# Risky Human Capital Investment, Income Distribution, and Macroeconomic Dynamics*

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**Abstract**

This paper demonstrates that the role of the personal income distribution for an economy's process of development through risky human capital accumulation critically depends on the shape of the saving function. Empirical evidence for the U.S. strongly suggests that the marginal propensity to save is increasing in income, a property which so far has not been allowed for in the literature on human capital, income distribution and macroeconomics. Doing so, the present analysis suggests that the impact of higher inequality on the aggregate human capital stock, and thus, on growth is positive under rather weak conditions. Results heavily rely on a positive impact of parents' income on children's human capital investments, which holds under standard assumptions on labor income risk and risk aversion in the model, and is largely supported by empirical evidence.

**Key words:** Growth, Income Distribution, Intergenerational Transfers, Risky Education, Saving Function.

**JEL classification:** I20; O11; O40.

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# 1 Introduction

This paper analyzes the interaction between intergenerational wealth transmission, human capital investments under uninsurable labor income risk, and economic growth in a small open overlapping-generations economy with heterogeneous agents. It demonstrates how the role of the personal income distribution for an economy's process of development through risky human capital accumulation depends on the shape of the saving function. The analysis suggests that when the *marginal propensity to save* (MPS) is increasing in income, the impact of higher inequality on the aggregate human capital stock and thus on growth is positive under rather weak conditions during the transition to a stationary equilibrium.

This result is novel to the literature on human capital, income distribution and macroeconomics, which so far has exclusively focussed on utility specifications which imply a non-increasing MPS (albeit a non-decreasing *average* propensity to save). However, empirical evidence strongly suggests that the MPS is increasing (not only in current but also in permanent income). For instance, in a recent paper, Dynan et al. (2004) show that this saving pattern is prevalent for the U.S. economy, allowing for various measures of savings and different time periods (see also Menchik and David, 1983, and Dynan et al., 2002, for similar evidence).

The mechanisms of the model heavily rely on a positive impact of parents' income on children's human capital investments, which holds under standard assumptions on labor income risk and risk aversion in the model. More specifically, individuals face idiosyncratic and nondiversifiable risk, like those associated with labor demand shocks for specific skills (e.g., Wildasin, 2000), health and disability risk, uncertainty about the quality of schooling, and uncertainty associated with social factors like the access to social networks.[1] Indeed, empirical studies find a positive and substantial effect

[1] The hypothesis that uninsurable labor income risk systematically affects incentives of risk-averse individuals to invest in human capital has received surprisingly little attention in the growth literature. See Gould et al. (2001), Bénabou (2002) and Krebs (2003) for notable exceptions, which, however, deal with different questions. Gould et al. (2001) are concerned with the evolution of wage inequality and its interaction with the rate of technical progress and its variance across sectors. Bénabou (2002)

of parents' income on human capital investments, even after controlling for parents' education, occupation and ability of both children and parents (e.g., Taubman, 1989; Sacerdote, 2002; Plug and Vijverberg, 2003).

The question how the personal income distribution affects an economy's process of development has stimulated the more recent macroeconomic theory and growth empirics like almost no other one (for a survey, see e.g. Aghion et al., 1999). Whereas empirical evidence is mixed,[2] in their pioneering theoretical contribution, Galor and Zeira (1993) show that inequality typically has an adverse effect on the process of development if credit markets are imperfect ("credit-market imperfections approach"). This is because poor individuals cannot borrow sufficiently high amounts to finance an indivisible level of schooling investments in their model.[3] Galor and Moav (2004) provide an important contribution for understanding historical development processes. They argue that inequality and growth are positively related in early stages of development when physical capital accumulation is the prime engine of growth. But when the return to capital falls over time, such that human capital investments become more attractive, the relationship between inequality and growth turns negative under binding borrowing constraints for the poor, as in Galor and Zeira (1993). In contrast to the credit-market imperfections approach, the positive relationship between parent's

examines the trade-offs of progressive income taxation for growth and efficiency, whereas Krebs (2003) studies the impact of labor income risk on growth in a framework with ex ante identical agents.

[2]Earlier empirical evidence suggests a negative link between inequality and growth (e.g., Alesina and Rodrik, 1994; Persson and Tabillini, 1994; Perotti, 1996). Using a new and comprehensive high-quality data set which allows to study panels, Deininger and Squire (1998) and Banerjee and Duflo (2003) find practically none, whereas Li and Zou (1998) and Forbes (2000) report a positive relationship.Allowing for a non-linear impact of inequality on GDP growth, Barro (2000) finds a negative relationship for developing countries and a positive relationship for more advanced countries. Finally, Banerjee and Duflo (2003) suggest that the change in inequality, in either direction, rather then its level is negatively associated with growth. Moreover, on basis of this finding, they argue that previous panel studies (particularly those relying on fixed effects) may have produced upward biased estimates of the effect of inequality on growth.

[3]As shown by Bénabou (1996) and Moav (2002), a negative relationship between inequaliy and growth can also be obtained by replacing this non-convexity in the education technology by the assumption of diminishing marginal returns to human capital investment. This implies that the aggregate human capital stock increases if educational investment is spread more equally. Another strand of literature deals with the role of imperfect capital markets for the relationship between wealth distribution and entrepreneurship, when project sizes (i.e., required physical capital investments to become entrepreneur) are fixed (e.g., Banerjee and Newman, 1991, 1993; Aghion and Bolton, 1997).

income and educational investment in the present model does not rely on individual constraints to borrow for educational investment. The proposed alternative micro-foundation based on labor income risk is consistent with the observation that family income is an important determinant of educational attainment even in advanced countries, where extensive provisions of college financial aid (like in the U.S.) or public education finance (prevalent in Europe) is supposed to remove credit constraints for human capital investments for the bulk of individuals. In fact, recent studies find no evidence for the relevance of educational borrowing constraints in the U.S. (see e.g. Cameron and Taber, 2004, and the references therein).[4]

Besides in the credit-market imperfections approach, it has been argued that high inequality is adversely related to growth because it induces high demand for redistributive taxation in the political process (e.g., Alesina and Rodrik, 1994; Persson and Tabellini, 1994), high fertility (e.g., Perotti, 1996), high social instability (e.g., Venieris and Gupta, 1986; Alesina and Perotti, 1996), low aggregate demand for R&D-intensive products (Zweimüller, 2000), and a low degree of the specialization of labor (Fishman and Simhon, 2002). In contrast, the classical view has suggested that wealth inequality is positively related to (investment-driven) growth. The foundation of this view by Bourguignon (1981) shows that under an increasing MPS, unegalitarian stable equilibria are even Pareto superior to an egalitarian stable equilibrium in the neoclassical growth model of Stiglitz (1969). The reason for this result is that physical capital accumulation, fueled by domestic savings, raises wages such that growth "trickles down" to less wealthy individuals.

This paper shows that the prediction of a positive inequality-growth relationship and a vital role of an increasing MPS, as hypothesized by the classical view, basically carries over when growth is driven by *human* capital accumulation (under idiosyncratic human capital risk) - albeit the mechanism is different. The analysis demonstrates that even in a small open economy, in which national savings are unrelated to physical cap-

[4] Of course, this does not mean that borrowing constraints play a minor role for developing countries as well.

ital investment (thus excluding the 'trickle-down' mechanism in e.g. Bourguignon, 1981), the relationship between inequality and an economy's the process of development critically depends on intergenerational wealth transmission and thus on savings behavior. According to the model, adults save in order to bequeath or to make inter vivos gifts, respectively,[5] and intergenerational transfers are optimally allocated to human capital investments and savings for future wealth of the young.[6] The idea of the theory proposed in this paper is that, with an increasing MPS, a decrease in inequality implies an average reduction in wealth transmission of the rich which may outweigh the average increase in wealth transmission of the poor. Consequently, if educational investments are increasing in family wealth, aggregate human capital investment may fall. Although some contributions have demonstrated that the relationship between inequality and human capital-based growth can be positive when there are poverty traps (e.g., Perotti, 1993; Galor and Tsiddon, 1997; Moav, 2002),[7] this paper shows that higher inequality may foster development even in advanced countries under empirically well-supported conditions.

The paper is organized as follows. Section 2 presents the basic structure of the model. Section 3 analyzes individual education and saving decisions. Section 4 examines the role of inequality for aggregate income dynamics. Section 5 discusses the main hypotheses which drive the results of this paper in the light of empirical evidence. The last section concludes. All proofs and an illustrative example are relegated to an appendix.

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[5] Such a "joy of giving" saving motive has received strong empirical support. See Carroll (2000) for an illuminating discussion of the empirical evidence.

[6] Thus overlapping-generations structure of the model builds on Galor and Moav (2004), as discussed in more detail below.

[7] For instance, if capital markets are imperfect and borrowing constraints are binding even for the rich (i.e., in very poor countries), redistribution to the rich enables more individuals to finance education (e.g., Perotti, 1993; Moav, 2002). Moreover, Galor and Tsiddon (1997) argue that for relatively poor economies, equality in the distribution of human capital may be an impediment to prosperity in the longer run under two conditions: first, the individuals' level of human capital positively depends on the parental level of human capital and, second, technological progress depends on the average level of human capital in the economy.

# 2 The Model

Consider a small open overlapping-generations economy with uninsurable risk of educational investments.

## 2.1 Production of Final Output

In every period, a single homogenous consumption good is produced according to a neoclassical, constant-returns-to-scale production technology. Output at time $t$, $Y_t$, is

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t), \quad k_t \equiv K_t/H_t, \tag{1}$$

where $K_t$ and $H_t$ are the amounts of physical capital and human capital employed in period $t$, the latter being measured in efficiency units. $f(\cdot)$ is a strictly monotonic increasing and strictly concave function which fulfills $\lim_{k\to\infty} f'(k) = 0$ and $\lim_{k\to 0^+} f'(k) = \infty$.

Output is sold to the world market in a perfectly competitive environment, with output price normalized to unity. The gross rate of return to capital, $R_t$, is internationally given and time-invariant, i.e., $R_t = \bar{R}$. Thus, profit maximization of the representative firm in any period $t$ implies that $k_t$ is given by $\bar{R} - 1 = f'(k_t)$. Thus, $k_t = (f')^{-1}(\bar{R} - 1) \equiv \bar{k}$. Consequently, the wage rate per efficiency unit of human capital, $w_t$, reads $w_t = f(\bar{k}) - \bar{k} f'(\bar{k}) \equiv \bar{w}$. Moreover, since $Y_t = H_t f(\bar{k})$ in this economy, $Y_t$ (i.e., the gross domestic product) grows at the same rate as the aggregate human capital stock $H_t$.[8]

## 2.2 Individuals and Education Technology

In each period, there is a unit mass of individuals with two-period lives. In the first period, individuals live by their parents and devote their entire time to acquire education. In the second period (adulthood), individuals supply their efficiency units of

[8] The capital-skill complementarity underlying production function (1) is empirically well supported; see e.g. Goldin and Katz (1998). This technology is common in the literature on income distribution, human capital and growth.

human capital to the labor market, and allocate their income between consumption and transfers to their offspring (i.e., bequests or inter vivos transfers, respectively). Intergenerational transfers (i.e., savings of adults) are optimally allocated (either by parent or child) between human capital investment and savings of the young for future wealth.[9] Individuals are identical with respect to their preferences and their ability to acquire human capital, but may differ in family wealth. So far, this overlapping-generations structure follows Galor and Moav (2004). However, in contrast to their model, individuals face idiosyncratic human capital risk,[10] which is uninsurable (e.g. Arrow, 1971).[11] As will become apparent below, this implies a positive relationship between human capital investments and parental income under empirically well-supported conditions. Moreover, to focus the analysis, suppose that individuals can freely borrow for educational purposes, e.g., due to public provision of financial college aid. This is not to deny that educational borrowing constraints are generally unimportant, although empirical evidence suggest that this is indeed the case in the U.S. (e.g. Cameron and Taber, 2004). However, when binding, they would just give an additional source for a positive relationship between educational investment and family wealth (for poorer households).

An individual $i$ born in period $t$ (a member $i$ of generation $t$) with investment $e_t^i$ (in units of the consumption good) in education obtains

$$h_{t+1}^i = h(e_t^i, \tilde{a}) \tag{2}$$

efficiency units of human capital. $\tilde{a}$ is a random variable which follows an i.i.d. process and is drawn each period from a (cumulative) distribution function $\Phi(\tilde{a})$ with support

[9] Human capital investments can be thought of both schooling and nonschooling forms of training.

[10] That is, the analysis employs the standard assumption that human capital investment is riskier than physical capital investment (Krebs, 2003). First, human capital risk is nondiversifiable since embodied in individuals, whereas diversified portfolios of financial capital can be held. Second, many forms of financial assets in advanced countries are indeed almost risk-free (e.g. government bonds).

[11] The present model also differs to Galor and Moav (2004) in that our small open economy assumption deliberately excludes the feedback mechanism from aggregate savings to factor prices (which underlies the results in Bourguignon, 1981) and preferences (specified below) allow for an increasing MPS.

$\mathcal{A} = [\underline{a}, \bar{a}] \subset \mathbb{R}$. The random shock realizes after investment decisions are made, i.e., at the end of the first period of life. The function $h(e, a)$ fulfills the following properties.

**A1.** For all $a \in \mathcal{A}$, $h_e(e, a) > 0$, $h_{ee}(e, a) \leq 0$, $h_a(e, a) > 0$, $h_{ea}(e, a) > 0$.

($h_e$ denotes the first partial derivative of $h$ with respect to $e$, etc.) $h_{ee} \leq 0$ implies that expected marginal returns to educational investment are non-increasing. The analysis explicitly allows for the case of non-diminishing returns to scholling, consistent with evidence from standard Mincer estimates. (See section 5.3 for a brief discussion.)[12] Moreover, given that $h_a > 0$, which merely serves as a convention, one can verify (available on request) that $h_{ea} > 0$ implies that the variance of earnings increases with human capital investment $e$.[13] This assumption is well-supported empirically, see e.g. Levhari and Weiss (1974) and, more recently, Pereira and Martins (2002, 2004).[14]

Denote by $s_t^i$ and $b_t^i$ the amount of savings invested in the financial market and the amount of wealth received by member $i$ of generation $t$, respectively, i.e., $s_t^i = b_t^i - e_t^i$. Thus, income of member $i$ of generation $t$ as an adult is given by

$$I_{t+1}^i = \bar{w}h_{t+1}^i + \bar{R}s_t^i = \bar{w}h(e_t^i, \tilde{a}) + \bar{R}\left(b_t^i - e_t^i\right) \equiv I(b_t^i, e_t^i, \tilde{a}). \tag{3}$$

Utility $U_t^i$ of member $i$ of generation $t$ is given by a function $u$ which is defined over

[12] This is not to deny that the return to schooling is ultimately diminishing due to physical constraints of human brain capacity, as sometimes argued in the literature. The relevant question, however, is whether this applies at the relevant range.

[13] A similar type of risk also underlies the model of Bénabou (2002). There are other notions of labor income risk. For instance, Gould et al. (2001) argue that an increasing variance of sectoral shocks increase educational attainment of workers because general education reduces the costs of moving across sectors.

