Factor Substitution and Factor Augmenting Technical Progress in the US: A Normalized Supply-Side System Approach

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Abstract: Using a normalized CES function with factor-augmenting technical progress, we estimate a supply-side system of the US economy from 1953 to 1998. Avoiding potential estimation biases that have occurred in earlier studies and putting a high emphasis on the consistency of the data set, required by the estimated system, we obtain robust results not only for the aggregate elasticity of substitution but also for the parameters of labor and capital augmenting technical change. We find that the elasticity of substitution is significantly below unity and that the growth rates of technical progress show an asymmetrical pattern where the growth of labor-augmenting technical progress is exponential, while that of capital is hyperbolic or logarithmic.
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Non Technical Summary

A remarkable element of recent advances in economic growth theory is the revival of the aggregate Constant Elasticity of Substitution (CES) function for the modeling of an economy's productive potential. This revival aims at linking considerations about changes in factor income distribution to particular constellations of the elasticity of substitution on the one hand and the growth rates of labor and capital efficiency on the other. Our study presents an innovative approach for the estimation of such a function, applies it successfully to US data and derives results that have important implications for our understanding of factor substitution in the medium and long run.

The workhorse of growth theory has tended to be the Cobb-Douglas production function whose elasticity of substitution is exactly unity. One reason for the general interest in this particular functional form is its accordance with the most prominent of the empirical stylized facts of long-term economic development: the approximate constancy of factor income shares during a steady increase in capital intensity (i.e., the capital/labor ratio) and per-capita income. Allowing for a non-unity elasticity of factor substitution (i.e., CES technology) would imply that the secular constancy in factor income shares has to be provided by another very strong assumption: technical progress has to be purely labor augmenting. Empirical research, moreover, has been hampered by the difficulties in identifying at the same time an aggregate elasticity of substitution and growth rates of labor and capital augmenting technical change from the available data.

The major contributions of this paper to the theoretical and empirical challenges mentioned above are the following. First, we propose that empirical research on aggregate CES functions can be extended and much improved by applying the normalization procedure of De La Grandville (1989) and Klump and De La Grandville (2000) in a supply-side system. Normalization implies the fixing of baseline values for output, the input factors, the factor shares, and in a growing economy, the growth rates of technical progress. The benefit of the supply-side system approach, in turn, is that it treats the first-order conditions of a profit maximizing firm *as a system*, containing cross-equation parameter constraints, which may essentially alleviate the identification of structural parameters as e.g. the elasticity of substitution and technical progress parameters. Second, using a normalized CES function with factor-augmenting technical progress, we estimate a supply-side system of the US economy from 1953 to 1998. Unlike in most empirical works, we do not, however, constrain technical progress to evolve at a constant rate but allow for a quite general functional form (namely, the Box-Cox transformation). Putting a high emphasis on data consistency, we obtain robust results not only for the elasticity of substitution but also for the parameters of labor- and capital-augmenting technical change. We find that the elasticity of substitution is significantly below unity (between 0.5 and 0.7) and that the growth rates of technical progress show an asymmetrical pattern where the growth of labor-augmenting technical progress is almost exponential, while that of capital is hyperbolic or logarithmic.

1. Introduction

A remarkable element of recent advances in economic growth theory is the revival of the aggregate Constant Elasticity of Substitution (CES) function for the modeling of an economy's productive potential. This revival that is evident in theoretical as well as in empirical contributions aims at linking considerations about changes in factor income distribution to particular constellations of the elasticity of substitution on the one hand and the growth rates of labor and capital efficiency on the other. Against this challenging background our study presents an innovative approach for the estimation of such a function, applies it successfully to US data and derives results that have important implications for our understanding of factor substitution in the medium and long run.

Neo-classical growth theory and the aggregate CES production function have a long common history, starting with the Solow's (1956) seminal contribution. However, the workhorse of growth theory became the Cobb-Douglas production function whose elasticity of substitution is exactly unity. One reason for the general interest in this particular functional form is its accordance with the most prominent of the empirical stylized facts of long-term economic development: the approximate constancy of factor income shares during a steady increase in capital intensity (i.e., the capital/labor ratio) and per-capita income. Allowing for a non-unity elasticity of factor substitution (and hence assuming CES technology) would imply that the secular constancy in factor income shares has to be provided by another very strong assumption: technical progress has to be purely labor augmenting. The ongoing competition between both alternatives has only recently become obvious in two important papers. Seeing no evidence for a fading away of capital augmenting technical change, Jones (2003) defends the view that the long-term production function is Cobb-Douglas. Acemoglu (2003) who relates the general existence of biased technical change to a non-unitary elasticity of substitution gives support to the idea that technical progress should be purely labor augmenting in the long-run.

Empirical research, moreover, has been unable to settle this dispute so far. This relates to the difficulties in identifying at the same time an aggregate elasticity of substitution and growth rates of labor and capital augmenting technical change from the available data. For more than a quarter of a century following Berndt (1976), it had been almost common knowledge that estimations for the US economy strongly supported Cobb-Douglas. This view, however, has now been challenged on empirical and theoretical grounds. Chirinko (2002), summarizing the results of recent estimate of the elasticity of substitution that made use of different data sets and various methodological approaches, finds little support for the unitary value. Antras (2004) suggests that the finding of the Cobb-Douglas result in many older econometric investigations may be due to an omitted-variable bias caused by the assumption of Hicks neutral technical change.

A still relatively rarely used framework for the estimation of aggregate CES production functions is the supply-side system approach. Its origin goes back to Marchak and Andrews (1947); in the context of cross-section analysis, and in the context of time series analysis it was introduced by Bodkin and Klein (1976) As presented by Willman (2002) or McAdam and Willman (2004b), the benefit of this approach is that it treats the first-order conditions of a profit maximizing firm *as a system*, containing cross-equation parameter constraints, which may fundamentally alleviate the identification of structural parameters as e.g. the elasticity of substitution and technical progress parameters. Applications of the supply-side system approach to European data (Willman, 2002), however, could not find sufficient support for a non-unitary elasticity of substitution, an application of this framework to US data, the quality and availability of which is better than that of euro-area data, is, notably, still missing. The major contributions of this paper to the theoretical and empirical challenges mentioned above are the following. First, we propose that the identification of technical level and distribution parameters from each other in the aggregate CES function can be much improved by applying the normalization procedure of De La Grandville (1989), Klump and De La Grandville (2000) and Klump and Preissler (2000) in a supply-side system. Normalization implies the fixing of baseline values for output, the input factors, the factor shares, and in a growing economy, the growth rates of technical progress. Second, using a normalized CES function with factor-augmenting technical progress, we estimate a supply-side system of the US economy from 1953 to 1998. Unlike in most empirical works, we do not, however, constrain technical progress to evolve at a constant rate but allow for a quite general functional form. Putting a high emphasis on data consistency, we obtain robust results not only for the elasticity of substitution but also for the parameters of labor- and capital-augmenting technical change. We find that the elasticity of substitution is significantly below unity and that the growth rates of technical progress show an asymmetrical pattern where the growth of labor-augmenting technical progress is almost exponential, while that of capital is hyperbolic or logarithmic. Third, our results are therefore supportive of Acemoglu's view on biased technological change and defend a more extensive use of aggregate CES production functions in modern growth theory.

The rest of the paper proceeds as follows. Section 2 reviews the theoretical and empirical disputes surrounding aggregate CES production functions. Section 3 discusses in detail the potential estimation biases that can occur in econometric estimations of the aggregate elasticity of substitution. Section 4 explains the normalization procedure and section 5 develops the normalized supply-side system that is estimated. Section 6 discusses the properties of the US data and their congruence with neo-classical growth theory. Section 7 discusses the estimation results. Section 8 draws conclusions for the theoretical debate and for further empirical research.

2. The revival of the CES production function

The continuous boom in endogenous growth theory since the mid-1980's led to a renewed interest in CES production functions (e.g., see the discussion in La Grandville and Solow, 2004). This special type of production functions rooted in the mathematical theory of elementary mean values (Hardy *et al.*, 1934, p. 13 ff.) was introduced into economics by Solow (1956) and has already been much debated in the early times of neoclassical growth theory during the 1960's and 1970's. [1] However, not only conceptual problems causing controversial and problematic results in theoretical growth models (Klump and Preissler 2000) but also ongoing difficulties in empirically testing the parameters of the CES function, notably the aggregate elasticity of substitution (s), made their use less attractive. If the pioneering work by Arrow *et al.*, (1961) stands for the hopeful beginning of empirical studies on the aggregate elasticity of substitution, the results derived by Berndt (1976) set perhaps a disappointing end to this debate. While in the former paper, estimate of the elasticity of substitution for the US was at a value of 0.57, the latter paper tended to prove that the elasticity of substitution in the US did not significantly deviate from one in the long run. Hence, the much less complicated Cobb-Douglas specification of the production function could be used for the modeling of the technological relationship between factor inputs and output.

[1] Though there are many plausible function forms, which may fit the data well – e.g., the *translog* function – here we concentrate on Cobb-Douglas and CES cases. This reflects the dominance of these two forms in the growth literature (e.g., Barro and Sala-i-Martin, 2003) and allows us to focus our discussion on key issues in the literature like the unitary or non-unitary value of the substitution elasticity and the nature of factor augmenting technical change.

One important property of the Cobb-Douglas function is the constancy of factor income shares. This property meets the essential condition for a steady state in neoclassical growth models and is in line with the most prominent of the empirical stylized facts of long-term economic development: the relative stability of factor income distribution despite a secular rise in capital intensity and per-capita income. It also follows immediately that the direction of technical change is irrelevant for income distribution in the Cobb-Douglas world. It is thus impossible to determine empirically any bias in the direction of technical change. In contrast, pronounced cycles in factor income distribution visible in many countries over what Blanchard (1997) called the "medium run" support the more general CES function and make possible biases of technical progress an important issue. It is an old insight that in the CES world a steady state with factor income shares is only possible, if exogenous technical progress is purely labor augmenting. Acemoglu (2002, 2003) was able to derive this same result in a model with endogenous innovative activities. He also demonstrates that over quite significant periods of transition growth of capital-augmenting progress can be expected resulting from endogenous changes in the direction of innovations.

Acemoglu's view that long-run production possibilities should be characterized by a CES production function with purely labor-augmenting technical change (and an elasticity of substitution below unity to avoid problems of stability) has recently been challenged by Jones (2003). He suggests that the long-run production function must be Cobb-Douglas under the crucial assumption that the underlying parameters of the production techniques, that basically stand for the emergence of new ideas, obey Pareto distributions. Short-run growth could then be modeled with the help of a CES production function (with an elasticity of substitution below unity) that is somehow nested in a Cobb-Douglas function for long-run growth. In a theoretical perspective the main difference between the two competing approaches is Acemoglu's "innovation possibilities frontier" on the one side, that describes the technological possibilities for transforming resources into blueprints for labor- and capital-augmenting innovations, and Jones' Pareto-distribution for the emergence of new ideas. From an empirical perspective, the difference can be seen in different results concerning not only the long run elasticity of substitution and the long run dominance of biased technological change. While Jones would exclude any technological bias in the long-run Cobb-Douglas world, Acemoglu underlined the coexistence of labor- and capital augmenting technical change, but with asymmetric long-term properties. To our knowledge, this asymmetry in the dynamics of factor-augmenting technical change has so far not been subject to any empirical testing.

Not only in the theory of endogenous growth, but also in other areas of dynamic macroeconomics, the concept of CES production functions could experience a revival. As it was already demonstrated by Solow (1956) in the standard neoclassical growth model, assuming an aggregate CES production function with an elasticity of substitution above unity is the easiest way to generate perpetual growth. Since scarce labor can be completely substituted by capital, the marginal product of capital remains bounded away from zero in the long run. Recently, it has been shown that integration into world markets is a feasible way for a country to increase the effective substitution between factors of production and pave the way for continuous growth (Ventura 1997, Klump 2001). On the other hand, it could be derived in several standard neoclassical growth models with aggregate CES production functions that with an elasticity of substitution below unity multiple growth equilibria and development traps can become possible (Azariadis 1996, Duffy and Papageorgiou 2000, Klump 2002, Kaas and von Thadden 2003).

Public finance and labor economics are other fields where the elasticity of substitution has been rediscovered as a crucial parameter for understanding the effects of policy changes. This has to do with the importance of factor substitution possibilities for the demand function of each input factor. As pointed out by Chirinko (2002), the lower the elasticity of substitution the smaller becomes the response of business capital formation to variations in interest rates that are caused by monetary or tax policy. In addition, the welfare effects of tax policy changes are highly sensitive to the assumed values of the elasticity of substitution. Rowthorn (1999) shows that with an elasticity of substitution below unity, fiscal incentives for investment become more effective for the creation of new jobs, whereas in a Cobb-Douglas world only wage policy affects employment. He concludes that with many estimates of σ lying below one, "capital investment does create employment even when benefits are upgraded in line with wages, whilst growth in the labor supply and technical progress with a labor-saving bias will cause a permanent rise in unemployment unless they are offset by additional investment. The policy implication is that measures to stimulate investment may have an important role to play in reducing unemployment." (Rowthorn, 1999, p. 414)

A whole series of papers have tried to explain the coincidence of rising unemployment and a hump-shaped behavior of factor income share in continental Europe with the help of models that incorporate particular assumptions about factor substitution and technological change. Caballero and Hammour (1998), Blanchard (1997) and Berthold *et al.*, (2002) assume a production technology with purely labor-augmenting technical progress and a relatively high elasticity of substitution with values of above unity in the long-run, while in the short run the possibilities of factor substitution are rather limited due to putty-clay characteristics of the ex-post production function. A wage-push shock would thus lead at first to only a small decline in employment and an increase in the labor-income share. In the long run, however, labor is replaced over-proportionally by capital and the labor-share will fall again. Critics of this line of explanation have argued that Europe has also experienced a decline in capital formation since the 1970's. A declining capital intensity, however, can cause a decline in employment and a rise in the capital income share only if the elasticity of substitution does not exceed unity (Rowthorn, 1999).

An intermediate position between the two views has been developed by Acemoglu (2002, 2003) who introduces changes in the direction of factor-augmenting technical progress as an important, and so far neglected, endogenous adjustment mechanism. In his model, technical progress is strictly labor augmenting along the long-term balanced growth path, but it also becomes capital-biased in periods of transition. The short-term response to a wage-push is now a short-term fall in employment and an increase in the labor-income share. In the long run, however, capital-biased technical change will reverse the trend in income distribution and lead to an increase of the capital income share, while employment falls even further. Capital-augmenting technical change has thus an important role to play in the medium run, even if it should not be dominant in the very long run.

3. Biases in earlier empirical studies

Tables 1 and 2 present an overview of the results that previous empirical investigation obtained for the elasticity of substitution. We concentrate on the results from time-series or panel studies on aggregate data. In the case of the US, which has been widely studied, it is possible to find values of the elasticity of substitution above unity (with Harrod-neutral technical progress), at unity (with Hicks-neutral progress) and below unity (with Hicks-neutral progress and with technical progress augmenting both factors). The situation for other countries is little better; for Germany, values of σ above, below and at unity have been estimated.

Using information about the degree of factor substitution from other sources does not resolve this puzzle, either. It has been recognized, for example, by Lucas (1967) that older time-series studies for the US have generally provided lower σ estimates than cross-section studies that are rather supportive of the Cobb-Douglas function. More recent cross section analysis based on micro data that where used to estimate the relationship between business capital formation and user costs (e.g., Chirinko *et al*., 1999) estimate very low elasticities of substitution ranging from 0.25-0.40. A drawback of these kinds of studies, however, is their inability to quantify any growth rate of technical progress.

Looking at **Tables 1 and 2**, there seem to be several reasons for systematic estimation biases. They are related not only to differences in data construction and measurement, but also to different à priori assumptions about the nature of technological change. Problems with data do not only concern the use of quality-adjusted measures for capital and labor inputs. In particular, they refer to the correct measurement of the user cost of capital. The papers by Berndt (1976) and Antras (2004) put a high emphasis on the selection of high quality, consistent data, but there are still weaknesses such as the non-regard of capital depreciation, of a possible mark-up, the treatment of indirect taxes and assumptions about self-employed labor income.

On the conceptual side there is also the problem how exactly the parameters of the CES functions are to be estimated. Single equation, two equations and three equations system approaches are competing. Single equation estimates usually concentrate either on the production function itself or on the first-order condition of profit maximization with respect to labor or capital. The estimation of the single production function, however, can only be accomplished with quite restrictive assumptions about the nature of technological progress. Those implications will be further discussed below. Furthermore, the elasticity of substitution estimated from the first-order condition with respect to labor seems to be systematically higher than that that derived from the first-order condition with respect to capital. Finally, it has been pointed out already by David and van de Klundert (1965, p. 369)**,** that single equation estimates (based on factor demand functions) are systematically biased, since factor inputs depend on relative factor prices that again depend on relative factor inputs.

Two-equation systems that estimate demand functions for both input factors as in Berthold *et al.*, (2002) should alleviate such a systematic simultaneous equation bias. However, since the two equation systems usually do not estimate explicitly a production function, the nature of technological progress is usually restricted by debatable à priori assumptions. Also, the estimated two equation systems have so far been unable to capture the existence of market imperfections that could be captured by the values of a possible mark-up on marginal production costs. This important empirical issue can only be adequately treated in three-equation system approaches.) The benefit of this approach is that it treats the first-order conditions of a profit maximizing firm *as a system*, containing cross-equation parameter constraints, which may fundamentally alleviate the identification of structural parameters as e.g. the elasticity of substitution and technical progress parameters. The estimation of the whole supply-side system not only contains demand functions for all factors of production but also an explicit aggregate CES production function. Applications of this framework to data from the Euro area by Willman (2002) and to German data by McAdam and Willman (2004a) demonstrated, however, that the high instability of most non-unitary elasticity of substitution estimates made it very difficult to reject the Cobb-Douglas hypothesis.

Furthermore, the significance of restrictions imposed on the estimation with regard to the direction of technical change is an issue. Antras (2004), for instance, showed that the assumption of Hicks neutrality of technical progress, so popular in studies of the elasticity of substitution, together with the observed development of factor income shares could lead to significant omitted-variable biases. As Antras demonstrated, with the à priori assumption of a common growth rate for labor and capital augmenting technical change, a relatively stable relation of factor share and a rising capital intensity in the long run, it is a logical conclusion that Berndt (1976) found a Cobb-Douglas function with an elasticity of substitution equal to one that should fit best for the US. Finally, it has to be noted that all earlier studies of the CES function imposed constant growth rates of factor efficiency. A notable exception can only be found in Ripatti and Vilmunen (2001) who besides assuming a constant labor augmenting technical change specify capital augmenting technical progress to follow a logistic pattern. However, the latter choice is not based on free estimation but it is chosen on à priori basis.

