Development Accounting in a Heckscher-Ohlin World

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Abstract

This paper tries to contribute to the strand of literature that investigates the question to what extend differences in per capita income between countries are due to differences in factor endowments like human- and physical capital on the one hand and due to differences in technology on the other hand. In particular, I am trying to assess to what extend structural transformation, ie the ability of a country to specialize in the production of goods that intensively use the factors with which it is abundantly endowed, has an important role in determining cross country income differences. I find that when productivities are country specific, for realistic parameter values structural transformation plays little role and productivity differences between countries remain large. However, when I allow for factor augmenting technology differences and factors are complementary in sectoral production, there seem to be large differences in the productivity of physical capital that are strongly correlated with per capita income, while human capital seems to have an inverse hump shape. This result is ad odds with Caselli (2005), who finds that poor countries use capital more efficiently than rich countries, while having a lower productivity of human capital. Finally, I use trade data and the Heckscher-Ohlin-Vanek equations to assess the plausibility of my calibrations and find a good fit for the model with factor specific productivities and complementary factors.

1 Introduction

This paper tries to contribute to the strand of literature that investigates the question to what extend differences in per capita income between countries are due to differences in factor endowments like human- and physical capital on the one hand and due to differences in technology on the other hand. In particular, I am trying to assess to what extend structural transformation, that is the ability of a country to specialize in the production of goods that intensively use the factors with which it is abundantly endowed, has an important role in determining cross country income differences. As Ventura (2005) notices there is a strong point on the theoretical side that trade in the Heckscher-Ohlin way is likely to increase the role of differences in factor endowments to explain differences in per capita income. The intuition is that as long as a country lies within a cone of diversification it does not experience decreasing returns because its factor prices are determined by the aggregate endowments of the countries in the cone. Consequently, the elasticity of substitution between factors becomes infinite, and unbalanced factor endowments can lead to a much higher per capita income than in a world without specialization. That implies that there may be a less prominent role for technology differences in explaining differences in per capita incomes. To study whether this theoretical channel really is of quantitative importance, I develop a generalization of the standard development accounting framework used by Hall and Jones (1999) and others that nests the standard framework as a special case. In particular, I view the world as one in which countries are connected by trade in goods, and in which comparative advantage is determined by differences in countries' factor endowments and in factor productivities.

In the paper I take two different viewpoints of the world economy: In the first one technology is country specific, while in the second one technology is specific to a factor located in a certain country.

I find that in the world with country specific productivities structural transformation is not quantitatively important and technology differences required to explain differences in income remain large if the model is calibrated to produce realistic labor shares. As a second overidentifying restriction I use trade data and the Heckscher-Ohlin-Vanek equations. Trade data reject any model with country specific productivities including including the model developed by Hall and Jones (1999). In the world with factor specific productivities results depend on whether inputs are complements or substitutes in production. If factors are substitutes there is a positive relationship between the productivity of human capital and income per worker, while the correlation between productivity of physical capital and income per worker is negative. These findings are very similar to Caselli's (2005). In this case there is no role for structural transformation because there is no clear pattern of relative factor abundance in efficiency units and income per worker. On the other hand, when factors are complements there is an inverse hump shaped relationship between the productivity of human capital and income per worker, so that medium income countries seem to be the most productive in the use of human capital. The correlation between the productivity of physical capital and income per worker is found to be strongly positive. In this case structural transformation important since rich countries relatively more abundant in efficient capital. This case is the preferred one because it is doing by far the best in matching measured and predicted flows of factors through trade in goods.

Let us now take a look at a very popular view of explaining cross country per worker income differences in order for the reader to have a clearly in mind where we are starting from. This view is a calibration view of explaining income differences which has been used by King and Levine (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998) and Hall and Jones (1999) and Caselli (2005). A common result of this approach is that factor endowments cannot explain a large part of income differences. This approach has been extended to factor specific productivity differences by Caselli (2005).

1.1 A Popular View of The World

1.1.1 The Hall and Jones World

a) A number of influential papers have adopted a calibration approach to explaining cross country income differences. In their viewpoint, the relationship between a countries' inputs and outputs can be described by an aggregate Cobb-Douglas production function. Let me call this way of thinking about cross country income differences the Hall and Jones world.

$$Y_c = A_c K_c^{\alpha} H_c^{1-\alpha} \tag{1}$$

Here Y_c is the Gross Domestic Product of country c, H_c and K_c are country c's endowments of human capital and physical capital, α is the capital share and A_c is total factor productivity in country c. Given data on endowments and GDP productivities required to make the left hand side equal to the right hand side are calibrated. Hall and Jones (and others following this approach) obtain the following results: 1) cross country differences in A_c are large and 2) strongly positively correlated with income per worker ($Corr(A_c, \frac{Y_c}{L_c}) >> 0$). This can be seen clearly from Figure 1 which plots countries' calibrated productivities against their incomes per worker for the year 1992 in constant 1987 International Dollars.[1] Consequently, Hall and Jones conclude that differences in country productivities are very important in explaining cross country income differences.

1.1.2 The Caselli World

In his chapter on Development Accounting in the Handbook of Economic Growth, Caselli (2005) presents a different model in order to describe the relationship

[1] Following Caselli (2005) I calibrate $A_c = \frac{Y_c}{K_c^{\alpha} H_c^{1-\alpha}}$ which is an accounting view point of explaining income differences, because some part of differences in capital stocks may actually be due to differences in productivity, a result that follows from any neoclassical growth model. Hall and Jones (1999) control for this by writing output as a function of the capital output ratio which is invariant to productivity in the steady state.

between a country's income and its endowments, namely he makes productivities factor specific. Let me call this model the Caselli world. In this world production functions have the following form

$$Y_c = [\alpha(A_{K,c}K_c)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)(A_{Hc}H_c)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where A_{Hc} is the efficiency of human capital in country c and A_{Kc} is the productivity of physical capital in the same country.

Caselli's main findings are the following ones: 1) differences in $A_{H,c}$ are large and every unit of human capital is much more efficient in rich countries than in poor countries ($Corr(A_{Hc}, \frac{Y_c}{L_c}) >> 0$). Figure 2 plots calibrated productivities of human capital from this model for $\epsilon = 0.8$[2] against income per worker under the assumption that the labor share is the empirical one.

2) Differences in $A_{K,c}$ are also significant and poor countries use each unit of physical capital more efficiently in absolute terms than rich countries ($Corr(A_{Kc}, \frac{Y_c}{L_c})$ $<< 0$). Figure 3 shows the negative relationship between the efficiency of physical capital and per capita income. It seems a bit counterintuitive, though, that placing a machine in Subsaharian Africa, should - all else equal - generate more output than in the United States.

In this paper I will present a somewhat more general view of the world, that includes the Hall and Jones world and the Caselli world as special cases and I will test the robustness of their findings to relaxing some of their assumptions. Especially, I allow for structural transformation as a mechanism in explaining income differences.

1.2 Outline of Paper

The outline of the paper is as follows: In the next section I develop a theoretical model of the world economy with Heckscher-Ohlin trade and factor specific productivities and show how factor productivities can be recovered from the model when data on countries' endowments and factor prices are fed in. In addition I show that standard models of development accounting obtain as special cases of the model, when the role of structural change is eliminated because all sectors have the same factor intensities. Subsequently I discuss how the Heckscher-Ohlin-Vanek equations can be used in this framework to evaluate calibrated productivities. This is followed by a presentation of the results of calibrating country- and factor-specific productivities in my framework. The last section concludes.

2 Theory

In order to formalize the idea that trade in the Heckscher-Ohlin form may be important for calibrating productivities I develop a flexible benchmark model of the world economy that relies on the following assumptions:

[2]The form of the productivity differences is not sensitive to the choice of ϵ.

Assumption 1): Countries are open to trade in goods and possess perfectly competitive goods and factor markets.

Assumption 2): Factors are immobile between countries and perfectly mobile within countries.

Assumption 3): Each country is endowed with human capital H_c and physical capital K_c.[3]

Assumption 4): Productivity is specific to a factor located in a country.

Assumption 5): All individuals in all countries have identical homothetic preferences that can be described by a Cobb-Douglas utility function.

Note that the existence of perfect factor markets is implicitly assumed both in the Hall and Jones and the Caselli world because this guarantees efficient allocation of inputs. [4] Neither Hall and Jones nor Caselli make an explicit statement about goods trade but nothing in their models excludes this possibility. Capital mobility is not assumed away explicitly in their models, while there must be some barriers to the movement of people in order to explain differences in per capita incomes. Finally, Assumption 4) is Caselli's assumption about technology and Assumption 5)is not necessary in the Hall and Jones or in the Caselli world.

The model is then easily described by Assumptions A1)-A5) and the specification of preferences and technology: There are $c = 1, ..., C$ countries, each of which possesses the technologies to produce $i = 1, ..., I$ goods. Consumers' preferences over the I goods are assumed to be represented by Cobb-Douglas utility functions:

$$U_c = \prod_{i=1}^{I} c_{ic}^{\sigma_i} \tag{3}$$

$$\sum_{i=1}^{I} \sigma_i = 1 \tag{4}$$

Sectors use inputs H_{ic} and K_{ic} and differ by their capital intensity for given factor prices. In each country productivity is specific to factors: A_{Kc}, A_{Hc}. Sectoral production functions are CES[5].

$$Q_{ic} = [\alpha_i (A_{Kc} K_{ic})^{\frac{\epsilon-1}{\epsilon}} + (1 - \alpha_i)(A_{Kc} H_{ic})^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \tag{5}$$

[3]Following the growth literature the factor "human capital" is measured as labor endowments in efficiency units, which is different from the convention used in the trade literature, where human capital is usually the amount of skilled labor. Because the model has only two factors this seems to be the adequate way to measure labor endowments.

[4]See Banerjee and Duflo (2005) for a model with imperfect factor markets which causes aggregate production to depend on the distribution of factor endowments within the economy.

[5]The assumption that elasticities of substitution are the same across sectors rules out factor intensity reversals. I have solved the model also for the more general assumption $\epsilon = \epsilon_i$ without getting any significantly different results. Hence, I maintain the hypothesis of equal sectoral elasticities for reasons that will become clear later.

For convenience, I define the following variables:

$$\hat{H}_c = A_{Hc} H_c \quad (6)$$

$$\hat{K}_c = A_{Kc} K_c \quad (7)$$

$$\hat{w}_c = \frac{w_c}{A_{Hc}} \quad (8)$$

$$\hat{r}_c = \frac{r_c}{A_{Kc}} \quad (9)$$

These are factor endowments in efficiency units and efficiency adjusted factor prices. So, for example, one unit of efficient capital is equivalent to A_{Kc} units of plain capital, and one unit of efficient capital, which is measured in common units across countries, costs $\frac{1}{A_{K,c}}$ as much as one unit of plain capital, which may differ in efficiency across countries. Capital prices in country c may be higher than in country c' because buying one unit of capital in country c provides ownership of more efficient units of capital or because capital is more scarce in country c.

With these renormalization of variables I am able to describe the world economy as an ordinary Heckscher-Ohlin model without productivity differences in which factor endowments are measured in efficiency units, while leaving the structure of the model formally equivalent to the one described by the production possibilities and preferences above. Because a country's production is not uniquely defined in a Heckscher-Ohlin model for any country lying within any set of countries that has common factor prices (a so called cone of diversification) and produces a number of goods greater than the number of factors, let me make the following definition:

Definition: Let D be a partition of C s.t. $\bigcap_{j=1}^{D} d_j = \emptyset$ with $d = \bigcup_{i \in C} c_i$ and $\bigcup_{i=1}^{C} c_i = \bigcup_{j=1}^{D} d_j$.

