Innovation, Trade, and Growth

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Abstract

We consider the interaction of trade and technology diffusion in a two-region model of innovation and imitation. We find that globalization, either in the form of lower trade barriers or in faster diffusion of technology between innovator and imitator spurs innovation, benefiting both regions.

1 Introduction

How does the degree of openness affect the incentives to innovate, and, thus, the ultimate level of income in the world? This question has been posed in a number of contexts in which openness has meant different things. It could refer to the absence of trade barriers, but also to the absence of barriers to the diffusion of ideas. A number of papers have looked at the effect of one type of openness taking the degree of openness of the other type as given. Examples are Helpman (1993), Eaton, Gutierrez, and Kortum (1998), Eaton and Kortum (1999), and Eaton and Kortum (2001). In some situations. Helpman (1993), for example, finds in a model with an innovating and imitating country, with costless trade and the absence of intellectual property protection in the imitating country, faster diffusion can spur innovation by reducing the wage, and hence the cost of innovation, in the innovating country. In a model with no diffusion, Eaton and Kortum (2001) find that the degree of openness to trade has no effect on innovative activity: While trade increases the size of the market that a successful innovator can hope to capture, it also means that an innovator faces a higher hurdle in terms of competition from abroad. In their model, unlike Helpman, all countries engage in innovative activity.[1]

To explore these issues further we develop a two-country model, like Helpman's, of innovation and diffusion. Initially we allow both countries to innovate,

[1] In multicountry models of innovation and diffusion that are fit to cross-country data on patenting and research activity, Eaton, Gutierrez, and Kortum (1998) and Eaton and Kortum (1999) find, like Helpman, that greater diffusion spurs growth. Their models, unlike Helpman's have no trade.

and for ideas to diffuse between them. Unlike Helpman, we allow for an arbitrary level of trade barriers, with costless trade a special case. We then explore the incentives to innovate in a special case in which only one country innovates. We find that faster diffusion and lower trade barriers spur innovation.

We proceed as follows.

Section 2 develops a static two-country model of technology, production, and trade along the lines of the Ricardian model developed in Eaton and Kortum (2002). In their many country model, the distribution of efficiencies is treated as independent from country to country. Such an outcome is consistent, for example, with a world in which each country relied on its own innovations for production, or one in which innovations applying to a particular good in one country applied to a different one where they diffused. Here we consider the much more complicated case in which innovations, when they diffuse, apply to the same goods. Hence we need to distinguish between innovations that are in the exclusive domain of the innovating country, and those that have diffused to a common pool, which other countries can access. Because of the many different situations that can arise, we limit ourselves to a two-country case. Even here we need to distinguish among situations in which: (i) one country produces using only those ideas that are exclusive to it, while only the other country uses ideas that have flowed into the common pool, (ii) both countries use ideas that have flowed into the common pool, with one exporting goods produced using those ideas to the other, and (iii) both countries use ideas that have flowed into the common pool, with goods produced using those ideas nontraded.

Section 3 introduces dynamics into the analysis. Each country innovates at an exogenous rate, and ideas diffuse from one to another at exogenous rates. The processes of innovation and diffusion generate a world steady-state growth rate in which the two countries, depending on their abilities to innovate and to absorb ideas from abroad, will, except by coincidence, have different relative income levels. As in Krugman (1979), the framework delivers "product cycles" in which the innovator initially exports the good using the technology it has developed, but later imports it once the technology has diffused abroad. In our model other outcome are possible, however. If the innovation is sufficiently small, before diffusion, the other country may continue to produce the good on its own using inferior technology rather than import the good from the innovator. In fact, its own technology could even be superior, so that the innovation is never useful outside the country of innovation.

Section 4 considers a special case in which only one of the two countries innovates, but ties the rate of innovation to the return on innovation. Here we consider the role of openness in the form of (i) lower trade barriers and (ii) faster diffusion on innovative activity.

