Optimal Conservation Policy under Imperfect Intergenerational Altruism

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Abstract

In this paper we study the optimal forest conservation policy by a hyperbolically discounting society. Society comprises a series of non-overlapping imperfectly altruistic generations each represented by its own government. Under uncertainty about future pay-offs we determine, as solution of an intergenerational dynamic game, the optimal timing of irreversible harvest. Earlier harvest occurs and the option value attached to the forest clearing decision is eroded under both the assumptions of naïve and sophisticated belief about future time-preferences. This results in a bias toward the current generation gratification which affects the intergenerational allocation of benefits and costs from harvesting and conserving a natural forest.

keywords: Imperfect altruism, Real Options, Hyperbolic Discounting, Time Inconsistency, Natural Resources Management
jel classification: D81,C70,Q23,Q58

1 Introduction

Land use has become increasingly important for public policy since several environmental policy goals, such as habitat conservation or carbon sequestration, crucially depends on land conversion decisions. In a meeting held in Copenaghen on December 2009 the United Nations Framework Convention on Climate Change (UNFCCC) has assigned a prominent role to forest conservation/avoided deforestation as a tool for balancing CO2 emissions.[3] At a society level, the decision to conserve natural forests by a must be framed accounting for several aspects. First, clearing may be an irreversible decision, second, pay-offs attached to such decision are often uncertain and, third, their weight in the decision objective depends on intergenerational time preferences.[4]

There is a vast literature dealing with optimal conservation policy and forest management under uncertainty and irreversibility.[5] An unifying aspect in this literature is the so called real option approach. This approach postulates that when an irreversible decision must be taken and payoffs attached to such decision are uncertain then the decision maker must account for the option value arising from the collection of additional information about future prospects (see Dixit and Pindyck, 1994). However, up to our knowledge, in the previous contributions the time preferences of the decision maker have always been considered constant and future pay-offs are consistently discounted exponentially. In a multigenerational frame, this assumption holds only if each generation is perfectly altruistic or equivalently if "each generation's preference for its own consumption relative to the next generation's consumption is no different from

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[3]The proposal is mainly based on the implementation of the Reducing Emissions from Deforestation and Degradation (REDD) program. The program proposes to consider forest conservation/avoided deforestation efforts under the Clean Development Mechanism (CDM) introduced by the Kyoto Protocol (Phelps et al., 2010). See also Fargione et al. (2008) on land clearing and biofuel carbon debt.

[4]The influence of intergenerational altruism on decisions regarding natural resource allocation and the solution of long-lived environmental problems has recently animated an interesting debate. See among others Li and Lofgren, 2000; Haurie, 2005; Karp, 2005; Saez-Marti and Weibull, 2005; Nowak, 2006.

[5]See among others Clarke and Reed (1989), Reed (1993), Conrad (1997), Bulte et al. (2002), Saphores (2003), Leroux et al. (2009), Di Corato et al. (2011).

their preference for any future generation's consumption relative to the succeeding generation" (Phelps and Pollack, 1968, p. 185).[6]

Unlike previous contributions, we study the optimal conservation policy set by a society composed by a sequence of non-overlapping imperfectly altruistic generations. We show that this is equivalent to view society as a sequence of hyperbolically discounting agents whose utility depends also on future agents' felicity. Harvest and conservation are evaluated by each generation in terms of welfare and a critical threshold triggering harvest is fixed on the basis of each generation time perspective. Once the forest is cleared, harvest revenues accrue to the generation alive while the harvest cost opportunity is beared by the future generations. Such cost includes the "sold" natural asset and the missing "dividend" represented by the flow of amenity services provided by the natural forest.[7] Under hyperbolic discounting future pay-offs are discounted at time-declining rates and decisions are consequently biased toward short-run gratification. In addition, another aspect characterizing these time-preferences is, as noted by Strotz (1956), the time-inconsistency of the planning. In other words, hyperbolic agents may wish to reconsider at a later date optimal plans previously defined. Depending on the awareness of her time-preferences, the decision maker may or may not anticipate such inconsistency (see Strotz, 1956 and Pollak, 1968). For instance, naive agents do not recognize it and set plans which they will disobey and revise according to the changed time perspective. On the contrary, sophisticated agents which anticipate the time-inconsistency set their plans strategically. They may follow a "strategy of precommitment" which consists in committing to a certain plan of action or a "strategy of consistent planning" which consists in not choosing the plans that are going to be disobeyed (Strotz, 1956).

In this frame, we set up a three-generation society model and determine for each generation the optimal conservation policy under both naive and sophisticated beliefs.[8] The optimal sequence of policies is obtained by solving a non-cooperative intergenerational dynamic game.[9] Each generation's life is random and regulated by a Poisson death process. Harvest returns are known and constant while amenity value is uncertain and follows a Geometric Brownian motion. We assume also that any commitment device is available since the presence of commitment would eliminate flexibility in the option exercise which is crucial in our analysis.