Given this, one can define a competitive equilibrium in the usual way:

Definition: A *Competitive Equilibrium* (*CE*) is a collection of goods prices $\{p_i\}_{i=1}^{I}$, efficiency adjusted wages $\{\hat{w}_c\}_{c=1}^{C}$, efficiency adjusted rental rates $\{\hat{r}_c\}_{c=1}^{C}$ and quantities $\{Q_{i,d}\}_{d=1}^{D}$ s.t. given a cross section of efficiency adjusted human capital endowments $\{\hat{H}_c\}_{c=1}^{C}$, efficiency adjusted physical capital endowments $\{\hat{K}_c\}_{c=1}^{C}$, and parameters ϵ, $\{\alpha_i\}_{i=1}^{I}$ and $\{\sigma_i\}_{i=1}^{I}$ the following system of equation holds for all $d \in D$:

$$p_i \leq [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (10)$$

(Profit maximization.)

$$\{p_i - [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\} Q_{i,d} = 0 \quad (11)$$

(A good is not produced in a set of countries with equal conditional factor prices whenever the price is lower than the unit production cost.)

$$\sum_i [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (1-\alpha_i)^\epsilon \hat{w}_c^{-\epsilon} Q_{i,d} = \sum_{c \in d} \hat{H}_c \tag{12}$$

(Labor market clearing)

$$\sum_i [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (\alpha_i)^\epsilon \hat{r}_c^{-\epsilon} Q_{i,d} = \sum_{c \in d} \hat{K}_c \tag{13}$$

(Capital market clearing)

$$p_i \sum_{d \in D} Q_{i,d} = \sigma_i \sum_{c \in C} (\hat{w}_c \hat{H}_c + \hat{r}_c \hat{K}_c) \tag{14}$$

(World goods market clearing for $i = 1, ..., I-1$ goods.)

$$\prod_i (\frac{p_i}{\sigma_i})^{\sigma_i} = 1; \tag{15}$$

(Optimal price index)

To illustrate what effects trade and structural change may have on income differences, and what can go wrong if one neglects these effects let us take a look at a simple example that can be solved analytically:

2.1 An Example

Assume that sectoral input ratios are sufficiently extreme and expenditure on sectors with extreme factor proportions is large enough in order for conditional factor price equalization to hold for the world economy, i.e. $\hat{w}_c = \hat{w}_{c'} = \hat{w}$ and $\hat{r}_c = \hat{r}_{c'} = \hat{r}\ \forall c \in C$. In this case, the open economy equilibrium replicates the equilibrium of the integrated economy, which is a hypothetical economy in which all impediments to factor movements between countries have been abolished. Hence, it suffices to look at the equilibrium of the integrated economy. For analytical tractability assume that $\epsilon \to 1$ so that sectoral production is Cobb-Douglas. From firms' first order condition for profit maximization we have:

$$\hat{w}\hat{H}_i = (1-\alpha_i) p_i Q_i \tag{16}$$

Since demand is Cobb-Douglas, the expenditure on good i is given by

$$p_i Q i = \sigma_i E_w = \sigma_i Y_w, \tag{17}$$

where E_w and Y_w are expenditure and income of the world economy. Then from factor market clearing for human capital we get that

$$\hat{w}\hat{H}_w = \hat{w}\sum_{i\in I}\hat{H}_i = \sum_{i\in I}(1-\alpha_i)p_i Q_i. \tag{18}$$

Combining these equations we obtain sectoral factor use in terms of the aggregate factor endowments of the world economy.

$$\hat{H}_i = \frac{(1-\alpha_i)\sigma_i}{\sum_{i\in I}(1-\alpha_i)\sigma_i}\hat{H}_w \tag{19}$$

Similarly,

$$\hat{K}_i = \frac{\alpha_i\sigma_i}{\sum_{i\in I}\alpha_i\sigma_i}\hat{K}_w \tag{20}$$

Using the definition of the price index and the fact that from demand

$$p_i = \sigma_i\frac{Y_w}{Q_i} \tag{21}$$

one obtains

$$Y_w = Q_w = \prod_{i\in I} Q_i^{\sigma_i}. \tag{22}$$

Now, plugging in the definition of the sectoral production functions, one obtains.

$$Q_w = B\hat{H}_w^{(1-\sum_{i\in I}\alpha_i\sigma_i)}\hat{K}_w^{\sum_{i\in I}\alpha_i\sigma_i}, \tag{23}$$

where

$$B = \prod_{i\in I}\left[\frac{(1-\alpha_i\sigma_i)}{\sum_{i\in I}(1-\alpha_i)\sigma_i}\right]^{(1-\alpha_i\sigma_i)}\left[\frac{\alpha_i\sigma_i}{\sum_{i\in I}\alpha_i\sigma_i}\right]^{\alpha_i\sigma_i} \tag{24}$$

This implies that the world economy behaves as if it had an aggregate Cobb-Douglas production function and there are decreasing returns to factor accumulation in efficiency units at the world level. World factor prices are given by

$$\hat{w} = (1-\sum_{i\in I}\alpha_i\sigma_i)B\left(\frac{\hat{K}_w}{\hat{H}_w}\right)^{\sum_{i\in I}\alpha_i\sigma_i} \tag{25}$$

and

$$\hat{r} = (\sum_{i\in I}\alpha_i\sigma_i)B\left(\frac{\hat{H}_w}{\hat{K}_w}\right)^{(1-\sum_{i\in I}\alpha_i\sigma_i)}. \tag{26}$$

Consequently, factor prices are determined at the world level and to the extend that they are given for individual countries, country production functions are linear and countries experience constant returns to factor accumulation. Relative incomes in the open economy with conditional factor price equalization are given by

$$\frac{Y_c}{Y_{c'}} = \frac{\hat{H}_c\hat{w} + \hat{K}_c\hat{r}}{\hat{H}_{c'}\hat{w} + \hat{K}_{c'}\hat{r}}, \tag{27}$$

Plugging in for the equilibrium factor prices this can be written as

$$\frac{Y_c}{Y_{c'}} = \frac{(1-\sum_{i\in I}\alpha_i\sigma_i)\hat{H}_c + \sum_{i\in I}\alpha_i\sigma_i\frac{\hat{H}_w}{\hat{K}_w}\hat{K}_c}{(1-\sum_{i\in I}\alpha_i\sigma_i)\hat{H}_{c'} + \sum_{i\in I}\alpha_i\sigma_i\frac{\hat{H}_w}{\hat{K}_w}\hat{K}_{c'}}, \tag{28}$$

while in the Hall and Jones and in the Caselli world relative incomes are determined by

$$\frac{Y_c}{Y_{c'}} = \left(\frac{\hat{K}_c}{\hat{K}_{c'}}\right)^{\alpha}\left(\frac{\hat{H}_c}{\hat{H}_{c'}}\right)^{(1-\alpha)}. \tag{29}$$

Now, several things can be learned from these formulas. Assume for a moment that the variables are defined in ordinary units and are not efficiency adjusted. First, to the extend that world factor prices are not affected by a single country, the elasticity of substitution between factors is infinite in the factor price equalization case because countries absorb additional factor inputs through structural transformation, i.e. a change of sectoral output holding constant the input ratios in each sector. Then, rich countries may escape decreasing returns to capital by producing above all capital intensive goods. In this case factor endowments would explain more of cross country income differences than in the Hall and Jones world. Second, relative incomes depend on the production and demand structure of the whole world economy and not only on the production technology of the two countries that are being compared. Third, if the correlation between expenditure shares σ_i and sectoral capital shares α_i is positive, differences in capital have more weight and will cause larger differences in income.

This example, of course, only gives some hints how trade may affect calibrated productivity differences because factor price equalization clearly does not hold for the world economy and productivity differences may be factor specific.

Given factor productivities for every country *CE* defines the equilibrium of the world economy. However, efficiency adjusted factor endowments are unknown, since they consist of factor endowments multiplied by the unknown factor efficiencies. Therefore, some additional information is required in order to calibrate country specific factor productivities. This information comes in form of data on country wages, w_c, and rental rates r_c, which allow me to solve for the $2C$ unknowns $\{A_{K,c}\}_{c=1}^{C}$, $\{A_{H,c}\}_{c=1}^{C}$. The procedure applied is analogue to the usual calibration exercise a la Hall and Jones or Caselli, just that on the way of calibrating productivities also the unknown equilibrium prices and production levels in each country have to be determined, because the relationship between a countries' inputs and outputs depends on endowments and preferences of the whole world economy. In order to formalize this idea a bit, let me make the following definition:

Definition: A *Productivity Calibration Problem with Factor Augmenting Productivities* (*FAP*) is a collection of goods prices $\{p_i\}_{i=1}^{I}$, efficiency adjusted wages $\{\hat{w}_c\}_{c=1}^C$, efficiency adjusted rental rates $\{\hat{r}_c\}_{c=1}^C$, quantities $\{Q_{i,d}\}_{d=1}^D$ and sectoral productivities $\{A_{H,c}\}_{c=1}^C$, $\{A_{H,c}\}_{c=1}^C$ such that given a cross section of human capital endowments $\{H_c\}_{c=1}^C$, physical capital $\{K_c\}_{c=1}^C$, wages $\{w_c\}_{c=1}^C$, rentals $\{r_c\}_{c=1}^C$ and parameters $\{\alpha_i\}_{i=1}^I$, ϵ and $\{\sigma_i\}_{i=1}^I$ the following system of equations holds for all $d \in D$:

$$p_i \leq [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{30}$$

with

$$\{p_i - [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\} Q_{i,d} = 0 \tag{31}$$

$$\sum_{i\in I} [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (1-\alpha_i)^\epsilon \hat{w}_c^{-\epsilon} Q_{i,d} = \sum_{c\in d} A_{H,c} H_c \tag{32}$$

$$\sum_{i\in I} [\alpha_i^\epsilon \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^\epsilon \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (\alpha_i)^\epsilon \hat{r}_c^{-\epsilon} Q_{i,d} = \sum_{c\in d} A_{K,c} K_c \tag{33}$$

$$p_i \sum_{d\in D} Q_{i,d} = \sigma_i \sum_{c\in C} Y_c \quad \forall i = 1, ..., I-1 \tag{34}$$

$$\prod_{i\in I} (\frac{p_i}{\sigma_i})^{\sigma_i} = 1; \tag{35}$$

$$A_{H,c} = \frac{w_c}{\hat{w}_c} \tag{36}$$

$$A_{K,c} = \frac{r_c}{\hat{r}_c} \tag{37}$$

Regarding uniqueness and the relationship between the *Competitive Equilibrium* and *Productivity Calibration Problem with Factor Augmenting Productivities* one can prove the following (for the proofs see Appendix):

Lemma 1: If given $\{H_c\}_{c=1}^C$, $\{K_c\}_{c=1}^C$, $\{w_c\}_{c=1}^C$,$\{r_c\}_{c=1}^C$,parameters $\{\alpha_i\}_{i=1}^I$, ϵ and $\{\sigma_i\}_{i=1}^I$, we have that $\{p_i\}_{i=1}^I$, $\{\hat{w}_c\}_{c=1}^C$, $\{\hat{r}_c\}_{c=1}^C$, $\{Q_{i,d}\}_{d=1}^D$, $\{A_{H,c}\}_{c=1}^C$, $\{A_{K,c}\}_{c=1}^C$ are a solution to the *FAP* then they are also a *CE* given $\{A_{H,c}H_c\}_{c=1}^C = \{\hat{H}_c\}_{c=1}^C$, $\{A_{K,c}K_c\}_{c=1}^C = \{\hat{K}_c\}_{c=1}^C$.