2 A Model of Technology, Production, and Trade

Following, for example, Dornbusch, Fischer, and Samuelson (1977, henceforth DFS) we consider a world with a unit continuum of goods, which we label by

$j \in [0,1]$. The production structure is Ricardian. There are two countries, which we label N (for North) and S (for South). Each country has a set of available technologies for making each of the goods on a continuum. Some technologies, denoted N, are available only to the North while another set, S, are available exclusively to the South. A third set C are commonly available. A technology is the ability to produce $z_i(j)$ units of good j with one worker, where, depending on which type of technology we are talking about, $i = N, S, C$. Here we treat the $z_i(j)$'s as realizations of random variables Z_i drawn independently for each j from the Frechet distributions:

$$F_i(z) = \Pr[Z_i \leq z] = \exp[-T_i z^{-\theta}].$$

which are independent across $i = N, S, C$. In this static context the T_i's reflect the differences in average efficiencies across the three sets of technologies. (We consider how these distributions arise from a dynamic process of innovation and diffusion below.)

The best technologies available in country i are thus

$$Z_i^* = \max[Z_i, Z_C] \; i = N, S$$

where Z_i^* has distribution:

$$F_i^*(z) = \Pr[Z_i^* \leq z] = \exp[-T_i^* z^{-\theta}]$$

where $T_i^* = T_i + T_C$.

Eaton and Kortum (2002) consider a case in which there is no common technology, so that $T_C = 0$. An implication is that the distribution of efficiencies available to each country is independent. here there is independence across the exclusive technologies, but the common technologies are perfectly correlated.

Because of this correlation between Z_N^* and Z_S^*, we will find it easier to work with the three independent technologies Z_N, Z_S,and Z_C, introduced above.

There are L_N workers in the North and L_S workers in the South. As is standard in a Ricardian setting, workers are identical and mobile across activities within a country, but cannot change countries. The wage is w_N in the North and w_S in the South. We take the wage in the South to be the numeraire, although we sometimes leave w_S in formulas for clarification. The labor market clearing conditions establish the relative wage. Without loss of generality we will impose restrictions on exogenous variables so that in equilibrium $w_N \geq w_S$.

As in DFS, demand is Cobb-Douglas. Hence expenditure in country i on good j is:

$$X_i(j) = Y_n.$$

where Y_n is total expenditure.[2]

Goods can be transported between the countries, but in order to deliver one unit in the destination $d \geq 1$ units need to be shipped from the source (the standard "iceberg" assumption). Unfortunately, even in low dimensional Ricardian

[2]Below we consider the case in which technologies and the labor forces evolve over time. Since in the next section we solve the static equilibrium given these magnitudes, We omit time subscripts for the time being.

problems, taxonomies are inevitable. There are three possible cases to consider: (1) If, in equilibrium, $w_N > w_S d$ then the commonly available technology will be used only in the South; the North will use only those technologies that it has unique access to. (2) If in equilibrium $w_N = w_S d$ then the commonly available technology may be used in both countries, but goods produced using this technology are exported only by the South. (3) If in equilibrium $w_S d \geq w_N \geq w_S/d$ then each country will use the commonly available technology. Goods produced using this technology are not traded since it will be more expensive to import the good than to make it oneself.

2.1 Cost Distributions, Market Structure, and Labor Demand

To mitigate the proliferation of special cases we introduce the notation $w_{NC} = \min\{w_S d, w_N\}$ for the wage paid to labor producing goods using the common technology and sold in the North (inclusive of transport cost),. so that $w_{NC}/z_C(j)$ is the cost of selling good j in the North if it is produced using the common technology. In the first case above $w_{NC} = w_S d$ while in the second two $w_{NC} = w_N$.

The least cost means of obtaining good j in the North is thus:

$$c_N(j) = \min\{w_N/z_N(j), w_{NC}/z_C(j), w_S d/z_S(j)\}$$

while in the South it is:

$$c_S(j) = \min\{w_N d/z_N(j), w_S/z_C(j), w_S/z_S(j)\}$$

since, under our assumptions, the South always buys goods using the common technology from itself.

