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Beyond Indifferent Players:

On the Existence of Prisoners Dilemmas in games
with amicable and adversarial preferences

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with amicable and adversarial preferences

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Abstract

Why don't agents cooperate when they both stand to gain? This question ranks among the most fundamental in the social sciences. Explanations abound. Among the most compelling are various configurations of the prisoner's dilemma (PD), or public goods problem. Payoffs in PD's are specified in one of two ways: as primitive cardinal payoffs or as ordinal final utility. However, as final utility is objectively unobservable, only the primitive payoff games are ever observed. This paper explores mappings from primitive payoff to utility payoff games and demonstrates that though an observable game is a PD there are broad classes of utility functions for which there exists no associated utility PD. In particular we show that even small amounts of either altruism or enmity may disrupt the mapping from primitive payoff to utility PD. We then examine some implications of these results.

JEL C7, D6, H4

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Résumé

Pourquoi des gens ne coopèrent-ils pas alors qu'ils auraient tous les deux à y gagner? Cette question se range parmi les plus fondamentales au sein des sciences sociales. Les explications abondent. Parmi les plus convaincantes se trouvent les diverses présentations du dilemme du prisonnier, ou le problème du bien public. Ceux qui paient dans le dilemme du prisonnier sont désignés de deux manières: comme des payeurs cardinaux primitifs ou comme visant une utilité finale ordinale. Cependant, comme l'utilité finale n'est pas observable de façon objective, seuls les jeux à payeurs primitifs au payeur d'utilité et démontre que, quoique le dilemme du prisonnier soit un jeu observable, il y a un grand nombre de fonctions d'utilité pour lesquelles il n'existe pas de dilemme du prisonnier d'utilité qui y soit associé. En particulier, nous montrons que même de petites doses d'altruisme ou d'inimitié peuvent venir rompre la cartographie du payeur primitif au dilemme du prisonnier d'utilité. Nous examinons alors quelques implications de ces résultats.

1. Introduction

Prisoner dilemmas (PDs) have been employed across the social and business sciences, philosophy, and biology as prime examples of the tension between individual and collective rationality.¹ They constitute powerful illustrations of the gains foregone when strategic structure precludes cooperation as an equilibrium strategy.

The payoffs in PD's have two forms. First, they may be cardinal observable payoffs (e.g., years in prison, nuclear warheads, or advertising budgets). We refer to such games as *Primitive Prisoner's Dilemmas* (PPDs). Alternatively, payoffs may be specified as final utility, which is inherently unobservable. We refer to these games as *Utility Prisoner's Dilemmas* (UPDs). In either case there is an implicit mapping between observable payoff and final utility that has received scant attention in the literature. Though this neglect may be innocuous for some mappings we show that when a player has amicable or adversarial inclination towards the other player there are broad classes of utility functions for which it is *impossible* for a PPD to map into a UPD. We identify classes of utility functions under which games that are *not* prisoners dilemmas in observable payoffs are, in fact, prisoner's dilemmas in the unobserved utility game.

Why our focus on amicable and adversarial preferences? First, there exists a large body of experimental evidence (see Fehr and Gächter 2000 for a survey) that casts doubt on the indifference of players with regard to the payoffs of other players. We will demonstrate that only in the case of truly indifferent players will a game that is a PD in observable payoffs necessarily be a PD in the unobserved utility game. In fact, the body of experimental evidence cited above indicates unambiguously that pure neutrality towards the welfare of the other players is the exception, rather the rule. Beyond the experimental literature, the potential for altruism in strategic environments has long been recognized. For example, strategic frameworks are frequently employed to model intra-household interactions (see Browning and Chiaporri 1998). Moreover, intra-household and kin altruism is implied by evolutionary biology.

Adversarial relationships, in the sense of competition, arise in virtually all economic environments. However in the typical strategic setting *adversarial* incentives are inherent in the payoff-structure rather than embodied in preferences. Thus, the incentive to adopt a particular strategy is typically governed by own payoff maximization rather than explicit consideration of rivals' payoff.² In contrast, we consider strategic behavior when a player's *utility* is decreasing in the other player's cardinal payoff. Such preferences may correspond to conventional notions of envy or malice. These terms, "envy" and "malice," have precise economic meanings (see Hammond 1987 and Brennan 1973), and though the related literature addresses issues tangentially related to this paper, it never discusses the implications of such utility mappings on the existence of the PD.

For those who remain skeptical of amicability or enmity in preferences per-se, there exists an alternative motivation that is also entirely consistent with the model and results. Namely, if the observable payoff of one player

¹ A nice survey of economic applications of the PD can be found in Rapoport (1987). In political science, Brams' (1994) "Theory of Moves" provides a novel analysis of the Prisoner's Dilemma (PD) and argues that mutual cooperation will typically emerge.

² In zero sum games these objectives would be equivalent. But as noted, our analysis does not concern zero-sum games.

yields an externality (in utils) to others, the analysis is identical. Formally, these externalities would create a wedge between the observable payoff game and the unobservable “welfare” game that is formally equivalent to either amicable or adversarial preferences.³ Such an interpretation opens a plethora of applications in economics as well as political science.

The remainder of the paper is organized as follows. Section 2 introduces notation and definitions necessary to analyze PDs with neutral, amicable, and adversarial players. Section 3 presents our most general existence results and specific congruence results for amicable, adversarial, indifferent, and asymmetric players. A Cobb-Douglas example is also provided in Section 3. Section 4 is somewhat of a digression on the possibilities of the approach, while Section 5 concludes.

2. Notation and Definitions

The Game

Consider a two-player game and call the players A and B and their cardinal (observable) payoffs α and β respectively. Each player has two strategies. Denote the players’ strategy sets and strategy choice as respectively: $S^p = \{1, 2\}$ and s^p for $p = A, B$. So the joint strategy space has four elements and denote the associated observable primitive (cardinal) payoff vectors as $\pi_{ij} = [\alpha_{ij}, \beta_{ij}]$ where $i = s^A$ and $j = s^B$ with the payoff space denoted as $\Pi \subset \mathbb{R}^2$. Let $r^p(s)$ denote the best response of player p to strategy s by the other player. Without loss of generality, payoffs are non-negative and when the clarity constraint permits we suppress the subscripts on α and β . The one-stage game defined by the above triplet $\Gamma = [P, S, \Pi]$ will be called *the primitive game*. All its elements are observable and fully known by both players.

A *primitive prisoner’s dilemma* (PPD) occurs when the Nash Equilibrium of the primitive game yields a payoff (π) that is vector dominated by some non-equilibrium payoff.⁴ Without loss of generality let $s^p = 1$ for ($p = A, B$) be the strategies that map to the vector dominated primitive payoff and $s^p = 2$ for ($p = A, B$) the strategies that map to the vector dominant payoff. Using the notation introduced above the payoff vectors are: $\pi_{22} > \pi_{11}$, where a vector inequality indicates vector dominance.