Lemma 1 means that any solution to the *FAP* is actually an equilibrium of the world economy if we had known the productivities beforehand, which is obviously a necessary condition for the concept of *FAP* to make sense.

Lemma 2: If given $\{H_c\}_{c=1}^C$, $\{K_c\}_{c=1}^C$, $\{\alpha_i\}_{i=1}^I$, ϵ and $\{\sigma_i\}_{i=1}^I$, we have that $\{p_i\}_{i=1}^I$, $\{w_c\}_{c=1}^C$, $\{r_c\}_{c=1}^C$, $\{Q_{i,d}\}_{d=1}^D$ are a *CE* then they also solve the *FAP* given $\{H_c\}_{c=1}^C$, $\{K_c\}_{c=1}^C$, $\{w_c\}_{c=1}^C$ and $\{r_c\}_{c=1}^C$ with $\{A_{H,c}\}_{c=1}^C = \{A_{K,c}\}_{c=1}^C = 1$.

Lemma 2 implies that prices and quantities of the *CE* of a Heckscher-Ohlin model in which all factor endowments are measured in efficiency units will also be a *FAP* and measured productivity differences will be zero. This (together with the following proposition) implies that when actual productivity differences are absent, *FAP* will also predict that all productivities are the same for all countries.

Proposition 1: The *Productivity Calibration Problem with Factor Augmenting Productivities* has a unique solution given given any cross section of human capital endowments $\{H_c\}_{c=1}^{C}$, physical capital endowments $\{K_c\}_{c=1}^{C}$, wages $\{w_c\}_{c=1}^{C}$, rental rates $\{r_c\}_{c=1}^{C}$ and parameters $\{\alpha_i\}_{i=1}^{I}$, ϵ and $\{\sigma_i\}_{i=1}^{I}$.

Now, after having made sure that the concept of FAP makes sense, it is insightful to explore a bit the relationship between this model and the standard development accounting framework. In fact, when all sectors use the same factor intensities, such that there is no possibility of adjusting production patterns to relative factor endowments through structural change, the model collapses to Caselli's model of factor specific productivities.

Assumption 6): All sectors have identical factor intensities: $\alpha_i = \alpha_j = \alpha$, $\forall i, j \in I$

Given this assumption it follows from the pricing equations (30) that $p_i = p_j \equiv p$. Then the equation for the price index (35) pins down $p = 1$. [6] Hence

$$1 = [\alpha^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{38}$$

and

$$[\alpha^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}(1-\alpha)^{\epsilon}\hat{w}_c^{-\epsilon}\sum_{i\in I} Q_{i,c} = A_{H,c}H_c \tag{39}$$

$$[\alpha^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}}\alpha^{\epsilon}\hat{r}_c^{-\epsilon}\sum_{i\in I} Q_{i,c} = A_{K,c}K_c \tag{40}$$

Consequently:

$$\frac{(1-\alpha)^{\epsilon}\hat{w}_c^{-\epsilon}}{\alpha^{\epsilon}\hat{r}_c^{-\epsilon}} = \frac{A_{H,c}H_c}{A_{K,c}K_c} \tag{41}$$

Then using (36) and (37) and substituting these expressions for $\hat{w}_c$, $\hat{r}_c$ we are left with two equations in $A_{H,c}$ and $A_{K,c}$ per country that can be easily solved for

$$A_{K,c} = \left(\frac{r_c K_c}{Y_c}\frac{1}{\alpha}\right)^{\frac{\epsilon}{\epsilon-1}}\frac{Y_c}{K_c} \tag{42}$$

[6] This shows that only in this special case the demand side of the economy does not matter because relative goods prices are pinned down from the production side and are equal to one. Hence, Assumption 4) becomes irrelevant. Otherwise terms of trade effects play a role in the determination of GDPs.

and

$$A_{H,c} = \left(\frac{w_c H_c}{Y_c} \frac{1}{1-\alpha} \right)^{\frac{\epsilon}{\epsilon-1}} \frac{Y_c}{H_c} \tag{43}$$

These two expressions are just Caselli's (2005) result for development accounting with factor augmenting productivities.

If one drops Assumption 6) and instead assumes that productivities are not factor- but country-specific, one obtains a another model.

Assumption 7) : $A_{H,c} = A_{K,c} = A_c$.

In this case one data point per country in addition to the factor endowments is sufficient to determine A_c and this is taken to be aggregate income Y_c. In this case the model is defined as follows:

A *Productivity Calibration Problem with Country Productivities (CAP)* is a collection of goods prices $\{p_i\}_{i=1}^{I}$, efficiency adjusted wages $\{\hat{w}_c\}_{c=1}^{C}$, efficiency adjusted rental rates $\{\hat{r}_c\}_{c=1}^{C}$, country productivities $\{A\}_{c=1}^{C}$ and quantities $\{Q_{i,d}\}_{d=1}^{D}$ s.t. given a cross section of human capital endowments $\{H_c\}_{c=1}^{C}$, physical capital $\{K_c\}_{c=1}^{C}$, Gross Domestic Products $\{Y_c\}_{c=1}^{C}$ and parameters $\{\alpha_i\}_{i=1}^{I}$, ϵ and $\{\sigma_i\}_{i=1}^{I}$ the following system of equation holds for all $d \in D$:

$$p_i \leq [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{44}$$

with

$$\{p_i - [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}}\} Q_{i,d} = 0 \tag{45}$$

$$\sum_{i \in I} [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (1-\alpha_i)^{\epsilon} \hat{w}_c^{-\epsilon} Q_{i,d} = \sum_{c \in d} A_c H_c \tag{46}$$

$$\sum_{i \in I} [\alpha_i^{\epsilon} \hat{r}_c^{1-\epsilon} + (1-\alpha_i)^{\epsilon} \hat{w}_c^{1-\epsilon}]^{\frac{\epsilon}{1-\epsilon}} (\alpha_i)^{\epsilon} \hat{r}_c^{-\epsilon} Q_{i,d} = \sum_{c \in d} A_c K_c \tag{47}$$

$$p_i \sum_{d \in D} Q_{i,d} = \sigma_i \sum_{c \in C} Y_c \quad \forall i = 1, ..., I-1 \tag{48}$$

$$\prod_i (\frac{p_i}{\sigma_i})^{\sigma_i} = 1; \tag{49}$$

$$A_c = \frac{Y_c}{\hat{w}_c H_c + \hat{r}_c K_c} \tag{50}$$

Proposition 2: The *Productivity Calibration Problem with Country Augmenting Productivities (CAP)* has a unique solution given any cross section of human capital endowments $\{H_c\}_{c=1}^{C}$, physical capital $\{K_c\}_{c=1}^{C}$, Gross Domestic Products $\{Y_c\}_{c=1}^{C}$, and parameters $\{\alpha_i\}_{i=1}^{I}$, ϵ and $\{\sigma_i\}_{i=1}^{I}$.

Again, if one adds Assumption 6) that all sectors have identical factor intensities: $\alpha_i = \alpha_j = \alpha \quad \forall i, j \in I$, it follows from the pricing equations and the price index that $p_i = p = 1$. Then, combining the factor market clearing conditions, we have that:

$$\frac{(1-\alpha)^{\epsilon}\hat{w}_c^{-\epsilon}}{\alpha^{\epsilon}\hat{r}_c^{-\epsilon}} = \frac{H_c}{K_c} \tag{51}$$

Combining this with the pricing equation

$$1 = [\alpha^{\epsilon}\hat{r}_c^{1-\epsilon} + (1-\alpha)^{\epsilon}\hat{w}_c^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{52}$$

we obtain two equations that can be solved for $\hat{w}_c$ and $\hat{r}_c$. Finally, using the fact that $A_c = \frac{Y_c}{\hat{w}_c H_c + \hat{r}_c K_c}$, we get that

$$A_c = \frac{Y_c}{[\alpha_i K_{i,c}^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha_i) H_{i,c}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}} \tag{53}$$

If one makes the additional assumption that production is Cobb-Douglas, one finally gets back to the Hall & Jones world.

Assumption 8): $\epsilon = 1$

Then it follows that:

$$A_c = \frac{Y_c}{K_c^{\alpha} H_c^{1-\alpha}} \tag{54}$$

3 Testing the Results with Trade Data

Both the CAP and the FAP provide unique productivity calibrations for given values $\{\alpha_i\}_{i=1}^{I}$, ϵ and $\{\sigma_i\}_{i=1}^{I}$. Hence, a way to determine reasonable values of those parameters is required. However, the model is too stylized for one wanting to associate the sectors in the model with empirical sectoral classifications, like SIC-aggregates. In addition, these empirically measured classifications aggregate goods according to similar uses (like textiles, electrical machinery) and not according to their factor intensities, which would be the relevant classification in a Heckscher-Ohlin model.

Therefore, in order to get an idea which parameter values may be reasonable, I use the Heckscher-Ohlin-Vanek (HOV) equations to evaluate the fit of the model. These equations, which can be derived from any Heckscher-Ohlin stile model state a theoretical identity between the predicted and the measured factor content of trade. The testable hypothesis is that a country should export (through trade in goods) the services of those factors with which it is abundantly endowed relative to the world average and export its relatively scarce factors.