For $i = N, S$, the lowest cost $c_i(j)$ is the realization of a random variable C_i whose distribution is determined by the distribution of the underlying technologies Z_i.. We denote the cost distribution in i by $H_i(c) = \Pr[C_i \leq c]$. The cost distribution in the North is:

$$\begin{aligned} H_N(c) &= 1 - \Pr[Z_N \leq w_N/c]\Pr[Z_S \leq w_S d/c]\Pr[Z_C \leq w_{NC}/c] \\ &= 1 - F_N(w_N/c)F_S(w_S d/c)F_C(w_{NC}/c) \\ &= 1 - \exp\left[-\Phi_N c^{\theta}\right] \end{aligned}$$

where $\Phi_N = T_N w_N^{-\theta} + T_S(w_S d)^{-\theta} + T_C w_{NC}^{-\theta}$. Similarly, for the South:

$$\begin{aligned} H_S(c) &= 1 - \Pr[Z_N \leq w_N d/c]\Pr[Z_S \leq w_S/c]\Pr[Z_C \leq w_S/c] \\ &= 1 - \exp\left[-\Phi_S c^{\theta}\right] \end{aligned}$$

where $\Phi_S = T_N(w_N d)^{-\theta} + T_S {w_S}^{-\theta} + T_C w_S^{-\theta} = T_N(w_N d)^{-\theta} + T_S^* {w_S}^{-\theta}$.

For now we assume that prices are proportional to costs as in the case of perfect competition. To solve for the labor market equilibrium we need to specify the demand for labor as a function of the relative wage.

As demonstrated in EK (2002), the fraction of goods j purchased in the North that are produced using its own exclusive technology is:

$$\pi_{NN} = \frac{T_N w_N^{-\theta}}{\Phi_N}$$

while the fraction the South purchases that are produced using the North's exclusive technology is:

$$\pi_{SN} = \frac{T_N (w_N d)^{-\theta}}{\Phi_S}$$

Hence π_{NN} is the fraction of the North's expenditure devoted to goods produced with exclusively Northern technology while π_{NS} is the fraction the South spends on goods produced with the exclusively Northern technology. For now we assume that all workers are engaged in production and that labor is the only source of income. Hence spending in country i is $w_i L_i$. With Cobb-Douglas technologies total spending on goods produced with the exclusively Northern technology is thus $\pi_{NN} w_N L_N + \pi_{SN} w_S L_S$.

Finally, as shown in EK (2002), the price index in country i is $P_i = \gamma \Phi_i^{-1/\theta}$, $i = N, S$ where γ is Euler's constant.[3]

To derive the wage we need to distinguish the three kinds of equilibria:

2.1.1 Case 1: The North uses only its Exclusive Technology

In this case, since only the South uses the commonly available technology, $w_{NC} = w_S d$. Since Northern labor is employed using only the North's exclusive technology, labor market clearing requires a value of w_N that solves:

$$w_N L_N = \frac{T_N w_N^{-\theta}}{T_N w_N^{-\theta} + T_S^* d^{-\theta}} w_N L_N + \frac{T_N \left(w_N d\right)^{-\theta}}{T_N \left(w_N d\right)^{-\theta} + T_S^*} w_S L_S.$$

which simplifies to:

$$w_N = \left[\left(\frac{T_N / L_N}{T_S^* / L_S} \right) \frac{T_N w_N^{-\theta} + T_S^* d^{-\theta}}{T_N (w_N d)^{-\theta} + T_S^*} \right]^{1/(1+\theta)}.$$

The solution needs to satisfy the condition that $w_N > d$ in order for the North not to use the commonly accessible technology. While the equation does not admit an analytic solution it is easy to solve numerically.

Since the North does not use any of the commonly produced technology, all goods produced are equally tradable regardless of which technology they employ. The fact that the North has access to the common technology is irrelevant since it doesn't use it. The outcome is isomorphic to one in which the North only knows the technologies that are exclusive to it, while the common technologies are exclusive to the South, as would be the case in EK (2002).

[3] Bernard, Eaton, Jensen, and Kortum (2003) show how these results generalize straightforwardly to preferences with constant elasticity of substitution (CES).

