The dynamics of wages and employment in a model of monopolistic competition and efficient bargaining

Preliminary version, not to be quoted

Pasquale Commendatore, University of Naples

Ingrid Kubin, Universit of Economics and Business Administration, Vienna

1. Introduction

Modern macroeconomic models with a Keynesian flavour usually involve nominal rigidities in wages and commodity prices. A typical microfoundation recurs to wage bargaining in the labour markets and monopolistic competition in the commodity markets (e.g. Blanchard and Giavazzi). A characteristic feature of those models is that deregulating the labour markets (i.e. reducing the bargaining power of workers and/or reducing the unemployment benefits) increases equilibrium employment; in a symmetric way, deregulating the commodity markets (i.e. reducing the market power of commodity suppliers) increases equilibrium employment as well.

However, those models are typically static models which do not specify explicitly the economic process in time. In the following paper, we develop a dynamic macroeconomic model in which commodity markets are characterised by monopolistic competition and labour markets by wage bargaining. In this first version, the number of firms is fixed. The incorporation of firm entry and exit is left for further research.

In our analysis the usual equilibrium solution is a fixed point of the dynamic model which exhibits the usual comparative static properties (deregulating the labour and/or the commodity market increases employment). However, depending upon the parameters the fixed point may loose stability through a Flip-bifurcation giving rise to cyclical solutions. We show analytically that commodity and labour market deregulation may lead to instability; in numerical simulation we even found cases in which deregulation leads to lower average employment. Both results, valid in a dynamic framework, contrast with the usual comparative static properties.

The paper is organised as follows. In the second section, we present the model the basic structure of which closely follows Blanchard/Giavazzi. Monopolistic competitive firms bargain with unions over employment and nominal wage rates on the basis of firms' anticipated demand functions and workers' nominal reservation wages. In the third section, we explicitly add two dynamic components. First, central to monopolistic competition is that firms neglect the reactions of other firms. They therefore base their decisions on too high a price elasticity and are necessarily surprised by the market results. As a consequence, they will adjust the position of their anticipated demand function over time. Second, the nominal reservation wage also adjusts through time on the basis of the realised employment rate and the realised nominal wage rate. Whereas for the second process several plausible specifications do exist, the specification of the first one is directly implied by the external effect at the core of the monopolistic competition model itself. We show, that the first process leads to diverging time paths: while it still holds that deregulation increases stationary employment rates, it leads to fundamental instability. In the fourth section, we study two different specifications of reservation wage adjustment, each of which mirrors a different institutional set up. We show analytically that this second process may dampen the sharp instability result: deregulation may still destabilise the economy; however, the time path does not diverge but is attracted to a period two cycle or eventually to a complex time path. Simulations complete the dynamic explorations. The last section is left for concluding remarks.

2. The economic framework

The analysis, being framed in a dynamic setting, requires dealing explicitly with time. We assume that during the time unit, which for expository purposes we call 'Week *t*', all events occur according to a well-defined sequence. On Monday of Week *t*, firm *i* and the associated union bargain over employment and the nominal wage rate on the basis of firms' anticipated demand and workers' nominal reservation wage. Production occurs during the week from Tuesday to Friday. On Friday commodities are delivered to the market. Market equilibrium determines the realised price. Finally, on Saturday firms and unions update the information relevant for decisions in the following period concerning the employment rate, the nominal wage rate and the price index.

2.1 Firms and Households

The economy comprises *m* firms, each firm producing one differentiated good in a regime of monopolistic competition; and *L* households which own the existing firms, each household supplying one unit of labour. Each firm faces a union composed of $M_i = \frac{L}{m}$ members.

All firms share the same production technology, which involves only one input, labour, used in a fixed proportion. Assuming that during Week *t* firm *i*' employs N_i^t workers, the production of good *i* is:

$$x_i^t = f(N_i^t) = N_i^t \tag{1}$$

for $i = 1 \ldots m$.