Assuming that there is no trade in intermediate goods, the factor content of trade can be written as follows: Let $f = 1, ...F$ denote factors, $i = 1, ..., I$ denote goods and let B_c be the F*I factor use matrix in country c, with elements $b_{f,i,c}$ determining the use of factor f in the production of one unit of good i in country c and rows $B_{f,c}$, that fix the use of factor f per unit of output in each sector. Let $D_c \equiv B_c(I - E_c)^{-1}$ be the matrix of direct plus indirect factor inputs, where E_c is country c's input-output matrix. Then in the above models, the use of human capital in the production of one unit of good i in country f is, for example

$$b_{f,i,c} = p_i^{\epsilon} \hat{w}_c^{-\epsilon}(1 - \alpha_i)^{-\epsilon}/A_{H,c}, \tag{55}$$

and the fth row of the factor use matrix of the United States is

$$B_{f,US} = \hat{w}_{US}^{-\epsilon} B_f / A_{f,US}, \tag{56}$$

where B_f is common to all countries because of free and costless trade. The factor use matrix in country c can be expressed as a function of the one of the US.

$$B_{f,c} = \hat{w}_c^{-\epsilon} B_f / A_{f,c} = \left(\frac{\hat{w}_{f,US}}{\hat{w}_{f,c}}\right)^{\epsilon} \left(\frac{A_{f,US}}{A_{f,c}}\right) B_{f,US} \tag{57}$$

Normalizing $A_{f,US} = 1$, the measured factor content of trade is defined as:

$$\begin{aligned} MFC^*_{f,c} = A_{f,c}D_{f,c}(Q_c - C_{cc}) - \sum_{c' \neq c} A_{f,c'}D_{f,c'}C_{cc'} = \\ \left(\frac{\hat{w}_{f,US}}{\hat{w}_{f,c}}\right)^{\epsilon} D_{f,US}X_c - \sum_{c' \neq c} \left(\frac{\hat{w}_{f,US}}{\hat{w}_{f,c'}}\right)^{\epsilon} D_{f,US}C_{cc'} \end{aligned} \tag{58}$$

where Q_c is country c's net production, $C_{cc'}$ is country c's consumption of goods produced in country c' (imports of c from c') and X_c are country c's exports. On the other hand, using the additional assumption that $C_{cc'} = s_c Q_{c'}$ [7], one can write the predicted factor content of trade as:

$$\begin{aligned} PFC^*_{f,c} = A_{f,c}D_{f,c}(Q_c - C_{cc}) - \sum_{c' \neq c} A_{f,c'}D_{f,c'}C_{cc'} = \\ = A_{f,c}D_{f,c}(Q_c - s_c Q_c) - \sum_{c' \neq c} s_c A_{f,c'}D_{f,c'}Q_{c'} \end{aligned} \tag{59}$$

Using factor market clearing, which implies $D_c Q_c = V_c$, we get

$$PFC^*_f = A_{f,c}V_{f,c} - s_c A_{f,c}V_{f,c} - \sum_{c' \neq c} s_c A_{f,c'}V_{f,c'} = A_{f,c}V_{f,c} - s_c \sum_{c' \in C} A_{f,c'}V_{f,c'} \tag{60}$$

[7] This relation is not strictly implied by the Heckscher-Ohlin model, where bilateral trade is usually not determined, but would follow for example from any increasing returns (see Helpman and Krugman (1985))or Armington-model. One could think of each sector as producing in fact a number of country specific varieties of goods and that sector demands are, for example, a CES- aggregate over sectoral varieties. The assumption is nevertheless usually made in the literature, because if there is no factor price equalization it is the only way to make the predicted factor content of trade depend only on factor endowments. Another way to justify this assumption would be to rely on random matching between importers and exporters(Deardorff (1998)).

Plugging the assumption $A_{f,c}D_{f,c} = \left(\frac{\hat{w}_{f,US}}{\hat{w}_{f,c}}\right)^{\epsilon} D_{f,US}$ into $MFC_{f,c}$ I get a relationship that can be checked on the data.

Since the HOV-equations apply to the FAP- and the CAP-models, it can be tested for which parameter values of technology and absorbtion they best fit observed trade data.

To test the fit of my productivity calibrations, I just plug the efficiency adjusted factor prices measured relative to the value of the US into (58) and the calibrated relative factor productivities into (60). I report the results of the following tests that are standard in the literature.[8] First, I report the Sign Test that measures the fraction of observations for which the predicted and the measured factor content of trade have the same sign. I also report results of the Weighted Sign Test which weights observation by the magnitude of factor flows, giving more weight to large flows. Third, I include slope coefficient and R^2 of a regression of the measured factor content on the predicted factor content of trade, which have a theoretical value of one in the case of perfect fit.Finally, I provide results of the "Missing Trade"-Test, that is the ratio of the variance of the measured factor content to the variance of the predicted factor content, which should be close to one if magnitudes of predicted and measured factor flows are similar.

Even though the Hall and Jones world and Caselli's Factor Augmenting Productivity-Model can be seen as special cases of the Heckscher-Ohlin model with country-/factor specific productivity differences in which all sectors use the same input ratios (see above), the HOV-equations cannot be used to test the productivity calibrations obtained from those models. When all sectors use exactly the same input ratios, trade patterns and hence factor trade are completely undetermined, because it does not matter whether a country produces a good or imports it, but infinitesimal trade costs should be enough to generate zero trade. Because the prediction of zero trade can be dismissed as counterfactual, since countries obviously trade a lot of goods, it does not seem too unreasonable, to assume that there is for example an underlying increasing return or Armington model, which predicts that every country imports a fraction of goods from every other country. Hence, the standard development accounting models cannot be tested with the HOV-equations, but they can be tested together with the assumption that every country imports a fraction of goods from every other country.

4 Data

In order to allow direct comparison of my results with the literature on development accounting, I construct data in the way suggested by Caselli (2005). Because I have trade data for the year 1992, I compute physical capital stocks in constant 1987 international Dollars from the Penn World Tables (Mark 6.1)[9].

[8]For a more detailed description of these tests see, for example, Davis and Weinstein (2003).
[9]For details see Caselli (2005).

GDPs in constant 1987 international Dollars are also computed from the same source. Endowments of human capital are taken directly from Caselli's dataset. Even though his dataset is for 1996 this should not matter much, since educational attainments change very slowly.

Since I need an additional data point per country in order to calibrate factor productivities, I construct estimates of average unskilled wages for the 94 countries in the sample. In order to get wage data, I proceed in the following way. As a first step I use data on country labor shares from Bernanke and Guerkaynak (2001). Following a procedure suggested by Gollin (2002), they have adjusted raw data on labor shares for the labor of self-employed workers, who make up a large fraction of the labor force in most developing countries. Because their dataset includes only 54 countries of my sample, I regress these labor shares on controls and predict labor shares out of sample for the rest of the countries. Right hand side variables include trade openness and regional dummies.

$$S_{H,c} = \beta_0 + \beta_1 OPEN_c + \sum_{i=2}^{n} \beta_i D_{i,c} \tag{61}$$

For the countries for which no observations of the labor share are available:

$$\hat{S}_{H,c} = \hat{\beta}_0 + \hat{\beta}_1 OPEN_c + \sum_{i=2}^{n} \hat{\beta}_i D_{i,c} \tag{62}$$

Once labor shares are constructed for all countries, wages are computed as $w_c = \frac{S_{H,c} Y_c}{H_c}$, where $S_{H,c}$ is the labor share in country C. Rental rates are then just computed as $r_c = \frac{Y_c - w_c H_c}{K_c}$. Figures 4) to 6) depict labor shares, wages and rentals for the countries in the sample. Labor shares are weakly positively correlated with income per worker, while wages are, as expected strongly positively correlated with per worker income. The country with the lowest observation for wages is 372 Dollars for Zaire, while the highest wage is observed for Italy with 12194 Dollars for workers with no education. Measured rentals, on the other hand, vary inversely with per capita income. The lowest rental, 6%, is observed for Romania, while the highest one, 138%, is calculated for Uganda.

For the trade data I use the dataset compiled by Antweiler & Trefler (2002) because it consists of a fairly large sample of countries at all stages of development (71). 64 of these countries form a subset of the countries for which I have data on human capital. The data include observations for bilateral trade at the level of 37 SIC sectors and the technology matrix of the US at intervals of five years, starting 1972 and ending 1992. I use 1992 trade data and adjust the factor use matrix of the US to fit my endowment data which are significantly different from Trefler's.

5 Calibrating Productivities

5.1 Country Specific Productivities

When productivities are specific to countries, GDPs are taken as additional data points in order to find unknown productivities.

To fix ideas, let me focus on the CAP with 2 industries, $i \in \{H, K\}$ with $\alpha_H < \alpha_K$. Hence, countries can lie either in the cone of diversification and have factor prices that are equalized in efficiency units, so that differences in factor prices reflect only differences in country productivities, or they specialize in the production of one of the goods. In this case their factor prices reflect both differences in productivity and differences in factor endowments. The model is solved simultaneously for specialization patterns, productions, factor- and goods prices and productivities. [10]

The examples shown in this section are supposed to be representative of the different cases that can arise in the model.

As a starting point let us have a look at an example where the role of structural transformation is potentially very large. This happens when sectoral input ratios are extreme, so that additional units of inputs will be absorbed mainly by an expansion of the output produced by the sector that uses the factor intensively. A second requirement for structural change to be important is that the demand for capital intensive goods is sufficiently large, so that capital abundant countries can concentrate on producing this good, while the small and capital scarce rest of the world delivers labor abundant products.

Example 1) $\alpha_H = 0.001$, $\alpha_K = 0.999$, $\sigma_H = 0.2$, $\epsilon = 1$

In this example 80% of income are spent on the capital intensive good, which use virtually only capital. It is obvious from figure ? that productivities are hardly related to per worker income and that middle income countries have on average the highest productivities. In this example conditional factor price equalization holds on the world level. Hence, differences in factor prices reflect only productivity differences. Rich countries do not require high productivities to be richer because they use their capital efficiently by specializing in capital intensive products, while their demand for labor intensive products is satisfied mainly by poorer countries. The model shows poor fit of the HOV-equations, since measured and predicted trade flows have the same direction only in around 40 per cent of observations (see table 1). Another major drawback of this calibration is, that it is unable to replicate factor shares. In fact, large trends in factor shares are predicted. Figure? plots labor shares against income per worker. With Cobb-Douglas production rich countries have very small labor shares because they produce mainly the capital intensive good and the reverse holds for poor countries. In order to fix the prediction of large trends in factor shares, either sectoral input ratios must be very similar, which eliminates the

[10] I have solved also models with more than 2 goods, that have potentially several cones of diversification. While these models are somewhat more cumbersome to solve because individual countries' productions may be indeterminate, results obtained from those models are very similar to the 2 goods case.

possibility of structural transformation or inputs must be complements which raises the price of the scarce factor and increases its income share.

Example 2) $\alpha_H = 0.3$, $\alpha_K = 0.4$, $\sigma_H = 0.2$, $\epsilon = 1$ In this example sectoral input ratios are more similar for given factor prices. This implies that trends in income shares become less pronounced (for countries that specialize income shares are given by the relevant sectoral factor shares). At the same time, however, the power of structural change is eliminated because even though the production structure differs across countries because of specialization, sectoral technologies are too similar for countries to be able to escape decreasing returns to capital accumulation.

The force of the law of decreasing returns can be mitigated when the sectoral elasticity of substution is allowed to be larger than one, but some trends in factor shares remain whenever structural change is powerfully at work.

Models with complementary inputs give rise to positive correlations between factor shares and per capita income that also can hardly be eliminated because with complementary inputs sectoral input ratios must differ more in order for structural change to work. Consequently, it must be concluded that the model with country specific productivities fits the data poorly.

But also the Hall and Jones model does not fit the data well. Of course, income shares fit by construction but this model does poorly in fitting trade data. As explained above, in the Hall and Jones world trade must arise for some reasons other than factor proportions. Given the assumption that each country imports a fraction of production of each other country, there will, however, still be trade in factors. The model does very poorly in fitting the HOV equations. Only for 25% of observations of capital flows predicted and measured flows have the same direction and also for human capital, the model does more poorly than a coin toss in predicting factor flows.

It seems, hence, that models with country specific productivities are not a very good description of the real world. Let us therefore have a look at the model with factor specific productivities.

		Sign	Weighted Sign	β	R^2	Missing Trade
	K	0.3906	0.3919	-0.0013	0.0013	0.0012
Example 1	H	0.3438	0.367	-0.0834	0.008	0.8674
	K	0.25	0.3769	-0.0101	0.0171	0.0059
Example 2	H	0.5625	0.4486	0.1892	0.024	1.4922
	K	0.25	0.3787	-0.0108	0.0163	0.0071
H.J.	H	0.5156	0.4376	0.1563	0.0149	1.6451

5.2 Factor Specific Productivities

When productivities are factor specific the model fits factor shares by construction for any parameter values.