Each player has unobservable preferences over the primitive payoff space that are complete, transitive, and reflexive. In a slight (but innocuous) abuse of notation that yields considerable notational economy we denote the unobservable utility functions as: $A(\alpha, \beta), B(\alpha, \beta)$. Let $U_{ij} = [A(\pi_{ij}), B(\pi_{ij})]$ be the vector of final utility payoffs when player A plays strategy i and B plays strategy j (where i may equal j). The functions $A(\pi_{ij})$ and $B(\pi_{ij})$ may map non-monotonically, for each respective player, from primitive-payoff vectors $\pi = (\alpha, \beta)$ to final own-utility due to either amicable or adversarial preferences. For a given U every *Primitive Game* maps to an associated *Utility Game* (UG) and we define the associated UG as $V(\Gamma) = [P, S, U(\Pi)]$. If U does not order payoffs as in the observable primitive game, V will be a weakly better predictor of players’ strategic behavior than Γ . For expositional convenience we

³ Yet another motivation is a game where joint strategies map into two-good payoffs with one of the primitive payoffs is a “good” and other is a “bad” for one player; while the second player has reverse preferences towards the payoffs. For example, we can imagine roommates who have contradictory preferences towards classical and rock music. For one roommate classical is a good and rock is a bad, while the reverse holds for the other roommate. Joint strategies yield quantities of both goods, and it is easy to construct a PD (i.e., Pareto Inferior equilibrium) in this environment.

⁴ For ease of exposition we consider Prisoners Dilemmas where the equilibrium is strictly inferior for both players. Naturally, the definition Pareto inferior would allow only one player to be worse off, while all other players might be indifferent. Focusing on strict PDs considerably streamlines the paper. However, it is critical to note that versions of all propositions and results can be obtained with the weaker PD definition – though at a considerable cost in tedium.

will assume henceforth that the utility functions are differentiable. Extension to well-behaved non-differentiable utility functions is straightforward for virtually the entire analysis.

A number of indifference curves will have special significance in our analysis and we employ the following notation: $A_{ij} = \{ \pi \in \Pi: \pi \sim \pi_{ij} \}$ for player A , while analogously B_{ij} denotes player B 's indifference set with π_{ij} where $i, j = 1, 2$. So A_{11} is the set of all joint payoffs that A finds indifferent to π_{11} . We use strong versions of the *upper* and *lower contour sets* of π_{ij} for player p , defining them respectively as follows:

$UCS_{ij}^p = \{ \pi \in \Pi: \pi \succ \pi_{ij} \}$, for player p , $LCS_{ij}^p = \{ \pi \in \Pi: \pi \preceq \pi_{ij} \}$, for player p . Again, all propositions hold with weak forms of the upper and lower contour sets, though the exposition is more tedious. The required modification of the proofs with weak contour sets is indicated subsequently.

Payoff Space Partitions

The following payoff space partitions are central to our analysis. We will subsequently provide graphically illustrations of these sets for amicable, adversarial, and indifferent players. Note that all sets are subsets of the primitive joint payoff space.

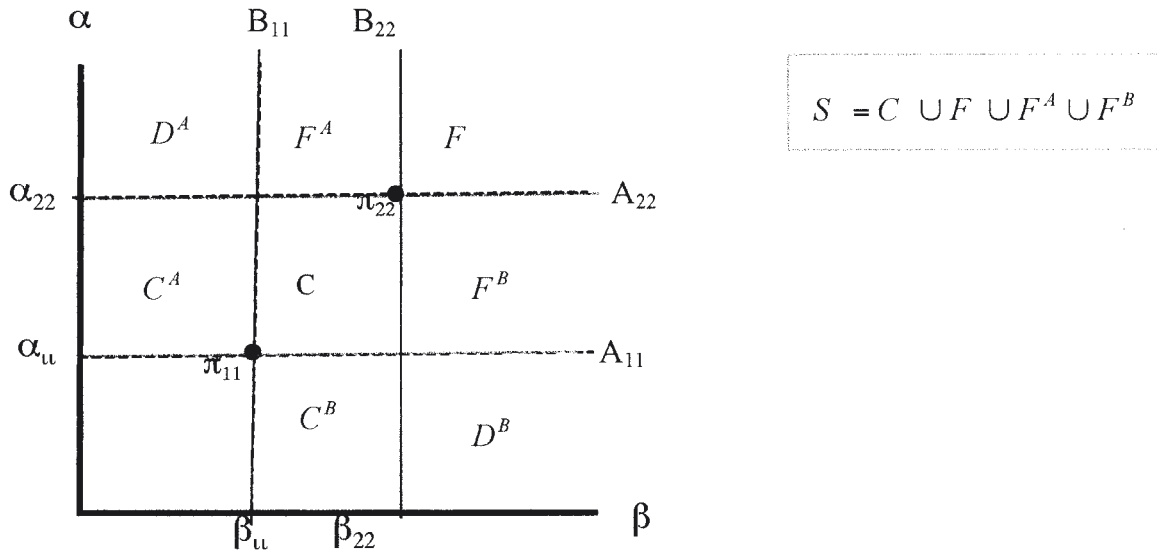
- (1) Superior Set (S)
 $S = UCS_{11}^A \cap UCS_{11}^B$
- (2) Far Set (F)
 $F = UCS_{22}^A \cap UCS_{22}^B$
- (3) Central Set (C)
 $C = S \cap LCS_{22}^A \cap LCS_{22}^B$
- (4) Dominant Set of player p (D^p)
 $D^p = UCS_{22}^p \cap LCS_{11}^{-p}$, where $-p$ indicates player "not p ."
- (5) Central Set of player p (C^p)
 $C^p = UCS_{11}^p \cap LCS_{11}^{-p} \cap LCS_{22}^p$
- (6) Far Set of player p (F^p)
 $F^p = UCS_{22}^p \cap LCS_{22}^{-p} \cap UCS_{11}^{-p}$

Payoff Partitions When Both Players are Indifferent

Since players' subjective amicable, adversarial, or indifferent attitude towards one another are not directly observable the standard assumption is one of indifference – that is, each player's strategy choices are governed by their own cardinal payoffs alone. Of course, it is also possible that such indifference is in fact a player's true preference towards others. Letting subscripts denote partials the indifferent player's preferences are: $A_\alpha > 0$, $A_\beta = 0$, $B_\beta > 0$, $B_\alpha = 0$, and indifference curves are linear in the joint-payoff space. Figure 1 below illustrates the payoff-space partition for indifferent players. These

sets have different topology for amicable or adversarial players and we will rigorously characterize the relationship between them under the various preferences in the following section.