Following Blanchard and Gavazzi, household *j*'s preferences towards good *i* (for $i = 1 \ldots m$ and $j = 1 \ldots n$) are represented by a CES utility function:

$$U_j^t = \left(m^{-\frac{1}{s}} \sum_{i=1}^{m} \left(c_i^t\right)^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}} \tag{2}$$

where $1 < s < \infty$ is the elasticity of substitution between goods and c_i^t is the consumption level of good *i* for the period. Note that the inclusion of the factor $m^{-\frac{1}{s}}$ into the utility function implies that the utility level depends only upon the overall consumption quantity and not on the number of different consumption goods. This specification therefore does not represent a preference for commodity variety and s is a parameter related only to the market power of a single commodity supplier. In Blanchard and Gavazzi, it is the central parameter for studying the effects of commodity market deregulation.

Household j's demand for good i is given by

$$d_j^t = \frac{Y_j}{mP_t}\left(\frac{p_i^t}{P_t}\right)^{-s}$$

where Y_j is the household j's income and where

$$P_t = \left(\frac{1}{m}\sum_{i=1}^{m}\left(p_i^t\right)^{1-s}\right)^{\frac{1}{1-s}} \qquad (3)$$

is a price index corresponding to the price of one unit of utility. Summing through j, we obtain the demand for commodity i during Week t

$$d_t = \frac{Y}{mP_t}\left(\frac{p_i^t}{P_t}\right)^{-s} \qquad (4)$$

where Y is the overall nominal income of the economy, which is taken as numéraire. Note that the elasticity of demand with respect to the own price is equal to $-s$.

If the price of each good is the same, $p_i^t = p_t$, the price index becomes $P_t = p_t$ and the demand for an individual good simplifies to

$$d_t = \frac{Y}{mp_t} \qquad (5)$$

In that case, the elasticity of demand with respect to the price is equal to -1.

Moving on to the description of the events occurring during 'Week *t*', most of our discussion will be devoted to the crucial ones, that is, the bargaining process and the determination of the market equilibrium.

2.2 Bargaining

During the Monday of week t the parties (firm i and the associated union) bargain over employment and the nominal wage rate on the basis of a known set of data relating to workers' reservation wage and firm i's anticipated demand function. In order to study this process we employ the efficient bargaining model. Our analysis differs from the text book treatment in that firms are monopolistically competitive (instead of perfectly competitive) in the commodity market.

Typically, in models of monopolistic competition of the Dixit-Stiglitz type each firm assumes that other firms will not react to its own price decisions.[1] Specifically, firm i anticipates a demand function $\tilde{d}_t$ with elasticity $(-s)$ and position K_t, which is considered as given for the period:

$$\tilde{d}_t = \frac{K_t}{\left(\tilde{p}_i^t\right)^s} \tag{6}$$

where $\tilde{p}_i^t$ is firm i's anticipated price.

[1] This is more plausible the larger the number of firms. See Currie and Kubin, 2002.

Using equations (1) and (6) we obtain

$$\tilde{p}_i^t = \left(\frac{K_t}{N_i^t} \right)^{\frac{1}{s}} \tag{7}$$

Firm *i*'s rent is given as its anticipated profit (with a fall back income equal to zero):

$$R_f^t = \tilde{p}_i^t x_i^t - W_t^i N_i^t = \left(\tilde{p}_i^t - W_t^i \right) N_i^t \tag{8}$$

with N_i^t denoting firm *i*' employees for Week *t* and W_i^t denoting the nominal wage rate for Week *t*.

Workers' rent is determined as in the standard Efficiency bargaining model. If the contract is concluded the parties agree on employment N_i^t and a nominal wage rate W_i^t. The remaining members of the union $M_i - N_i^t$ have to find employment in other firms. The income of hired workers is

$$N_i^t W_i^t \tag{9}$$

The expected income of the remaining members is

$$\left(M_i - N_i^t \right) W_R^t \tag{10}$$

where W_R^t is the nominal reservation wage, i.e. the wage rate expected to prevail outside the firm under consideration.

The overall workers' compensation if the contract is concluded is given by summing (9) and (10):

$$N_i^t W_i^t + \left(M_i - N_i^t\right) W_R^t \tag{11}$$

If no contract is concluded, the reservation wage would be the fall back income of the trade union members

$$M_i W_R^t \tag{12}$$

Workers' rent is given by the difference between (11) and (12):

$$\begin{aligned} R_w^t &= N_i^t W_i^t + \left(M_i - N_i^t\right) W_R^t - M_i W_R^t \\ &= N_i^t \left\{W_i^t - W_R^t\right\} \end{aligned} \tag{13}$$

Efficient (Nash) bargaining between firm's i and the associated union corresponds to choosing N_i^t (p_i^t) and W_i^t so as to maximise the following expression subject to (7):

$$\left(R_w^t\right)^b \left(R_f^t\right)^{1-b} = \left(W_i^t - W_R^t\right)^b \left(\tilde{p}_i^t - W_i^t\right)^{1-b} N_i^t \tag{14}$$

where $0 < b < 1$ represents the relative bargaining power of the unions. In Blanchard and Gavazzi, b is the only parameter for studying the effects of labour market deregulation.