[14] In contrast, if $h_{ea} < 0$ for all $a$, in addition to $h_a > 0$, then the variance of $\bar{w}h(e, \tilde{a})$ would decrease with $e$. To illustrate the intution for an *in*creasing variance of earnings in the data, suppose that there are two groups of individuals: a highly educated individual from the first group earns $9,000 with probability 1/2 and $11,000 with 1/2, whereas a less educated individual from the other group earns $1 with probability 1/2 and $1,999 with 1/2. Although most would agree that income from the latter type is riskier than that of the former (with a percentage fluctuation around the mean of 99.9 compared to 10 percent), the earnings variance of less educated individuals is lower ($999^2$ compared to $1000^2$). Thus, the increasing variance property employed here is consistent with the notion that less educated workers face considerably higher risk under various notions of risk.

consumption $c^i_{t+1}$ as an adult and transfer $b^i_{t+1}$ to her offspring ("joy of giving"), i.e.,

$$U^i_t = u(c^i_{t+1}, b^i_{t+1}). \tag{4}$$

**A2.** $u_c > 0$, $u_b > 0$, $u_{cc} < 0$, $u_{cc}u_{bb} - (u_{cb})^2 > 0$, $u_{cb} \geq 0$ and $\lim_{I\to\infty} u_c(I, 0) < \lim_{I\to\infty} u_b(I, 0)$.

According to assumption A2, $u$ is strictly monotonic increasing and strictly concave, which, as will become apparent below, implies risk aversion of individuals. Moreover, the latter two relations in A2 imply normality (for income levels which exceed some threshold) of intergenerational transfers, $b^i_{t+1}$, which is the empirically relevant case. Note that, in order to study the role of savings behavior for the relationship between inequality and growth, no particular functional form on utility is imposed.[15]

Finally, assume that there are two groups of dynasties in the initial period $t = 0$. A fraction $\lambda \in (0, 1)$ of ("rich") young individuals in $t = 0$ receives a transfer $b^R_0 > 0$ and a fraction $1 - \lambda$ of ("poor") individuals receives a transfer $b^P_0 \in [0, b^R_0)$ from their parent. Thus, in the aggregate, an amount $B_0 \equiv \lambda b^R_0 + (1-\lambda)b^P_0$ is initially transferred. Adult individuals possess an aggregate human capital stock $H_0$ in the initial period, i.e., initial output is $Y_0 = H_0 f(\bar{k})$.[16]

# 3 Individual Decisions

Note that income $I^i_{t+1} = I(b^i_t, e^i_t, \tilde{a})$ of an adult member $i$ of generation $t$ is a random variable ex ante, but is known in period $t+1$ (i.e., after realization of the shock) when

[15] Allowing for different types of saving behavior by employing a non-parametric utility function follows Bourguignon (1981).

[16] Introducing endogenous growth, e.g. by assuming that the aggregate human capital stock $H_t$ enters the education technology as positive externality (following Glomm and Ravikumar, 1992, among others), i.e., letting $h^i_{t+1} = h(e^i_t, \tilde{a}, H_t)$, does not alter the main insights of this paper. The main focus lies on the impact of initial inequality (in $t = 0$) on subsequent growth averaged over a longer period (i.e. on $Y_t/Y_0 - 1$, and thus on $Y_t$, $t \geq 1$), as usually examined in the empirical literature on inequality and growth. Introducing endogenous growth through human capital accumulation simply implies that initial inequality also affects the period-by-period growth rate $Y_t/Y_{t-1} - 1$ in a qualitatively similar fashion as $Y_t$ both during the transition to a stationary equilibrium and in the long run.

allocating income to consumption and transfer to her offspring. The budget constraint of such an individual in $t+1$ reads $c^i_{t+1} + b^i_{t+1} \leq I^i_{t+1}$. Thus, under the additional constraint $b^i_{t+1} \geq 0$, her optimal transfer in $t+1$ is given by

$$b(I^i_{t+1}) \equiv \arg\max_{b^i_{t+1} \geq 0} u(I^i_{t+1} - b^i_{t+1}, b^i_{t+1}). \tag{5}$$

$b(I)$ is called "saving function" (as $b^i_{t+1}$ equals forgone consumption of an adult) and has the following properties. (All results are proven in Appendix A.)

**Lemma 1.** *Under A2, there exists an income level* $\underline{I} \geq 0$ *such that* $b(I^i_{t+1}) > 0$ *and* $b'(I^i_{t+1}) > 0$ *for all* $I^i_{t+1} > \underline{I}$.

Lemma 1 shows that savings are increasing in income above a threshold income level. How the MPS, $b'(I)$, is affected by income, however, is generally ambiguous. To demonstrate how the dynamical system depends on the shape of the saving function, the following analysis will distinguish between the cases $b'' \leq 0$ and $b'' > 0$. It is important to note the difference between $b'' > 0$ on the one hand and an increasing *average* propensity of adults to save (which holds if $b(I)/I$ is increasing in $I$) on the other hand. For instance, following Galor and Zeira (1993), Moav (2002) and Galor and Moav (2004), among others, consider the saving function $b(I) = \delta[I - \vartheta]$ if $I > \vartheta$ and $b(I) = 0$ otherwise, $\delta > 0$, $\vartheta \geq 0$.[17] Thus, if $I \geq \vartheta > 0$, the average propensity of adults to save, $b(I)/I = \delta - \vartheta/I$, is increasing in $I$. However, as will become apparent below, since $b''(I) = 0$ for $I > \vartheta$, this particular functional form rules out a potentially positive effect of inequality on growth during the transition to the stationary state. In contrast, for instance, suppose $u(c, b) = \alpha c - \beta c^2 + \ln b$, an example which is discussed for illustrative purposes in more detail in Appendix B. It is easy to verify that, in addition to Lemma 1, $b'' > 0$ holds for all $I \geq 0 (= \underline{I})$ (as long as $u_c > 0$) under this utility specification, i.e., the MPS is strictly increasing in income. (See section 5.1 for a discussion of empirical evidence on saving behavior.)

[17]This can be derived by assuming that utility takes the form $u(c, b) = (1 - \delta)\ln c + \delta \ln(\gamma + b)$, $\gamma \geq 0$, $0 < \delta < 1$, where $\vartheta \equiv \gamma(1 - \delta)/\delta$.

From the optimal allocation of income earned as an adult, we can derive the following properties of indirect life-time utility,

$$v(I^i_{t+1}) \equiv u(I^i_{t+1} - b(I^i_{t+1}), b(I^i_{t+1})). \tag{6}$$

**Lemma 2.** *Under A2. $v(I)$ is a strictly monotonic increasing and strictly concave function.*

According to Lemma 2, individuals are risk-averse. Throughout the remainder of the paper, the assumption of "decreasing absolute risk aversion" is maintained, which has gained overwhelming empirical support.[18]

**A3.** $A(I) \equiv -v''(I)/v'(I)$ is strictly decreasing in $I$.

An amount of transfers $b^i_t$ received by a member $i$ of generation $t$ is allocated to savings for future wealth, $s^i_t$ (with a safe gross rate of return $\bar{R}$), and the risky investment in education, $e^i_t$, to maximize expected life-time utility $E\left[v(I^i_{t+1})\right] = E\left[v(I(b^i_t, e^i_t, \tilde{a}))\right]$, where $E$ is the expectation operator. Thus, using (3), the optimal human capital investment is given by

$$e(b^i_t) \equiv \arg\max_{e^i_t \geq 0} E\left[v\left(\bar{w}h(e^i_t, \tilde{a}) + \bar{R}\left(b^i_t - e^i_t\right)\right)\right]. \tag{7}$$

For simplicity, the analysis exclusively focusses on an interior solution of this optimization problem. One can then derive the following result.

**Proposition 1.** (Human capital investment). *Under A1 and A3, if $e^i_t > 0$, the human capital investment is strictly increasing in family wealth, i.e., $e'(b^i_t) > 0$.*

[18] Decreasing absolute risk aversion is consistent with observed behavior in the context of portfolio decisions in financial markets (e.g. Carroll, 2002), occupational choice, demand for insurance and other household decisions (e.g. Gollier, 2001). In an interesting empirical study, Guiso and Paiella (2001) present survey evidence which clearly rejects the hypothesis that the degree of absolute risk aversion is non-decreasing.

Proposition 1 coincides with a result derived in the pioneering work on risky education by Levhari and Weiss (1974), who consider a two-period model with exogenous wealth. (See also Eaton and Rosen, 1980.) The intuition for the result is the following. Under convention $h_a > 0$, the assumption $h_{ea} > 0$ implies that risk (i.e., the variance of earnings) is increasing with the level of investment in human capital. In contrast, investing in physical capital (i.e., financial assets) is risk-free. Hence, if the degree of absolute risk aversion, $A(I)$, is decreasing in income $I$, such that $A(I^i_{t+1}) = A(I(b^i_t, \cdot, \cdot))$ is decreasing in $b^i_t$, then individuals with larger $b^i_t$ invest more in risky education.

Proposition 1 is well-supported empirically even in advanced economies in which credit constraints seem to play a negligible role for human capital investments.[19] (See section 5.2 for a brief review of empirical evidence.) There is neither a theoretical prediction nor (to my knowledge) empirical evidence, however, on how the marginal propensity to invest in education, $e'(b^i_t)$, changes with $b^i_t$.[20] The following assumption focusses the analysis on the critical hypotheses in this paper (in addition to Proposition 1), i.e., the relationship between earnings and human capital investments on the one hand and the shape of the saving function, $b(I)$, on the other hand.

**A4.** The impact of a change in family wealth $b^i_t$ on the magnitude of the marginal propensity to invest in education is negligible, i.e., $|e''(b^i_t)| \approx 0$ for all $b^i_t \in \mathbb{R}_+$.

The role of Assumption A4 is discussed throughout. It turns out that relaxing A4 only affects the impact of a change in inequality on the process of development in the

[19] Note that without uncertainty, i.e., if $\underline{a} = \bar{a} \equiv a$, the optimal schooling investment in an interior solution (which requires $h_{ee} < 0$ under certainty) is given by $Wh_e(e^i_t, a) = R$, according to (7). Thus, under certainty, $e^i_t$ is independent of $b^i_t$. For instance, this coincides with a result by Galor and Moav (2004) when credit constraints are not binding in their model.

[20] There exist some estimates for the impact of parental income on children's earnings which allow for non-linearity. Whereas Becker and Tomes (1986) suggest that the marginal impact is diminishing, if anything, Behrman and Taubman (1990) find a positive marginal impact. To see how these findings relate to the present model, first, define earnings of a member $i$ of generation $t$ as function of her parent's income (suppressing $\tilde{a}$): $E(I^i_t) \equiv \bar{w}h(e(b(I^i_t)), \cdot)$, where the relationships $e^i_t = e(b^i_t)$ and $b^i_t = b(I^i_t)$ have been used. Thus, $E'(I) = \bar{w}h_e(e(b(I)), \cdot)e'(b(I))b'(I)$ and $E''(I) = \bar{w}\left[h_{ee}e'(b)^2 b'(I)^2 + h_e e''(b)b'(I)^2 + h_e e'(b)b''(I)\right]$. Hence, for instance, $E''(I) \approx 0$ if $e''(b) \approx 0$ and the effects arising from $h_{ee}$ and $b''(I)$ approximately cancel.

subsequent period ('short run') - but not beyond that - in a systematic way.[21]

# 4 The Role of Inequality for Income Dynamics

This section studies the aggregate behavior of the economy which results from individual decisions analyzed in the preceding section. In particular, we examine how the distribution of initial family wealth, for a given initial aggregate transfer $B_0$ (and conditional on initial GDP, $Y_0 = H_0 f(\bar{k})$), affects the process of development. For this purpose, it is useful to recall the dynamical system of the considered economy.

A given transfer $b_t^i$ to a member $i$ of generation $t$ is optimally allocated to savings when young, $s_t^i$, and educational investments, $e_t^i = e(b_t^i)$, in period $t$ (Proposition 1). According to (2), this leads to an individual amount

$$h_{t+1}^i = h(e(b_t^i), \tilde{a}) \equiv \hat{h}(b_t^i, \tilde{a}) \tag{8}$$

of efficiency units of human capital, supplied during adulthood (which is a random variable). Thus, denoting the economy's c.d.f. of family wealth in period $t$ by $\Psi_t(b)$, with support $\mathcal{B}_t \subset \mathbb{R}_+$,[22] the aggregate human capital stock at $t+1$ is given by

$$H_{t+1} = \int_{\mathcal{A}} \int_{\mathcal{B}_t} \hat{h}(b, \tilde{a}) d\Psi_t(b) d\Phi(\tilde{a}). \tag{9}$$

Aggregate income is given by $Y_{t+1} = H_{t+1} f(\bar{k})$.[23]

[21] It is interesting to see assumption A4 in the light of a model in which there is no uncertainty and a credit market is fully absent (e.g. Galor and Moav, 2004). In this case, one additional unit of wealth is entirely devoted to education when human capital investment is below the level which would be optimal if there were no educational borrowing constraint. That is, $e' = 1$ for poor households (whereas $e' = 0$ when a household is not credit-constrained), i.e., (when $e(b)$ is differentiable) $e'' = 0$ holds exactly in such a framework.

[22] Note from the assumptions on initial conditions that $\mathcal{B}_0 = \{b_0^P, b_0^R\}$ and $\Psi_0(b) = 0$ for $0 \leq b < b_0^P$, $\Psi_0(b) = 1 - \lambda$ for $b_0^P \leq b < b_0^R$, and $\Psi_0(b) = 1$ for $b \geq b_0^R$.

[23] Note that the human capital risk considered in the model is consistent with risk associated with skill specificity in the following sense. Suppose individuals acquire skills which are applicable in a single "industry" only (which may also be interpreted as specific task) and there are ideosyncratic productivity shocks across industries (Wildasin, 2000). To see that this is consistent with the risk considered here, suppose there is a continuum $[0, 1]$ of intermediate goods industries, indexed by $j$. Output

According to (3) and (8), given realization $a$ of the random variable $\tilde{a}$ after educational investments are made, income in $t+1$ of an adult individual $i$ reads

$$I_{t+1}^{i}=\bar{w}h(e(b_t^i),a)+\bar{R}\left(b_t^i-e(b_t^i)\right)=I(b_t^i,e(b_t^i),a)\equiv\hat{I}(b_t^i,a). \tag{10}$$

$I_{t+1}^{i}$ is then optimally allocated to consumption, $c_{t+1}^{i}$, and transfers to the offspring,

$$b_{t+1}^{i}=b(I_{t+1}^{i})=b(\hat{I}(b_t^i,a))\equiv\hat{b}(b_t^i,a). \tag{11}$$

According to (11), the wealth transfer within each dynasty $i$ follows a discrete time Markov process defined by the difference equation $b_{t+1}^{i}=\hat{b}(b_t^i,\tilde{a})$.

Due to the small open economy assumption, there are no feedback effects through factor price changes from aggregate variables to individual behavior. This not only simplifies the analysis but also excludes the mechanism suggested by the classical view: i.e., that higher inequality fosters growth fueled by domestic accumulation of physical capital if the MPS is increasing. It will turn out, however, that wealth accumulation nevertheless plays a fundamental role for the inequality-growth relationship by affecting resources available for human capital accumulation at the individual level.

For simplicity, let us restrict attention to the case in which even for the worst realizations of $\tilde{a}$, income of an adult increases without bound with the amount of transfer received as child.[24] Formally, this means the following.