2.1.2 Case 2: The North and the South both use the Common Technology, with Trade in Some Goods Produced using that Technology

In this case $w_{NC} = w_N = w_S d$, so that $w_N = d$. The demand for labor L_N^E to work on technologies that are exclusively Northern, relative to the North's total labor force is:

$$\frac{L_N^E}{L_N} = \frac{T_N}{T_W} + \frac{T_N d^{-2\theta}}{T_N d^{-2\theta} + T_S^*} \frac{L_S}{dL_N}.$$

For this case to emerge parameter values must be such that this ratio does not exceed one. Otherwise we are in case 1 above. We also need that the demand for workers using the South's exclusive technology L_S^E not exceed the supply of Southern workers. This condition requires that the ratio

$$\frac{L_S^E}{L_S} = \frac{T_S}{T_W} \frac{dL_N}{L_S} + \frac{T_S}{T_N d^{-2\theta} + T_S^*}$$

not exceed one. Otherwise we are in case 3 below.

In this case the range of goods produced using the common technology in the North are not traded. Hence technology diffusion results in less trade than would otherwise occur.

2.1.3 Case 3: Goods Produced with the Common Technology are not Traded

In this case $w_{NC} = w_N \leq w_S d$. Labor market equilibrium requires a wage w_N that solves:

$$w_N = \left[\left(\frac{T_N/L_N}{T_S/L_S} \right) \frac{T_N^* w_N^{-\theta} + T_S d^{-\theta}}{T_N (w_N d)^{-\theta} + T_S^*} \right]$$

Again, there is no analytic solution but solving for the wage numerically is straightforward.

All goods produced using the common technology are not traded. In this case technology diffusion reduces the scope for trade even further.

2.2 Trade and Prices

What is the relationship between technology, wages, and prices in each of these cases? In case 1 and 2 the wage in the North is higher than that in the South by a factor of at least d while the prices of goods produced using the common technology are higher by a factor of exactly d. In case 2, some of these goods are produced in both countries using the same technology. In contrast, goods produced with the Northern technology are more expensive in the South by a factor d. In this case the model delivers the implication that nontraded goods are cheaper in the South.

3 Technology Dynamics

We have so far considered the static equilibrium in which labor forces and levels of technology are given. Over time, however, we can envisage processes of innovation and diffusion governing the evolution of T_{Nt}, T_{St}, and T_{Ct} over time (introducing a time subscript). Following the specification in Krugman (1979), for example, we can imagine that each country innovates at a rate that is proportional to its current knowledge, and that ideas flow from the exclusive to the common pool at rates that are proportional to the stocks of exclusive ideas. We introduce four parameters, ι_N, the rate at which the North innovates, ι_S, the rate at which the South innovates, ϵ_N, the rate at which the South learns about exclusively Northern ideas, and ϵ_S, the rate at which the North learns about exclusively Southern ideas: Thus T_{Nt}, T_{St}, and T_{Ct} evolve over time according to:

$$\begin{aligned}
\frac{dT_{Nt}}{dt} &= (\iota_N - \epsilon_N)T_{Nt} + \iota_N T_{Ct} \\
\frac{dT_{St}}{dt} &= (\iota_S - \epsilon_S)T_{St} + \iota_S T_{Ct} \\
\frac{dT_{Ct}}{dt} &= \epsilon_N T_{Nt} + \epsilon_S T_{St}.
\end{aligned}$$

While the analytic solution to this dynamic system is complex, it is straightforward to show that as long as the innovation and diffusion parameters are strictly positive and the initial value of at least one T_i is positive, the system evolves to a steady state in which all three types of knowledge grow at the same rate.

In general, the resulting growth rate is the solution to an unpleasant cubic equation. It can be shown, however, that the steady-state growth rate is increasing in both innovation and diffusion parameters. In the special case of symmetry, $\iota_N = \iota_S = \iota$, and $\epsilon_N = \epsilon_S = \epsilon$, the steady-state growth rate is merely quadratic:

$$g = \frac{\iota - \epsilon + \sqrt{(\iota - \epsilon)^2 + 8\iota\epsilon}}{2},$$

strictly increasing in ι and ϵ. A world with more innovation but also more diffusion grows faster.