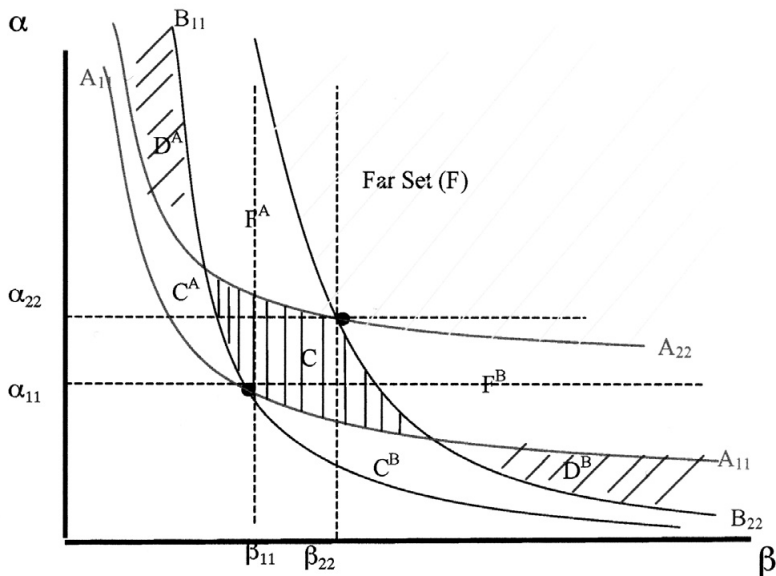
Figure 1 The Payoff Partition – Indifferent Players



Payoff Partitions When Both Players are Amicable

We say player A is *amicable* at π , if $A_\alpha(\pi) > 0$ and $A_\beta(\pi) > 0$ and a *globally amicable* if the inequalities hold at all π . A similar definition applies for player B. An extreme form of amicability is altruism. Player A is an *altruist* at π if and only if $\partial \ln A(\pi) / \partial \ln \beta > \partial \ln A(\pi) / \partial \ln \alpha$ and a *global altruist* if the condition holds at all π . When comparing preferences $A^\circ(\pi)$ and $A^*(\pi)$ we say that A° is more amicable than A^* at π if $-A^\circ_\alpha / A^\circ_\beta > -A^*_\alpha / A^*_\beta$. Given our definitions an *amicable* player's indifference curves are downward sloping in the joint payoff-space. Figure 2 illustrates a payoff-space partition for amicable players, with indifferent players' partitions indicated by the dashed lines.

Figure 2 The Payoff Partition – Amicable Players

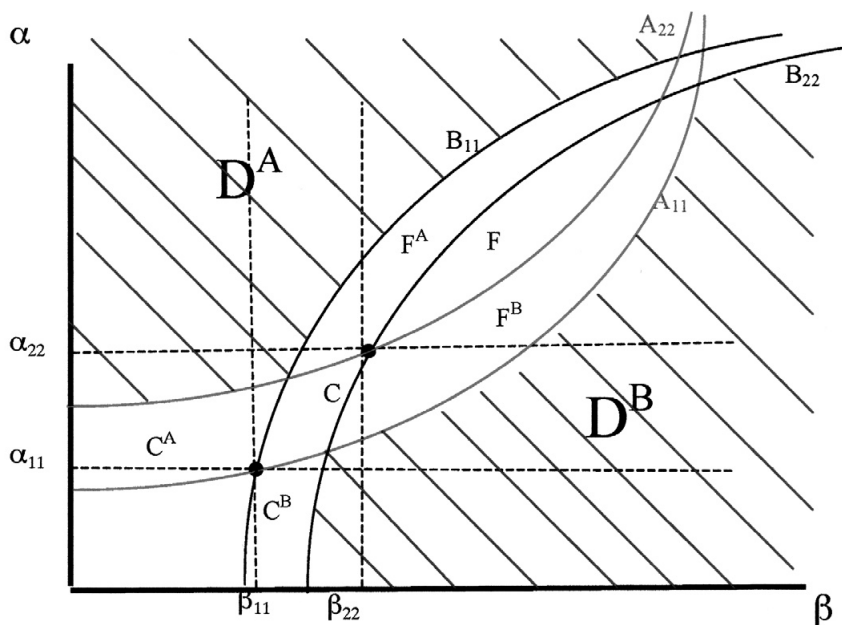


The juxtaposition of Figures 2 and 1 provides a striking illustration of the distortions of the payoff-partitions vis-à-vis the indifferent player. We will demonstrate that this non-congruence has critical implication for the interpretation and existence of PDs in the unobserved utility game.

Payoff Partitions When Both Players are Adversarial

We say player A is an *adversary* (or has enmity) at π if $A_\alpha(\pi) > 0$ and $A_\beta(\pi) < 0$ and a *global adversary* if the inequalities hold at all π . Indifference curves of a player with enmity are upward sloping (with finite slope) in the joint payoff space. We say that player A has strong enmity for the other player at π if: $|\partial \ln A(\pi) / \partial \ln \beta| > \partial \ln A(\pi) / \partial \ln \alpha$, and that preferences A° display less enmity than A^* at π if $-A^\circ_\alpha / A^\circ_\beta > -A^*_\alpha / A^*_\beta$ at π . Figure 3 below illustrates a payoff-space partition for adversaries, with indifferent players' partitions again in the background.

Figure 3 The Payoff Partition – Adversaries



As in the case of amicable preferences, Figure 3 reveals dramatic “distortions” in the payoff-space partitions – though they are markedly different. Note that for these adversaries, as opposed to indifferent and amicable players, the Far Sets (F , F^A , F^B) are now bounded. Also note that the Central Set (C) has remained bounded in all scenarios, though, together with the two other players’ central sets, it is not connect any more. We now move to consideration of the existence of PD under these various preferences.

3. Prisoners Dilemmas under Alternative Preferences

3.1 Necessary and Sufficient Conditions for Prisoners Dilemmas in the Utility Game

We are now in a position to connect the existence of PDs with the payoff-space partitions. We begin by defining two forms of PDs: Strong and Weak.

- **Strong Prisoners Dilemma (SPD)**

A game is a SPD if the strategy yielding the Pareto Inferior payoff is a dominant strategy for both players.

- **Weak Prisoners Dilemma (WPD)**

A game is a WPD if the strategy yielding the Pareto Inferior payoff is a dominant strategy for only one player.

The following Propositions provide the necessary and sufficient conditions for the various forms of PDs, for multiple equilibrium, and for no equilibrium in our framework. We note that Propositions 1-5 hold for any types of attitudes between the players: amicable, adversarial, and indifferent.

