  is:

$$\ln \psi = \frac{-1 - \sum_{m=1}^M \frac{\partial \ln D(\mathbf{x}^0, \mathbf{y}^0)}{\partial \ln y_m}}{\sum_{m=1}^M \sum_{l=1}^M \alpha_{ml}} \quad (6)$$

Using this term in (4) provides the following solution to  $\ln \theta$ :

$$\ln \theta = -(\ln \psi) \sum_{m=1}^M \frac{\partial \ln D(\mathbf{x}^0, \mathbf{y}^0)}{\partial \ln y_m} - \frac{1}{2} (\ln \psi)^2 \left( \sum_{m=1}^M \sum_{l=1}^M \alpha_{ml} \right) \quad (7)$$

## 4 Data and estimation

The data used are farm account data from the database of individual farm accounts collected by the Institute of Food and Resource Economics (FOI), University of Copenhagen. The farms included in the database are selected annually using stratified random sampling from the total Danish farm population to obtain representativity concerning farm size, geographical location and economic size (FOI, 2006). The data used in the present analysis cover the 23-year period 1985-2007 and comprise 43,806 observations. The number of observations per year is around 1,900 farms and each observation has a weight describing the number of farms it represents.<sup>3</sup> Around 70- 80 % of the farms remain in the sample the following year. Hence, farms are, on average, represented in the sample for 3-5 subsequent years making the dataset an unbalanced, rotating panel dataset.

Table 1. The data sample for the 23-year period 1985-2007

	Number of obs.	Number of farms	Average years per
Crop farms	5,481	1,852	2.96
Dairy farms	13,370	3,133	4.27
Pig farms	9,274	2,379	3.90

The data used in the present paper only include *full-time farms*, i.e. farms with a standard labour requirement of 1,665 hours or more and comprise three independent sub-sets of the special-

<sup>3</sup> In the following, all averages within years are calculated as weighted averages.

ised farm types<sup>4</sup>, *cash crop*, *dairy* and *pig farms*. The number of observations and the number of farms included in each of the three sub-sets are shown in Table 1.

For each of the three sub-sets (cash crop, dairy and pig farms), the individual outputs were aggregated into two or three *main outputs*. For *crop farms*, two outputs are distinguished: 1) cash crop products (Y2) and 2) other products (Y9), which includes all cattle products, pigs and other animal products. For *dairy farms*, three outputs are distinguished: 1) cash crop products (Y2), 2) cattle products (beef and milk) (Y3), and 3) other products (Y7), which includes pigs and other animal products (except cattle products). For *pig farms*, three outputs are distinguished: 1) cash crop products (Y2), 2) pigs (Y4) and 3) other products (Y8), which includes cattle products and other animal products (except pig products).

The aggregation of outputs into the above mentioned product categories was performed by dividing the total revenue of all the outputs in question with Törnqvist price indices for the output elements in question. *Inputs* were aggregated into *six categories of aggregate inputs*, crop inputs (fertilizers etc.) (X1), feedstuff (X2), land (X3), labour (X4), machinery (X5) and other capital (X6). ‘Land’ (X3) is the hectares of land registered in the accounts multiplied by a quality index. ‘Labour’ (X4) is the number of working hours of the farmer, his family members and the paid labour registered in the accounts. The quantities of the remaining four inputs (crop inputs, feedstuff, machinery and other capital) were calculated by dividing the total cost of each of the four input types by the Törnqvist price index for the input elements involved.<sup>5</sup>

For cash crop farms, a large number of observations had a zero value for the output variable Y9 (animal products) and the input variable X2 (feedstuff). This is also the case with dairy farms (output variable Y7) and pig farms (output variable Y8). To avoid missing observations we used the methods described in Battese (1997).