In this section there are again 2 traded goods, the H-good and the K-good, with $\alpha_H < \alpha_K$. In this model it is important to distinguish whether physical and human capital are complements or substitutes in sectoral production. Let us first take a look of substitutable inputs ($\epsilon > 1$)

5.2.1 Substitutable Inputs

Example 1) $\alpha_H = 0.1$, $\alpha_K = 0.9$, $\sigma_H = 0.5$, $\epsilon = 1.5$. In this case there is hardly any difference between the open economy and Caselli's way to calibrate factor productivities. There is a strong positive correlation between the productivity of human capital, which varies between 2 and 140 per cent of US-human capital productivity, and per capita GDP. On the other hand, there is a negative correlation between the productivity of physical capital and per capita GDP. The poorest countries seem to utilize capital up to 12 times more efficiently than the US. In equilibrium all countries are inside the cone and the relative abundance of physical capital of rich countries disappears when factors are measured in efficiency units. If anything, poor countries are somewhat more capital abundant on average (see figure ??). Consequently, structural change has no power to explain income differences because production patterns do not vary systematically across countries. Since all countries produce both goods, conditional factor prices are equalized. Because w_c is much higher in rich countries than in poor ones and because $A_{Hc} = w_c/\hat{w}$, a high A_H is needed in rich countries in order to match wage data. Exactly the opposite is true for poor countries. In addition, as r_c is lower in rich countries and because $A_{Kc} = r_c/\hat{r}_c$, A_{Kc} needs to be relatively low in rich countries. The reason for conditional factor price equalization is that relative efficient factor abundance does not vary much across countries in equilibrium so that rich countries are just upscaled versions of poor countries. When $\epsilon > 1$ the wage is a positive function of A_H, and the rental a positive function of A_K because the efficient factor has a high value as input demand shifts towards it (from the FOC for profit maximization: $w_c = p_i \left(\frac{Q_{i,c}}{H_{i,c}}\right)^{\frac{1}{\epsilon}} (1-\alpha_i) A_{H,c}^{(\frac{\epsilon-1}{\epsilon})}$.) This explains the positive correlation between A_H and per worker income and the negative correlation between AK and the same variable. The fact that productivities are so close to the ones in Caselli's world can also be explained by the way the data are constructed: Since for all countries in the cone $\frac{A_{H,c}}{A_{H,US}} = \frac{\frac{w_c}{\hat{w}}}{\frac{w_{US}}{\hat{w}}}$ and $\frac{w_c}{w_{US}} = \frac{\frac{s_{H,c} Y_c}{H_c}}{\frac{s_{H,US} Y_{US}}{H_{US}}} \approx \frac{\frac{Y_c}{H_c}}{\frac{Y_{US}}{H_{US}}}$ while in Caselli's world $\frac{A_{H,c}}{A_{H,US}} = \frac{\left(\frac{s_{H,c}}{1-\alpha}\right)^{\frac{\epsilon}{1-\epsilon}} \frac{Y_c}{H_c}}{\left(\frac{s_{H,US}}{1-\alpha}\right)^{\frac{\epsilon}{1-\epsilon}} \frac{Y_{US}}{H_{US}}}$.

The fit of the HOV-equations of this model is poor. Only in 37,5% of observations for capital flows do measured and predicted capital flows have the same direction and also all the other measures of fit do very poorly. The picture remains the same for flows of human capital, for which the value of the sign test is roughly 0.3. The reason for the poor fit is that because of conditional factor price equalization rich countries are now measured to export hardly any capital, while they are predicted to export a lot. The measured human capital content of trade, on the other hand is larger than the predicted one, even though it remains small in absolute terms.

The result that productivities are very similar to the ones in the Caselli world obtains as long as conditional factor price equalization holds. In over to overturn it, some specialization is needed. Specialization occurs when sectoral input ratios are not too different and the elasticity of substitution is relatively small.

Example 2) $\alpha_H = 0.5$, $\alpha_K = 0.6$, $\sigma_H = 0.5$, $\epsilon = 1.2$ Now 47 countries specialize in the capital intensive good, 42 countries lie in the cone and 4 countries specialize in the human capital intensive good. The general pattern of productivities remains similar but human capital productivity is somewhat less correlated with per worker income and the correlation between per worker income and the productivity of physical capital is less negative. Countries specializing in the capital intensive good have higher efficient wages and lower efficient rentals and therefore have lower A_{Hc}s and higher A_{Kc}s than with conditional factor price equalization. The fit of the HOV equations improve for the direction of trade flows for the factor physical capital, while remaining much too small in magnitude. For human capital the fit remains poor. Now many poor countries have lower efficient rentals and higher efficient wages, so that their exports are predicted to contain more capital and their imports human capital than in example 1).

Summarizing, one can say that if factors are relatively good substitutes in sectoral production, rich countries seem to be relatively abundant in efficient human capital and there is a clear positive correlation between per capita GDP and the efficiency of human capital, while there is a negative relation between per capita GDP and the efficiency of physical capital, that may be weakened as the elasticity decreases. Note however, that when the productivity of physical capital is higher in poor countries and human capital is more efficiently used in rich countries, the fit of the HOV equations is unsatisfying because poor countries are measured to export labor and import capital and the predicted factor content has just the opposite sign. Also structural change (in the sense that as countries accumulate physical capital, their production patterns switch from labor to capital intensive goods) plays no role in explaining income differences because the model predicts that poor countries are capital abundant in efficiency units, implying that they export capital intensive goods while importing labor intensive ones. Let us now turn to the case in which sectoral inputs are complements in production ($\epsilon < 1$).

5.2.2 Complementary Inputs

Example 3) $\alpha_H = 0.2$, $\alpha_K = 0.8$, $\sigma_H = 0.4$, $\epsilon = 0.6$ In this case, the 19 richest countries specialize in the capital intensive good, 11 middle income countries are in the cone and 64 countries produce only the human capital intensive good.

With complementary inputs results are very different from Caselli's world. Now the relationship between A_{Hc} and income per worker is inverse hump shaped, so that the productivity of human capital is highest for medium income countries. On the other hand, the correlation between A_{Kc} and per capita income becomes strictly positive, just the opposite of Caselli's result!

In this world, rich countries are abundant in efficient capital, while poor countries are abundant in efficient human capital. The simple link between factor productivities and factor prices that obtains within a conditional factor price equalization set, where factor prices and productivities are positively related, is broken by specialization. Instead, the sectoral prediction that factor prices and factor efficiencies are inversely related, prevails. The intuition is, that with complementary inputs, the inefficient input is expensive because it is scarce in efficiency units and required in production. So high wages imply relatively low efficiencies of human capital. Conversely, low rentals imply relatively higher productivities of physical capital. Hence, the relationship between factor prices and factor productivities is non-monotonic and depends on in which of the 3 possible sets of specialization countries lie.

The fit of the HOV of this particular model is relatively poor for physical capital but reasonably good for human capital. Since poor countries have relatively low efficient wages and high efficient rentals, their exports are measured to contain a lot of human capital and little physical capital. The reverse holds for rich countries. On the other hand, predicted factor trade, does not seem to coincide in magnitude, even though given productivities, poor countries are more human capital abundant and less abundant in physical capital than rich ones.

Example 4) $\alpha_H = 0.1$, $\alpha_K = 0.6$, $\sigma_H = 0.4$, $\epsilon = 0.8$

This example is similar to the previous one in qualitative terms, just the magnitudes of productivity differences are much larger in this example. This improves the fit of the HOV-equations significantly, as the size of the predicted factor content of trade becomes larger. The fit for physical capital is reasonably good, even though the model still measures to little trade in capital, while the fit for human capital is very good with respect to all tests. The intuition for the good fit of the HOV equations is the following. When factors are complements rich countries have high efficient wages and low efficient rentals, because they specialize in the capital intensive good. Consequently, they use a lot of capital and little human capital per unit of output and are measured to export a lot of capital and to import human capital. When rich countries have high capital productivities they are very capital abundant in efficiency units and therefore are also predicted to export capital. The reverse holds for poorer countries. For human capital, on the other hand, rich countries are measured to import a lot of the factor, while with relatively low productivities of human capital they are also predicted to do so. At the same time, middle income countries are measured to be very abundant in human capital, exporting a lot of this factor, which they are also measured to do because of relatively low efficient wages.

At this point it is also interesting to check how Caselli's productivities fit the trade data. As can be seen in table two where I report results for $\epsilon = 0.3$, the parameter for which Caselli's productivities have the best fit, this model does very poorly in predicting trade flows. The reason is that in this model rich countries have relatively high efficient wages and low efficient rentals, so that their exports are measured to contain lots of capital and little human capital, while on the other hand, rich countries are predicted to export little capital because they are not so abundant in this factor due to low productivities, and to import little human capital because they are abundant in efficiency units.

Summarizing, when factors are complements, two stable patterns emerge: The productivity of human capital seems to be hump shaped when plotted against income per worker, with medium income countries exhibiting highest productivity levels of this factors. Second, there is a clear positive correlation between the productivity of physical capital and income per worker. In this environment structural change is important because countries that are capital abundant when capital is measured in PPP dollars are also capital abundant in efficiency units and specialize in the good that uses their abundant factor intensively. The magnitude of productivity differences depends on parameter values and gets larger as ϵ approaches one from below. The best fit of the HOV equations is achieved with implausibly large productivity differences. This may be due to the fact, that the model predicts too much trade, because in deriving the HOV equations it was assumed that every country imports a fraction of every other country's output that is proportional to its relative size in the world economy.

		Sign	Weighted Sign	β	R^2	Missing Trade
	K	0.375	0.3801	-0.0006	0.0012	0.0004
Example 1	H	0.2969	0.3078	-0.1404	0.0057	3.6406
	K	0.5469	0.8763	0.0242	0.3839	0.0015
Example 2	H	0.2813	0.2689	-0.8458	0.1190	6.0150
	K	0.3594	0.4079	-0.1544	0.3204	0.0744
Example 3	H	0.6406	0.6369	0.8654	0.4606	1.63
	K	0.8125	0.5503	0.1068*	0.0656*	0.2644
Example 4	H	0.8125	0.7614	0.783	0.5332	1.1504
Caselli	K	0.5000	0.5407	0.0042	0.1090	0.0002
$\epsilon = 0.3$	H	0.2969	0.2990	-0.5805	0.0710	4.7520

6 Conclusion

This paper has developed a quantitative Heckscher-Ohlin model of the world economy with country and factor specific productivities in order to address the question how much of cross country income differences are due to differential factor endowments and what kind of technology differences are necessary to explain the observed per capita income differences. Theoretically, productivity differences necessary to explain cross country income differences may be smaller in an open economy framework, because countries may be able to adjust their production patterns to their factor endowments through structural transformation, thereby escaping decreasing returns. Quantitatively, I find however that when productivities are country specific, structural change is not important in explaining cross country income differences for most parameter values that imply realistic labor shares because rich countries are too large to specialize in capital intensive goods and to use their capital endowments more efficiently.

I also introduce an overidentifying restriction based on the Heckscher-Ohlin-Vanek equations to check if calibrated productivities help to explain trade in factors. I find that neither the Heckscher-Ohlin model nor the Hall and Jones model are able to explain the observed trade in factors.