To have a benchmark and a measure for intergenerational Pareto optimality we first solve for the optimal conservation policy under perfect altruism.[10] Then, we show that under imperfect altruism harvest occurs earlier in expected terms with respect to the benchmark. This result holds under both naive and sophisticated beliefs and the intuition behind it is straightforward. Under both beliefs, the bias for current generation's gratification relative to the future generations' gratification erodes the option value attached to the decision to conserve and lowers the cost opportunity of harvesting. Interestingly, comparing optimal harvest strategies, we show that under sophistication harvest occurs earlier than under naiveté. Under sophistication, the time-inconsistency is perfectly anticipated by the current generation and the optimal conservation policy is set accounting for the cost of sub-optimal (from the current time perspective) future conservation plans. Finally, we show that the optimal policy under perfect altruism is a Pareto superior equilibrium which, however, imperfectly altruistic generations cannot achieve in the absence of a commitment mechanism.

[6]Ramsey (1928) defines discounting at a constant rate of time-preference "a practice which is ethically indefensible and arises merely from the weakness of the imagination". Despite this statement, Ramsey (1928) assumes perfect altruism. This assumption is later discussed by Phelps and Pollak (1968) studying optimal saving policies in a setting where the government acts on behalf of an imperfectly altruistic current generation.

[7]The value of a conserved forest may include the value attached to the provision of services such as flood control, carbon sequestration, erosion control, wildlife habitat, biodiversity conservation, recreation and tourism. See for instance Reed (1993).

[8]The solution to the more general problem with a finite number I and an infinite number of generations is provided in Di Corato (2008).

[9]The same solution concept is applied by Grenadier and Wang (2007) to determine investment timing under the assumption of a hyperbolic discounting entrepreneur.

[10]This is equivalent to the standard solution under exponential discounting. See Reed (1993) and Conrad (1997).

The paper is structured as follows. In section 2, the set-up of the model is presented. In section 3 we determine the optimal conservation policy under perfect altruism. In section 4, we account for imperfect altruism and solve the problem under naive and sophisticated beliefs. In this section we also discuss our results. In Section 5 we conclude presenting some remarks. All the proofs are available in the appendix.

2 The basic set-up

Consider an area of primary forest which can be totally conserved or irreversibly harvested.[11] If conserved, at each time period t a flow of amenity value, A_t, accrues to Society. We assume that such flow randomly fluctuates according to the following geometric Brownian motion

$$dA_t = \mu A_t dt + \sigma A_t dz_t \tag{1}$$

where μ and σ^2 are expected growth rate and variance and $\{z_t\}$ is a standard Wiener process with $E[dz_t] = 0$ and $E[(dz_t)^2] = dt$.[12]

We denote by R the net revenue realized if the forest is harvested. This includes the timber value net of harvest costs plus the value of the cleared land under some alternative possible destinations. For the sake of simplicity, we set $R = 1$ and use harvest revenue as numeraire.[13]

The forest can be seen as an asset paying a dividend represented by the flow A_t and Society as holding an option to harvest. Benefits and costs of forest conservation must be considered to determine an optimal harvesting strategy maximizing the social welfare. In this respect, harvesting is irreversible and an option value is attached to the decision to conserve since harvest postponement allows the collection of information about the uncertain future flow of amenity value. Clearly, harvesting at a later date has a cost represented by the foregone harvest revenues.

Let's now present a model of Society. As in Winkler (2006), Society is composed of three generations which live at times $i = 0$ (the present), $i = 1$ (near future), and $i = 2$ (distant future). Even if composed by different generations Society behaves as a single intertemporal entity and decisions are taken considering not only the welfare of the present generation but also the discounted welfare of future generations. In this respect, note that when harvest occurs only the generation living at that time benefits from it. On the contrary, the cost opportunity of harvesting, i.e. harvest revenue plus the flow of amenity value, is faced by the successive generations. We assume that each generation, G_i, is risk neutral and that, unlike Winkler (2006), lives for a random time period, $t_{i+1} - t_i$, where t_i and t_{i+1} are the birth dates of the current and the subsequent generation and intergenerational transition is regulated by a Poisson death process with intensity $\lambda \in (0, \infty)$. Each generation discounts exponentially at a constant time preference rate ρ and gives weight β to future generations' welfare relative to its own. By $\beta = 1$ and $\beta \in (0, 1)$ we characterize a perfectly altruistic and imperfectly altruistic generation, respectively. Note that this is equivalent to assume that

[11] Irreversibility makes sense considering that the recovery the forest could require a substantially long time period. See for instance Hilbert and Wiensczyk (2007). We do not consider the possibility of partial and incremental irreversible harvesting. Such problem is treated by Bulte et al. (2002). See also Di Corato et al., (2010) for an equilibrium model of habitat conservation.

[12] The assumption of a geometric Brownian motion is quite common in the literature. Conrad (1997, p. 98) uses a geometric Brownian motion to capture uncertainty over preferences for habitat conservation. Up to Bulte et al. (2002, p.152) the expected trend μ "can be positive (e.g., reflecting an increasingly important carbon sink function as atmospheric CO2 concentration rises), but it may also be negative (say, due to improvements in combinatorial chemistry that lead to a reduced need for primary genetic material)".