Proposition 1. Given any π_{11} and $\pi_{22} \in S$:

$\pi_{1j} \in D^A$ and $\pi_{j1} \in D^B$ are necessary and sufficient conditions for the unique Nash Equilibrium to be a SPD.

Proof: Sufficiency: First consider player A's best responses. Given the above conditions: $\pi_{12} \in UCS_{22}^A$ and $\pi_{21} \in LCS_{11}^A$, therefore $s^A = 1$ is a dominant strategy for A. An analogous argument holds for B.

Necessity: Again first consider player A. Suppose the conditions of the Proposition are not satisfied. If $\pi_{12} \notin D^A$ then either $\pi_{12} \notin UCS_{22}^A$ or $\pi_{12} \notin LCS_{11}^B$. If $\pi_{12} \notin UCS_{22}^A$, $\pi_{12} \in LCS_{22}^A$ (recall our strong definitions of UCS and LCS) and $r^A(2) = 2$ so that $s^A = 1$ is no longer a dominant strategy. If $\pi_{12} \notin LCS_{11}^B$, $\pi_{12} \in UCS_{11}^B$ so that $r^B(1) = 2$ and $s^B = 1$ is no longer a dominant strategy. A similar argument holds for $\pi_{21} \notin D^B$. \therefore

Theorem 1: Given any π_{11} and $\pi_{22} \in S$, a necessary and sufficient condition for the possibility of existing a SPD (WPD) is that (at least one of the following hold)

$$B_{11} \cap A_{22} \neq \phi \text{ and } B_{22} \cap A_{11} \neq \phi.$$

Proof: Necessity: Suppose that, given preferences A and B, a utility game is exhibited in which a unique Nash WPD takes place. The result is then immediate as, given Proposition 1, if $B_{11} \cap A_{22} = \phi$, D_V^A is empty.

Sufficiency: Let preferences A and B be given and suppose the intersections occur. We work out first the B_{11}, A_{22} case. Call π^* the intersection point and let O be an open ball centred in π^* . The Jordan Curve Theorem (JCT) and the fact that π^* lies on both curves allow to write that:

$$\begin{aligned} O &= O_{B_l} + O_{B_u} + O \cap B_{11} \quad \text{and} \\ O &= O_{A_l} + O_{A_u} + O \cap A_{22} \quad ; \end{aligned}$$

where O_{B_l} and O_{B_u} are, respectively, the intersections of player B's lower and upper contour sets at π^* (which, by the JCT, curve B_{11} subdivides two-dimensional space) with O; sets O_{A_l} and O_{A_u} having a similar definition with respect to A_{22} . As the curves are supposed not to have a segment in common, the above identities imply that

$$\begin{aligned} O &= O_{B_l} \cap O_{A_l} + O_{B_l} \cap O_{A_u} + O_{B_u} \cap O_{A_l} + O_{B_u} \cap O_{A_u} + \{\pi^*\} + \\ &\quad + O_{B_l} \cap A_{22} + O_{B_u} \cap A_{22} + O_{A_l} \cap B_{11} + O_{A_u} \cap B_{11} \end{aligned}$$

none of the above subsets being the null set. This means that $\exists \pi^o \in O_{B_l} \cap O_{A_u}$, but set $O_{B_l} \cap O_{A_u}$ is contained in the (utility) dominant set for the first player, which, thus, results non-empty. Working, in a similar way, around the intersection of A_{11} and B_{22} , a corresponding point can be found in the second player's utility dominant set. As both dominant sets are non void, it is possible to choose off-diagonal payoffs in them that would give way to utility games with a SPD.

Proposition 2: Given any π_{11} and a $\pi_{22} \in S_v$, a sufficient condition for a unique Nash Equilibrium which is a WPD is: $\pi_{ij} \in D^p$ and $\pi_{ji} \in C^{-p}$ for $i, j = 1, 2$ where $i \neq j$.

Proof: First consider the case where $\pi_{12} \in D^A$ and $\pi_{21} \in C^B$. Then $\pi_{12} \in UCS_{22}^A$ and $\pi_{21} \in LCS_{11}^A$, therefore $s^A = I$ is a dominant strategy for A. For player B, $\pi_{12} \in LCS_{11}^B$ so $r^B(1) = 1$ and $\pi_{21} \in LCS_{22}^B$ so $r^B(2) = 2$. So B has no dominant strategy and $\underline{s} = \{1, 1\}$ is the unique Nash Equilibrium. An analogous argument holds for the case of $\pi_{21} \in D^B$ and $\pi_{12} \in C^A$, in which case player B is the one with the dominant strategy.

Proposition 3: Given any π_{11} and a $\pi_{22} \in S_v$ if $\pi_{ij} \in D^p$ and $\pi_{ji} \in F^B$ $i \neq j, i, j = 1, 2$, then (j, i) is the unique Nash equilibrium of the game.

Proof: First consider the case where $\pi_{12} \in D^A$ and $\pi_{21} \in F^B$. Then $\pi_{12} \in UCS_{22}^A$ and $\pi_{21} \in UCS_{11}^A$, so $r^A(2) = I$ and $r^A(1) = 2$. For player B, $\pi_{12} \in LCS_{11}^B$ so $r^B(1) = 1$ and $\pi_{21} \in UCS_{22}^B$ so $r^B(2) = 1$. So B has a dominant strategy and $\underline{s} = \{2, 1\}$ is the unique Nash Equilibrium – which is not a PD. An analogous argument holds for the case of $\pi_{21} \in D^B$ and $\pi_{12} \in F^A$, in which case player A has the dominant strategy.

Proposition 4: Given any π_{11} and a $\pi_{22} \in S_v$ if $\pi_{ij} \in D^p$ and $\pi_{ji} \in C$ $i \neq j, i, j = 1, 2$ then the game has no Nash equilibrium.

Proof: First consider the case where $\pi_{12} \in D^A$ and $\pi_{21} \in C$. Then $\pi_{12} \in UCS_{22}^A$ and $\pi_{21} \in UCS_{11}^A$, so $r^A(2) = 1$ and $r^A(1) = 2$. For player B, $\pi_{12} \in LCS_{11}^B$ so $r^B(1) = 1$ and $\pi_{21} \in UCS_{11}^B$ so $r^B(2) = 1$. So there is no Nash Equilibrium. An analogous argument holds for the case of $\pi_{21} \in D^B$ and $\pi_{12} \in C$.

Proposition 5: Given any π_{11} and a $\pi_{22} \in S_{\nu}$, both $\underline{s} = \{1, 1\}$ and $\underline{s}' = \{2, 2\}$ are equilibrium if: $\pi_{12} \in C^A$ and $\pi_{21} \in C^B$.