The outcome from the bargaining is:

$$W_i^t = \left(\frac{s - 1 + b}{s - 1}\right) W_R^t \tag{15}$$

$$M\tilde{R} = \frac{s - 1}{s} \tilde{p}_i^t = W_R^t \tag{16}$$

$$N_i^t = K_t \left[\left(\frac{s}{s-1} \right) W_R^t \right]^{-s} \tag{17}$$

Note the familiar result from the efficient bargaining problem, namely that a position is chosen off the profit maximum. In equation (16), the marginal revenue is equated to the nominal reservation wage and not to the marginal cost (corresponding to the nominal wage rate). From equation (16) it follows that the price is set as a mark-up over the reservation wage, $\tilde{p}_i^t = (1+m) W_R^t$, where $m \equiv 1/(s-1)$.

Following Blanchard and Gavazzi, we interpret a decrease of the mark-up m (i.e. an increase in s) as commodity market deregulation. It reduces the size of firm i's anticipated surplus per unit of output, $\left(\tilde{p}_i^t - W_R^t\right) = mW_R^t$. Labour market deregulation is reflected in Blanchard an Gavazzi by a decrease the relative bargaining power of the union, b. As equation (15) shows, b determines the distribution of the anticipated surplus between wages and profits. Decreasing b reduces the workers' share in the surplus, bmW_R^t. In addition, our model as developed in the following sections allows for labour market deregulation in the form of changing the unemployment insurance scheme. Also this form of labour market deregulation may impinge on the bargaining outcome (and on the size of the surplus) through its effects on the nominal reservation wage.

Note finally that equations (16) and (17) are not independent results, equation (17) can be obtained from equation (16) taking constraint (7) into account.

During Week t, at the end of contracting, firm i employs N_i^t workers paying the nominal wage rate W_i^t and it anticipates the price $\tilde{p}_i^t$. Figure 1 depicts the bargaining or partial equilibrium. Given the reservation wage W_R^t and the position of the anticipated demand function K_t, the bargain determines the wage rate and the employment level in firm *i*.

2.3 Short term commodity markets equilibrium

The characteristics of the general equilibrium follow from the assumption that all firms behave identically (what is called the symmetry assumption in Blanchard and Giavazzi). Each firm pays the same wage $W_i^t = W_t$, hires the same number of workers, $N_i^t = N_t$, to produce the same quantity $x_i^t = x_t$, which is equal to their respective supplies $s_i^t = s_t = x_t$. They envisage to sell these quantities at an identical anticipated price $\tilde{p}_i^t = \tilde{p}^t$. At the end of the production period, on Friday, each firm supplies s_t to the (identical) true demand function. In contrast to the anticipated demand functions, the true demand functions take into account the reactions of all other firms; each of them therefore has an elasticity of -1 (instead of $-s$). Assuming flexible prices and commodity market-clearing, the realized market price $\hat{p}_t$ will differ from the anticipated one.[2] Using (5) it is determined as:

$$\hat{d}_t = s_t = x_t = N_t \qquad \hat{p}_t = \frac{Y}{mx_t} = \frac{Y}{mN_t} \tag{18}$$

---

[2] Note that the difference $\hat{p}_t - \tilde{p}_i^t$ determines the difference between firm i's realised and anticipated rent per unit of output $\left(\hat{p}_t - W_R^t\right) - \left(\tilde{p}_i^t - W_R^t\right)$ and also the difference between realised and anticipated profit given the residual nature of the former.

The short run general equilibrium is depicted in Figure 2.

Finally, on Saturday firms and unions update their information. The position of firm *i*'s anticipated demand curve changes on account of the realised price $\hat{p}_t$ and the realised demand $\hat{d}_t$. Inserting this information in equation (6) determines the position of the anticipated demand for the period *t*+1:

$$K_{t+1} = \hat{d}_t \hat{p}_t^s \tag{19}$$

Similarly, workers' reservation wage adjusts in the light of the realised nominal wage W_t and of the realised employment rate $e_t = \frac{mN_t}{L}$.