[13] Using R as numeraire we may, by invoking the homogeneity of option value, also allow for stochastic harvest revenues. See Dixit and Pindyck (1994, pp. 207-211).

Definition 1 *For any $\beta \in (0,1]$ and $\lambda \in (0,\infty)$ the generation i discount function is given by*

$$D_i(t,s) = \begin{cases} e^{-\rho(s-t)} & \text{if} \quad s \in [t_i,\ t_{i+1}] \\ \beta e^{-\rho(s-t)} & \text{if} \quad s \in [t_{i+1},\ \infty] \end{cases} \tag{2}$$

for $s > t$ and $t_i \le t \le t_{i+1}$.

The stochastic discount functional form in (2) is equivalent to the one introduced by Harris and Laibson (2004) to model a hyperbolic discounting agent.[14] Generation i discounts exponentially at rate ρ pay-offs occurring over its lifespan while it additionally discounts by the factor β pay-offs occurring once dead. This implies, as one can easily note,[15] that pay-offs are discounted at a rate declining with time and that consequently a higher weight is put on outcomes occuring over the short run. In addition, as noted by Strotz (1956), this class of time-preferences leads to time inconsistent planning. That is, since each generation has its own time preferences then the optimal conservation policy fixed by the previous generation may be disobeyed and revised according to its own time perspective. Finally, we assume that each generation is not able to commit future generations to any plan. This implies that each generation is free to define its optimal conservation plan on the basis of its expectations about future generations' behavior. In this respect, we will allow for two different types of beliefs: sophisticated or naive (see Strotz, 1956 and Pollak, 1968). A generation is sophisticated or naive on the basis of its ability to anticipate or not that future generations will stick to their plans.

3 Optimal conservation policy under intergenerational altruism

Let's start by determining the optimal policy under perfect altruism ($\beta = 1$). This is equivalent to solve a standard optimal stopping problem in continuous time. At each t the value of harvesting (stopping) must be compared with the expected value of conserving over the next dt (continuation) given the information available at that point in time and the knowledge of the process in (1). Note that with $\beta = 1$ the discount function $D_i(t,s)$ reduces to the standard exponential form. This implies that the optimal harvest policy is time-consistent and that each G_i must solve the same problem.

We denote by $V(A)$ the value function and solve for G_i the following maximization problem:[16]

$$V(A) = \max_A \left\{1,\ Adt + e^{-\rho dt} E\left[V(A + dA)\right]\right\} \tag{3}$$

By using Ito's lemma it follows that

Definition 2 *In the continuation region, $A \ge A^*$, the value function, $V(A)$, solves the following second-order non-homogenous differential equation*

$$\Delta V(A) + A = 0, \text{ for } A \ge A^* \tag{4}$$

where $\rho > \mu$,[17] and $\Delta = \frac{1}{2}\sigma^2 A^2 \frac{\partial^2}{\partial A^2} + \mu A \frac{\partial}{\partial A} - \rho$ and A^* are a differential operator and the level of amenity value delimiting the continuation region, respectively. At A^* conserving or harvesting is indifferent and as soon as this level is hit the option is exercised. Note that for $A \to \infty$ the option to harvest is never exercised. Then, the solution of the differential equation (4) requires the boundary condition:

$$\lim_{A \to \infty} V(A) = 0 \tag{4.1}$$

[14] Grenadier and Wang (2007) uses the same functional form to model investment choices undertaken by an hyperbolic agent.

[15] As $\frac{1-e^{-\rho}}{e^{-\rho}} < \frac{1-\delta e^{-\rho}}{\delta e^{-\rho}}$, the discount rate between two consecutive periods t and $t+1$ increases as date t comes close.

[16] We drop the time subscript for notational convenience.

[17] Note that if $\rho \le \mu$ conserving forever is the optimal plan. To account for an appropriate adjustment for risk, we should have taken the expectation with respect to a distribution of A adjusted for risk neutrality. See Cox and Ross (1976) for further details.

and must meet the following value-matching and smooth-pasting conditions:[18]

$$V(A^*) = 1,\ V'(A^*) = 0 \qquad (4.2\text{-}4.3)$$

Solving for A^* and $V(A)$ yields the following proposition

Proposition 1 *Under constant time-preference the solution to the optimal stopping problem in (3) is given by*

$$A^* = \frac{\theta}{\theta - 1}(\rho - \mu) \qquad (5)$$

$$V(A) = \begin{cases} \left(1 - \frac{A^*}{\rho-\mu}\right)\left(\frac{A}{A^*}\right)^\theta + \frac{A}{\rho-\mu} & \text{for } A > A^* \\ 1 & \text{for } A \leq A^* \end{cases} \qquad (6)$$

where θ is the negative root of the characteristic equation $Q(\theta) = \frac{1}{2}\sigma^2\theta(\theta-1) + \mu\theta - \rho = 0$.