4. Normalizing a CES function with biased technological change

The idea of normalizing CES functions was explicitly developed by de La Grandville (1989) and further explored by Klump and De La Grandville (2000) and Klump and Preissler (2000). It starts from the observation that a family of CES functions whose members are distinguished only by different elasticities of substitution needs a common fix point. Since the elasticity of substitution is defined as a point elasticity, one needs to fix baseline values for per capita production, capital intensity and factor income shares (or the marginal rate of substitution). If technical progress is biased in the sense that factor income shares change over time the nature of this bias can only be classified with regard to the baseline values at the given fix point. This important observation has already been pointed out by Kamien and Schwartz (1967) for the special case that the capital intensity at the given fix point, is equal to one. A principle contribution of this paper, therefore, lies with merging the normalization method with the empirical system approach. Furthermore, we model technical progress with a very flexible functional form which allows the data to discriminate between the different forms of technical progress.

Since the focus of our analysis is on identifying possible biases in technical change, we concentrate on the following specification of the CES production function specification that was introduced by David and van de Klundert (1965, p. 75 ff.). This is a linear homogeneous CES production function with technological change that is augmenting the efficiency of both factors of production and can be written as:

$$Y_t = [(E_t^N * N_t)^{-r} + (E_t^K * K_t)^{-r}]^{-\frac{1}{r}} \tag{1}$$

In this notation, N_t and K_t represent conventional measures of the physical flow of labor and capital inputs. The coefficients E_t^N, E_t^K represent the levels of efficiency of both input factors and $r = \frac{1-s}{s}$ is the substitution parameter (with s the elasticity of substitution).

The relationship between the CES production function (1) and the traditional Arrow *et al.*, (1961) form which, instead of the two efficiency levels contains a distribution and a single efficiency parameter, has been explored by Klump and Preissler (2000, p. 43 f.). Both specifications can be regarded as two members of one family of normalized CES productions functions as long as they share the same baseline values of capital (K_0) and labor input (N_0), output (Y_0) and the marginal rate of substitution ($m_0 = \frac{\partial Y_0 / \partial N_0}{\partial Y_0 / \partial K_0}$), respectively. This implies automatically, that under imperfect competition, two members of one family also share the same baseline values for the distribution parameter $p_0 = \frac{q_0 K_0}{w_0 N_0 + q0 K_0}$, where w and q refer to the wage rate and the rental price of capital, respectively.[2]

Whereas the Arrow *et al.* (1961) specification seems to imply that technological change is always Hicks-neutral, the specification (1) allows for different growth rates of factor efficiency. In order to circumvent problems related to the non-identification theorem by Diamond *et al.*, (1978), we assume a certain functional form for the growth rates of both efficiency levels and define:

$$E_t^N = E_{t_0}^N e^{g_N(t)} \qquad ; \; g_N(t = t_0) = 0 \tag{2}$$

$$E_t^K = E_{t_0}^K e^{g_K(t)} \qquad ; \; g_K(t = t_0) = 0 \tag{3}$$

$g_N(t)$ and $g_K(t)$ define the growth rates of labor-augmenting and capital-augmenting technical progress, respectively. Following the recent theoretical discussion about possible biases in technical progress, it is not clear that these growth rates should always be constant; patterns of logarithmic or hyperbolic growth seem plausible. This is why in our empirical investigations we will work with the hypothesis of constant growth rates ($\frac{\partial g_N(t)}{\partial t} = g_N$) and ($\frac{\partial g_K(t)}{\partial t} = g_K$) as well as with a transformation that gives more flexibility on the actual functional form and nests exponential, logarithmic and hyperbolic growth patterns as special cases. The Box-Cox (1964) transformation leads to the expressions $g_N(t) = \frac{g_N}{l_N}\left(t^{l_N} - 1\right)$, $g_K(t) = \frac{g_K}{l_K}\left(t^{l_K} - 1\right)$ with $t > 0$. While l_i equals unity (zero) {less than zero}, technical progress functions, g_i are linear (long-linear) {hyperbolic} functions in time.

$E_{t_0}^N$ and $E_{t_0}^K$ are the baseline values of the two efficiency levels, taken at the common baseline time $t = t_0$. Again, normalization of the CES function implies that members of the same CES family should all share the same baseline values and should in this point and at that time of reference only be characterized by different elasticities of substitution. In order to assure that this property holds also in the presence of growing factor efficiencies it follows that (see appendix One for an extended explanation of the normalization procedure):

$$E_{t_0}^N = \frac{Y_0}{N_0} * \left(\frac{1}{1 - p_0}\right)^{\frac{1}{r}} \tag{4}$$

$$E_{t_0}^K = \frac{Y_0}{K_0} * \left(\frac{1}{p_0}\right)^{\frac{1}{r}} \tag{5}$$

[2] Under perfect competition distribution parameter is equal to the capital income share but, under imperfect competition with zero markup, it equals the share of capital income over total factor income.

$$e^{\;g_N(t_0)} = e^{\;g_K(t_0)} = 1 \tag{6}$$

The last expression assures that in the common point of reference the factor shares are not biased by the growth of factor efficiencies but are just equal to the distribution parameters p_0 and $1-p_0$ (see also Appendix 1).

Inserting the assumptions (2) and (3) and the normalized values (4), (5) and (6) into function (1) leads to a normalized CES function that can be rewritten in the following form that resembles again the Arrow *et al.*, (1961) variant:

$$\begin{aligned} Y_t &= \left[\left(E_t^N \cdot N_t\right)^{-r} + \left(E_t^K \cdot K_t\right)^{-r} \right]^{-\frac{1}{r}} \\ &= \left\{ (1-p_0)\left[\frac{Y_0}{N_0} \cdot e^{g_N(t,t_0)} \cdot N_t\right]^{-r} + p_0\left[\frac{Y_0}{K_0} \cdot e^{g_K(t,t_0)} \cdot K_t\right]^{-r} \right\}^{-\frac{1}{r}} \\ &= Y_0\left\{ (1-p_0)N_0^r\left[N_t \cdot e^{g_N(t,t_0)}\right]^{-r} + p_0 K_0^r\left[K_t \cdot e^{g_K(t,t_0)}\right]^{-r} \right\}^{-\frac{1}{r}} \end{aligned} \tag{7}$$

In this specification of the normalized CES function, with factor augmenting technical progress the growth of efficiency levels is now measured by the expressions $N_0\, e^{\,g_N\,(t-t_0)}$ and $K_0\, e^{\,g_K\,(t-t_0)}$, respectively. As a test of consistent normalization, we see from (7) that for $t = t_0$ we retrieve $Y = Y_0$. This also holds for the Box-Cox transformation of the efficiency growth variables which can be written as $g_N(t,t_0) = \frac{g_N}{l_N} t_0\left(\left[\frac{t}{t_0}\right]^{l_N} - 1\right)$ and $g_K(t,t_0) = \frac{g_K}{l_K} t_0\left(\left[\frac{t}{t_0}\right]^{l_K} - 1\right)$.[3]

Earlier theoretical and empirical work on CES functions used to assume exponential growth of both efficiency levels so that rates are given by $\frac{\partial g_N(t)}{\partial t} = g_N$, $\frac{\partial g_K(t)}{\partial t} = g_K$. Special cases of (7) are the specifications used by Rowthorn (1999), Bentolila and Saint-Paul (2003) or Acemoglu (2002, 2003), where $N_0 = K_0 = Y_0 = 1$ is implicitly assumed, or by Antras (2004) who sets $N_0 = K_0 = 1$. Caballero and Hammour (1997), Blanchard (1997) and Berthold *et al.*, (2002) work with a version of (7) where in addition to $N_0 = K_0 = 1$, $\frac{\partial g_K(t)}{\partial t} = g_K = 0$ is also assumed so that technological change is only of the labor-augmenting variety. It is also worth noting that for constant efficiency levels $g_N(t) = g_K(t) = 0$ our normalized function (7) is formally identical with the CES function that Jones (2003, p. 12) has proposed for the characterization of the "short term". In his terminology, the baseline parameters N_0, K_0 and Y_0 are "appropriate" values of the fundamental production technology that determines long run dynamics. This long-run production function is then considered to be of a Cobb-Douglas form with constant factor shares equal to

[3] Note we scaled (divided) the original g_i and time t by the fixpoint value t_0. This rescaling allows us to interpret g_N and g_K directly as the rates of labour and capital augmenting technical change at the fixpoint period t_0.

p_0 and $1-p_0$ and with a constant exogenous growth rate. Actual behavior of output and factor input is thus modeled as permanent fluctuations around "appropriate" long-term values.

For empirical estimations of the normalized CES production function (7), it becomes an interesting question how the baseline values should be determined. One should be aware that the choice of the baseline values fixes also a reference level for factor income shares that is considered as "normal" – or "appropriate" in the sense of Jones (2003) – and is then used for the measurement of biased technological change. We think that this "normal" level of factor shares can only be detected from the data and should make use of as much information as possible. This is why we suggest that baseline values should be calculated on the basis of sample geometric averages, because over a longer period of time cyclical variations have netted out and even longer-term fluctuations have compensated.

The choice of sample geometric average values for a practical implementation of normalization can imply a problem of scaling, however, since the geometric average of each time series is calculated independently. Hence, fix points calculated as the geometric averages of inputs correspond to the geometric average of output only if the production function is log-linear i.e. the Cobb-Douglas case. Therefore, we capture and measure the possible emergence of a scaling problem by introducing and estimating an additional parameter A. Its role is to capture the effects of the deviation of the CES from the log linear function on the fix point output corresponding to the geometric averages of inputs.

With treating sample averages as baseline values at the common point (and time) of reference and introducing an additional scaling parameter A so that $Y_0 = A \cdot \bar{Y}$, $K_0 = \bar{K}$, $N_0 = \bar{N}$, $p_0 = \bar{p}$ and $t_0 = \bar{t}$. The scaling parameter A deviates from unity, when the estimated CES production function deviates from the log liner Cobb-Douglas function. Under perfect competition, the distribution parameter could be calculated directly, pre-recursively, from the data but, when associated with unobservable markup, it can be estimated jointly with the other parameters of the model. Hence, we arrive at the final econometric specification of our normalized CES function with factor augmenting technological change. Per-capita output can be written in logarithmic form as:

$$\log\left(\frac{Y_t}{N_t}\right) = -\frac{1}{r}\cdot\log\left[\bar{p}e^{r[g_N(t,t_0)-g_K(t,t_0)]}\left(\frac{K_t}{N_t}\frac{\bar{N}}{\bar{K}}\right)^{-r} + \left(1-\bar{p}\right)\right] + \log\left(\frac{\bar{Y}}{\bar{N}}\right) + g_N(t,t_0) + \log A \qquad (8)$$

From the point of view of estimation, the advantage of normalized equation (8) over the un-normalized case, is that all parameter have clear economic interpretations with well-defined, plausible ranges. For comparison, we can re-write an un-normalized counterpart of (8):

$$Y_t = B\left\{b\left[e^{g_K(t,t_0)}K_t\right]^{-r} + (1-b)\left[e^{g_N(t,t_0)}N_t\right]^{-r}\right\}^{-\frac{1}{r}} \qquad (8')$$

where $b = \dfrac{\bar{p}\bar{K}^r}{\bar{p}\bar{K}^r + (1-\bar{p})\bar{N}^r}$ and $B = A\bar{Y}\left[b\bar{K}^r + (1-b)\bar{N}^r\right]^{-\frac{1}{r}}$.

As discussed in Klump and Preissler (2000), an important feature of the above un-normalized formulation is that the parameters B and b have no clear theoretic interpretation. They are composite parameters conditional on, besides the selected fixed points, the elasticity of substitution.

In the following, we use expression (8) for the estimation of an aggregate supply-side system as in Willman (2002) or in McAdam and Willman (2004b). Hence, the main merit in using it, instead of the un-normalized form, is that all parameters have clear empirical interpretation.

5. The normalized supply-side system

Firms are assumed to maximize their profits in imperfectly competing markets under the production function constraint (8). The first-order maximization conditions can be presented by the following three-equation system (which incorporates the generalized Box-Cox technical progress terms),

$$\log\left(\frac{w_t N_t}{p_t Y_t}\right)=\log\left(\frac{1-\bar{p}}{1+m}\right)+\frac{1-s}{s}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right)-\log A-\frac{\bar{t}g_N}{l_N}\left(\left(\frac{t}{\bar{t}}\right)^{l_N}-1\right)\right] \tag{9}$$

$$\log\left(\frac{q_t K_t}{p_t Y_t}\right)=\log\left(\frac{\bar{p}}{1+m}\right)+\frac{1-s}{s}\left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right)-\log A-\frac{\bar{t}g_K}{l_K}\left(\left(\frac{t}{\bar{t}}\right)^{l_K}-1\right)\right] \tag{10}$$

$$\log\left(\frac{Y_t}{N_t}\right)=\log\left(\frac{A\cdot\bar{Y}}{\bar{N}}\right)+\frac{\bar{t}g_N}{l_N}\left(\left(\frac{t}{\bar{t}}\right)^{l_N}-1\right)-\frac{1-s}{s}\log\left[\bar{p}e^{\frac{1-s}{s}\left[\frac{\bar{t}g_N}{l_N}\left(\left(\frac{t}{\bar{t}}\right)^{l_N}-1\right)-\frac{\bar{t}g_K}{l_K}\left(\left(\frac{t}{\bar{t}}\right)^{l_K}-1\right)\right]}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{1-s}{s}}+(1-\bar{p})\right] \tag{11}$$

where the substitution parameter r is presented in terms of the elasticity of substitution s and where parameter $m \geq 0$ measures the size of the markup determined by the price elasticity of demand for goods. Equations (9) and (10) are the first-order condition of profit maximization with respect to labor and capital, respectively, and (11) is the production function.

Equations (9) and (10) are normalized for the respective factor-income shares. This normalization demonstrates problems coupled with the identification of the Cobb-Douglas production function and the CES production function from each other, when the two-stage approach is used in estimating the parameters of the underlying production function. In most of the studies referred to in **Tables 1 and 2**, the elasticity of substitution has been estimated as a first stage result, i.e. the estimates are based on single equation estimation of equation (9) or equation (10) or on the difference of (9) and (10).

To demonstrate the identification problem, let us assume that the available data is compatible with the implications of standard neo-classical growth model, i.e. that factor income shares – the lhs variables of equations (9) and (10) – as well as the capital-output ratio are stationary. If technical progress is not purely labor-augmenting, i.e. $g_K(t,\bar{t})\neq 0$ and is non-stationary, then the square bracket term on the right-hand side of equation (10) must be non-stationary since, by assumption, the capital-output ratio was found to be stationary. Now by necessity, the elasticity of substitution, s must be unity (implying Cobb-Douglas). However, as technical progress is unobservable, data compatibility with neo-classical growth model and the Cobb-Douglas production function, implies that factor augmentation is not estimable from the system (9) and (10). Another alternative is to take labor-augmentation as the maintained hypothesis, i.e. $g_K(t,\bar{t})=0$. Now the data compatibility with neo-classical growth model implies that, besides the left hand terms, the right hand terms in square brackets are also stationary. This, in turn, implies that independently from the magnitude of s the estimation of equations (9) and (10) gives stationary residuals, which is a necessary condition for data compatibility. Accordingly, the sign and size of the deviation of s from unity depends on the correlation between stationary variables. If the squared bracket terms in (9) and (10) are un-correlated with the left-hand terms, we end up with unit elasticity of substitution. The higher positive (negative) correlation is found, the more below (above) unity the estimate of the elasticity of substitution is. However, in the real world with frictions, one can question whether these current-period correlations measure only the elasticity of substitution with no effects from the speeds of adjustments of factor and output. [4]

The identification of the augmentation of technical change and the elasticity of substitution becomes easier, if the production function (11) is estimated jointly with the equations (9) and (10). In this respect, the non-linearity of the CES function alleviates the identification. The role of non-linearity can be illustrated by applying the Kmenta-approximation (Kmenta, 1967) around the fix-points $N_t=\bar{N}$, $K_t=\bar{K}$ and $t=\bar{t}$ to separate the total factor productivity (TFP) term from the rest of the production function. We obtain:

$$\log\left(\frac{Y_t}{N_t}\right)=\overbrace{\bar{p}\,g_K(t,\bar{t})+(1-\bar{p})g_N(t,\bar{t})-\left(\frac{1-s}{s}\right)\frac{\bar{p}(1-\bar{p})}{2}[g_N(t,\bar{t})-g_K(t,\bar{t})]^2}^{TFP}$$
$$+\log\left(\frac{A\cdot\bar{Y}}{\bar{N}}\right)-\frac{s}{1-s}\log\left[\bar{p}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{1-s}{s}}+(1-\bar{p})\right] \tag{12}$$

A useful feature of approximation (12) is that it separates the output contribution of the total factor productivity from the output contribution of inputs. Equation (12) shows clearly that, when the elasticity of substitution $s\neq 1$, the factor augmentation introduces additional curvature into the estimated production function i.e., the squared bracket term in power two. When estimating equations (9)-(11) [or (9)-(10) and (12)] as a system, the stationarity of the estimation residual of equation (11) [or (12)] also requires the inclusion of this curvature term ($s\neq 1$), if the true production function is CES with labor-augmenting technical change. If the curvature term is not needed, the observed data compatibility with the neo-classical growth model implies that the underlying production function is Cobb-Douglas. One reservation to the above argument can be made. An estimated system can account for non-linear curvature effect also with close to unity values of the elasticity of substitution, if the difference $g_N(t,\bar{t})-g_K(t,\bar{t})$, in power two in (12), is sufficiently high. However, in that case one would expect that the estimates for factor augmenting technical change components $g_N(t,\bar{t})$ and $g_K(t,\bar{t})$ are unreasonable in economic sense and, therefore, these results can be rejected.

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[4] Apparently, identification becomes easier, if factor income shares as well as the capital-output ratio are non-stationary, because in that case the estimation of the elasticity of substitution depend on the co-integration I(1) variables. On the other hand, then the time-series properties of the data indicate inconsistency with the requirements of the balanced growth path.

6. Data

Our principle data source for the US (annual) series was the NIPA Tables (National Income and Product Accounts) for production and income, whose series may be found at http://www.bea.doc.gov/bea/dn/nipaweb/index.asp, Ho and Jorgenson (1999) for labor input, Herman (2000) for current cost and real capital stock and Auerbach (1983, 2003) for the data of the rental price of capital.[5] Our data series runs from 1953 until 1998: the data span is explained by the availability of Auerbach's user cost series. In estimation, we use the following series and data transformations.

The output series is calculated as Private non-residential Sector Output – this is total output minus Indirect Tax Revenues, Public-Sector output and Housing-Sector Output. After these adjustments the output concept we use, corresponds to the concept of the private non-residential capital stock.