3. The dynamic system

<u>*3.1. Outline*</u>

The dynamic behaviour of the model, therefore, involves two processes. First, since producers do not know the true elasticity of the demand function, the position of the anticipated demand function (and thus of the anticipated marginal revenue) shifts over time. For a given nominal reservation wage and marginal revenue curve, the efficient bargaining outcome, $MR = W_R^t$, determines employment. Since all firms are identical, equation (17) can be written as:

$$e_t = \frac{m}{L} K_t \left\{ \left(\frac{s}{s-1} \right) W_R^t \right\}^{-s} \tag{20}$$

and, taking into account equations (18) and (19), also as

$$e_t = \left(\frac{Y}{L}\right)^s e_{t-1}^{1-s} \left(\frac{s}{s-1}\right)^{-s} \left(W_R^t\right)^{-s} \tag{21}$$

Second, the nominal reservation wage, W_R^t, is also adjusted over time.

For expository purposes, in what follows we begin our study with the dynamics of the first process in isolation by assuming a nominal reservation wage invariant over time. After that we specify the adjustment of the reservation wage change explicitly and explore the dynamic properties of the full system.

3.2. Commodity market dynamics in isolation

If we assume a reservation wage fixed at an arbitrarily chosen level, $W_R^t = \overline{W}_R$, the implied dynamic process is

$$e_t = \left(\frac{Y}{L}\right)^s e_{t-1}^{1-s} \left(\frac{s}{s-1}\right)^{-s} \left(\overline{W}_R\right)^{-s} \tag{22}$$

Equation (22) is a one-dimensional first-return map the following fixed point and first derivative

$$\overline{e} = \left(\frac{Y}{L}\right)\left(\frac{s}{s-1}\right)^{-1} \left(\overline{W}_R\right)^{-1} \tag{23}$$

$$\frac{\partial e_t}{\partial e_{t-1}} = \left(\frac{Y}{L}\right)^s \left(\frac{s}{s-1}\right)^{-s} \left(\overline{W}_R\right)^{-s} (1-s)\, e_{t-1}^{-s} \overset{FP}{=} 1-s \tag{24}$$

As long as $s > 1$, the fixed point, given by equation (23), increases with s, $\frac{\partial \overline{e}}{\partial s} = \frac{\overline{e}}{s\,(s-1)} > 0$, but looses stability at $s = 2$. Eq. (24) shows that the derivative of the first return map is negative for all values of the employment rate; therefore, the time path diverges for $s > 2$. Deregulating the commodity market increases the stationary employment but may eventually lead to instability.

Figure 3 illustrates the period 2 cycle occurring precisely at $s = 2$. The employment rate alternates between e_1 and e_2; the anticipated demand and anticipated marginal revenue functions shift accordingly. Starting with the anticipated demand function 1 and the corresponding anticipated marginal revenue function 1, the bargaining process results in the higher employment rate e_1. The realised market price is below the anticipated one. The anticipated demand and marginal revenue curves shift downward to the position 2. The next bargaining results in the lower employment rate e_2. The market price is now above the anticipated one inducing an upward shift of the anticipated demand and marginal revenue back to their respective position 1.

3.3. The nominal reservation wage as a function of the employment rate

Next we introduce a positive dependence on the employment rate of the *nominal* reservation wage, $W_R^t = W_R(e_{t-1})$: A higher employment rate is considered as increasing the expected employment probabilities outside the firm under consideration and thus as increasing the reservation wage. As a consequence, the adjustment of the nominal reservation wage affects the commodity market dynamics. The resulting dynamic system is still one-dimensional:

$$e_t = \left(\frac{Y}{L}\right)^{s} e_{t-1}^{1-s} \left(\frac{s}{s-1}\right)^{-s} \left(W_R(e_{t-1})\right)^{-s} \tag{25}$$

The fixed point is

$$\bar{e} = \left(\frac{Y}{L}\right)\left(\frac{s}{s-1}\right)^{-s} \left(W_R(\bar{e})\right)^{-1} \tag{26}$$

