Growth, sectoral composition, and the wealth of nations*

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Abstract

We characterize the dynamic equilibrium of a two-sector endogenous growth model with constant returns to scale. We assume that both sectors produce consumption and investment goods, and we introduce a minimum consumption requirement. In this model, economies with the same fundamentals but different endowments of capitals will converge to a common growth rate, although the long run level and sectoral composition of GDP will be different. Because total factor productivity depends on sectoral composition, capital endowments will also contribute to GDP by means of changing the sectoral composition. This suggests that the development accounting exercises should consider the endogeneity of total factor productivity when measuring the contribution of capital to GDP. Along the transition, the slope of the policy functions depends on the initial values of the capital stocks and of the minimum consumption requirement. This implies that the minimum consumption is a barrier to development and that economies initially similar may diverge along the transition.

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1. Introduction

New growth theory has provided increasing evidence suggesting that the accumulation of production factors alone cannot explain the observed cross-country differences in per capita gross domestic product (see, for instance, McGrattan and Schmitz, 1999; and Parente and Prescott, 2004). Authors like Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) argue that differences in per capita gross domestic product (GDP, henceforth) are mainly explained by differences in total factor productivity (TFP, henceforth). Simultaneously, the recent development literature explains international differences in the growth rates of GDP as the result of differences in the sectoral composition of GDP (see Echevarria, 1997; and Laitner, 2000). Recently, Caselli (2004), Cordoba and Ripoll (2004), and Chanda and Dalgaard (2005) unify these two lines of research by showing that sectoral change contributes not only to output growth, but also to productivity growth without any true technological change. By using multisector growth models as the basis of growth accounting exercises, these works demonstrate that the aggregate level of TFP can be decomposed into a contribution from sectoral composition and a contribution from the level of technology. In this way, they show that the composition effect can explain a large part of the differences in aggregate TFP levels across countries. At this point, the open question is to explain sustained cross-country differences in sectoral composition of output. In this paper, we assert that the accumulation of capital is a candidate to explain these differences. For that purpose, we characterize the equilibrium dynamics of an endogenous growth model, where the long run sectoral composition of GDP depends on the initial stocks of capital. As TFP depends on the sectoral composition of GDP, we will conclude that the contribution of capital to explain GDP is larger when TFP is endogenous.[1]

Economies experiment meaningful changes in the structure of the production activity along the process of economic development. On the one hand, empirical evidence has shown that there is a relationship between the level and the sectoral composition of GDP. Baumol and Wolf (1989), Chenery and Syrquin (1975) and Kuznets (1971), among others, show that the process of development is related to the process of structural change. On the other hand, as Chari et al. (1997) point out, "the recent literature emphasizes that a broad measure of capital is needed to account for at least some of the regularities in the data". In particular, the process of development is related to the growth of human capital, which explains the existence of a strong accumulation of human capital along the development process. Galor (2005), Galor and Moav (2004) and Pereira (xxxx) have shown the link between human capital accumulation and GDP growth. Therefore, according to the data, the process of development is linked to structural change and the accumulation of human capital. However, while empirical evidence provides a strong support to the previous relations, most growth models do not simultaneously take into account structural change and the accumulation of human capital. The aim of this paper is to construct a growth model

[1] Cordoba and Ripoll (2005) also stress that the results of development accounting may be modified when TFP is assumed to be endogenous and depend on capital stocks. However, in their model, TFP is endogenous because they impose a particular accumulation law of TFP that depends on capital. In contrast, in our model, TFP is endogenous because it depends on the sectoral composition of GDP that is determined by capital accumulation.

that while takes into account the relationship between human capital accumulation, structural change and development, it also satisfies the Kaldor's facts. Therefore, the equilibrium of this model will converge to a balanced growth path (BGP, henceforth) along which the GDP to capital ratio will remain constant.

For our purpose, we extend the two-sector model of endogenous growth with constant returns to scale and with physical and human capital accumulation, introduced by Uzawa (1965) and Lucas (1988). Apart from the absence of external effects, the main departure from Lucas (1988) is in the modeling of preferences. On the one hand, we assume that agents derive utility from the consumption of two goods. On the other hand, we assume that consumption is subject to a minimum requirement, which makes the utility function be non-homothetic. Because of this non-homotheticity, the sectoral composition of consumption changes as the economy develops, which drives the change in both the sectoral composition of GDP and in the sectoral allocation of production factors along the development process. The later in turn affects the level of GDP by modifying TFP. Thus, in our model there is a double causality between growth and structural change. Moreover, contrary to standard development literature, we assume that the minimum consumption requirement does not vanish as the economy grows, which means that the utility function is asymptotically non homothetic. This implies that the long run sectoral composition of consumption will depend on the income level. Therefore, we do not interpret the consumption requirement as a minimum subsistence level but as an aspiration in consumption, because utility rises only when consumption grows faster than its minimum requirement.

Many others papers analyzing economic growth and sectoral composition have also considered multiple consumption goods or minimum consumption requirements. For instance, Rebelo (2001) and Steger (2000) assume that consumption is subject to a minimum subsistence level in an endogenous growth model. Ngai and Pissaridis (2004) consider an exogenous growth model with heterogenous consumption goods, whereas Roberson (1999, 2000) and Steger (2006) introduce heterogenous consumption goods in endogenous growth models. Finally, Echevarria (1997), Laitner (2000) and Kongsamunt et al. (2001) introduce the two assumptions in exogenous growth models to analyze the interrelationship between sectoral composition and growth. In the present paper, we introduce these two assumptions in a model of endogenous growth, which yields important changes in the growth patterns. As in the standard two-sector growth model (see, for instance, Caballé and Santos, 1993), the set of BGPs is a linear manifold of dimension one along which the relative prices and the growth rate are constant. This means that there is a continuum of BGPs and that the initial conditions on the two capital stocks determines the BGP the economy converges to. However, in contrast with the standard two-sector growth model, the manifold of BGPs does not emanate from the origin when the following conditions hold: (i) individuals derive utility from the consumption of the two heterogenous goods; (ii) consumption is subject to a minimum consumption requirement; and (iii) the technologies used by the two sectors exhibit different capital intensities. When these conditions are satisfied, the GDP level, the ratio between physical to human capital and the sectoral structure, given by the allocation of resources across sectors, depends on the BGP the economy converges to. Thus, our model predicts that economies with the same fundamentals but different endowments of human and physical capital will converge to a common price level and growth rate, although the long-run levels of the capital stocks, the ratio of physical to human capital and the GDP to capital ratio will remain being different.

These results imply that economies with different endowments will converge to different sectoral capital allocations and different sectoral compositions of consumption and GDP. To see this, note that when capital endowments are small, the economies are forced to devote a large amount of their resources to consume the good that is subject to the minimum consumption requirement. Thus, the composition of consumption depends on the stocks of capital and, obviously, this makes the composition of GDP depend on these stocks. In particular, the ratio between the output of the sector producing the good that is subject to the minimum consumption requirement and the output of the other sector will be larger in those economies with smaller capital stocks and, moreover, this ratio will decrease as capital stocks rise. As a consequence of this different composition of GDP, when the stock of physical capital is low, the ratio of physical to human capital will be large (small) if the sector producing the good that is subject to the minimum consumption requirement is more (less) intensive in physical capital than the other sector.

The non-convergence to a common long-run sectoral composition has interesting consequences for the conclusions derived from the exercises of development accounting. In fact, TFP in a multisector growth model depends on the sectoral structure, which in our model is endogenous and depends on the stock of capital. Our results then imply that the level of physical capital is a source of differences in TFPs across economies. Thus, we assert that, because of the minimum consumption requirement, capital accumulation also affects the level of GDP by means of the induced changes in the sectoral composition and TFP. We can then interpret this minimum consumption as a barrier to riches, as it forces a particular sectoral composition of GDP that limits the value of aggregate production that can be attained given the capital stocks.[2] Therefore, according to our model, the empirical studies of development accounting that assume an exogenous TFP obtain biased measures of the contribution of capital endowments to explain the observed cross-country differences in GDP.

However, we show numerically that the contribution of capital to explain differences in the long-run values of TFP is relatively small in our model. This negative result arises because the differences in capital stocks generate small disparities in the sectoral allocation of physical and human capital at the BGP. This contribution of capital may instead be large when economies are assumed to be off the BGP, as the process of sectoral change in the composition of GDP occurs mainly along the transition. In particular, we show that two economies with different initial levels of physical and human capital exhibit significatively different sectoral structures when there is a negative relationship between the accumulation of the two capital stocks along the transition. In this case, the contribution of capital endowments to explain differences in GDP across economies is larger along the transition than at the BGP. From this analysis, we conclude that the nature of the development process depends on the slope of the policy functions driving the accumulation of the capital stocks. In particular, by limiting the analysis to the plausible case where the sector producing the good subject to

[2] Cordoba and Ripoll (2004) postulate that economies would be poor because they specialize in a sector with a low level of TFP. We argue that they specialize in these sectors because they must satisfy a minimum consumption requirement.

the minimum consumption requirement is relatively more intensive in physical capital, we obtain that the value and even the sign of the slope of these policy functions depends on the initial levels of physical and human capital, which is in stark contrast with the standard two-sector model of endogenous growth (see Caballé and Santos, 1993). This means that the sectoral structure of two economies with the same fundamentals but different initial conditions on the capital stocks may diverge along the transition.

The plan of the paper is as follows. Section 2 presents the model. Section 3 characterizes the steady-state equilibrium. In Section 4, we explain differences in GDP per capita across countries. Section 5 characterizes the transition towards the BGP and its implications for development accounting. Section 6 concludes the paper and presents some possible extensions to the present research. All the proofs and lengthy computations are in the Appendix.

2. The economy

Let us consider a two-sector growth model in which there are two types of capital k and h, that we denote physical and human capital, respectively. One sector produces a commodity Y according to technology $Y = A\,(sk)^{\alpha}\,(uh)^{1-\alpha} = Auhz_Y^{\alpha}$, where, respectively, s and u are the shares of physical and human capital allocated in this sector, and $z_Y = \frac{sk}{uh}$ is the capital ratio in this sector. The commodity Y can either be consumed or added to the stock of physical capital. The law of motion of the physical capital stock is thus given by

$$\dot{k} = A\,(sk)^{\alpha}\,(uh)^{1-\alpha} - c - \delta k, \tag{2.1}$$

where c denotes the amount of Y devoted to consumption, and $\delta = (0,1)$ is the depreciation rate of the physical capital stock. The other sector produces a commodity H by means of the production function $H = \gamma\left[(1-s)\,k\right]^{\beta}\left[(1-u)\,h\right]^{1-\beta} = \gamma(1-u)hz_H^{\beta}$, where $z_H = \frac{(1-s)k}{(1-u)h}$. This commodity can also be either devoted to consumption or to increase the stock of human capital. The evolution of human capital is thus given by

$$\dot{h} = \gamma\left[(1-s)\,k\right]^{\beta}\left[(1-u)\,h\right]^{1-\beta} - x - \eta h, \tag{2.2}$$

where x denotes the amount of H devoted to consumption, and $\eta \in (0,1)$ is the depreciation rate of the human capital stock. Note that this model can be interpreted as a generalization of the two-sector growth model, in which the two sectors produce both consumption and investment goods. Because the two sectors produce final goods, we define the Gross Domestic Product (GDP) as follows:

$$Q = Y + pH, \tag{2.3}$$

where p is the relative price of good H in terms of good Y.

The economy is populated by an infinitely lived representative agent characterized by the following utility function:

$$U(c,x) = \frac{\left[(c-\overline{c})^{\theta}\,x^{1-\theta}\right]^{1-\sigma}}{1-\sigma},$$

where $\theta \in [0,1]$ is the share parameter for good c in the composite consumption good $(c-\overline{c})^{\theta} x^{1-\theta}$, $\sigma > 0$ is the constant elasticity of marginal utility with respect to this composite consumption good, and $\overline{c}$ is a minimum consumption requirement. We also assume that the growth rate of the minimum consumption coincides with the long-run growth rate of consumption g^*, i.e.

$$\overline{c} = c_0 e^{g^* t}, \tag{2.4}$$

in order to guarantee that the equilibrium converges to a BGP.[3] Observe that, given a constant value of x, the utility rises when the growth rate of consumption c is larger than g^*. We can then interpret this increasing path of consumption requirements as an aspiration, as the utility rises only when consumption grows faster than the minimum consumption requirement. Note that the introduction of the minimum consumption using this additive functional form implies that the constraint $c > \overline{c}$ must hold for all t.