In addition, three alternative measures are used for labor input, i.e. total private sector employment, corresponding number of hours worked and the constant quality index of labor input taken from Ho and Jorgenson (1999). They argue that total number of hours is not appropriate measure of the flow of labor services because it ignores significant differences in the quality of the labor services provided by different workers. A constant quality index of labor input, which they have constructed, captures substitution among different types of labor inputs by weighting the hours of differentiated labor groups by their marginal products. They argue that this helps identify correctly the contribution of productivity to output growth

As discussed by, for example, Krueger (1999) and Gollin (2002), a problem in calculating labor-income is that it is unclear how the income of proprietors (self-employed) should be categorized in the labor-capital dichotomy. Some of the income earned by self-employed workers clearly represents labor income, while some represents a return on investment or economic profit. In this study, two alternative approaches to account for also self-employed workers' labor income are applied. First, following e.g. Krueger (1999) and Antras (2004) we add two-thirds of proprietors' income to the private sector compensation to employees. Although blunt, since Johnson (1954), this has been a common convention to account for self-employed labor income. Second, a straightforward approach – although apparently better founded in economic terms – is to use compensation per employee as a shadow price of labor of self-employed workers. Hence, labor-income is calculated also as:

$$\left(1+\frac{\text{Self Employed}}{\text{Total Private Employment}}\right)\cdot \text{Compensation to Private Sector Employees} \qquad (13)$$

Recently the latter approach has been applied e.g. by Blanchard (1997), Gollin (2002) and Bentolila and Saint-Paul (2003).

The construction of the capital income data is most problematic. This is due to the fact that the pure profit (or the markup) component cannot be separated from the rest of non-labor income in national accounting. However, as part of constructing the national income and product accounts, the Bureau of Economic Analysis (BEA) calculates also estimates of fixed assets and consumer goods including estimates of net capital stocks in real and nominal terms (Herman, 2000). This information is needed in calculating national account figures for consumption. Accordingly, a consistent estimate for the (non-profit) capital income should be obtained as the

[5] We are grateful to Alan Auerbach for providing us with his data on the rental price of capital.

product of real rental price of capital, as e.g. constructed by Auerbach (1983, 2003), and BEA figures of current-cost fixed capital. We chose this practice.

Before estimation, it is useful, first, to check the internal consistency of our data set and, second, to evaluate how compatible it is with the implications of the standard neo-classical growth model.

Conventionally, we believe in empirical applications little (or too little) attention has been paid to the internal consistency of the data, especially regarding the distribution of the non-labor income into the capital income and the implied markup components. For that purpose, the accounting identity of the non-housing private sector provides a useful framework:

$$\Pr ofit = pY - (wN + qK) \tag{14}$$

Internal consistency would require that the sample average of the implied markup component is non-negative. Further, if demand functions of goods are isoelastic and either competition in different sectors is the same or sectoral output shares remain stable, then the implied markup component should be stationary.[6] These requirements are fulfilled by our data. As shown in the lowest panel of **Figure 1**, the markup share, although exhibiting temporarily negative values, is for the most of the sample period positive. In addition, it is also stationary as the augmented Dickey-Fuller test statistic shows ADF (1)= -3.65, where the number inside brackets refer to the number of lags.

How compatible is our data with the balanced growth path implied by the standard neo-classical growth model? Are factor-income ratios stationary, as the theory would imply? Based on the "eyeball" econometrics the three panels of **Figure 1** show that these stationarity requirements are not, at least, strongly violated. The upper panel of **Figure 1** shows that, over the sample, the trend growth rates of production and the capital stock are around the same whilst the growth of employment has been slower. A closer statistical examination, however, shows that the capital-output ratio, with ADF (1) = -1.80, is non stationary. Interestingly, two of our three alternative measures for labor input have around the same longer-run trend, i.e. total private sector employment and the constant quality index of the labor input. This implies that improvements in the quality of labor have roughly compensated for the effects of shortened average weekly working hours on labor input and that both measures of labor input imply around the same average contribution from labor-augmented technical change. The development of factor income shares seems to be quite well in line with the implications of the neo-classical growth model (the middle and lower panel of **Figure 1**). The ADF (0) = -3.61 for the capital income share implies stationarity. Our two ways to account for self-employed workers' labor income results in quite similar developments for the labor-income share. In both cases, the rejection of the null hypothesis of unit root is just in the border of 5 per cent significance level. ADF (0) = -2.90, when two-thirds of proprietors' income is included by the labor income, and ADF (2) = -3.00, when compensation per employee is used as a shadow price of labor of self-employed workers.

We can conclude, therefore, that although the deviations are not dramatic, the properties of our data do not fully coincide with the implications of the standard neo-classical growth model. The most prominent deviation is the non-stationarity of the capital-output ratio although factor-income shares are, at least, borderline stationary. Do these deviations have any implications concerning the underlying production technology and the nature of technical progress? The first implication is that technical progress cannot be only labor augmenting. There

[6] See the discussion in Willman (2002) or McAdam and Willman (2004a, 2004b)

must be also a capital augmenting component in technical progress. However, as in our sample, the capital-output ratio has no clear trend, in line with the endogenous growth model of Acemoglu (2003), capital augmenting technical progress may be a transitory phenomenon. The second implication is that the Cobb-Douglas production technology is not fully compatible with these properties of the data. Under the Cobb-Douglas technology, independently from the augmentation of the technical change, the capital-output share as well as factor income shares should be stationary.

These anticipations are confirmed by the estimation results, which we present in the following section.

7. Results

Our main results are presented in **Tables 3** to **6,** referring, as before, to the estimation of the system (9)-(11).

Alongside the numerical results are the associated **Graphs 1.1 to 4.4** corresponding to each case in the tables. The tables show the parameter estimates and their standard errors. As can be seen, most parameters are significant at the 1% level. Also shown are the changes in technical progress (evaluated at the fixed point), total Technical Factor Progress (calculated as defined in equation (12)), as well as the Log Likelihood of each specification and the Augmented Dickey-Fuller tests for stationarity for the residuals of the Labor, Capital and Output equations. **Tables 3 and 4** present estimation results when labor input is measured in employed persons, **Table 5** when labor input is measured in hours, whereas in **Table 6** we use the quality-adjusted series of Ho and Jorgenson (1999). As already discussed (Section 6) a problem in calculating labor-income is that it is unclear how the income of proprietors (self-employed) should be categorized in the labor-capital dichotomy. First, we add two-thirds of proprietors' income to the private sector compensation to employees. Second, we use compensation per employee as a shadow price of labor of self-employed workers. In **Table 3**, we use the former approach and the latter in **Tables 4 to 6**. Results, however, are not overly sensitive to which definition is used.

Furthermore, the graphs display the residuals from each of the equations, technical progress and its growth contributions. The box on the bottom right hand side of each graph shows the sensitivity of the Log-Likelihood with respect to the initial condition for the key s parameter; given, that we estimate a non-linear system, the issue of identifying global optima is a primary concern.[7]

In the first two columns of each table, constant factor augmenting technical progress is assumed (i.e., $l_N = l_K = 1$). Examining the Log-Likelihood in each case, we see that the second column embodies the global minimum implying a s around one and the first column contains a local minimum with a value around 0.5-0.6. Nevertheless, a closer examination of second columns of tables 3-6 reveals some weakness in these (constant-growth) estimates (i.e., the

[7] This consideration arises because non-linear estimation can be sensitive to starting parameter values. Accordingly, variations help identify the global maximum. In our case, the results were only sensitive to different starting values in the s parameter. This may be expected since our system has a singularity at s =1. When we start with s <1, it would essentially require a stroke of luck for the estimation process to skip over the singularity into the above-unity zone; similarly starting with s >1, we will be limited to above-unity solution territory. Thus, there may be separate optima with below and above unity values for s . Consequently, we performance a fine grid search of initial guesses: $s(0) \in [0, 2.5]$. Given this range and for the various cases, we find that estimated s 's cluster around unity or 0.6. In each of the graphs (lower right-hand box) therefore, we present the range of s 's estimated against this starting-value grid search and the associated log-likelihood.

apparent global minimum). For example, although the implied total Technical Factor Progress over the sample on average is reasonable, its time profile is less so. More importantly, factor-specific technical progress parameters, though insignificant, appear economically unreasonable: yielding large negative (positive) labor (capital) progress and an implied growth in total factor productivity (i.e., second right-hand panel box in **Graphs 1.2**) that is strongly decreasing and at the end is close to intersecting the zero line. These unreasonably high estimates for factor augmenting technical progress indeed results in a strong curvature effect also with close to unity elasticity of substitution in the TFP-component, without any reasonable economic interpretation, as discussed in Section 5, in the context of equation (12).

The first column, in turn, although not representing a global minimum is, in an economic sense, more plausible. This well-below-unity elasticity (around 0.5-0.6) is coupled with reasonable labor and capital augmenting technical progress. According to these results, labor-augmenting technical progress is dominating (i.e., 1.7% annual versus 0.4 for capital and 1.4 for TFP when labor input is measured in terms of employed persons or quality adjusted index; when labor input is measured in terms of hours both labor augmented technical progress and TFP are somewhat faster, i.e. 2.1% and 1.7%, respectively). Furthermore, although improved, the residual properties of production function (associated with the local minimum) are not fully satisfactory – see the hump in the labor share in the **Graphs 1.1, 2.1, 3.1** and **4.1** (bottom-left box), which suggests some underlying (and missing) non-linear form for the equations.

Consequently, we proceeded to freely estimate the technical progress parameters in a time-varying manner (i.e. $l_N, l_K \neq 1$). This yields appreciably better results: in all cases, the Log Likelihood was superior and s was always found to be significantly well below unity (as before, around 0.6). Note, that this s value was always associated with the global minimum, as can be verified by inspecting the lower right box in the Graphs (although as this box shows there still remain well-defined local minimum for s around unity). Likewise, we see in these time-varying cases that residual properties are now satisfactory (i.e., stationary) when the Ho and Jorgenson (1999) quality-adjusted labor input is used. This therefore represents our preferred case. However in all tables, we derive economically-reasonable values for augmented technical progress for labor and capital and broadly in line with the first column, the elasticity of substitution are around 0.6.

Of the unconstrained time-varying results (column 1.3 in **Table 3**, column 2.3 in **Table 4**; column 3.3 in **Table 5;** column 4.3 in **Table 6**), we see that labor-augmenting technical progress shows an exponential pattern (also seen in the top right box of the corresponding graphs) although with growth rates slightly decelerating (i.e., middle right boxes). Further, since point estimates for l_K are negative, this implies that capital-augmenting technical progress shows a hyperbolic pattern (i.e., top right box) although with growth rates asymptoting towards zero (i.e., middle right boxes). Since l_K is not significantly different from zero (except in Table 3), we impose a value of zero and therefore estimate the more parsimonious form in **Tables 3-6** (columns 1.4, 2.4, 3.4 and 4.4). There, all parameters are highly significant and residual properties are roughly similar to the corresponding unconstrained case.

Furthermore, we present in the tables cases which explicitly uses the Kmenta approximation for the TFP component (columns 2.4 KMENTA, 3.4 KMENTA and 4.4 KMENTA in **Tables 3-6**). We find quite similar technical progress as before. Estimates of the value for s, though still well below unity, are somewhat higher than before (of the order of 0.05, 0.1 extra); this apparent bias arises from the fact that the Kmenta approximation is itself an approximation linearised around a value of unity for s.

Let us summarize. We have seen that where we do not allow for time-varying technical change (i.e., we impose the conventional case of constant factor-augmenting technical progress), that a global minimum is associated (like so many other studies) with a unitary elasticity of substitution. However, that case is not characterized by stationary residuals or by plausible technical parameters. A local minimum (associated with an elasticity of 0.6) displays similarly poor residual properties but more economically reasonable technology parameters. When we allowed the data to choose the functional form (via the Box-Cox transformation) we found not only that labor-augmenting technical progress exceeds that of capital but that labor-augmenting technical progress shows an exponential pattern whilst that of capital displays a hyperbolic (or a most logarithmic) dynamic. This is in line with Acemoglu's growth model, which allows temporary but persistent capital-augmenting technical progress.

Regarding other parameters, (A and m), estimated values are well in line with our priors. The scale parameter is in the neighborhood of one, the capital share parameter corresponds well to the sample average of capital income share. The mark-up is significantly different from zero and appears relatively robust at 4%.[8] Furthermore, as discussed in Section 4, the distribution parameter is robustly identified by the normalized system (at 0.2), and is not, for example, sensitive to different sigma estimates and is well in line with observed average capital-income share (accounting for the markup).

Finally, in Appendix Two, to provide further evidence on the empirical application of the Normalization approach, we perform recursive estimation of our supply-side system with the various fixed point estimated first within the rolling and then imposed at their full-sample values. Unsurprisingly, we find the second, full-information approach, to, provide the more robust results.

8. Conclusions

Our analysis was motivated by both theoretical and empirical contributions that have recently challenged the dominance of the Cobb-Douglas production for modeling the aggregate production function in models of economic growth. The natural alternative to the Cobb-Douglas function being the CES function with non-unitary elasticity of substitution we analyzed the many problems that occur with estimations of the respective parameters of this functional form. We propose that by using normalized CES functions and more flexible functional form for the growth rate of factor efficiencies the estimation results can be much improved.

Applying a normalized CES function with factor-augmenting technical progress, we estimate a supply-side system of the US economy from 1953 to 1998. Avoiding potential estimation biases that have occurred in earlier estimations and putting a high emphasis on the consistency of the data set, required by the estimated system, we obtain robust results not only for the aggregate elasticity of substitution but also for the parameters of labor and capital augmenting technical change. We find that the elasticity of substitution is significantly below unity (between 0.5 and 0.7) and that the growth rates of technical progress show an interesting asymmetrical pattern where the growth of labor-augmenting technical progress is exponential, while the growth of capital augmenting progress is hyperbolic or logarithmic.

Our results are therefore supportive of Acemoglu's view on biased technological change where labor efficiency growth is dominant in the long run while capital efficiency growth must fade away. Given a non-unitary elasticity of substitution this pattern of technical growth rates

[8] It is worth noting that estimates of the markup parameter, m, are very sensitive to possible measurement errors and aggregation errors in the level of the capital stock and the user cost of capital.

guarantees the secular stability of income shares whereas they can fluctuate in the medium run. We think that with these properties the CES production function has still (or again) a prominent role to in the theory of economic growth.

References

Acemoglu, D. (2002) "Directed technical change", *Review of Economic Studies*, 69, 781-809.

Acemoglu, D. (2003) "Labor- and capital-augmenting technical change", *Journal of the European Economic Association*, 1, 1-37.

Andersen, P. S., Klau, M. and Yndgaard, E. (1999) "Higher profits and lower capital prices: Is factor allocation optimal?" BIS Working Paper No. 65, Bank of International Settlements.

Antràs, P. (2004) "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution", *Contributions to Macroeconomics*, 4, 1, Article 4.

Arrow, K. J., Chenery, H. B., Minhas, B. S. and Solow, R. M. (1961) "Capital-labor substitution and economic efficiency", *Review of Economics and Statistics*, 43, 225-250.

Auerbach, A. J. (1983) "Corporate Taxation in the United States", *Brookings Papers on Economic Activity*, 2, 451-513.

Auerbach, A. J. (2003) "Fiscal Policy, Past and Present", *Brookings Papers on Economic Activity*, 1, 75-138.

Azariadis, C. (1996) "The economics of poverty traps – Part One: Complete markets", *Journal of Economic Growth*, 1, 449-486.

Barro, R. J., and X. Sala-i-Martin (2003) *Economic Growth*, Second Edition, MIT Press.

Berndt, E. R. (1976) "Reconciling alternative estimates of the elasticity of substitution", *Review of Economics and Statistics*, 58, 59-68.

Berthold, N., Fehn, R. and Thode, E. (2002) "Falling labor share and rising unemployment: Long-run consequences of institutional shocks?", *German Economic Review*, 3, 431-459.

Bertolila, S. and Saint-Paul, G. (2003) "Explaining movements in the labor share", *Contributions to Macroeconomics*, 3, Art. 9.

Blanchard, O. (1997) "The medium run", *Brookings Papers on Economic Activity*, 2, 89-158.

Bodkin, R. G. and Klein, L. R. (1967) "Nonlinear estimation of aggregate production functions", *Review of Economics and Statistics*, 49, 28-44.

Bolt, W. and van Ells, P. J. A. (2000) "Output and inflation in the EU", Staff Reports No. 44, De Nederlandsche Bank.

Box, G. and Cox, D. (1964) "An analysis of transformations", *Journal of the Royal Statistical Society*, 26, Series B, 211-243.

Brown, M. and De Cani, J. (1963) "Technological change and the distribution of income", *International Economic Review*, 4, 289-309.

Caballero, R. J. and Hammour, M. (1998) "Jobless growth: Appropriablity, factor substitution and unemployment", *Carnegie-Rochester Conference Proceedings*, 48, 51-94.

Chirinko, R. S., Fazzari, S. M. and Meyer, A. P. (1999) "How responsive is business capital formation to its user cost?", *Journal of Public Economics*, 74, 53-80.

Chrininko, R. S. (2002) "Corporate taxation, capital formation, and the substitution elasticity between labor and capital", *National Tax Journal*, 60, 339-355.

David, P. A. and van de Klundert, T. (1965) "Biased efficiency growth and capital-labor substitution in the US, 1899-1960", *American Economic Review*, 55, 357-394.

De La Grandville, O. (1989) "In quest of the Slutzky diamond", *American Economic Review*, 79, 468-481.

De La Grandville, O. and R. M. Solow (2004) "On the determinants of economic Growth: Is something missing?", mimeo, April 14th.

Diamond, P., Mc Fadden, D. and Rodriguez, M. (1978) "Measurement of the elasticity of factor substitution and bias of technical change", in: Fuss, M., and Mc Fadden, D. (Eds.) *Production economics: A dual approach to theory and application*, 2, Amsterdam and New York, p. 125-147.

Duffy, J. and Papageorgiou, C. (2000) "A cross-country empirical investigation of the aggregate production function specification", *Journal of Economic Growth*, 5, 86-120.

Easterly, W. and Fischer, S. (1995) "The soviet economic decline", *World Bank Economic Review*, 9, 341-371.

Gollin, D. (2002) "Getting Income Shares right", *Journal of Political Economy*, 110, 2, 458-474.

Hardy, G. H., Littlewood, J. E., Polya, G. (1934) "Inequalities", 2nd ed. 1952, reprinted 1967, Cambridge.

Herman, S. (2000) "Fixed Assets and Consumer Durable Goods", *Survey of Current Business*, April.

Ho, Mun S. and Jorgenson, Dale W. (1999) "The Quality of the U.S. Workforce 1948-95", Harvard University, mimeo with data update through 2000.

Johnson, D. G. 1954) "The functional Distribution of Income in the United States, 1850-1952", *Review of Economics and Statistics*, 35, 2, 175-182.

Jones, C. I. (2003) "Growth, capital shares, and a new perspective on production functions", Working Paper, Version 1.0, June

Kaas, L. and von Thadden, L. (2003) "Unemployment, factor substitution, and capital formation", *German Economic Review*, 4, 475-495.

Kalt, J. P. (1978) "Technological change and factor substitution in the United States: 1929-1967", *International Economic Review*, 19, 761-775.