Proof: $\pi_{12} \in LCS_{22}^A$ so $r^A(2) = 2$ and $\pi_{21} \in LCS_{11}^A$ so $r^A(1) = 1$. For player B $\pi_{12} \in LCS_{11}^B$ so $r^B(1) = 1$ and $\pi_{21} \in LCS_{22}^B$ so $r^B(1) = 1$ and $r^B(2) = 2 \therefore$

Discussion

Propositions 1 through 5 make clear that it is the membership of the π_{ij} payoffs in the various partitions of the joint-payoff space that determine the nature of the equilibrium, or lack thereof. Theorem 1 is a fundamental result, whose relevance is linked to the cases of non-indifferent players. They all show that of critical relevance to the existence of Prisoner's Dilemmas is the membership of at least one of the π_{ij} payoffs in a player's Dominant Set. The Figures in Section 2 suggest that under amicable or adversarial preferences the Players' Dominant Sets contract and expand respectively. An immediate implication is that though the observable payoff structure of a game suggests a Prisoners Dilemma equilibrium, unobserved amicable or adversarial attitudes of the players may transform the utility game to one with a different equilibrium. The mechanism of this transformation is the "migration" of π_{ij} payoffs between Payoff-space Partitions as we move from the primitive game, with the implied indifference of players, to a utility game with amicable or adversarial preferences. Only in the case of truly indifferent player can we be certain that the Dominant Sets in the observable game and utility games are congruent.

3.2 Payoff-Space Partition Congruence with non-Indifferent Players

The Figures in Section 2 were merely suggestive of the types of Payoff-space Set transformation that may occur when non-indifferent players are present. We now formally characterize these transformations. Note that in the prior and proceeding analysis, multiple-crossing of an indifference curve of an *amicable player* with a particular indifference curve of another amicable player would complicate the analysis. To keep the paper of manageable length we focus on single crossing indifference curves of amicable players and note that the results would be modified in fairly obvious ways in the presence of multiple crossing curves. To facilitate presentation of the next results we introduce the following additional notation: for each set of the payoff-space partition let the subscripts Γ or V indicate respectively the primitive or utility game partition. For example, D_{Γ}^p is player p's Dominant Set in observable payoffs while D_V^p is player p's Dominant Set in the utility game.

Both Players are Adversaries

Proposition 6. With adversarial preferences a player's Primitive Dominant Set is a strict sub-set of their Utility Dominant Set: $D_{\Gamma}^p \subset D_V^p$.

Proof. Consider the point $\pi' = (\alpha_{22}, \beta_{11})$. With adversarial preferences $\pi' \in UCS_{22}^A$ and $\pi' \in LCS_{11}^B$, so $\pi' \in D_V^A$. With regard to the primitive game $\pi' \notin D_{\Gamma}^A$, since $\pi' \in A_{22}$ and $\pi' \in B_{11}$. A similar argument holds for player B. To see that every element of D_{Γ}^i must be an element of D_V^i simply note that because of the finite upward slope of indifference curves with adversarial preferences $\forall \pi \in D_{\Gamma}^A, \pi \in UCS_{22}^A$ and $\pi \in LCS_{11}^B$.

Note that even if the upper and lower contour sets were defined weakly we could find a point in an open ball centered on π' that is an element of D_V^i but not D_{Γ}^i . This general argument holds for all subsequent propositions, and will not be repeated.

Proposition 7. If both players have adversarial preferences the Utility Central Set is a strict sub-set of the Primitive Central Set: $C_V \subset C_{\Gamma}$

Proof: C_{Γ} is the quadrilateral defined by $\{\pi \mid \pi_{11} < \pi < \pi_{22}\}$. Given that B_{11} and A_{22} have finite positive slopes and pass through π_{11} and π_{22} respectively, they must intersect in the interior of C_{Γ} since B_{11} cannot intersect B_{22} which also passes through π_{22} . Likewise for A_{11} and B_{22} . Therefore $C_V \subset C_{\Gamma}$.

Proposition 8. With adversarial preferences the Superior Set of the utility game is a strict sub-set of the primitive Superior Set: $S_V \subset S_{\Gamma}$.

Proof. S_{Γ} is the quadrant defined by $\pi > \pi_{11}$. Since adversarial indifference curves have finite positive upward slope A_{11} and B_{11} are contained in S_{Γ} for all $\pi > \pi_{11}$. The intersection of the upper contour sets for $\pi > \pi_{11}$ must therefore be empty or contained in S_{Γ} . So every element of S_V must also be an element of S_{Γ} . To see that not every element of S_{Γ} is an element of S_V let $B_e^c(\pi')$ be a closed ball of radius e centered on $\pi' \in A_{11}$ for some $\pi' > \pi_{11}$, where we choose e such that $B_e^c(\pi') \subset S_{\Gamma}$. By the Jordan Curve Theorem the indifference curve through π' divides the ball into two distinct domains, one a subset of UCS_{11}^A and the other a subset of LCS_{11}^A where by definition if $\pi'' \in B_e^c(\pi')$ and $\pi'' \in LCS_{11}^A, \pi'' \notin S_V$.

We can make the following stronger characterization of S_V when both players are strong adversaries.

Proposition 9. S_V is either empty or bounded if players are strong global adversaries.

Proof: With α on the ordinate and β the abscissa, as in Figure 3, A's and B's indifference curves are respectively convex and concave with positive finite slope.

(i). If the indifference curves are tangent at π_{11} the intersection of the upper contour sets is empty.

(ii). If the slope of A_{11} exceeds that of B_{11} at π_{11} the intersection of the upper contour sets must lie to the southwest of π_{11} and is contained in the bounded set: $\{\pi : \alpha \leq \alpha_{11}, \beta \leq \beta_{11}\}$.

(iii.) If the slope of B_{11} exceeds that of A_{11} at π_{11} the indifference curves must intersect again (since B_{11} is strictly concave and A_{11} strictly convex). Call this intersection π' . In this case the contour sets intersection must lie to the southwest of π' : in the bounded set $\{\pi : \alpha \leq \alpha' \text{ and } \beta \leq \beta'\}$ \therefore

Corollary to Proposition 9. If players are strong adversaries F^V is either empty or bounded.

Proof: Simply repeat the above proof substituting π_{22} for π_{11} , A_{22} for A_{11} , and B_{22} for B_{11} .

The following very strong proposition is the principal non-existence results of our analysis.

Theorem 2. If players are strong adversaries at π_{11} a game which is PD in observable payoffs can never be a PD in the utility game.