The representative agent maximizes the discounted sum of utilities

$$\int_0^{\infty} e^{-\rho t} U(c,x) dt,$$

subject to (2.1) and (2.2), where $\rho > 0$ is the subjective discount rate. Let μ_1 and μ_2 be the shadow prices of k and h, respectively. Appendix A provides the first order conditions of this maximization problem, and derives the system of dynamic equations that fully characterize the equilibrium paths by following the standard procedure used in the two-sector model of endogenous growth (see, for example, Bond et al., 1996). In the remainder of this section, we only provide the equations that compose this dynamic system defining the equilibrium dynamics. Define z as the aggregate ratio from physical to human capital, i.e., $z = \frac{k}{h}$. First, because by definition $p = \frac{\mu_2}{\mu_1}$, we get the equation that drives the growth of prices

$$\frac{\dot{p}}{p} = -(1-\beta)\gamma z_H^{\beta} + \beta\gamma z_H^{\beta-1} p + \eta - \delta, \tag{2.5}$$

with

$$z_H = \phi \left[\frac{\beta(1-\alpha)}{\alpha(1-\beta)}\right] p^{\frac{1}{\alpha-\beta}}, \tag{2.6}$$

where

$$\phi = \left(\frac{\gamma}{A}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{1-\beta}{\alpha-\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\beta}{\alpha-\beta}}.$$

As follows from (2.5), the growth of the price is driven by the standard non-arbitrage condition that states that the returns on physical and human capital must coincide. Note that (2.5) is a function of p alone and only depends on technology parameters. In contrast, the price level is driven by the marginal rate of substitution, i.e.,

$$p = \left(\frac{1-\theta}{\theta}\right)\left(\frac{c-\overline{c}}{x}\right), \tag{2.7}$$

[3] If we instead assumed that the minimum consumption grows at a rate lower than the long-run growth rate of consumption c, then the equilibrium would converge to an asymptotic BGP. This means that the equilibrium would converge to a stationary solution only when consumption diverges to infinite.

which shows that the price level depends on the parameter θ and on the composition of consumption. This yields an important difference with respect to the standard two-sector growth model with a unique consumption good (see, as an example, Bond et al., 1996), where the price level is related with the level of consumption.

We now proceed to characterize the growth rate of consumption expenditure. In this economy with two consumption goods, we define consumption expenditure as $w = c + px$. Moreover, we denote the fraction of consumption expenditures devoted to purchases of good c by w_c, where

$$w_c = \frac{1}{1 + \left(\frac{1-\theta}{\theta}\right)\left(1 - \frac{\overline{c}}{c}\right)}, \tag{2.8}$$

as follows from (2.7). Note that w_c provides a measure of the composition of consumption expenditures, which depends on θ and on the ratio $\frac{\overline{c}}{c}$.[4] It follows that the composition of consumption changes along the development process because of the introduction of the minimum consumption requirement. In fact, the minimum consumption makes the utility function be non-homothetic, which implies that the composition of consumption depends on the level of income. In Appendix A, we obtain that

$$\frac{\dot{w}}{w} = g^* + \left(\frac{w - \overline{c}}{\sigma w}\right)\left[\beta\gamma p z_H^{\beta-1} - \delta - \rho - \sigma g^* - (1-\theta)(1-\sigma)\frac{\dot{p}}{p}\right]. \tag{2.9}$$

As follows from (2.9), the two assumptions introduced on preferences have interesting implications on convergence. On the one hand, the introduction of the minimum consumption requirement implies that the dynamic adjustment is driven by the dynamic behavior of the intertemporal elasticity of substitution (IES, henceforth), that we will denote by χ. If we define the IES as a measure of the sensitivity of the growth rate of consumption expenditure to the annual net discount rate, we obtain that $\chi = \frac{w-\overline{c}}{\sigma w}$. Given that we have assumed that the growth rate of the minimum consumption requirement is equal to the long-run growth rate of consumption c, the IES is constant in the long run even for finite values of consumption. However, during the transition, and unlike the case of homothetic preferences, the IES is not constant. On the other hand, the existence of two consumption goods implies that the convergence is not only driven by the diminishing returns to scale but also by the change in the relative price. The growth effect of a variation in the growth rate of prices depends on the IES and on the elasticity of the marginal utility of c with respect to x, which is given by $(1-\theta)(1-\sigma)$. The combination of these two facts makes the model suitable to explain some interesting features of the growth patterns along the development process.[5] However, this issue is beyond the scope of the present paper.

Finally, we characterize the growth rate of the two capital stocks. For that purpose, we use (2.7) and the definition of w to rewrite the ratios $\frac{c}{k}$ and $\frac{x}{h}$ as functions of p, w, k and h. Given these functions, we get in Appendix A that

[4] Note that if $\overline{c} = 0$ then $w_c = \theta$, whereas if $\overline{c} = c$ then $w_c = 1$. Note also that if $\theta = 0$ then $w_c = 0$, whereas if $\theta = 1$ then $w_c = 1$.

[5] As an example, note that the IES decreases with $\frac{\overline{c}}{w}$, which implies that the minimum consumption deters growth. Thus, the model may explain the low growth rates in those economies, where agents must devote most resources to consume in order to satisfy the minimum consumption requirement.

$$\frac{\dot{k}}{k} = A\left(\frac{uh}{k}\right) z_Y^{\alpha} - \delta - \frac{\theta w + (1-\theta)\,\overline{c}}{k}, \tag{2.10}$$

and

$$\frac{\dot{h}}{h} = \gamma\,(1-u)\,(z_H)^{\beta} - \eta - (1-\theta)\left(\frac{w-\overline{c}}{ph}\right), \tag{2.11}$$

with

$$z_Y = \phi p^{\frac{1}{\alpha-\beta}}, \tag{2.12}$$

and

$$u = \frac{z - z_H}{z_Y - z_H}. \tag{2.13}$$

We can now define the dynamic equilibrium as a set of paths $\{w, p, k, h, \overline{c}\}$ that, given the initial levels of the two capital stocks k_0 and h_0 and the initial level of minimum consumption requirement $\overline{c}_0$, solves (2.5), (2.9), (2.10), and (2.11), satisfies (2.6), (2.12), (2.13), the exogenous law of motion of the minimum consumption constraint (2.4) and the usual transversality conditions

$$\lim_{t\to\infty} \mu_1 k = 0, \tag{2.14}$$

and

$$\lim_{t\to\infty} \mu_2 h = 0. \tag{2.15}$$

Note that the equilibrium will be characterized by three state variables, k, h and $\overline{c}$, and two control variables, w and p. Because there are three state variables, the transition will be driven not only by the imbalances between the two capital stocks, as occurs in the standard two-sector growth model, but also by the initial levels of the capital stocks.

3. The balanced growth path

A steady-state equilibrium or BGP in our economy is an equilibrium path along which both capital stocks, both consumption goods and consumption expenditures grow at a constant rate, and capital allocation between sectors, relative prices and the ratio from aggregate output to physical capital are constant. This section lays down the properties of a BGP and the conditions for its existence.

Proposition 3.1. *Assume that a BGP exists. Then, the relative price remains constant along the BGP and, moreover, there is a unique long-run value of the relative price p^* that solves*

$$-(1-\beta)\,\gamma z_H^{\beta} + \beta\gamma z_H^{\beta-1} p^* + \eta - \delta = 0. \tag{3.1}$$

The two capital stocks and consumption expenditure grow at the same constant growth rate

$$g^* = \frac{\beta\gamma z_H^{\beta-1} p^* - \delta - \rho}{\sigma}. \tag{3.2}$$

We have shown the existence and uniqueness of a long-run price level and growth rate. Obviously, this does not imply the existence of a BGP, but it implies that if a BGP exists then the price level and growth rates are unique. Note that these long-run price level and growth rate neither depend on the weight of consumption goods in the utility function, θ, nor on the minimum consumption requirement, $\overline{c}_0$. Thus, the assumptions made on preferences do not affect the long-run value of these two variables that, as in the standard two-sector growth model, only depends on technology. We show next that the long-run level of the variables depends on the assumptions made on preferences. For that purpose, we normalize the variables w, k, and h as follows

$$\widehat{w} = we^{-g^* t}, \tag{3.3}$$

$$\widehat{k} = ke^{-g^* t}, \tag{3.4}$$

and

$$\widehat{h} = he^{-g^* t}. \tag{3.5}$$

Note that the normalized variables $\widehat{w}$, $\widehat{k}$ and $\widehat{h}$ will remain constant along a BGP, and $\widehat{w}^*$, $\widehat{k}^*$, and $\widehat{h}^*$ will denote the respective steady-state values of these variables. The following proposition characterizes a steady-state equilibrium in terms of the normalized variables $\widehat{w}$, $\widehat{k}$ and $\widehat{h}$.

Proposition 3.2. *Given $\widehat{k}^*$, a BGP is a set $\left\{g^*, p^*, \widehat{h}^*, \widehat{w}^*\right\}$ that satisfies $(1-\sigma)\, g^* < \rho$ and $\widehat{k}^* \geq k^c$, and solves (3.1), (3.2), and*

$$\widehat{h}^* = m + n\widehat{k}^*, \tag{3.6}$$

$$\widehat{w}^* = l + j\widehat{k}^*, \tag{3.7}$$

where

$$m = \left(\frac{1-\theta}{\theta}\right)\left(\frac{\overline{c}_0}{bp^*}\right),$$

$$n = -\left(\frac{1}{b}\right)\left\{\left(\frac{1-\theta}{\theta p^*}\right)\left[\left(\frac{Az_Y^{\alpha}}{z_Y - z_H}\right) - (\delta + g^*)\right] + \frac{\gamma z_H^{\beta}}{z_Y - z_H}\right\},$$

$$l = -\left(\frac{1}{\theta}\right)\left[m\left(\frac{Az_H z_Y^{\alpha}}{z_Y - z_H}\right) + (1-\theta)\,\overline{c}_0\right],$$

$$j = \left(\frac{1}{\theta}\right)\left[(1 - nz_H)\left(\frac{Az_Y^{\alpha}}{z_Y - z_H}\right) - (\delta + g^*)\right],$$

and

$$b = (\eta + g^*) - z_H\left(\frac{Az_Y^{\alpha}}{z_Y - z_H}\right)\left(\frac{1-\theta}{\theta p^*}\right) - \frac{\gamma z_H^{\beta} z_Y}{z_Y - z_H}.$$

Moreover, the following statements hold:

(i) the slopes n and j are positive;

(ii) when $\overline{c}_0 > 0$, $\theta \in (0,1)$ and $\alpha \neq \beta$, then $m > 0$ if $\alpha < \beta$, whereas $m < 0$ if $\alpha > \beta$. Otherwise, $m = 0$.

The previous result states the conditions on the fundamentals for the existence of an interior BGP. On the one hand, the condition $(1-\sigma)\,g^* < \rho$ guarantees that the transversality conditions hold and the objective function in the representative agent's problem takes a bounded value. On the other hand, the condition $\widehat{k}^* \geq k^c$ ensures that the value of the physical capital stock at the BGP satisfies $\widehat{h}^* > 0$ and $\widehat{c}^* > \overline{c}_0$. Proposition 3.2 also shows that the set of steady-state values is a linear manifold of dimension one. This means that there is a continuum of BGPs, which we will index by $\widehat{k}^*$. Along this manifold there is a positive relationship between $\widehat{w}^*$ and $\widehat{k}^*$, and between $\widehat{h}^*$ and $\widehat{k}^*$. However, the ratios $\frac{\widehat{h}^*}{\widehat{k}^*}$ and $\frac{\widehat{w}^*}{\widehat{k}^*}$ can either change or remain constant from one BGP to another. More precisely, the linear manifold of BGPs does not emanate from the origin when the independent terms in (3.6) and (3.7) are different from zero. This is an important difference with respect to the standard two-sector growth model, where the set of BGPs forms a linear manifold emanating from the origin.[6] This difference yields the main results of the paper. The next corollary provides conditions for this difference to hold.

Corollary 3.3. *The manifold of BGPs does not emanate from the origin if and only if the following statements hold:*
(i) individuals derive utility from the consumption of the two goods, i.e. $\theta \in (0,1)$*;*
(ii) the minimum consumption requirement is strictly positive, i.e., $\overline{c}_0 > 0$; *and*
(iii) capital intensity is different across sectors, i.e., $\alpha \neq \beta$.

The main implication of the previous result for the purpose of this paper is that the sectoral composition can change along the set of BGPs, which is in stark contrast with the standard two-sector model of endogenous growth without externalities. In order to show this conclusion, we first proceed to characterize in some detail the relationship between the values of $\widehat{h}^*$ and $\widehat{k}^*$ derived from Proposition 3.2. Note first that the value $\widehat{h}^*$ depends positively on $\widehat{k}^*$, since the function (3.6) has a positive slope. However, the stationary ratio between both capital stocks, that we will denote by z^*, can increase, decrease or remain constant after an increase in $\widehat{k}^*$. By using the results on Proposition 3.2, we next characterize this dependence of z^* on $\widehat{k}^*$.

Proposition 3.4. *Assume that* $\overline{c}_0 > 0$, $\theta \in (0,1)$ *and* $\widehat{k}^* > k^c$. *If* $\alpha > \beta$ *the ratio* z^* *is a decreasing function of* $\widehat{k}^*$, *whereas* z^* *is an increasing function of* $\widehat{k}^*$ *when* $\alpha < \beta$. *The ratio* z^* *is constant if either* $\overline{c}_0 = 0$, $\theta = 1$, *or* $\alpha = \beta$.

From the previous result, we conclude that different BGPs can exhibit different physical to human capital ratios, although these ratios will be constant in each BGP.[7] The minimum consumption requirement implies that if the normalized stock of capital $\widehat{k}^*$ at a BGP is small and, thus, GDP is small, then the economies are forced to devote more resources to consume c than x. This means that the composition of consumption expenditure depends on the value of $\widehat{k}^*$. Obviously, this dependence also makes the

[6] See Caballé and Santos (1993) for an analysis of the BGP in the standard two-sector growth model.