Kamien, M. I. and Schwartz, N. L. (1968) "Optimal 'Induced' technical change", *Econometrica*, 36, 1-17.

Kendrick, J. and Sato, R. (1963) "Factor prices, productivity and economic growth", *American Economic Review*, 53, 974-1003.

Klump, R. (2001) "Trade, money and employment in intertemporal optimising models of growth", *Journal of International Trade and Development*, 10, 411-428.

Klump, R. (2002) "Verteilung und Wirtschaftswachstum: Eine neoklassische Verallgemeinerung des Wachstumsmodell von Kaldor", in Menkhoff, L. and Sell, F. (Eds.) *Zur Theorie, Empirie und Politik der Einkommensverteilung*, Berlin-Heidelberg-New York, 11-26.

Klump, R. and De La Grandville, O. (2000) "Economic growth and the elasticity of substitution: Two theorems and some suggestions", *American Economic Review*, 90, 282-291.

Klump, R. and Preissler, H. (2000) "CES production functions and economic growth", *Scandinavian Journal of Economics*, 102, 41-56.

Kmenta, J. (1967) "On Estimation of the CES Production Function", *International Economic Review*, 8, 180-189.

Krueger, A. B. (1999) "Measuring Labor's Share", *American Economic Review*, Papers and Proceedings, 89, 45-51.

Lewis, P. E. T. and Kirby, M. G. (1988) "A new approach to modelling the effects of income policies", *Economics Letters*, 28, 81-85.

Lucas, R. E. (1969) “Labor-capital substitution in U.S. manufacturing”, in Harberger, A. C. and Bailey, M. J. (Eds.) *The taxation of income from capital*, Washington, D. C.: The Brookings Institution, 223-274.

Marschak, J. and W. Andrews (1947) “Random simultaneous equations and the theory of production”, *Econometrica*, 12, 143-53.

McAdam, P. and Willman, A. (2004a) “Production, supply and factor shares: An application to estimating German long-run supply”, *Economic Modelling*, 21, 2, 191-215.

McAdam, P. and Willman, A. (2004b) “Supply, Factor Shares And Inflation Persistence”, *Oxford Bulletin of Economics and Statistics*, forthcoming.

Panik, M. (1976) “Factor learning and biased factor efficiency growth in the United States, 1929-1966”, *International Economic Review*, 17, 733-739.

Ripatti, A. and Vilmunen, J. (2001) “Declining labour share – Evidence of a change in the underlying production technology?”, Bank of Finland Discussion Papers 10/2001.

Rowthorn, R. (1999) “Unemployment, wage bargaining and capital-labour substitution”, *Cambridge Journal of Economics*, 23, 413-425.

Sato, R. (1970) “The estimation of biased technical change and the production function”, *International Economic Review*, 11, 179-208.

Ventura, J. (1997) “Growth and interdependence”, *Quarterly Journal of Economics*, 62, 57-84.

Wilkinson, M. (1968) “Factor supply and the direction of technological change”, *American Economic Review*, 58, 210-217.

Willman, A. (2002) “Euro area production function and potential output: A supply-side system approach”, ECB Working Paper No. 153.

Table 1.
Empirical studies of aggregate elasticity of substitution and technological change in the US

Study	Sample (Annual Frequency)	Assumption on Technological Change	Estimated Elasticity of Substitution: σ	Estimated Annual Rate Of Efficiency Change		
				Neutral: $g_N = g_K$	Labor-augmenting: g_N	Capital-Augmenting: g_K
Arrow *et al.* (1961)	1909-1949	Hicks-Neutral	0.57	1.8	-	-
Kendrick and Sato (1963)	1919-1960	Hicks-Neutral	0.58	2.1	-	-
Brown and De Cani (1963)	1890-1918	Factor Augmenting	0.35	Labor saving ($g_N - g_K = 0.48$)		
	1919-1937		0.08	Labor saving ($g_N - g_K = 0.62$)		
	1938-1958		0.11	Labor saving ($g_N - g_K = 0.36$)		
	1890-1958		0.44	?		
David and van de Klundert (1965)	1899-1960	Factor Augmenting	0.32	-	2.2	1.5
Bodkin and Klein (1967)	1909-1949	Hicks-neutral	0.5-0.7	1.4-1.5		
Wilkinson (1968)	1899-1953	Factor Augmenting	0.5	Labor saving ($g_N - g_K = 0.51$)		
Sato (1970)	1909-1960	Factor Augmenting	0.5 – 0.7	-	2.0	1.0
Panik (1976)	1929-1966	Factor Augmenting	0.76	Labor saving ($g_N - g_K = 0.27$)		
Berndt (1976)	1929-1968	Hicks-neutral	0.96-1.25	?	-	-
Kalt (1978)	1929-1967	Factor Augmenting	0.76	-	2.2	0.01
Antras (2003)	1948-1998	Hicks-neutral	0.94-1.02	1.14	-	-
		Factor-augmenting	0.8	Labor saving ($g_N - g_K = 3.15$)		

Table 2.
Recent empirical studies of aggregate elasticity of substitution in different countries

Study	Countries	Sample (Frequency)	Assumption For Technological Change	Estimated Elasticity Of Substitution: σ
Lewis and Kirby (1988)	Australia	1967-1987 (Weekly)	Hicks-Neutral	0.78
Easterly and Fischer (1995)	Soviet Union	1950-1987 (Annual)	Hicks-Neutral	0.37
Andersen *et al.* (1999)	Panel of 17 OECD countries	1966-1996 (Annual)	Hicks-Neutral	1.12
Bolt and van Els (2000)	Austria Belgium Germany Denmark Spain Finland France Italy Netherlands Sweden UK US Japan	1971-1996 (Quarterly)	Hicks-Neutral	0.24 0.78 0.53 0.61 1 0.34 0.73 0.52 0.27 0.68 0.6 0.82 0.3
Duffy and Papageorgiou (2000)	82 developed and developing countries	1960-1987 (Annual)	Hicks-Neutral	1.4
Ripatti and Vilmunen (2001)	Finland	1975-1999 (Quarterly)	Factor Augmenting	0.6
Willman (2002)	Euro area	1970-1997 (Quarterly)	Harrod Neutral Hicks Neutral Solow Neutral	0.37-Infinity 0.66-2.23 0.95-1.05 [(a)]
Berthold *et al.* (2002)	US Germany France	1970-1995 (Semi-Annual)	Harrod-Neutral	1.15 1.45 2.01
Bertolila and Saint-Paul (2003)	Panel of 13 industries in 12 OECD countries	1972-1993 (Annual)	Harrod-Neutral	1.06
McAdam and Willman (2004a)	Germany	1983-1999 (Quarterly)	Hicks Neutral	0.7-1.2

Note:

(a) In the light of statistical criteria, Solow Neutral was preferred since, for Hicks- and Harrod-Neutral cases elasticity estimates were highly unstable (being strongly sample dependent).

Table 3. Estimation of Supply-Side
(Labor Input: Employed persons, Labor Income: Proprietors' income set at 67%)

	Constant Factor-Augmenting Technical Growth		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)	
GRAPHS	**1.1** (Local Optimum)	**1.2** (Global Optimum)	**1.3** (Global Optimum)	**1.3 KMENTA** (Global Optimum)
A	1.000 (0.014)	1.030 (0.017)	1.027 (0.011)	1.027 (0.011)
p	0.221 (0.010)	0.219 (0.009)	0.219 (0.010)	0.219 (0.010)
g_N	0.017 (0.001)	-0.248 (0.141)	0.016 (0.001)	0.016 (0.001)
l_N	1.000 (—)		0.538 (0.120)	0.609 (0.137)
g_K	0.004 (0.001)	0.944 (0.488)	0.002 (0.001)	0.003 (0.001)
l_K	1.000 (—)		-1.028 (0.460)	-0.835 (0.502)
s	0.600 (0.014)	0.999 (0.001)	0.639 (0.032)	0.707 (0.036)
m	1.039 (0.014)	1.041 (0.014)	1.042 (0.014)	1.040 (0.015)
$\frac{d\, g_N}{d\, t}\Big\vert t = t_0$	0.017	-0.248	0.016	0.016
$\frac{d\, g_K}{d\, t}\Big\vert t = t_0$	0.004	0.944	0.002	0.003
TFP	0.014	0.014	0.013	0.013
Log Lik.	-18.039	-18.157	-18.714	-18.718
ADF_N	-2.880	-3.350	-3.340	-3.460
ADF_K	-3.550	-3.520	-3.530	-3.520
ADF_Y	-2.440	-2.000	-2.480	-2.470

Note: Standard Errors in parenthesis.

Table 4. Estimation of Supply-Side
(Labor Input: Employed persons, Labor income: Self-employed labor share)

	Constant Factor-Augmenting Technical Growth		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case) With Logarithmic Capital-Augmenting Technical Growth	
GRAPHS	**2.1** (Local Optimum)	**2.2** (Global Optimum)	**2.3** (Global Optimum)	**2.3 KMENTA** (Global Optimum)	**2.4** (Global Optimum)	**2.4 KMENTA** (Global Optimum)
A	1.000 (0.014)	1.039 (0.016)	1.029 (0.012)	1.029 (0.012)	1.029 (0.012)	1.028 (0.013)
p	0.221 (0.009)	0.220 (0.009)	0.220 (0.010)	0.220 (0.010)	0.221 (0.010)	0.220 (0.010)
g_N	0.017 (0.001)	-0.198 (0.113)	0.015 (0.001)	0.015 (0.001)	0.015 (0.001)	0.015 (0.001)
l_N	1.000 (—)		0.461 (0.153)	0.531 (0.188)	0.428 (0.142)	0.557 (0.131)
g_K	0.004 (0.001)	0.768 (0.384)	0.004 (0.001)	0.005 (0.002)	0.004 (0.001)	0.005 (0.001)
l_K	1.000 (—)		-0.253 (0.371)	0.219 (0.653)	0.00 (a) (—)	
s	0.467 (0.007)	0.997 (0.003)	0.592 (0.021)	0.677 (0.028)	0.579 (0.021)	0.695 (0.027)
m	1.039 (0.012)	1.042 (0.012)	1.042 (0.012)	1.040 (0.013)	1.042 (0.012)	1.040 (0.013)
$\frac{d\,g_N}{d\,t}\vert t=t_0$	0.017	-0.198	0.015	0.015	0.015	0.015
$\frac{d\,g_K}{d\,t}\vert t=t_0$	0.004	0.768	0.004	0.005	0.004	0.005
TFP	0.014	0.014	0.013	0.013	0.013	0.013
Log Lik.	-18.131	-18.414	-18.836	-18.835	-18.825	-18.833
ADF_N	-2.460	-3.140	-2.810	-2.840	-3.670	-2.850
ADF_K	-3.610	-3.540	-3.570	-3.560	-3.580	-3.560
ADF_Y	-2.230	-1.660	-2.400	-2.370	-2.380	-2.380

Note: Standard Errors in parenthesis. (a) We imposed a value of –0.001 for l_K which proved, within our sample, to give a close enough approximation to logarithmic function since a value of zero renders the equation indeterminate; thus, we employ the approximation of the function $g_K \ln\left(t-\bar{t}\right)$ by $\frac{g_K \bar{t}}{-0.001}\left[\left[\frac{t}{\bar{t}}\right]^{-0.001}-1\right]$.

Table 5. Estimation of Supply-Side
(Labor Input: Hours, Labor Income: Self-employed labor share)

	Constant Factor-Augmenting Technical Growth		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case) With Logarithmic Capital-Augmenting Technical Growth	
GRAPHS	**3.1** (Local Optimum)	**3.2** (Global Optimum)	**3.3** (Global Optimum)	**3.3 KMENTA** (Global Optimum)	**3.4** (Global Optimum)	**3.4 KMENTA** (Global Optimum)
A	1.000 (0.012)	1.040 (0.011)	1.029 (0.008)	1.030 (0.008)	1.029 (0.008)	1.029 (0.009)
p	0.221 (0.009)	0.219 (0.009)	0.221 (0.009)	0.221 (0.010)	0.221 (0.009)	0.219 (0.009)
g_N	0.021 (0.001)	-0.198 (0.116)	0.019 (0.001)	0.019 (0.001)	0.019 (0.001)	0.019 (0.001)
l_N	1.000 (—)		0.545 (0.075)	0.611 (0.100)	0.533 (0.079)	0.671 (0.080)
g_K	0.004 (0.001)	0.785 (0.397)	0.003 (0.001)	0.005 (0.001)	0.004 (0.000)	0.004 (0.001)
l_K	1.000 (—)		-0.175 (0.317)	0.880 (0.873)	0.00 (a) (—)	
s	0.485 (0.009)	0.997 (0.003)	0.544 (0.019)	0.651 (0.028)	0.541 (0.019)	0.694 (0.029)
m	1.039 (0.012)	1.043 (0.012)	1.043 (0.012)	1.039 (0.012)	1.043 (0.012)	1.039 (0.012)
$\frac{d\,g_N}{d\,t}\Big\vert t=t_0$	0.021	-0.198	0.019	0.019	0.019	0.019
$\frac{d\,g_K}{d\,t}\Big\vert t=t_0$	0.004	0.785	0.003	0.005	0.004	0.004
TFP	0.017	0.017	0.016	0.016	0.016	0.016
Log Lik.	-18.341	-18.982	-19.338	-19.328	-19.331	-19.298
ADF_N	-2.504	-3.152	-3.621	-3.474	-3.653	-3.399
ADF_K	-3.612	-3.542	-3.581	-3.571	-3.588	-3.553
ADF_Y	-2.317	-2.338	-3.175	-3.105	-3.163	-3.133

Note: Standard Errors in parenthesis. (a) We imposed a value of –0.001 for l_K which proved, within our sample, to give a close enough approximation to logarithmic function since a value of zero renders the equation indeterminate; thus, we employ the approximation of the function $g_K \ln(t-\bar{t})$ by $\frac{g_K \bar{t}}{-0.001}\left[\left[\frac{t}{\bar{t}}\right]^{-0.001} - 1\right]$.

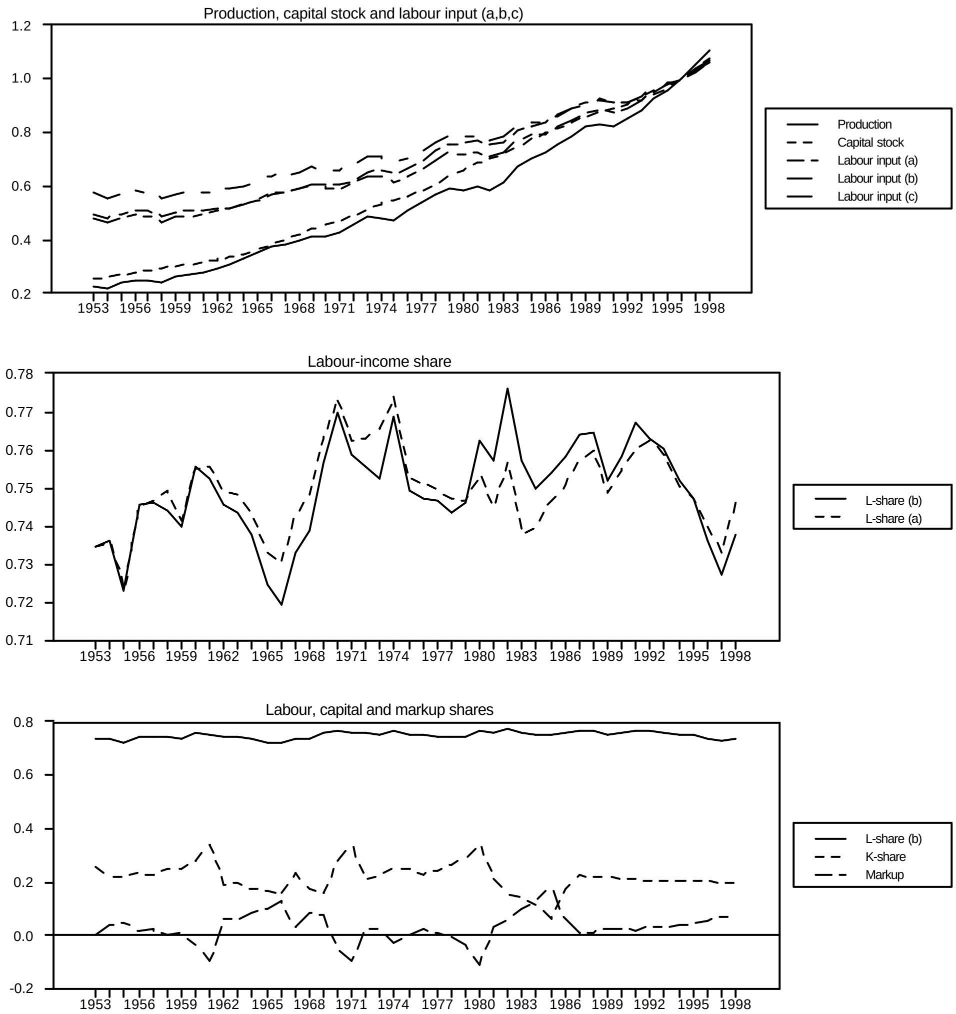
Table 6. Estimation of Supply-Side
(Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

	Constant Factor-Augmenting Technical Growth		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)		Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case) With Logarithmic Capital-Augmenting Technical Growth	
GRAPHS	**4.1** (Local Optimum)	**4.2** (Global Optimum)	**4.3** (Global Optimum)	**4.3 KMENTA** (Global Optimum)	**4.4** (Global Optimum)	**4.4 KMENTA** (Global Optimum)
A	1.000 (0.012)	1.040 (0.007)	1.029 (0.006)	1.029 (0.006)	1.029 (0.006)	1.029 (0.007)
p	0.222 (0.009)	0.219 (0.009)	0.221 (0.009)	0.222 (0.009)	0.222 (0.009)	0.221 (0.009)
g_N	0.017 (0.001)	-0.210 (0.121)	0.015 (0.000)	0.015 (0.000)	0.015 (0.000)	0.016 (0.000)
l_N	1.000 (—)		0.439 (0.076)	0.499 (0.090)	0.427 (0.083)	0.562 (0.082)
g_K	0.004 (0.001)	0.813 (0.416)	0.004 (0.001)	0.004 (0.001)	0.004 (0.000)	0.004 (0.001)
l_K	1.000 (—)		-0.118 (0.336)	0.866 (0.583)	0.00 (a) (—)	
s	0.509 (0.012)	0.998 (0.002)	0.556 (0.018)	0.605 (0.019)	0.557 (0.018)	0.642 (0.024)
m	1.038 (0.012)	1.043 (0.012)	1.042 (0.011)	1.037 (0.012)	1.042 (0.012)	1.038 (0.012)
$\frac{d\,g_N}{d\,t}\vert t=t_0$	0.017	-0.210	0.015	0.015	0.015	0.016
$\frac{d\,g_K}{d\,t}\vert t=t_0$	0.004	0.813	0.004	0.004	0.004	0.004
TFP	0.014	0.014	0.013	0.013	0.013	0.013
Log Lik.	-18.365	-19.408	-19.618	-19.633	-19.614	-19.577
ADF_N	-2.500	-3.170	-4.310	-4.050	-4.360	-3.880
ADF_K	-3.610	-3.540	-3.580	-3.570	-3.580	-3.550
ADF_Y	-2.150	-3.140	-3.960	-3.880	-3.970	-3.840

Note: Standard Errors in parenthesis. (a) We imposed a value of –0.001 for l_K which proved, within our sample, to give a close enough approximation to logarithmic function since a value of zero renders the equation indeterminate; thus, we employ the approximation of the function $g_K \ln(t-\bar{t})$ by $\frac{g_K \bar{t}}{-0.001}\left[\left[\frac{t}{\bar{t}}\right]^{-0.001}-1\right]$.