Proof: Suppose the existence of a game which is a PD in cardinal payoffs and is also a PD in the utility game. Then $\pi_{22} \in S_v$ and $A(\pi_{22}) > A(\pi_{11})$ and $B(\pi_{11}) < B(\pi_{22})$. As both preference functions are continuously differentiable, there exists an open ball $B_e(\pi_{11})$, and a $\pi' = (\alpha_{11} + d\alpha, \beta_{11} + d\beta) \in B_e(\pi_{11})$, with $d\alpha, d\beta > 0$, such that $A_\alpha(\pi_{11}) d\alpha + A_\beta(\pi_{11}) d\beta > 0$ and $B_\alpha(\pi_{11}) d\alpha + B_\beta(\pi_{11}) d\beta > 0$. Taking into account the signs of the partial derivatives these inequalities yield: $A_\alpha(\pi_{11}) / A_\beta(\pi_{11}) < B_\alpha(\pi_{11}) / B_\beta(\pi_{11})$. By the definition of strong adversaries at π_{11} , however, $A_\alpha(\pi_{11}) / A_\beta(\pi_{11}) > B_\alpha(\pi_{11}) / B_\beta(\pi_{11})$, a contradiction \therefore .

Theorem 2 extends an important implication of Proposition 9. That is, when players are global strong adversaries a primitive Prisoner's Dilemma can never be a UPD. It also provides a dramatic example of a more general result which holds for all forms of adversarial preferences. Namely, adversarial behaviour reduces from an infinite to a finite (Lebesgue) measure the set of primitive payoffs that could possibly be associated with a utility prisoner's dilemma. Moreover, in the strong adversary case of Theorem 2, the bounded set F^V will never include π_{22} . Since all prisoners dilemmas (primitive or utility) require a Pareto dominant payoff, the non-existence of the utility prisoners dilemma follows. This non-congruence of the observable primitive game and the inherently unobservable utility game has profound implications for the interpretation of a wide range of economic applications – including the public goods problem.

Both Players are Amicable

Proposition 10. If both players are amicable the Dominant Sets of the utility game (if they exist) are strict sub-sets of their primitive game

Dominant Sets: $D_V^i \subset D_\Gamma^i$.

Proof. Since both A_{22} and B_{11} have negative finite slope their intersection must occur in D_Γ^A , if at all. Thus every element of D_V^A is also an element of D_Γ^A . By Corollary 1 A_{22} and B_{11} must intersect for D_V^A to be non-empty. If it occurs call the intersection point $\pi' \in D_\Gamma^A$.

Now consider an $\epsilon > 0$ such that the closed ball $B_\epsilon^c(\pi') \subset D_\Gamma^A$, and A_{11} partitions $B_\epsilon^c(\pi')$ into distinct domains one of which contains elements of LCS_{22}^A , which are not members of D_V^A but are elements of D_Γ^A \therefore .

Proposition 11. If both players have amicable preferences the Primitive Central Set is a strict sub-set of the Utility Central Set:

$C_\Gamma \subset C_V$

Proof: C_Γ is the quadrilateral defined by $\{\pi \mid \pi_{11} < \pi < \pi_{22}\}$. Given that B_{11} and A_{22} have finite negative slopes and pass through π_{11} and π_{22} respectively, they must intersect in the interior of D_Γ , if at all.

Likewise for A_{11} and B_{22} . Therefore $C_\Gamma \subset C_V$ \therefore .

Proposition 12. With amicable preferences the Superior Set of the primitive game is a strict sub-set of the Superior Set of the utility game: $S_\Gamma \subset S_V$

Proof. Immediate. With amicable preferences the indifference curves A_{11} and B_{11} are support functions for S_Γ \therefore .

Figure 4 provides a summary of the principal results of this sub-section.

<u>Indifferent</u>	<u>Adversaries</u>	<u>Amicable</u>
$D_\Gamma^p = D_V^p$	$D_\Gamma^p \subset D_V^p$	$D_V^p \subset D_\Gamma^p$
$S_\Gamma = S_V$	$S_V \subset S_\Gamma$	$S_\Gamma \subset S_V$
$C_\Gamma = C_V$	$C_V \subset C_\Gamma$	$C_\Gamma \subset C_V$
↓	↓	↓
PP ↔ UP	PP ↔ UP	PP ↔ UPD

Note that the likelihood of a PPD having an associated UPD is reduced in different ways for adversaries and amicable players. In the case of adversaries the likelihood that the Pareto Superior payoff in the primitive game is also Pareto Superior in the utility game is reduced. For amicable players the likelihood that the “off diagonal” payoff provides defection incentive from the Pareto Superior payoff is reduced.

Asymmetric Attitudes and the Central Set as a Measure of Amicability

Asymmetric attitudes across players generate a rich set of possibilities. In this sub-section we briefly explore congruence properties of the Central Set when players have asymmetric attitudes. We focus on the Central Set since its boundedness properties and sensitivity to alternative preferences render it a good measure of general amicability.

Proposition 13. Suppose player B is neutral. If player A is amicable $C_T \subset C_V$ while if player A is an adversary: $C_V \subset C_T$.

Proof. Immediate given slight adaptations of Propositions 7 and 11. As attitudes move from indifference to amicability, the defection incentive that gives rise to the PPD is attenuated and the potential for no-equilibrium increases. By Propositions 7 and 11 the Central Set expansion comes at the expense of the Dominant Sets. The following result, which combines amicable and adversarial players, is also immediate:

Proposition 14. Suppose player A is amicable and B adversarial. Ceteris paribus,

- i). the *larger* the ratio $-B_\alpha / B_\beta$ on the interval $[\alpha_{11}, \alpha_{22}]$, the smaller C_V
- ii). the *smaller* the ratio $-A_\alpha / A_\beta$ on the interval $[\alpha_{11}, \alpha_{22}]$, the smaller C_V .

Proof. Again immediate given slight modifications to Propositions 7 and 11.

Together Propositions 7, 11, 13, and 14 suggest that the area of the Central Set is an intriguing metric of the aggregate “friendliness” of the players. Recall that the Central Set is always bounded when both players are indifferent or adversarial. As preferences move from adversarial to indifference to amicability the area of Central Set increases monotonically. When at least one player is amicable, the Central Set is no longer necessarily bounded, though it remains bounded under many well behaved utility functions and its area increases uniformly with increasing amicability as defined earlier. The Cobb-Douglas example of the following sub-section will further illustrate this property.

3.3. A Cobb-Douglas Example

We begin with amicable preferences and to simplify exposition express B’s preferences in terms of primitive payoffs “a” for player A and “b” for own primitive payoff. The utility functions are then:

$$(1) \quad \begin{aligned} A(\alpha, \beta) &= \alpha^x \beta^{1-x} \\ B(a, b) &= a^{1-y} b^y, \end{aligned}$$

where x and y are non-negative. Note that if $x \in (0,1)$, Player A is amicable, and is an altruist if $x \in (0,1/2)$.