Individual estimations were carried out for *cash crop farms*, *pig farms* and *dairy farms*. Estimation of the model was performed using the BC-model in LIMDEP version 9.0 (Greene, 2007) and using the same estimation model and specification of the inefficiency term as in Rasmussen (2010). Before estimation, all the variables were normalised by their respective overall averages, i.e. all the output quantity indices were normalised by dividing the indices by the overall averages. The input variables were treated in the same way, except land (X3) and labour (X4), which were normalised by dividing the quantities by the overall quantity averages. Finally, the input variables were normalised by dividing the normalised input variables by normalised land (X3).

## 5 Results

### 5.1 Test of model specification

Monotonicity assumptions were tested for the entire sample. Monotonicity assumptions are not violated if input elasticities are positive and output elasticities are negative.

There are only very few violations for all inputs and the main outputs. The three estimated distance-functions therefore seem quite robust in fulfilling the theoretical conditions of being non-decreasing in  $x$  and non-increasing in  $y$ .

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<sup>4</sup> The classification of farm systems is according to the definition of types of farming used in the EU agricultural statistics (FADN).

<sup>5</sup> Data are further described in Rasmussen (2010)

The curvature conditions were tested at the sample mean. None of the models fulfil the necessary conditions of being concave in  $x$  at the sample mean. However, this is not surprising in the present empirical context, in which the convexity property of the input set may not be satisfied locally because of increasing returns to scale around the sample mean. Tests for curvature conditions in  $y$  were also carried out, but no clear results were achieved.<sup>6</sup>

## 5.2 Elasticity of scale

Predicted values of elasticity of scale (EOS) according to (2) were calculated for each year based on the weighted average for each year. The average elasticity of scale (EOS) is greater than 1 in all years indicating increasing returns to scale. For pig farms, the elasticity of scale has declined over time (from 1.24 to 1.14) suggesting that the farms – on average – have moved towards a more efficient scale of production. For dairy farms, the elasticity of scale remained at almost the same level (around 1.27) until the turn of the century. However, from 2001 it has declined to the present level of around 1.20. Crop farms have had the same average elasticity of scale around 1.35 through the whole period.

If we consider the individual observations in the first year (1985), 0.3 % of the crop farms, 0.6 % of the dairy farms and 0 % of the pig farms had a predicted EOS less than 0.95, while 2.9 % of the crop farms, 6.5 % of the dairy farms and 1.4 % of the pig farms had a predicted EOS less than 1.05. This means that if we consider farms with an elasticity of scale in the interval  $0.95 < \text{EOS} < 1.05$  as having a technical optimal scale of production, 2.6 % of the crop farms, 5.9 % of the dairy farms and 1.4 % of the pig farms had an optimal scale in 1985. The corresponding numbers in 2007 are that: 0.1 % of the crop farms, 0.6 % of the dairy farms and 0.2 % of the pig farms had a predicted EOS less than 0.95, while 0.8 % of the crop farms, 7.1 % of the dairy farms and 8.2 % of the pig farms had a predicted EOS less than 1.05. Using the same definition as before, 0.7 % of the crop farms, 6.5 % of the dairy farms and 8.0 % of the pig farms had a technical optimal scale in 2007.

The very low number of farms with an EOS less than 0.95 indicates that only very few farms are larger than the technical optimal scale.

The estimated technical efficiency (TE) is relatively constant over time for all three farm types. The average technical efficiency is lower on crop farms (0.82) than on dairy (0.88) and pig farms (0.90). However, one should be careful when making comparisons, as the estimated technical efficiency on crop, dairy and pig farms does not refer to the same production frontier. Furthermore, it is likely that the predicted mean efficiency of pig farms is higher because the sample of pig farms is more homogeneous than the other farm types.