When productivities are factor specific, two types of results emerge: If factors are substitutes in sectoral production, rich countries are calibrated to have higher productivities of human capital, while their productivities of physical capital seem to be lower. This finding is basically a replication of Caselli's (2005)result. In the model rich countries are not the most capital abundant in efficiency units, so that structural change plays no role for explaining income differences. When using the Heckscher-Ohlin-Vanek (HOV) equations to evaluate calibrated productivities I find very poor fit of the model. The same poor fit is also found for Caselli's model.

When factors are complements at the sectoral level, on the other hand, structural change does play a role and calibrated productivities fit the HOV equations surprisingly well for some specifications. Rich countries are predicted to specialize in the production of capital intensive goods and have far higher productivities of physical capital than poor countries. The productivity of human capital is inverse U shaped, being low for the poorest countries, reaching its maximum in the middle income range and becoming again lower in rich countries. The conclusion is hence that productivity differences remain important even with trade, but they are of a different form. The results also suggest that a model with country specific technology differences may be a too restrictive picture of the world.

The form of productivity differences is not too different from Trefler (1993) who finds in a Heckscher-Ohlin-Vanek framework that rich countries have both higher productivities of labor and physical capital. There are, however, important differences between Trefler (1993) and this paper. Trefler (1993) assumes that conditional factor price equalization holds at the world level and then solves for the factor productivities that make the Heckscher-Ohlin-Vanek equations fit exactly. He finds little trade in factors and, as Gabaix shows, with zero measured factor trade, the formulas for factor productivities in his model converge to the inverse of the factor-output ratio. In this paper, no assumption of conditional factor price equalization has been made. I have also shown that it is actually very unlikely to hold at the level of the world economy. Contrary to Trefler (1993) but similar to Trefler and Zhu (2006) I find a lot of trade in factors.

There may also be need to explain why the productivity of physical capital seems to be larger in rich countries. An approach focusing on institutions may relate capital productivity to factors like financial development that affect the efficiency of the distribution of capital within the economy. Another viewpoint may be to interpret the results as productivity differences being larger in capital intensive sectors than in labor intensive sectors.

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7 Appendix

7.1 Solution Method

Since both the *Productivity Calibration Model with Country Productivities* and the *Productivity Calibration Model with Factor Productivities* form a large set of nonlinear equations with inequality constraints I use numerical methods to solve for the equilibrium prices, quantities and productivities. In order to be able to solve the problem, I exploit the properties of the Heckscher-Ohlin multiple cone model. Let me discuss the solution procedure of the two models in turn, starting with the *Productivity Calibration Model with Country Productivities.*

Algorithm for solving the *Productivity Calibration Problem with Country Productivities*

In this model there are two sectors, call them the K- and the H-sector, with $\alpha_K > \alpha_H$. With two sectors and two factors production levels of individual countries are determined which simplifies the solution of the problem. Finding the equilibrium of the world economy amounts to finding the set of countries that lie in the cone. The CAP is in mathematical terms a nonlinear complementarity problem. While some numerical algorithms for solving this type of problem exist (for example the PATH solver developed by Ferris et al.), the size of the problem makes a direct application of these methods intractable. Instead, I choose to guess a particular specialization pattern and verify if it is a solution to the CAP.

The algorithm works as follows:

1) Guess a specialization pattern, starting with all countries in the cone.

2) Aggregate all countries that are assumed to be in a cone.

3) Solve the resulting nonlinear system of equations.

4) Whenever a country in the cone produces a negative quantity of a good, update the guess by setting its production of that good to zero and go to step 2).

5) If the quantities produced are positive for all countries in the cones, check if with the new guess production of the countries that have been moved from a cone into a set of countries that specialize satisfies the condition that firms in that country would make a loss if they were to produce any of the other goods. If this is true stop, otherwise go to step 6).

6) If this is not the case, put those countries back into the cone.

7) Iterate on this procedure until convergence.

Alternatively, one can also order countries by K_c/H_c and just check all possible specialization patterns. This procedure becomes necessary when solving models with more than two goods, because production will then only be determined at the level of a cone and not at the country level, where production is undetermined for countries lying in any cone. In this case the Lens Condition (Deardorff 1994) can be used to determine whether a set of countries can constitute a cone.

Algorithm for solving for the *Productivity Calibration Equilibrium with Factor Productivities*

In the case in which productivities are factor specific, matters are more complicated because countries cannot ex ante be ordered according to their effective capital to human capital ratio as $\frac{A_{K,c}K_c}{A_{H,c}H_c}$ is only obtained, once the productivities have been solved for. For this reason I limit my analysis to the two-goods-case in which individual countries' production levels are determined. Let there be two sectors, the K- and the H-sector with $\alpha_K > \alpha_H$. Then there can be at most one cone of diversification and countries with extreme $\frac{A_{K,c}K_c}{A_{H,c}H_c}$ will specialize in the K-good or the H-good. The algorithm works quite similarly to the previous case:

1) Guess a specialization pattern starting with the guess that all countries are in the cone.

2) If for any country the production of a good under this specialization pattern is negative, update the guess by assuming that equilibrium production of this particular good in that country is zero.

3) Check if under the new guess firms in all countries that specialize would make a loss in producing the other good at equilibrium prices of this specialization pattern. If not put those countries back into the cone.

4) Repeat steps 2) and 3) until convergence, i.e. until a production pattern is found such that all equilibrium prices and quantities are strictly positive and it would not pay for specializing countries to produce the other good.