By Theorem 1, A_{22} and B_{11} must intersect for D^A to exist and membership of a π_{12} in D^A is a necessary condition for the existence of a SPD. Let \bar{B}_{11} and \bar{A}_{22} be the utility levels associated with indifference curves B_{11} and A_{22} . For an arbitrary $\alpha = a$, $b \neq \beta$, the ordinates in these indifference sets will be different, except for a point of intersection. Considering an arbitrary $\alpha = a$, substituting α for “a” in (1), defining $\frac{\beta}{b} |_{\alpha=a} \equiv R(\alpha; x, y)$, and rearranging (1) yields:

$$(2) \quad R(\alpha; x, y) = (\bar{A}_{22}^y / \bar{B}_{11}^{1-x}) \alpha^{1-(x+y)}.$$

Proposition 15. The Dominant Sets are non-empty if either: i). both players are amicable but not altruists; ii). one is an altruist and one merely amicable, with $x+y > 1$.

Proof: For either i) or ii) above at $\alpha = \alpha_{11}$, $\beta > b$ so $R(\alpha_{11}; x, y) > 1$.

Moreover if either i) or ii) are satisfied $x + y > 1$ and

$\lim_{\alpha \uparrow \infty} R(\alpha; x, y) = 0$ so the indifference curves cross at some α , with the intersection obtained by solving $R(\alpha; x, y) = 1 \quad \therefore$

Proposition 16. The Dominant Sets are empty if either: i). both players are altruists; ii). one is an altruist and the other merely amicable, with $x+y < 1$; iii). players are amicable with $x+y = 1$.

Proof: Reasoning similar to the previous proof implies for i) and ii)

the curves never intercept at $\alpha > \alpha_{11}$. If $x+y = 1$, the β is independent

of α , and $\lim_{\alpha \uparrow \infty} R(\alpha; x, y) = \lim_{\alpha \downarrow 0} R(\alpha; x, y) = \bar{A}_{22}^y / \bar{B}_{11}^{1-x}$, a constant. Thus the curves are either parallel or coincide, but never cross.

These Propositions indicate that when both players are amicable and at least one is sufficiently *altruistic*, a utility Prisoner’s Dilemma will *never occur*. With these explicit utility functions we can also compute the “gains and losses” in the Dominant Set from different attitudes. For example, for simplicity letting $x=y$ we can derive a lower bound for the reduction in the dominant sets when both players are amicable. Using (8) the intersection occurs at: $\alpha^* = (\bar{A}_{22})^{x/1-2x} (\bar{B}_{11})^{1-x/1-2x}$. Recalling that A_{22} and B_{22} pass through π_{22} one can also write:

$$(3) \quad \alpha_{22} = (\bar{A}_{22})^{x/1-2x} (\bar{B}_{22})^{1-x/1-2x}, \quad \text{so that}$$

$$\alpha^* / \alpha_{22} = (\bar{B}_{11} / \bar{B}_{22})^{1-x/1-2x} > 1 \quad .$$

Therefore player A’s utility dominant set relative to primitive dominant set, is reduced by at least the area: $\beta_{11} \cdot \alpha_{22} [(\bar{B}_{22} / \bar{B}_{11})^{1-x/2x-1} - 1]$. To this it must be added the area beyond point α^* and below the indifference curve A_{22} . Computing this integral yields:

$$(4) \quad (\bar{A}_{22})^{1/1-x} (1-x / 2x-1) \alpha^* 1-2x/1-x =$$

$$(1-x / 2x-1) (\bar{A}_{22})^{1/1-x} (\bar{B}_{11} / \bar{B}_{22}) \alpha_{22}^{1-2x/1-x} \quad .$$

In spite of curve B_{11} going to zero, Player's A utility dominant set (which remains unbounded) does not have finite measure. A more precise bound may be obtained by computing the area outside the UDS between B_{11} and the vertical line passing through π_{11} until an $\alpha' > \alpha^*$. We shall not pursue this computation here. The above results suggest the following proposition.

Proposition 17. If $x=y$ and $2x > 1$, the lower bound to the reduction in Player's A utility dominant set is increasing with the ratio $r_B = B_{22} / B_{11}$ whenever $r_B > (A_{22})^{1/1-x} (1/\beta_{11}) (\alpha_{22})^{1-x/1-2x}$ and decreasing if the reverse occurs.

Proof. The increasing result is immediate. For the decreasing it suffices to compute the derivative, w.r.t. r_B , of the combined area.

The main importance of Proposition 17 is to emphasize that it is the normalized utility values at points π_{11} and π_{22} that are crucial in determining the distortions in the relevant PD sets.

Adversarial Preferences

For adversaries additional flexibility is obtained if we re-specify the Cobb-Douglas utility functions as follows:

$$(5) \quad \begin{aligned} A(\alpha, \beta) &= \alpha^x \beta^{-y} \\ B(\alpha, \beta) &= a^w b^z, \end{aligned}$$

with all exponents non-negative and x and z less than 1. Note that if $y > x$ and $w > z$ both players have strong global enmity. In this case *all* indifference curves emanate from origin and have a single-crossing property. If $x > y$ and $w < z$, the concavities of indifference curves are reversed, though all indifference curves still emanate from the origin. Finally, if $x > y$ and $w > z$, or $x < y$ and $w < z$, multiple crossing of indifference curves are possible and a set of complex possibilities arise.

Now return to the strong global enmity case ($y > x$ and $w > z$). Proposition 9 states that F_V is either empty or bounded. It follows that the area of F can be obtained.

Proposition 18. With strong global enmity the area of F_V shrinks from an infinite (Lebesgue-measure) value to $\alpha_{22} \cdot \beta_{22} [(wy - zx)/(w+z)(x+y)]$.

Proof: The A_{22} and B_{22} curves intercept at $(0, 0)$ and $(\alpha_{22}, \beta_{22})$. With α on the ordinate A_{22} is "below" B_{22} on the interval $(0; \alpha_{22})$ and we have the function: $F(\alpha) = (\bar{A}_{22})^{-1/y} \alpha^{x/y} - (\bar{B}_{22})^{1/z} \alpha^{w/z}$. Integrating this function on the interval and recalling that: $(\bar{A}_{22})^{-1/y} \alpha_{22}^{x/y} = \beta_{22} = (\bar{B}_{22})^{1/z} \alpha_{22}^{w/z}$, one arrives at the result. Notice that the relationship between the exponents ensures that $wy - zx > 0$.

4. A discussion on the scope of the results.

We think that, in practice, three important consequences can be drawn from our approach. The first, of a global nature, is that consideration of the primitive game is not enough to understand PD outcomes in real life – a point also present at the origin of the Theory of Moves.