FIGURE 1.

Production, inputs, factor-income shares and markup

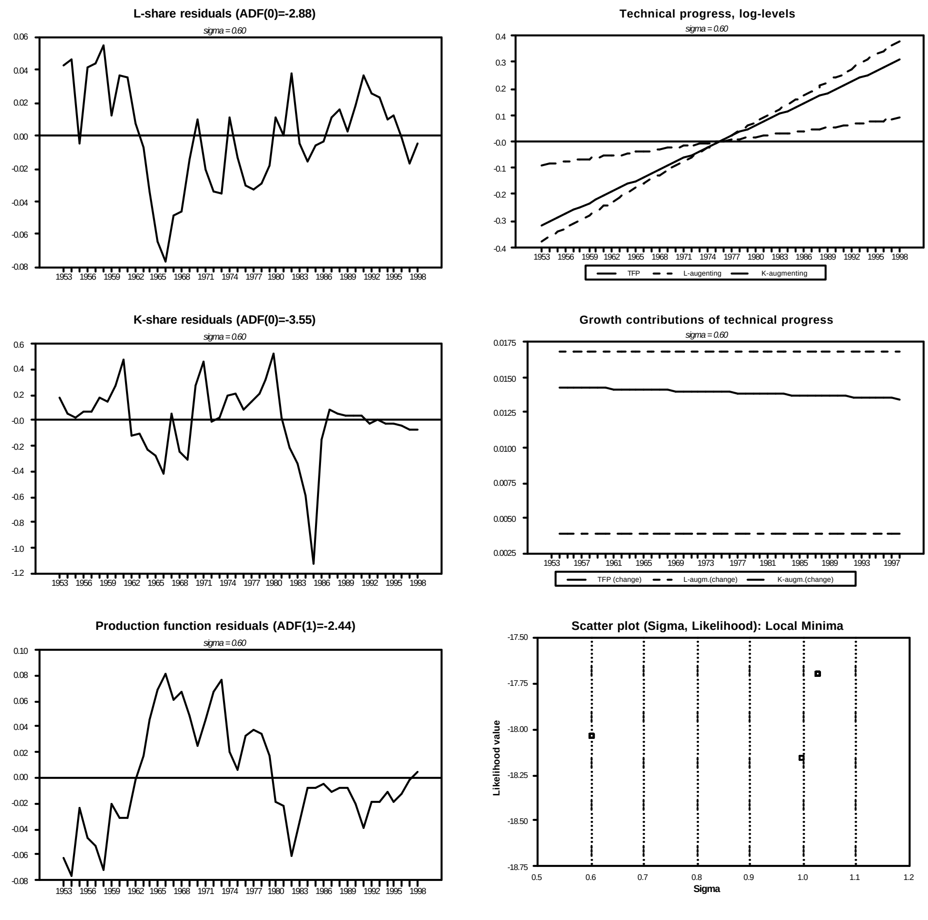


Notes: For the employment series we have three measures: (a) employed persons, (b) hours worked, (c) constant labor quality index.

Graphs 1.1: Constant Factor-Augmenting Technical Growth (Labor Input: Employed persons, Labor Income: Proprietors' income set at 67%)

(Local Optimum)

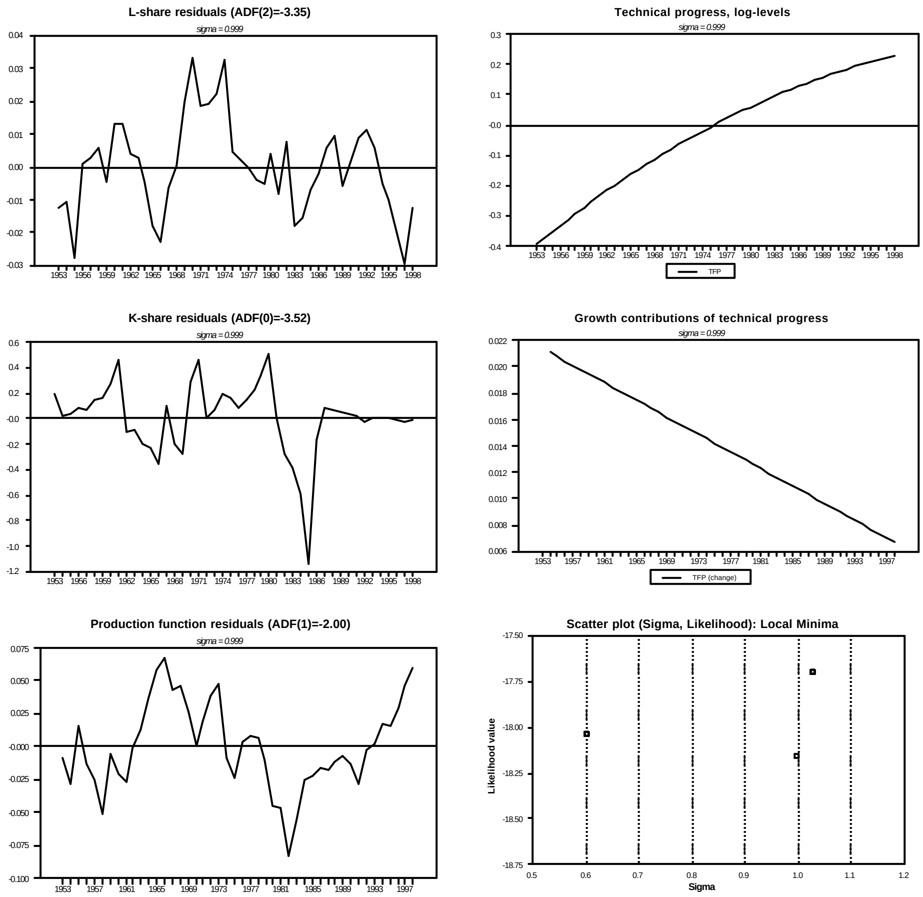
Properties of the supply-side system



Graphs 1.2: Constant Factor-Augmenting Technical Growth
(Labor Input: Employed persons, Labor Income: Proprietors' income set at 67%)

(Global Optimum)

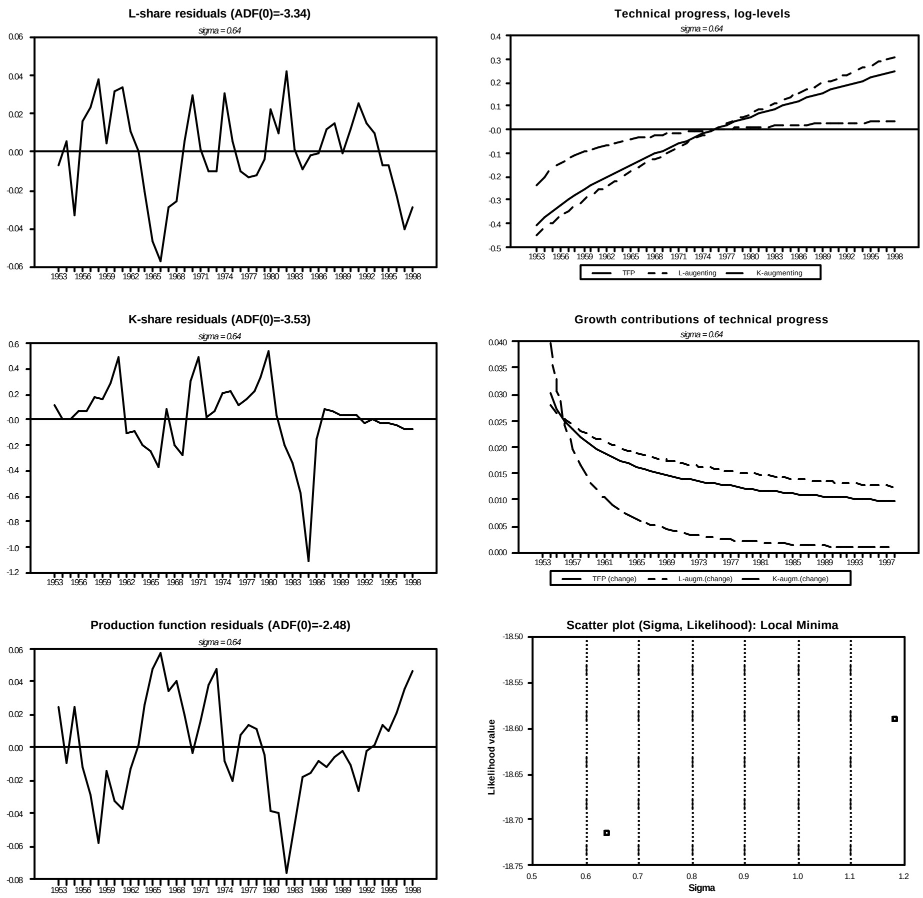
Properties of the supply-side system



Graphs 1.3.: Time -Varying Factor-Augmenting Technical Growth (Box-Cox Case) (Labor Input: Employed persons, Labor Income: Proprietors' income set at 67%)

(Global Maximum)

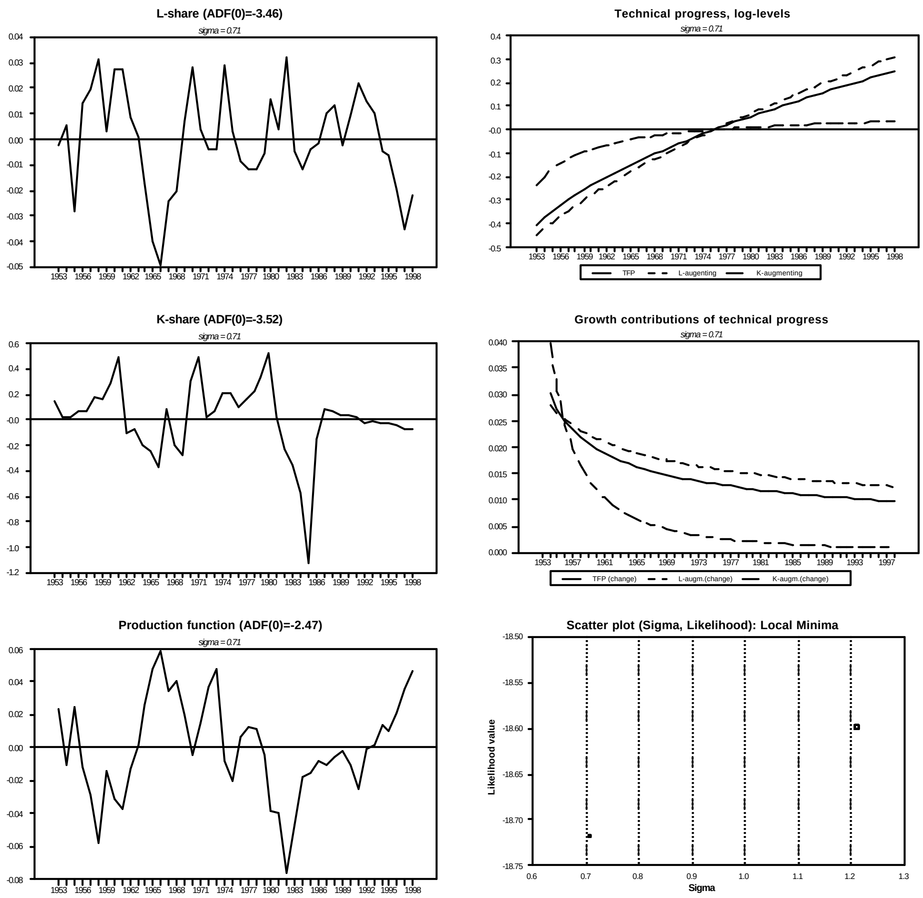
Properties of the supply-side system



Graphs 1.3. (Kmenta Case): Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)
(Labor Input: Employed persons, Labor Income: Proprietors' income set at 67%)

(Global Maximum)

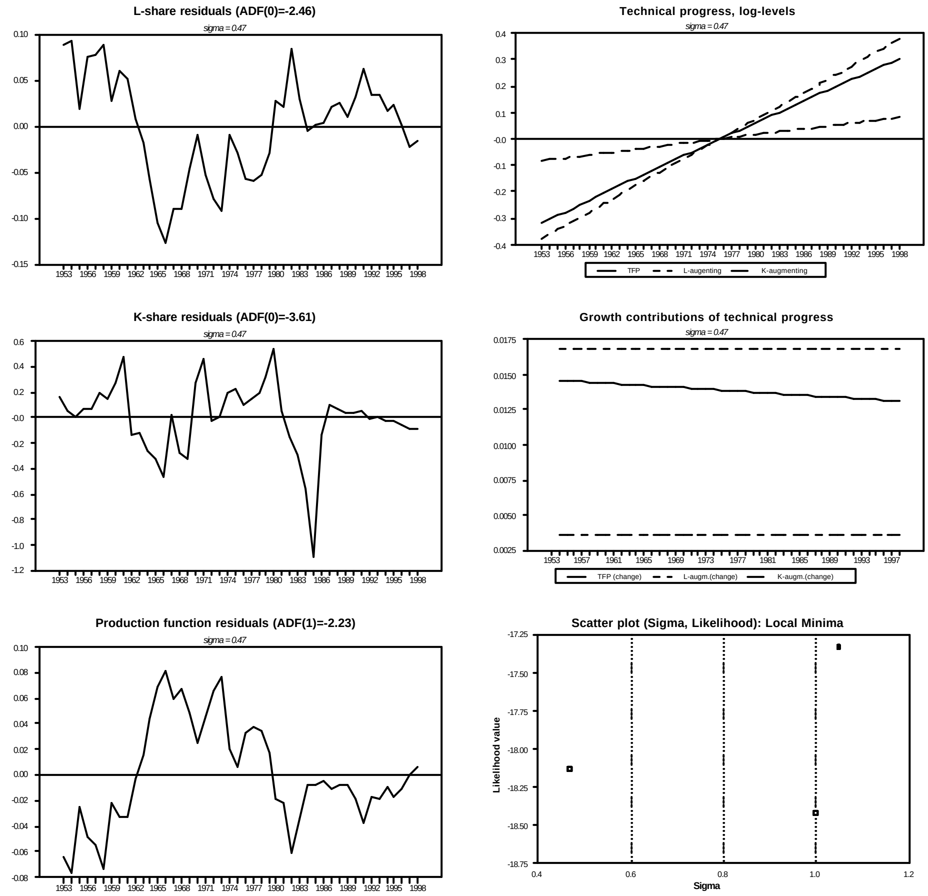
Properties of the supply-side system



Graphs 2.1: Constant Factor-Augmenting Technical Growth
(Labor Input: Employed persons, Labor income: Self-employed labor share)

(Local Optimum)

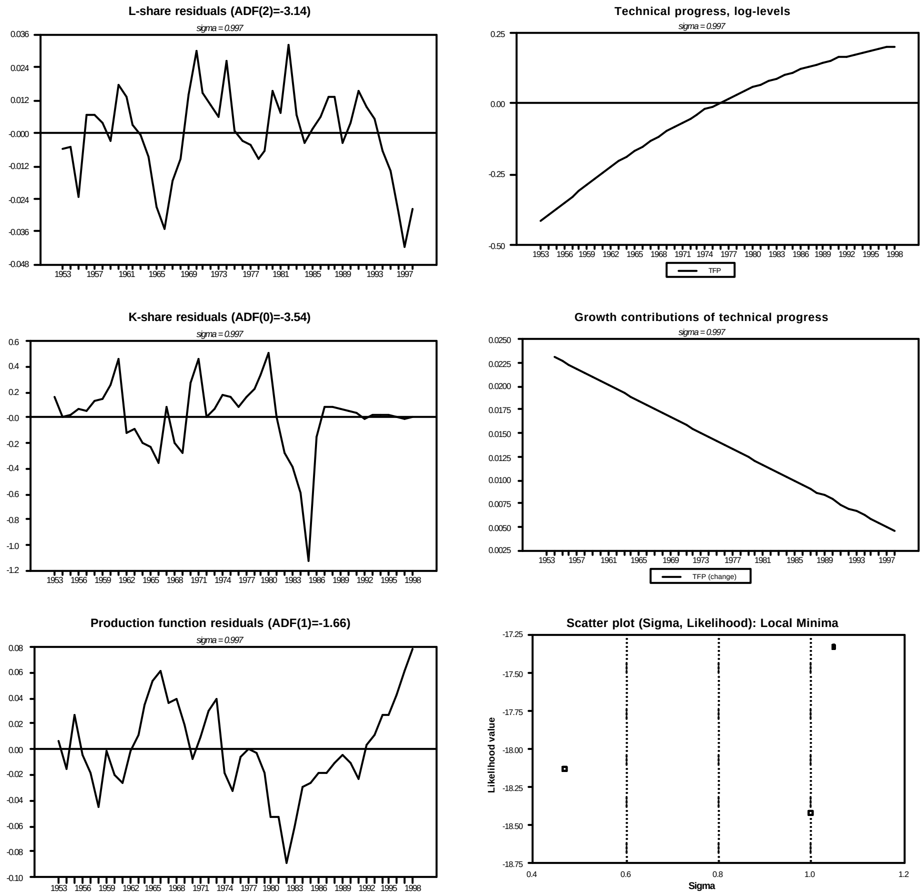
Properties of the supply-side system



Graphs 2.2: Constant Factor-Augmenting Technical Growth (Labor Input: Employed persons, Labor income: Self-employed labor share)

(Global Optimum)