$q_t(j)$ in industry $j$ at $t$ is produced with industry-specific human capital $\hat{H}_t(j)\equiv\int_{\mathcal{B}_t}\hat{h}(b,\theta(j))d\Psi_t(b)$, where where $\theta(j)$ is the realization of an i.i.d. shock $\tilde{\theta}$. (Note that because industries are symmetric and $b$ and $\tilde{\theta}$ are independently distributed, skill supply across industries is fully symmetric.) Suppose this production technology simply reads $q_t(j)=\hat{H}_t(j)$. Thus, the the aggregate stock of human capital in period $t+1$, $H_{t+1}=\int_{j\in[0,1]}\hat{H}_t(j)dj=\int_{j\in[0,1]}q_t(j)dj$ can be thought of a composite input of (perfectly substitutable) intermediate goods. Let $\tilde{\theta}$ and $\tilde{a}\in\mathcal{A}$ (still with c.d.f. $\Phi(\tilde{a})$) be related such that $\theta(j)\equiv\Phi^{-1}(j)$ is the realization of the shock in industry $j\in[0,1]$ (i.e., after realization of shocks, industries are ordered such that $j=\Phi(a)$, where $a$ is a realization of $\tilde{a}$). Thus, one can write $H_{t+1}=\int_{j\in[0,1]}\int_{\mathcal{B}_t}\hat{h}(b,\tilde{\theta}(j))d\Psi_t(b)dj=\int_{\mathcal{A}}\int_{\mathcal{B}_t}\hat{h}(b,\tilde{a})d\Psi_t(b)d\Phi(\tilde{a})$, which coincides with (9).

[24] Moreover, it is implicitly assumed throughout the paper that any young individual with zero wealth is able to pay back the loan $\bar{R}e(0)$, which equals such an individual's optimal amount of lending, by her labor income even for the worst realization of the shock $\underline{a}$, i.e., $\hat{I}(0,\underline{a})=\bar{w}h(e(0),\underline{a})-\bar{R}e(0)\geq 0$.

**A5.** For all $a \in \mathcal{A}$, $\hat{I}_b(b,a) > 0$, $b \in \mathbb{R}_+$, and $\lim_{b\to\infty} \hat{I}(b,a) = \infty$.[25]

The next two results characterize the Markov process $b^i_{t+1} = \hat{b}(b^i_t, \tilde{a})$.

**Lemma 3.** *Under A2 and A5, for all $a \in \mathcal{A}$. There exists $\underline{b}_a \geq 0$ such that $\hat{b}(b,a) > 0$ and $\hat{b}_b(b,a) > 0$ for all $b > \underline{b}_a$.*

Lemma 3 is easy to understand. Since the saving function $b(I)$ is strictly increasing above a threshold income level $\underline{I}$ (Lemma 1) and income as an adult is higher when having received a higher transfer (from A5), the transfer left to the offspring is positive and (above a threshold level) strictly increasing in that received as child. Since intergenerational transfers are increasing in income, which itself depends on the realization of the random shock, $a$, this threshold level, $\underline{b}_a$, also depends on $a$.

For notational simplicity in what follows, suppress the random variable $\tilde{a}$ by defining wealth and income one period after the level of wealth started in $b^i_t$ (indicated by subscript '1') as

$$\tilde{b}_1(b^i_t) \equiv \hat{b}(b^i_t, \tilde{a}), \tag{12}$$

$$\tilde{I}_1(b^i_t) \equiv \hat{I}(b^i_t, \tilde{a}). \tag{13}$$

We can then derive the following.[26]

**Lemma 4.** *Under A1-A5, for any realization $a \in \mathcal{A}$ of the random shock $\tilde{a}$. (i) If $b''(I) \leq 0$, then $\tilde{b}''_1(b) \leq 0$. (ii) If, by contrast, $b''(I) > 0$ and $h_{ee} = 0$, then $\tilde{b}''_1(b) > 0$. (iii) Otherwise, the sign of $\tilde{b}''_1(b)$ is ambiguous.*

According to part (i) of Lemma 4, if the saving function $b(I)$ is concave (i.e., if $b''(I) \leq 0$), then the transfer of an adult to her offspring will be concave in the

[25] Using (10), it is easy to verify that $e'(b) \leq 1$ is sufficient for $\hat{I}_b(b,a) > 0$ to hold. That is, if a marginal increase in $b$ does not lead to a decline of investment in the financial market, then $\hat{I}_b > 0$. However, although plausible, $e'(b) \leq 1$ is not ensured by the assumptions made so far. One can show, for instance, that $e'(b) < 1$ if $-\frac{A'(\hat{I}(b,a))\bar{w}}{A(\hat{I}(b,a))} \geq \frac{h_{e\tilde{a}}(e(b),a)}{h_{\tilde{a}}(e(b),a)h_e(e(b),a)}$ for all $a \in \mathcal{A}$.

[26] Note that in the case $b(I) = 0$ for $I \leq \underline{I}$ and $b(I) > 0$ for $I > \underline{I}$, $\underline{I} \geq 0$, the function $b(I)$ is not differentiable at $I = \underline{I}$ (and $b'(I) > 0$ for $I > \underline{I}$ under A2, according to Lemma 1). The subsequent analysis neglects this for simplicity, implicitly stating results for $I \neq \underline{I}$ only.

transfer she received as a child (i.e., $\tilde{b}_1''(b) \leq 0$). However, if the MPS (for adults) is increasing (i.e., if $b''(I) > 0$), the transfer of an adult to her offspring may be convex in the transfer received herself as a child (i.e., $\tilde{b}_1''(b) > 0$ is possible). Indeed, if the return to education is non-diminishing, then $\tilde{b}_1''(b) > 0$ will typically (e.g., under A4) hold under the empirically supported case of an increasing MPS (part (ii)). This is because an individual which received a higher transfer $b$ as child has higher (expected) income as adult (recall $\hat{I}_b > 0$), which positively affects the MPS if $b'' > 0$. As will become apparent, this is the crucial argument which gives rise to a potentially positive relationship between inequality and growth. However, if $h_{ee} < 0$ (diminishing returns to education), the marginal increase in income from a higher $b$ received is typically diminishing in $b$ (i.e., $\hat{I}_{bb} < 0$). This is a counteracting effect to the one arising from an increasing MPS, which explains part (iii) of Lemma 4.

The consequences of Lemma 4 for the process of development are analyzed in the following.

## 4.1 Inequality and the Process of Development

Recall that, initially, there are two groups of individuals, rich and poor, and the aggregate initial transfer is $B_0 = \lambda b_0^R + (1-\lambda) b_0^P$. To study the role of inequality for the growth process, suppose the distribution of initial transfers changes (in a lump-sum fashion) to

$$\breve{b}_0^R \equiv b_0^R - \varepsilon, \quad \breve{b}_0^P \equiv b_0^P + \varepsilon\lambda/(1-\lambda), \tag{14}$$

i.e., aggregate family wealth at $t = 0$, $B_0$, is held constant. Under restriction $\varepsilon < (1-\lambda)(b_0^R - b_0^P)$, i.e., as long as $\breve{b}_0^P < \breve{b}_0^R$, the economy is said to be more equal, the higher $\varepsilon$.[27] This subsection derives comparative-static results with respect to changes in $\varepsilon$ in the transition path.[28]

[27] Note that income inequality and inequality of family wealth are closely related, according to Lemma 1.

[28] Examining the transition path is consistent with the usual modelling approach in growth empirics which relies on some hypothesis of conditional convergence (e.g., Barro, 1991, 2000). That is, regression analysis regarding the determinants of economic growth, like inequality of income, usually control for

### 4.1.1 Short Run Impact of Higher Equality

Let $g_{s,t} \equiv (Y_s - Y_t)/Y_t$ be the growth rate of aggregate output (or GDP, respectively) between periods $s$ and $t$, $s > t \geq 0$. First, consider the impact of an increase in $\varepsilon$ on the aggregate human capital stock $H_1$, which gives us the *short run* effect of higher equality on aggregate (or per capita) income, $Y_1 = H_1 f(\bar{k})$, and thus, on the initial growth rate $g_{1,0} = H_1/H_0 - 1$ of the economy (recall $Y_0 = H_0 f(\bar{k})$). Again, suppressing the random variable for notational simplicity, let

$$\tilde{h}_1(b_t^i) \equiv \hat{h}(b_t^i, \tilde{a}) \tag{15}$$

denote the (risky) human capital level of an individual which has received a transfer $b_t^i$. The aggregate human capital stock in period 1, $H_1$, can be written as

$$\begin{aligned} H_1 &= E\left[\lambda \tilde{h}_1(\breve{b}_0^R) + (1-\lambda)\tilde{h}_1(\breve{b}_0^P)\right] \\ &= E\left[\lambda \tilde{h}_1\left(b_0^R - \varepsilon\right) + (1-\lambda)\tilde{h}_1\left(b_0^P + \varepsilon\lambda/(1-\lambda)\right)\right] \equiv \hat{H}_1(\varepsilon), \end{aligned} \tag{16}$$

according to (9) and (14). From this, we obtain the following result.

**Proposition 2.** (Impact of higher equality in the short run). *Suppose A1, A3 and A4 hold. Then a reduction in inequality of initial family wealth is associated with higher aggregate income $Y_1$ (and thus, faster growth $g_{1,0}$) when the return to education is diminishing, i.e., $\hat{H}_1'(\varepsilon) > 0$ if $h_{ee} < 0$. If $h_{ee} = 0$, then $\hat{H}_1'(\varepsilon) \approx 0$.*

Proposition 2 is in line with a standard result in the literature on inequality and growth when growth is driven by human capital investments (e.g., Galor and Zeira, 1993; Bénabou, 1996; Moav, 2002). Intuitively, if individual human capital investment is an increasing function of wealth, which holds according to $e'(\cdot) > 0$ (Proposition 1), (initial) educational investment is spread over the population more equally when wealth

the level of per capita income in some base year (properly instrumented) to account for the stage of development of an economy, like initial GDP, $Y_0$. Usually, $Y_0$ has significant effects, suggesting that observed economies are not yet close to stationary equilibria.

is distributed more equally. Thus, under diminishing marginal returns to education (i.e., $h_{ee} < 0$), the aggregate human capital stock typically increases in the short run when wealth is redistributed lump sum. In contrast, if $h_{ee} = 0$, then inequality is basically unrelated to short-run growth under A4.

If we would relax A4 and suppose $h_{ee} = 0$, then $\hat{H}_1'(\varepsilon) > (<)0$ if $e'' > (<)0$. That is, if the marginal propensity to invest in education is decreasing in $b$, then (lump-sum) redistribution to the poor would imply faster short-run growth. That is, $e'' < 0$ has the same qualitative effect as diminishing returns to education in the short run. In the medium run, however, the role of $e''$ becomes unsystematic, as can be deduced from the subsequent analysis (and is illustrated in Appendix B). The remainder of this section thus focusses on the case in which A4 holds.

#### 4.1.2 Medium Run Impact of Higher Equality

As shown next, the impact of a change in inequality in the medium run may be rather different to that in the short run, when $b'' > 0$ is allowed for.

Given family wealth $b_t^i$ in period $t$, human capital of a member $i$ of generation $t+1$ in period $t+2$, for some realizations of random shocks regarding the own level of human capital and that of her parent, respectively, is

$$h_{t+2}^i = \tilde{h}_1(b_{t+1}^i) = \tilde{h}_1(\tilde{b}_1(b_t^i)) \equiv \tilde{h}_2(b_t^i), \tag{17}$$

according to (15) and (12), respectively. The following auxiliary result is established.

**Lemma 5.** *For any realizations of random shocks.*

(a) *Under A1-A3 and A5,* $\tilde{h}_2'(b) > 0$ *if* $b'(I) > 0$.

(b) *Under A1-A5:* (i) *If* $b''(I) \leq 0$, *then* $\tilde{h}_2''(b) \leq 0$*;* (ii) *If, by contrast,* $b''(I) > 0$ *and* $h_{ee} = 0$, *then* $\tilde{h}_2''(b) > 0$.*(iii) Otherwise, the sign of* $\tilde{h}_2''(b)$ *is ambiguous.*

Regarding Lemma 5 (a), if the MPS is positive, then a higher transfer, $b$, received as child raises the own child's level of human capital, for given realizations of random shocks. Lemma 5 (b) concerns the question how this marginal effect of $b$ in Lemma 5 (a) changes with $b$. As is intuitive from Lemma 4, if the transfer $\tilde{b}_1(b)$ left as adult is strictly convex in the own transfer $b$ received and educational investments are not subject to diminishing returns, the marginal impact of an increase in $b$ on the human capital level of the own child is typically increasing in $b$ (part (ii) of Lemma 5 (b)), for given realizations of random shocks. This can never happen if the saving function, $b(I)$, is concave (part (i)). If $h_{ee} < 0$, then an increase in education of an individual and her child which is triggered by an increase in $b$ received by this individual is subject to diminishing returns. This counteracts the effect which arises if the MPS is increasing in income, which explains part (iii) of Lemma 5.

Using (14) and (17), the aggregate human capital stock in period 2 may be written as

$$H_2 = E\left[\lambda E\left[\tilde{h}_2\left(b_0^R - \varepsilon\right)\right] + (1-\lambda)E\left[\tilde{h}_2\left(b_0^P + \varepsilon\lambda/(1-\lambda)\right)\right]\right] \equiv \hat{H}_2(\varepsilon). \qquad (18)$$

This leads to the following result.

**Lemma 6.** *Under A1-A5. (i) If $b''(I) \leq 0$, then $\hat{H}_2'(\varepsilon) \geq 0$. (ii) If, by contrast, $b''(I) > 0$ and $h_{ee} = 0$, then $\hat{H}_2'(\varepsilon) < 0$. (iii) Otherwise, the sign of $\hat{H}_2'(\varepsilon)$ is ambiguous.*

In view of Lemma 5, the intuition of Lemma 6 is straightforward. In contrast to the short-run impact of a change in inequality, the impact of an increase in $\varepsilon$ on the aggregate human capital stock in period 2, $H_2$, also depends on the intergenerational wealth transmission as function of parents' income. When $\varepsilon$ increases in the initial period, members of generation 0 which belong to a poor dynasty (endowed with $\breve{b}_0^P$) invest more in education and thus (when $b'(\cdot) > 0$) transmit more wealth to their children (in period 1), on average, whereas those from rich dynasties transmit less. If $b'' > 0$ and $h_{ee} = 0$, then the average reduction in wealth transmission of the rich outweighs the average increase in wealth transmission of the poor. The resulting effect on the aggregate level of human capital of members of generation 1 (in period 2) is typically negative (part (ii) of Lemma 6). By contrast, if $h_{ee} < 0$, then the short run effect of higher equality on the aggregate human capital stock which underlies Proposition 2 also applies two periods after a lump-sum redistribution, and may dominate the effect arising from an increasing MPS. Thus, if $b'' > 0$ and $h_{ee} < 0$, the overall impact of a change in inequality on the aggregate human capital level two periods after this change is ambiguous. (part (iii) of Lemma 6). Roughly spoken, the "more convex" $b(I)$ and the "less concave" $h(e, \cdot)$ in $e$ is, the "more likely" is $\hat{H}_2'(\varepsilon) < 0$, which reflects a positive relationship between inequality and growth. As argued in section 5, for the U.S., for instance, the evidence for $b'' > 0$ is strong but that for $h_{ee} < 0$ is not, which suggests that part (ii) of Lemma 6 refers to the relevant case.