Note that rising s still increases the equilibrium employment rate

$$\frac{\partial \bar{e}}{\partial s} = \frac{\bar{e}}{s(s-1)}\left(1 + \frac{\bar{e}}{\bar{W}_R}\frac{\partial \bar{W}_R}{\partial \bar{e}}\right) > 0$$

The first derivative of (25) is

$$\begin{aligned}\frac{\partial e_t}{\partial e_{t-1}} &= \left(\frac{Y}{L}\right)^{s} \left(\frac{s}{s-1}\right)^{-s} \left(W_R^t\right)^{-s} (1-s) e_{t-1}^{-s} + \left(\frac{Y}{L}\right)^{s} e_{t-1}^{1-s} \left(\frac{s}{s-1}\right)^{-s} (-s)\left(W_R^t\right)^{-s-1} \frac{\partial W_R}{\partial e_{t-1}} \\ &\overset{FP}{=} 1 - s - s\,\frac{\bar{e}}{W_R(\bar{e})}\frac{\partial W_R}{\partial e_{t-1}}\end{aligned} \tag{27}$$

The fixed point now looses stability at $s < 2$; beyond that value the time path diverges. Therefore, a positive influence of the employment rate on the nominal reservation wage destabilises the economy.[3]

---

[3] Blanchard and Giavazzi consider the *real* reservation wage to be an implicit function of the employment rate. However, it is not possible to assess the stability properties of their

Looking at Figure 3, this destabilising effect can be illustrated. Start from the market result 1 and consider the determination of position 2. The anticipated demand function and the anticipated marginal revenue shift again towards their position 2; but now, in addition, the nominal reservation wage is increased because of the high realised employment rate e_1. Therefore, the new employment rate 2 will be below e_2 in the figure. The increase in the nominal reservation wage thus increases the fluctuations. The process is destabilised.

3.4. The nominal reservation wage as a function of the employment rate and of its lagged value

Finally we allow also for a positive influence of the lagged reservation wage upon its current value:

$$W_R^t = W_R\left(e_{t-1}, W_R^{t-1}\right) \tag{28}$$

with positive partial derivatives; that is, the function $W_R(\bullet)$ is monotonically increasing in both arguments. The rationale for the dependence of the nominal reservation wage on the employment rate is as sketched above; for the second one it runs along the following lines: a high reservation wage in t-1 will result in a high bargained nominal wage rate in t-1 which in turn is expected to raise the nominal reservation wage in t.

The central dynamic system is now two-dimensional and given by equations (21) and (28).

---

model without specifying a relationship between the reservation wage and the employment rate. In what follows we put forward such a specification.

In the fixed point the following holds

$$\overline{W}_R = q\left(\overline{e}, \overline{W}_R\right) \tag{29}$$

$$\overline{e} = \left(\frac{Y}{L}\right)\left(\frac{s}{s-1}\right)^{-1}\left(\overline{W}_R\right)^{-1} \tag{30}$$

The Jacobian evaluated at the fixed point is given by

$$J_E = \begin{pmatrix} \dfrac{\partial W_R^t}{\partial W_R^{t-1}} & \dfrac{\partial W_R^t}{\partial e_{t-1}} \\ -s\,\dfrac{\overline{e}}{\overline{W}_R}\dfrac{\partial W_R^t}{\partial W_R^{t-1}} & 1-s-s\,\dfrac{\overline{e}}{\overline{W}_R}\dfrac{\partial W_R^t}{\partial e_{t-1}} \end{pmatrix} \tag{31}$$

The trace and the determinant are

$$\mathrm{tr}\, J_E = 1-s-s\,\frac{\overline{e}}{\overline{W}_R}\frac{\partial W_R^t}{\partial e_{t-1}} + \frac{\partial W_R^t}{\partial W_R^{t-1}} \tag{32}$$

$$\det J_E = (1-s)\frac{\partial W_R^t}{\partial W_R^{t-1}} \tag{33}$$

Figure 3 can be used to illustrate that the additional effect introduced by equation (28) is potentially stabilising. Start again with the market position realised in period 1 and consider the determination of the position 2. The anticipated demand function and the anticipated marginal revenue shift to their respective position 2. The nominal reservation wage will be changed by two factors. As in the previous case, the high employment rate e_1 tends to increase it. At the same time, the high employment rate e_1 implies that the bargained nominal wage in period 1 was comparatively low. This would reduce the nominal reservation wage, thus introducing a stabilising element.