Krugman (1979) considers what happens when only the North innovates, so that $i_S = 0$.and the growth rate is just ι_N while $T_S = 0$. This case is the one we pursue from now on. Since ι_S and ϵ_S are irrelevant we set $\iota_N = \iota$ and $\epsilon_N = \epsilon$. In steady state T_N and T_C each grow at rate ι and $T_N/T_C = \iota/\epsilon$.However, we wish to endogenize ι to see how it reacts to more rapid diffusion (higher ϵ) and lower trade barriers (lower d).

4 Endogenizing Innovation

We now amend the model to endogenize the Northern innovation rate ι. Following, for example, the two-country analysis of Grossman and Helpman (1991), we can introduce an endogenous innovation process. Since only the North innovates we eliminate case 3 above from the range of possibilities. We continue to assume that exclusive Northern ideas flow into common knowledge at an exogenous rate ϵ (suppressing the subscript since it is now unnecessary).

As in Kortum (1997), we model innovation as the production of ideas. An idea is a way to produce a good j with output per worker q. We assume that an idea is equally likely to apply to any good in the unit interval, and that q is the realization of a random variable Q drawn from the Pareto distribution:

$$F(q) = \Pr[Q \leq q] = 1 - q^{-\theta}.$$

Only an idea that lowers the cost of serving a market will be used. Initially, ideas will only be usable in the North. Hence to lower the cost of serving the Northern market an idea q must satisfy:

$$w_N/q \leq c_N(j) = \min[w_N/z_N(j), dw_S/z_C(j)]$$

where $z_N(j)$ and $z_C(j)$ represent the states of the art in the exclusively Northern and commonly available technologies, respectively. To lower the cost of serving the Southern market it must satisfy:

$$w_N d/q \leq c_S(j) = \min[w_N d/z_N(j), w_S/z_C(j)]$$

Note that the second criterion is more stringent. Hence some innovations will initially be used only for the Northern market while others will be sold to the world market. Moreover, to be useful in either market the innovation must exceed the Northern state of the art.

The probability that the idea constitutes an innovation in the North is thus:

$$\Pr[w_N/Q \leq c_N(j)] = \Pr[Q \geq w_N/c_N(j)] = [w_N/c_N(j)]^{-\theta}$$

while the probability that it is useful in the South is:

$$\Pr[w_N d/Q \leq c_S(j)] = \Pr[Q \geq w_N d/c_S(j)] = [w_N d/c_S(j)]^{-\theta}$$

We need to introduce an incentive for the North to innovate. We follow the quality ladders framework (Grossman and Helpman, 1991, Aghion and Howitt, 1992) an posit that the owner of an innovation has the ability to use it to produce and sell a product at the highest price that keeps the competition at bay. Thus an innovation for producing good j with worker productivity $z(j)$ allows the owner to produce the good at unit cost $c^{(1)}(j) = w/z(j)$ the next cheapest source has unit cost $c^{(2)}(j)$ then the innovation allows the seller to charge a markup $c^{(2)}(j)/c^{(1)}(j)$.

4.1 The Distribution of the Mark Up

We first derive the distribution of the markup in the North. For an idea that is exclusively available to its Northern inventor, the markup for producing a good with an efficiency of Q (drawn from the distribution LINK), which would otherwise cost some amount C_{Nt} in the North (drawn from the distribution LINK) is $M_{Nt} = C_{Nt}/(w_N/Q)$ (where capital letters denote random variables). The probability that the markup exceeds some value m is:

$$\begin{aligned} b_{NNt}(m) &= \Pr[M_{Nt} \geq m] = \Pr[C_{Nt} \geq w_N m/Q] \\ &= \int_1^\infty \Pr[C_{Nt} \geq m w_N/Q | Q = q] \theta q^{\theta-1} dq \\ &= \int_1^\infty \exp\left[-\Phi_{Nt} \left(m w_N\right)^\theta q^{-\theta}\right] \theta q^{\theta-1} dq \\ &= \frac{m^{-\theta}}{\Phi_{Nt} w_N^\theta} \\ &= \frac{1}{T_{Nt} + T_{Ct} \left(w_N/w_{NC}\right)^\theta} m^{-\theta}. \end{aligned}$$

