On Taxation in a Two-Sector Endogenous Growth Model with Endogenous Labor Supply

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Abstract

This paper studies the effects of taxation on long-run growth in a two-sector endogenous growth model with (i) physical capital as an input in the education sector and (ii) leisure as an additional argument in the utility function. Due to the flexibility of labor supply, taxation of income may induce agents to spend more or less time on leisure activities. Income taxation - the same rate applies for capital and labor income - reduces the growth rate. The contribution of endogenous leisure in this case is confined to reducing or increasing the size of the effect on the growth rate. The same is true if only labor income is taxed. However, if only capital income is taxed, the sign of the effect may reverse. In that case, the positive effect of the increase in total non-leisure time dominates the direct negative effect, implying that capital taxation increases the long-run growth rate.

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1 Introduction

Income taxation affects the incentives to invest in different sectors of the economy, potentially affecting the long-run growth rate. This paper stud-

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ies the impact of taxation on long-run growth in a two-sector endogenous growth model. The model features both physical capital as an input in the education sector as well as leisure as an additional argument in the utility function. Endogenizing labor supply through leisure-dependent utility in models of economic growth has significant implications for the dynamics of these models, see e.g. Ladrón-de-Guevara et al. (1997) and De Hek (1998, 1999). The present analysis shows that the model may exhibit multiple balanced growth paths, and establishes a necessary condition for the existence of multiple balanced growth paths. The main purpose of this paper, however, is to show that the introduction of both features has significant and novel results on the impact of taxation on long-run growth. Most notably, this set-up implies that capital income taxation may lead to faster long-run growth.

The taxation analysis focuses on the case of a unique stationary equilibrium. It distinguishes three tax regimes: income taxation (equal rates on labor and capital income), capital income taxation and labor income taxation. Like in the case without a labor-leisure choice, income taxation has a negative effect on the long-run growth rate. However, depending on the relative importance of substitution and income effects, a tax on income leads to a rise or fall in the time spent on leisure activities, which in turn increases respectively reduces the size of the negative effect on growth. The same is true if only labor income is taxed. However, contrary to the case without a labor-leisure choice, a tax on capital income may have a *positive* effect on the long-run growth rate. This occurs mainly when the elasticity of intertemporal substitution is small, since in that case the income effect is (relatively) strong, implying that agents tend to work more and/or invest more time in human capital accumulation - the main engine of growth - in response to an increase in the tax rate. This reduction in leisure time then is reinforced as, compared to a tax on income, a tax on capital income induces more time spent on production and human capital accumulation, because labor (time) is not taxed. The positive impact of capital income taxation on growth is shown to hold for sensible parameter values and is robust. In an extension of the model, the implications of introducing productive government expenditures are studied. The main conclusion from this analysis is that improvements in the productivity (in the goods sector as well as in the education sector) affect the growth rate positively.

The effects of income taxation in the context of a two-sector endogenous growth model have been examined before by many authors. Some of these studies use numerical simulations of calibrated models to calculate the effect of tax reform on growth, e.g. Lucas (1990), Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1995) and Hendricks (1999). Others, like Chamley (1992) and Mino (1996), examine analytically the effect of (capital) income taxation on growth. Almost all these studies conclude that a (capital) income tax is bad for growth. An exception is Uhlig and Yanagawa (1996) who show that higher capital income taxes may lead to faster growth in an overlapping generations economy with endogenous growth. The reason for this positive effect is however entirely different from the reason in the present paper. They assume that labor income is paid mostly to the young while capital income accrues mostly to the old. This implies that a higher capital income tax, accompanied with a lower labor income tax, leaves the young with more income out of which to save. If the interest elasticity of savings is sufficiently low, the net effect on savings and, therefore, on growth is positive.

The analysis in the present paper is closely related to the analyses in Rebelo (1991), Ladrón-de-Guevara et al. (1997) and Ortigueira (1998). In fact, the model in the present paper is the same as Rebelo's model with an endogenous leisure choice (Rebelo, 1991, section III). Due to analytical difficulties, Rebelo confines his analysis of the effect of income taxation on the rate of growth to numerical simulations. These simulations indicate that taxing income has a negative effect on the growth rate. The analysis in this paper shows analytically that this is true. Ladrón-de-Guevara et al. (1997) establishes that there could be multiple balanced growth paths in the human capital accumulation model of Lucas (1988) if leisure is endogenously determined. The present analysis extends this result to the more general two-sector endogenous growth model, where physical capital is included as an input in the education sector. Ortigueira (1998) studies the impact of labor and capital income taxation on the transitional dynamics to the balanced growth path. He considers both the case of physical capital in the education sector and leisure as an additional argument in the utility function as two *separate* extensions of the basic model. That model is different from the present model in the sense that the tax revenues are remitted as lump-sum transfers to the households.

From a modelling point of view, the main innovation of this paper relative to the previous literature is to combine physical capital as an input in the educational sector *and* leisure as an additional argument in the utility function. Previous work has studied models with either the former or the latter feature, but not with both. The finding that endogenous leisure may strengthen or weaken the effect of income taxation on growth, but does not reverse the sign, depends on this combination. Furthermore, compared to the model with endogenous leisure but without capital in the education sector, the present study shows that the combination of both features leaves the possibility of multiple balanced growth paths intact. The interesting and novel finding that capital income taxation may have a positive impact on the long-run growth rate is however shown to be a consequence of the labor-leisure choice. The inclusion of capital as an input in the education sector actually reduces this possibility.

The paper is organized as follows. Section 2 describes the model and its solution, including the possibility of multiple balanced growth paths. Section 3 analyses the effects of taxation on long-run growth, including income, capital-income and labor-income taxation, in the unique BGP case. Section 4 discusses the implications of introducing productive government expenditures in the model. A summary is given in section 5.

2 The Model

Following Rebelo (1991, section III), the model consists of two sectors with different technologies for production and education. A fraction ϕ of physical capital K together with NH efficiency units of labor, where N is the fraction of time allocated to labor and H the stock of human capital, are used for the production of goods, i.e.,

$$Y(t) = A\left(\phi(t)K(t)\right)^{1-\gamma}\left(N(t)H(t)\right)^{\gamma}, \tag{1}$$

where $A > 0$ and $0 < \gamma < 1$ are parameters. Profit maximization implies that in equilibrium firms must pay each production factor its marginal product:

$$r(t) = (1-\gamma)A\left(\phi(t)K(t)\right)^{-\gamma}\left(N(t)H(t)\right)^{\gamma}, \tag{2}$$

$$w(t) = \gamma A\left(\phi(t)K(t)\right)^{1-\gamma}\left(N(t)H(t)\right)^{\gamma-1}, \tag{3}$$

where r is the interest rate and w the wage rate. The government imposes flat-rate taxes on capital income, τ_r, and labor income, τ_w. The analysis will be undertaken in a closed economy context, but, as noted by Rebelo (1991), is valid in a world of open economies connected by international capital markets if all countries follow the worldwide tax system. Furthermore, to focus on the effects of taxation, government revenues do not affect the marginal utility of private consumption and leisure or the production possibilities of the private sector[1]. Under these assumptions, capital accumulation takes place according to

$$\dot{K}(t) = (1-\tau_r)r(t)\phi(t)K(t) + (1-\tau_w)w(t)N(t)H(t) - \delta_k K(t) - C(t), \tag{4}$$

[1] The implications of productive government expenditures are examined in section 4.

where δ_k is the depreciation rate of capital and C aggregate consumption.

Human capital accumulation takes place by combining the remaining fraction $(1-\phi)$ of the capital stock with $(1-N-L)H$ efficiency units of labor, where L is the fraction of the time used for leisure activities. To focus on the impact of income taxation, the production of human capital is not included in the definition of the tax base (whereas income generated from human capital is included). In essence, human capital is viewed as a nonmarket activity whose inputs are not subject to factor income taxation.[2] The human capital stock depreciates at the rate δ_h. This leads to the following human capital accumulation equation:

$$\dot{H}(t) = B\left[(1-\phi(t))\,K(t)\right]^{1-\beta}\left[(1-N(t)-L(t))\,H(t)\right]^{\beta} - \delta_h H(t), \tag{5}$$

where $B > 0$ and $0 < \beta < 1$ are parameters.

The government is restricted to run a balanced budget. That is, the government can neither finance deficits by issuing debt nor run surpluses by accumulating assets. Government expenditures, $G(t)$, therefore equal tax revenues in each period,

$$G(t) = \tau_r r(t)\phi(t)K(t) + \tau_w w(t)N(t)H(t). \tag{6}$$

Inserting the expressions for $r(t)$ and $w(t)$ in the government budget constraint implies that $G(t) = [\tau_r(1-\gamma) + \tau_w\gamma]Y(t)$.

In the presence of an endogenous leisure choice, the preferences of the households have to be such that in equilibrium the rate of growth of consumption and the allocations of time between work, leisure and human capital accumulation are constant. The following utility function is used throughout the paper[3]:

$$U(C, L, G) = \frac{\left(C^{\alpha}L^{1-\alpha}\right)^{1-\sigma} - 1}{1-\sigma} + v(G), \tag{7}$$

[2] In support of this case, we can argue that the individual time input (and implicit labor income) used in the production of human capital - think of student's time spent on school - is not taxed. Lucas (1990) and Rebelo (1991) are examples of this case. On the other hand, human capital accumulation can also be thought of as (partly) a market activity whose inputs are taxed and/or subsidized (e.g. Pecorino, 1993, Stokey and Rebelo, 1995). The salaries of teachers, for example, are taxed inputs into the production of human capital. However, in many countries public spending on education is large and/or human capital accumulation is subsidized or has favorable tax treatments, limiting the impact of taxation in the education sector.

[3] See King, Plosser and Rebelo (1988) for a derivation of the class of utility functions from which this function is taken.

for $\sigma > 0$ ($1/\sigma$ is the elasticity of intertemporal substitution) and $0 < \alpha \leq 1$. The function $v(.)$ satisfies the usual regularity conditions with $v\prime(.) > 0$ and $v\prime\prime(.) < 0$. Tax revenues are used to provide services that enter into households' utility functions (see e.g. Barro, 1990). Here we impose an additive structure, ensuring that G does not affect the marginal utility of private consumption and leisure. Because households take the government expenditures as given, government expenditures do not affect the choices of the households. (However, they do affect the utility or welfare level of the households, implying that production will be lower than optimal.)