Without specifying explicitly the dynamic adjustment process for the nominal reservation wage, it is difficult to assess the stability properties. We provide such a specification in the following section and explore the dynamics of the full model.

4. Fully specified dynamics and numerical simulations

4.1 . The dynamic adjustment process for the nominal reservation wage

The reservation wage in the bargaining process represents the income expected by the trade union for members who do not find employment in the firm under consideration. It therefore depends on the expected probability of finding employment in other firms, on the wage rate expected to be paid by other firms and on the unemployment benefit B_t. Consonant with the monopolistic competition set up, we assume that the trade unions do not take into consideration reactions of other firms and the impact of their own decisions on the aggregate variables. The expected probability of finding employment in other firms and the expected wage rate outside the firm under consideration is therefore given by the respective values realised in the previous period.

$$W_R^t = e_{t-1} W_{t-1} + (1 - e_{t-1}) B_t \tag{34}$$

For the unemployment benefit we consider two different specifications (see e.g. Layard et al., 1991, and Pissarides, 1998) mirroring two possible institutional set-ups. The first is close to a social assistance scheme according to which the compensation for the unemployed is fixed in real terms and the nominal payment is adjusted each period to the realised price, $B_t = w\,\hat{p}_{t-1}$. In the second, the compensation for the unemployed worker corresponds to a fraction of the nominal wage rate she was receiving in the previous period, $B_t = f\,W_{t-1}$, where $0 < f < 1$ is the replacement ratio. Therefore, taking into consideration the symmetry assumption the reservation wage is given by

$$W_R^t = W_R\left(e_{t-1},\, W_{t-1},\, \hat{p}_{t-1}\right) \tag{35}$$

Note that, using equ. (15) and equ. (18), this equation can be transformed into equ. (28).

Combing equations (15) and (34), results in

$$W_R^t = e_{t-1}\left(\frac{s+b-1}{s-1}\right)W_R^{t-1} + \left(1-e_{t-1}\right)B_t \tag{36}$$

At this stage we have to introduce explicitly the two alternative specifications of the unemployment benefit mentioned above. Such specifications will allow us to clarify the role of changes in parameters, which relate to the unemployment benefit, as measures of (de)regulation in the labour market.

4.2 Unemployment benefit fixed in real terms

We consider first the institutional set up for the unemployment benefit close to a social assistance system. The real unemployment benefit consists of a commodity bundle w, its nominal counterpart is given by $w\,\hat{p}_{t-1}$. The dynamic system is

$$W_R^t = e_{t-1}\left(\frac{s-1+b}{s-1}\right)W_R^{t-1} + \left(1-e_{t-1}\right)\frac{Y}{Le_{t-1}}w \tag{37}$$

$$e_t = \left(\frac{Y}{L}\right)^{s} e_{t-1}^{1-s} \left\{\left(\frac{s}{s-1}\right) W_R^t\right\}^{-s} \tag{38}$$

A crucial feature of this specification is that the realised market position impacts on the dynamics both through the determination of the nominal reservation wage, as shown in equation (37), and through shifts of the demand function, implicit in equation (38).

The fixed point, the partial derivatives, the trace and determinant are given by

$$\bar{e} = \frac{s-1-sw}{s-1+b-sw} \quad \text{and} \quad \bar{W}_R = \frac{Y}{\bar{e}L}\frac{s-1}{s} \tag{39}$$

$$\frac{\partial W_R^t}{\partial W_R^{t-1}}^{FP} = \bar{e}\frac{s-1+b}{s-1} > 0 \qquad \text{and} \qquad \frac{\partial W_R^t}{\partial e_{t-1}}^{FP} = \frac{s-1+b}{s-1}\bar{W}_R - \frac{Y}{L}w\frac{1}{\bar{e}^2} \tag{40}$$

$$\text{tr}\, J_E = 1 - s + \frac{s}{s-1}sw - \left(s-1+b\right)\bar{e} \qquad \det J_E = -\left(s-1+b\right)\bar{e} \tag{41}$$

Note that the fixed point solutions and the stability properties only depend upon three parameters: the parameter reflecting the degree of monopoly power in the commodity markets, s or *m*; and the two parameters related with the extent of labour market regulation, namely with the bargaining power of trade unions b and with the parameter concerning the specification of the real unemployment benefit *w*. These parameters are subject to the following boundaries: $0 < b < 1$, $0 < w < \bar{w}_R$ and $1 < s$, where $\bar{w}_R = \frac{s-1}{s}$ represents the stationary real reservation wage. Choosing an upper limit for s therefore allows to study the entire parameter space numerically. Later on, we assume $s \leq 5$ (or $m \geq 0.25$).