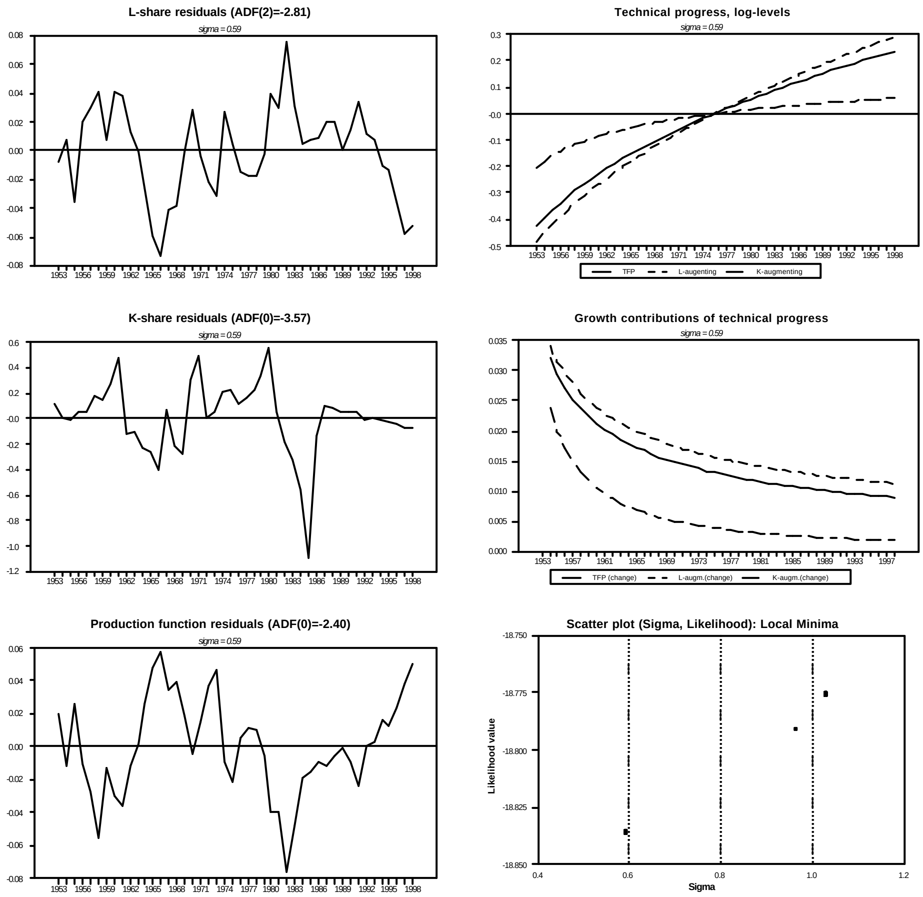
Properties of the supply-side system



Graphs 2.3: Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)
(Labor Input: Employed persons, Labor income: Self-employed labor share)

(Global Optimum)

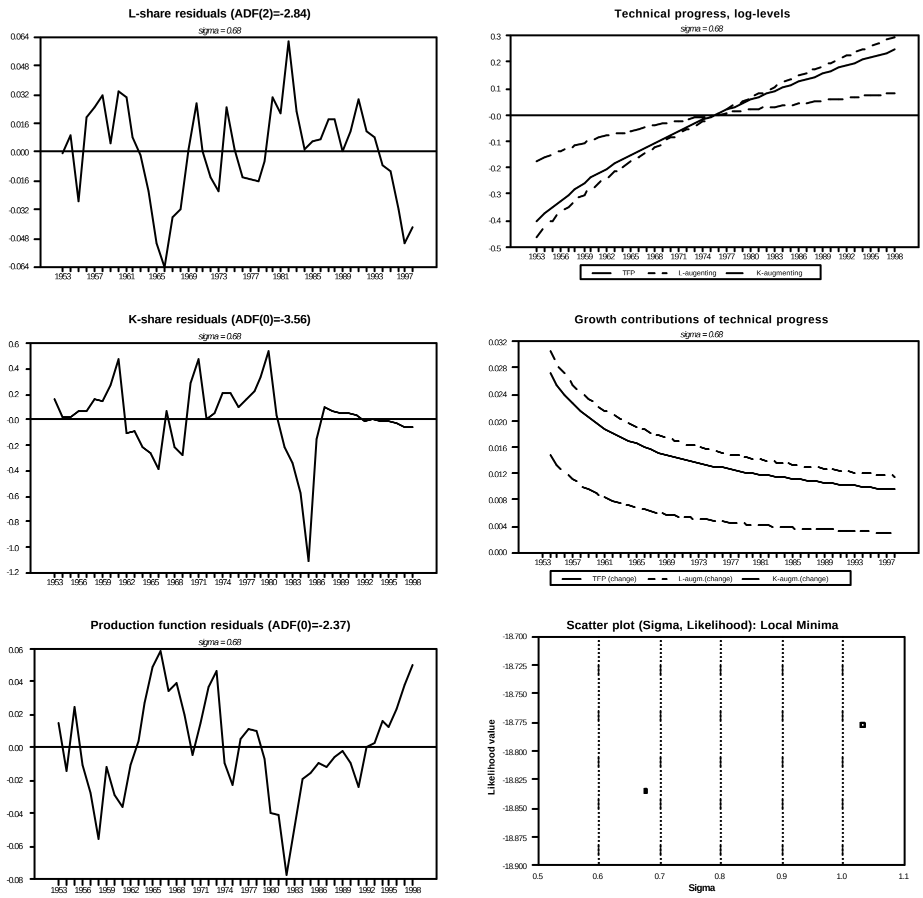
Properties of the supply-side system



Graphs 2.3 (Kmenta Case): Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case)
(Labor Input: Employed persons, Labor income: Self-employed labor share)

(Global Optimum)

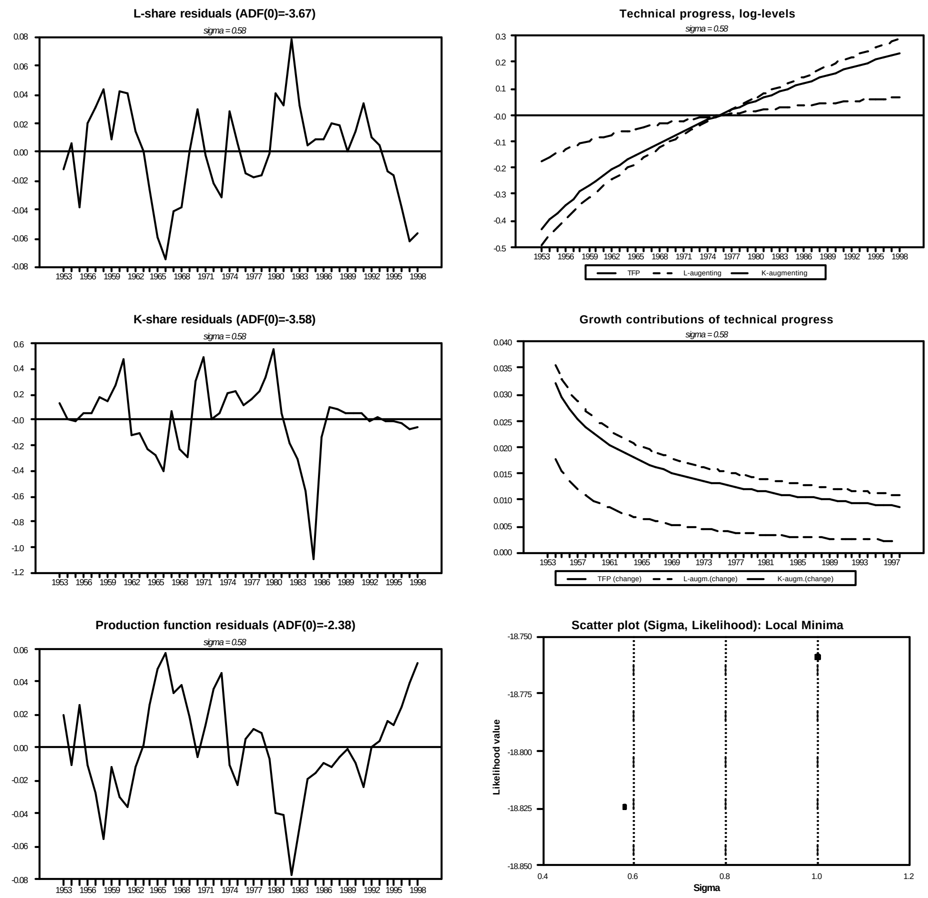
Properties of the supply-side system



Graphs 2.4: Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case) With Logarithmic Capital-Augmenting Technical Growth (Labor Input: Employed persons, Labor income: Self-employed labor share)

(Global Optimum)

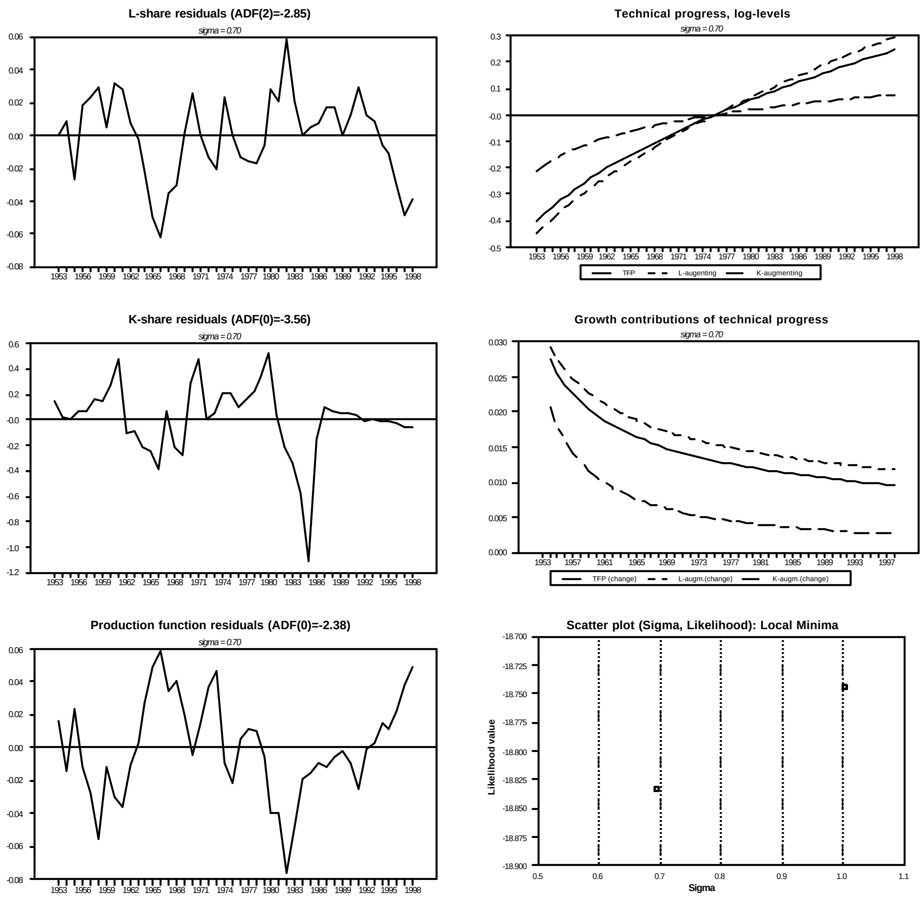
Properties of the supply-side system



Graphs 2.4 (Kmenta Case): Time-Varying Factor-Augmenting Technical Growth (Box-Cox Case) With Logarithmic Capital- Augmenting Technical Growth (Labor Input: Employed persons, Labor income: Self-employed labor share)

(Global Optimum)

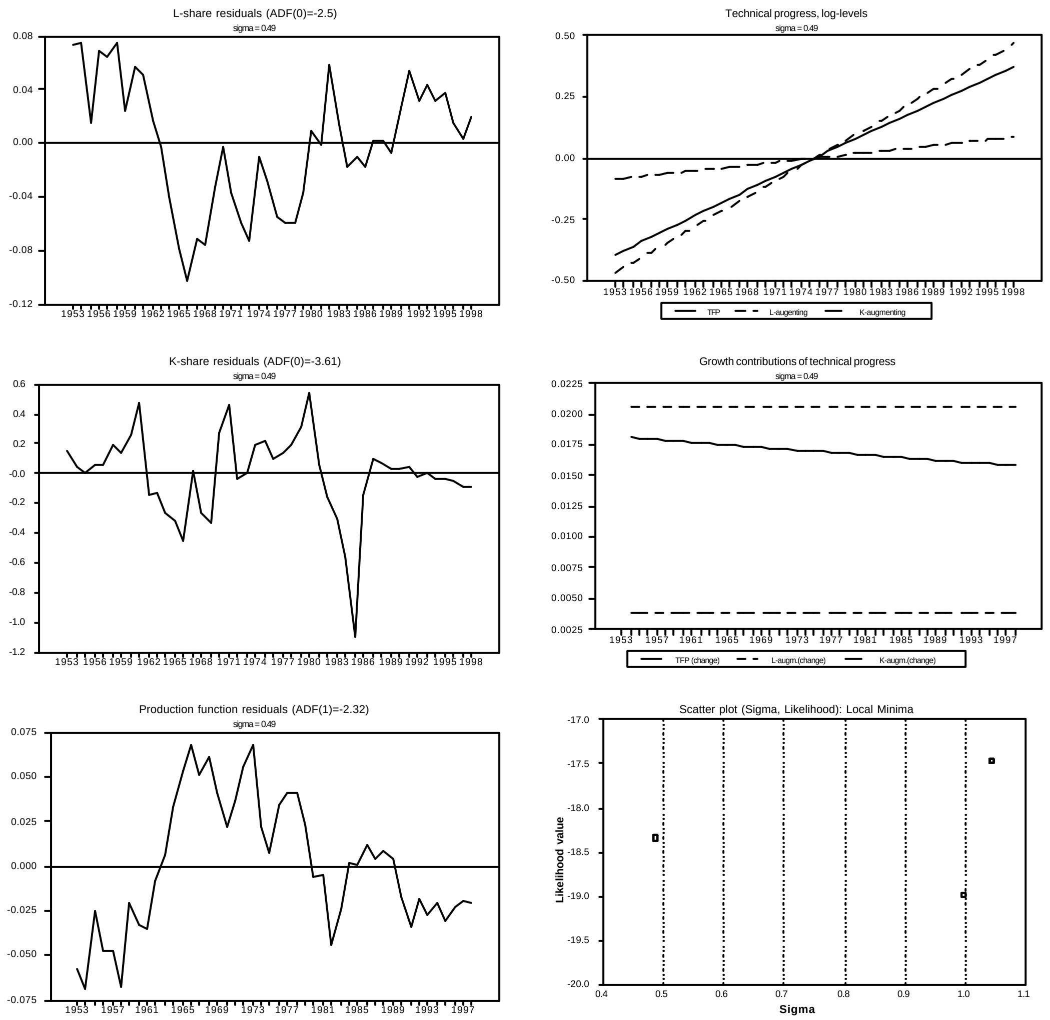
Properties of the supply-side system



Graphs 3.1: Constant factor-augmenting technical growth (Labor Input: Hours, Labor Income: Self-employed labor share)

(Local Optimum)

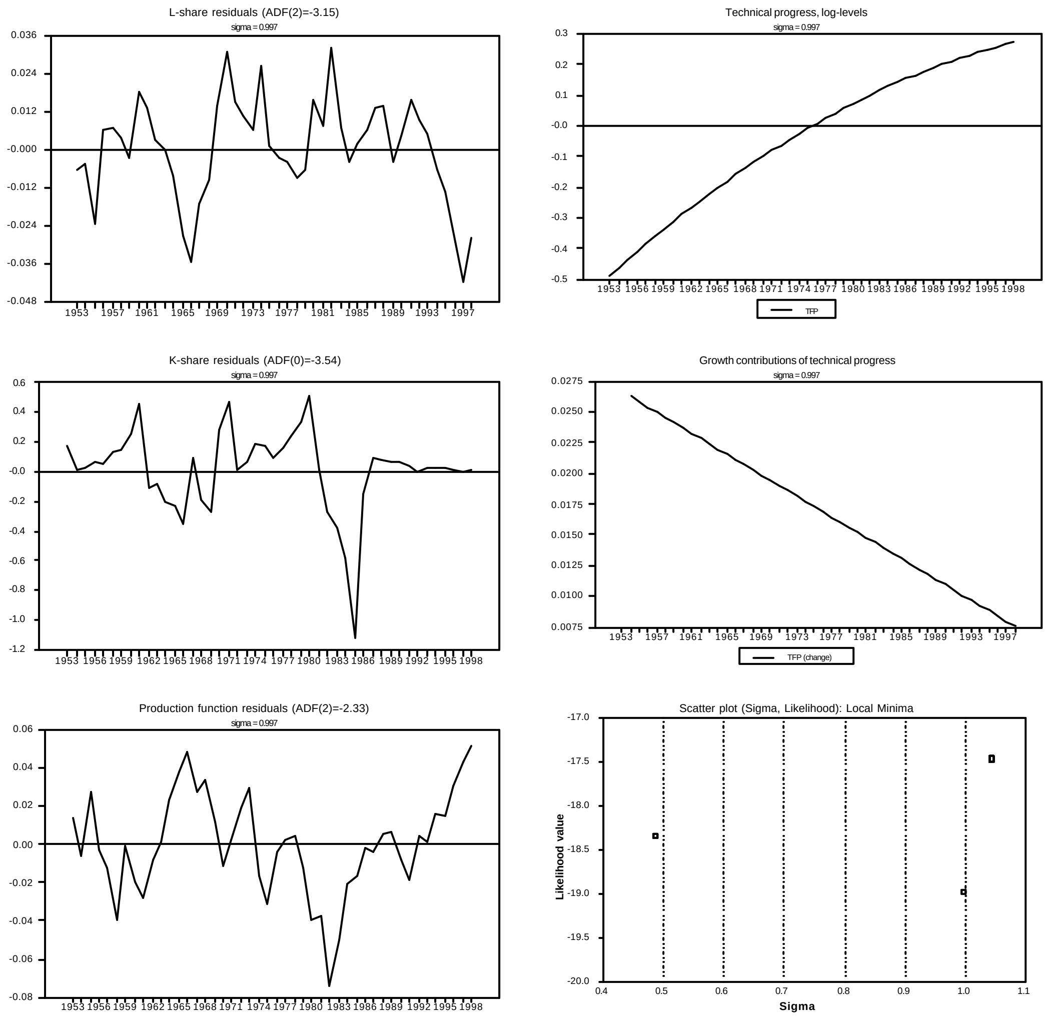
Properties of the supply-side system



Graphs 3.2: Constant Factor-Augmenting Technical Growth (Labor Input: Hours, Labor Income: Self-employed labor share)

(Global Optimum)