A competitive equilibrium for this economy is a set of infinite sequences for the quantities $\{C(t), L(t), N(t), \phi(t), K(t), H(t), G(t)\}$, prices $\{r(t), w(t)\}$ and constant tax rates τ_r and τ_w such that, taken the prices and $G(t)$ as given, the tuple $\{C(t), L(t), N(t), \phi(t)\}$ maximizes

$$\int_0^\infty e^{-\rho t} U(C(t), L(t), G(t)) dt \tag{8}$$

subject to equations (4) and (5), and such that $C(t) \geq 0$, $K(t) \geq 0$, $H(t) \geq 0$, $0 \leq L(t) \leq 1$, $0 \leq N(t) \leq 1$, $0 \leq L(t) + N(t) \leq 1$, $0 \leq \phi(t) \leq 1$, $K(0)$, $H(0)$ given, while the path $\{L(t), N(t), \phi(t), K(t), H(t), G(t), r(t), w(t)\}$ satisfies equations (2), (3) and (6). The parameter ρ reflects the time preference of the households.

In order to have different technologies for production and education, γ and β should be different. The most plausible case is that the goods sector is relatively intensive in physical capital while the educational sector is relatively intensive in human capital, i.e. $\gamma < \beta$. In most models with human capital accumulation β is chosen to be one, as in Lucas (1988) and Ladrón-de-Guevara et al. (1997).

First, in a competitive equilibrium, the allocation of both physical and human capital across the two sectors is such that the marginal products of the two types of capital (measured in terms of units of physical capital) should be equated in the two sectors, i.e.,

$$(1-\tau_r)r = q(1-\beta)B\left[(1-\phi)K\right]^{-\beta}\left[(1-N-L)H\right]^{\beta} \tag{9}$$

and

$$(1-\tau_w)w = q\beta B\left[(1-\phi)K\right]^{1-\beta}\left[(1-N-L)H\right]^{\beta-1}, \tag{10}$$

where q is the relative value of human capital in terms of physical capital. Second, the return from investing one unit in physical capital should be the same as the return from investing $1/q$ units in human capital. Hence,

$$(1-\tau_r)r-\delta_k = \beta B\left[(1-\phi)K\right]^{1-\beta}\left[(1-N-L)H\right]^{\beta-1}(1-L)-\delta_h+\frac{\dot{q}}{q}. \tag{11}$$

Third, the optimal growth rate of consumption, given the interest rate, is

$$\frac{\dot{C}}{C} = \frac{(1-\tau_r)r - \delta_k - \rho}{\sigma_\alpha} \tag{12}$$

with $\sigma_\alpha = 1 - (1-\sigma)\alpha$. Last, the optimal allocation of time between leisure and non-leisure activities requires that the marginal utility of leisure equals the marginal productivity of non-leisure time measured in terms of utility of forgone consumption,

$$\left(C^\alpha L^{1-\alpha}\right)^{-\sigma}(1-\alpha)C^\alpha L^{-\alpha} = \lambda q \beta B\left[(1-\phi)K\right]^{1-\beta}\left[(1-N-L)H\right]^{\beta-1}H, \tag{13}$$

where λ is the current-value shadow price of the capital stock. The derivation of equations (12) and (13) is given in appendix 6.1.

2.1 The balanced growth path

In a balanced growth path C, K and H grow at constant rates, while ϕ, N and L remain constant. In particular, it follows that $\dot{C}/C = \dot{K}/K = \dot{H}/H \equiv g$. If we define $x \equiv K/H$ and $z \equiv C/H$, then this implies that, along a balanced growth path, $\dot{x} = 0$ and $\dot{z} = 0$. Using equations (4) and (5), the $\dot{x} = 0$ equation is given by

$$(1-\tau_r)r\phi + (1-\tau_w)w\frac{N}{x} - \frac{C}{K} - \delta_k = B(1-\phi)^{1-\beta}(1-N-L)^\beta x^{1-\beta} - \delta_h. \tag{14}$$

Similarly, by equations (5) and (12), the $\dot{z} = 0$ equation can be written as

$$\frac{(1-\tau_r)r - \delta_k - \rho}{\sigma_\alpha} = B(1-\phi)^{1-\beta}(1-N-L)^\beta x^{1-\beta} - \delta_h. \tag{15}$$

Furthermore, equations (9), (10), (11) and (13), together with the fact that $\dot{q} = 0$ in a balanced growth path, imply that

$$\frac{1-\tau_w}{1-\tau_r}\frac{\gamma}{1-\gamma}\frac{\phi}{N} = \frac{\beta}{1-\beta}\frac{1-\phi}{1-N-L}, \tag{16}$$

$$(1-\tau_r)(1-\gamma)A\left[\frac{\phi}{N}x\right]^{-\gamma} - \delta_k = \beta B\left[\frac{(1-\phi)}{1-N-L}x\right]^{1-\beta}(1-L) - \delta_h, \tag{17}$$

$$(1-\alpha)z = \alpha(1-\tau_w)\gamma A\left[\frac{\phi}{N}x\right]^{1-\gamma}L. \tag{18}$$

This system of equations, consisting of the equations (14)-(18), characterizes a balanced growth path. To solve this system of equations, we need to impose $\delta_k = \delta_h$. Then, as is shown in appendix 6.2, the solution of this system involves the solution of the next two equations in the two unknown variables N and L:

$$\begin{aligned}&(1-\gamma)(1-T\psi)N^2 + \\ &+(1-L)\{\beta(1-\gamma+T\gamma)-(1-\gamma)(1-2T\psi)\}N + \\ &-T(1-L)\left\{(1-\gamma)\psi(1-L)+\frac{\alpha\beta\gamma}{1-\alpha}L\right\}=0,\end{aligned} \tag{19}$$

$$N=\frac{\sigma_\alpha-\beta}{\sigma_\alpha}(1-L)+\frac{(\rho+(1-\sigma_\alpha)\delta)(T\psi)^{-(1-\nu)(1-\beta)}\beta^\nu}{\sigma_\alpha B^{1-\nu}\left[(1-\tau_r)A(1-\gamma)\right]^\nu}(1-L)^\nu, \tag{20}$$

with $\psi \equiv \frac{\gamma(1-\beta)}{\beta(1-\gamma)} < 1$, $T \equiv \frac{1-\tau_w}{1-\tau_r}$ and $\nu \equiv \frac{1-\beta}{1-\beta+\gamma} < 1$. The first equation derives from the $\dot{x} = 0$ equation, whereas the second equation derives from the $\dot{z} = 0$ equation. To solve this reduced system of two equations, notice that the first equation can be written as $aN^2 + b(L)N + c(L) = 0$, with $c(L) < 0$. To keep the analysis clear, we impose the following condition:

Condition 1 $T\psi < 1$.

The reason for imposing this condition is that this ensures that $a > 0$, which implies that (i) the discriminant $(b^2 - 4ac)$ is always positive (hence, there are only real solutions) and (ii) only one of the two solutions is positive. Hence - disregarding the negative solution - N can be expressed as a function of L, say $N = f(L)$. Moreover, imposing this condition is not only a device to establish a unique positive solution, it is also an empirically plausible condition. As ψ lies between 0 and 1, the condition requires T to be less than some value larger than one. E.g. if $\gamma = 2/3$ (consistent with the usual data on labor's share of income) and $\beta = 3/4 (> \gamma)$, the condition requires T to be less than 3/2. Concerning the empirical evidence, Carey and Tchilinguirian (2000) have constructed average effective tax rates on both capital and labor income for the OECD countries, which are reported in table 1. The last column shows the resulting values of T. It follows that 14 out of 21 countries have a T smaller than 1 and all reported countries have a T that is smaller than 3/2. Hence, the condition is likely to be satisfied in reality. Notice that this condition is automatically satisfied if $\beta = 1$ (as this implies that $\psi = 0$).

Similarly, equation (20) expresses N as a function of L, say $N = h(L)$. Therefore we have a balanced growth path if $f(L) = h(L)$.

2.2 The possibility of multiple balanced growth paths

In this section we shortly review the possibility of having multiple balanced growth paths in this model. First, we establish a necessary condition for the existence of more than one balanced growth path. This necessary condition derives from the fact that a balanced growth path is characterized by the equality of two functions of L, $f(L)$ and $h(L)$. First it is shown that $f(L)$ is strictly concave. Since a strictly concave function has at most two intersections with a convex function, and because $f(1) = h(1)$ is one of the intersections, there will be at most one interior solution if $h(L)$ is convex. Hence, convexity of $h(L)$ excludes the possibility of multiple balanced growth paths. This leads to the next proposition.

Proposition 1 *Let Condition 1 hold. A necessary condition for the existence of multiple balanced growth paths is that* $\sigma < 1 + \frac{\rho}{\alpha\delta}$.

Proof. See appendix 6.5.1. ■

Is it likely that this condition holds in reality, or not? Given plausible values of the relevant parameters - $0 \leq \rho \leq 0.1$, $0 < \delta \leq 0.1$ and $0 < \alpha \leq 1$ - and the empirically estimated range of values for σ - usually σ is estimated somewhere between 1 and 5 - the comparison could come out either way.

However, if the condition holds, this does not imply that there are multiple balanced growth paths, as it is only a necessary condition. Given the complexity of the model, to find out whether it is actually possible to have multiple BGP's we resort to a numerical analysis. It turns out that it is possible to have more than one balanced growth path, as can be seen in figure 1[4]. This figure shows the existence of three BGP's (all of which satisfy the transversality condition as given in appendix 6.1). BGP 1 and BGP 2 are interior, while BGP 3 is a non-interior balanced growth path (in which $N + L = 1$). See appendix 6.3 for a proof that the non-interior BGP is characterized by $f(L) = 1 - L$ (i.e., it lies on the point of intersection of $N = f(L)$ and $N = 1 - L$).