[7] In Lucas (1988), the presence of externalities in production generates a non-linear locus of capital pairs $\left(\widehat{k}^*, \widehat{h}^*\right)$ along which capital ratio z^* changes. Klenow and Rodriguez (2004) obtain a similar result.

composition of GDP depend on this capital stock $\widehat{k}^*$. In particular, the ratio between the output of sector Y and the output of sector X will be larger in those economies with a smaller value of $\widehat{k}^*$ and decrease as this value rises. As a consequence of this different composition of GDP, when $\widehat{k}^*$ is small, the required stock of physical capital is larger (smaller) than the required stock of human capital if sector Y is more (less) intensive in physical capital than sector X, i.e. when $\alpha > (<)\, \beta$. Note that this explains that if $\alpha > (<)\, \beta$ then the ratio z^* is large (small) in poor economies, where the minimum consumption requirement forces agents to devote most resources to sector Y, and it also explains the reduction (increase) in this ratio as the normalized stock of physical capital increases. Observe also that if $\alpha = \beta$ the factor intensity is the same in the two sectors and, thus, the relative requirements of both capital stocks do not change as $\widehat{k}^*$ rises. This means that the ratio z^* is constant when $\alpha = \beta$. Finally, if either $\theta = 1$ (there is a unique consumption good) or $\overline{c}_0 = 0$, a rise in the normalized stock of physical capital does not change the composition of consumption and, thus, it does not change the capital ratio z^*. Therefore, it follows that only when $\theta \in (0, 1)$, $\overline{c}_0 > 0$, and $\alpha \neq \beta$, the ratio z^* changes with the normalized stock of physical capital $\widehat{k}^*$.

We next characterize the dependence of the stationary values of u and w_c on $\widehat{k}^*$. For that purpose, we use the definition of u and w_c and the results in Proposition 3.2.

Proposition 3.5. *Let u^* and w_c^* be the steady-state values of u and w_c, respectively. If $\overline{c}_0 > 0$, $\theta \in (0, 1)$, and $\alpha \neq \beta$, then u^* and w_c^* are an increasing and a decreasing function of $\widehat{k}^*$, respectively. Otherwise, both variables do not depend on $\widehat{k}^*$.*

The previous result implies that the composition of consumption and the sectoral structure at the BGP also depends on the value of $\widehat{k}^*$. In particular, if the conditions in Corollary 3.3 hold, then these two variables will change from one BGP to another. This result will be crucial to understand the mechanics that underlines the endogenerity of TFP in our model. This result is a consequence of the introduction of a minimum consumption requirement and of heterogenous consumption goods. However, the result does not depend on the relative factor intensity ranking. The intuition is as follows. In economies with a low normalized stock of physical capital at the BGP (poor economies), the minimum consumption requirement forces agents to devote a large amount of resources to consume good c. Thus, these economies are also forced to produce a large amount of the commodity Y in order to satisfy the minimum consumption requirement, which explains that u^* decreases with the value of $\widehat{k}^*$. Thus, the long-run sectoral composition of consumption and GDP depends on the normalized stock of physical capital. Note that the long-run sectoral composition does not depend on the actual level of capital k but on the normalized level of capital $\widehat{k}^*$. This implies that two economies exhibit a different sectoral composition of GDP for a given level of capital stock k if they attain this level at different periods. The economy that reaches a given level of capital stock later has accumulated more aspirations, so that a larger fraction of GDP must be devoted to satisfy the larger minimum consumption requirement at that moment. Therefore, in our model the level and the composition of GDP is not directly determined by capital stocks, but by the relationship between these stocks and the stock of aspirations.

At this point, it seems convenient to analyze the stability of the set of BGPs in order to determine if the initial conditions determine the BGP. The standard duality between

Rybczynski and Stolper-Sumuelson effects determines the stability property.[8] Thus, this property neither depends on the factor intensity ranking nor on the assumptions made on preferences.

Proposition 3.6. *The manifold of BGPs is saddle path stable.*

Saddle path stability means that there is a unique equilibrium path that converges to a BGP in the manifold. Thus, we conclude that two economies with the same fundamentals, but different initial endowments of capital stocks and aspirations, will end up with the same relative prices and growing at the same rate, although the levels of capital stocks, the physical to human capital ratio and the sectoral composition remain being different. Therefore, this model provides an explanation of the differences across countries in the long-run sectoral composition of GDP. Note that this explanation is not present in the standard two-sector growth model, in which the economies share the same long-run sectoral structure. In this sense, the version of the two-sector growth model we consider is a theory of both economic growth and structural change because long-run economic growth and long-run sectoral composition are endogenous.

Echevarria (1997), Rebelo (1991), Steger (2000) and Kongsamunt et al. (2001), among many others, also consider growth models with endogenous sectoral composition that changes as the economy develops along its transitional path. However, in these papers the sectoral composition is constant along the set of BGPs. Thus, although economies converge to different BGPs, they exhibit the same long-run sectoral composition. By the contrary, in our model the economies can converge into BGPs with different long-run sectoral composition depending on the initial conditions. Once in a BGP, each economy grows at a constant rate, and its sectoral composition remains constant. The dependence of the long-run sectoral composition on capital stocks, and so on the initial conditions, is a result that emerges from the fact that the minimum consumption requirement is an aspiration that grows permanently. This increasing aspiration forces agents to devote a permanently growing amount of resources to consume good Y, which sets a limit to structural change.

The non-convergence to a common sectoral composition of GDP will generate cross-country differences in TFP without any change in technology levels. In fact, the endogeneity of sectoral composition makes TFP depend on capital stocks. To see this, we follow Cordoba and Ripoll (2004) and we use (2.1), (2.2) and (2.3) to rewrite GDP as follows

$$Q = TFPk^{\alpha}h^{1-\alpha}, \tag{3.8}$$

where

$$TFP = A\left[\frac{u(\alpha-\beta)+1-\alpha}{(1-\beta)u}\right](s)^{\alpha}(u)^{1-\alpha}.$$

This decomposition of GDP between production factors and TFP shows that the later depends on the sectoral allocations of capital stocks. Therefore, as in any multisector growth model, the level of TFP is endogenous in the sense that it depends on sectoral composition. However, while in the standard two-sector growth model the long-run

[8] The role of the factor intensity ranking in the transitional dynamics of multi-sector growth models is extensively presented in Bond et al. (1996).

values of these capital shares are equal across countries with the same fundamentals, as they do not depend on the value of the capital stock $\widehat{k}^*$, in our model they depend on this value. Therefore, in our version of the two-sector growth model, TFP is endogenous in the sense that it depends on the capital stock. In particular, in poor economies the value of u is large and the TFP will be lower than in richer economies where u is smaller. Note that this result has interesting consequences on development accounting. In particular, by taking TFP as exogenous, several authors have concluded that differences in capital stocks cannot explain the observed disparities in the levels of GDP per capita (see, for instance, Hall and Jones, 1999). According to our model, taking TFP as exogenous introduces a bias in the results from the accounting analysis because the differences in capital stocks also imply differences in TFP.[9] In other words, our model implies that the contribution of capital to explain GDP differences is underestimated when TFP is assumed to be exogenous.

4. Development accounting

We will now show that the differences in capital stocks yield larger differences in GDP levels when TFP is endogenous. For that purpose, in this section we will focus on the set of BGPs. We first show this result analytically and then we provide several numerical exercises. Note first that using (3.8) to explain GDP differences requires a measure of human capital, which is a difficult variable to be measured. To avoid this problem, we use the long-run relationship between physical and human capital implied by (3.6) to rewrite the long-run value of GDP as a function of the long-run value of the normalized stock of physical capital $\widehat{k}^*$. By combining (3.6) and (3.8), and using Proposition 3.2, the following result characterizes the relationship between the long-run values of GDP and of physical capital implied by the model.

Proposition 4.1. *Let us define the normalized level of GDP as* $\widehat{Q} = Qe^{-g^*t}$ *and its steady-state value as* $\widehat{Q}^*$. *The value of* $\widehat{Q}^*$ *is the following linear function of the steady-state value of* $\widehat{k}^*$:

$$\widehat{Q}^* = \widetilde{b} + \widetilde{a}\widehat{k}^*, \tag{4.1}$$

where

$$\widetilde{b} = (1-\alpha)\, mAz_Y^{\alpha},$$

and

$$\widetilde{a} = \alpha A z_Y^{\alpha-1} + (1-\alpha)\, nAz_Y^{\alpha}.$$

Moreover, the following statements hold: (i) $\widetilde{a} > 0$; *and (ii) if* $\theta \in (0,1)$, $\overline{c}_0 > 0$, *and* $\alpha > (<)\, \beta$ *then* $\widetilde{b} < (>)\, 0$, *whereas* $\widetilde{b} = 0$ *otherwise.*

[9] Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999) rewrite (3.8) as $Q = TFP^{\frac{1}{1-\alpha}} \left(\frac{k}{Q}\right)^{\frac{\alpha}{1-\alpha}} h$. They use this transformation because they want to take into account that the impact of the difference in technology between economies is larger than the one measured by TFP, as it also affects the accumulation of capital. We do not have this problem since we do not consider differences in technologies across economies. In contrast, we assume that economies exhibit different TFP values only because they have different initial capital stocks. This means that, in this paper, in order to capture the actual effect of differences in capital stocks we must take into account that TFP is endogenous.

As it is usual, in our model the ratio from GDP to physical capital is constant at a steady-state equilibrium. However, this ratio may change along the set of BGPs. In fact, when either $\theta = 1$ or $\overline{c}_0 = 0$, our version of the two-sector constant returns to scale growth model coincides with the standard two-sector growth model. In this case, the relation between the steady-state values of GDP and of physical capital stock is the same for all BGPs under the assumption of constant returns to scale. This result also arises when $\alpha = \beta$ because in this case there is a unique production technology, so that the model coincides with a one sector constant returns to scale growth model. On the contrary, if $\theta \in (0,1)$, $\overline{c}_0 > 0$ and $\alpha \neq \beta$, then the GDP to physical capital ratio is not constant along the set of BGPs since $\widetilde{b}$ is different from zero. In this case, this ratio is increasing (decreasing) in the stock of capital as $\widetilde{b} < (>)\, 0$ when $\alpha > (<)\, \beta$. The intuition is as follows. The minimum consumption requirement makes poor economies devote a relatively large fraction of resources to sector Y. This implies that in these economies the physical to human capital ratio is larger (smaller) when sector Y is more (less) intensive in physical capital than sector X, i.e. when $\alpha > (<)\, \beta$. Note that this explains that if $\alpha > (<)\, \beta$, then the ratio of GDP to capital is initially large (small) in poor economies and this ratio decreases (increases) as the economy develops.

Note that the minimum consumption requirement affects the sectoral structure of GDP and, in fact, it prevents capital accumulation. In this sense, the minimum consumption requirement can be interpreted as a barrier to riches.[10] To see this, note that when $\alpha > (<)\, \beta$, a rise in the stock of physical (human) capital that reduces the strength of the minimum consumption requirement allows the economy to change the sectoral structure, which results into a higher level of normalized GDP; that is, the barrier to growth is surpassed when the physical capital stock rises.[11] This implies that $\widehat{Q}^*$ increases more (less) than proportionally with the level of $\widehat{k}^*$ when $\alpha > (<)\, \beta$. In fact, in the plausible case $\alpha > \beta$ an increase in the stock of capital causes a larger increase in GDP in this model with endogenous TFP than in a model with exogenous TFP.

Furthermore, the disparities observed across countries may be partially explained by means of the differences in their initial level of minimum consumption. In fact, economies with the same fundamentals, except for the initial level of the minimum consumption requirement, will diverge permanently in their GDP to physical capital ratio, in the capital ratio and in the sectoral structure, even though these countries will exhibit the same stationary growth rate and relative price level. Figure 1 shows how the long run ratios between GDP and physical capital and between the two capital stocks depend on the value of $\overline{c}_0$ when $\alpha > \beta$. We observe that the minimum consumption requirement reduces the human capital stock and the level of GDP that can be attained with a given stock of physical capital. This clearly shows that the

[10] Parente and Prescott (2000) introduce the notion of barrier to riches. In their paper, economies start growing when a constraint that societies set to the adoption of new technologies is surpassed. In contrast, in our paper, the economy starts growing when the stock of physical capital is large enough to surpass the minimum consumption requirement, so that the society can start accumulating human capital.

[11] When $\alpha < \beta$, a rise in the stock of human capital allows to surpass the barrier to growth, whereas a rise in the stock of physical capital has the opposite effect. The reason is that if $\alpha < \beta$ a rise in h (k) results into a lower (higher) ratio z that reduces (increases) u. Thus, if $\alpha < \beta$, a rise in h (k) increases (decreases) TFP by means of reducing (rising) u.

minimum consumption is a barrier to riches that deters development by means of modifying the sectoral composition.