Development and growth policy, as well as trade negotiations, are two areas where PD settings usually take place. Cost-benefit analyses, CGE (computable general equilibrium) or partial equilibrium modelling, as well as economic-ecological welfare measurements are common tools for supplying negotiators and policy makers, i.e. players, with reasonably good evaluations of the different pay-offs. The nowadays widespread use of such techniques qualifies them nearly as common knowledge. Together with the ever widening information society devices and possibilities, all this makes for admitting that a unique, common agreed pay-off matrix is a highly acceptable assumption in the majority of situations. However, if this matrix fits into our definitions, it will be far from bringing out a socially desirable outcome.

Indeed, in real-life, even if both players are apparently better off through co-operation, the temptation of defection often precludes the co-operative equilibrium. One way society found to avoid this dilemma was by creating institutions that can support the Pareto-superior pay-offs. In the trade negotiations arena, Kyle and Bagwell (2002), among others, explain the *raison d'être* of the WTO exactly via this line. However, though we also observe other situations where such institutions, once created, do allow the gains from co-operation to be realised, we also see instances where the very institutions – apparently well conceived and designed – do not work at all, and players keep on reaping inefficiently low returns. The Kyoto Protocol, the 2003 “WTO Cancun Meeting”, and so many unsuccessful development aid programmes could be examples of events close to this kind of outcome. Our framework proposes a new explanation for such phenomena: either the joint-strategy Pareto-superior *utility pay-offs* do not in fact exist, or the institution's format, instead of reducing, enhanced the possibility of UPDs. On the flip side, equilibriums which appear Pareto optimal in primitive pay-offs may in fact be UPDs in the preferences game. Anyhow, the key to tackling the dilemma lies in the interaction between the players, as we formally showed.

Moreover, contrary to the co-operative games approach, avoiding or lowering the probability of a Nash solution does not necessarily require that both players co-operate with each other. In fact, as exemplified in the previous section, the “negative” effect of an aggressive, competitive player can be counterbalanced by a friendly – or rather tolerant – attitude from the other player. This is a strong call for a careful understanding of the opponent's tactics and way of playing *together with a judicious choice of one's own behaviour, given the circumstances and possibilities available*. Responsible, tough and experienced players have been keenly aware of this. Many US Trade Representatives, for instance, are (or were) well-known for going to important meetings only after having had a thorough briefing on the other negotiator(s)'s likes and dislikes, i.e., preferences. Still in the trade negotiations context, key

developing countries, like India or Brazil, are known for usually being, in spite of respectable negotiators, friendlier to the “three big ones” (Japan, the EU and the US) than these are to them. As powerful economies are naturally more aggressive, without a friendlier stance at the other side of the table, the number of existing trade agreements would be much inferior. Unfortunately, this “care with the opponent” seems less frequent in the design of development strategies. On the other hand, ironically, the fact that both players behave like foes does not necessarily exclude avoiding the PD. In other words, tough behaviour in the face of a tough opponent is riskier, but may pay.

This last point adds also extra support to experimental economics efforts in game theory. In forums like the WTO, an international development agency, or in a sectoral negotiation involving government officials and class representatives, players can many times amass – in an as-scientific-as-possible way – previous evidences on the other’s behaviour in order to make an educated guess on his utility function. From this they can, in a stepwise manner, and drawing from experimental economics results, build up their optimal (utility) behaviour.

The third point has to do with an old, well-known saying, that has appeared in different versions through time, a popular one being “it takes two to tango”. Even in the event of a very aggressive opponent, one can display such an altruistic behaviour that will rule out completely the possibility of a PD. In the language of our Theorem 1 (section 3), this means that the altruistic player will dispose his (highest and lowest) iso-utilities curves in pay-off space in such a way that the (inversely) corresponding ones from his opponent will never be able to cross them. Actually, as in practice pay-off space is a bounded set, it suffices that the intersections take place at points, in the first quadrant, “outside” pay-off space.

In modern history, a major example of this kind of behaviour is Gandhi’s non-violence campaign against the British Government, for the independence of India. Though not without sacrifice, Gandhi – through his “an angry answer to an angry action only heightens the level of anger” - eventually managed to avoid the Pareto-inferior outcome and get India’s independence. In a different field, religion, one finds in nearly every creed the story of the altruistic and pacific fellow, who answered with generosity and love to his savage foes. In the Catholic tradition, for instance, St Francis is the emblematic representative of such behaviour. Avoiding the risk to get into areas and considerations which, beyond their complexity, would be entirely outside the scope of our paper, we stop this digression calling attention to the fact that our proposal incorporates in the formal framework of economic theory this kind of behaviour, usually grounded on (much more) subjective or moral arguments. A wider scope of its application, in sheer economic instances, seems to be waiting ahead.

5. Conclusion.

Prisoner's dilemmas provide a fundamental paradigm of the tension between individual and collective rationality. Analysis of their structure and operation has provided insight into issues ranging from the public goods problem to arms races. Yet the predictive power of the paradigm depends critically on implicit assumptions on the nature of the mapping from observable primitive payoff to unobservable final utility. When unobservable final utility depends only on own-primitive-payoff the equilibrium of a primitive-payoff-game and the associated utility-games are identical. Under this circumstance, the specific properties of the unobservable utility function are immaterial for predictions of strategy choice and a primitive game with a PD equilibrium is a perfect proxy for the unobservable final utility game. However, when linkages exist between the primitive payoff of one player and the utility of another, PD equilibrium in the observable game may not correspond to equilibrium in the utility game. Moreover, as discussed previously, a large body of experimental evidence is generally inconsistent with pure indifference of players to the payoffs of others.

This paper explored the implications of two types of linkages between the players' final utility and the other player's primitive payoff: adversarial and amicable preferences. We demonstrated that such non-indifference generates specific non-congruencies of the "primitive-dominant-set" and "utility-dominant-sets," which has the consequence of mapping apparent PD's into other (non-PD) equilibrium. On the other hand, utility PDs may arise in games that do not exhibit PD structure in primitive payoffs.

To appreciate the implications of this non-congruence consider a standard two-person PD in observable payoffs. Both players are *apparently* better off through cooperation than competition, though the temptation of defection precludes cooperation as a non-cooperative equilibrium. It would therefore seem that players have incentive to create institutions that can support the Pareto-superior payoffs. Indeed, there exist many instances where institutions supporting the Pareto Superior outcome are created and the gains from cooperation can be realized. However, as said in the previous section, we also observe many situations where such institutions do not emerge and inefficiency prevails. This paper proposes a new explanation for such phenomenon. Namely, that the joint-strategy Pareto superior *utility-payoffs* do not in fact exist. Also, equilibrium that appears Pareto optimal in primitive payoffs may in fact be PDs in utility payoffs. Ongoing research will apply this theory to the successes and failures of a variety of trade and development situations.

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