The existence of multiple balanced growth paths raises the question of the stability properties of the BGP's. Due to the complexity of the dynamic system, we investigated the stability of the BGP's numerically (see appendix 6.4). That analysis implies, similar to Ladrón-de-Guevara et al. (1997), that BGP 2 is unstable, while BGP 1 and BGP 3 are stable. Let x_i $(i = 1, 2, 3)$

[4] There are many combinations of the parameters that result in three BGP's. Another example, with a higher (more realistic) value of σ, is: $\alpha = 0.3$, $\beta = 0.97$, $\gamma = 0.5$, $A = 1.7$, $B = 0.385$, $\rho = 0.07$, $\sigma = 1$, $\delta = 0.05$, $\tau_r = \tau_w = 0$.

denote the capital-human capital ratio at BGP i. Then, given some initial value x_0 ($\neq x_2$) there are z_0, N_0, L_0 and ϕ_0 such that $\{x_0, z_0, N_0, L_0, \phi_0\}$ lies on a stable manifold and x_0 converges to either x_1 or x_3, depending on the specific value of x_0. The next lemma implies that along the curve of $f(L)$ a higher value of L is accompanied with a higher capital-human capital ratio.

Lemma 2 *Let Condition 1 hold and let $x(L, N)$ denote the optimal capital-human capital ratio as a function of L and N. Then* $\frac{\partial x(L,f(L))}{\partial L} > 0$.

Proof. See appendix 6.5.2. ■

From this Lemma we may conclude that - given the stability properties implied by the numerical analysis - if x_0 is smaller than x_2 it converges to x_1 and if x_0 is larger than x_2 it converges to x_3. Furthermore, as will be apparent from the equation describing the growth rate of the economy as a function of leisure (see equation 21), more leisure lowers the growth rate on a balanced growth path. Thus, an economy with a relatively high human capital stock will converge to the 'high-growth' BGP, while an economy endowed with a relatively low human capital stock will end up in the 'low-growth' BGP (in which the growth rate is actually negative if the depreciation rate δ is positive).

As a last remark, it should be noted that the existence of multiple balanced growth paths relies on the presence of a labor-leisure choice, i.e. $0 < \alpha < 1$. If $\alpha = 1$, the model reduces to the basic two-sector endogenous growth model as described by Rebelo (1991, section III A), which has a unique interior solution.

3 The effects of taxation

This section analyses the effects of taxation - income taxation, labor income taxation and capital income taxation - on the long-run growth rate. We will restrict the analysis to the case of a unique interior balanced growth path. The analysis of taxation in the case of multiple balanced growth paths[5] does not add much to the economic intuition. In particular, the taxation analysis is practically the same for the stable interior equilibrium (the most plausible one) as for the unique balanced growth case.

[5]If $h(0) < f(0)$ it may happen that there is one interior BGP. However, that BGP will be unstable and the economy will converge to one of the two non-interior BGP's. This situation, therefore, falls into the category of multiple BGP's.

To find the effect of taxation on the long-run growth rate we first write the growth rate, g, as a function of L,

$$g(L) = Q(1-\tau_r)^{(1-\gamma)\nu}(1-\tau_w)^{\gamma\nu}(1-L)^{1-\nu} - \frac{\rho+\delta}{\sigma_\alpha}, \tag{21}$$

with $Q \equiv \psi^{(1-\beta)(1-\nu)}(\beta B)^{1-\nu}\left((1-\gamma)A\right)^{\nu}/\sigma_\alpha$. See appendix 6.6 for a derivation of this result. Notice that this expression for the growth rate reduces to the growth rate in Rebelo (1991, equation 14) if $\alpha = 1$.

3.1 Income taxation

Taxing (total) income implies that both tax rates are equal, i.e. $\tau_w = \tau_r \equiv \tau$ and $T = 1$. The above equation for the growth rate shows that, if both tax rates are equalized to τ, the tax rate has a direct negative effect on the growth rate of $(1-\tau)^{\nu}$. This direct effect consists of two opposing effects. First, taxation takes resources away from a productive sector (the capital production sector) to use it in some unproductive (but utility enhancing) way. Second, income taxation reduces the marginal products of capital and labor in the production of capital, inducing the economy to shift resources from capital production to human capital production.

Moreover, the tax rate has an *indirect* effect on the growth rate through its impact on leisure. This indirect effect consists of a substitution effect and an income effect. On the one hand, due to the reduced return to labor in the production of capital, more time will be directed towards leisure activities. On the other hand, taxation reduces the agent's available income, urging him to devote more time to the production of physical and human capital.

The determination of the overall effect of an income tax on the long-run growth rate thus involves the effect of the tax rate on the equilibrium value of leisure. To find this latter effect, notice that, since $T = 1$, the function $f(L)$ is not affected by a change in the tax rate τ. The function $h(L)$, on the contrary, is affected by a change in the tax rate. In particular, the nature of this effect (positive or negative) depends on the earlier encountered comparison between σ and $1 + \rho/(\alpha\delta)$: An increase in τ induces a positive shift of $h(L)$ if $\sigma < 1 + \rho/(\alpha\delta)$ and a negative shift if $\sigma > 1 + \rho/(\alpha\delta)$. (See figure 2 for an example of the latter effect.) Hence, higher income taxes lead to a new balanced growth path with more or less time spent on leisure activities, depending on the relative value of σ, the inverse of the elasticity of intertemporal substitution (EIS). Intuitively, the higher σ, the lower the EIS and the more anxious agents are to smooth their consumption over time and, hence, the stronger the income effect. As a result, at a relatively high level of σ the income effect dominates the substitution effect, leading to less time spent on leisure activities.

Spending less time on leisure activities and hence more time on working, either to produce output (and capital) or to produce human capital, has obviously a positive effect on the growth rate, as is also clear from equation (21). This raises the question whether this (indirect) positive effect could be stronger than the (direct) negative effect. Analyzing the derivative of $g(L)$ with respect to the tax rate τ, however, reveals that this situation cannot arise.

Proposition 3 *Let Condition 1 hold. Suppose that there is a unique interior BGP. Let $\tau_w = \tau_r \equiv \tau$. Then an increase in the tax rate induces a decline the long-run rate of growth.*

Proof. See appendix 6.5.3. ■

Taxing both labor income and capital income equally reduces the long-run growth rate. Compared to the situation with a fixed labor supply, the negative effect of income taxation on the long-run growth rate is either stronger (i.e. more negative) in the case of a relatively large EIS, which leads to more time spent on leisure activities, or weaker (i.e. less negative) in the case of a relatively small EIS, which leads to less time spent on leisure activities. Smith (1996) finds a similar but opposite effect of introducing uncertainty into a one-sector AK model. He finds that: (i) If the EIS is small, an increase in the tax rate reduces growth more than predicted by non-stochastic models; (ii) If the EIS is large, the long-run growth rate decreases by less than predicted by non-stochastic models.[6] The reason behind this result is that in his model, by construction, an increase in the tax rate reduces both the mean and variance of after-tax income. The resulting reduction in uncertainty (as measured by the variance) might increase or decrease the growth rate, depending on the value of the EIS. E.g., if the EIS is small, the income effect dominates the substitution effect, implying that a reduction in uncertainty leads to less (precautionary) savings and, consequently, less growth.

If $\beta = 1$, that is, if human capital accumulation is independent from physical capital, the expression for the growth rate, as given by equation (21), changes to

$$g(L) = \frac{B}{\sigma_\alpha}(1 - L) - \frac{\rho + \delta}{\sigma_\alpha}. \tag{22}$$

[6]The comparison between the effects of uncertainty in Smith (1996) and the effects of endogenous labor supply in the present paper on the impact of tax rates on long-run growth can de drawn even further. Both analyses show that it is actually possible for a tax increase to *increase* growth (see section 3.2 in the present paper).

This implies that taxes have no direct effect on the long-run growth rate. Moreover, it can easily be shown that in this case the function $h(L)$ is independent from any tax rate (see appendix 6.2). As a result, income taxation has no indirect effect on the growth rate either. Hence, the finding that endogenous leisure may strengthen or weaken the effect of income taxes on growth, but does not reverse the sign, depends on the combination of both physical capital in the educational sector and leisure as an additional argument in the utility function.

3.2 Capital income taxation

In this section we analyze the effect of a capital income tax on the long-run growth rate. Let us first start with the case in which capital is not an input in the education sector, i.e. $\beta = 1$. In this case, the two depreciation rates on physical and human capital need not be equal. In fact, the analysis is feasible[7] for $\delta_k \geq \delta_h$. The growth rate, obtained from equations (11) and (12), is equal to

$$g(L) = \frac{B}{\sigma_\alpha}(1-L) - \frac{\rho + \delta_h}{\sigma_\alpha}, \tag{23}$$

implying that the capital income tax rate has no direct effect on the growth rate. A change in the time spent on leisure, then, directly translates into a change in the growth rate. To find the indirect effect of the tax rate through the effect on leisure, we need to examine, as in the previous section, the effect of the tax rate on the functions $f(L)$ and $h(L)$, which are (implicitly) given in appendix 6.2, equations (38) and (39). First notice that $f(L)$ only depends on the capital income tax rate, τ_r, through T. The derivative of $f(L)$ with respect to T can be derived from total differentiation of equation (38). As a result,

$$\frac{\partial f(L)}{\partial T} \propto \frac{\alpha}{1-\alpha}L - f(L) > 0. \tag{24}$$

The positive sign of this derivative follows directly from equation (38) by noticing that $(1-\gamma)BN^2 > 0$. A higher capital income tax rate therefore shifts the function $f(L)$ upwards.

Second, the capital income tax rate has no effect on $h(L)$. This implies that, given the existence of a unique equilibrium, the effect of the tax rate on the equilibrium value of leisure is negative. Since $h(0) > f(0) = 0$ (see footnote 4), $h(L)$ - which is monotone in L - intersects $f(L)$ from above, which implies that an upward shift of $f(L)$ decreases the equilibrium value of leisure. A rise in the capital income tax rate consequently leads to less

[7] See Appendix 6.2.

time spent on leisure activities, and hence to a *rise* in the rate of economic growth.

Proposition 4 *Given Condition 1, let a unique interior BGP exist. If human capital is the only input in the education sector, an increase in the tax rate on (physical) capital income induces a rise in the long-run rate of growth.*

Proof. This follows immediately from the consideration preceding the proposition. ∎

Let us now examine the case in which capital is an input in the education sector, i.e. $0 < \beta < 1$, with $\delta_k = \delta_h$. Equation (21) reveals that in this case the capital income tax rate has a direct negative effect on the growth rate of $(1-\tau_r)^{(1-\gamma)\nu}$. The indirect effect is now more complicated as the sign of the derivative of $f(L)$ with respect to T cannot be established analytically. However, extensive numerical computations[8] show that $\partial f(L)/\partial T$ is positive for all $L \in (0,1)$. Then a higher τ_r, which implies a higher T, shifts $f(L)$ upwards.