[Insert Figure 1]

We have shown that capital stocks in our model have an indirect effect on TFP and GDP by changing the sectoral structure. In what follows, we use the output decomposition in (3.8) to illustrate by means of numerical simulations how shocks in the steady-state level of the normalized stock of physical capital alter GDP. These shocks will affect GDP through three channels: (i) the direct contribution of physical capital, that we denote by C_k; (ii) the indirect contribution derived from the induced change in the human capital stock, that we denote by C_h; and (iii) the indirect contribution derived from the induced change in TFP, that we denote by C_{TFP}. For our purpose, we consider two economies that only differ in their values of $\widehat{k}^*$. The parameter values are chosen so that the economy with the larger stock of physical capital (rich economy) replicates some facts from US economy. We first set arbitrarily the values of $\widehat{k}^*$ and A equal to unity. We then proceed to choose the other parameter as follows: we set $\alpha = 0.42$ from Perli and Sakelliaris (1998); we fix $\delta = 5.6\%$ to replicate the investment/capital ratio and, moreover, we assume that $\eta = \delta$; the value of the preference parameter σ is equal to 2, which implies that the IES would be 0.5 if there were no minimum consumption requirement; the value of γ is such that the net interest rate equals to 5.2%; the value of ρ is such that $g^* = 2\%$; the value of $\overline{c_0}$ is such that $\chi = 0.21$; and θ is such that $w_c = 0.6$.[12] Finally, we take alternative values for the technological parameter β to illustrate how differences in the capital intensities across sectors alters the results from the accounting exercises. In Table 1 we assume that $\beta = 0.32$, whereas in Table 2 we assume that $\beta = 0$. Once the parameters have been calibrated, we fix the value of $\widehat{k}^*$ for the poor economy such that the value of w_c in this economy is equal to 0.95.[13]

[Insert Tables 1 and 2]

As we have mentioned, in Tables 1 and 2 the only difference between the rich and poor economies is the value of $\widehat{k}^*$. It follows that these economies have a common long-run growth rate, interest rate and relative price. However, the levels of the other variables, including the sectoral composition, are different. In fact, the differences in $\widehat{k}^*$ yield a sectoral adjustment that is made in terms of both the sectoral composition of consumption and the sectoral composition of GDP. The differences in the sectoral composition occur because in the poor economy aspirations are stronger and, thus, affect at a larger extend the composition of consumption. In Tables 1 and 2, these stronger aspirations are shown in the ratio $\frac{\overline{c_0}}{c^*}$, which is larger in the poor economy. Note that these stronger aspirations result into a lower IES and a larger value of w_c^*. This

[12] The variables that provide information on the sectoral composition are the sectoral structure (u), the composition of consumption (w_c), and the composition of output. In a BGP, these three variables are related, so that they cannot be calibrated simultaneously. In this paper we use w_c as an object of calibration.

[13] Our definition of poor economy then includes those economies whose aspiration forces to allocate a large amount of resources to consume good Y. This happens when the fraction w_c is large.

different composition of consumption affects the sectoral structure, which is measured by u^*. The effect of these differences in sectoral structures on GDP is measured by C_{TFP}, which is clearly higher in Table 2 than in Table 1. This means that the effect of sectoral composition on GDP is larger when the difference between the technologies is larger, i.e., when the difference between α and β is larger. However, as can be checked from Table 2, the large contribution of physical capital through the TFP when α and β are very different is obtained at the cost of having an unreasonably low labor share in sector Y.[14] This implies that this contribution C_{TFP} would be clearly small if the value of β is set such that u^* takes empirically plausible values.[15]

Note also that while there is a strong difference between the poor and rich economies in terms of w_c^*, the difference in terms of u^* is small. This dissimilar response of w_c^* and u^* to the difference in the values of $\widehat{k}^*$ is explained as follows. A larger w_c^* implies that the ratio between the production of sectors Y and X must also be larger. This can be satisfied either by rising the amount of human capital devoted to sector Y or by reducing the consumption of the good produced in sector X in order to raise the stock of human capital, which results into a higher production in sector Y. Note that only the first effect rises TFP as it only depends on sectoral allocation of resources given by u^* and s^* (see (3.8)). However, as follows from the previous numerical examples, it seems that the second effect is more important than the first one as large differences in w_c^* translates into a large value of C_h and a small value of C_{TFP} (it is particularly small when we consider the plausible case $\beta = 0.32$).[16] These low values of C_{TFP} imply that the endogeneity of TFP does not rise significatively the ability of capital to explain GDP differences across countries. In the following section, we show that this negative result arises in part because we have assumed that both economies are in their BGPs. We will also show that if we instead assume that the poor economy is in its transition to the BGP, then the contribution of physical capital through the TFP may be large. Moreover, we must finally point out that our model is too much stylized to closely match particular observations; we merely use the parameterized model to illustrate the qualitative features of our theory.

5. The dynamic equilibrium

In the previous sections we have proved that the sectoral composition and the TFP at the BGP are determined by the initial conditions. In this section, we will show that the sectoral composition of economies with the same fundamentals but different

[14] Tables 1 and 2 are based on the assumption that the initial stock of aspirations of the two economies coincide. We could have assumed instead that the rich economy has an initial larger stock of aspirations, which drives larger differences between the poor and rich economy. In fact, by introducing differences in the initial stock of aspirations we can introduce large differences between the levels of GDP of the two economies.

[15] Our analysis underestimates the contribution of TFP as it only considers the shift in TFP due to differences in sectoral composition. In other words, our analysis does not take into account that economies are also different in their technologies, which obviously results into differences in TFP. Temple (2001) provides an indirect measure of the magnitude of this underestimation. He shows cross-country evidence that structural change explains between 20% and 70% of the growth in TFP.

[16] In Tables 1 and 2 the contribution of human capital is large. Manuelli and Seshadri (2005) show that qualified human capital explains most of the differences in GDP. Pereira (xxxx) also stresses the relevance of the accumulation of human capital to explain modern economic growth.

initial conditions may diverge along the process of dynamic adjustment to the BGP . This increases the capacity of differences in capital endowments to explain disparities in TFP across countries. Thus, in this section, we show that the contribution of capital to TFP is larger when we consider that economies are out of their BGPs.

For the purpose in this section, we first linearly approximate the policy functions around the set of BGPs. These policy functions depend on the assumptions made on preferences and on the relative factor intensity ranking. In this section, we limit our analysis to the plausible case with $\alpha > \beta$. In Appendix C, we characterize the linear approximation of the policy functions in this case. From this approximation, we first observe that $\widehat{k}^*$, which we have used to index the set of BGPs, is a function of the initial values of both capital stocks. This means that, in contrast with the standard two-sector model of endogenous growth (i.e., as $\overline{c}_0 = 0$ and $\theta = 1$), it is not only the initial value of the physical to human capital ratio, but also the levels of both capital stocks that determine the BGP the economy converges to when our assumptions on preferences hold (i.e., when $\theta \in (0, 1)$ and $\overline{c}_0 > 0$). Therefore, we have proved that economies with different initial endowments of capitals will converge to BGPs with different values of physical to human capital ratio, of consumption composition, of sectoral structure and of GDP composition if $\theta \in (0, 1)$, $\overline{c}_0 > 0$ and $\alpha \neq \beta$, eventhough these economies have a common initial capital ratio.

At this point, we must also note that the stock of aspirations $\overline{c}$ is another state variable in our model and, moreover, the initial value of this stock , $\overline{c}_0$, also affects the value of $\widehat{k}^*$. Thus, economies with different initial stocks of aspirations will converge to long-run equilibria with different levels of GDP and different sectoral compositions of consumption and GDP. Table 3 compares the BGPs of economies that are different only in the initial value of the stock of aspirations. This example shows that a larger $\overline{c}_0$ implies a smaller long-run value of GDP and affects the composition of consumption and the sectoral structure. This shows up that the aspirations force individuals to consume the good c, which forces a particular sectoral structure that limits economic development.

[Insert Table 3]

In what follows we use the linear approximation to the policy functions to compare, by means of numerical examples, two economies that are only different in their initial stock of physical capital. We assume that the rich economy (i.e. the economy with a higher stock of physical capital) is in its BGP, whereas the poor economy is in the transition to the BGP. Table 4 shows the results of this simulation exercise. As follows from this table, when we assume that both economies are in the BGP, the differences in sectoral structure given by u^* are small, whereas the differences in the composition of consumption given by w_c^* are large. In contrast, if we assume that the poor economy is outside of the BGP, there are mainly no differences between the rich and poor economies in terms of w_c, whereas there are huge differences in terms of u.[17] Obviously, this means that the contribution of physical capital to the differences in TFP is larger when we assume that the poor economy is outside of the BGP. It then follows

[17] The dynamic adjustment in u is diven by the capital ratio z and the relative prices.

that if we consider the process of dynamic adjustment to the BGP, the endogeneity of TFP rises meaningfully the ability of capital endowments to explain GDP differences.

[Insert Table 4]

The difference between the results of Table 4 and those in Table 1 and 2 arises from the fact that TFP depends on sectoral composition in terms of sectoral structure and not in terms of consumption composition. Note that w_c^*, is a decreasing function of $\widehat{k}^*$ as follows from Proposition 3.5, whereas u^* is an increasing function of z^* as follows from equation (2.13). Because along the manifold of BGPs the relationship between the two capital stocks is positive, the difference in $\widehat{k}^*$ between BGPs is larger than the differences in the values of z^*, as it can be seen from Figure 2. This means that the difference between two economies at the BGP is larger in terms of consumption composition than in terms of the sectoral structure. This explains the numerical results obtained in Tables 1 and 2, that show a large difference in w_c^* between the two economies and a small difference in u^*. In contrast, as shown in Figure 2, along the transition the capital ratio changes more rapidly than the normalized stock of physical capital if the policy function is downward sloping. It then follows that in this case economies adjust their sectoral structures along the transition in a larger extend than their consumption compositions. Thus, it is now obvious the explanation on the results in Table 4, where the differences in TFP across economies with different endowments of capital stocks are larger along the transition than these differences at the BGP.

[Insert Figure 2]

As we have mentioned, the results in Table 4 depend crucially on the negative sign of the policy function relating the two capital stocks along the transition to the BGP. This downward-sloping policy function ensures that the capital ratio z and the sectoral allocations of capitals u and s change more rapidly along the transition than the normalized stock of physical capital $\widehat{k}$, which is the mechanism generating the larger differences in TFP across economies reported by Table 4. Obviously, these results would be the opposite if the policy function relating the two capital stocks were upward-sloping. The sign of the slope of the policy functions then determines the nature of the transition the economy follows, i.e., it determines the patterns of development and of structural change, as well as the long run level of TFP. In order to illustrate this fact, let us assume that an economy is initially in the BGP and a sudden injection of physical capital occurs. If the slope of the policy function is negative the economy will converge after this shock to a new BGP with a larger stock of human capital and a smaller physical to human capital ratio. In this case, the structural change induced by the increase in physical capital has a positive effect on the long-run level of TFP, as the later depends negatively on the capital ratio (see equation (3.8) and the definitions of u and s). In contrast, if policy function is upward-sloping, then the aforementioned shock in the stock of physical capital leads the economy to another BGP with a smaller stock of human capital and a larger capital ratio than the initial one. In this case, the injection of physical capital changes the sectoral structure in a way that reduces the long-run level of TFP. Clearly, the opposite conclusions would be derived from studying the effects of a negative shock in the stock of physical capital.

At this point, we characterize the relationship between the two capital stocks along the equilibrium path and we show how our assumptions on preferences affect it. The analysis of the sign of the slope of the policy functions was first addressed by Caballé and Santos (1993) in the framework of a two-sector endogenous growth model without minimum consumption requirement and a unique consumption good. They claim that: (i) the economy belongs to the normal growth case when an increase in physical capital stock results into an increase in human capital stock; (ii) the economy belongs to the exogenous growth case when human capital stock does not increase when physical capital stocks increases; and (iii) the economy belongs to the paradoxical growth case when an increase in physical capital stock results into a decrease in human capital stock. Therefore, the economy belongs to the normal growth case when the policy function relating both capital stocks are downward-sloping, whereas the economy belongs to the paradoxical growth case if this policy function is upward-sloping. The next proposition characterizes the growth cases in our model and show how the introduction of a minimum consumption requirement and of the second consumption good modifies the relationship between the two capital stocks along the equilibrium path. In order to simplify the analysis and to facilitate the comparison with the results from Caballé and Santos (1993), we impose that $\beta = 0$, as these authors do.

Proposition 5.1. *Assume that $\beta = 0$, and let us denote*

$$v = 1 + \left[\frac{(1-\sigma)(1-\theta) z_Y}{\gamma p^*}\right]\left(A z_Y^{\alpha-1} - \delta - g^*\right),$$

and

$$\kappa = v\left[1 - (1-\theta)\frac{\overline{c}_0}{\widehat{c}^*}\right].$$

Then, the equilibrium corresponds to the normal growth case when $\alpha\kappa\chi < 1$, to the exogenous growth case when $\alpha\kappa\chi = 1$, and to the paradoxical growth case when $\alpha\kappa\chi > 1$.

Note that v may be either positive or negative when $\sigma > 1$, whereas if $\sigma \leq 1$ then v is larger than or equal to one. In particular, if v is negative then κ is also negative and the equilibrium always belongs to the normal growth case. Note also that if $\theta = 1$ then $\kappa = 1$ and we obtain the result in Caballé and Santos (1993), i.e. the equilibrium belongs to the normal growth case when $\alpha\chi < 1$, to the exogenous growth case when $\alpha\chi = 1$, and to the paradoxical growth case when $\alpha\chi > 1$. In contrast, if $\theta < 1$ then κ can be either larger or smaller than one. This means that the introduction of the minimum consumption requirement when $\theta < 1$ may change the relationship between the two capital stocks along the transition. Thus, economies with the same fundamentals but different stocks of aspirations, that make them belong to different growth cases, will exhibit different patterns of growth and structural change. Furthermore, the next result shows that economies with the same fundamentals but different initial values of capital stocks may also belong to different growth cases if $\theta \in (0,1)$ and $\overline{c}_0 > 0$.