To draw conclusions for the role of (initial) inequality for the process of development in the medium run, i.e., to examine the impact of an increase in $\varepsilon$ on the growth rate $g_{t,0}(= Y_t/Y_0 - 1 = H_t/H_0 - 1)$ for $t \geq 2$, let

$$h_{t+3}^i = \tilde{h}_2(b_{t+1}^i) = \tilde{h}_2(\tilde{b}_1(b_t^i)) \equiv \tilde{h}_3(b_t^i). \tag{19}$$

Thus, recalling (14), the aggregate human capital stock in period 3 may be written as

$$H_3 = E\left[E\left[\lambda E\left[\tilde{h}_3\left(b_0^R - \varepsilon\right)\right] + (1-\lambda)E\left[\tilde{h}_3\left(b_0^P + \varepsilon\lambda/(1-\lambda)\right)\right]\right]\right] \equiv \hat{H}_3(\varepsilon). \tag{20}$$

As can be seen from comparing (20) with (18), the structure of the development process through wealth transmission and human capital investments remains similar from period 2 onwards. In fact, in line with Lemma 6, the following can be concluded.

**Proposition 3.** (Impact of higher equality in the medium run). *Under A1-A5, for all finite $t \geq 2$. (i) If $b''(I) \leq 0$, a reduction in inequality of initial family wealth has a non-negative impact on the subsequent growth rate $g_{t,0}$. (ii) If, by contrast, $b''(I) > 0$ and $h_{ee} = 0$, then the relationship between inequality and $g_{t,0}$ is positive. (iii) Otherwise, the relationship between inequality and $g_{t,0}$ is ambiguous.*

Proposition 3 establishes that the medium run impact of higher inequality on human capital accumulation and growth critically depends on the properties of the saving function, $b(I)$, and whether the return to educational investment is diminishing. The existing literature has focussed on specifications such that $b''(I) \leq 0$ and $h_{ee} < 0$ (apart from a different microfoundation of the relationship between intergenerational transfers and human capital investment, $e(b)$, which usually relies on credit constraints rather than on labor income risk as pursued here). The analysis has made clear that these hypotheses are not innocuous by identifying the critical forces which drive the inequality-growth relationship, further discussed in section 5 in light of the empirical evidence.

Appendix B illustrates part (ii) of Proposition 3 by specifying $u(c, b) = \alpha c - \beta c^2 + \ln b$, which gives an example such that $b'' > 0$, and, in addition, assumptions A3 and A4 typically hold.

## 4.2 Inequality and Aggregate Income in Stationary Equilibrium

What is the impact of a change in initial inequality on the aggregate human capital stock and per capita income in the long run (i.e., as $t \to \infty$), denoted by $H_\infty$ and $Y_\infty = H_\infty f(\bar{k})$, respectively? Answering this question requires an analysis of the long run behavior of wealth transfers within dynasties, which are governed by the Markov process $b^i_{t+1} = \hat{b}(b^i_t, \tilde{a})$, defined by (11). Due to the uncertainty in the model, these transfers never reach steady state points as known from deterministic models. Therefore, the goal is to find stationary equilibria in the sense that, as $t \to \infty$, the *distribution* of $b^i_t$ within dynasty $i$ is time-invariant. The following discussion of such *stationary equilibria*, and its consequences for the relationship between inequality and per capita income in the long run deals, in a rather informal way, with simple cases which capture the relevant mechanisms. A more formal and general treatment is provided in a supplement to this paper (available on request).

To focus the discussion on an empirically relevant case, suppose intergenerational transfers are zero for low levels of income.[29] Moreover, to prevent infinite wealth accumulation of rich dynasties, suppose that for *high* wealth levels $b^i$ and for $a = \underline{a}, \bar{a}$, $\hat{b}(b^i, a)$ is strictly concave as function of $b^i$.[30] Simple cases which meet these two criteria are depicted in Fig. 1.[31]

**<Figure 1>**

Panel (a) shows a situation in which, irrespective of initial wealth holdings, for any dynasty $i$, $b_t^i = 0$ as $t \to \infty$ with probability one. This is called a *trivial* stationary equilibrium. (In panel (a), this equilibrium is also globally stable.) Obviously, in this case, a change in the inequality of initial wealth has no impact on per capita income in the long run, $Y_\infty$.

In panel (b), if $b_0^i \leq c_{\bar{a}}$, again, wealth levels within dynasty $i$ become zero with probability one in the long run (which now is a locally stable stationary equilibrium). If $b_0^i \geq c_{\underline{a}}$, then the distribution of $b_t^i$ converges with probability one to a locally unique stable stationary equilibrium on the interval $[d_{\underline{a}},\, d_{\bar{a}}]$. Thus, if $b_0^i \leq c_{\bar{a}}$ for $i = R, P$ or if $b_0^i \geq c_{\underline{a}}$ for all $i$, a reduction in initial inequality does not affect $Y_\infty$. If, however, $b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$, then, as $t \to \infty$, both $b_t^i = 0$ or $b_t^i \in [d_{\underline{a}}, d_{\bar{a}}]$ is possible with positive probability.[32] Thus, according to the law of large numbers, if $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$, some fraction $q \in (0, 1)$ of initially poor dynasties will end up with zero wealth in the long run. Also by the law of large numbers, note that this fraction $q$ is increasing in the distance of $b_0^P$ to $c_{\underline{a}}$ (or decreasing in $\left(b_0^P - c_{\bar{a}}\right)$, respectively). Thus, the lower $\left(c_{\underline{a}} - b_0^P\right)$

[29] For the U.S., for instance, it has been frequently confirmed that the mean savings rate of households in the lowest quintile of the income distribution is non-positive (e.g., Browning and Lusardi, 1996; Dynan et al., 2004).

[30] It is easy to verify that, even if $b''(I) > 0$, this occurs under weak conditions if the return to education is eventually diminishing, i.e., $h_{ee} < 0$ for high levels of $e$, triggered by high intergenerational transfers.

[31] Note that for $b_{t+1}^i > 0$, the $\hat{b}-$curves for $\underline{a}$ and $\bar{a}$ do not intersect because $\hat{b}_a(b, a) > 0$ in this case. Also note that, under A2 and A5, $\hat{b}(b^i, a)$ is strictly increasing in $b^i$ for $a = \underline{a}, \bar{a}$ if $b^i > \underline{b}_{\underline{a}}, \underline{b}_{\bar{a}}$, according to Lemma 3.

[32] For illuminating discussions of this stationary equilibrium indeterminacy in stochastic models, see, e.g., Laitner (1981) and Wang (1993).

is, the larger the fraction $1-q$ of initially poor dynasties which transmit positive wealth levels in the long run. Hence, if $b_0^R > c_{\underline{a}}$ and $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$, then any lump-sum redistribution to the poor which leaves wealth levels of initially rich individuals sufficiently high,[33] unambiguously leads to higher average human capital investments, $H_\infty$, if $e'(b) > 0$ (as ensured under A1 and A3, according to Proposition 1), and thus, to a higher $Y_\infty$. This case may be relevant in advanced countries, i.e., if there is a large group of sufficiently rich individuals. Whereas in this situation there is a negative link between inequality and $Y_\infty$, there may be a positive link in other situations. To see this, suppose $b_0^i \in (0, c_{\bar{a}}]$ for $i = R, P$, such that the distribution of wealth levels of all dynasties converge with probability one to the trivial stationary equilibrium.[34] In this case, sufficient redistribution to the rich may result in a situation in which wealth levels of at least some initially rich dynasties converge to the stationary equilibrium on the interval $[d_{\underline{a}}, d_{\bar{a}}]$, without affecting the long run wealth distribution of the initially poor (who end up with zero wealth anyway). Thus, $Y_\infty$ is raised. Finally, if $b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$ for $i = R, P$, the impact of a change in inequality on $Y_\infty$ is ambiguous. In sum, we may conclude the following from this discussion.

**Proposition 4.** (Impact of higher equality in the long run). *Suppose a stationary equilibrium of the Markov process $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$ exists. Then the relationship between equality and long run income, $Y_\infty$, is generally ambiguous, irrespective of the shape of the saving function, $b(I)$.*

Thus, even if there is a systematic relationship between initial inequality and aggregate income in the short run and medium run (see Propositions 2 and 3 in section 4.1), one cannot draw general conclusions regarding the relationship between initial inequality and $Y_\infty$ from the shape of the saving function. It might be concluded, how-

[33] More precisely, this refers to any increase in $\varepsilon$ small enough such that $\breve{b}_0^R = b_0^R - \varepsilon \geq c_{\underline{a}}$, implying that the distribution of wealth holdings of initially rich dynasties still converges with probability one to a locally unique stationary equilibrium on the interval $[d_{\underline{a}}, d_{\bar{a}}]$.

[34] This reflects a poverty trap of the kind often encountered in the literature on inequality and growth (e.g., Perotti, 1993; Galor and Tsiddon, 1997; Moav, 2002), which here is derived from a stochastic model.

ever, that the relationship between initial inequality and long run income, if anything, is negative for advanced countries, whereas higher inequality may help to overcome a poverty trap in poor economies.

# 5 Empirical Evidence on the Critical Hypotheses

This section reviews empirical evidence regarding the three critical determinants of the inequality-growth relationship identified in this paper: how the marginal propensitiy to save and bequeath, respectively, changes with income (section 5.1), the impact of higher family wealth (or income) on children's human capital investment (section 5.2), and whether the marginal return to education is diminishing (section 5.3).

## 5.1 Saving Behavior and Intergenerational Transfers

In the model, the amount of intergenerational transfers equals the amount of savings of adult individuals. In fact, empirically, savings seem to be strongly related to inter vivos gifts and bequests, thus lending support for a crucial assumption about preferences in the model. For instance, Kotlikoff and Summers (1981) emphasize the importance of intergenerational transfers for capital accumulation in the U.S., a hypothesis which has been frequently confirmed by later evidence. In particular, as argued by Menchik and David (1983) and Dynan et al. (2002, 2004), observed saving behavior in the U.S. is empirically consistent with models hypothesizing a "joy of giving" motive for intergenerational transfers.[35] De Nardi (2003) calibrates an overlapping-generations model with voluntary and accidental bequests, arguing that voluntary bequests play a

[35] In contrast, the standard altruism (dynastic) model á la Barro (1974) seems to be inconsistent with the data. For instance, unlike predicted by the dynastic model, inheritances do not seem to compensate for earnings differences among siblings (e.g. Wilhelm, 1996). Moreover, as discussed in Carroll (2000), there does not seem to be an indication that the size of bequest is an increasing function of the ratio of parent's to child's lifetime income. Finally, whereas a one-dollar reduction in income of a recipient should raise inter vivos transfers from parents to child by one dollar according to the dynastic model, evidence by Altonji et al. (1997) suggests that transfers increase by just 13 cent on average, conditional on the event of a positive transfer having occurred.

crucial role for explaining observed wealth concentration patterns not only in the U.S. but also in Sweden.

According to the preceding analysis, if the MPS for adults, $b'(\cdot)$, is increasing in income, then inequality may be positively related to growth in the medium run even if marginal returns to human capital investments are diminishing (Proposition 3). This is because under individual uncertainty of returns to human capital investments, (initial) wealth inequality affects human capital accumulation through intergenerational transfers.

Evidence on U.S. saving and bequest behavior suggests that saving rates are strongly increasing in *lifetime* income.[36] For instance, Dynan et al. (2004) provide estimates from median regressions which imply that a \$10,000 increase in (permanent) income is associated with an increase in the median saving rate in a range from 2 to 7 percent, depending on the database and instruments used.[37] This strong correlation between saving rates and permanent income can neither be led back to the saving behavior of high-income entrepreneurs nor is there evidence that the relationship changes for older households. Most importantly for the results of the theoretical model developed in the present paper, their estimates strongly suggest that the MPS is increasing in income. For instance, the MPS rises from 0.08-0.09 in the lowest quintile to around 0.18-0.23 in quintile 4, depending on the saving measure used. In a less recent study, Menchik and David (1983) directly focus on bequest behavior. Their evidence suggests that the marginal propensity to bequeath is increasing in lifetime earnings. In sum, at least for the U.S., the evidence clearly supports the case $b''(\cdot) > 0$.

[36] There is overwhelming evidence for the hypothesis that saving rates rise strongly with *current* income (e.g., Browning and Lusardi, 1996). However, as pointed out by Friedman (1957), this finding may just reflect a response of savings behavior to changes in transitory income. That is, if income is temporarily high, savings increase and, analogously, if income is temporarily low, savings are reduced. Over the life-cycle, however, a positive relationship between current income and saving rates may still be consistent with a constant saving rate as function of lifetime (or permanent) income.

[37] Dynan et al. (2004) use three different databases to account for different measures of savings and use different instruments for permanent income like consumption, lagged and/or future earnings, and education.

## 5.2 Educational Investment and Parental Income

Another crucial element in the model is the positive relationship between family wealth and educational investments, $e'(\cdot) > 0$ (Proposition 1). This result has been derived from the hypothesis that the variance of earnings increases with educational investments, consistent with evidence by Pereira and Martins (2002, 2004) for advanced European countries (in a period between 1980 and 1995) who confirm somewhat less recent evidence discussed in Levhari and Weiss (1974).

In fact, there is overwhelming evidence for Proposition 1. First, although credit constraints to finance higher education seem to be negligible in advanced countries (e.g., Cameron and Taber, 2004), there is a strong positive relationship between parental social background and children's investment (or participation) in higher education. For instance, Manski (1992) finds that the percentage of children from low-income families in the U.S. who graduate from high school is substantially lower than among high school graduates from other families. Similar patterns can be found in Germany. Egeln et al. (2003) report that in 1996 even among those children who were eligible for university education (not more than roughly a third of all high school graduates in Germany), only 24 percent with a less favorable social background went to university, in contrast to 86 percent with a favorable social background.[38] This comparison is striking, as it is rather implausible that heterogeneity in intellectual ability (which may partly be shaped by the social background) can account for this difference among those who have already acquired eligibility ("*Hochschulreife*").

In his review of U.S. evidence based on econometric studies, Taubman (1989) concludes that estimates for the elasticity of years of schooling with respect to parental income are generally positive and range from 3 to 80 percent, after controlling for parents' education, father's occupation, and/or children's test scores on mental ability tests. Accounting for similar controls, also the correlation between children's adult

[38] In Germany, eligibility to attend university is exclusivley determined by high school performance. Egeln et al. (2003) also find that among *all* high school graduates only 8 percent with a less favorable social background went to university, in contrast to 72 percent of high school graduates with a favorable social background.

earnings and their parents' income is highly positive (e.g. Behrman and Taubman, 1990). As concluded in a recent survey article by Solon (1999, p. 1789): "Most of the evidence [...] indicates that intergenerational earnings elasticities are substantial and are larger than we used to think." An important question which arises from this evidence is to which degree these findings are due to genetic factors. For instance, Sacerdote (2002) finds that the effect of socioeconomic status on children's college attendance is just as large for adoptees as for children raised by biological parents, suggesting no significance of genetic factors. Plug and Vijverberg (2003) report higher effects of genetic factors (measured by parents' IQ) on the children's years of schooling and college attainment, although family income still has a large effect, consistent with Proposition 1.

### 5.3 Diminishing Returns to Education?

The question whether or not the marginal return to human capital investments is diminishing at the individual level is perhaps most debatable. For relevant ranges, standard Mincer estimates do not seem to support the diminishing-returns hypothesis. For instance, empirical evidence for the returns to schooling suggests that (mean) log-earnings log-hourly wages are approximately linear in the (mean) years of education (e.g., Card, 1999), thus implying a strictly convex mapping between the years of schooling and earnings. (Recall that in the present model, individual earnings are given by $\bar{w}h$.) This does not necessarily suggest $h_{ee} > 0$, since the cost of an additional year of schooling may increase with the years of schooling (e.g., think of college versus high-school education). However, it certainly does not suggest $h_{ee} < 0$ either.