Numerical Result 1 $\partial f(L)/\partial T > 0$ *for all* $L \in (0,1)$.

The effect of the capital income tax rate on $h(L)$ can be deduced from rewriting equation (20) as

$$\begin{aligned} h(L) &= \frac{\sigma_\alpha - \beta}{\sigma_\alpha}(1-L) + [\rho + (1-\sigma_\alpha)\delta] \times \\ &\quad \times \Delta (1-\tau_r)^{-\nu(1-\gamma)}(1-\tau_w)^{-(1-\nu)(1-\beta)}(1-L)^\nu, \qquad (25) \end{aligned}$$

with $\Delta = \frac{\psi^{-(1-\nu)(1-\beta)}\beta^\nu}{\sigma_\alpha B^{1-\nu}[A(1-\gamma)]^\nu}$. Therefore, the effect of the capital income tax rate on $h(L)$, like in the case of the (total) income tax rate, depends on the relative values of σ, α, δ and ρ: An increase in τ_r induces a positive shift of $h(L)$ if $\sigma < 1 + \rho/(\alpha\delta)$ and a negative shift if $\sigma > 1 + \rho/(\alpha\delta)$.

Again we can ask ourselves the question whether the positive effect on the growth rate through a fall in the time spent on leisure activities could dominate the direct negative effect. While this was not possible in the case of income taxation, it is possible in the case of capital income taxation to construct examples in which the tax rate has a positive effect on the long-run growth rate. Moreover, it is possible for sensible or empirically plausible

[8] The numerical computations depend on the values of α, β, γ and T, such that $0 < \alpha < 1$, $0 < \gamma < \beta < 1$ and $0 < T < 1/\psi$.

parameter values. To show this, consider the following benchmark economy:

$$\begin{aligned} \gamma &= 0.67, \quad \rho = 0.05, \quad \delta = 0.1, \quad \sigma = 3, \quad \beta = 0.95, \quad \alpha = 0.5, \\ A &= 0.5, \quad B = 0.4, \quad \tau_w = 0. \end{aligned}$$

The value of $\gamma = 0.67$ is consistent with data on labor's share of income; the time preference parameter ρ and the depreciation rate δ are usually set between 0 and 0.1; the elasticity of intertemporal substitution, $1/\sigma$, is usually estimated between 0.2 and 1, see e.g. Vissing-Jørgenson (2002), Mulligan (2002); labor's share in the accumulation of human capital is restricted to be higher than γ, i.e. $\beta \in (\gamma, 1]$; the parameter α, representing the relative importance of consumption versus leisure in utility, should lie between 0 and 1; the parameters A and B are chosen to get reasonable values in equilibrium for working and leisure time and the growth rate. Table 2 shows the impact of increasing τ_r from 0 to 0.5 on leisure L and the growth rate g. In the benchmark economy - see table 2a with $\sigma = 3$ - the increase in the capital income tax rate induces a decline in the time spent on leisure, from 0.325 to 0.306, which is enough to offset the direct negative effect of taxation on the growth rate as the growth rate *increases* from 3.71% to 3.82%. Tables 2a - 2i examine the robustness of this result by varying the parameters. The general picture that arises from these exercises is that the positive effect of capital income taxation on the growth rate is robust to (small) changes in the parameters. The most critical parameters are α, β and σ. Table 2a shows that, according to intuition, the positive effect on the growth rate declines as σ becomes smaller. Actually, the critical value of σ, below which the effect on the growth rate turns negative, is approximately 1.54. This shows that the positive effect on the growth rate is not restricted to values of σ above $1 + \rho/(\alpha\delta)$ (which is equal to 2 in the benchmark economy). The restriction on β is more severe, as is shown in table 2b. Here, the turning point lies just below 0.92, indicating that the accumulation of human capital should be highly human capital intensive to exhibit a positive effect. Varying the parameter α (table 2d) shows a positive effect on the growth rate for $\alpha < 0.7$, i.e., as long as the appreciation for leisure (compared to consumption) is not too small. Finally, table 2g shows that starting from a distorted economy ($\tau_w > 0$) does not change the result qualitatively.

Notice that the finding that capital income taxation can have positive growth effects depends on the endogeneity of the leisure choice. With a fixed labor supply this result cannot be obtained.

Proposition 5 *Let capital be an input in the education sector. Then there exist configurations of the parameters such that an increase in the tax rate on capital income induces a rise in the long-run rate of growth.*

Proof. This follows from numerical simulation of the model (see table 2). ■

Taxation of physical capital in the output sector reduces the marginal products of capital and labor (the latter through a fall in ϕ, the share of capital used in final goods production) in the production of capital, inducing the economy to shift resources from capital production to human capital production and to leisure activities. This substitution effect concerns substitution of resources between the two *sectors* of the economy. With respect to the time allocated to leisure, this substitution effect is opposed by the income effect. These effects are similar to the effects of income taxation. Contrary to total income taxation, taxing only physical capital makes human capital accumulation even more attractive because the income generated from capital is taxed, while the income generated from human capital is not taxed. This extra substitution effect, which concerns substitution between the *factors* physical and human capital, allows capital taxation to raise the long-run growth rate.

In the case of $\beta = 1$, the growth rate is independent of the tax rate other than through leisure. This independence arises from the fact that the rate of return to investment in human capital is equal to $B(1-L) - \delta_h$. In an efficient production plan the capital-labor intensity in the output sector is chosen so that the rate of return to physical capital is also $B(1-L)-\delta_h$. The resulting effect of capital income taxation on leisure time is unambiguously negative, giving rise to the positive impact of capital income taxation on the growth rate.

Decreasing β, however, introduces physical capital into the human capital accumulation function, causing the rate of return to investment in human capital and therefore the growth rate to be directly dependent of the tax rate. The lower β, the higher the direct impact of τ_r on the growth rate. As a result, in the case of $\beta < 1$, a positive effect of capital income taxation on growth needs the extra effect of a strong income effect (a relatively low value of the EIS), causing agents to work more and/or invest more in human capital as a response to the introduction of capital income taxation.

To get an idea about the robustness of this result with respect to the functional forms of the production function and the human capital accumulation function[9], it is easy to show (see appendix 6.7) that the growth rate in equation (23) requires human capital accumulation to be a linear function of the human capital input only but is independent from the functional

[9]The functional form of the utility function is restricted to the CES-type (see also footnote 2).

form of the production function. Thus, given the linearity in the human capital accumulation function, for any production function, a change in the time spent on leisure directly translates into a change in the growth rate. A positive effect of capital income taxation on the growth rate, then, requires that an increase in the tax rate induces agents to spend *less* time on leisure activities. This is a realistic possibility given the clear and plausible reasons for a possible reduction in leisure time, and is unlikely to be the result of the functional forms being of the Cobb-Douglas type. Moreover, suggested by the results above, in the case of physical capital as an input in human capital accumulation the reduced amount of time spent on leisure may still dominate the direct negative effect as long as the human capital sector is human capital intensive enough.

A last robustness check concerns a situation in which the education sector is included in the tax base. If capital in the education sector is taxed too, this would lower the growth rate as human capital accumulation is made less attractive, thereby reducing the scope for capital taxation to have a positive impact on growth. In fact, in the benchmark economy, if we tax physical capital in both sectors equally (at 50%) the long-run growth rate actually declines, compared to no taxation. However, a positive impact on the growth rate is regained if e.g. the tax rate on capital in the education sector is reduced (due to subsidies or favorable tax treatments) to ca. 25% or less. Another possibility to regain the positive impact is to increase β, human capital's share in knowledge production, to ca. 0.97 or higher.

3.3 Labor income taxation

Let us again first examine the case of $\beta = 1$. Given the growth rate in equation (23) and the functions $f(L)$ and $h(L)$ (equations 38 and 39 in appendix 6.2), it is easy to see that the effect of a labor income tax is exactly the opposite of the effect of a capital income tax. A tax on labor income, therefore, induces a downward shift in $f(L)$, while leaving $h(L)$ unchanged. Under the same conditions as in the previous subsection, a higher tax rate on labor income will increase the time spent on leisure.

Proposition 6 *Given Condition 1, let a unique interior BGP exist. If human capital is the only input in the education sector, an increase in the tax rate on labor income induces a decline in the long-run rate of growth.*

Proof. Follows immediately from Proposition 4. ■

Considering the more general case in which capital is an input in the education sector, equation (21) directly shows that the labor income tax rate has a direct negative effect on the growth rate of $(1-\tau_w)^{\gamma\nu}$. The indirect effect of the labor income tax rate through leisure is very similar to the indirect effect of the capital income tax rate. First, the effect of τ_w on $f(L)$ is exactly the opposite from the effect of τ_r, i.e., a higher τ_w shifts $f(L)$ downwards (given Numerical Result 1). Second, the effect of τ_w on $h(L)$ is qualitatively similar to the effect of τ_r and follows from equation (25): An increase in τ_w induces a positive shift of $h(L)$ if $\sigma < 1+\rho/(\alpha\delta)$ and a negative shift if $\sigma > 1+\rho/(\alpha\delta)$.

What will be the effect of the labor income tax rate on the long-run rate of growth? If we compare labor income taxation with income taxation, the use of physical capital in the capital accumulation process is not taxed, implying that more capital is used to accumulate physical capital and less to accumulate human capital. This has a negative effect on growth. This suggests that the effect of the labor income tax rate on the long-run rate of growth is negative. This is confirmed in the next proposition.

Proposition 7 *Let Condition 1 and Numerical Result 1 hold. Suppose that there exists a unique interior BGP. Then an increase in the tax rate on labor income induces a decline in the long-run rate of growth.*

Proof. See appendix 6.5.4. ■

Hence, increasing the tax rate on labor income reduces the long-run rate of growth. Like in the case of income taxation, the negative effect of income taxation on the long-run growth rate, compared to the situation with a fixed labor supply, is either stronger (i.e. more negative) in the case of more time spent on leisure activities or weaker (i.e. less negative) in the case of less time spent on leisure activities.