Proposition 5.2. *Assume that $\beta = 0$. Let us define*

$$k^n = \frac{\alpha v \overline{c}_0}{(\alpha v - \sigma)\left[(1 - n z_H)\left(\frac{A z_Y^{\alpha}}{z_Y - z_H}\right) - (\delta + g^*)\right]}.$$

If $\alpha v < \sigma$, then the equilibrium belongs to the normal growth case. If $\alpha v > \sigma$, then those equilibrium paths converging to a BGP with $\widehat{k}^* < k^n$ belong to the normal growth case, whereas those equilibrium paths converging to a BGP with $\widehat{k}^* > k^n$ belong to the paradoxical growth case.

Because $\widehat{k}^*$ depends on the initial conditions, the previous result states that, for those vectors of parameters satisfying that $\alpha v > \sigma$, the economy belongs to any growth case depending on the initial conditions on the capital stocks. Therefore, the result in Proposition 5.2 is an important difference with respect to the result in Caballé and Santos (1993), where the economy belongs to a particular growth case regardless of the initial conditions. This difference arises because the slope of the policy functions in our model depends on the BGP the economy converges to (see Appendix C) and, thus, it is determined by the initial conditions, whereas in the standard model ($\theta = 1$ and $\overline{c}_0 = 0$) the slope of the policy functions is only determined by the vector of parameters.

Our results on the growth cases have interesting implications on convergence and development accounting. In particular, because the slope of the policy functions depends on the initial conditions, two economies with the same fundamentals but different initial endowments of capitals and aspirations may diverge along the transition to their BGPs. In fact, the capital ratio, the sectoral composition and the TFP of these two economies may diverge along the equilibrium path. This divergence would be meaningfully larger if these economies belong to different growth cases. Figure 3 illustrates this extreme case. This figure shows the equilibrium path of two economies that belong to different growth cases because they have different initial stocks of human capital. Note that these two economies are initially close and they diverge along the transition. On the one hand, the poor economy belongs to the normal growth case. Therefore, along the transition, the stock of physical capital rises, while the stock of human capital decreases. The rise in $\widehat{k}$ implies that consumption composition w_c decreases, and the reduction in $\widehat{h}$ implies that the ratio z increases strongly, which yields a strong increase in the labor share u. On the other hand, the rich economy belongs to the paradoxical growth case. Therefore, along the transition, both capital stocks rise. This economy converges to a BGP with a value of $\widehat{k}^*$ larger than in the poor economy, which means that the change in w_c is much larger than in the poor economy. In contrast, the rise in $\widehat{h}$ implies that the increase in z is small compared to the increase in this ratio in the poor economy and, thus, the change in u is lower than in the poor economy.

[Insert Figure 3]

Table 5 presents a numerical exercise that compares two economies as the ones illustrated in Figure 3. The table shows that these two economies diverge along the transition in terms of both their sectoral composition and also in their GDP levels. The divergence in terms of sectoral composition implies that the indirect contribution of capital endowments to explain the differences in GDP through their effects on TFP rises along the transition.

[Insert Table 5]

The main result of this section is that the contribution of capital to TFP may be large when we assume that the economies are out of BGPs. This result arises because the structural change occurs mainly along the transition to the BGP and, moreover, because the initial conditions modify the slope of the policy functions. On the one hand, when $\theta < 1$, the initial stock of aspirations modifies this slope, so that aspirations are a barrier to economic development that may explain divergence in the patters of economic development. On the other hand, when $\theta \in (0, 1)$ and $\overline{c}_0 > 0$, the initial value of the capital stocks also changes the slope of the policy functions, which also explains that economies may diverge in terms of sectoral composition, TFP and GDP.

6. Concluding remarks and extensions

In this paper we have analyzed the dynamic equilibrium of an extended version of the two-sector, constant returns to scale and endogenous growth model, in which both sectors produce consumption and investment goods. We have shown that the introduction of a second consumption good modifies the patterns of growth both in the long run and during the transition if we assume that preferences are non-homothetic. In the model, preferences are non-homothetic because we introduce a minimum consumption requirement. Under these assumptions, two economies with the same fundamentals but different initial endowments will converge to a BGP with the same relative prices and growth rates, although the capital ratio, the output-capital ratio and the sectoral composition will be different. Furthermore, the initial conditions determine the relationship between capital stocks along the equilibrium path leading the economy to the BGP. Thus, according our model, identical economies except for initial endowments may diverge along the transition. In this model, aggregate TFP depends on the sectoral composition of GDP. As a rise in the stock of physical capital modifies the sectoral composition of GDP, it increases TFP which is thus endogenous in the model.

The theoretical results in this paper extends the debate on development accounting initiated by Mankiw et al. (1992). These authors show that the accumulation of capital explains most of GDP differences between rich and poor economies. Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999) argue that this analysis was wrong because differences in technology yield differences in the accumulation of capitals. When this is taken into account, it follows that differences in GDP are mainly explained by differences in technology that result into differences in TFP. Our contribution, as those of Cordoba and Ripoll (2005), shows that the previous analysis is also wrong when the TFP is endogenously determined. In our paper, TFP is endogenous because sectoral composition depends on the capital stocks. This means that a rise in the stock of capital changes the sectoral composition and rises TFP. We show that the contribution of the capital stocks to explain GDP differences is larger when this endogeneity is taken into account. This suggests that an appropriate analysis of the contributions of technology and of the stock of capital to explain GDP differences should take into account this interaction between the capital stock and TFP.

We should point out that our results do not reduce the role of technology in explaining international differences in GDP. On the contrary, we understand that they reinforce this contribution, because in our model an increase in the technology level of one sector, say for instance A, has the following effects in TFP (see equation (3.8)): (i) a direct effect because this technology level is a primary component of our decomposition of TFP; (ii) an indirect effect because technology directly determines the sectoral structure; and (iii) an indirect effect because technology indirectly affects the sectoral structure by means of the induced changes in capital accumulation. The first two effects have already been computed in the development accounting exercises. However, to the best of our knowledge, the third effect has not been considered before. Note that our results reconcile the two sides of the debate on development accounting. On the one hand, technology has a crucial role in explaining the observed disparities in GDP across countries. On the other hand, part of the contribution of technology to explain these international differences comes from its interaction with capital accumulation.

Our model may also have other applications to explain some macroeconomic facts, that should be incorporated to the research agenda. For instance, it provides a possible explanation to the following puzzle on the share of labor income on GDP: cross-section data show that richer countries have a larger labor income share, whereas time-series data shows that the labor income share remains constant along the development process of each country. Because labor income shares depends on the sectoral structure, our model explains this puzzles. According to or model, labor income shares change out of the BGP, whereas they remain constant in the BGP. However, different BGPs have different sectoral composition and labor income shares. In particular, richer countries have a higher share of labor in sector X, which has a higher labor income share. This explains that the labor income share is larger in richer economies.

We have mentioned that there are different forces driving the growth of GDP in this economy. This means that the growth rate may exhibit a non-monotonic behavior along the development process. Therefore, the analysis of convergence seems a promising line of research, that may show up the Kuznets' facts concerning the relation between sectoral composition of GDP and development. Another line of research is the study of the effects of fiscal policy in the environment proposed in this paper. In our model, fiscal policy also affects economic development by means of modifying sectoral composition. In this environment, it seems interesting to study the level effects of fiscal policy. In fact, some policies, that may not have effects on long run growth, may modify the sectoral composition and, thus, the level of GDP and TFP. As an example, note that consumption taxes, by modifying the composition of consumption, would affect the long-run level and composition of GDP.

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A. The Equilibrium path

The first order conditions of the representative agent's maximization problem are:

$$\left(\frac{\theta\,(1-\sigma)\,U}{c-\overline{c}}\right)e^{-\rho t}=\mu_1, \tag{A.1}$$

$$\left(\frac{(1-\theta)\,(1-\sigma)\,U}{x}\right)e^{-\rho t}=\mu_2, \tag{A.2}$$

$$\left(\frac{\alpha Y}{s}\right)\mu_1=\left(\frac{\beta H}{1-s}\right)\mu_2, \tag{A.3}$$

$$\left(\frac{(1-\alpha)\,Y}{u}\right)\mu_1=\left(\frac{(1-\beta)\,H}{1-u}\right)\mu_2, \tag{A.4}$$

$$\left(\frac{\alpha Y}{k}-\delta\right)+\left(\frac{\beta H}{k}\right)\left(\frac{\mu_2}{\mu_1}\right)=-\frac{\dot{\mu}_1}{\mu_1}, \tag{A.5}$$

$$\left(\frac{(1-\alpha)\,Y}{h}\right)\left(\frac{\mu_1}{\mu_2}\right)+\left(\frac{(1-\beta)\,H}{h}-\eta\right)=-\frac{\dot{\mu}_2}{\mu_2}. \tag{A.6}$$

We now proceed to obtain the system of dynamic equations that characterize the equilibrium. First, combining (A.3) and (A.4), we obtain the expressions (2.12) and (2.6). Moreover, using the definitions of the ratios z, z_Y, and z_H, we obtain the expression of u given by (2.13) and

$$s=\left(\frac{z_Y}{z}\right)\left(\frac{z-z_H}{z_Y-z_H}\right). \tag{A.7}$$

Second, from the definition of p, and using equations (A.3), (A.4), (A.5) and (A.6), we get the equation that drives the growth of prices given by (2.5). Moreover, note that (A.1) and (A.2) imply that (2.7) holds.

Third, log-differentiating the definition of w with respect to time, we get that the growth rate of this variable is

$$\frac{\dot{w}}{w}=(1-w_c)\left(\frac{\dot{p}}{p}+\frac{\dot{x}}{x}\right)+w_c\left(\frac{\dot{c}}{c}\right), \tag{A.8}$$

where, using (2.7), w_c can be rewritten as

$$w_c=\frac{1}{1+\left(\frac{1-\theta}{\theta}\right)\left(1-\frac{\overline{c}}{c}\right)}. \tag{A.9}$$

Differentiating with respect to time (A.1), (A.2) and (2.7), we obtain the growth rate of c and x. Using these growth rates, equation (A.8) yields the growth rate of w given by (2.9).

Finally, we use (2.7) to rewrite $\frac{c}{k}$ and $\frac{x}{h}$ as functions of p, w, k and h. Given these functions, we obtain from (2.1) and (2.2) the growth rate of k and h as is given by (2.10) and (2.11), respectively.

B. The steady-state equilibria

B.1. Proof of Proposition 3.1

By using (2.4), (2.5), (2.9), (2.10), and (2.11), it can be shown that the variables w, p, k, h and $\overline{c}$ grow at a constant rate only if the price level is constant and satisfies (3.1) and the growth rates of w, k, h and $\overline{c}$ coincide and satisfy (3.2). The uniqueness of the price level and of the long-run growth rate follow by noticing that the left hand side of (3.1) is a monotonic function of the price. Finally, the existence of a price level follows from continuity of (3.1) and by noticing that the left hand side of this equation changes its sign as the price rises from zero. ■

B.2. Proof of Proposition 3.2

We first use the definition of the normalized variables to rewrite the equations (2.9), (2.10), and (2.11) as follows:

$$\frac{\dot{\widehat{k}}}{\widehat{k}} = A\left(\frac{u\widehat{h}}{\widehat{k}}\right) z_Y^{\alpha} - \frac{\theta\widehat{w} + (1-\theta)\,c_0}{\widehat{k}} - (\delta + g^*), \tag{B.1}$$

$$\frac{\dot{\widehat{h}}}{\widehat{h}} = \gamma\,(1-u)\,z_H^{\beta} - (\eta + g^*) - (1-\theta)\left(\frac{\widehat{w} - \overline{c}_0}{p\widehat{h}}\right), \tag{B.2}$$

$$\frac{\dot{\widehat{w}}}{\widehat{w}} = g^* + \left(\frac{\widehat{w} - \overline{c}_0}{\sigma\widehat{w}}\right)\left[\beta\gamma p z_H^{\beta-1} - \delta - \rho - \sigma g^* - (1-\theta)\,(1-\sigma)\left(\frac{\dot{p}}{p}\right)\right]. \tag{B.3}$$

Given the initial levels of the two capital stocks, k_0 and h_0, and of the minimum consumption, $\overline{c}_0$, we define an equilibrium path of the normalized variables $\left\{p, \widehat{k}, \widehat{h}, \widehat{w}\right\}_{t=0}^{\infty}$ as a path that solves the system of differential equations (2.5), (B.1), (B.2), and (B.3), satisfies (2.12), (2.6), (2.13), the definition of the long-run growth rate (3.2) and the two transversality conditions (2.14) and (2.15).