(Since we consider a steady state in which w_N is constant, we do not index it by t.) For the good to be sold, of course, requires $M \geq 1$. which occurs with probability:

$$b_{NNt}(1) = \frac{1}{T_{Nt} + T_{Ct} \left(w_N/w_{NC}\right)^\theta} = \frac{1}{\Phi_N w_N^\theta}$$

The equivalent derivation for using a Northern technology to sell in the South yields:

$$\begin{aligned} b_{SNt}(m) &= \Pr[M_{St} \geq m] \\ & \quad \frac{m^{-\theta}}{\Phi_{St} \left(w_N d\right)^\theta} \\ &= \frac{1}{T_{Nt} + T_{Ct} \left(w_N d/w_S\right)^\theta} m^{-\theta}. \end{aligned}$$

Turning to an idea that has passed into the common technology, the markup for producing a good with an efficiency of Q , which would otherwise cost some amount C_{Nt} in the North (drawn from the distribution LINK) is $M_{Nt} = C_{Nt}/(w_{NC}/Q)$. The probability it exceeds some value m is:

$$\begin{aligned} b_{NCt}(m) &= \Pr[M_{Nt} \geq m] = \Pr[C_{Nt} \geq w_{NC} m/Q] \\ &= \frac{m^{-\theta}}{\Phi_{Nt} \left(w_{NC}\right)^\theta} \\ &= \frac{1}{T_{Ct} + T_{Nt} \left(w_{NC}/w_N\right)^\theta} m^{-\theta}. \end{aligned}$$

Finally, the distribution of the markup for selling a good using the common technology in the South is:

$$\begin{aligned} b_{SCt}(m) &= \Pr[M_{St} \geq m] \\ &= \frac{m^{-\theta}}{\Phi_{St} w_S^{\theta}} \\ &= \frac{1}{T_{Ct} + T_{Nt} \left[w_S / (w_N d) \right]^{\theta}} m^{-\theta}. \end{aligned}$$

In any of these cases, an idea will be used if and only if $m \geq 1$. The distribution of the markup conditional on $m \geq 1$ is just:

$$G(m) = \frac{b_{ii't}(1) - b_{ii't}(m)}{b_{ii't}(1)} = 1 - m^{-\theta} \quad i = N, S; \; i' = N, C.$$

the simple Pareto distribution with parameter θ.

With these expressions in hand we are now armed to confront the profit calculations.

4.2 Profits

We are now in a position to derive the expected profit stream from a new invention. An issue we need to confront is what happens when an idea diffuses to the common technology. There are many possible cases to consider. At one extreme this diffusion could mark the end of the inventor's exclusive use of the idea anywhere, so that her profit streams end. At the other the inventor could retain the right to use the idea anywhere, in which case diffusion allows the possibility of relocating production to the South in order to exploit the lower wage there. In between are situations in which she keeps her rights in the North but not the South. We begin with the case of universal intellectual property protection.

Let's assume that we are in Case 2, in which the North and the South both use the common technology, so that $w_S d = w_N = w_{NC}$. In this case diffusion does not affect the profit stream earned from sales in the North, since the South has no cost advantage for selling in the North. Diffusion will allow greater revenue in the South, however. We assume that the labor forces in each country grow at rate n, and denote the ratio of the Southern to Northern labor force as λ.

We first calculate the expected profit stream from selling in the North. At any time s the distribution of the markup for a technology still in use is $G(m)$. For a markup m the profit is $(1 - m^{-1})Y_N$, where Y_N is total spending by Northerners. Since we use the Southern wage as numeraire, as we show below, in steady state Y_N grows at rate n.