3.4 Optimal taxation

So far, we have discussed the impact of various tax rates on the economy's long-run growth rate. One of the key results shows that increasing the capital income tax rate can raise the growth rate. The implication for welfare, however, is not clear. Obviously, the welfare implications depend on the utility provided by government services G, i.e. on the function $v(.)$ in equation (7). If government services do not provide any utility ($v(.) \equiv 0$), implying that the tax revenue is wasted, any taxation will decrease welfare in this model, as there is no externality or any other friction to block the First Fundamental Theorem of Welfare Economics. If the marginal utility from government services on the other hand is extremely high for small values of

G (e.g. $\lim_{G\downarrow 0} v'(G) = \infty$) taxing capital income (or labor income) to some degree will improve welfare.

In the first situation where tax revenues are wasted it is obviously optimal to abstain from taxation, either on capital or labor income. The second situation is more complicated. What are the welfare-maximizing tax rates on capital and labor income when the (benevolent) government can choose any mix of (linear) capital and labor income tax rates? Answering this question would imply the comparison of different paths - corresponding to different tax rates - given initial conditions. Such an analysis would involve transitional dynamics. Combined with the fact that we cannot explicitly solve the model, finding the optimal tax rates would involve an extensive numerical analysis well beyond the scope of this paper. What we can do, however, is to point out two reasons why precisely capital income or labor income should be taxed. In favor of a positive tax on capital income is the fact that, since the households take the government services as given, too much time is spent on leisure compared to the social optimum. This fact is favorable to capital income taxation because capital income taxation leads to less time spent on leisure (on the balanced growth path) than labor income taxation. In favor of a positive tax on labor income is the fact that a tax on labor income will generate higher tax revenues since labor's share of income, γ, is higher than capital's share of income, $1-\gamma$.

4 Productive government expenditures

In the preceding analysis we have assumed that tax revenues are used to provide services that enter into households' utility functions, without affecting the marginal utility of consumption and leisure or the production possibilities of the private sector. In this section we will examine the implications of introducing government expenditures that do influence the production possibilities of the private sector (infrastructure, education, etc.). See e.g. Barro (1990) and Glomm and Ravikumar (1997) for related analyses.

Suppose that government expenditures can be used for either improving the productivity of goods production, G_Y, or facilitating the accumulation of human capital, G_H, with $G_Y + G_H = G$, without generating utility directly (i.e., $v(.) \equiv 0$ in equation 7). In fact, let the production function change to

$$Y(t) = A\left(\frac{G_Y(t)}{Y(t)}\right)(\phi(t)K(t))^{1-\gamma}(N(t)H(t))^{\gamma}, \tag{26}$$

in which the productivity in the goods sector, $A(.)$, is an increasing function of government expenditures in the goods sector as a fraction of output[10], with $G_Y(t) = \xi[\tau_r(1-\gamma) + \tau_w\gamma]Y(t)$, $0 < \xi < 1$. Similarly, let human capital accumulation take place according to

$$\dot{H}(t) = B\left(\frac{G_H(t)}{Y(t)}\right)[(1-\phi(t))\,K(t)]^{1-\beta}\,[(1-N(t)-L(t))\,H(t)]^{\beta} - \delta_h H(t), \tag{27}$$

in which the productivity in the education sector, $B(.)$, is an increasing function of government expenditures in the education sector as a fraction of output, with $G_H(t) = (1-\xi)[\tau_r(1-\gamma) + \tau_w\gamma]Y(t)$. The advantage of this formulation is that the effect of a change in the capital and/or labor income tax rate(s) is essentially the same as its effect in the model without productive government expenditures *combined with* a change in the parameters A and B.

The effect of an increase in A and/or B on the growth rate (through both the direct as well as the indirect effect) runs opposite to the effect of an increase in the tax rate on income, τ. First, by equation (21), increasing A and/or B increases the growth rate directly. Second, by equation (20), increasing A and/or B induces more or less time spent on working depending on the relative value of σ compared to $1+\rho/(\alpha\delta)$. Since raising the productivity of goods production or human capital accumulation is income enhancing, it will reduce working time and stimulate leisure time if the income effect is relatively strong (σ relatively large), and vice versa in the case of a relatively weak income effect. However, since the productivity improvements affect both capital and labor in one of the two sectors, the direct positive effect on growth is always stronger than the indirect effect through leisure[11] Thus, productivity improvements in the output sector or in the education sector unambiguously raise the long-run growth rate.

The overall effect of an increase in the tax rate therefore depends on the size of the government-induced productivity improvements. For example, in the original model with $B = 0.25$ (and a relatively small σ)[12] increasing the tax rate (on total income) from 0 to 0.165 (which creates the same amount

[10] If e.g. $A(x) = Ax^{\theta}$, the production function can be rewritten as $Y(t) = A^{\frac{1}{1+\theta}} G(t)^{\frac{\theta}{1+\theta}} (\phi(t)K(t))^{\frac{1-\gamma}{1+\theta}} (N(t)H(t))^{\frac{\gamma}{1+\theta}}$. This shows that this formulation encompasses the one in which G, K and H together display constant returns to scale. However, this particular functional form implies that G is an essential factor of production, complicating the comparison with the model without productive government expenditures.

[11] An increase in A has the same effects as a decrease in τ, implying that the direct effect on growth is stronger than the indirect effect through leisure (see Proposition 3). The same result can be derived (along the lines of the proof of Proposition 3) for an increase in B.

[12] The other parameter values are: $\gamma = 0.67$, $\rho = 0.05$, $\delta = 0.05$, $\sigma = 1$, $\beta = 0.8$, $\alpha = 0.5$, $A = 0.5$.

of government revenues as $\tau_r = 0.5$) raises leisure time from 0.361 to 0.375 and reduces the growth rate from 2.21% to 1.51%. However, if the government revenues G are used to boost productivity in the education sector to $B = 0.264$, both leisure time and the growth rate stay roughly constant. Similarly, it takes productivity in the goods sector, A, to rise from 0.5 to ca. 0.6 to offset the negative effect on the growth rate from taxation.[13] Hence, compared to these growth-neutral productivity improvements, smaller productivity increases will (only) weaken the effect of taxation, while higher productivity increases will reverse the sign. The latter situation makes it possible that increasing tax rates and (productive) government expenditures simultaneously has a net positive effect on the growth rate. This is similar to what Kocherlakota and Yi (1997) find, using time series data for the U.S. and U.K., when they regress per capita income on public policy variables. Their results suggest that a simultaneous increase in public equipment capital and marginal tax rates has a net positive effect on growth.[14]

Similar results apply for the case of a relatively high σ. Although leisure time now increases as a response to higher productivity in both sectors, the resulting effect is an increase in the growth rate which may or may not overtake the initial negative effect of increased tax rates.

The extension of above observations to the cases of capital income and labor income taxation is straightforward. Improvements in the productivity in the goods sector as well as in the education sector affect the growth rate positively. This increases the set of parameter values for which capital income taxation raises the growth rate, and, depending on the size of the government-induced productivity improvements, opens up the possibility that even labor income taxation results in a higher growth rate.

5 Summary

The present study analyses the effects of taxation on long-run growth in a two-sector endogenous growth model with (i) physical capital as an input in the education sector and (ii) leisure as an additional argument in the utility function. Due to the flexibility of labor supply, taxation of (capital and/or labor) income may induce agents to spend more or less time on leisure activities, depending on the relative sizes of the substitution and income

[13] Notice that the parameters A and B in both equations (20) and (21) are raised to the power ν resp. $(1-\nu)$. As β is relatively high ($\gamma < \beta < 1$), ν is relatively small, implying that increases in B are more effective than increases in A.

[14] For more empirical evidence regarding the impact of public capital/expenditures on output/growth see Glomm and Ravikumar (1997, section 4).

effects. In the case of income taxation, where capital and labor income are taxed equally, the resulting effect on the growth rate is negative, as in the case without endogenous leisure. The finding that endogenous leisure may strengthen or weaken the effect of income taxation on growth (without reversing the sign) is, however, the result of including both endogenous leisure as well as capital as an input in the education sector.

Capital income taxation increases both the attractiveness of the education sector relative to the output sector as well as the factor human capital relative to the factor physical capital. In this case the direction of the effect may also change, in particular when the elasticity of intertemporal substitution is (relatively) small, implying a strong income effect. In that case, the positive effect of the increase in total non-leisure time dominates the direct negative effect of the tax rate on growth, implying that capital income taxation increases the long-run growth rate. The possibility of a positive effect of capital income taxes on growth is shown to be a consequence of the labor-leisure choice. The inclusion of capital as an input in the education sector actually reduces this possibility.

Labor income taxation, on the contrary, has a negative effect on the growth rate, with or without capital as an input in the education sector. In general, with capital in the education sector, endogenous leisure may strengthen or weaken the effect of labor income taxes on growth, but does not reverse the sign.

If the tax revenues are used to improve the productivity in the goods sector and/or the education sector, the impact on the growth rate is positively influenced. The effect of taxation on the long-run growth rate then depends on the size of the productivity improvements. Relatively small productivity improvements leave the qualitative results unchanged, while relatively large productivity improvements convert a negative effect on growth to a positive one. Hence, even labor income taxation may positively influence the growth rate as long as the productivity improvements - financed with the tax - are high enough.

Besides the effects of taxation on growth, this study establishes a necessary condition for the existence of multiple balanced growth paths and shows that, for certain parameter configurations, the model actually exhibits three balanced growth paths. This is due to the labor-leisure choice. Whether capital is included in the education sector or not does not change this result.

6 Appendix

6.1 The consumer optimization problem

The Hamiltonian associated with the representative consumer's optimization problem reads

$$\mathcal{H} = \frac{\left(C^{\alpha} L^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} + \lambda \dot{K} + \mu \dot{H}, \tag{28}$$

where $\dot{K}$ and $\dot{H}$ are given by equations (4) and (5). The first-order conditions are:

$$\left(C^{\alpha} L^{1-\alpha}\right)^{-\sigma} \alpha C^{\alpha-1} L^{1-\alpha} = \lambda, \tag{29}$$

$$\dot{\lambda} = \lambda \rho - \frac{\partial \mathcal{H}}{\partial K}, \tag{30}$$

$$\left(C^{\alpha} L^{1-\alpha}\right)^{-\sigma} (1-\alpha) C^{\alpha} L^{-\alpha} = \mu \beta B \left[(1-\phi) K\right]^{1-\beta} \left[(1-N-L) H\right]^{\beta-1} H, \tag{31}$$

$$\dot{\mu} = \mu \rho - \frac{\partial \mathcal{H}}{\partial H}. \tag{32}$$

If we define $q \equiv \frac{\mu}{\lambda}$, and make use of equations (9), (10) and (11), we can show that

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu} = \rho + \delta_k - (1-\tau_r) r, \tag{33}$$

which implies, by equation (29), that the growth rate of consumption is given by equation (12). Furthermore, by the definition of q, equation (31) directly turns into equation (13). Notice that equation (33) implies that q is constant.