The estimated parameters of the inefficiency term show that technical efficiency decreases with farmer age and farm size. Old farmers have a significantly lower technical efficiency than middle aged and young farmers except for dairy farms, where middle-aged farmers have a significantly higher efficiency than young and old farmers. Large farms have a significantly lower efficiency than small farms for all farms types, whilst for crop and dairy farms, large farms also have a significantly lower efficiency than middle-sized farms. The results suggest that small farms, on average, are more careful producers and put more effort into the efficient use of inputs than large farms. This may be their way of compensating for not (being able to) producing at the optimal scale. Apparently, young farmers are more careful producers than old farmers, maybe because their

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<sup>6</sup> Due to space limits it was not possible to include the estimated parameter values in this paper. However lists of the estimated parameters are available from the author, and will be available at the congress.



education is more up to date, or because their economic situation is more vulnerable than old farmers.

### 5.3 The technical optimal scale of production

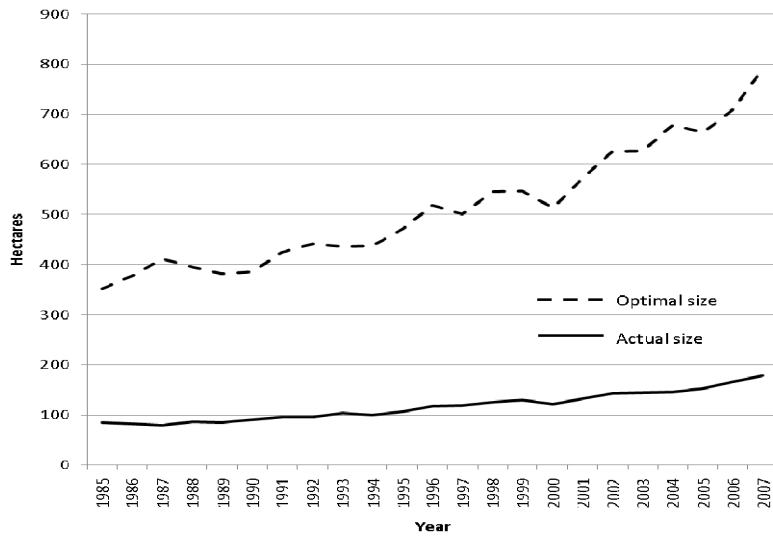
To identify how far the actual scale of production ( $\mathbf{x}^0, \mathbf{y}^0$ ) is from the technical optimal scale of production, the values of  $\theta$  and  $\psi$  in (6) and (7) have been estimated and the results are shown in Table 2.

Table 2. Factors by which to multiply the input vector  $\mathbf{x}^0$  ( $\theta$ ) and the output

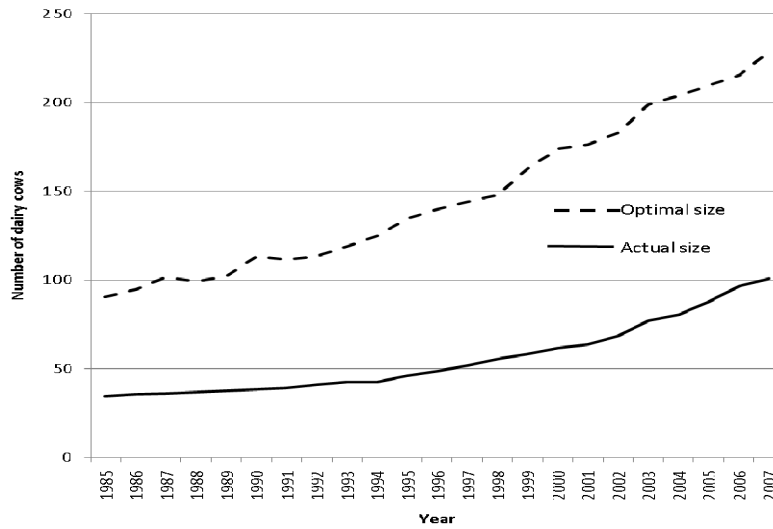
Year	Crop farms		Dairy farms		Pig farms	
	$\theta$	$\psi$	$\theta$	$\psi$	$\theta$	$\psi$
1985	4.2	5.1	2.6	2.9	5.4	6.5
1986	4.6	5.9	2.7	3.0	5.4	6.5
1987	5.2	7.0	2.8	3.2	5.2	6.2
1988	4.6	5.9	2.7	3.0	4.9	5.7
1989	4.5	5.7	2.7	3.0	4.4	5.0
1990	4.2	5.3	3.0	3.4	4.8	5.7
1991	4.4	5.6	2.9	3.3	4.5	5.2
1992	4.6	5.9	2.8	3.1	4.4	5.1
1993	4.2	5.2	2.8	3.1	4.2	4.7
1994	4.4	5.5	2.9	3.3	4.2	4.8
1995	4.4	5.6	2.9	3.3	4.3	4.9
1996	4.4	5.6	2.9	3.3	4.4	5.0
1997	4.2	5.2	2.8	3.2	4.4	5.0
1998	4.4	5.5	2.7	3.0	4.0	4.5
1999	4.2	5.2	2.8	3.2	3.9	4.4
2000	4.2	5.2	2.8	3.2	3.8	4.2
2001	4.3	5.4	2.8	3.1	3.8	4.2
2002	4.4	5.5	2.7	3.0	3.8	4.2
2003	4.3	5.5	2.6	2.9	3.6	4.0
2004	4.6	6.0	2.5	2.8	3.5	3.8
2005	4.3	5.4	2.4	2.6	3.2	3.5
2006	4.2	5.3	2.2	2.4	3.0	3.3
2007	4.4	5.6	2.3	2.4	3.0	3.2
Average	4.4	5.6	2.7	3.0	4.2	4.8