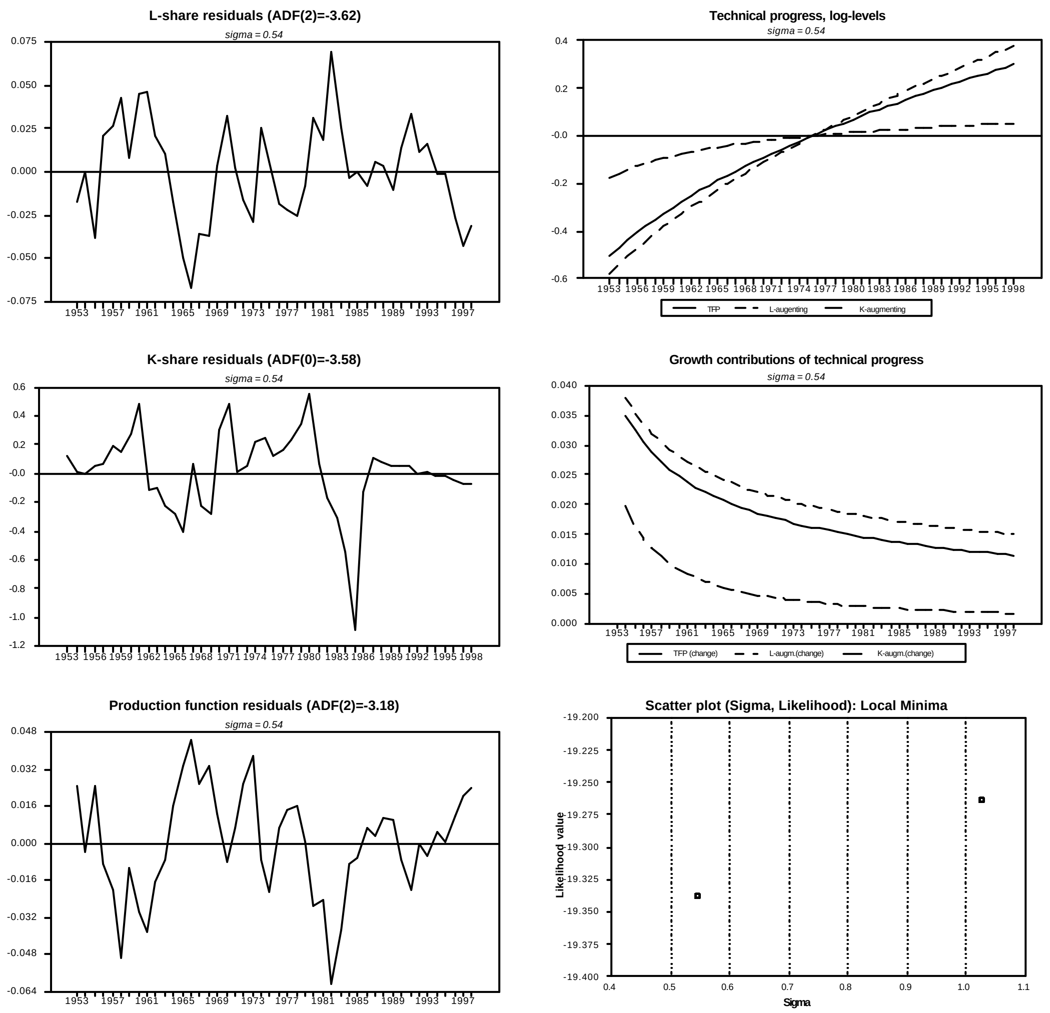
Properties of the supply-side system



Graphs 3.3. : Time -varying factor-augmenting technical growth (Box-Cox case)
(Labor Input: Hours, Labor Income: Self-employed labor share)

(Global Optimum)

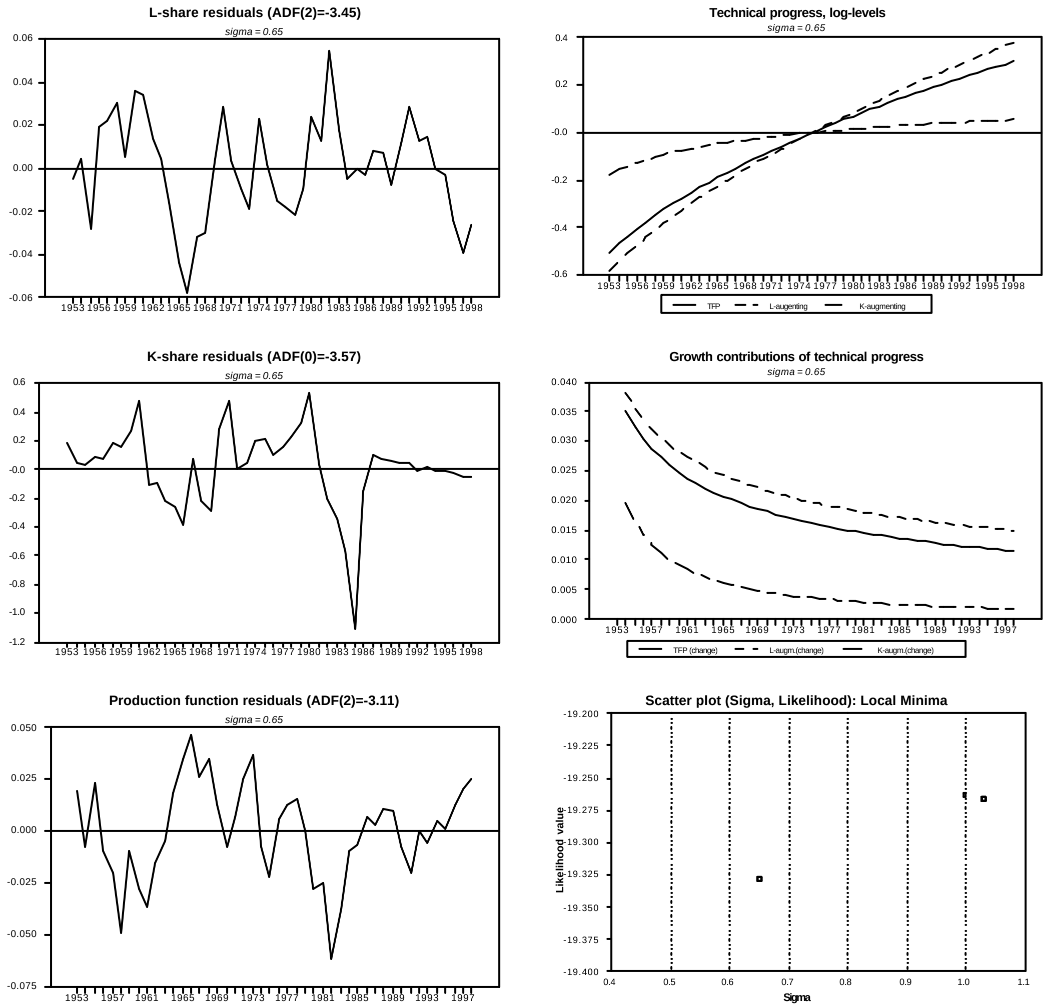
Properties of the supply-side system



Graphs 3.3. (Kmenta Case): Time -varying factor-augmenting technical growth (Box-Cox case)
(Labor Input: Hours, Labor Income: Self-employed labor share)

(Global Optimum)

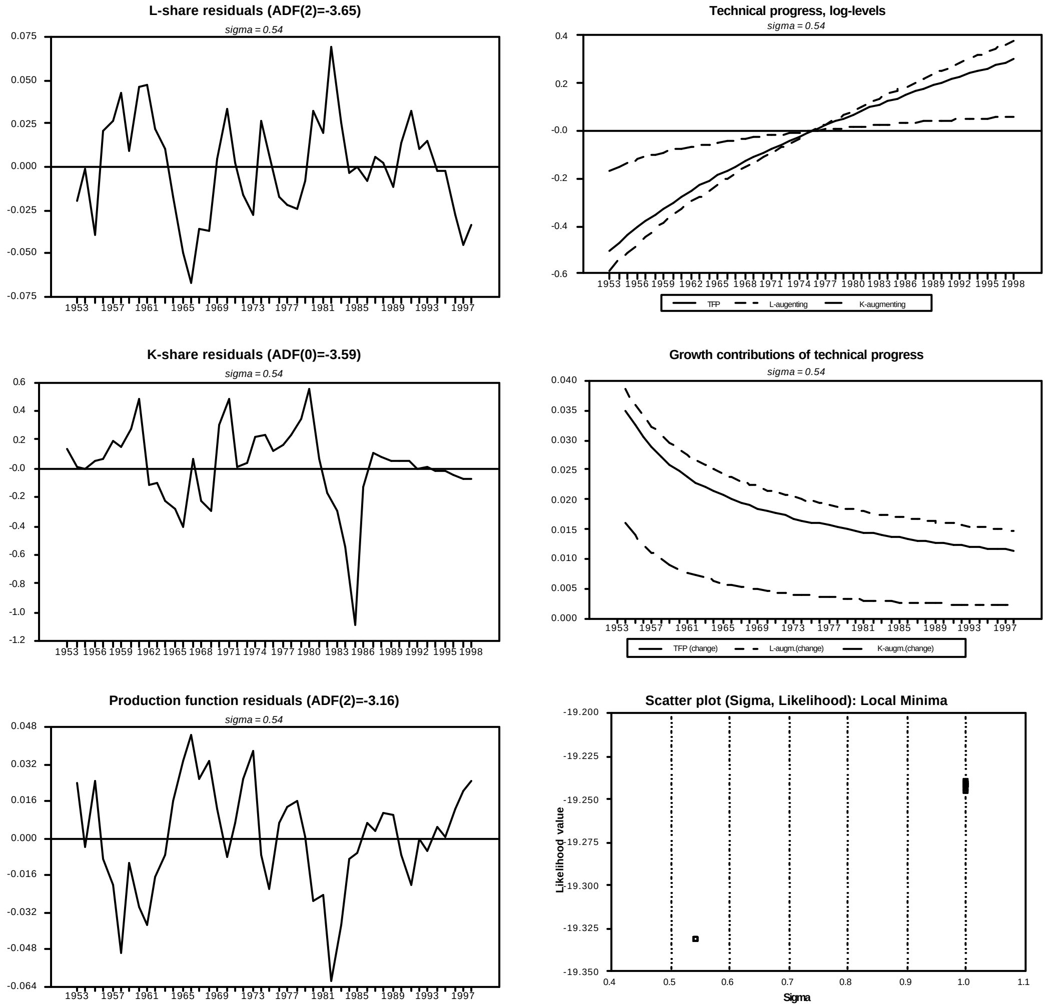
Properties of the supply-side system



Graphs 3.4. : Time-varying factor-augmenting technical growth (Box-Cox case) with logarithmic capital- augmenting technical growth.
(Labor Input: Hours, Labor Income: Self-employed labor share)

(Global Optimum)

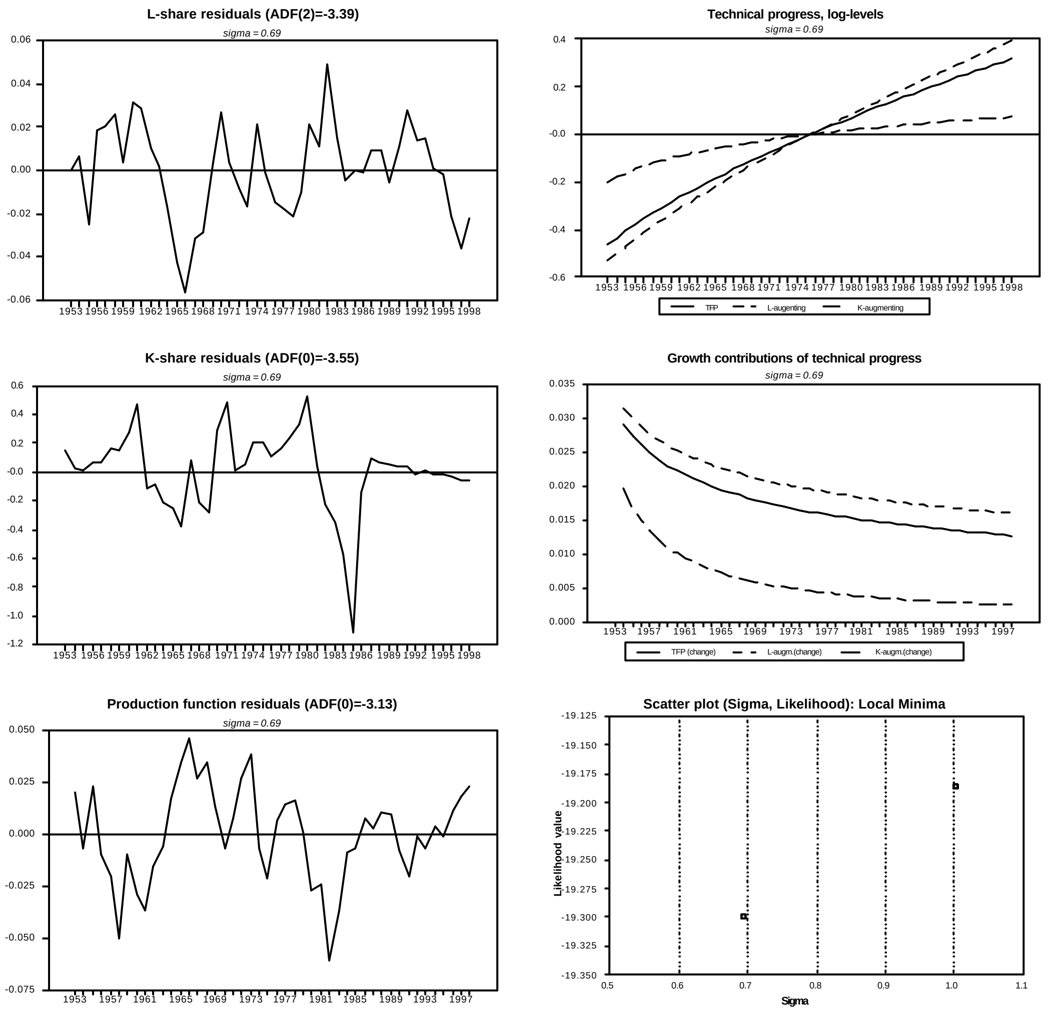
Properties of the supply-side system



Graphs 3.4. (Kmenta case): Time -varying factor-augmenting technical growth (box-cox case) with logarithmic capital- augmenting technical growth.
(Labor Input: Hours, Labor Income: Self-employed labor share)

(Global Optimum)

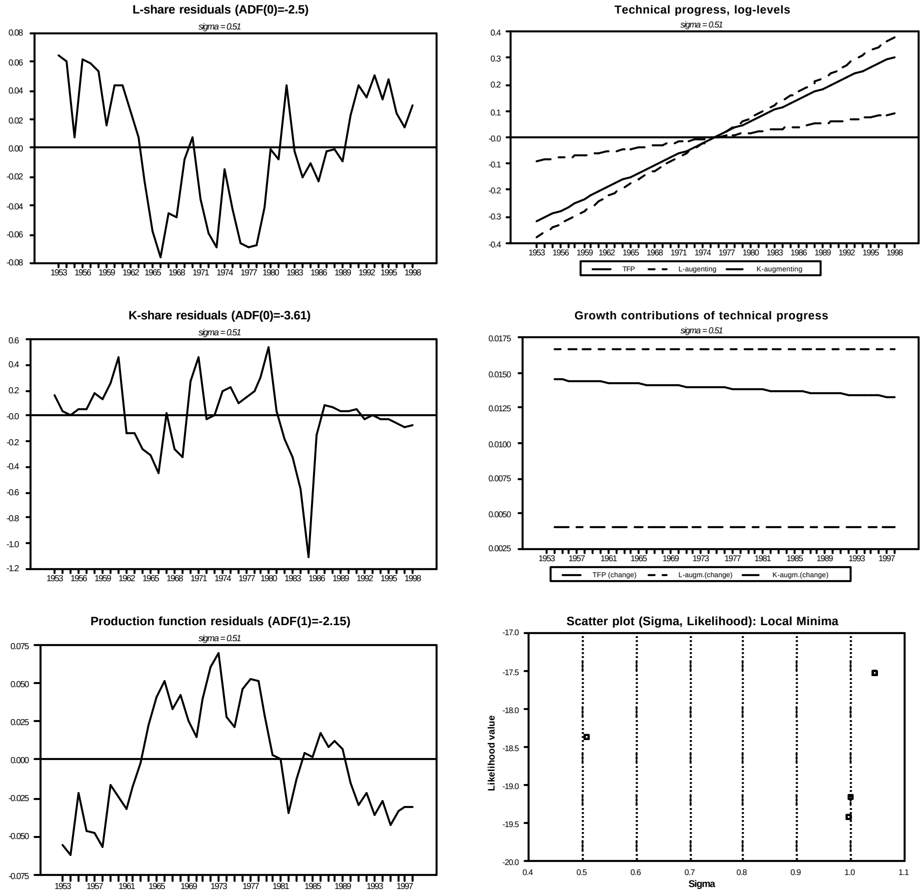
Properties of the supply-side system



Graphs 4.1: Constant factor-augmenting technical growth
(Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Local Optimum)

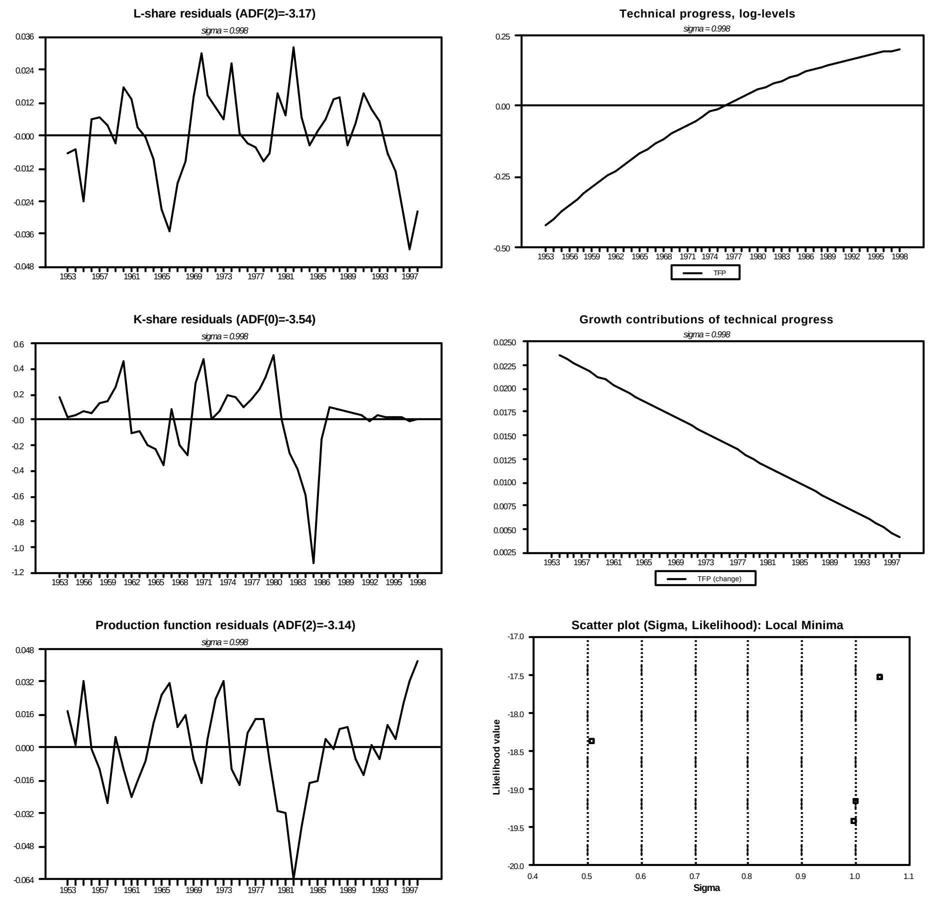
Properties of the supply-side system



Graphs 4.2: Constant Factor-Augmenting Technical Growth (Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Global Optimum)