Since μ is growing at the same rate as λ and H is growing at the same rate as K, the transversality condition is given by

$$\lim_{t \to \infty} \left[e^{-\rho t} \lambda(t) K(t)\right] = 0. \tag{34}$$

It is easy to show that the transversality condition holds if

$$(1-\sigma)\alpha \left[(1-\tau_r) r - \delta_k\right] < \rho. \tag{35}$$

Notice that the transversality condition is automatically satisfied if $\sigma > 1$.

6.2 Solution to the system of equations (14)-(18)

Let $\delta_k = \delta_h$. Using equations (2) and (3), equation (14) can be rewritten as

$$\begin{aligned} &(1-\tau_r)(1-\gamma)A\left(\frac{\phi}{N}\right)^{-\gamma} x^{1-\gamma}\phi + (1-\tau_w)\gamma A\left(\frac{\phi}{N}\right)^{1-\gamma} x^{1-\gamma}N - z \\ = \; & B\left(\frac{1-\phi}{1-N-L}\right)^{1-\beta} x^{2-\beta}(1-N-L). \end{aligned} \tag{36}$$

Note that equation (16) implies that

$$\frac{1-\phi}{1-N-L} = (T\psi)\frac{\phi}{N}$$

and

$$\phi = \frac{N}{T\psi(1-N-L)+N}.$$

Then, from equation (17), we can derive an expression of x, i.e.,

$$x = \left[\frac{(1-\tau_r)(1-\gamma)A}{\beta B(T\psi)^{1-\beta}(1-L)}\right]^{\frac{1}{1-\beta+\gamma}} \frac{N}{\phi}. \tag{37}$$

Moreover, it is easy to show that $\frac{N}{\phi}(1-N-L) = T\psi(1-L)^2 + (1-2T\psi)(1-L)N + (T\psi-1)N^2$. These observations, together with equation (18), imply that equation (36) transforms to equation (19).

To derive equation (20), we rewrite equation (15) with the help of equation (11):

$$\begin{aligned} &\beta B\,(1-\phi)^{1-\beta}\,(1-N-L)^{\beta-1}\,x^{1-\beta}(1-L) - \delta - \rho \\ = \; & \sigma_\alpha B\,(1-\phi)^{1-\beta}\,(1-N-L)^{\beta}\,x^{1-\beta} - \sigma_\alpha\delta, \end{aligned}$$

or,

$$\begin{aligned} &B\,(1-\phi)^{1-\beta}\,(1-N-L)^{\beta-1}\,x^{1-\beta}\,[\beta(1-L) - \sigma_\alpha(1-N-L)] \\ = \; & \rho + (1-\sigma_\alpha)\delta. \end{aligned}$$

Inserting the expression for x and simplifying yields equation (20).

If $\beta = 1$ and $\delta_k \neq \delta_h$, it is easy to show that equations (19) and (20) change to

$$(1-\gamma)BN^2 + T\gamma[B(1-L)+\delta_k-\delta_h]N - T\frac{\alpha\gamma}{1-\alpha}[B(1-L)+\delta_k-\delta_h]L = 0, \tag{38}$$

$$N = \frac{\sigma_\alpha - 1}{\sigma_\alpha}(1-L) + \frac{\rho + (1-\sigma_\alpha)\delta_h}{\sigma_\alpha B}. \tag{39}$$

To ensure the existence of a unique positive solution to equation (38), the depreciation rates are restricted to $\delta_k \geq \delta_h$. (This restriction implies that $B(1-L)+\delta_k-\delta_h$ is positive for all L, such that (i) the discriminant is always positive (implying that there are only real solutions) and (ii) only one of the two solutions is positive.)

6.3 The non-interior balanced growth path

The Hamiltonian associated with the restricted optimization problem (in which $N = 1 - L$) reads

$$\mathcal{H} = \frac{\left(C^\alpha L^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} + \theta \dot{K}, \tag{40}$$

where $\dot{K}$ is given by equation (4) (with $\phi = 1$ and $N = 1-L$). The first-order conditions are:

$$\left(C^\alpha L^{1-\alpha}\right)^{-\sigma} \alpha C^{\alpha-1} L^{1-\alpha} = \theta, \tag{41}$$

$$\dot{\theta} = \theta\rho - \frac{\partial \mathcal{H}}{\partial K}, \tag{42}$$

$$\left(C^\alpha L^{1-\alpha}\right)^{-\sigma} (1-\alpha) C^\alpha L^{-\alpha} = \theta(1-\tau_w)wH. \tag{43}$$

Equations (41) and (43), together with (3) (with $\phi = 1$ and $N = 1 - L$) imply that

$$z = \frac{\alpha}{1-\alpha}(1-\tau_w)\gamma A x^{1-\gamma}(1-L)^{\gamma-1}L,$$

while the $\dot{x} = 0$ and $\dot{z} = 0$ equations are now given by

$$(1-\tau_r)(1-\gamma)A(1-L)^\gamma x^{1-\gamma} + (1-\tau_w)\gamma A(1-L)^\gamma x^{1-\gamma} = z \tag{44}$$

and

$$(1-\tau_r)r - \delta - \rho = -\sigma_\alpha \delta. \tag{45}$$

It is easy to show that the solution for L to this set of equations is implicitly given by

$$(1-\tau_r)(1-\gamma) + (1-\tau_w)\gamma = \frac{\alpha}{1-\alpha}(1-\tau_w)\gamma\frac{L}{1-L}, \tag{46}$$

which is exactly equal to equation (19) with $N = 1 - L$.

6.4 Stability of the balanced growth paths

To investigate the local stability of the balanced growth paths, we first derive the law of motion of the variables $\{x, z, L, N, \phi\}$. Elimination of the costate variables leads to the following system of five differential equations:

$$\dot{x} = A\,(\phi x)^{1-\gamma} N^{\gamma} - B(1-\phi)^{1-\beta}(1-N-L)^{\beta} x^{2-\beta} - z,$$

$$\dot{L} = \frac{L}{1-\frac{(1-\sigma)(1-\alpha)}{\sigma_\alpha}}\left[\eta(x,L,N,\phi) + \frac{1-\gamma}{\beta-\gamma}\mu(x,L,N,\phi)\right],$$

$$\dot{z} = z\left[\eta(x,L,N,\phi) + \frac{(1-\sigma)(1-\alpha)}{\sigma_\alpha L}\dot{L}\right],$$

$$\dot{N} = N\left[\frac{1-\phi}{\phi(1-L)-N}\dot{L} + \frac{1-N-L}{\phi(1-L)-N}\left(\frac{\dot{x}}{x} + \frac{1}{\beta-\gamma}\mu(x,L,N,\phi)\right)\right],$$

$$\dot{\phi} = \phi(1-\phi)\left[\frac{1-L}{(1-N-L)N}\dot{N} + \frac{1}{1-N-L}\dot{L}\right],$$

where $\eta(.)$ and $\mu(.)$ are given by

$$\begin{aligned}\eta(x,L,N,\phi) &= \frac{(1-\gamma)A}{\sigma_\alpha}(\phi x)^{-\gamma} N^{\gamma} - B((1-\phi)x)^{1-\beta}(1-N-L)^{\beta} \\ &\quad -\frac{\rho + (1-\sigma_\alpha)\delta}{\sigma_\alpha},\end{aligned}$$

$$\begin{aligned}\mu(x,L,N,\phi) &= (1-L)\beta B((1-\phi)x)^{1-\beta}(1-N-L)^{\beta-1} \\ &\quad -(1-\gamma)A\,(\phi x)^{-\gamma} N^{\gamma}.\end{aligned}$$

By differentiating each differential equation with respect to x, z, L, N and ϕ in a steady state, we obtain the coefficients matrix of the linearized system around a steady state. The eigenvalues of this matrix determine the local stability of the steady state. If all eigenvalues have negative real parts the steady state is locally stable. If all eigenvalues have positive real parts the steady state is unstable. If some of the eigenvalues have negative and others positive real parts, there exists a stable manifold, which is defined as the set of points on which convergence to the steady state takes place. The dimension of the stable manifold is equal to the number of eigenvalues with negative real parts.

In the case of the three equilibria of figure 1 the (approximate) values of the eigenvalues, which are given in table 3, imply that BGP 2 is unstable, while BGP's 1 and 3 are stable, i.e. both have a stable manifold of dimension one.

6.5 Proofs

6.5.1 Proof of Proposition 1

As written in the paragraph preceding Proposition 1, the first step in the proof is to prove that $f(L)$ is strictly concave. Solving equation (19) for $N = f(L)$ yields

$$2(1-\gamma)(1-T\psi)f(L) + (1-L)\Omega = (1-L)Q \tag{47}$$

with $Q = \left(\Omega^2 + 4(1-\gamma)(1-T\psi)T\{(1-\gamma)\psi + \frac{\alpha\beta\gamma}{1-\alpha}\frac{L}{1-L}\}\right)^{1/2} > 0$ and $\Omega = \{\beta(1-\gamma+T\gamma)-(1-\gamma)(1-2T\psi)\}$. Differentiating this equation with respect to L gives

$$2(1-\gamma)(1-T\psi)f'(L) = \Omega - Q + \frac{2(1-\gamma)(1-T\psi)T\alpha\beta\gamma}{(1-\alpha)Q(1-L)}.$$

Replacing $\Omega - Q$ with the expression implied by equation (47) and simplifying yields

$$(1-L)f'(L) = -f(L) + \frac{\alpha\beta\gamma T}{(1-\alpha)Q}, \tag{48}$$

which, since $Q > 0$, implies that

$$-(1-L)f'(L) < f(L). \tag{49}$$

Using equation (48), the second derivative of $f(L)$ can be written as

$$f''(L) = \frac{-2(1-\gamma)(1-T\psi)(\alpha\beta\gamma T)^2}{(1-\alpha)^2(1-L)^3Q^3} < 0. \tag{50}$$

This proves that $f(L)$ is strictly concave.