The benchmark ( $\mathbf{x}^0, \mathbf{y}^0$ ) is the average scale of production in each year. The estimated coefficients are thus the factors by which the average scale of production has to be multiplied in order to achieve the technically optimal scale for the current year. To illustrate, take year 1985. In 1985, the average size of full time crop farms was 85 hectares using 3,100 hours of labour and producing a gross output of €135,758 (not shown in the table). As the estimated value of  $\theta$  and  $\psi$  for 1985 is 4.2 and 5.1, respectively, this means that the estimated optimal size of crop farms in 1985, measured in the land dimension, is a farm of  $85 \cdot 4.2 = 352$  hectares producing a gross output of  $135.758 \cdot 5.1 = \text{€}698.261$ . For dairy farms, which in 1985 had a size of 38 hectares, a stock of 34 dairy cows, using 3,277 hours of labour and producing a gross output of €106,483, the optimal scale of production in 1985,

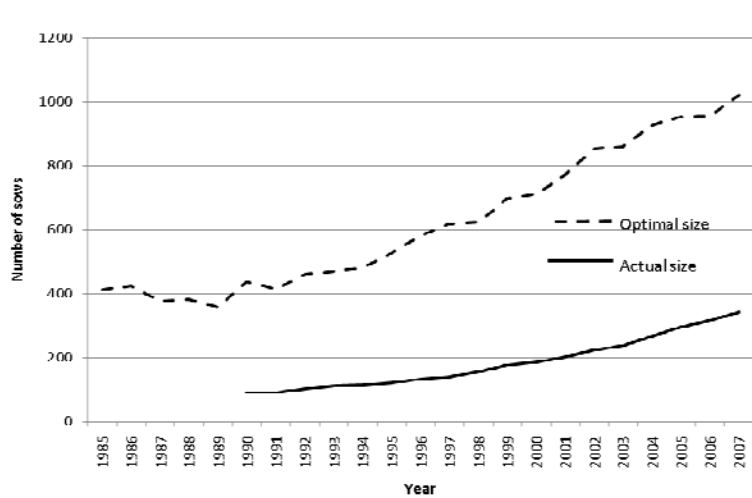
**Figure 2. Size of crop farms**



**Figure 3. Size of dairy farms**



**Figure 4. Size of farms with sows**



measured in the land and cow dimension, is a farm of  $38 \times 2.6 = 99$  hectares with a stock of  $34 \times 2.6 = 90$  dairy cows producing a gross output of  $106,483 \times 2.9 = \text{€}308,595$ .