From the previous dynamic system we can obtain the stationary values of the normalized variables $\widehat{k}^*$, $\widehat{h}^*$, $\widehat{w}^*$. In a BGP $\dot{\widehat{k}} = \dot{\widehat{h}} = 0$, and then (B.1) and (B.2) can be rewritten as

$$A\left(\frac{u^*\widehat{h}^*}{\widehat{k}^*}\right) z_Y^{\alpha} - \frac{\theta\widehat{w}^* + (1-\theta)\,\overline{c}_0}{\widehat{k}^*} - \delta - g^* = 0, \tag{B.4}$$

$$\gamma\,(1-u^*)\,z_H^{\beta} - (\eta + g^*) - (1-\theta)\left(\frac{\widehat{w}^* - \overline{c}_0}{p^*\widehat{h}^*}\right) = 0. \tag{B.5}$$

Let us define $z^* = \frac{\widehat{k}^*}{\widehat{h}^*}$ and $q^* = \frac{\widehat{c}^*}{\widehat{h}^*}$. Using the definition of $\widehat{w}$ and (B.4), we obtain

$$\frac{\widehat{c}^*}{\widehat{h}^*} = (z^* - z_H)\left(\frac{Az_Y^{\alpha}}{z_{Y} - z_H}\right) - (\delta + g^*)\,z^*. \tag{B.6}$$

Combining (B.6) with (B.5), we obtain

$$\widehat{c}^* = c(z^*) = \frac{\overline{c}_0}{1 - \frac{\frac{\gamma z_H^\beta (z_Y - z^*)}{z_Y - z_H} - (\eta + g^*)}{\left(\frac{1-\theta}{\theta p^*}\right)\left[(z^* - z_H)\left(\frac{A z_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) z^*\right]}}. \tag{B.7}$$

By introducing (B.7) in (B.6), and after some algebra we get the following equation:

$$a\ z^* + b = 0, \tag{B.8}$$

where

$$a = \left(\frac{1-\theta}{\theta p^*}\right)\left[\left(\frac{A z_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*)\right] + \frac{\gamma z_H^\beta}{z_Y - z_H} - \left(\frac{1-\theta}{\theta p^*}\right)\left(\frac{\overline{c}_0}{\widehat{k}^*}\right), \tag{B.9}$$

and

$$b = (\eta + g^*) - z_H \left(\frac{A z_Y^\alpha}{z_Y - z_H}\right)\left(\frac{1-\theta}{\theta p^*}\right) - \frac{\gamma z_H^\beta z_Y}{z_Y - z_H}. \tag{B.10}$$

The following lemma proves that there exists a positive root of equation (B.8). This root is a function of $\widehat{k}^*$ and it is given by

$$z^* = \widetilde{z}\left(\widehat{k}^*\right) = -\frac{b}{a}. \tag{B.11}$$

Lemma B.1. *If $\alpha > \beta$ then $a > 0$ and $b < 0$, whereas $a < 0$ and $b > 0$ when $\alpha < \beta$.*

Proof. On the one hand, (B.4) implies that

$$\left(\frac{A z_Y^\alpha}{z_Y - z_H}\right) - (\delta + g^*) = \left(\frac{z_H}{z}\right)\left(\frac{A z_Y^\alpha}{z_Y - z_H}\right) + \frac{\widehat{c}^*}{\widehat{k}^*}, \tag{B.12}$$

and a simplifies as follows

$$a = \left(\frac{1-\theta}{\theta p^*}\right)\left[\left(\frac{z_H}{z^*}\right)\left(\frac{A z_Y^\alpha}{z_Y - z_H}\right) + \left(\frac{\widehat{c}^* - \overline{c}_0}{\widehat{k}^*}\right)\right] + \frac{\gamma z_H^\beta}{z_Y - z_H},$$

which is positive when $\alpha > \beta$ (i.e., $z_Y > z_H$) and $\widehat{c} - \overline{c}_0 > 0$. In this case, b is negative since the equation (B.2) requires that in the BGP the following inequality is satisfied:

$$\frac{\gamma z_H^\beta z_Y}{z_Y - z_H} > \eta + g^*.$$

On the other hand, when $\alpha < \beta$ (i.e., $z_Y < z_H$) one can directly see from (B.10) and (B.9) that $b > 0$ and $a < 0$, respectively. ■

At this point, we can also express the values of $\widehat{h}^*$, $\widehat{c}^*$ and $\widehat{w}^*$ as a function of $\widehat{k}^*$. First, note that $\widehat{h}^* = \frac{\widehat{k}^*}{z^*}$, so that from (B.11) we get

$$\widehat{h}^* = \widetilde{h}\left(\widehat{k}^*\right) = m + n\widehat{k}^*, \tag{B.13}$$

where

$$m = \left(\frac{1-\theta}{\theta}\right)\left(\frac{\overline{c}_0}{bp^*}\right), \tag{B.14}$$

and

$$n = -\left(\frac{1}{b}\right)\left[\left(\frac{1-\theta}{\theta p^*}\right)\left[\left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right) - (\delta + g^*)\right] + \frac{\gamma z_H^\beta}{z_{Y-}z_H}\right]. \tag{B.15}$$

Lemma B.2. *The function (B.13) satisfies the following properties: (i) $n > 0$; and (ii) if $\alpha > \beta$ then $m \leq 0$, whereas $m \geq 0$ when $\alpha < \beta$.*

Proof. The part (i) follows from Lemma B.1, the condition (B.12) and from the fact that the sign of $z_{Y-}z_H$ coincide with that of $\alpha - \beta$. The part (ii) is directly proved by noting that the sign of m coincides with that of b. ■

Second, given the stationary value of z^*, and by using (B.6), we get

$$\frac{\widehat{c}^*}{\widehat{h}^*} = (z^* - z_H)\left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right) - (\delta + g^*)\, z^*. \tag{B.16}$$

Thus, using (B.16) and (B.13), we get

$$\widehat{c}^* = \widetilde{c}\left(\widehat{k}^*\right) = d + f\widehat{k}^*, \tag{B.17}$$

where

$$d = -m\left(\frac{Az_H z_Y^\alpha}{z_Y - z_H}\right), \tag{B.18}$$

and

$$f = (1 - nz_H)\left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right) - (\delta + g^*). \tag{B.19}$$

Lemma B.3. *The function (B.17) satisfy that $d > 0$ and $f > 0$.*

Proof. First, from Lemma B.2 and from the fact that the sign of $z_{Y-}z_H$ coincides with that of $\alpha - \beta$, it can be shown that $d > 0$. Second, observe that

$$f = \underbrace{\left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right) - (\delta + g^*)}_{\xi} - nz_H\left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right),$$

and using (B.15) and (B.10), we obtain

$$bf = \xi\left[(\eta + g^*) - \frac{\gamma z_H^\beta z_Y}{z_{Y-}z_H}\right] + \left(\frac{\gamma z_H^\beta}{z_{Y-}z_H}\right) z_H \left(\frac{Az_Y^\alpha}{z_{Y-}z_H}\right). \tag{B.20}$$

Observe that (B.2) implies that

$$(\eta + g^*) - \frac{\gamma z_Y z_H^\beta}{z_{Y-}z_H} < -\gamma z_H^\beta\left(\frac{z}{z_{Y-}z_H}\right). \tag{B.21}$$

Using the previous inequality, we can already show the sign of f. On the one hand, if $z_{Y-}z_H > 0$, then (B.20) and (B.21) implies that

$$bf < \left(\frac{\gamma z_H^{\beta}}{z_{Y-}z_H}\right) z_H \left(\frac{Az_Y^{\alpha}}{z_{Y-}z_H}\right),$$

so that $f > 0$ since in that case $b < 0$. On the other hand, if $z_{Y-}z_H < 0$, then we get from (B.20) and (B.21) that

$$bf > -\xi\gamma z_H^{\beta}\left(\frac{z}{z_{Y-}z_H}\right) + \left(\frac{\gamma z_H^{\beta}}{z_{Y-}z_H}\right) z_H \left(\frac{Az_Y^{\alpha}}{z_{Y-}z_H}\right).$$

Thus, using the definition of u^* and ξ, the previous inequality can be written as

$$bf > \left(\frac{\gamma z_H^{\beta}}{z_{Y-}z_H}\right)\left(-u^* Az_Y^{\alpha} + z^*\left(\delta + g^*\right)\right).$$

Therefore, given that (B.4) implies that $z^*\left(\delta+g^*\right) - u^* Az_Y^{\alpha} < 0$, we obtain that in this case $f > 0$ since $b > 0$ as $z_{Y-}z_H < 0$. ■

Finally, we use the definition of $\widehat{w}^*$ and (B.17) to obtain

$$\widehat{w}^* = l + j\widehat{k}^* \tag{B.22}$$

where

$$l = \frac{d - (1-\theta)\,\overline{c}_0}{\theta},$$

and

$$j = \frac{f}{\theta}.$$

From Lemma A.3 is obvious that $j > 0$. However, the sign of l can not be characterized, at least, analytically.

To close the proof of Proposition 3.2, we must show that a stationary solution $\left\{g^*, p^*, \widehat{h}^*, \widehat{w}^*\right\}$ of the system of equations (2.5), (B.1), (B.2) and (B.3) for a given $\widehat{k}^* > 0$ is a BGP. First, this stationary solution must satisfy that $\widehat{h}^* > 0$ and $\widehat{c}^* > \overline{c}_0$. Thus, we must impose constraints on the value of $\widehat{k}^*$. On the one hand, by using (3.6), it can be shown that $\widehat{h}^* \geq 0$ if and only if $\widehat{k}^* \geq k^h$ where

$$k^h = -\frac{m}{n}. \tag{B.23}$$

On the other hand, by using (B.17) it can be shown that $\widehat{c}^* \geq \overline{c}_0$ if and only if $\widehat{k}^* \geq k^c$ where

$$k^c = \left(\frac{\overline{c}_0}{bf}\right)\left[b + \left(\frac{1-\theta}{\theta}\right)\left(\frac{Az_H z_Y^{\alpha}}{p\left(z_Y - z_H\right)}\right)\right]. \tag{B.24}$$

Moreover, it can be shown that $k^c \geq \max\left\{0, k^h\right\}$. First, after some algebra we get that $k^c > k^h$ when

$$1 > -\left(\frac{1-\theta}{\theta}\right)\left(\frac{1}{bp^*}\right)\left(\frac{\left(\frac{Az_Y^{\alpha}}{z_{Y-}z_H}\right) - \left(\delta + g^*\right)}{n}\right).$$

Using the definition of n in (B.15), the previous inequality implies that

$$-\left(\frac{1}{b}\right)\left[\frac{\gamma z_H^{\beta}}{z_{Y-}z_H}\right]>0,$$

which is satisfied as $b\,(z_{Y-}z_H)<0$. Second, using (B.10), we get

$$k^c=\left(\frac{\overline{c}_0}{bf}\right)\left[\underbrace{(\eta+g^*)-\frac{\gamma z_H^{\beta}z_Y}{z_{Y-}z_H}}_{\vartheta}\right].$$

Note that if $z_Y>(<)\,z_H$ then $b<(>)\,0$ and (B.2) implies that $\vartheta<(>)\,0$. Now, it is obvious that $k^c>0$. Therefore, $\widehat{h}^*>0$ and $\widehat{c}^*>\overline{c}_0$ if and only if $\widehat{k}^*\geq k^c$.

We must also prove that the transversality conditions (2.14) and (2.15) are satisfied at steady-state equilibrium. Note that using (A.1) and (2.7), we get that the transversality condition (2.14) is satisfied when

$$\lim_{t\to\infty}-\rho-\sigma\left(\frac{\dot{c}-\overline{c}g^*}{c-\overline{c}}\right)+(1-\theta)(1-\sigma)\frac{\dot{p}}{p}+\frac{\dot{k}}{k}<0,$$

which holds if $(1-\sigma)\,g^*<\rho$. Following a similar procedure, it can be shown that the transversality condition (2.15) is also satisfied when $(1-\sigma)\,g^*<\rho$.