Proof. See appendix A.1. ■

The first term on the RHS of (7) represents the value of the option to harvest. As one can easily check it vanishes as $A \to \infty$. The second term is the expected present value of the flow of amenity value accruing intertemporally to society if the forest is conserved. Harvest occurs if $A \leq A^*$ and the generation living at that time benefits from net revenue 1. The relevant comparative statics are $\frac{dA^*}{d\rho} > 0$, $\frac{dA^*}{d\mu} < 0$ and $\frac{dA^*}{d\sigma} < 0$. That is, harvest occurs earlier if the decision maker adopts a higher discount rate since future pay-offs from conservation have less weight for the current decision. On the contrary, as standard in the real option literature, a higher expected growth and volatility in the amenity value delay the harvest.

4 Optimal conservation policy under imperfect intergenerational altruism

4.1 Optimal conservation policy under naiveté

Let's relax the assumption of perfect altruism ($0 < \beta < 1$) and assume naive beliefs. The generation, G_i, when alive, may exercise the option to harvest and obtain 1 as net return or, by conserving the forest, benefit from the flow of amenity value during its life and from the value attached to the forest if managed by future generations. If harvest does not occur during its life, the forest is left as a legacy to the succeeding G_{i+1} which in turn may or may not harvest. Here, unlike the optimal stopping problem in the previous section, the optimal harvest policy results from the game played over an infinite horizon by G_0, G_1 and G_2 for the definition, under their own time perspective, of the optimal conservation strategies.

The present generation, G_0, is naive and believes that G_1 and G_2 will set their policies according to its time preferences, i.e. $D_0(t,s)$. This implies that G_1 and G_2 are considered by G_0 as perfect altruistic generations discounting future pay-offs exponentially at the rate ρ. By $D_0(t,s)$, G_0 discounts by $e^{-\rho(s-t)}$ the pay-offs occurring at $t_0 < s < t_1$ and by $\beta e^{-\rho(s-t)}$ the pay-offs occurring at $s \geq t_1$. The optimal harvest strategy for G_0 is completely characterized by a critical threshold, A_n, at which harvesting is triggered. If the next generation, G_1, is born before A_n is met then G_0 enjoys the flow of amenity value, A, for the period $[t_0,\ t_1)$ and the continuation value, $V_c{}^n(A)$, which is given by the expected present value of the pay-offs attached to future Governments' conservation strategies. Otherwise, G_0 exercises the option to harvest and earns 1. Note that if, as incorrectly believed by G_0, all future Governments discounts at rate ρ then their optimal stopping problem is equivalent to the one solved in the previous section. Hence, their critical threshold and value function are given by A^* and $V(A)$, respectively. Finally, since G_0 lowers by β all pay-offs occurring at $s \geq t_1$ then $V_c{}^n(A) = \beta V(A)$.

[18] On optimality conditions see Dixit and Pindyck (1994).

Denoting by $V^{\,n}(A)$ the value function for G_0, we solve the following maximization problem

$$V^{\,n}(A) = \max_{A}\left\{1,\ Adt + e^{-\rho dt}\left[e^{-\lambda dt}E[V^{\,n}(A+dA)] + \left(1-e^{-\lambda dt}\right)E[V_c^{\,n}(A+dA)]\right]\right\} \tag{7}$$

By following the usual steps, we can state that

Definition 3 *In the continuation region, $A \geq A_n$, the value function, $V^{\,n}(A)$, solves the following second-order non-homogenous differential equation*

$$\Delta V^{\,n}(A) + A + \lambda(V_c^{\,n}(A) - V^{\,n}(A)) = 0, \text{ for } A \geq A_n \tag{8}$$

Solving (9) by imposing (4.1) and the following value-matching and smooth-pasting conditions

$$V^{\,n}(A_n) = 1,\ V^{\,n\prime}(A_n) = 0 \tag{8.1-8.2}$$

yields

Proposition 2 *Under imperfect altruism and naïve belief the solution to the optimal stopping problem in (7) is given by*

$$A_n = \frac{\gamma - \beta\frac{\theta-\gamma}{\theta-1}(\frac{A_n}{A^*})^\theta}{\gamma-1}(\frac{\rho-\mu}{\eta}) \tag{9}$$

$$V^{\,n}(A) = \begin{cases} [1+\frac{\beta}{\theta-1}(\frac{A_n}{A^*})^\theta - \eta\frac{A_n}{\rho-\mu}](\frac{A}{A_n})^\gamma - \frac{\beta}{\theta-1}(\frac{A}{A^*})^\theta + \eta\frac{A}{\rho-\mu} & \text{for } A > A_n \\ 1 & \text{for } A \leq A_n \end{cases} \tag{10}$$

where $\eta = \frac{\rho+\lambda\beta-\mu}{\rho+\lambda-\mu} \leq 1$ and $\gamma \leq \theta$ is the negative root of the characteristic equation $Q(\gamma) = \frac{1}{2}\sigma^2\gamma(\gamma-1) + \mu\gamma - (\rho+\lambda) = 0$.