Appendix 1 investigates analytically the properties of the dynamic system (37)-(38). It is shown that the fixed point exhibits the usual comparative static results: Commodity and labour market deregulation – as reflected in a higher value of s and in lower values of b and w, respectively – engender a higher employment rate. The analysis further shows that the fixed point is stable for low values of s and for high values of b and w. However, in contrast to the one-dimensional models studied previously, the stability loss now occurs through a Flip bifurcation giving rise to attracting cyclical and eventually to chaotic fluctuations. Therefore, commodity and labour market deregulation – though expected to increase the stationary employment rate – may destabilize the economy, but does not lead to diverging time paths.

Figures 4 to 6 collect several bifurcation diagrams illustrating the possible complex dynamics; on the bottom of each figure, the average employment rate is compared with its stationary couterpart. Figure 4 explores the implications of varying b: On the fixed points of higher order the average employment rate still increases with a reduction in b. However, it is considerably lower than the equilibrium employment rate and even on a period two cycle both the upper and the lower value may lie well below the equilibrium value.

Figure 5 shows the dynamic effects of changing w. In this case – in stark contrast to the fixed point result – average employment declines on the higher period fixed points with a reduction in w.

Deregulating the labour market – that is, reducing b and/or w –not only endangers stability but may even reduce average employment.

Figure 6 investigates the implications of commodity market deregulation, as reflected in a increase of s. Also in this case, deregulation increases the stationary employment rates, but may eventually lead to instability. On the higher order fixed points average employment rates may even decline with higher values of s.

So far we have explored analytically the dynamics properties of the system (37)-(38) and illustrated the results by simulations. We now turn to a calibration exercise: we search for values of b, w and s such that the fixed point approximates some macroeconomic facts and we ask which dynamic properties the time path exhibits close to that fixed point. We use 1990s continental Europe economic evidence for the employment rate (which typically assumes values above 0.8) and the wage share (which typically assumes values slightly below 0.6)[4]. From the calibration exercise we find that there are parameter constellations resulting in plausible fixed points which exhibit stability, which loose stability with commodity or labour market deregulation giving rise to cyclical time paths with relatively small fluctuations. An example are the time paths for the employment rate $\left(e_{t+1}\right)$ and the wage share $\left(\frac{W_{t+1}e_{t+1}L}{Y}\right)$ plotted in Figure 7 in which $w = 0.506$, $s = 2.15$ and the values of b correspond to stable fixed points ($b = 0.02$ and $b = 0.015$) and to an unstable one ($b = 0.01$). Below the bifurcation value of the parameter referring to the bargaining power of workers, $b_{bif} \cong 0.016$, the time path loses stability and the resulting time paths exhibit

[4] Taking the average over a significant group of European Countries, Giammarioli et al. (2002) estimate that in the 1990s the labour share was 58.60 pct in the Business sector, 58.01 pct in Industry and 53.61 pct in the Tradable Services.

fluctuations with a comparatively small amplitude (smaller that 0.15 for the employment rate and smaller than 0.1 for the wage share).

4.3. The unemployment benefit as a fixed proportion of the nominal wage rate

We now consider another institutional set up according to which the unemployment benefit is a fixed replacement ratio f of the nominal wage rate earned in the lost job, $B_t = f W_{t-1}$. The system (36) and (21) can be written as

$$W_R^t = e_{t-1}\left(\frac{s-1+b}{s-1}\right)W_R^{t-1} + \left(1-e_{t-1}\right) f \left(\frac{s-1+b}{s-1}\right)W_R^{t-1} \tag{42}$$

$$e_t = \left(\frac{Y}{L}\right)^{s} e_{t-1}^{1-s} \left\{\left(\frac{s}{s-1}\right)W_R^t\right\}^{-s} \tag{43}$$