The condition $(1-\sigma)\,g^*<\rho$ also implies that the utility function is bounded. To see this, note that the utility function is bounded when $\lim_{t\to\infty}e^{-\rho t}U(c-\overline{c},x)=0$. By using, (2.7), we get that the previous limit holds if

$$\lim_{t\to\infty}-\rho+(1-\sigma)\left(\frac{\dot{c}-\overline{c}g^*}{c-\overline{c}}\right)<0\ ,$$

which is satisfied if $-\rho+(1-\sigma)\,g^*<0$. Note that, even though $c\to\overline{c}$, the utility function is bounded when this condition is satisfied. ■

B.3. Proof of Proposition 3.5

On the one hand, from the definition of u^* in equation (2.13), we bet that the variable only depends z^* and p^*. Since, p^* is constant for all BGP (i.e., does not depend on $\widehat{k}^*$), we get directly from Proposition 3.4 the relationship between u^* and $\widehat{k}^*$.On the other hand, the proof of Proposition 3.2 states that $\widehat{c}^*$ rises with $\widehat{k}^*$. Moreover, from (2.8) we observe that fraction of consumption expenditures w_c decreases with $\widehat{c}^*$ if $\overline{c}_0>0$ and $\theta\in(0,1)$. Therefore, the dependence of w_c^* on $\widehat{k}^*$ follows directly from the two previous facts. ■

C. The dynamic equilibrium

C.1. Proof of Proposition 3.6

Using (2.5), (B.1), (B.2), and (B.3), we obtain the following Jacobian matrix evaluated at a BGP:

$$J = \begin{pmatrix} a_{11} & a_{12} & -1 & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \\ 0 & 0 & 0 & a_{44} \end{pmatrix},$$

where

$$a_{11} \equiv \frac{\partial \dot{\widehat{k}}}{\partial \widehat{k}} = \frac{Az_Y^{\alpha}}{z_Y - z_H} - (\delta + g^*),$$

$$a_{12} \equiv \frac{\partial \dot{\widehat{k}}}{\partial \widehat{h}} = -\frac{Az_H z_Y^{\alpha}}{z_Y - z_H},$$

$$a_{14} \equiv \frac{\partial \dot{\widehat{k}}}{\partial p} = \left[\frac{A\widehat{h}^* z_Y^{\alpha}}{(\alpha - \beta)\, p^*}\right] \left(\alpha u^* - \frac{z^*}{z_Y - z_H}\right),$$

$$a_{21} \equiv \frac{\partial \dot{\widehat{h}}}{\partial \widehat{k}} = -\frac{\gamma z_H^{\beta}}{z_Y - z_H},$$

$$a_{22} \equiv \frac{\partial \dot{\widehat{h}}}{\partial \widehat{h}} = \left(\frac{1-\theta}{\theta}\right) \left(\frac{\widehat{c}^* - \overline{c}_0}{p^* \widehat{h}^*}\right) - \frac{\gamma z^* z_H^{\beta}}{z_Y - z_H},$$

$$a_{23} \equiv \frac{\partial \dot{\widehat{h}}}{\partial \widehat{c}} = -\left(\frac{1-\theta}{\theta}\right) \left(\frac{1}{p^*}\right),$$

$$a_{24} \equiv \frac{\partial \dot{\widehat{h}}}{\partial p} = \left[\frac{\gamma \widehat{h}^* z_H^{\beta}}{(\alpha - \beta)\, p^*}\right] \left[\beta(1-u^*) - \frac{z^*}{z_Y - z_H}\right] + \left(\frac{1-\theta}{\theta}\right) \left(\frac{\widehat{c}^* - \overline{c}_0}{p^2}\right),$$

$$a_{34} \equiv \frac{\partial \dot{\widehat{c}}}{\partial p} = \left(\frac{\widehat{c}^* - \overline{c}_0}{\sigma}\right) \left(\beta\gamma z_H^{\beta-1} + \frac{(\beta - 1)\,\beta\gamma z_H^{\beta-1}}{(\alpha - \beta)} - (1-\theta)(1-\sigma)\frac{a_{44}}{p^*}\right),$$

$$a_{44} \equiv \frac{\partial \dot{\widehat{p}}}{\partial p} = -\frac{\beta\gamma p^* z_H^{\beta-1}}{\alpha - \beta} \left[1 - \alpha + (1-\beta)\left(\frac{z_H}{p^*}\right)\right].$$

It is immediate to see that the eigenvalues λ_i, $i = 1, 2, 3, 4$, of J are $\lambda_1 = 0$, $\lambda_2 = a_{44}$ and the two roots λ_3 and λ_4 are the solution of the following equation:

$$Q(\lambda) = \lambda^2 - \lambda(a_{11} + a_{22}) + a_{11}a_{22} - a_{12}a_{21} = 0.$$

Note that $\lambda_2 < (>) 0$ if $\alpha > (<) \beta$. In order to obtain the sign of λ_3 and λ_4, we characterize the elements of the polynomial $Q(\lambda)$. First, using (B.2), and rearranging terms, we get

$$\begin{aligned} a_{11}a_{22} - a_{12}a_{21} &= \\ &= \left(\frac{Az_Y^\alpha}{z_Y - z_H} - (\delta + g^*) \right) \left(\frac{\gamma z_Y z_H^\beta}{z_Y - z_H} - \eta - g^* \right) \\ &\quad + \left(\frac{Az_Y^\alpha z_H}{z_Y - z_H} - (\delta + g^*) z \right) \left(\frac{\gamma z_H^\beta}{z_Y - z_H} \right). \end{aligned}$$

By manipulating the previous equation by using (B.10), (B.15) and (B.19), we obtain $a_{11}a_{22} - a_{12}a_{21} = -bf$, which is positive when $z_Y > z_H$ and negative otherwise. Next, we consider the other element of $Q(\lambda)$. After some manipulation, where we basically replace $\delta + g^*$ from (B.6), we get

$$a_{11} + a_{22} = \frac{Az_Y^\alpha z_H}{(z_Y - z_H) z^*} + \frac{\widehat{c}^*}{\widehat{k}^*} + \left(\frac{1-\theta}{\theta} \right) \left(\frac{\widehat{c}^* - \overline{c}_0}{p^* \widehat{h}^*} \right) + \frac{\gamma z_H^\beta z^*}{(z_Y - z_H)},$$

which is positive when $z_Y > z_H$. It follows that one of the roots of the polynomial, for example λ_3 is always positive and the other one, λ_4, is positive if $\alpha > \beta$ and negative otherwise. Note that, regardless of the relation between α and β, there is a unique negative root, which means that the manifold of BGPs is saddle path stable. ■

C.2. Linear approximation of the policy functions

We proceed to approximate the policy functions around the set of BGPs. Because there are two control variables and two positive roots, using Proposition (3.6) we get the following the linear approximation of the equilibrium saddle path:

$$E(t) = V + A + Be^{\widehat{\lambda} t}. \tag{C.1}$$

where $E(t) = (\widehat{k}(t), \widehat{h}(t), \widehat{c}(t), p(t))$, $\widehat{\lambda}$ is the negative eigenvalue of J, $V = (V_k, V_h, V_c, V_p)$ is a vector of constant terms, $A = (A_k, A_h, A_c, A_p)$ is the eigenvector associated to the null root and $B = (B_k, B_h, B_c, B_p)$ is the eigenvector associated to $\widehat{\lambda}$. We proceed to find some properties of the vectors V, A and B. First, the relation between the elements of the eigenvector A follows from matrix relationship $JA = 0$. Solving this system of ordinary equations, we get that $A_p = 0$ and

$$\begin{aligned} A_h &= \underbrace{-\left(\frac{a_{21} + a_{23}a_{11}}{a_{22} + a_{23}a_{12}} \right)}_{a_h} A_k, \\ A_c &= \underbrace{\left(\frac{a_{11}a_{22} - a_{12}a_{21}}{a_{22} + a_{23}a_{12}} \right)}_{a_c} A_k. \end{aligned}$$

By using (B.2) and (B.15), we obtain that $a_h = n$ and $a_c = f$.

Second, the relation between the elements of the eigenvector B follows from the system of equations $(J-\lambda)B=0$, where I is the identity matrix. Assume that $\alpha>\beta$, then $\widehat{\lambda}=a_{44}$. Thus, we obtain from the previous matrix relationship that

$$
\begin{aligned}
B_h &= \underbrace{\frac{a_{21}\left(a_{14}-\frac{a_{34}}{a_{44}}\right)-(a_{11}-a_{44})\left(\frac{a_{23}a_{34}}{a_{44}}+a_{24}\right)}{(a_{22}-a_{44})(a_{11}-a_{44})-a_{21}a_{12}}}_{b_h} B_p, \\
B_k &= \underbrace{\left(\frac{a_{12}\left(a_{24}+a_{23}\frac{a_{34}}{a_{44}}\right)-(a_{22}-a_{44})\left(a_{14}-\frac{a_{34}}{a_{44}}\right)}{(a_{22}-a_{44})(a_{11}-a_{44})-a_{21}a_{12}}\right)}_{b_k} B_p, \\
B_c &= \underbrace{\left(\frac{a_{34}}{a_{44}}\right)}_{b_c} B_p,
\end{aligned}
$$

Finally, since $\widehat{\lambda}<0$ then the set of paths given by (C.1) converges to the following stationary solution:

$$
\begin{aligned}
k^* &= V_k + A_k, \\
h^* &= V_h + nA_k, \\
c^* &= V_c + fA_k, \\
p^* &= V_p.
\end{aligned}
$$

This stationary solution corresponds to the manifold of BGPs obtained in Section 3 when $V_k=0$, $V_h=m$, $V_c=d$, $V_p=p^*$ and $A_k=\widehat{k}^*$.

Therefore, the linear approximation of the policy functions is given by

$$
\begin{aligned}
\widehat{k}(t) &= \widehat{k}^* + b_k B_p e^{\widehat{\lambda} t}, \\
\widehat{h}(t) &= m + n\widehat{k}^* + b_h B_p e^{\widehat{\lambda} t}, \\
\widehat{c}(t) &= d + f\widehat{k}^* + b_c B_p e^{\widehat{\lambda} t}, \\
p(t) &= p^* + B_p e^{\widehat{\lambda} t}.
\end{aligned}
$$

Finally, using the initial conditions $\widehat{k}(0)$ and $\widehat{h}(0)$, we get

$$
\begin{aligned}
\widehat{k}^* &= \frac{b_h\widehat{k}(0)-b_k\widehat{h}(0)+b_k m}{b_h-nb_k}, \\
B_p &= \frac{\widehat{h}(0)-n\widehat{k}(0)-m}{b_h-nb_k}.
\end{aligned}
$$

C.3. Proof of Proposition 5.1

We study the relationship between the two capital stocks when $\beta=0$. This slope is characterized by the sign of $\frac{b_h}{b_k}$. In order to find this sign we must redefine the equilibrium dynamics of our economy under the assumption $\beta=0$. First, note that if

$\beta = 0$ then $s = 0$ and, thus, $z_H = 0$. Moreover, it can be proved that z_H^β converges to unity as β tends to zero. Finally, observe that both marginal products of physical capital in each sector equalize the rental rate of this capital in the aggregate economy in the general case with $\beta \neq 0$. Thus, to characterize the equilibrium dynamics in the case with $\beta = 0$ we replace $\alpha A z_Y^{\alpha-1}$ by $\gamma \beta p z_H^{\beta-1}$. Given this properties of the economy when $\beta = 0$, and using the elements of the Jacobian matrix in the proof of Proposition 3.6, it can be shown that the eigenvector simplifies as follows

$$\frac{b_h}{b_k} = -\frac{a_{21}\left(a_{14} - \frac{a_{34}}{a_{44}}\right) - (a_{11} - a_{44})\left(\frac{a_{23}a_{34}}{a_{44}} + a_{24}\right)}{(a_{22} - a_{44})\left(a_{14} - \frac{a_{34}}{a_{44}}\right)},$$

since $a_{12} = 0$ when $\beta = 0$. The denominator is negative since

$$a_{22} - a_{44} = \left(\frac{1-\theta}{\theta}\right)\left(\frac{\widehat{c}^* - \overline{c}_0}{p\widehat{h}^*}\right) + \gamma u^* + (1-\alpha) A z_y^{\alpha-1} > 0,$$

and

$$a_{14} - \frac{a_{34}}{a_{44}} = \frac{(\alpha-1)A}{\alpha p^*} z_Y^{\alpha-1}\widehat{k}^* - \left(\frac{\widehat{c}^* - \overline{c}_0}{\sigma}\right)\left(\frac{1-(1-\theta)(1-\sigma)}{p^*}\right) < 0.$$

It then follows that the sign of the eigenvector coincides with the sign of numerator, that we will denote by N. We get that

$$\begin{aligned} N \;\; = \;\; & -\left(\frac{\gamma}{z_Y}\right)\left[\frac{(\alpha-1)Az_Y^{\alpha-1}\widehat{k}^*}{\alpha p^*} - \left(\frac{\widehat{c}^* - \overline{c}_0}{p^*}\right)\left(\frac{1-(1-\theta)(1-\sigma)}{\sigma}\right)\right] - \\ & \left[\frac{\widehat{c}^*}{\widehat{k}^*} - (\alpha-1)Az_y^{\alpha-1}\right]\left[\frac{\gamma\widehat{h}^* z^*}{\alpha z_Y p^*} - \left(\frac{\widehat{c}^* - \overline{c}_0}{(p^*)^2}\right)\left(\frac{(1-\theta)(1-\sigma)}{\sigma}\right)\right]. \end{aligned}$$

Since the marginal products of human capital equalize across sectors, the following condition holds:

$$\gamma p^* = (1-\alpha) A z_y^\alpha.$$

By using the previous condition, we get after some manipulation that

$$N = \left[\frac{\widehat{c}^*\gamma}{\alpha p^* z_Y}\right]\left[\left(\frac{\alpha(\widehat{c}^* - \overline{c}_0)}{\sigma\widehat{c}^*}\right)\left[1 + \left(\frac{(1-\sigma)(1-\theta)z_Y}{p^*\gamma}\right)\left(\frac{\widehat{c}^*}{\widehat{k}^*}\right)\right] - 1\right].$$

Using (B.4), and since $u^* = \frac{\widehat{k}^*}{\widehat{h}^* z_Y}$, we get

$$N = \left[\frac{\widehat{c}^*\gamma}{\alpha p^* z_Y}\right]\left[\left(\frac{\alpha(\widehat{c}^* - \overline{c}_0)}{\sigma\widehat{c}^*}\right)\underbrace{\left[1 + \left(\frac{(1-\sigma)(1-\theta)z_Y}{p^*\gamma}\right)\left(Az_y^{\alpha-1} - \delta - g^*\right)\right]}_{v} - 1\right]. \tag{C.2}$$

Observe that $v < 1$ when $\sigma < 1$, whereas if $\sigma > 1$ then either $v \in (0,1)$ or $v < 0$.