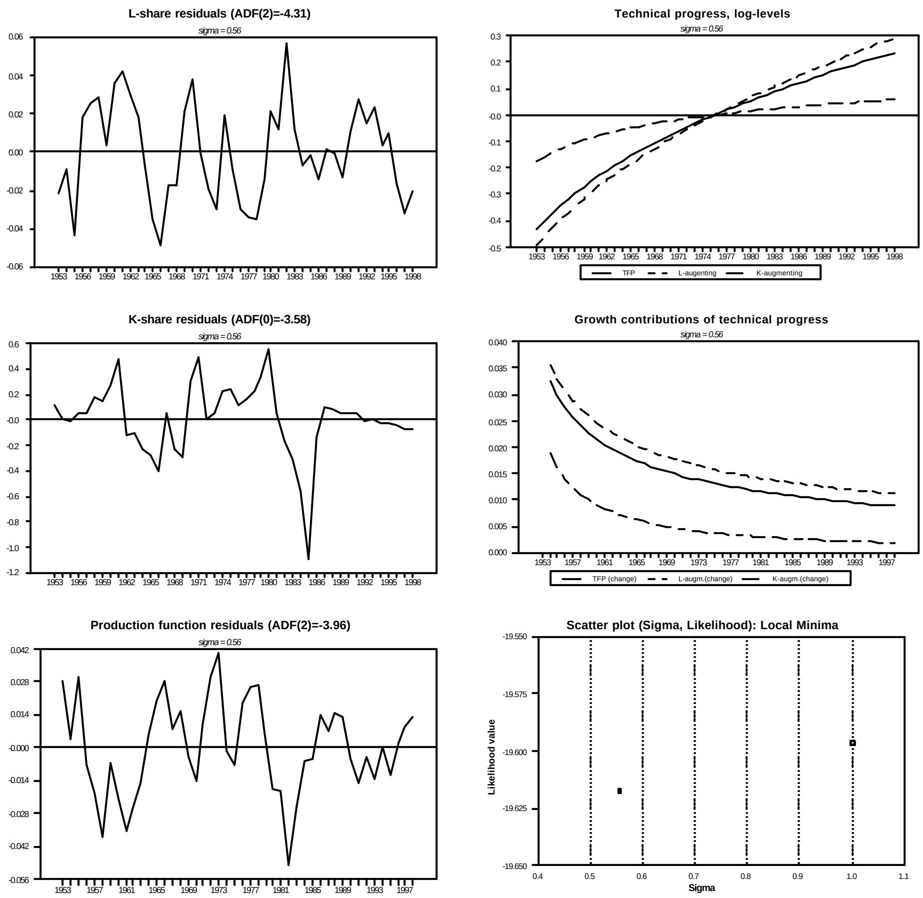
Properties of the supply-side system



Graphs 4.3. : Time -varying factor-augmenting technical growth (Box-Cox case)
(Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Global Optimum)

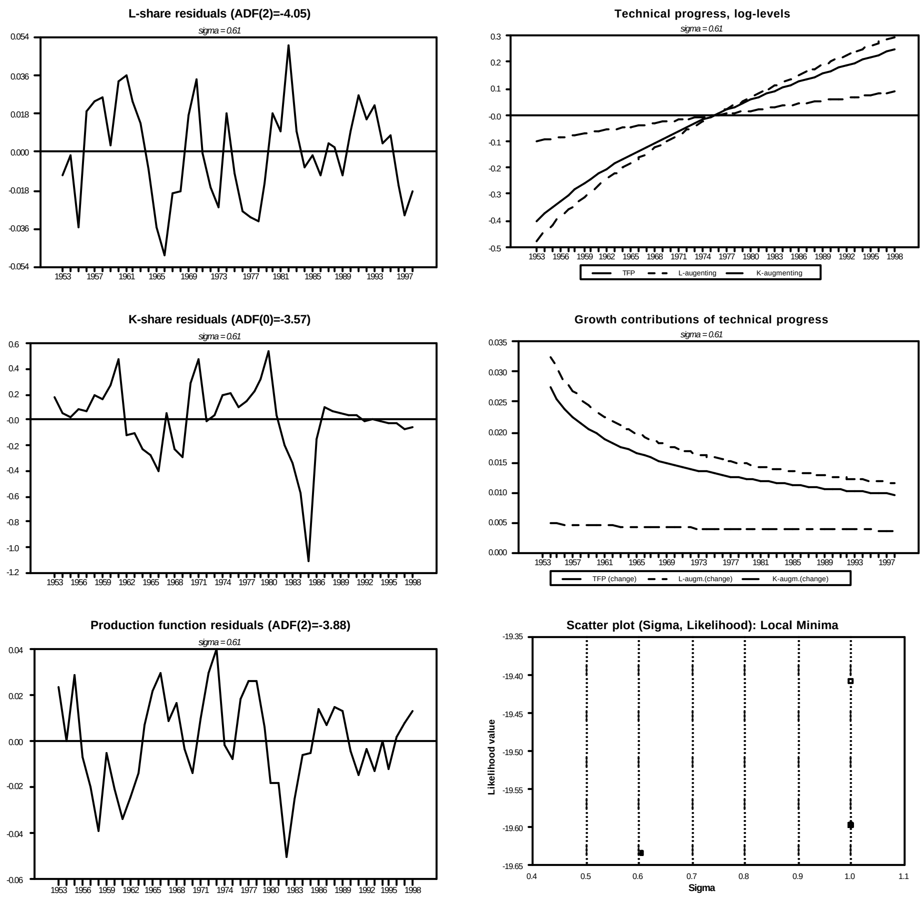
Properties of the supply-side system



Graphs 4.3. (Kmenta Case): Time -varying factor-augmenting technical growth (Box-Cox case)
(Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Global Optimum)

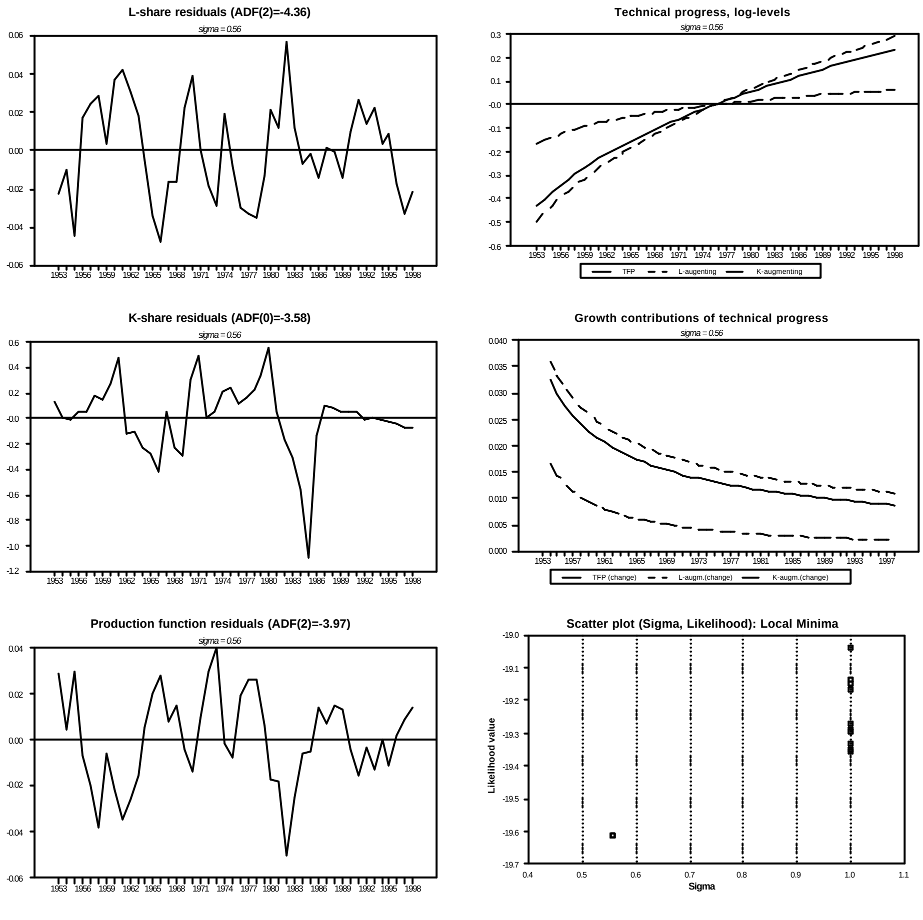
Properties of the supply-side system



Graphs 4.4. : Time-varying factor-augmenting technical growth (Box-Cox case) with logarithmic capital- augmenting technical growth.
(Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Global Optimum)

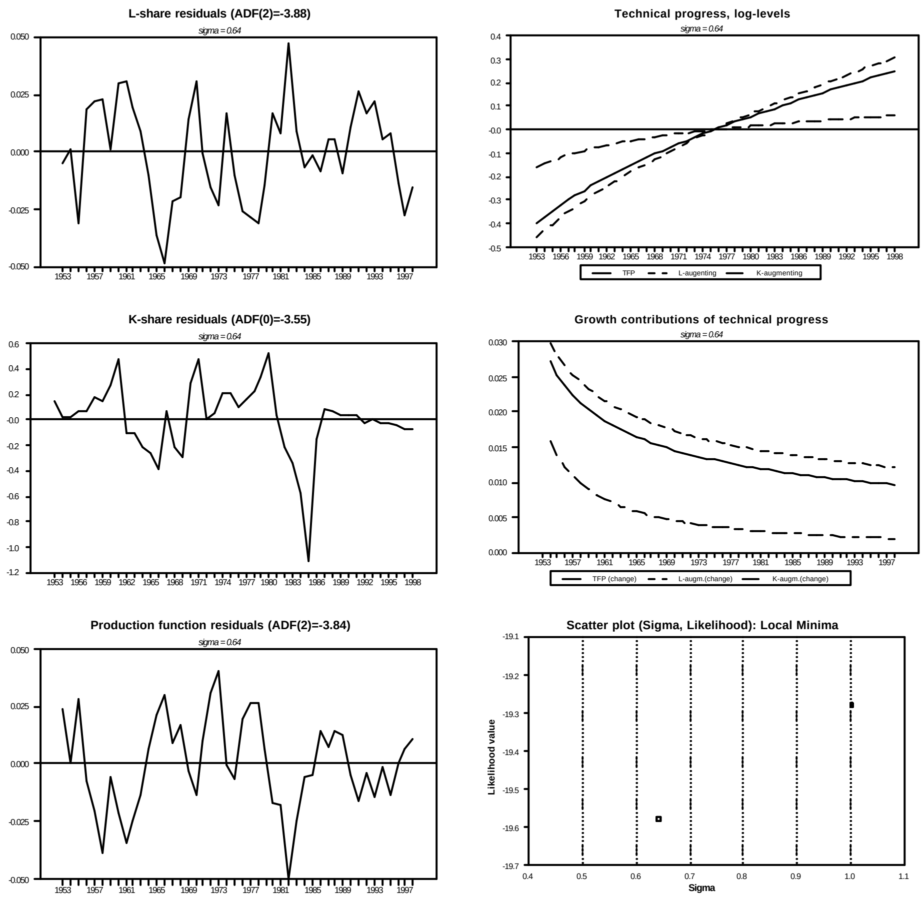
Properties of the supply-side system



Graphs 4.4. (Kmenta case): Time -varying factor-augmenting technical growth (box-cox case) with logarithmic capital- augmenting technical growth. (Labor Input: Quality-adjusted hours, Labor income: Self-employed labor share)

(Global Optimum)

Properties of the supply-side system



Appendix One: Normalization

From (2) and (3), it follows immediately that at the point in time of reference $t = t_0$ the expression (6) must hold.

Next, the factor income share ratio is calculated and at the point of reference set equal to the baseline values:

$$\frac{\partial F/\partial N_{t=t_0}}{\partial F/\partial K_{t=t_0}} = \frac{w_0}{q_0} \quad =>. \left(\frac{E_{t_0}^N N_0}{E_{t_0}^K K_0} \right)^{-r} = \frac{w_0 N_0}{q_0 K_0} = \frac{1-p_0}{p_0} .$$

This leads to the following expression for $E_{t_0}^N : E_{t_0}^N = \left(\frac{1-p_0}{p_0} \right)^{-\frac{1}{r}} \frac{K_0 E_{t_0}^K}{N_0}$.

Finally, total output according to (1) is calculated at the point of reference and set equal to the baseline value, leading to:

$$Y_0 = [(E_{t_0}^N * N_0)^{-r} + (E_{t_0}^K * K_0)^{-r}]^{-\frac{1}{r}} = \{[\left(\frac{1-p_0}{p_0} \right)^{-\frac{1}{r}} K_0 E_{t_0}^K]^{-r} + (K_0 E_{t_0}^K)^{-r}]^{-\frac{1}{r}} \text{ and}$$

$$Y_0^{-r} = \left(\frac{1-p_0}{p_0} \right) (K_0 E_{t_0}^K)^{-r} + (K_0 E_{t_0}^K)^{-r} .$$

With the help of this expression, one obtains (4) and (5).

Appendix Two: Recursive Estimation

To provide further evidence on the empirical application of the Normalization approach, we perform recursive estimation of our supply-side system (specifically, the following results refer to our preferred specification 2.4). We consider two exercises: first, to estimate incrementally (i.e., recursively) specification 2.4 where the fixed points and the scale parameter (*A*) are determined or estimated within each rolling sample; second, where these values are fixed at their full-sample values.

Fig1A embodies the time-varying sub-sample fixed points. We can see that the system embodies good convergence properties – that is to say, the recursive values converge very quickly to their long-run values, especially the technical progress parameters. For example, σ starts at 0.9 but exhibits thereafter a smooth downward profile towards a value of around 0.7 in the 1980s and then around mid-1990s it starts to dip to a lower value, this may represent a new regime or some protracted mean-reversion. So it may indicate that the late 1990s was somehow exceptional (either permanent or temporarily).

Notably the Scale parameter (*A*) has an upward trend, which mirrors inversely σ. A plausible explanation is that this relates to the non-linearity which becomes stronger the more sigma estimates deviates from unity. When σ in the early part of the estimation sample (close to unity) then also as expected scale parameter (*A*) is close to unity.

Now we look at **Fig 2A** (single full sample fixed-point average). Here, we derive more precise estimates from this procedure. Initial estimates closer to their full-sample ones: unsurprising since we use more information. The parameter σ converges from 0.9 to 0.6 albeit in a rather bumpy manner which appears to relate to the first oil crisis. A plausible explanation is that the data features around the first oil shock did not reflect developments in the rest of the sample. Nevertheless, the recursive procedure tries hard to fit this movement in the data (this appears inconsistent with this full-sample fixed point which captures (as desired in modeling the economy's long-run productive potential) the truly long-run features. This movement can also be seen in the other parameters.

The names in the below graphs correspond to the symbols as: *beta* (π), *eta* (μ), *gamma* (γ_N), *kamma* (γ_K), *laml* (λ_N), *scale* (*A*) and *sigma* (σ).

Figure. 2A. Recursive Estimates
(Time-Varying Sub-Sample Dependent Fixed-Point)

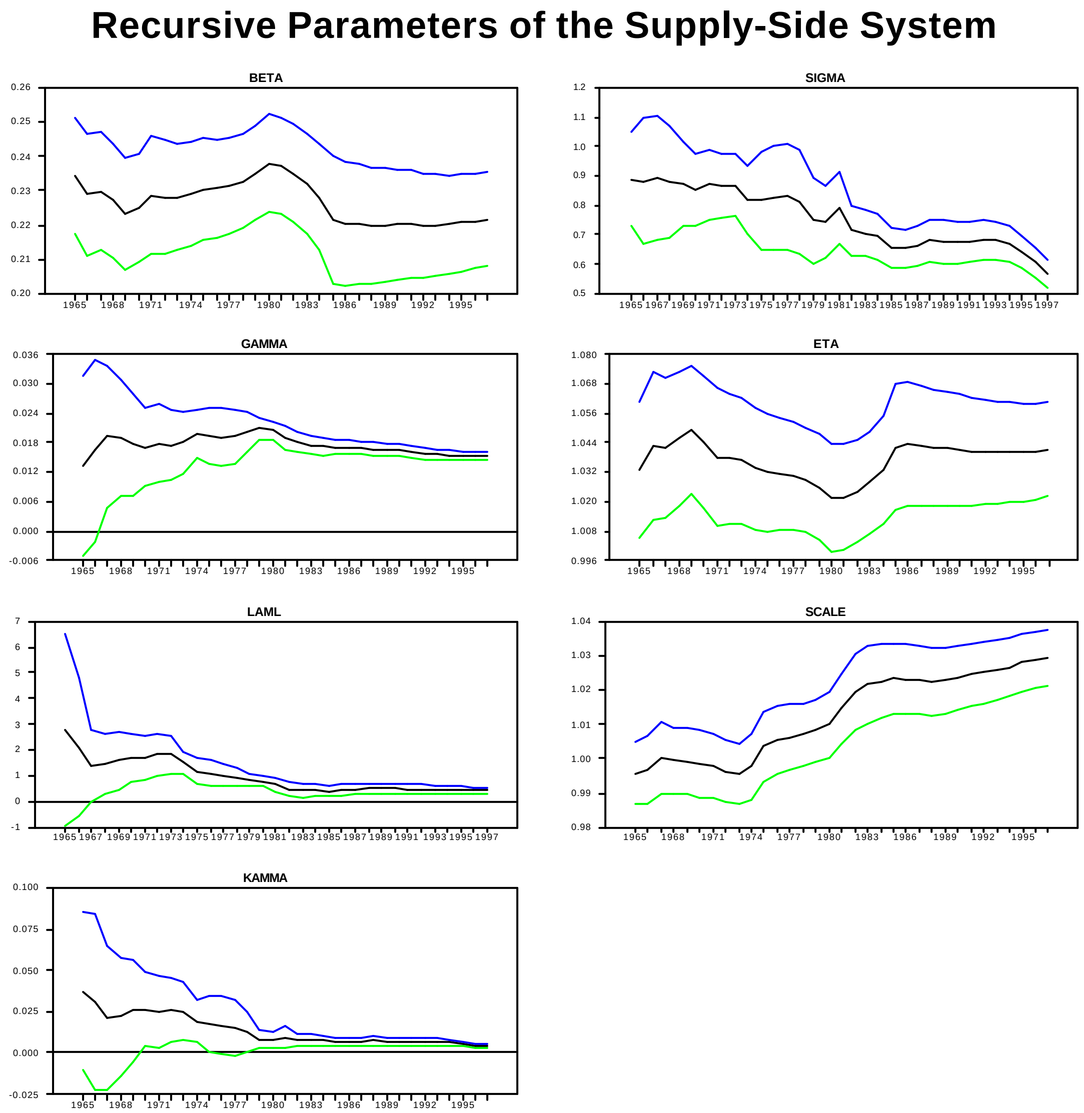


Figure. 2B— Recursive Estimates
(Single Full-Sample Dependent Fixed-Point)