Proof. See appendix A.2. ■

In the appendix we show also that

Proposition 3 *Under imperfect altruism and naïve belief $A_n > A^*$.*

Proof. See appendix A.3. ■

The intuition behind Proposition (3) is that the value of keeping open the option to harvest is lower under imperfect altruism since the expected present value of the utility from the decisions of the future generations is lower ($0 < \beta < 1$, $\eta < 1$). Due to its short-run biased time preferences G_0 rushes in order to anticipate the exercise of the option to harvest by future G_i. Note that the less altruistic is G_0 the higher is the threshold triggering an earlier harvest ($\frac{\partial A_n}{\partial \beta} < 0$). In addition, also a faster intergenerational transition rate may induce rush since the present generation a shorter expected life ($\frac{\partial A_n}{\partial \lambda} > 0$). Clearly, G_0's plan is irrational in that it is based on the false belief of having the subsequent G_i defining their optimal policy according to $D_0(t,s)$. On the contrary, as soon as G_1 will be born the harvest threshold adopted will not be A^* but higher and fixed according to the discount function $D_1(s,t)$. This does not happen when G_2 steps in but note that this is independent from G_0's false beliefs. In fact, G_2 discounts exponentially only because it is the last generation potentially managing the forest. The remaining comparative statics confirms the effects discussed in section 3 ($\frac{dA_n}{d\rho} > 0$, $\frac{dA_n}{d\mu} < 0$ and $\frac{dA_n}{d\sigma} < 0$). Finally, considering G_2's plan we will show in the next section that independently from its belief G_1 harvest at $A_n > A^*$.

4.2 Optimal conservation policy under sophistication

In this section we assume imperfect altruism and sophisticated beliefs. Under sophistication, each generation anticipates future time inconsistency, recognizes the sub-optimality of future generations' conservation policy with respect to its time perspective and accounts for it in the welfare maximization problem. Let's start at the generic time period t with G_0. On the next time interval dt G_1 will be born with probability λdt . Once G_1 has replaced G_0 then G_2 will take control according to the same process. Once in charge G_2 will rule forever. Given the structure of the problem we determine by playing backward a subgame-perfect equilibrium sequence of harvest time thresholds.

Now, consider G_2 and denote by $A_{s,2}$ and ${V_2}^s(A)$ the critical threshold and its value function. G_2 is the last generation managing the forest and then its maximization problem is equivalent to the one solved in section 3. Thus, $A_{s,2} = A^*$ and ${V_2}^s(A) = V(A)$. Let's skip to G_1. Its plan must be defined considering that G_2 exercises the option to harvest at $A_{s,2}$. Due to its present-biased preferences G_2's value function is worth for G_1 only β times its value. The problem for G_1 is then equivalent to the one solved in section 4.1 and then $A_{s,1} = A_n$ and ${V_1}^s(A) = V^n(A)$. However, note that in this case the underlying beliefs are rationally formed. Finally, it is time for G_0 to formulate its optimal harvest plan. Denote respectively by $A_{s,0}$ and ${V_0}^s(A)$ its value function and harvest threshold and let $V_{c,1}^s(A)$ represent its valuation of the exercise decisions that could be taken by G_1 and G_2 ($A_{s,1}$, $A_{s,2}$). The continuation value, $V_{c,1}^S(A)$, must be determined recursively as follows. If G_1 is alive when the threshold $A_{s,1}$ is hit then the option is exercised and the payoff for G_0 is equal to β. Instead if G_2 replaces G_1 before $A_{s,1}$ is met, then the G_0 continuation value is equal to G_1's continuation value $V_{c,2}^s(A) = \beta {V_2}^s(A)$.
The optimal stopping problem for G_0's is then represented by the following Bellman equation

$$ {V_0}^s(A) = \max_A \left\{1,\ Adt + e^{-\rho dt}\left[e^{-\lambda dt}E[{V_0}^s(A+dA)] + \left(1-e^{-\lambda dt}\right)E[V_{c,1}^s(A+dA)]\right]\right\} \tag{11} $$

Definition 5 *In the continuation region, $A \geq A_{s,1}$, the value function, ${V_0}^s(A)$, solves the following second-order non-homogenous differential equation*

$$ \Delta {V_0}^s(A) + A + \lambda(V_{c,1}^s(A) - {V_0}^s(A)) = 0, \text{ for } A \geq A_{s,0} \tag{12} $$

where $V_{c,1}^s(A) = \beta[(1 - {V_2}^s(A_{s,1}))(\frac{A}{A_{s,1}})^\gamma + {V_2}^s(A)]$.[19]

We can now solve (12) by requiring that at $A_{s,0}$ the following standard optimality conditions hold:

$$ {V_0}^s(A_{s,0}) = 1,\ {V_0}^{s\prime}(A_{s,0}) = 0 \tag{12.1-12.2} $$

Once solved, we can state that

Proposition 4 *Under imperfect altruism and sophisticated belief the solution to the optimal stopping problem in (11) is given by*

$$ A_{s,0} = \frac{\gamma - \beta\frac{\theta-\gamma}{\theta-1}(\frac{A_{s,0}}{A_{s,2}})^\theta - Z_1 A_{s,0}^\gamma}{\gamma - 1}(\frac{\rho-\mu}{\eta}) \tag{13} $$

$$ {V_0}^s(A) = \begin{cases} (\frac{Z_0}{A_{s,0}^\gamma} - \frac{Z_1}{A_{s,1}^\gamma}\ln A)A^\gamma - \frac{\beta}{\theta-1}(\frac{A}{A_{s,2}})^\theta + \eta\frac{A}{\rho-\mu} & \textit{for } A > A_{s,0} \\ 1 & \textit{for } A \leq A_{s,0} \end{cases} \tag{14} $$

where $Z_0 = [1 - \eta\frac{A_{s,0}}{\rho-\mu} + \frac{\beta}{\theta-1}(\frac{A_{s,0}}{A_{s,2}})^\theta + Z_1(\frac{A_{s,0}}{A_{s,1}})^\gamma \ln A_{s,0}]$ *and* $Z_1 = \lambda\beta\frac{1-{V_2}^s(A_{s,1})}{\sigma^2\gamma - (\frac{\sigma^2}{2}-\mu)}$.