## 6 Concluding Remarks

This paper has proposed a theory which suggests that the impact of higher (wealth) inequality on growth fueled by risky human capital investments during the transition to the steady state may be positive under empirically supported conditions. In contrast to the classical view which argues that higher inequality enhances growth through *physical* capital accumulation, this alternative theory does not require any connection between national savings and physical capital investment in an economy. Instead, it rests on the role of intergenerational wealth transmission for individual incentives to invest in risky and uninsurable *human* capital.

The hypotheses which drive the results are well testable empirically. First, it has been shown that, under the fairly weak and empirically supported requirements of decreasing absolute risk aversion and labor income risk, individual human capital investment is increasing in parental income. This prediction is consistent with empirical evidence even in advanced countries, where credit-market imperfections seem to play a minor (or no) role. Starting from this hypothesis of a positive relationship between parental income and human capital investments, the analysis has demonstrated that saving behavior critically determines how the personal income distribution affects human-capital based growth in an economy's process of development through the effects of intergenerational wealth transmission. The analysis suggests that, under non-diminishing marginal returns to education, the impact of higher inequality on medium run growth is typically positive if the MPS is increasing in income. In contrast, if the MPS is non-increasing, then the opposite holds, which in line with the existing literature on inequality, human capital, and macroeconomics (provided there is no poverty trap). However, at least for the U.S., empirical evidence supports the hypothesis on a strictly convex saving function.[39]

Contrary to the transition path, how inequality affects per capita income in the *long run* (i.e., in stationary equilibrium) does not critically depend of the properties of the saving function. One can conclude that, on the one hand, higher inequality

[39] Regarding an empirical test of the inequality-growth relationship suggested by the proposed theory, first, one has to be cautious to interpret the terms short run and medium run, which have an obviously different meaning in an OLG model than conventionally used in empirical studies (which may refer to the medium run as 10 years or so, whereas here it refers to at least two generations). Thus, a strict test requires long panels, which yet may not be available. Second, a rigorous cross-country study of the proposed theory requires good data about the critical determinants for the inequality-growth relationship identified here (discussed primarily for the U.S. in section 5) in sufficiently many countries.

may contribute to overcome poverty traps. On the other hand, the analysis suggests that inequality may discourage long run human capital investment for initially poorer dynasties, without affecting the rich, which is an adverse long run effect of inequality.

It is important to note, however, that even if the relationship between inequality and human-capital based growth turns out to be positive, the proposed theory does not suggest a rationale for inegalitarian policies. For instance, under uninsurable human capital risk, distortionary redistribution through the tax system may enhance risk-taking by providing insurance, as suggested by the literature on portfolio choice and taxation.[40] To examine the role of the shape of the saving function for implications of redistributive taxation in a similar context as analyzed in this paper is left for future research.

# Appendix

## A. Proofs

**Proof of Lemma 1:** Note that, according to (5), $b^i_{t+1}$ is implicitly given by the first-order condition

$$\Omega(b^i_{t+1}, I^i_{t+1}) \equiv -u_c(I^i_{t+1} - b^i_{t+1}, b^i_{t+1}) + u_b(I^i_{t+1} - b^i_{t+1}, b^i_{t+1}) \leq 0, \quad \text{(A.1)}$$

which holds with equality if $b^i_{t+1} > 0$. According to (A.1), $\Omega_b = u_{cc} - 2u_{cb} + u_{bb}$ and $\Omega_I = -u_{cc} + u_{cb}$, i.e., assumption A2 implies $\Omega_I > 0$ and $\Omega_b < 0$.[41] First, suppose $b^i_{t+1} = 0$. Since $\Omega_I > 0$, the left-hand side of the inequality in (A.1) is strictly increasing in $I^i_{t+1}$. Hence, using $\lim_{I\to\infty} u_c(I, 0) < \lim_{I\to\infty} u_b(I, 0)$ from A2, eventually, $b^i_{t+1} > 0$ if income $I^i_{t+1}$ exceeds some level $\underline{I} \geq 0$. Second, for $b^i_{t+1} > 0$, applying the implicit function

[40] Moreover, since the model does not contain any "trickle-down" mechanism, a standard equity-growth trade-off arises such that optimal policy will crucially hinge on the social welfare function chosen.

[41] To confirm $\Omega_b < 0$, use $u_{cc} < 0$ and $u_{cc}u_{bb} - (u_{cb})^2 > 0$ from A2 to obtain $u_{bb} < (u_{cb})^2/u_{cc}$. Thus, $\Omega_b = u_{cc} - 2u_{cb} + u_{bb} < u_{cc} - 2u_{cb} + (u_{cb})^2/u_{cc} = (u_{cc} - u_{cb})^2/u_{cc} < 0$. Of course, this is nothing else than showing that strict concavity of $u$ implies strict quasiconcavity of $u$.

theorem reveals

$$b'(I) = -\frac{\Omega_I}{\Omega_b} = \frac{u_{cc} - u_{cb}}{u_{cc} - 2u_{cb} + u_{bb}} > 0 \tag{A.2}$$

under A2. ■

**Proof of Lemma 2:** First, if $b^i_{t+1} = b(I^i_{t+1}) = 0$, we have $v(I^i_{t+1}) = u(I^i_{t+1}, 0)$; thus, $v'(I) = u_c(I, 0) > 0$ and $v''(I) = u_{cc}(I, 0) < 0$. If $b(I) > 0$, then $v'(I) = u_c(I - b(I), b(I)) > 0$, according to (6), (A.1) and the envelope theorem. Thus, $v''(I) = u_{cc} + b'(I)(u_{cb} - u_{cc})$. Substituting (A.2) into the latter expression, we obtain $v''(I) = u_{cc} - (u_{cb} - u_{cc})^2/(u_{cc} - 2u_{cb} + u_{bb})$. Manipulating the latter expression implies $v''(I) = \left[u_{cc}u_{bb} - (u_{cb})^2\right]/\Omega_b$ (recall $\Omega_b = u_{cc} - 2u_{cb} + u_{bb} < 0$). Hence, $v''(I) < 0$, according to the strict concavity of $u(c, b)$ presumed (assumption A2). This concludes the proof. ■

**Proof of Proposition 1:** First, note that, in an interior solution, $e^i_t = e(b^i_t)$ is given by the first-order condition

$$\begin{aligned} 0 &= E\left[v'\left(I(b^i_t, e^i_t, \tilde{a})\right)\left(\bar{w}h_e(e^i_t, \tilde{a}) - \bar{R}\right)\right], \text{ i.e.,} \\ \Xi(b^i_t, e^i_t) &\equiv E\left[v'\left(\bar{w}h(e^i_t, \tilde{a}) + \bar{R}\left(b^i_t - e^i_t\right)\right)\left(\bar{w}h_e(e^i_t, \tilde{a}) - \bar{R}\right)\right] = 0, \end{aligned} \tag{A.3}$$

according to (3) and (7). (An interior solution is ensured by assumption A1.) Due to $v''(I) < 0$ (recall Lemma 2) and $h_{ee} \leq 0$, we have $\Xi_e < 0$. Thus, according to the implicit function theorem, $e'(b^i_t) > 0$ if and only if $\Xi_b(b^i_t, e^i_t)|_{e^i_t = e(b^i_t)} > 0$. For notational simplicity, indices $t$ and $i$ are suppressed in the remainder of this proof. Moreover, define $\hat{I}(b, \tilde{a}) \equiv I(b, e(b), \tilde{a})$. Then, according to (A.3),

$$\begin{aligned} \Xi_b(b, e)|_{e=e(b)} &= E\left[v''\left(\hat{I}(b, \tilde{a})\right)\left(\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right)\right]\bar{R} \\ &= E\left[A\left(\hat{I}(b, \tilde{a})\right)v'\left(\hat{I}(b, \tilde{a})\right)\left(\bar{R} - \bar{w}h_e(e(b), \tilde{a})\right)\right]\bar{R}, \end{aligned} \tag{A.4}$$

where $A(I) = -v''(I)/v'(I)$ has been used for the latter equation. Define $a'$ as the realization of $\tilde{a}$ such that $\bar{w}h_e(e, a') - \bar{R} = 0$. We can write

$$\begin{aligned} & E\left[A\left(\hat{I}(b,\tilde{a})\right) v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right)\right] \\ = & \int_{\underline{a}}^{a'} A\left(\hat{I}(b,\tilde{a})\right) v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}) + \\ & \int_{a'}^{\bar{a}} A\left(\hat{I}(b,\tilde{a})\right) v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}). \end{aligned} \tag{A.5}$$

Recall from assumption A1 that $h_{ea} > 0$. Thus, by the definition of $a'$, the first integral in (A.5) is positive, whereas the second one is negative. Moreover, note that since $h_a > 0$, $\hat{I}(b, a)$ is increasing in $a$, according to (3). Thus, $A\left(\hat{I}(b,a)\right)$ is strictly decreasing in $a$ under assumption A3. Hence, under A3,

$$\begin{aligned} & A\left(\hat{I}(b,a')\right) \int_{\underline{a}}^{a'} v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}) \\ < & \int_{\underline{a}}^{a'} A\left(\hat{I}(b,\tilde{a})\right) v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}), \end{aligned} \tag{A.6}$$

$$\begin{aligned} & A\left(\hat{I}(b,a')\right) \int_{a'}^{\bar{a}} v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}) \\ < & \int_{a'}^{\bar{a}} A\left(\hat{I}(b,\tilde{a})\right) v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right) d\Phi(\tilde{a}). \end{aligned} \tag{A.7}$$

Adding up (A.6) and (A.7) and using $A(I)v'(I) = -v''(I)$ yields

$$A\left(\hat{I}(b,a')\right) E\left[v'\left(\hat{I}(b,\tilde{a})\right)\left(\bar{R} - \bar{w}h_e(\cdot,\tilde{a})\right)\right] < E\left[v''\left(\hat{I}(b,\tilde{a})\right)\left(\bar{w}h_e(\cdot,\tilde{a}) - \bar{R}\right)\right]. \tag{A.8}$$

Under the optimal human capital investment, $e(b)$, the left-hand side of (A.8) is zero, according to (A.3). Thus, $E\left[v''(\hat{I}(b,\tilde{a}))\left(\bar{w}h_e(e(b),\tilde{a}) - \bar{R}\right)\right] > 0$, implying $\Xi_b(b,e)|_{e=e(b)} > 0$, according to (A.4). Hence, $e'(b) > 0$. This concludes the proof. ■

**Proof of Lemma 3:** Recall from Lemma 1 (which is implied by A2) that there exists $\underline{I} \geq 0$ such that $b(I) > 0$ and $b'(I) > 0$ if $I > \underline{I}$. Also recall from (11) that

$\hat{b}(b,a) = b(\hat{I}(b,a))$, which implies $\hat{b}_b(b,a) = b'(\hat{I}(b,a))\hat{I}_b(b,a)$. Thus, under A5, for all $a \in \mathcal{A}$, a gradual increase in $b$ eventually must lead to a level of $b$, denoted by $\underline{b}_a$, such that $\hat{b}(b,a) > 0$ and $\hat{b}_b(b,a) > 0$ for all $b > \underline{b}_a$. This concludes the proof. ■

**Proof of Lemma 4:** First, note that (whenever differentiable)

$$\tilde{b}_1'(b) = b'(\tilde{I}_1(b))\tilde{I}_1'(b), \tag{A.9}$$

according to (11)-(13). Hence,

$$\tilde{b}_1''(b) = b''(\tilde{I}_1(b))\tilde{I}_1'(b)^2 + b'(\tilde{I}_1(b))\tilde{I}_1''(b). \tag{A.10}$$

Moreover, according to (10) and (13), we have

$$\tilde{I}_1'(b) = \left(\bar{w}h_e(e(b),a) - \bar{R}\right)e'(b) + \bar{R}, \tag{A.11}$$

$$\tilde{I}_1''(b) = \bar{w}h_{ee}(e(b),a)e'(b)^2 + \left(\bar{w}h_e(e(b),a) - \bar{R}\right)e''(b), \tag{A.12}$$

$a \in \mathcal{A}$. Under A4, we can neglect the second summand on the right-hand side of (A.12). Thus, under A1, A3 and A4, we have $\tilde{I}_1''(b) < (=)0$, if $h_{ee} < (=)0$, since $e'(b) > 0$ (Proposition 1). Hence, under A2 (which implies $b'(I) \geq 0$, according to Lemma 1), we have $\tilde{b}_1''(b) \leq 0$ if $b''(I) \leq 0$, according to (A.10). This confirms part (i). To confirm part (ii), note that, if $b''(I) > 0$, the sign of the first summand on the right-hand side of (A.10) is strictly positive for all $a \in \mathcal{A}$ under A5. Observing that, under A4, $\tilde{I}_1''(b) = 0$ if $h_{ee} = 0$ proves part (ii). However, since $\tilde{I}_1''(b) < 0$ if $h_{ee} < 0$ under A4, the sign of $\tilde{b}_1''(b)$ is generally ambiguous if $b''(I) > 0$ and $h_{ee} < 0$, according to (A.10). This confirms part (iii). ■

**Proof of Proposition 2:** According to (16), differentiating $\hat{H}_1(\varepsilon)$ with respect to

$\varepsilon$ yields

$$\begin{aligned} \hat{H}_1'(\varepsilon) &= \lambda E\left[\tilde{h}_1'(\breve{b}_0^P) - \tilde{h}_1'(\breve{b}_0^R)\right], \text{ where} && \text{(A.13)} \\ \tilde{h}_1'(b) &= h_e(e(b), a)e'(b), && \text{(A.14)} \end{aligned}$$

$a \in \mathcal{A}$, according to (8) and (15). Since $\breve{b}_0^P < \breve{b}_0^R$, (A.13) implies that $\hat{H}_1'(\varepsilon) > 0$ if, for instance, for all $a \in \mathcal{A}$ and for all $b \in \mathbb{R}_{++}$, $\tilde{h}_1''(b) < 0$. Note that

$$\tilde{h}_1''(b) = h_{ee}(e(b), a)e'(b)^2 + h_e(e(b), a)e''(b), \tag{A.15}$$

according to (A.14). The first summand on the right-hand side of (A.15) is negative (zero) if $h_{ee} < (=)0$, since $e'(b) > 0$, according to Proposition 1 (which holds under A1 and A3). Thus, if $e''(b) \leq 0$, or if $e''(b)$ is positive but small in magnitude as supposed in A4, we have $\tilde{h}_1''(b) < 0$ if $h_{ee} < 0$. In constrast, if $h_{ee} = 0$ and $e'' = 0$, then $\tilde{h}_1'' = 0$. This confirms the result. ■

**Proof of Lemma 5:** First, note that

$$\tilde{h}_2'(b) = \tilde{h}_1'(\tilde{b}_1(b))\tilde{b}_1'(b) = h_e(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))\tilde{b}_1'(b) \tag{A.16}$$

according to (8), (15) and (17). Substituting (A.9) into (A.16) leads to

$$\tilde{h}_2'(b) = h_e(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))b'(\tilde{I}_1(b))\tilde{I}_1'(b). \tag{A.17}$$

Under A2, (whenever differentiable) $b'(\cdot) \geq 0$, according to Lemma 1, and, under A1 and A3, $e'(\cdot) > 0$, according to Proposition 1. Moreover, $\tilde{I}_1'(b) > 0$ under A5. Thus, recalling $h_e > 0$, Lemma 5 (a) follows from (A.17).