Integrating across the markup distribution $G(m)$, the expected flow of profit from an idea conditional on selling is:

$$Y_{Nt}\int_1^\infty (1-m^{-1})dG(m)=\frac{Y_{Nt}}{1+\theta}$$

The probability that the technology is in use at time s is $b_{NNs}(1)$. Since we have made the Southern wage the numeraire, and the Northern wage is proportional to it, technical progress lowers the price level over time. As indicated above, the price level is $P_{Nt}=\gamma\Phi_{Nt}^{-\theta}$. The expected real discounted profit from an idea at date t (unconditional on realized quality) is thus:

$$\begin{aligned}V_{Nt} &= \frac{P_{Nt}Y_{Nt}}{1+\theta}\int_t^\infty \frac{1}{P_{Ns}}e^{-(\rho-n)(s-t)}b_{NNs}(1)ds \\ &= \frac{\Phi_{Nt}^{-1/\theta}Y_{Nt}w_N^{-\theta}}{1+\theta}\int_t^\infty e^{-(\rho-n)(s-t)}\Phi_{Ns}^{-1+1/\theta}ds\end{aligned}$$

where ρ is the discount factor. Since in steady state $\Phi_{Nt}=T_{Nt}w_N^{-\theta}(1+\epsilon/\iota)$, Φ_{Nt} grows at rate ι. Thus in steady state:

$$\begin{aligned}V_{Nt} &= \frac{Y_{Nt}w_N^{-\theta}}{\Phi_{Nt}\,(1+\theta)}\int_t^\infty \exp[-(s-t)(\rho+\iota-\iota/\theta-n)]ds \\ &= \frac{Y_{Nt}w_N^{-\theta}}{\Phi_{Nt}\,(1+\theta)\,(\rho+\iota-\iota/\theta-n)} \\ &= \frac{Y_{Nt}}{T_{Nt}(1+\epsilon/\iota)\,(1+\theta)\,(\rho+\iota-\iota/\theta-n)}.\end{aligned}$$

By analogy to the North, expected profit conditional on selling is in the South is:

$$Y_{St}\int_1^\infty (1-m^{-1})dG(m)=\frac{Y_{St}}{1+\theta}$$

where Y_S is total spending by Southerners, which also grows at rate n in steady state.

In calculating the value of an idea in serving the Southern market, we need to distinguish between profits earned while the idea remains exclusively Northern and those earned after it has transited into the common technology. Given our assumptions the first would be produced in the North and the second in the South.

The probability that an idea created at time t remains exclusively Northern at time s is $\exp[-\epsilon(s-t)]$, while otherwise it has transited into the common technology pool. The probability that a Northern technology is used for serving the South at time s is $b_{SNs}(1)$ and the probability that a common technology is serving the South at time s is $b_{SCs}(1)$.The price level in the South is $P_{St}=\gamma\Phi_{St}^{-\theta}$. Putting these items together, the expected real discounted profit from serving the South with an idea created at date t (unconditional on realized quality) is:

$$\begin{aligned}
V_{St} &= \frac{P_{St}Y_{St}}{1+\theta}\int_t^{\infty}\frac{e^{-(\rho-n)(s-t)}}{P_{Ss}}\left[e^{-\epsilon(s-t)}b_{SNs}(1)+(1-e^{-\epsilon(s-t)})b_{SCs}(1)\right]ds \\
&= \frac{Y_{St}}{\Phi_{St}\left(1+\theta\right)}\left[\frac{1}{w_S^{\theta}(\rho+\iota-\iota/\theta-n)}+\frac{1-d^{2\theta}}{\left(w_N d\right)^{\theta}\left(\rho+\epsilon+\iota-\iota/\theta-n\right)}\right] \\
&= \frac{Y_{St}}{\left(T_{Nt}+d^{2\theta}\epsilon/\iota\right)\left(1+\theta\right)}\left[\frac{d^{2\theta}}{\rho+\iota-\iota/\theta-n}+\frac{1-d^{2\theta}}{\rho+\epsilon+\iota-\iota/\theta-n}\right]
\end{aligned}$$