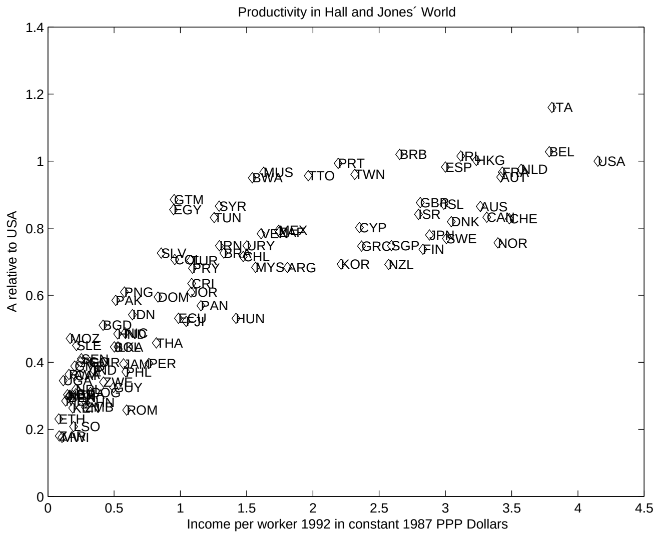

Figure 1: TFP in Hall & Jones' world

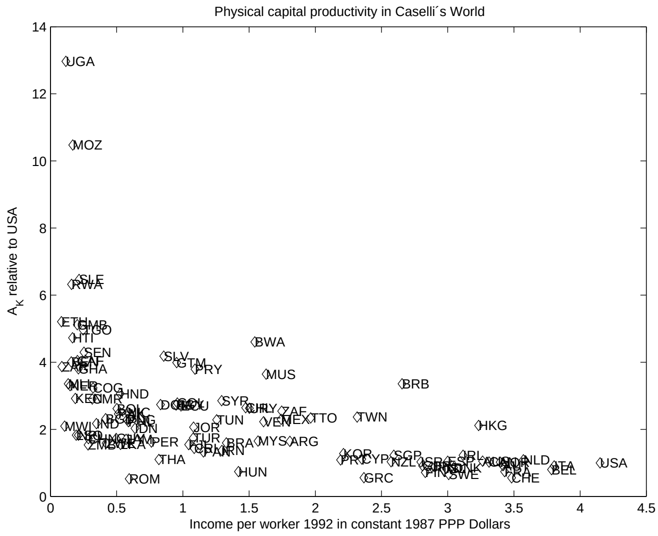

Figure 2: A_K in Caselli's world for $\epsilon = 0.8$

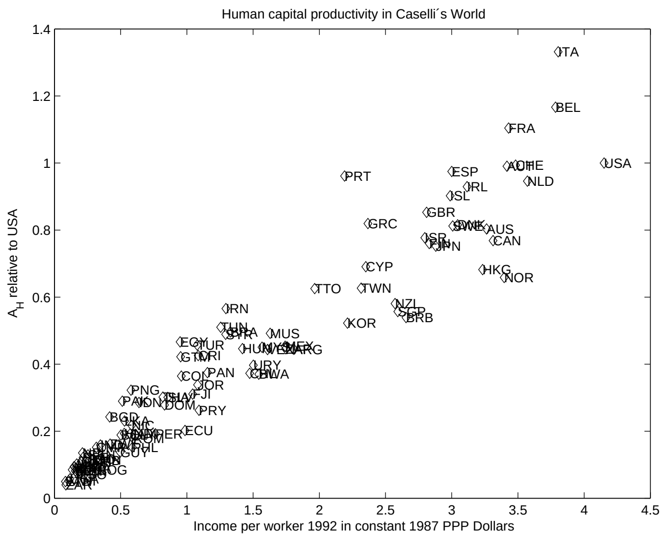

Figure 3: A_H in Caselli's world for $\epsilon = 0.8$

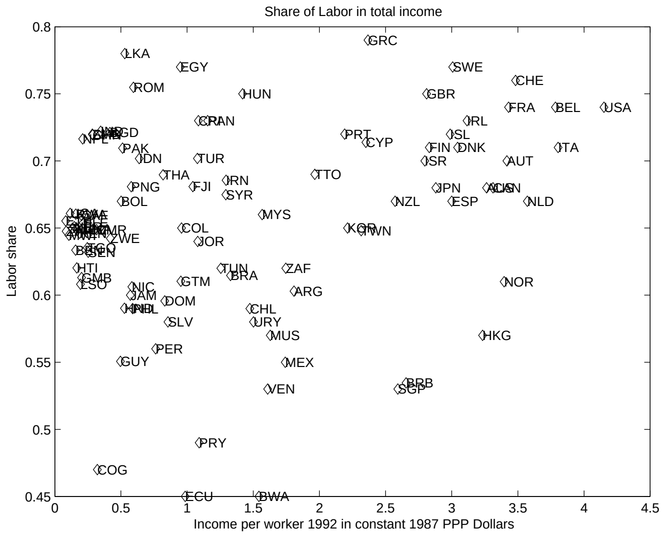

Figure 4: Labor Share

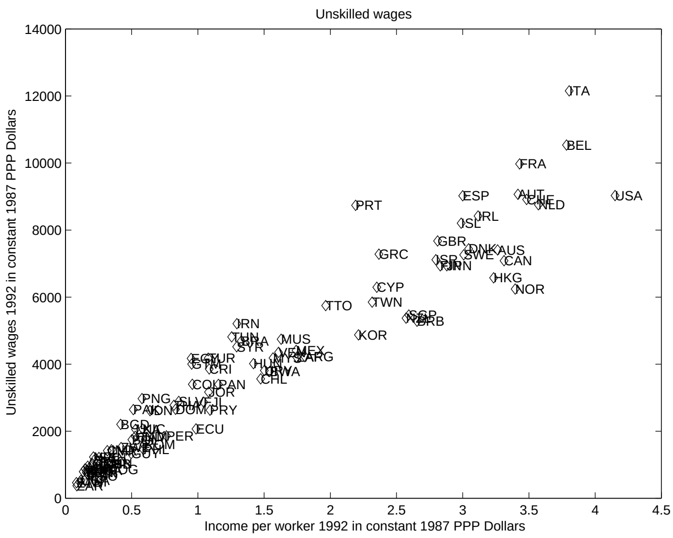

Figure 5: unskilled wages

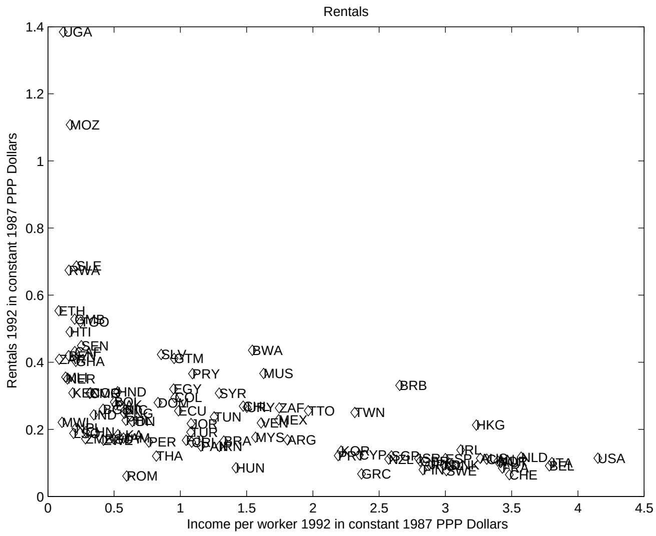

Figure 6: rental rates

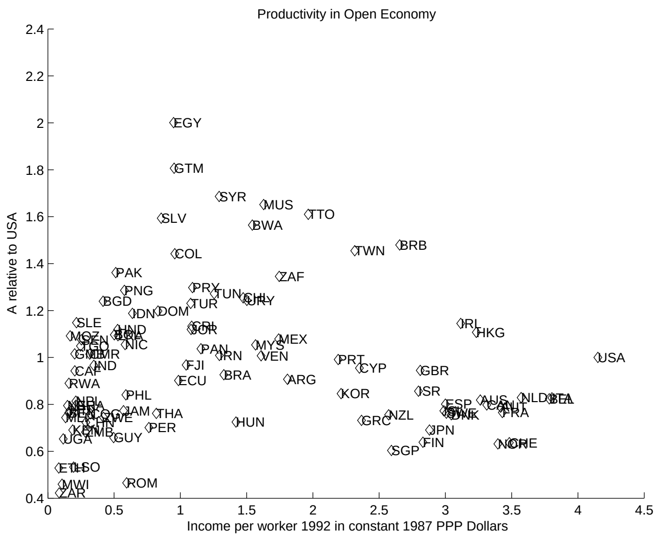

Figure 7: $\alpha_H = 0.001, \alpha_K = 0.999, \epsilon = 1, \sigma_H = 0.2, \sigma_K = 0.8$

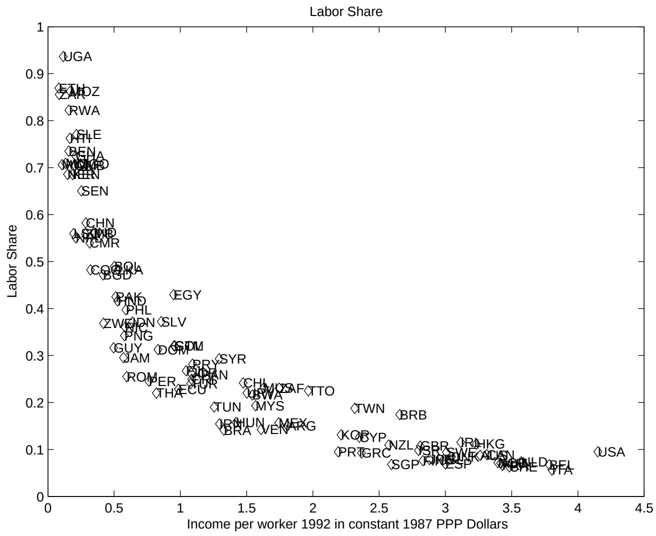

Figure 8: $\alpha_H = 0.001, \alpha_K = 0.999, \epsilon = 1, \sigma_H = 0.2, \sigma_K = 0.8$

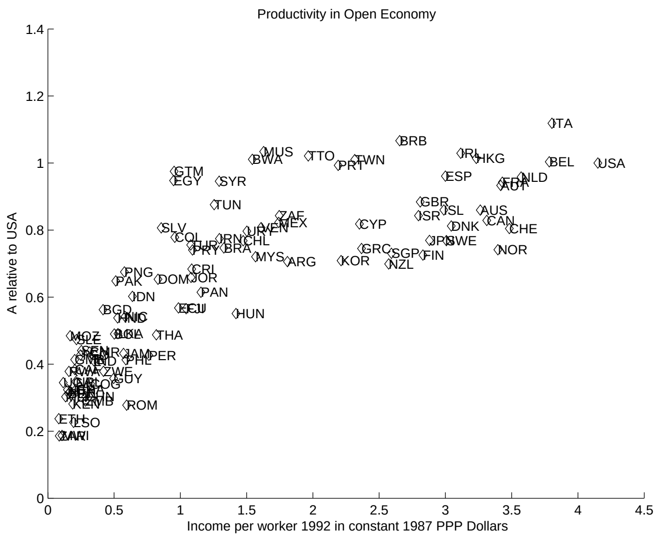

Figure 9: $\alpha_H = 0.3, \alpha_K = 0.4, \epsilon = 1, \sigma_H = 0.2, \sigma_K = 0.8$

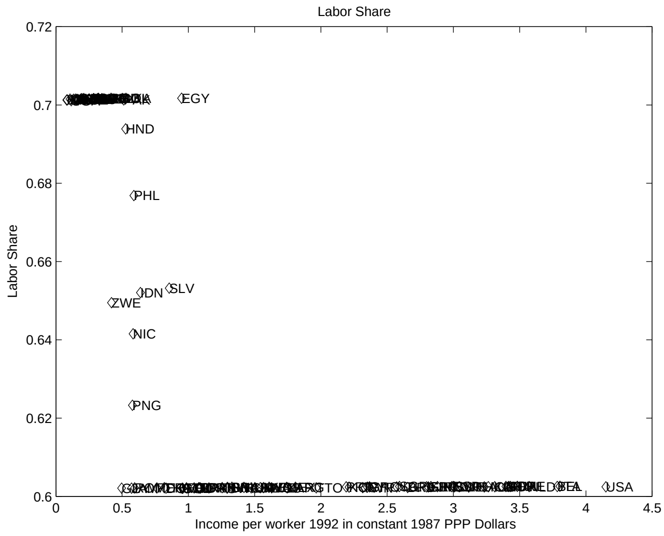

Figure 10: $\alpha_H = 0.3, \alpha_K = 0.4, \epsilon = 1, \sigma_H = 0.2, \sigma_K = 0.8$