Proof. See appendix A.4 ■

[19] See appendix A.4.

Proposition 5 *Under imperfect altruism and sophisticated belief*

1) $A_{s,0} > A_{s,1} > A_{s,2}$,

and

2) ${V_2}^s(A) > {V_1}^s(A) > {V_0}^s(A)$.

Proof. See appendix A.5 ■

By Proposition 5 an additional effect emerges under sophistication. Harvest timing and the value functions are affected by the anticipation of conservation policies set by the following generations. Note in fact that G_2 which is not followed by any generation does not anticipate the harvest with respect to the benchmark ($A_{s,2} = A^*$). On the contrary, both G_1 and G_0 rush on harvesting since as explained above their time preference are "present-biased" and lowers the cost opportunity of their drastic choices. In addition, comparing G_1 and G_0, we note that G_0 fixes a higher threshold for harvesting. With respect to a naive generation, the current sophisticated generation accounts for the "burden" represented by the sub-optimality (from its time perspective) of future policies and a clear "sophistication" effect emerges. As discussed by Winkler (2006), without commitment each generation could indirectly influence the choices of the successive ones by limiting the set of available choices. Similarly, in our model the present sophisticated generation rushes on harvesting to reduce the probability of the future "sub-optimal" forest management. Finally, in the second part of Proposition 5, we show that for both G_1 and G_0 the optimal policy under perfect altruism, $A_{s,2}$, is a Pareto superior equilibrium. However, without commitment this outcome cannot be achieved by imperfectly altruistic generations.[20] Finally the effect of changes in the relevant parameters is confirmed ($\frac{dA_s}{d\beta} < 0$, $\frac{dA_s}{d\lambda} > 0$, $\frac{dA_s}{d\rho} > 0$, $\frac{dA_s}{d\mu} < 0$ and $\frac{dA_s}{d\sigma} < 0$).

5 Conclusions

Before concluding, some considerations are in order. In our analysis, we study the allocation of a natural resource from an intergenerational welfare maximizing perspective modeling Society as an hyperbolic decision maker. However, we note that the discount function in (2) may also be used to represent:

1. a Society composed by two individuals, a utilitarian and a conservationist, discounting at a positive rate and at a rate tending to zero, respectively. In fact, it suffices to assign to their utilities weight β and $1-\beta$ in the welfare function and rates $\rho + \lambda$ and ρ with $\rho \to 0$;[21]

2. an individual having a taste for immediate gratification as firstly proposed by Harris and Laibson (2004);[22]

3. a Government accounting for political turnover as suggested by Amador (2004). That is, consider a risk-neutral party,[23] say X, and assume that it is currently governing the country at the generic time t. Suppose that it discounts exponentially at rate ρ the pay-offs occurring over all future periods but that undervalues pay-offs occurring in the future periods to account for the probability of being in charge in those future periods.[24] Hence, let $\beta = p \cdot 1 + (1-p)\omega \leq 1$ where $0 \leq p \leq 1$ is the exogenous probability of winning an electoral round and $\omega \leq 1$ is the weight given to social welfare when

[20] Commitment mechanisms set by the present generation to overcome the time-inconsistency issue are problematic in the context of intergenerational decisions. First, they would denote the *dictatorship* of the present generation on the future ones; second, even taking apart these ethical considerations, the present generation would have *limited control* on future generations' decisions (see Winkler, 2006, p. 580).

[21] See Li and Lofgren (2000, p.238).

[22] Discussing the use of exponential discounting, Strotz argues that there is "*no reason why an individual should have such a special discount function*" (Strotz, 1956, p.172). Experimental evidence in psychology has later supported his conjecture (Loewenstein and Prelec, 1992). See Frederick et al., 2002 for a review.

[23] Assume that also rival parties not at the government have the same time preferences.

[24] Note that Brocas and Carrillo (1998) justifies imperfect intergenerational altruism on this basis.

party X is not at the government.[25] Finally, suppose that due to populist or rival parties pressure and/or other unexpected events the government may suddenly fall over the next time period dt with probability λdt where $\lambda \in [0, \infty)$ is the intensity of a Poisson process.

Based on these considerations we believe that our contribution may be useful not only to give a rationale for governments fostering natural resource depletion and for the time inconsistency of environmental policies[26] but also in that it provides a frame for addressing other economic problems where the decision maker may hold a real option on technological innovation, land development, etc.