To prove Lemma 5 (b), note that (A.16) implies

$$\tilde{h}_2''(b) = h_{ee}(e(\tilde{b}_1(b)), \tilde{a})e'(\tilde{b}_1(b))^2\tilde{b}_1'(b)^2 + h_e(e(\tilde{b}_1(b)), \tilde{a})\left[e''(\tilde{b}_1(b))\tilde{b}_1'(b)^2 + e'(\tilde{b}_1(b))\tilde{b}_1''(b)\right]. \tag{A.18}$$

The first summand on the right-hand side of (A.18) is non-positive since $h_{ee} \leq 0$. Moreover, if $|e''(b)|$ is small in magnitude as supposed in A4, the first term in square brackets of (A.18) is negligible. Regarding the second term in square brackets, if $b''(I) \leq 0$, then $\tilde{b}_1''(\cdot) \leq 0$ according to part (i) of Lemma 4. In addition, recall $e'(\cdot) > 0$ from Proposition 1 (which holds under A1 and A3). Thus, if $b''(I) \leq 0$, then the second term in square brackets of (A.18) is non-positive. Hence, $\tilde{h}_2''(b) \leq 0$ if $b''(I) \leq 0$, confirming part (i) of Lemma 5 (b). However, if $b''(I) > 0$ and $h_{ee} = 0$, then $\tilde{b}_1''(b) > 0$, according to part (ii) of Lemma 4. Consequently, $\tilde{h}_2''(b) > 0$ if $b''(I) > 0$, $h_{ee} = 0$ and A4 hold, according to (A.18). This confirms part (ii) of Lemma 5 (b). Part (iii) follows from the fact that the sign of $\tilde{b}_1''(b)$ is ambiguous if $b''(I) > 0$ and $h_{ee} < 0$ (part (iii) of Lemma 4). This concludes the proof. ■

**Proof of Lemma 6:** First, note that (18) implies

$$\hat{H}_2'(\varepsilon) = \lambda E\left[E\left[\tilde{h}_2'(\breve{b}_0^P) - \tilde{h}_2'(\breve{b}_0^R)\right]\right] \tag{A.19}$$

Since $\breve{b}_0^P < \breve{b}_0^R$, (A.19) implies that $\hat{H}_2'(\varepsilon) > (=,<)0$ if, for instance, for all realizations $a_0, a_1 \in \mathcal{A}$ at $t = 0, 1$ and for all $b \in \mathbb{R}_+$, $\tilde{h}_2''(b) < (=,>)0$. Finally, use Lemma 5 (b). This confirms Lemma 6. ■

**Proof of Proposition 3:** Note that (20) implies

$$\begin{aligned} \hat{H}_3'(\varepsilon) &= \lambda E\left[E\left[E\left[\tilde{h}_3'(\breve{b}_0^P) + \tilde{h}_3'(\breve{b}_0^R)\right]\right]\right], \text{ where} && \text{(A.20)} \\ \tilde{h}_3'(b) &= \tilde{h}_2'(\tilde{b}_1(b))\tilde{b}_1'(b), && \text{(A.21)} \end{aligned}$$

according to the definition $\tilde{h}_3(b) = \tilde{h}_2(\tilde{b}_1(b))$ in (19). Since $\breve{b}_0^P < \breve{b}_0^R$, (A.20) implies that $\hat{H}_3'(\varepsilon) > (=,<)0$ if, for instance, for all realizations $a_0, a_1, a_2 \in \mathcal{A}$ at $t = 0, 1, 2$, and for all $b \in \mathbb{R}_+$, $\tilde{h}_3''(b) < (=,>)0$. Also note that (A.21) implies

$$\tilde{h}_3''(b) = \tilde{h}_2''(\tilde{b}_1(b))\tilde{b}_1'(b)^2 + \tilde{h}_2'(\tilde{b}_1(b))\tilde{b}_1''(b). \tag{A.22}$$

Recall that $\tilde{h}_2'(\cdot) \geq 0$, according to Lemma 5 (a). First, suppose $b''(I) \leq 0$. Thus, $\tilde{h}_3''(b) \leq 0$, since $\tilde{b}_1''(\cdot) \leq 0$ and $\tilde{h}_2''(b) \leq 0$ in this case, according to part (i) of Lemma 4 and part (i) of Lemma 5 (b), respectively. Using (A.22), this confirms part (i) of Proposition 3. In an analogous fashion, parts (ii) and (iii) of Proposition 3 follow from (A.22) together with parts (ii) and (iii) of Lemma 4 and parts (ii) and (iii) of Lemma 5 (b), respectively. Thus, the impact of an increase in $\varepsilon$ on $H_3$ is similar to its impact on $H_2$, where the latter has been established in Lemma 6. The impact of an increase in $\varepsilon$ on $H_4$ and higher can be established in a completely analogous fashion, employing a very similar structure, which yields similar results. This concludes the proof. ■

## B. An Example (With an Increasing MPS)

This appendix provides a simple illustration of the analysis (particularly, part (ii) of Proposition 3) by specifying preferences in a way that the MPS is strictly increasing with income. As claimed in the main text and as will become apparent soon, $b'' > 0$ if $u(c, b) = \alpha c - \beta c^2 + \ln b$, as long as $u_c > 0$. (It is easy to see that assumption A2 and thus also Lemma 1 and 2 hold under this specification.) Also note, by recalling $c = I - b$, that $u_c > 0$ if and only if $b > I - \alpha/(2\beta)$, which is the exclusive focus in the following discussion. First-order condition (A.1) for optimal savings as adult implies that $b_{t+1}^i$ is given by the positive root of

$$(b_{t+1}^i)^2 + \left( \frac{\alpha}{2\beta} - I_{t+1}^i \right) b_{t+1}^i - \frac{1}{2\beta} = 0, \tag{B.1}$$

i.e., $b_{t+1}^i = b(I_{t+1}^i) > 0$ for all $I_{t+1}^i \geq 0$. From (B.1), it is easy to verify that

$$b'(I) = \frac{b(I)}{2b(I) + \frac{\alpha}{2\beta} - I} > 0, \quad b''(I) = \frac{2b(I) \left[ b(I) + \frac{\alpha}{2\beta} - I \right]}{\left( 2b(I) + \frac{\alpha}{2\beta} - I \right)^3}. \tag{B.2}$$

(B.2) confirms that $b'' > 0$ holds whenever $u_c > 0$. Since $b(I) > 0$ is given by $u_c = u_b$, according to (A.1), we have $v' = u_c = u_b$, according to (6), and thus $v'(I) = 1/b(I)$ in the considered utility specification. Thus, $v''(I) = -b'(I)/b(I)^2$. From this and (B.2), it is easy to verify that

$$v'''(I) = \frac{2}{\left(2b(I) + \frac{\alpha}{2\beta} - I\right)^3} > 0, \tag{B.3}$$

which is a necessary condition for assumption A3, $A'(I) < 0$, to hold.[42] The degree of absolute risk aversion, $A = -v''/v'$, is given by

$$A(I) \left[= \frac{b'(I)}{b(I)}\right] = \frac{1}{2b(I) + \frac{\alpha}{2\beta} - I}, \tag{B.4}$$

according to (B.2). That is, assumption A3 holds (i.e., $A(I)$ is strictly decreasing in $I$) if $2b'(I) - 1 > 0$, which is equivalent to $I > \alpha/(2\beta)$, according to (B.2). In sum, $u_c > 0$ and $A'(I) < 0$ require

$$b > I - \frac{\alpha}{2\beta} > 0. \tag{B.5}$$

Hence, under assumption A1, $e'(b) > 0$ if (B.5) holds, according to Proposition 1.

In the remainder of this appendix, we focus on a range of $e$ in which $h_{ee}(e, a) = 0$ for all $a \in \mathcal{A}$. We now examine whether assumption A4 and $\tilde{b}_1''(b) > 0$ hold, which give rise a positive medium run effect of inequality on per capita income (part (iii) of Proposition 3). According to first-order condition (A.3), if $h_{ee} = 0$,

$$e'(b) = -\left.\frac{\Xi_b(b, e)}{\Xi_e(b, e)}\right|_{e=e(b)} = -\frac{E\left[v''\left(\hat{I}(b, \tilde{a})\right)\left(\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right)\right]\bar{R}}{E\left[v''\left(\hat{I}(b, \tilde{a})\right)\left(\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right)^2\right]}. \tag{B.6}$$

Using (B.3) and (B.6), it is tedious but straightforward to show that (if $h_{ee} = 0$)

$$e''(b) = -\frac{\int_{\mathcal{A}} v'''\left(\hat{I}(b, \tilde{a})\right)\left[\left(\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right)e'(b) + \bar{R}\right]^2\left[\bar{w}h_e(e(b), \tilde{a}) - \bar{R}\right]d\Phi(\tilde{a})}{\Xi_e(b, e(b))}. \tag{B.7}$$

Recall from (B.3) that $v''' > 0$ in the considered specification. Since, in addition,

[42] Note that $A'(I) < 0$ if and only if $-v'''(I)/v''(I) > A(I)$, i.e., $-v'(I)$ is "more concave" that $v(I)$.

$\bar{w}h_e(e(b), a) < \bar{R}$ for small $a$ and $\bar{w}h_e(e(b), a) > \bar{R}$ for high $a$, according to first-order condition (A.3) and assumption $h_{ea} > 0$, the sign of $e''(b)$ is generally ambiguous (and can be zero), lending some justification to $|e''(b)| \approx 0$ (assumption A4). This implies $\hat{H}_1'(\varepsilon) \approx 0$ (if $h_{ee} = 0$), i.e., the impact of a change in inequality on short-run growth is negligible (compare with Proposition 2). Substituting (A.11), (A.12) and (B.2) into (A.10), and recalling that $\tilde{b}_1(b) = b(\tilde{I}_1(b))$, it is easy to verify that (if $h_{ee} = 0$)

$$\tilde{b}_1''(b) = \tilde{b}_1(b) \cdot \frac{\left(\bar{w}h_e - \bar{R}\right) e''(b) + \frac{2\left[\tilde{b}_1(b) + \frac{\alpha}{2\beta} - \tilde{I}_1(b)\right]\left[\left(\bar{w}h_e - \bar{R}\right)e'(b) + \bar{R}\right]^2}{\left[2\tilde{b}_1(b) + \frac{\alpha}{2\beta} - \tilde{I}_1(b)\right]^2}}{2\tilde{b}_1(b) + \frac{\alpha}{2\beta} - \tilde{I}_1(b)}. \tag{B.8}$$

According to (B.5), the second summand in the numerator is strictly positive. The first summand may have either sign, and is negligble under A4. If, for instance, $e'' < (>) 0$, then it is positive (negative) for small realizations of $\tilde{a}$, and vice versa for large realizations of $\tilde{a}$. This demonstrates that for the medium run impact of a change in inequality (see Proposition 3), the sign of $e''$ plays a rather unsystematic role (in contrast to the short run effect). Relaxing A4 beyond the short run analysis thus does not yield further economic insights.

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# *Supplement to:* Risky Human Capital Investment, Income Distribution, and Macroeconomic Dynamics

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**Abstract**

This supplement provides a more formal treatment of stationary equilibria and generalizes the cases discussed in section 4.2 (based on Fig. 1). Moreover, a simple example which is based on a standard specification of preferences in the literature is provided. Finally, it is shown that, if $h_a > 0$ and $h_{ea} > (<)0$, then the variance of earnings is strictly increasing (strictly decreasing) in educational investment $e$.

## 1 Stationary Equilibria (ad Section 4.2)

Let $P(b, \cdot)$ be the *transition function* of the Markov process $b^i_{t+1} = \hat{b}(b^i_t, \tilde{a})$, i.e., $P(b^i, \mathcal{Z})$ is the probability that $b^i$ is in the set $\mathcal{Z}$ one period after it started in $b^i$. That is,

$$P(b^i, \mathcal{Z}) \equiv \Pr\{\tilde{a} : \hat{b}(b^i, \tilde{a}) \in \mathcal{Z}\} \equiv \Pr\{\tilde{a} \in \mathcal{Z}_{b^i}\} = \int_{\mathcal{Z}_{b^i}} \Phi(\tilde{a}), \tag{C.1}$$

where $\mathcal{Z}_{b^i} \equiv \{\tilde{a} : \hat{b}(b^i, \tilde{a}) \in \mathcal{Z}\}$ and $\mathcal{Z}$ is a Borel set in $\mathbb{R}_+$. Moreover, let $\mu^i_t(\mathcal{Z}) \equiv \Pr\{b^i_t \in \mathcal{Z}\}$ for all $\mathcal{Z} \subset \mathbb{R}_+$, $t = 0, 1, 2, ...$, be the *probability measure* associated with $b^i_t$. Thus, given initial wealth $b^i_0$ of dynasty $i$, $\mu^i_0$ is given by $\mu^i_0([0, b^i)) = 0$ for all $b^i \leq b^i_0$ and $\mu^i_0([b^i, \infty)) = 1$ otherwise. Starting from $\mu^i_0$, the distribution of family wealth evolves according to $\mu^i_{t+1}(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu^i_t(db^i)$ for all $\mathcal{Z} \subset \mathbb{R}_+$. From this,

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we can define a stationary equilibrium as follows. (Definitions 1 and 2 closely follow Wang, 1993).

**Definition 1.** (Stationary equilibrium). A *stationary equilibrium* for family wealth $b^i$ of dynasty $i$ is a probability measure $\mu^i$ such that $\mu^i(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu^i(db^i)$ for all $\mathcal{Z} \subset \mathbb{R}_+$. A *trivial stationary equilibrium* is a stationary equilibrium which is associated with a distribution of $b^i$ such that all mass is concentrated on zero (i.e., $lim_{t\to\infty} \Pr\{b_t^i = 0\} = 1$).

Together with Definition 1, the next definition leads to an important existence result.

**Definition 2.** (Stable set). An interval $[x, y] \subset \mathbb{R}$ is called a *stable set* of the stochastic process $\hat{b}$ if (i) $\hat{b}(b^i, x) = x$, $\hat{b}(b^i, y) = y$, and (ii) $\hat{b}(b^i, x) < b^i$, $\hat{b}(b^i, y) > b^i$ for all $x < b^i < y$.

**Lemma C.1.** (Wang, 1993). *There is a unique stable stationary equilibrium on a stable set $\mathcal{I}$. Moreover, the convergence to the stationary equilibrium is uniform on $\mathcal{I}$.*

**Proof.** Brock and Mirman (1972), Wang (1993). ■

Given these preliminaries, let $S_a \equiv \{b \in \mathbb{R}_{++} \Big| \hat{b}(b, a) = 0\}$ be the set of strictly positive transfers received by a young individual, such that the optimal transfer as adult to her offspring is zero, $a \in \mathcal{A}$. Moreover, let $\underline{b}_a \equiv \max S_a$ be the largest element of such a set. Suppose that the following holds.