The second step is to find out when $h(L)$ is convex and/or concave. It is straightforward to show that the second derivative of $h(L)$ is given by

$$h''(L) = -\nu(1-\nu)\frac{(\rho+(1-\sigma_\alpha)\delta)(T\psi)^{-(1-\nu)(1-\beta)}\beta^\nu}{\sigma_\alpha B^{1-\nu}\left[(1-\tau_r)A(1-\gamma)\right]^\nu}(1-L)^{\nu-2}. \tag{51}$$

Hence, $h(L)$ is strictly concave if $\sigma < 1+\frac{\rho}{\alpha\delta}$ and strictly convex if $\sigma > 1+\frac{\rho}{\alpha\delta}$.

6.5.2 Proof of Lemma 2

From equation (37) we derive that

$$x(L,N) = X_0\left[T\psi(1-L) + (1-T\psi)N\right](1-L)^{\frac{-1}{1-\beta+\gamma}}, \tag{52}$$

with $X_0 \equiv \left[\frac{(1-\tau_r)(1-\gamma)A}{\beta B(T\psi)^{1-\beta}}\right]^{\frac{1}{1-\beta+\gamma}} > 0$. This implies that

$$x(L, f(L)) = \eta_1(1-L)^{\frac{\gamma-\beta}{1-\beta+\gamma}} + \eta_2 f(L)(1-L)^{\frac{-1}{1-\beta+\gamma}}, \tag{53}$$

with $\eta_1 \equiv X_0 T\psi > 0$ and $\eta_2 \equiv X_0(1-T\psi) > 0$ (by Condition 1). Differentiating with respect to L yields

$$\begin{aligned} \frac{\partial x(L, f(L))}{\partial L} &= \eta_1 \frac{\beta-\gamma}{1-\beta+\gamma}(1-L)^{\frac{-1}{1-\beta+\gamma}} + \eta_2 f'(L)(1-L)^{\frac{-1}{1-\beta+\gamma}} \\ &\quad + \eta_2 \frac{1}{1-\beta+\gamma} f(L)(1-L)^{\frac{-1}{1-\beta+\gamma}-1}. \end{aligned} \tag{54}$$

This derivative is positive iff

$$\eta_1 \frac{\beta-\gamma}{1-\beta+\gamma} + \eta_2 f'(L) + \eta_2 \frac{1}{1-\beta+\gamma} f(L)(1-L)^{-1} > 0, \tag{55}$$

which is true since equation (49) implies that

$$-f'(L) < \frac{1}{1-\beta+\gamma} f(L)(1-L)^{-1}, \tag{56}$$

since $\frac{1}{1-\beta+\gamma} > 1$.

6.5.3 Proof of Proposition 3

Inserting $\tau_w = \tau_r \equiv \tau$ into equation (21) implies that the growth rate as a function of L reads

$$g(L) = Q(1-\tau)^{\nu}(1-L)^{1-\nu} - \frac{\rho+\delta}{\sigma_\alpha}. \tag{57}$$

The derivative with respect to τ is, then, given by

$$\frac{\partial g(L)}{\partial \tau} = -\nu Q(1-\tau)^{\nu-1}(1-L)^{1-\nu} - (1-\nu)Q(1-\tau)^{\nu}(1-L)^{-\nu}\frac{\partial L}{\partial \tau}, \tag{58}$$

which implies that $\frac{\partial g}{\partial \tau} < 0$ iff

$$\frac{\partial L}{\partial \tau} > -\frac{\nu}{1-\nu}\left(\frac{1-L}{1-\tau}\right) \equiv -\frac{1-\beta}{\gamma}\left(\frac{1-L}{1-\tau}\right). \tag{59}$$

To prove the proposition we will show that this inequality is satisfied.

The total derivative of $f(L) = h(L)$, which applies in a balanced growth path, can be written as

$$(f'(L) - h'(L)) \frac{\partial L}{\partial \tau} = \frac{\partial h(L)}{\partial \tau}. \tag{60}$$

Let $h(L) = H_0(1-L)+H_1(1-\tau)^{-\nu}(1-L)^{\nu}$, where H_0 and H_1 can be deduced from equation (20). Then

$$\frac{\partial h(L)}{\partial \tau} = \nu H_1(1-\tau)^{-\nu-1}(1-L)^{\nu} \tag{61}$$

and

$$-h'(L) = H_0 + \nu H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}. \tag{62}$$

This implies that

$$\left(\frac{1-\tau}{1-L}\right)\frac{\partial L}{\partial \tau} = \frac{\nu H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}}{f'(L) + H_0 + \nu H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}}, \tag{63}$$

which should be higher than $-\frac{\nu}{1-\nu}$ to satisfy equation (59). Hence, equation (59) is satisfied iff

$$\frac{(1-\nu)H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}}{f'(L) + H_0 + \nu H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}} > -1. \tag{64}$$

In the case of a unique interior BGP, $h(L)$ intersects $f(L)$ from above, implying that $f'(L) - h'(L) > 0$. This allows us to multiply both sides of equation (64) with $f'(L) - h'(L)$ which yields

$$f'(L) + H_0 > -H_1(1-\tau)^{-\nu}(1-L)^{\nu-1}, \tag{65}$$

or

$$-f'(L) < H_0 + H_1(1-\tau)^{-\nu}(1-L)^{\nu-1} = \frac{h(L)}{1-L} = \frac{f(L)}{1-L}. \tag{66}$$

According to equation (49) this inequality is satisfied.

6.5.4 Proof of Proposition 7

This proof proceeds along the same lines as the proof of proposition 3. First, it is easy to show that $\frac{\partial g}{\partial \tau_w} < 0$ iff

$$\frac{\partial L}{\partial \tau_w} > -\frac{\gamma \nu}{1-\nu}\left(\frac{1-L}{1-\tau_w}\right) \equiv -(1-\beta)\left(\frac{1-L}{1-\tau_w}\right). \tag{67}$$

Then we take the total derivative of $f(L) = h(L)$, which can be written as

$$(f'(L) - h'(L)) \frac{\partial L}{\partial \tau_w} = \frac{\partial h(L)}{\partial \tau_w} - \frac{\partial f(L)}{\partial \tau_w}. \tag{68}$$

Two observations need to be made at this point. First, by Numerical Result 1, $\frac{\partial f(L)}{\partial \tau_w} < 0$. Second, in the case of a unique interior BGP, $h(L)$ intersects $f(L)$ from above, which implies that $f'(L) - h'(L) > 0$. These two observations imply that

$$\frac{\partial L}{\partial \tau_w} > \frac{\partial h(L)/\partial \tau_w}{f'(L) - h'(L)}, \tag{69}$$

which can be rewritten as

$$\left(\frac{1-\tau_w}{1-L}\right) \frac{\partial L}{\partial \tau_w} > \frac{(1-\nu)(1-\beta)H_1(1-\tau_w)^{-(1-\nu)(1-\beta)}(1-L)^{\nu-1}}{f'(L) + H_0 + \nu H_1 (1-\tau_w)^{-(1-\nu)(1-\beta)}(1-L)^{\nu-1}}. \tag{70}$$

If the right-hand side of this inequality is higher than $-(1-\beta)$, i.e.,

$$\begin{aligned} & (1-\nu)(1-\beta)H_1(1-\tau_w)^{-(1-\nu)(1-\beta)}(1-L)^{\nu-1} \\ > \; & -(1-\beta)\left[f'(L) + H_0 + \nu H_1(1-\tau_w)^{-(1-\nu)(1-\beta)}(1-L)^{\nu-1}\right], \end{aligned} \tag{71}$$

equation (67) is satisfied and the proof is done. It is easy to show that this inequality can be transformed to

$$-f'(L) < \frac{h(L)}{1-L} = \frac{f(L)}{1-L}, \tag{72}$$

which is satisfied according to equation (49).

6.6 Derivation of the growth rate as a function of L

Equation (2) implies that

$$\begin{aligned} (1-\tau_r)r &= (1-\tau_r)(1-\gamma)A\left[\frac{\phi}{N}x\right]^{-\gamma} \\ &= \beta B\left[\frac{(1-\phi)}{1-N-L}x\right]^{1-\beta}(1-L) \\ &= \beta B(T\psi)^{1-\beta}\left(\frac{\phi}{N}\right)^{1-\beta} x^{1-\beta}(1-L) \end{aligned}$$

where the second and third equations follow from equations (17) - with $\delta_k = \delta_h$ - and (16) respectively. Inserting the expression for x as given by equation (37), yields

$$(1-\tau_r)r = \psi^{(1-\beta)(1-\nu)}(\beta B)^{1-\nu}\left((1-\gamma)A\right)^{\nu}(1-\tau_r)^{\nu}T^{(1-\beta)(1-\nu)}(1-L)^{1-\nu}. \tag{73}$$

Inserting this equation into equation (12) yields the growth rate as a function of L as given in equation (21).

6.7 A general set-up

For any functions $F(.,.)$ and $G(.,.)$ - ignoring the time indications t - let

$$Y = F(\phi K, NH),$$

and

$$\dot{H} = G\left((1-\phi)\,K, (1-N-L)\,H\right) - \delta_h H,$$

replace equations (1) and (5) respectively. The first-order conditions, then, read

$$(1-\tau_r)r = qG_1((1-\phi)\,K, (1-N-L)\,H), \tag{74}$$

$$(1-\tau_w)w = qG_2((1-\phi)\,K, (1-N-L)\,H), \tag{75}$$

$$(1-\tau_r)r - \delta_k = G_2((1-\phi)\,K, (1-N-L)\,H)(1-L) - \delta_h, \tag{76}$$

$$\sigma_\alpha g = (1-\tau_r)r - \delta_k - \rho, \tag{77}$$

$$z = \frac{\alpha}{1-\alpha} qG_2((1-\phi)\,K, (1-N-L)\,H)L. \tag{78}$$

Equations (76) and (77) together directly imply that

$$g = \frac{G_2((1-\phi)\,K, (1-N-L)\,H)(1-L)}{\sigma_\alpha} - \frac{\delta_h + \rho}{\sigma_\alpha}. \tag{79}$$

If $G(.,.) = B(1-N-L)H$, i.e. a linear function of the human capital input only, the growth rate reduces to

$$g = \frac{B(1-L)}{\sigma_\alpha} - \frac{\delta_h + \rho}{\sigma_\alpha}. \tag{80}$$

References

[1] Barro, R.J. (1990), "Government spending in a simple model of endogenous growth", Journal of Political Economy 98, S103-S125.