A Appendix

A.1 Proposition 1

By the linearity of (4) and the boundary condition (4.1), the solution takes the following functional form:[27]

$$V(A) = KA^{\theta} + \frac{A}{\rho - \mu} \tag{A.1.1}$$

where θ is the negative root of the characteristic equation $Q(\theta) = \frac{1}{2}\sigma^2\theta(\theta - 1) + \mu\theta - \rho = 0$ and K a constant to be determined.

By plugging (A.1.1) into (4.1) and (4.2) and solving for A^* and K we obtain (5) and (6).

A.2 Proposition 2

By substituting (6) into (8) and rearranging we obtain

$$\Gamma V^{\,n}(A) = -[A(1 + \frac{\lambda\beta}{\rho - \mu}) + \lambda\beta(1 - \frac{A^*}{\rho - \mu})(\frac{A}{A^*})^{\theta}], \text{ for } A \geq A_n \tag{A.2.1}$$

where Γ is the differential operator $\Gamma = \frac{1}{2}\sigma^2 A^2 \frac{\partial^2}{\partial A^2} + \mu A \frac{\partial}{\partial A} - (\rho + \lambda)$.

The solution takes the form[28]

$$V^{\,n}(A) = NA^{\gamma} + \beta(1 - \frac{A^*}{\rho - \mu})(\frac{A}{A^*})^{\theta} + \eta\frac{A}{\rho - \mu} \tag{A.2.2}$$

where γ is the negative root of the characteristic equation $Q(\gamma) = \frac{1}{2}\sigma^2\gamma(\gamma - 1) + \mu\gamma - (\rho + \lambda) = 0$, $\eta = \frac{\rho+\lambda\beta-\mu}{\rho+\lambda-\mu} \leq 1$ and N is a constant to be determined.

Conditions (8.1) and (8.2) must hold at A_n. Using (A.2.2) we obtain the following system

$$\begin{cases} NA_n^{\gamma} + \beta(1 - \frac{A^*}{\rho-\mu})(\frac{A_n}{A^*})^{\theta} + \eta\frac{A_n}{\rho-\mu} = 1 \\ N\gamma A_n^{\gamma-1} + \beta\theta(1 - \frac{A^*}{\rho-\mu})(\frac{A_n}{A^*})^{\theta-1}\frac{1}{A^*} + \eta\frac{1}{\rho-\mu} = 0 \end{cases}$$

By solving it we determine (9) and (10).

[25] This could be due for instance to the fact that political parties are aware that people when voting takes into account only their conduct when in charge.

[26] See for instance Brocas and Carrillo (1998) suggesting that "slash and burn" agricultural practices may be justified on the basis of hyperbolic time preferences or Hepburn (2003) showing how under naive hyperbolic planning a renewable resource may be poorly managed and an unintended collapse may occur. A striking example of hyperbolic time preferences induced by political turnover is given by the management of publicly owned natural forests in Indonesia where despite targeting a sustainable exploitation of these natural assets there is evidence of a faster depletion rate and of time-inconsistent environmental policy (Atje and Roesad, 2004). For other examples see Winkler (2006).

[27] The solution for the homogeneous part of (4) is $V(A) = K_1A^{\theta_1} + K_2A^{\theta_2}$ where $\theta_1 > 0$ and $\theta_2 < 0$ are the roots of $Q(\theta) = 0$ and K_1 and K_2 are two constants to determined. However, since $\lim_{A\to\infty} V(A) = 0$ then we must drop the first term by setting $K_1 = 0$. The same discussion will apply also in the next sections. The particular solution $\frac{A}{\rho-\mu}$ is found using the method of undetermined coefficients.

[28] The general solution is given by the homogeneous solution for the complementary function in (A.2.1), $V_h{}^n(A) = NA^{\gamma}$, plus the particular solution $V_p{}^n(A) = M_1A^{\theta} + M_2A$ where the undetermined coefficients c_1 and c_2 are set to satisfy (A.2.1).

A.3 Proposition 3

Define $g(x) = \frac{\gamma - \beta\frac{\theta-\gamma}{\theta-1}(\frac{x}{A^*})^\theta}{\frac{\eta(\gamma-1)}{\rho-\mu}}$ and $f(x) = g(x) - x$. Note that $g'(x) > 0$ and $g'(x) < 0$. Since $f(A_n) = 0$ and $f(A^*) = \frac{\gamma(1-\eta)-(\beta-\eta)\frac{\theta-\gamma}{\theta-1}}{\frac{\eta(\gamma-1)}{\rho-\mu}} > 0$ then it follows that $A_n > A^*$.[29]

A.4 Proposition 4

Continuation Value Function - In the $A \geq A_{s,1}$, the continuation value function, $V_{c,1}^{\;s}(A)$, solves the following second-order non-homogenous differential equation

$$\Gamma V_{c,1}^{\;s}(A) = -\beta(A + \lambda V_2{}^s(A)), \text{ for } A \geq A_{s,1} \tag{A.4.1}$$