Hence the total value of an idea at time t is:

$$V_t = V_{Nt}+V_{St} = \frac{1}{T_{Nt}(1+\theta)}\left\{\frac{1}{\rho+\iota-\iota/\theta-n}\left[\frac{Y_{Nt}}{1+\epsilon/\iota}+\frac{Y_{St}}{1+d^{2\theta}\epsilon/\iota}\right]+\frac{Y_{St}(1-d^{2\theta})}{\left(1+d^{2\theta}\epsilon/\iota\right)\left(\rho+\epsilon+\iota-\iota/\theta-n\right)}\right\}.$$

Having solved for the value of an idea, we can now examine the amount of innovation that the North will engage in.

4.3 The Rate of Innovation

Northern workers engaging in R&D produce ideas. We assume that a Northern worker can produce ideas at a Poisson rate α. Hence the return to research is αV_t. An equilibrium in which the North is both producing goods and undertaking research thus requires that the return to research equal the Northern wage, i.e.:

$$\alpha V_t = d.$$

Since all Southern workers produce, and the South earns no income from inventions, $Y_{St} = L_{St}$. Since Northern workers engage in research and earns profits both in the North and in the South, the expression for its income is more complicated. Denoting the number of Northern workers engaged in production as L_{Nt}^P:

$$\begin{aligned}
Y_{Nt} &= w_N L_{Nt}^P + \frac{Y_{Nt}+Y_{St}}{1+\theta} \\
&= \frac{L_{Nt}}{\theta}[(1+\theta)(1-r)d+\lambda].
\end{aligned}$$

We define the ratio of Northern technology to Northern workers at time t as $t_{Nt} = T_{Nt}/L_{Nt}$. It evolves according to:

$$\begin{aligned}
\frac{\dot{t}_{Nt}}{t_{Nt}} &= \frac{\dot{T}_{Nt}}{T_{Nt}}-\frac{L_{Nt}}{L_{Nt}} \\
&= \frac{\alpha r_t L_{Nt}}{T_{Nt}}-(n+\epsilon).
\end{aligned}$$

where r_{Nt} is the fraction of the Northern labor force doing research. In steady state both r_{Nt} and t_{Nt} are constant, so that $t_N = \alpha r/(n+\epsilon)$ and $\iota = n$.

Substituting these expressions into the condition for labor market equilibrium allows us to solve for the steady state share of Northern workers who engage in research. Solving we get:

$$r = \frac{n}{\rho\theta}\left\{1 + \frac{\lambda}{(1+\theta)d}\left\{1 + \theta\Lambda\left[\Gamma + d^{2\theta}(1-\Gamma)\right]\right\}\right\}$$

where:

$$\Lambda = \frac{n+\epsilon}{n + d^{2\theta}\epsilon}$$

and:

$$\Gamma = \frac{\rho\theta - n}{(\rho+\epsilon)\theta - n}$$

The expression for research intensity is complicated, but in three special cases it reduces to something simple and intuitive:

1. When S is small relative to N ($\lambda = 0$) it becomes $r = n/\rho\theta$, what Eaton and Kortum (2001) get for the case of the closed economy and for symmetric research economies. Not surprisingly, globalization in the form of lower transport costs and faster diffusion has no effect, since the South doesn't matter.

2. When trade is costless, so that $d = 1$, .it becomes $r = n(1+\lambda)/\rho\theta$. A larger south scales up research in the North in proportion to its size. Since the wage is the same in each country, the speed of diffusion doesn't matter.

3. When research diffuses to the common pool instantaneously, as $\epsilon \mapsto \infty$, $r = n(1 + \lambda/d)/\rho\theta$. The effect of the South on research is diminished by the extend of the transport cost. In this case globalization in the form of lower trade barriers raises research effort.

What about more difficult cases? Given the complexity of the algebra, an analytic answer is hard to discern. However, a numerical evaluation using a wide range of parameter values suggested that, both increases in the rate of diffusion ϵ and reductions in trade barriers d, spur innovation.

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