**A6.** $S_{\bar{a}}$ is non-empty, i.e., there exists $\underline{b}_{\bar{a}} > 0$.

Note that $\hat{b}_a(b, a) = b'\left(\hat{I}(b, a)\right)\hat{I}_a$, according to (10) and (11). Thus, using $\hat{I}_a > 0$ and Lemma 1, we have $\underline{b}_a > \underline{b}_{\bar{a}}$ for all $a \in [\underline{a}, \bar{a})$. That is, if an individual which has received a transfer $b$ when young does not save as adult in the best state $\bar{a}$, neither she does in any other state. Also note that, if $\underline{b}_a > 0$ exists for some $a$, then A2 and A5 imply that $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$, according to Lemma 3.

We are now ready to give a formal characterization of the result corresponding to panel (a) of Fig. 1.

**Proposition C.1.** *Under A2, A5 and A6. If* $\hat{b}(b^i, \bar{a}) < b^i$ *for all* $b^i > 0$*, the distribution of* $b^i$ *globally converges to a trivial stationary equilibrium.*

**Proof.** First, note that A6, which says that $S_{\bar{a}}$ is non-empty, also implies that $S_{\underline{a}}$ is non-empty. Moreover, recall $\underline{b}_{\underline{a}} > \underline{b}_{\bar{a}} > 0$. Also recall that A2, A5 and A6 ensure $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$, according to Lemma 3. Thus, Proposition C.1 corresponds to the case in panel (a) of Fig. 1. From this figure, it is clear that there no stable set of the process $\hat{b}$ exists (recall Definition 2), and global convergence to a trivial stationary equilibrium (recall Definition 1) is obvious. ■

To analyze more general situations than global convergence to a trivial stationary equilibrium, the next assumption prevents infinite wealth accumulation of rich dynasties.

**A7.** $lim_{b \to \infty} \hat{b}_b(b, a) = 0$ for all $a \in \mathcal{A}$.

Moreover, let $\Theta_a \equiv \{b \in \mathbb{R}_{++} \,\big|\, \hat{b}(b, a) = b\}$ be the set of strictly positive fixed points of $\hat{b}(b, a)$, $a \in \mathcal{A}$. In the remainder of this appendix, we focus on situations in which $\Theta_{\underline{a}}$ and $\Theta_{\bar{a}}$ have the following properties.

**A8.** (i) $\Theta_{\underline{a}}$ is non-empty.[1] (ii) $\Theta_{\underline{a}}$ and $\Theta_{\bar{a}}$ are finite. (iii) Let $c_{\bar{a}}, d_{\bar{a}}$ be two adjacent elements of $\Theta_{\bar{a}}$ such that $c_{\bar{a}} < d_{\bar{a}}$ and $\hat{b}_b(d_{\bar{a}}, \bar{a}) \leq 1$. Then there exists $c_{\underline{a}} \in \Theta_{\underline{a}}$ such that $c_{\underline{a}} \in (c_{\bar{a}}, d_{\bar{a}})$. (iv) Let $c_{\underline{a}}, d_{\underline{a}}$ be two adjacent elements of $\Theta_{\underline{a}}$ such that $c_{\underline{a}} < d_{\underline{a}}$ and $\hat{b}_b(d_{\underline{a}}, \underline{a}) \geq 1$. Then there exists $c_{\bar{a}} \in \Theta_{\bar{a}}$ such that $c_{\bar{a}} \in (c_{\underline{a}}, d_{\underline{a}})$.

<**Figure 2**>, <**Figure 3**>

It is easy to check that the case depicted in panel (b) of Fig. 1 is consistent with A6-A8.[2] Fig. 2 shows situations, which are consistent with A6 and A7, but inconsistent

[1] Note that, under A6, part (i) of A8 implies that $\Theta_{\bar{a}}$ is non-empty as well.

[2] Note that part (iv) of A8 is not relevant for panel (b) of Fig. 1, since $\hat{b}_b(d_{\underline{a}}, \underline{a}) < 1$.

with some parts of A8, whereas panels (a)-(c) of Fig. 3 are, like panel (b) of Fig. 1, consistent with A6-A8.

Note that, by applying Definition 2, $I_3$ and $I_6$ in panels (a) and (b) of Fig. 3 are both stable sets, whereas in panel (c) $I_3$ is the unique stable set. Moreover, by replicating the arguments in Laitner (1981; section III), the following can be concluded from Fig. 3, starting with panels (a) and (b). In panel (a) of Fig. 3, if $b_0^i \in I_2 \cup I_3 \cup I_4$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1$. In panel (b), the same is true if $b_0^i \in I_2 \cup I_3$. In panel (a), if $b_0^i \in I_6 \cup I_7$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_6\} = 1$. In panel (b), the same is true if $b_0^i \in I_5 \cup I_6 \cup I_7$. In both panels (a) and (b), if $b_0^i \in I_0$, then $q \equiv \lim_{t\to\infty} \Pr\{b_t^i = 0\} = 1$, and, if $b_0^i \in I_1$, then $q \in (0,1)$ and $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1 - q$. Finally, if $b_0^i \in I_5$ in panel (a), then $b_t^i \in I_3$ or $b_t^i \in I_6$ with probability one. In panel (b), the same is true if $b_0^i \in I_4$. Now consider panel (c) of Fig. 3. If $b_0^i \in I_0 \cup I_1$, then $q = 1$. If $b_0^i \in I_2$, then $q \in (0,1)$ and $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1 - q$. Finally, if $b_0^i \in I_3 \cup I_4$, then $lim_{t\to\infty} \Pr\{b_t^i \in I_3\} = 1$. Using this discussion, one can generalize the conclusions drawn from panel (b) of Fig. 1 discussed in the main text, in the following sense.

**Proposition C.2.** *Under A2, A5-A8. Let* $c = \min \Theta_{\bar{a}}$ *and* $d = \min \Theta_{\underline{a}}$ *(i.e.,* $c < d$*).*

*(i) For all* $b_0^i \in [0, c]$*, as* $t \to \infty$*, the distribution of* $b_t^i$ *converges to a locally stable trivial stationary equilibrium.*

*(ii) For all* $b_0^i \in (c, d)$*, there is a positive probability* $q \in (0,1)$ *that the distribution of* $b_t^i$ *converges to a trivial stationary equilibrium, whereas with probability* $1 - q$ *it converges to a (unique and stable) stationary equilibrium on a stable set.*

*(iii) For all* $b_0^i \in [d, \infty)$*, the distribution of* $b^i$ *converges to a (unique and stable) stationary equilibrium on a stable set.*

**Proof.** Part (i) of Proposition C.2 can be deduced by similar arguments as in the proof of Proposition C.1. To prove parts (ii) and (iii), recall Definitions 1 and 2 and verify from A2 and A5 (which ensure $\hat{b}(b, a) > 0$ and $\hat{b}_b(b, a) > 0$ for all $b > \underline{b}_a$) as well as A6-A8, that the result can directly be deduced by replicating the discussion of Fig. 3 above. ■

## 2 An Additional Example

The following example provides a simple illustration of the additional assumptions in this supplement by specifying preferences, and discusses the some properties of the analysis in the paper. Following Galor and Zeira (1993), Moav (2002) and Galor and Moav (2004), among others, the utility is given by

$$u(c_{t+1}^i, b_{t+1}^i) = (1-\alpha)\ln c_{t+1}^i + \alpha \ln(\gamma + b_{t+1}^i), \quad 0 < \alpha < 1,\, \gamma \geq 0. \tag{C.2}$$

It is easy to check that assumption A2 and thus Lemma 1 and 2 holds. Using the first-order condition (A.1) for optimal savings as adult, specification (C.2) implies a saving function $b_{t+1}^i = b(I_{t+1}^i) = \alpha[I_{t+1}^i - \vartheta]$ if $I_{t+1}^i > \vartheta \equiv \gamma(1-\alpha)/\alpha$ and $b_{t+1}^i = 0$ otherwise. Moreover, indirect utility is given by $v(I_{t+1}^i) = 2\ln(I_{t+1}^i + \gamma) + \eta$, where $\eta \equiv \alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)$. Thus, the degree of absolute risk aversion is given by $A(I) = -v''(I)/v'(I) = 1/(I+\gamma)$. That is, $A(I)$ is strictly decreasing in $I$, in line with assumption A3. Hence, $e'(b) > 0$, according to Proposition 1, where $e(b)$ is given by

$$\int_{\mathcal{A}} \frac{\bar{w}h_e(e,\tilde{a}) - \bar{R}}{\bar{w}h(e,\tilde{a}) + \bar{R}(b-e) + \gamma} d\Phi(\tilde{a}) = 0, \tag{C.3}$$

according to the first-order condition (A.3) for optimal educational investment. Using (C.3), tedious derivations reveal that the sign of $e''(b)$ is ambiguous, as in Appendix B, lending some justification to $|e''(b)| \approx 0$ (assumption A4). Regarding the short run, this means that $\hat{H}_1'(\varepsilon) > 0$ (Proposition 2) is likely to hold. For the remainder of this supplement, suppose $\gamma > 0$, i.e., $\vartheta > 0$. For the medium run and long run analysis, using (10), one then obtains

$$b_{t+1}^i = \hat{b}(b_t^i, a) = \tilde{b}_1(b_t^i) = \begin{cases} \alpha[\bar{w}h(e(b_t^i), a) + \bar{R}\,(b_t^i - e(b_t^i)) - \vartheta] & \text{if } \; b_t^i \geq \underline{b}_a, \\ 0 \quad \text{otherwise}, \end{cases} \tag{C.4}$$

where $\underline{b}_a$ is given by $\bar{w}h(e(\underline{b}_a), a) + \bar{R}\,(\underline{b}_a - e(\underline{b}_a)) = \vartheta$, $a \in \mathcal{A}$. Note that A5 implies that $\underline{b}_a$ is unique, and A6 implies that $\underline{b}_a > 0$. Thus, if $\alpha\left[\bar{w}h(e(b), a) + \bar{R}\,(b - e(b)) - \vartheta\right] < b$

for all $a \in \mathcal{A}$ and for all $b > 0$, Proposition C.1 applies, i.e., the distribution of $b^i$ globally converges to a trivial stationary equilibrium. Moreover, it is easy to check that, under assumption A4, (C.4) implies

$$\hat{b}_{bb}(b_t^i, a) = \tilde{b}_1''(b) < 0 \text{ for all } b > \underline{b}_a,\ a \in \mathcal{A}. \tag{C.5}$$

Since $b''(I) = 0$ for $I > \vartheta$ under utility specification (A.8), $\tilde{b}_1''(b) < 0$ illustrates part (i) of Lemma 4. Thus, the impact of higher equality on medium run growth is positive, according to part (i) of Proposition 3. For the long run analysis, note that (C.5) is consistent with assumption A7. Moreover, if there exists a $b > 0$ such that $\alpha[\bar{w}h(e(b), \underline{a}) + \bar{R}\,(b - e(b)) - \vartheta] = b$, i.e., if part (i) of A8 holds, (C.5) implies that also parts (ii)-(iv) of A8 hold. This illustrates that, for the long run, the discussion of Fig. 1 in the main text applies (Proposition 4).

# 3 Increasing Variance of Earnings

The remainder of this supplement proves the following claim made in section 2.2.

**Lemma C.2.** *If $h_a > 0$ and $h_{ea} > (<)0$, then the variance of earnings is strictly increasing (strictly decreasing) in the level of educational investment $e$.*

**Proof.** Recall that earnings are given by $\bar{w}h(e, \tilde{a})$. Denote the mean human capital level of an individual with education investment $e$ as $\bar{h}(e)$, which reads

$$\bar{h}(e) = \int_{\underline{a}}^{\bar{a}} h(e, \tilde{a}) d\Phi(\tilde{a}), \tag{C.6}$$

and note that the variance is given by

$$\Sigma(e) \equiv \int_{\underline{a}}^{\bar{a}} \left[h(e, \tilde{a}) - \bar{h}(e)\right]^2 d\Phi(\tilde{a}). \tag{C.7}$$

Thus, $\Sigma'(e) = 2\int_{\underline{a}}^{\bar{a}} \left[h(e, \tilde{a}) - \bar{h}(e)\right]\left[h_e(e, \tilde{a}) - \bar{h}'(e)\right] d\Phi(\tilde{a})$. Using (C.6), this reduces to $\Sigma'(e) = 2\int_{\underline{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a})$, which can be decomposed as

$$\Sigma'(e) = 2\left\{\int_{\underline{a}}^{\hat{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a}) + \int_{\hat{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a})\right\}, \tag{C.8}$$

where $\hat{a}$ is defined as the realization of $\tilde{a}$ such that $h(e,\hat{a}) = \bar{h}(e)$. Thus, according to assumption $h_a > 0$, the first integral in (C.8) is negative, whereas the second integral is positive. Hence, $h_{ea} > (<)0$ together with the definition of $\hat{a}$ implies

$$h_e(e,\hat{a})\int_{\underline{a}}^{\hat{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] d\Phi(\tilde{a}) < (>) \int_{\underline{a}}^{\hat{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a}) \tag{C.9}$$

$$h_e(e,\hat{a})\int_{\hat{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] d\Phi(\tilde{a}) < (>) \int_{\hat{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a}) \tag{C.10}$$

Adding up (C.9) and (C.10) yields

$$h_e(e,\hat{a})\int_{\underline{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] d\Phi(\tilde{a}) < (>) \int_{\underline{a}}^{\bar{a}} \left[h(e,\tilde{a}) - \bar{h}(e)\right] h_e(e,\tilde{a})d\Phi(\tilde{a}). \tag{C.11}$$

Since, the left-hand side of (C.11) is zero, according to (C.6), we have $\Sigma'(e) > (<)0$ iff $h_{ea} > (<)0$, according to (C.8) and (C.11). This proves Lemma C.2. ■

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**Figure 1:** Stationary equilibrium and convergence.