By the continuity of $V_{c,1}^{\;s}(A)$ it follows that $V_{c,1}^{\;s}(A_{s,1}) = \beta$. Note that we take $A_{s,1}$ as optimally determined by maximizing $V_1^S(A)$ and then we do not need to impose smooth-pasting at this point. Using standard arguments, the general solution is given by

$$V_{c,1}^{\;s}(A) = HA^\gamma + \beta[(1 - \frac{A^*}{\rho-\mu})(\frac{A}{A_{s,2}})^\theta + \frac{A}{\rho-\mu}] = HA^\gamma + \beta V_2{}^s(A) \tag{A.4.2}$$

Solving (A.4.2) subject to the value-matching condition $V_{c,1}^{\;s}(A_{s,1}) = \beta$ we obtain

$$V_{c,1}^{\;s}(A) = \beta[(1 - V_2{}^s(A_{s,1}))(\frac{A}{A_{s,1}})^\gamma + V_2{}^s(A)] \tag{A.4.3}$$

Value Function - Equation (12) can be restated as follows:

$$\Gamma V_0{}^s(A) = -\{A + \lambda\beta[(1 - V_2{}^s(A_{s,1}))(\frac{A}{A_{s,1}})^\gamma + V_2{}^s(A)]\}, \text{ for } A \geq A_{s,0} \tag{A.4.4}$$

The solution to the homogenous part is standard. On the contrary, we must be more careful on the guess candidate functional form for the particular solution since it contains a term in A^γ which is present also in the homogenous solution.[30] We choose for our guess the following functional form:

$$V_{0,\,p}^{\;s}(A) = J_1 A + J_2 A^\theta + J_3 A^\gamma \ln A + J_4 A^\gamma \tag{A.4.5}$$

We substitute (A.4.5) and its first two derivatives into (A.4.4). Solving for the coefficients of each power of A yields

$$J_1 = \frac{\eta}{\rho-\mu},\ J_2 = -\frac{\beta}{\theta-1}A_{s,2}^{-\theta},\ J_3 = -\lambda\beta\frac{1 - V_2{}^s(A_{s,1})}{\sigma^2\gamma - (\frac{\sigma^2}{2} - \mu)}A_{s,1}^{-\gamma},\ J_4 = 0$$

The general solution is then given by

$$V_0{}^s(A) = SA^\gamma + \eta\frac{A}{\rho-\mu} - \frac{\beta}{\theta-1}(\frac{A}{A_{s,2}})^\theta - \lambda\beta\frac{1 - V_2{}^s(A_{s,1})}{\sigma^2\gamma - (\frac{\sigma^2}{2} - \mu)}(\frac{A}{A_{s,1}})^\gamma \ln A \tag{A.4.6}$$

Solving (A.4.4) subject to conditions (12.1-12.2) we obtain $S = (1 - J_1 A_{s,0} - J_2 A_{s,0}^\theta - J_3 A_{s,0}^\gamma \ln A_{s,0})A_{s,0}^{-\gamma}$ and then (13-14).

[29] Note that $f(A_n) = 0$ may have two roots, $A_{n,1}$ and $A_{n,2}$ with $A_{n,1} < A^* < A_{n,2}$. However, as one can easily check, at $A_{n,1}$ the second order optimality condition do not hold.

[30] The method of undetermined coefficients must be modified when dealing with a system in resonance. See Simon and Blume (1994, pp. 654-656).

A.5 Proposition 5

We remind that $A_{s,2} = A^*$, ${V_2}^s(A) = V(A)$, $A_{s,1} = A_n$ and ${V_1}^s(A) = V^n(A)$. To prove this result we exploit the similarity between the option to harvest and an American put option. Note in fact that G_1 may be seen as holding a put option paying a dividend equal to $A + \lambda\beta {V_2}^s(A)$ if kept and a strike price equal to 1 if exercised. Similarly, G_0 holds a put option paying $A+\lambda\beta[(1-{V_2}^s(A_{s,1}))(\frac{A}{A_{s,1}})^\gamma + {V_2}^s(A)]$ as a dividend and 1 as strike price. Note from (A.2.1) and (A.4.4) that both G_0 and G_1 are discounting at the same rate, i.e. $\rho + \lambda$. Then, comparing the two assets, we note that they differ only in the dividend. In particular, G_0 receives a lower dividend in that $1 - {V_2}^s(A_{s,1}) < 0$ for $A_{s,2} < A_{s,1}$. Since an option paying a lower dividend is exercised earlier it follows that $A_{s,0} > A_{s,1}$. In addition, if ${V_0}^s(A)$ and ${V_1}^s(A)$ are seen as the function expressing the value of the assets characterized above then it must be ${V_0}^s(A) < {V_1}^s(A)$. Finally, we rearrange (4) as follows

$$\Gamma {V_2}^s(A) + A + \lambda {V_2}^s(A) = 0, \text{ for } A \geq A_{s,2} \tag{A.5.1}$$

That is, G_2 holds a put option paying $A + \lambda {V_2}^s(A)$ as a dividend and 1 as strike price. Using the same arguments, it follows that as proved in section (A.3) $A_{s,1} > A_{s,2}$ and ${V_1}^s(A) < {V_2}^s(A)$.

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