[2] Carey, D. and H. Chilinguirian (2000), "Average effective tax rates on capital, labor and consumption", OECD Economics Department Working Paper no. 258.

[3] Chamley, C. (1992), "The welfare cost of taxation and endogenous growth", Institute for Economic Development Discussion Paper No. 30, Boston University.

[4] De Hek, P.A. (1998), "An aggregative model of capital accumulation with leisure-dependent utility", Journal of Economic Dynamics and Control 23/2, 255-276.

[5] De Hek, P.A. (1999), "Endogenous technological change with leisure-dependent utility", Economic Theory 14, 669-684.

[6] Glomm, G. and B. Ravikumar (1997), "Productive government expenditures and long-run growth", Journal of Economic Dynamics and Control 21, 183-204.

[7] Hendricks, L. (1999), "Taxation and Long-Run Growth", Journal of Monetary Economics, 43, 411-434.

[8] Jones, L.E., R. Manuelli and P. Rossi (1993), "Optimal taxation in models of endogenous growth", Journal of Political Economy 101, 485-517.

[9] King, R.G., Plosser, C.I. and S.T. Rebelo (1988), "Production, growth and business cycles: I. The basic neoclassical model", Journal of Monetary Economics 21, 195-232.

[10] Kocherlakota, N.R. and K.-M. Yi (1997), "Is there endogenous long-run growth? Evidence from the United States and the United Kingdom", Journal of Money, Credit, and Banking 29, 235-262.

[11] Ladrón-de-Guevara, A., Ortigueira, S. and M.S. Santos (1997), "Equilibrium dynamics in two-sector models of endogenous growth", Journal of Economic Dynamics and Control 21, 115-143.

[12] Lucas, R.E., Jr. (1988), "On the mechanics of economic development", Journal of Monetary Economics 22, 3-42.

[13] Lucas, R.E., Jr. (1990), "Supply-side economics: An analytical review", Oxford Economic Papers 42, 293-316.

[14] Mino, K. (1996), "Analysis of a two-sector model of endogenous growth with capital income taxation", International Economic Review 37, 227-251.

[15] Mulligan, C.B. (2002), "Capital, interest, and aggregate intertemporal substitution", NBER Working Paper 9373.

[16] Ortigueira, S. (1998), "Fiscal policy in an endogenous growth model with human capital accumulation", Journal of Monetary Economics 42, 323-355.

[17] Pecorino, P. (1993), "Tax structure and growth in a model with human capital", Journal of Public Economics 52, 251-271.

[18] Rebelo, S. (1991), "Long-run policy analysis and long-run growth", Journal of Political Economy 99, 500-521.

[19] Smith, W.T. (1996), "Taxes, uncertainty, and long-term growth", European Economic Review 40, 1647-1664.

[20] Stokey, N.L. and S. Rebelo (1995), "Growth effects of flat-rate taxes", Journal of Political Economy 103, 519-550.

[21] Uhlig, H. and N. Yanagawa (1996), "Increasing the capital income tax may lead to faster growth", European Economic Review 40, 1521-1540.

Table 1. Average Effective Tax Rates[1], 1991-1997

Per cent

	capital income (τ_r)	labor income (τ_w)	$T \equiv \frac{1-\tau_w}{1-\tau_r}$
Australia	28.0	22.6	1.21
Austria	18.9	41.8	0.72
Belgium	30.8	39.7	0.87
Canada	38.6	28.7	1.16
Denmark	29.1[2]	42.8	0.81
Finland	19.6	44.5	0.69
France	23.6	40.2	0.78
Germany	19.9	35.9	0.80
Greece	26.8	24.3	1.03
Ireland	18.7	25.1	0.92
Italy	31.0	36.3	0.92
Japan	32.6	24.0	1.13
Netherlands	24.7	41.0	0.78
New Zealand	34.9	24.2	1.16
Norway	20.2	35.5	0.81
Portugal	18.3	22.7	0.95
Spain	20.6	30.4	0.88
Sweden	30.5	48.5	0.74
Switzerland	25.1	30.2	0.93
U.K.	38.4	21.0	1.28
U.S.A.	31.1	22.6	1.12
Simple average	26.7	32.5	0.92

1. The average effective tax rates on capital and labor income are taken from Carey and Tchilinguirian (2000, Table 4) using the revised methodology.

2. Denmark's capital income tax rate is for 1991-1996.

Table 2. The effect of capital income taxation on leisure time, L, and the growth rate, g

a. Varying σ

	$\sigma = 2$		$\sigma = 3$		$\sigma = 4$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.288	0.273	0.325	0.306	0.343	0.323
g	0.0570	0.0577	0.0371	0.0382	0.0275	0.0285

b. Varying β

	$\beta = 0.90$		$\beta = 0.93$		$\beta = 1$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.325	0.306	0.325	0.306	0.328	0.311
g	0.0265	0.0260	0.0322	0.0325	0.0594	0.0629

c. Varying γ

	$\gamma = 0.5$		$\gamma = 0.6$		$\gamma = 0.75$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.356	0.323	0.336	0.312	0.314	0.301
g	0.0322	0.0338	0.0352	0.0365	0.0392	0.0399

d. Varying α

	$\alpha = 0.4$		$\alpha = 0.6$		$\alpha = 0.7$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.399	0.381	0.257	0.240	0.193	0.178
g	0.0284	0.0298	0.0432	0.0438	0.0479	0.0479

e. Varying ρ

	$\rho = 0$		$\rho = 0.02$		$\rho = 0.07$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.261	0.248	0.344	0.329	0.437	0.415
g	0.0719	0.0720	0.0547	0.0553	0.0107	0.0128

f. Varying δ

	$\delta = 0.05$		$\delta = 0.07$		$\delta = 0.15$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.391	0.366	0.364	0.342	0.261	0.248
g	0.0519	0.0541	0.0460	0.0477	0.0219	0.0220

g. Varying τ_w

	$\tau_w = 0.1$		$\tau_w = 0.25$		$\tau_w = 0.5$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.328	0.306	0.335	0.311	0.353	0.321
g	0.0361	0.0382	0.0341	0.0359	0.0293	0.0323

h. Varying A

	$A = 0.1$		$A = 0.3$		$A = 0.7$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.317	0.299	0.322	0.304	0.326	0.308
g	0.0263	0.0271	0.0336	0.0345	0.0396	0.0406

i. Varying B

	$B = 0.2$		$B = 0.3$		$B = 0.5$	
	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$	$\tau_r = 0$	$\tau_r = 0.5$
L	0.267	0.253	0.305	0.288	0.337	0.317
g	−0.0115	−0.0114	0.0131	0.0137	0.0607	0.0622

Table 3. The (approximate) eigenvalues of the three BGP's

BGP 1	BGP 2	BGP 3
0.055	0.052	0.073
0.027	0.027+0.033i	-0.006
0	0.027-0.033i	0
-0.028	0	
0	0	

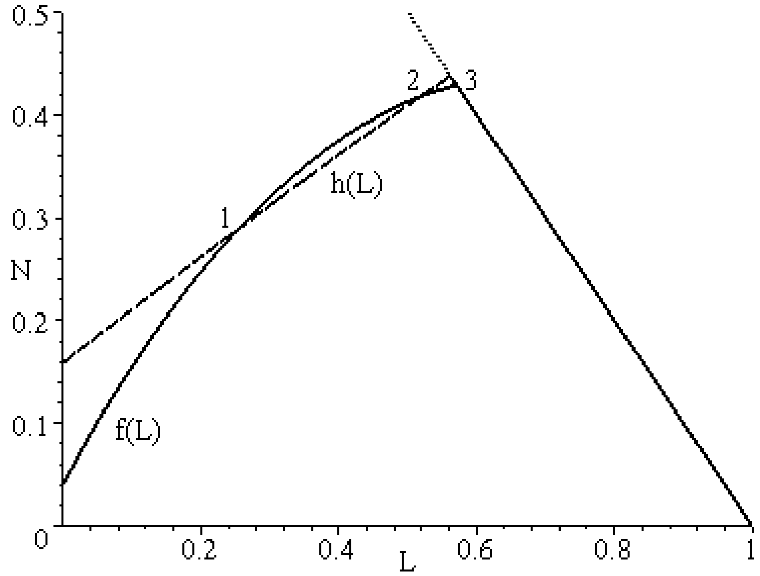

Figure 1: An example of multiple equilibria ($\alpha = 0.6$, $\beta = 0.96$, $\gamma = 0.5$, $A = 1.8$, $B = 0.106$, $\rho = 0.03$, $\sigma = 0.35$, $\delta = 0.05$, $\tau_r = \tau_w = 0$).

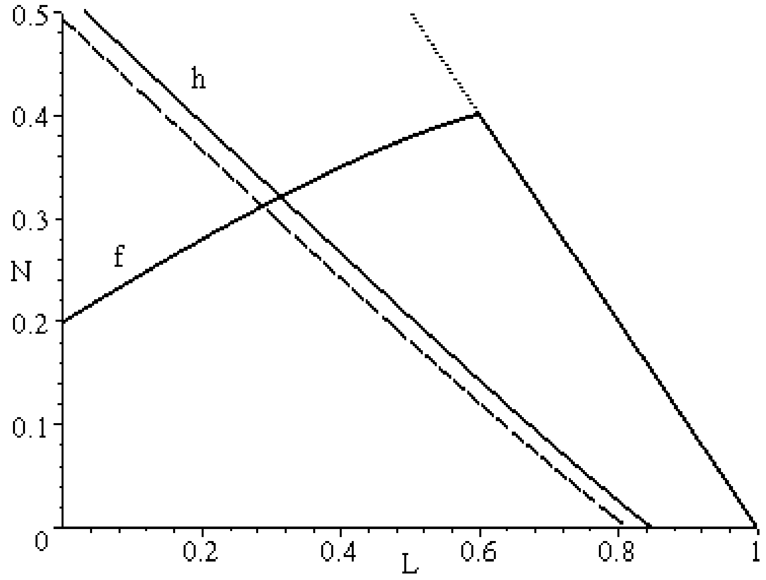

Figure 2: An example of the effect of income taxation on the unique BGP ($\alpha = 0.5$, $\beta = 0.8$, $\gamma = 0.67$, $A = 0.2$, $B = 0.4$, $\rho = 0.05$, $\sigma = 4$, $\delta = 0.1$, $\tau_r = \tau_w = 0$, $\tau_r' = \tau_w' = 0.5$).