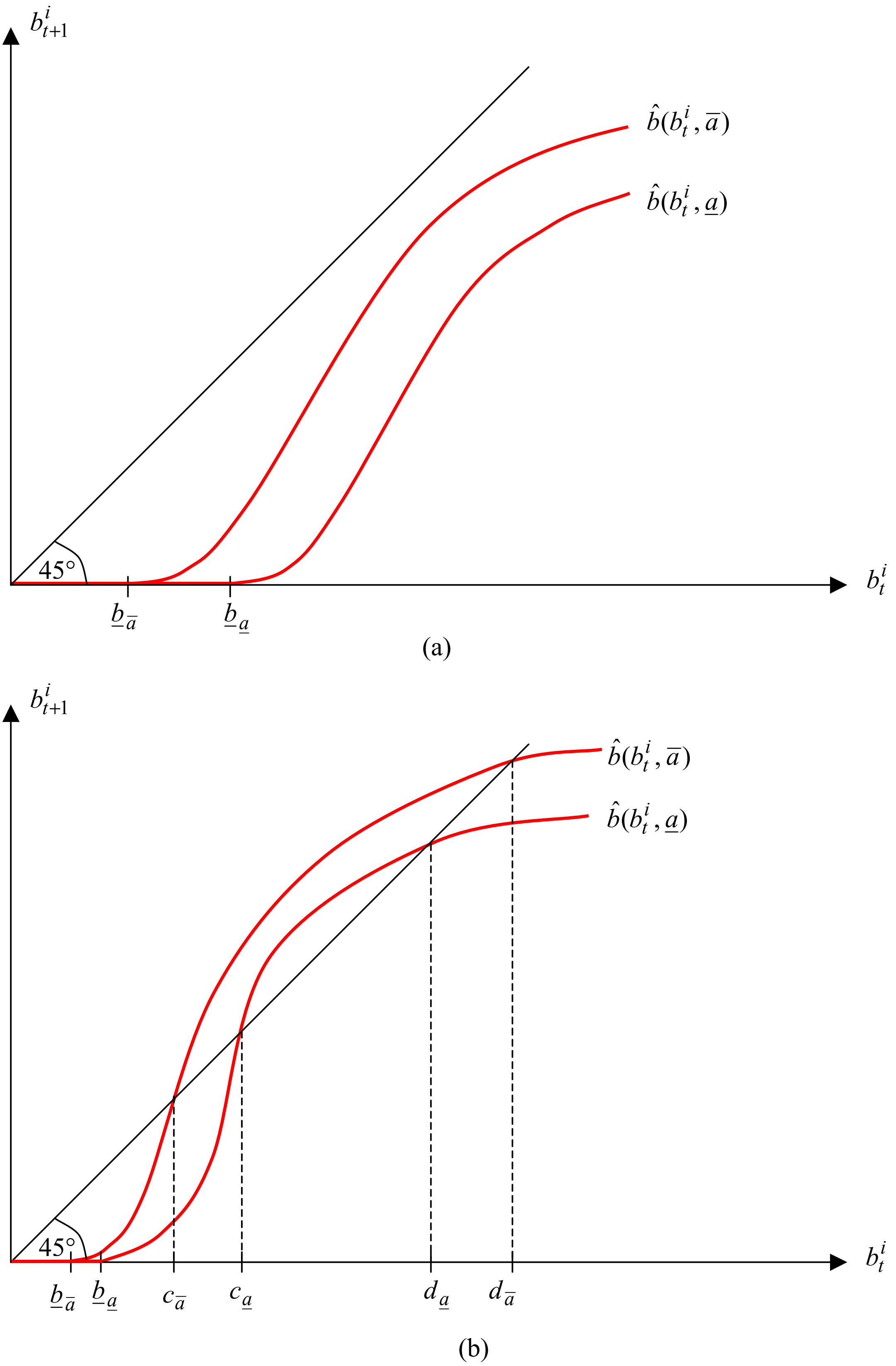
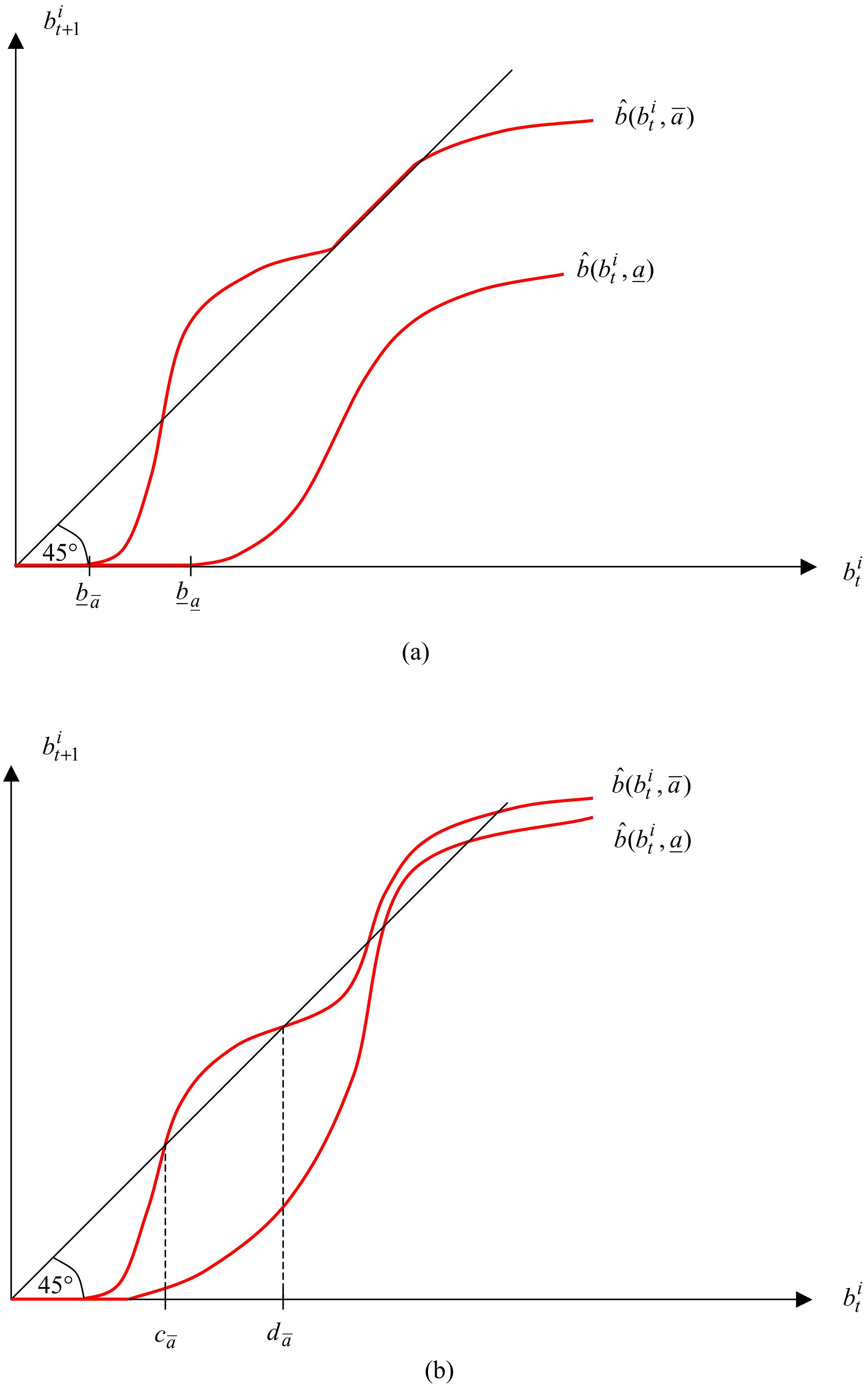
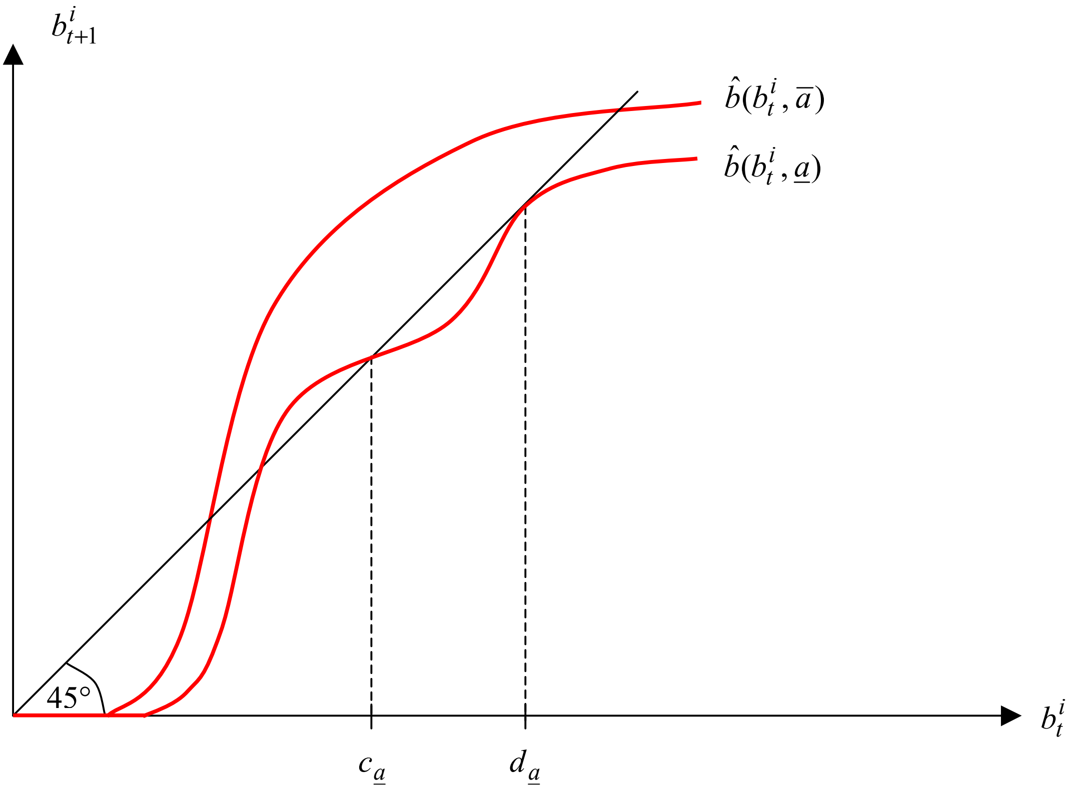


**Figure 2:** Inconsistency with assumption A8.

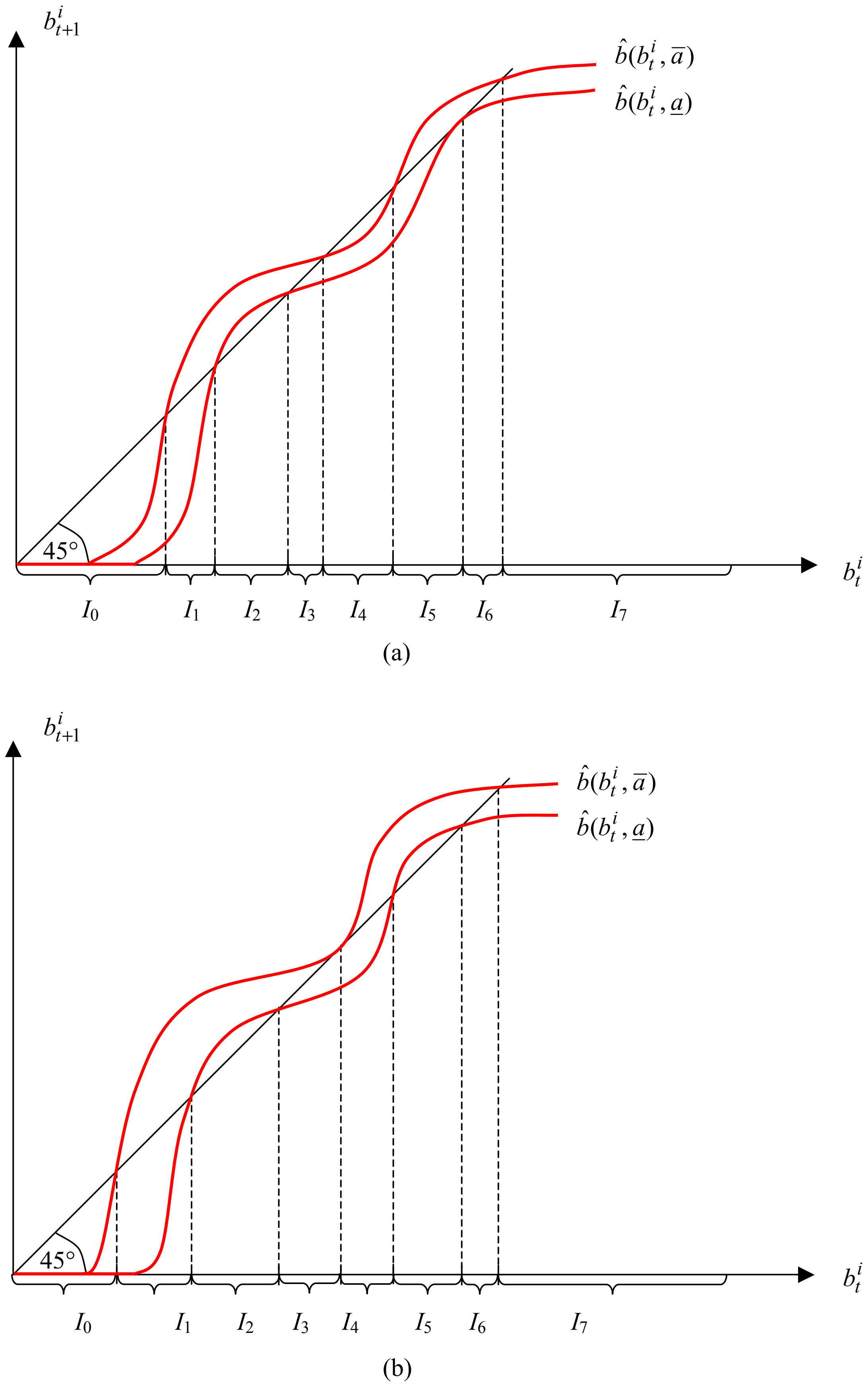


(Figure 2 continued)

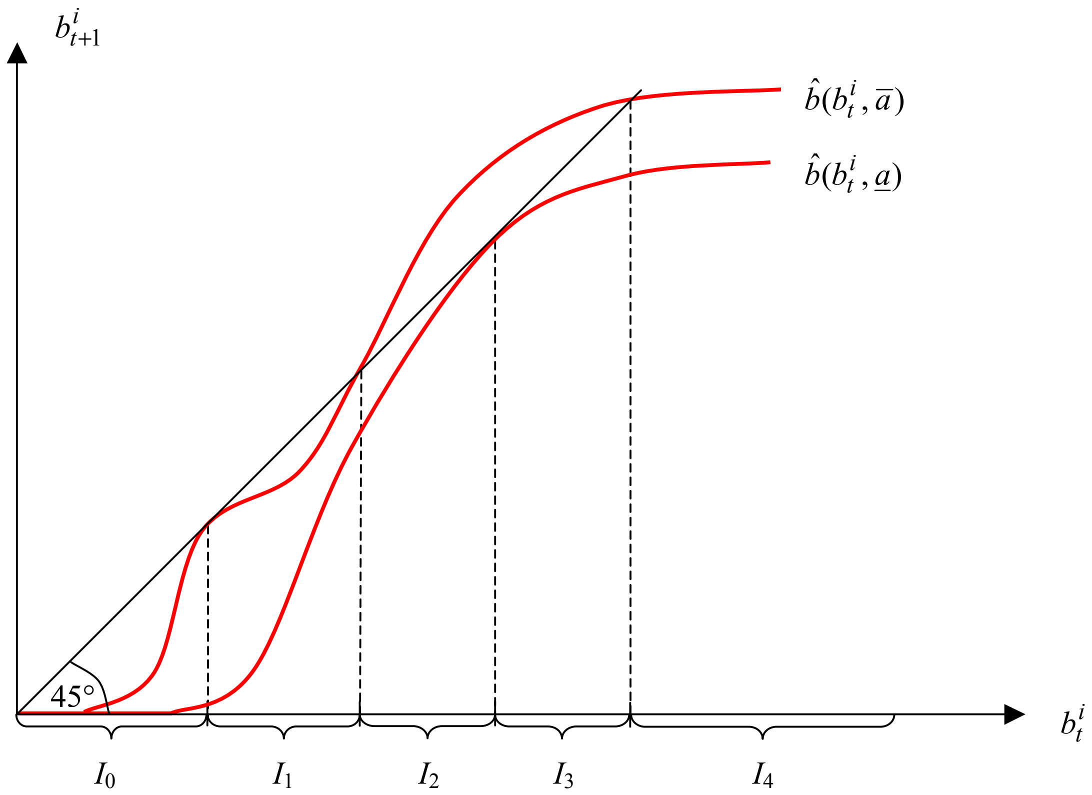



(c)

**Figure 3:** Consistency with assumptions A6-A8.



(Figure 3 continued)




(c)