The estimated technical optimal scale of production for crop farms in the last year (2007) is a farm with almost 800 hectares of land and a yearly turnover of more than €1.7 million. For dairy farms, the technical optimal scale is a farm with around 250 hectares, 230 dairy cows and a yearly turnover of €1 million. For pig farms, the technical optimal scale is a farm with around 425 hectares, 1,000 sows and a yearly turnover of €2.2 million.

The relation between the average size of full time farms, as illustrated earlier in Figures 1, 2 and 3, and the technical optimal scale of production, according to table 6, is shown in Figures 5, 6 and 7.

It is striking that, even though the actual average size of farms has increased considerably over the time period, the technical optimal scale of production has increased even more, increasing the gap between actual and the technical optimal scale of production.

## 6 Discussion and conclusions

The results presented in this paper illustrate the profit potential of increasing the farm size in Danish agriculture. Although the results do not identify the optimal farm size as such, they show that there is potential for amalgamating average farms into larger units, i.e. by increasing inputs along a scale line. Further benefits may be accomplished by changing the composition of inputs, which means that the potential profit potential gained from increasing the scale of operation by amalgamating farms is probably a lower than the actual benefits of allowing farmers to increase the size of their farms.

The validity of the results depends on a number of conditions, including the validity of the estimated models. The estimation of the individual input distance-function models for crop, dairy and pig farms performed well, and the fact that these individual estimations provided comparable results adds to the confidence that the data and the model are well chosen and provide reliable results. The test results for monotonicity and curvature conditions also provided reasonable results.

To further evaluate the validity of the results, the size of the individual farms in the sample was compared to the estimated technical optimal size. This comparison showed that in 1985, 2.2 % of the crop farms, 2.0 % of the dairy farms and 0.5 % of the pig farms were larger than, or equal to, the estimated technical optimal size (measured in hectares, number of dairy cows, and number of sows, respectively). In 2007, 0.8 % of the crop farms, 5.2 % of the dairy farms and 2.0 % of the pig farms were larger than, or equal to, the estimated technical optimal size. Thus, a small number of farms of a size corresponding to the estimated optimal size do in fact exist, which supports the validity of the results.

The results support the hypothesis that restrictions concerning the acquisition of farm land severely restrict the ability of crop farms to adjust the farm size. During the whole period, the estimated technical optimal scale of production measured on the input side is 4-5 times higher than the actual average scale of production. The corresponding estimated potential increase of production is 5-6 times the actual average production. The relation between these figures indicates the potential advantage of increasing the scale of production.<sup>7</sup> The almost unchanged elasticity of scale in the dairy sector during the 1990s indicates that during this period, dairy farmers were just able to adjust

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<sup>7</sup> Notice that the estimated factors in Table 2 are based on the assumption that the scale is increased by using inputs in unchanged *proportions*. Further returns to increasing the scale of production may be achieved by changing the input composition.

their farm size to the change in technology and the increasing optimal scale of production in dairy farming. The estimated optimal scale of production has been 2-3 times higher than the actual average scale of production through the whole period. At the end of the period, dairy farms seem to move even closer to the optimal scale of production. The reason for this change could very well be due to the introduction of the milk quota exchange market in 1999 which improved flexibility regarding structural development in the dairy sector.

Pig farms are different from cash crop and dairy farms in the sense that pig farms show a systematic trend towards a relatively more optimal scale of production through the period. While at the beginning of the period, the pig farms were behind the optimal scale of production by a factor of 5, the optimal scale of production in 2007 was only 3 times the actual average scale of production. This supports the hypothesis that pig farms have been in a better position to increase the scale of operation towards the optimal scale of production because this industry has been the least regulated. The reason that the advantage of increasing the farm size has been highest on pig farms could be also partially explained by the financial constraints and the land market. With cash crops, farmers need land which is regulated according to the Danish Agricultural Act. The development of pig farms has been less dependent on the land market.