The intertemporal elasticity of substitution satisfies

$$\chi = \frac{\widehat{c}^* - \overline{c}_0}{\sigma\left[\widehat{c}^* - (1-\theta)\overline{c}_0\right]}. \tag{C.3}$$

Using (C.3) to replace $\widehat{c}^* - \overline{c}_0$ in (C.2), we get that

$$N = \left[\frac{\widehat{c}^*\gamma}{\alpha p^* z_Y}\right]\left[\alpha v \chi\left(\frac{\widehat{c}^* - (1-\theta)\,\overline{c}_0}{\widehat{c}^*}\right) - 1\right].$$

The results in Proposition 5.1 then follows. ■

C.4. Proof of Proposition 5.2

Note from (C.2) that if $v < 0$ then $N < 0$ and if $v > 0$ then $N < (>)\, 0$ when

$$1 - \frac{\sigma}{\alpha v} < (>)\, \frac{\overline{c}_0}{\widehat{c}^*}. \tag{C.4}$$

Using the definition of $\widehat{c}^*$ as a function of $\widehat{k}^*$ established by Lemma B.3 in the proof of Proposition 3.2 (see Appendix B), Proposition 5.2 directly follows from (C.4). ■

Tables

Table 1.[18] $\beta = 0.32$

Rich Economy	Poor Economy	Comparison	Accounting[19]
$\widehat{k}^R = 1$	$\widehat{k}^P = 0.63$	$\frac{\widehat{k}^R}{\widehat{k}^P} = 1.58$	$C_k = 39.5\%$
$\widehat{h}^R = 0.12$	$\widehat{h}^P = 0.073$	$\frac{\widehat{h}^R}{\widehat{h}^P} = 1.66$	$C_h = 60\%$
$TFP^R = 1.007$	$TFP^P = 1.004$	$\frac{TFP^R}{TFP^P} = 1.002$	$C_{TFP} = 0.5\%$
$\widehat{Q}^R = 0.29$	$\widehat{Q}^P = 0.18$	$\frac{\widehat{Q}^R}{\widehat{Q}^P} = 1.63$	
$\frac{\overline{c}}{c^R} = 0.96$	$\frac{\overline{c}}{c^P} = 0.99$		
$w_c^R = 0.6$	$w_c^P = 0.95$		
$u^R = 0.4$	$u^P = 0.5$		
$g^R = 2\%$	$g^P = 2\%$		
$\chi^R = 0.21$	$\chi^P = 0.026$		

Table 2.[20] $\beta = 0$

Rich Economy	Poor Economy	Comparison	Accounting
$\widehat{k}^R = 1$	$\widehat{k}^P = 0.97$	$\frac{\widehat{k}^R}{\widehat{k}^P} = 1.03$	$C_k = 3\%$
$\widehat{h}^R = 0.58$	$\widehat{h}^P = 0.33$	$\frac{\widehat{h}^R}{\widehat{h}^P} = 1.76$	$C_h = 67\%$
$TFP^R = 1.35$	$TFP^P = 1.17$	$\frac{TFP^R}{TFP^P} = 1.16$	$C_{TFP} = 30\%$
$\widehat{Q}^R = 0.99$	$\widehat{Q}^P = 0.6$	$\frac{\widehat{Q}^R}{\widehat{Q}^P} = 1.63$	
$\frac{\overline{c}}{c^R} = 0.96$	$\frac{\overline{c}}{c^P} = 0.99$		
$w_c^R = 0.6$	$w_c^P = 0.95$		
$u^R = 0.17$	$u^P = 0.3$		
$g^R = 2\%$	$g^P = 2\%$		
$\chi^R = 0.21$	$\chi^P = 0.026$		

[18] The other parameters are $\alpha = 0.42$, $A = 1$, $\gamma = 0.086$, $\eta = \delta = 0.056$, $\sigma = 2$, $\rho = 0.012$, $\theta = 0.0476$ and $\overline{c}_0 = 0.051$.

[19] The contributions of the different factors to explain GDP differences is obtained from (3.8) as follows: $C_{TFP} = ln\left(TFP^R/TFP^P\right)/\, ln\left(Q^R/Q^P\right)$, $C_h = (1-\alpha)\left[ln\left(\widehat{h}^R\Big/\widehat{h}^P\right)\Big/\, ln\left(Q^R/Q^P\right)\right]$, and $C_k = \alpha\left[ln\left(\widehat{k}^R\Big/\widehat{k}^P\right)\Big/\, ln\left(Q^R/Q^P\right)\right]$, where the superscripts R and P indicate the rich and poor economies, respectively.

[20] The other parameters are $\alpha = 0.42$, $A = 1$, $\gamma = 0.112$, $\eta = \delta = 0.056$, $\sigma = 2$, $\rho = 0.016$, $\theta = 0.0475$ and $\overline{c}_0 = 0.18433$.

Table 3.[21]

$\overline{c}_0 = 0.017$	$\overline{c}_0 = 0.025$	$\overline{c}_0 = 0.05$	$\overline{c}_0 = 0.1$	$\overline{c}_0 = 0.15$
$\widehat{k}^* = 1.951$	$\widehat{k}^* = 1.955$	$\widehat{k}^* = 1.975$	$\widehat{k}^* = 2$	$\widehat{k}^* = 2.03$
$\widehat{h}^* = 0.242$	$\widehat{h}^* = 0.241$	$\widehat{h}^* = 0.240$	$\widehat{h}^* = 0.238$	$\widehat{h}^* = 0.236$
$\widehat{Q}^* = 0.5864$	$\widehat{Q}^* = 0.5862$	$\widehat{Q}^* = 0.5860$	$\widehat{Q}^* = 0.5858$	$\widehat{Q}^* = 0.5856$
$u^* = 0.351$	$u^* = 0.360$	$u^* = 0.387$	$u^* = 0.442$	$u^* = 0.498$
$w_c^* = 0.458$	$w_c^* = 0.488$	$w_c^* = 0.576$	$w_c^* = 0.753$	$w_c^* = 0.930$

[21] The parameters are $\alpha = 0.42$, $\beta = 0.32$, $A = 1$, $\gamma = 0.086$, $\delta = \eta = 0.056$, $\rho = 0.01185$, $\sigma = 2$, $\theta = 0.4$, $\widehat{h}_0 = 0.1$ and $\widehat{k}_0 = 4$. These values coincide with the ones of Table 1, with the exception of θ. We consider here a smaller value of θ to avoid that small differences in the minimum consumption yield huge differences in w_c.

Table 4. [22]

Comparison at BGP

Rich Economy	Poor Economy	Comparison	Accounting
$\widehat{k}^R = 1.95$	$\widehat{k}^P = 1$	$\frac{\widehat{k}^R}{\widehat{k}^P} = 1.95$	$C_k = 16\%$
$\widehat{h}^R = 2.12$	$\widehat{h}^P = 0.32$	$\frac{\widehat{h}^R}{\widehat{h}^P} = 6.65$	$C_h = 63\%$
$TFP^R = 1.636$	$TFP^P = 1.1367$	$\frac{TFP^R}{TFP^P} = 1.44$	$C_{TFP} = 21\%$
$\widehat{Q}^R = 3.35$	$\widehat{Q}^P = 0.586$	$\frac{\widehat{Q}^R}{\widehat{Q}^P} = 5.7$	
$u^R = 0.1$	$u^P = 0.34$	$\frac{u^R}{u^P} = 0.377$	
$w_c^R = 0.34$	$w_c^P = 0.99$	$\frac{w_c^R}{w_c^P} = 0.34$	
$\frac{\overline{c}}{c^R} = 0.51$	$\frac{\overline{c}}{c^P} = 0.997$		
$g^R = 2\%$	$g^P = 2\%$		
$\chi^R = 0.41$	$\chi^P = 0.00625$		

Comparison along the transition

Rich Economy	Poor Economy	Comparison	Accounting
$\widehat{k}^R = 1.95$	$\widehat{k}_0^P = 1.2125$	$\frac{\widehat{k}^R}{\widehat{k}_0^P} = 1.61$	$C_k = 13\%$
$\widehat{h}^R = 2.12$	$\widehat{h}_0^P = 0.532$	$\frac{\widehat{h}^R}{\widehat{h}_0^P} = 3.99$	$C_h = 54\%$
$TFP^R = 1.636$	$TFP_0^P = 1.0013$	$\frac{TFP^R}{TFP_0^P} = 1.634$	$C_{TFP} = 33\%$
$\widehat{Q}^R = 3.35$	$\widehat{Q}_0^P = 0.753$	$\frac{\widehat{Q}^R}{\widehat{Q}_0^P} = 4.46$	
$u^R = 0.1$	$u_0^P = 0.9$	$\frac{u^R}{u_0^P} = 0.11$	
$w_c^R = 0.34$	$w_0^P = 0.326$	$\frac{w_c^R}{w_0^P} = 1.035$	
$\frac{\overline{c}}{c^R} = 0.51$	$\frac{\overline{c}}{c_0^P} = 0.48$		

[22] The parameters are $\beta = 0$, $\alpha = 0.42$, $A = 1$, $\gamma = 0.116$, $\eta = \delta = 0.056$, $\sigma = 2$, $\rho = 0.02$, $\theta = 0.2$ and $\overline{c}_0 = 0.19968$.

Table 5.[23]

Comparison at BGP

Rich Economy	Poor Economy	Comparison	Accounting
$\widehat{k}^R = 2$	$\widehat{k}^P = 1$	$\frac{\widehat{k}^R}{\widehat{k}^P} = 2$	$C_k = 21.1\%$
$\widehat{h}^R = 0.28$	$\widehat{h}^P = 0.069$	$\frac{\widehat{h}^R}{\widehat{h}^P} = 4.1$	$C_h = 59.4\%$
$TFP^R = 2.22$	$TFP^P = 1.69$	$\frac{TFP^R}{TFP^P} = 1.31$	$C_{TFP} = 19.5\%$
$\widehat{Q}^R = 1.43$	$\widehat{Q}^P = 0.36$	$\frac{\widehat{Q}^R}{\widehat{Q}^P} = 3.97$	
$u^R = 0.0439$	$u^P = 0.09$	$\frac{u^R}{u^P} = 0.488$	
$w_c^R = 0.498$	$w_c^P = 0.989$	$\frac{w_c^R}{w_c^P} = 0.5$	
$\frac{\overline{c}}{c^R} = 0.497$	$\frac{\overline{c}}{c^P} = 0.995$		
$g^R = 2\%$	$g^P = 2\%$		
$\chi^R = 7.5$	$\chi^P = 0.15$		
$\frac{b_h}{b_k} = 0.0058$	$\frac{b_h}{b_k} = -0.0043$		

Comparison at the initial period

Rich Economy	Poor Economy	Comparison	
$\widehat{k}_0^R = 1.3$	$\widehat{k}_0^P = 0.945$	$\frac{\widehat{k}_0^R}{\widehat{k}_0^P} = 1.38$	$C_k = 16.2\%$
$\widehat{h}_0^R = 0.278$	$\widehat{h}_0^P = 0.09$	$\frac{\widehat{h}_0^R}{\widehat{h}_0^P} = 3.1$	$C_h = 78.4\%$
$TFP_0^R = 2.22$	$TFP_0^P = 2.127$	$\frac{TFP_0^R}{TFP_0^P} = 1.04$	$C_{TFP} = 5.4\%$
$\widehat{Q}_0^R = 1.18$	$\widehat{Q}_0^P = 0.515$	$\frac{\widehat{Q}_0^R}{\widehat{Q}_0^P} = 2.3$	
$u_0^R = 0.044$	$u_0^P = 0.049$	$\frac{u_0^R}{u_0^P} = 0.89$	

[23] The parameters are $\beta = 0$, $\alpha = 0.42$, $A = 1$, $\gamma = 0.022$, $\eta = \delta = 0$, $\sigma = 0.1$, $\rho = 0.02$, $\theta = 0.333$ and $\overline{c}_0 = 0.032217$.

Figures

Figure1. $\alpha > \beta, \bar{c}_0' > \bar{c}_0, u*' > u*$

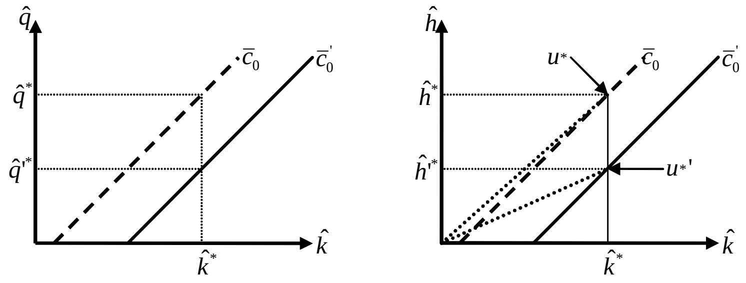
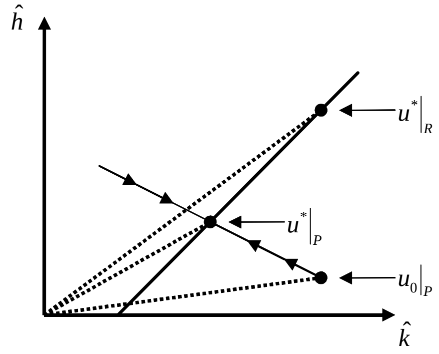


Figure 2. $u^*\big|_R < u^*\big|_P < u_0\big|_P$



$Figure 3. u_0|_R = u_0|_P < u^*|_R < u^*|_P$