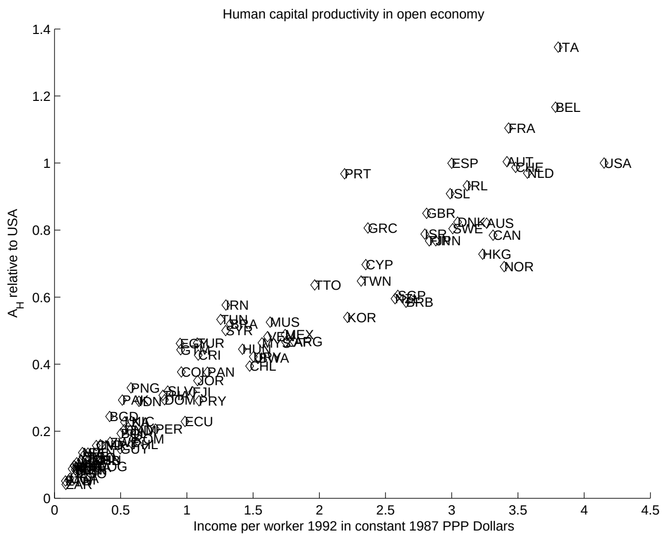

Figure 11: $\alpha_H = 0.1, \alpha_K = 0.9, \epsilon = 1.5, \sigma_H = 0.5$

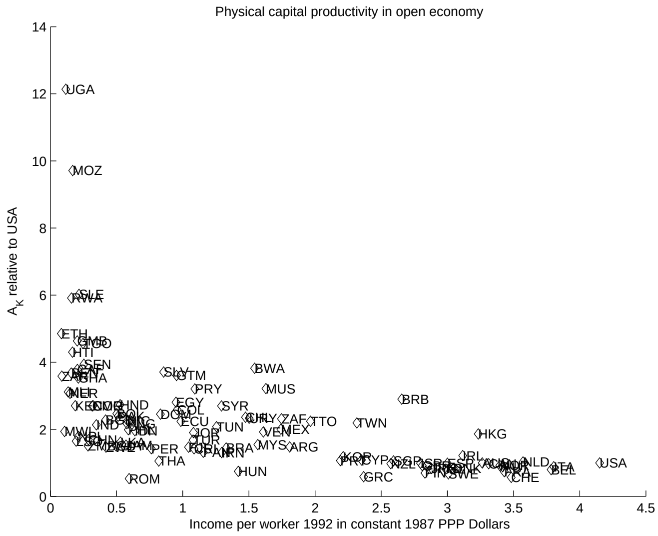

Figure 12: $\alpha_H = 0.1, \alpha_K = 0.9, \epsilon = 1.5, \sigma_H = 0.5$

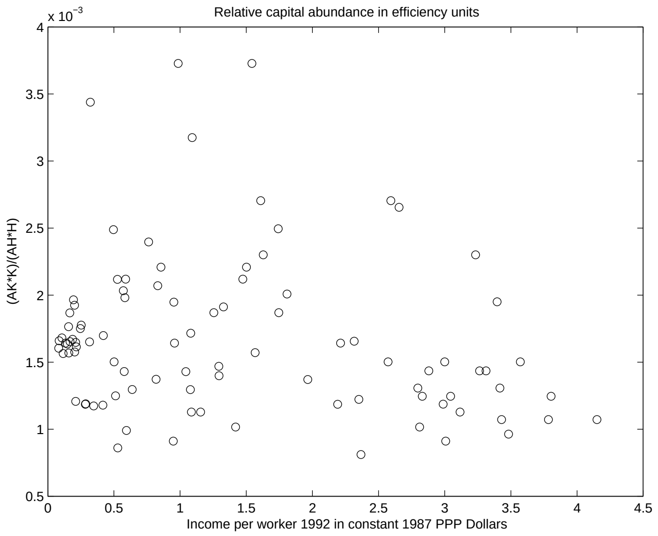

Figure 13: $\alpha_H = 0.1, \alpha_K = 0.9, \epsilon = 1.5, \sigma_H = 0.5$

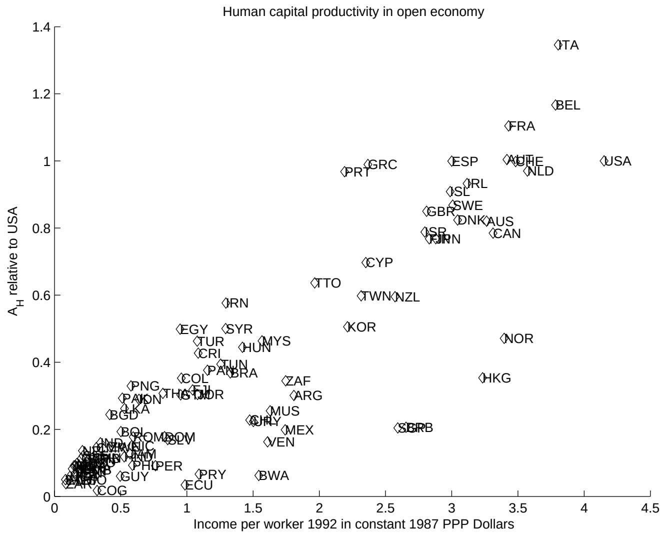

Figure 14: $\alpha_H = 0.5, \alpha_K = 0.6, \epsilon = 1.2, \sigma_H = 0.5$

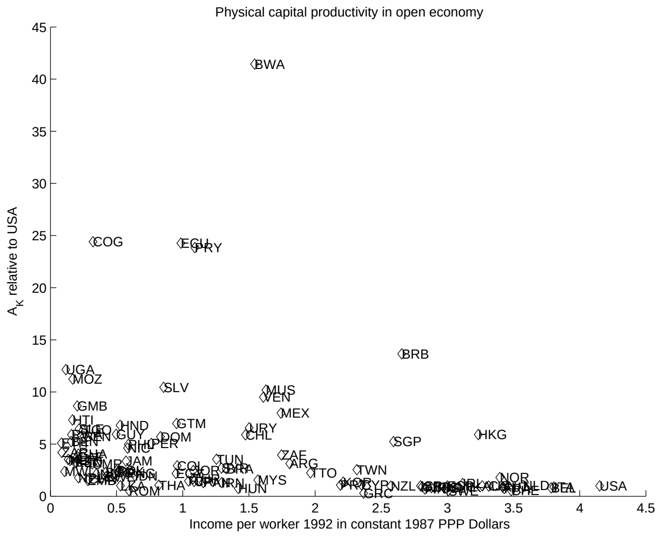

Figure 15: $\alpha_H = 0.5, \alpha_K = 0.6, \epsilon = 1.2, \sigma_H = 0.5$

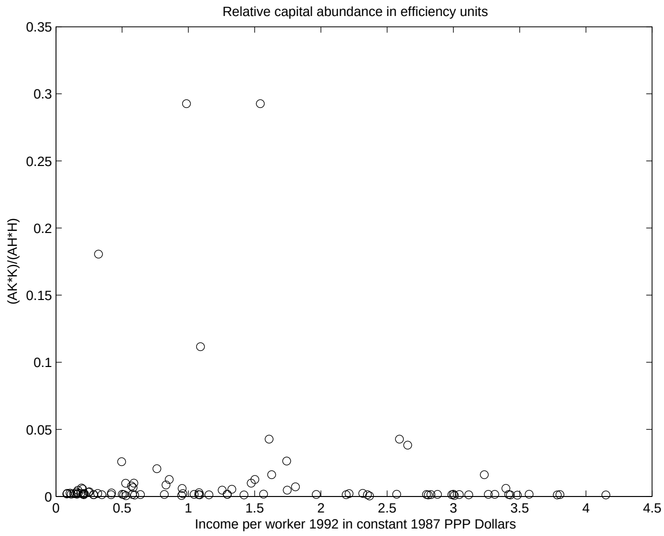

Figure 16: $\alpha_H = 0.5, \alpha_K = 0.6, \epsilon = 1.2, \sigma_H = 0.5$

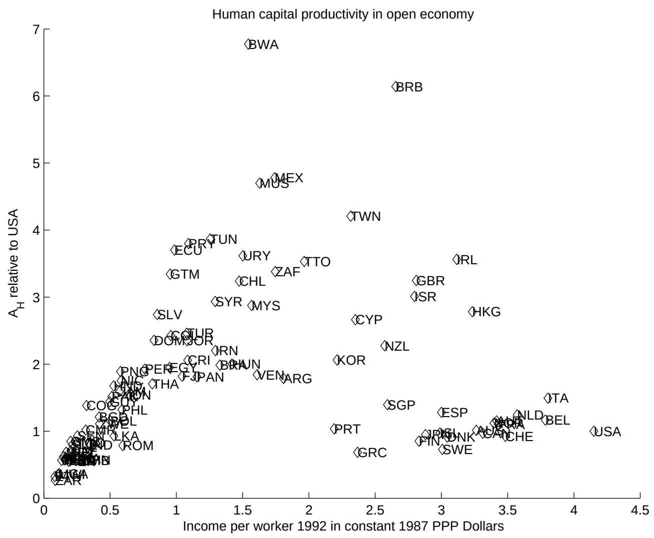

Figure 17: $\alpha_H = 0.2, \alpha_K = 0.8, \epsilon = 0.6, \sigma_H = 0.4$

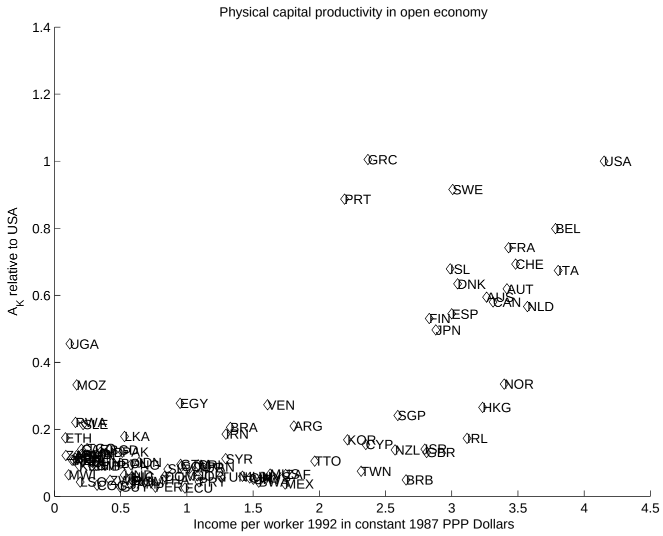

Figure 18: $\alpha_H = 0.2, \alpha_K = 0.8, \epsilon = 0.6, \sigma_H = 0.4$

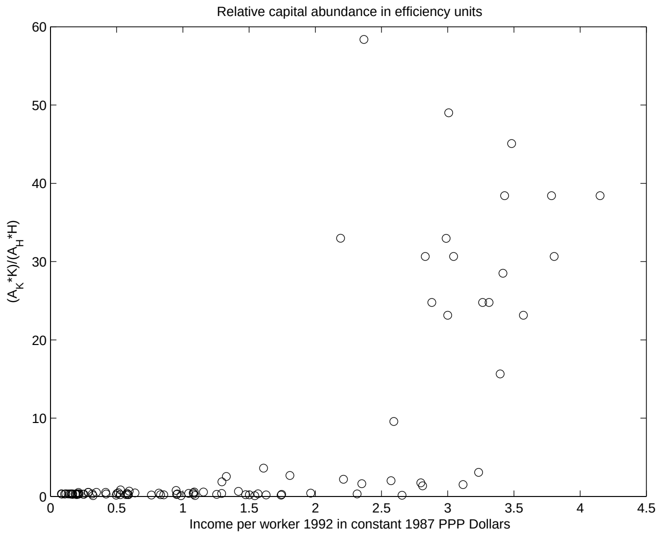

Figure 19: $\alpha_H = 0.2, \alpha_K = 0.8, \epsilon = 0.6, \sigma_H = 0.4$

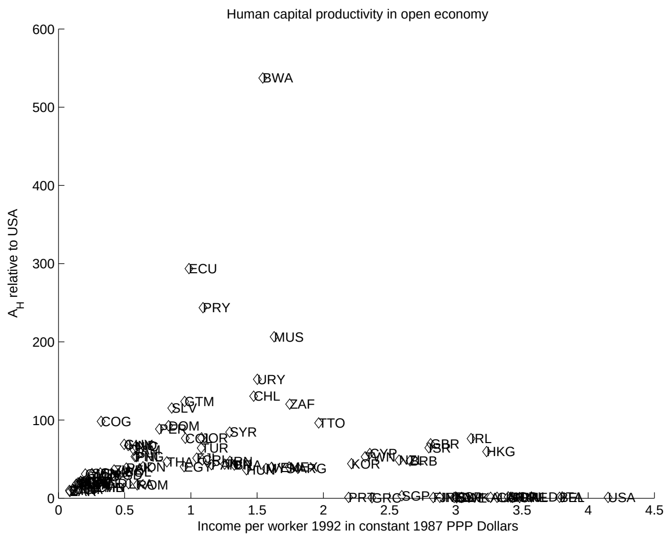

Figure 20: $\alpha_H = 0.1, \alpha_K = 0.6, \epsilon = 0.8, \sigma_H = 0.4$

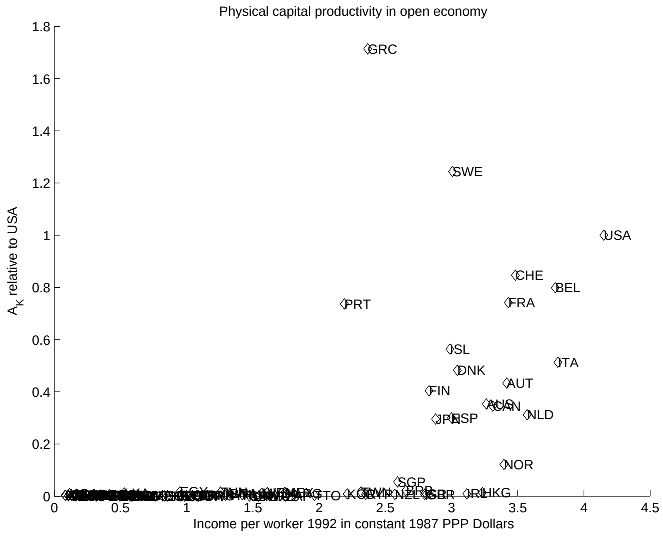

Figure 21: $\alpha_H = 0.1, \alpha_K = 0.6, \epsilon = 0.8, \sigma_H = 0.4$

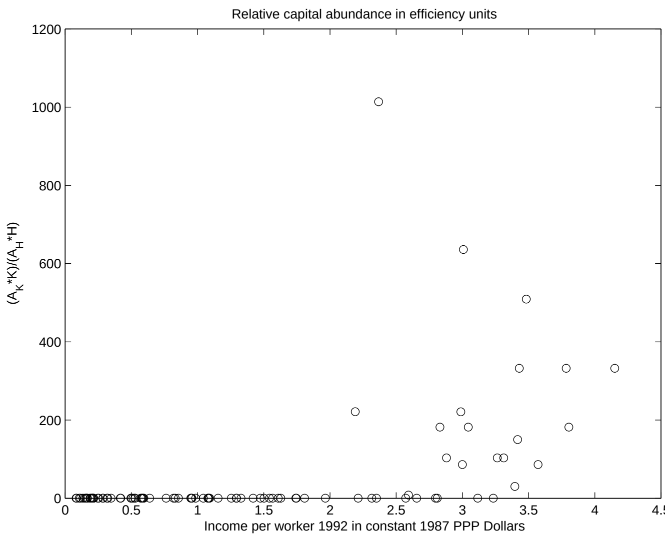

Figure 22: $\alpha_H = 0.1, \alpha_K = 0.6, \epsilon = 0.8, \sigma_H = 0.4$