Two alternative measures have been used to quantify the differences between the actual size of farms and the technical optimal farm size. The “pragmatic” measure (number of farms with an elasticity of scale in the interval 0.95-1.05) shows that the proportion of crop farms having a technical optimal scale has decreased from 2.6 % in 1985 to 0.7 % in 2007. For dairy farms, the proportion of farms having a technical optimal scale has increased from 5.9 % in 1985 to 6.5 % in 2007. For pig farms, the proportion of farms having a technical optimal scale has increased from 1.4 % in 1985 to 8.0 % in 2007. The alternative measure (number of farms with a size equal to, or larger than, the technical optimal size) shows a similar pattern i.e. that crop farms are behind dairy and pig farms.

The general conclusion is that, although the size of Danish farms has increased considerably during the last 20-30 years, more than 90 % of Danish full time farms are still far below the technical optimal scale of production. The reason is that the technical optimal size of farms has also increased considerably over time and only pig farms have managed to move relatively closer to the frontier of a technically optimal scale of production. Cash crop farms, on the other hand, have moved relatively further away from the optimal scale of production, despite that fact that the average sized cash crop farm has doubled during the 23 year period from 1985 to 2007. This supports the hypothesis that the restrictions concerning the amalgamation of farms and buying farm land has seriously prevented Danish farmers, and especially cash crop farmers, from taking full advantage of scale economies.

In 2010, the Danish Parliament changed the Agricultural Act. Earlier restrictions have been eased considerably so that it is now possible to amalgamate farms into larger units without any restrictions. Based on the results in this paper, the forecast is that this will increase the amalgamation of Danish farms into larger units, and improve the productivity of Danish agriculture.

## References

Alvarez A, Arias C (2003). Diseconomies of Size with Fixed Managerial Ability. *American Journal of Agricultural Economics* 85(1): 134-142.

Balk BM (2001). Scale Efficiency and Productivity Change. *Journal of Productivity Analysis* 15: p. 159-183.

- Battese G (1997). A Note on the Estimation of Cobb-Douglas Production Functions when some Explanatory Variables have Zero Values. *Journal of Agricultural Economics* 48: 250-252.
- Chambers RG (1988). *Applied Production Analysis*. Cambridge University Press, Cambridge.
- Chavas J-P (2001). Structural Change in Agricultural Production: Economics, Technology and Policy. p. 263-285 In: *Handbook of Agricultural Economics*, Volume 1 edited by B. Gardner and G. Rausser, Elsevier Science.
- Färe RS, Grosskopf S, Lovell CAK (1986). Scale Economics and Duality. *Journal of Economics* 46:175-182.
- Färe R, Primont D (1995). *Multi-Output Production and Duality: Theory and Applications*. Kluwer Academic Publishers, Boston.
- Greene W (2007). LIMDEP, Version 9. Econometric Software Inc., New York.
- FOI (Year). *Agricultural Account Statistics, Yearly*. Institute of Food and Resource Economics, Copenhagen, Denmark
- Kislev Y, Peterson W (1996). Economies of Scale in Agriculture: A Re-examination of the Evidence. In: *Papers in Honour of D. Gale Johnson, The Economics of Agriculture* 2: 156-170.
- Morrison Paul C, Nehring R, Banker D, Somwaru A (2004). Scale Economies and Efficiency in U.S. Agriculture: Are traditional Farms History? *Journal of Productivity Analysis* 22:185-205.
- Mosheim R, Lovell CAK (2009). Scale Economies and Inefficiency of U.S. Dairy Farms. *American journal of Agricultural Economics* 91 (3): 777-794.
- Rasmussen, S. (2010). Scale Efficiency in Danish Agriculture. *European Review of Agricultural Economics*. 37 (3): 335-367
- Shephard R. (1970). *Theory of Cost and Production Functions*. Princeton University Press. Princeton.