Note that the analytical structure of this specification is simpler than the previous one given that the realised market position only affects the dynamics through shifts of the demand function implicit in equation (43). The fixed point, the partial derivatives, the trace and the determinant are given by

$$\bar{e} = \frac{(s-1)(1-f)-fb}{(s-1+b)(1-f)} \quad \text{and} \quad \bar{W}_R = \frac{Y}{\bar{e}L}\frac{s-1}{s} \tag{44}$$

$$\left.\frac{\partial W_R^t}{\partial W_R^{t-1}}\right|^{FP} = 1 \qquad \text{and} \qquad \frac{\partial W_R^t}{\partial e_{t-1}} = \frac{s-1+b}{s-1}\bar{W}_R(1-f) > 0 \tag{45}$$

$$\operatorname{tr} J_E = 2(1-s) + \frac{s}{s-1}\left(s-1+b\right) f \qquad \det J_E = \left(1-s\right) \tag{46}$$

The major conclusions from the previous case carry over: As is shown in Appendix 2, deregulation in the labour or the commodity market increases the stationary employment rate, but may eventually lead to a Flip bifurcation of the fixed point giving rise to attracting period two cycles and eventually to complex time paths. Therefore, the basic trade off remains present: Deregulation increases employment rates but may destabilise the economy.

First computer simulations (not presented here) show that in the unstable region further deregulation usually increase average employment rates. The amplitude of the cyclical time path, however, is much higher than in the previous case, hitting quite often the boundary of one.

5. Conclusion

In the previous paper, we have analysed the dynamics of a model following closely the prototype specification put forward by Blanchard and Gavazzi combining monopolistic competition in the commodity market and efficient bargaining in the labour market. The dynamics results from two sources: First, inherent to monopolistic competition is that each single supplier overestimates the price-elasticity of his demand function; each single supplier is therefore necessarily surprised by the market outcome and will adapt his anticipated demand function accordingly. Second, efficient bargaining processes are based on a reservation wage indicating the expected income outside the firm under consideration. Also this anticipation is adjusted in the light of realised market results.

While the second process may be specified in various forms, the first one is directly implied by the external effect at the core of the monopolistic competition model itself. There are no alternative specifications without leaving the assumed market structure. In the paper it is shown, that the first necessary process may destabilise the economy: Deregulation does increase the stationary employment rate, but engenders diverging time paths. Introducing various plausible specifications for the adjustment process of the reservation wage dampens this sharp result: The stability loss occurs through a Flip bifurcation giving rise not to diverging time paths but to attracting period-two cycles and eventually to complex time paths. However, the basic trade-off remains: Deregulation increases the stationary employment but may lead to instability with time paths exhibiting in some cases even lower average employment rates.

References:

Blanchard, Olivier, Gavazzi, Francesco (2001): Macroeconomic effects of regulation and deregulation in goods and lybor market. NBER-Working Paper 8120.

Giammarioli, Nicola et al. (2002): European Labor Share Dynamics: An Institutional Perspective. European University Institute, Working Paper Eco 2002-13.

Kubin, Ingrid, Currie, Martin (2002): Complex Dynamics and Monopolistic Competition. Beiträge zur Wirtschaftsforschung, Nr. 02-01, Johannes Gutenberg-Universität Mainz, Fachbereich Rechts- und Wirtschaftswissenschaften.

Layard, Richard, Nickell, Stephen, Jackman, Richard (1991): Unemployment.

Pissarides, Christopher A. (2000): Equilibrium Unemployment Theory. 2nd edition. MIT Press.

Appendix 1

In this appendix we study the dynamic system related to the case in which the employment benefit is fixed in real terms and its nominal counterpart is adjusted to the price index, $B_t = wP_{t-1}$. The dynamic system is

$$W_R^t = \left(\frac{s-1+b}{s-1}\right) e_{t-1} W_R^{t-1} + \frac{1-e_{t-1}}{e_{t-1}} \frac{Y}{L} w \tag{A1.47}$$

$$e_t = \left(\frac{Y}{L}\right)^s e_{t-1}^{1-s} \left\{\left(\frac{s}{s-1}\right) W_R^t\right\}^{-s} \tag{A1.48}$$

A1.1 Fixed point and comparative statics

The stationary real reservation wage and the stationary real wage are

$$\overline{w}_R = \frac{s-1}{s} \qquad \overline{w} = \frac{s-1+b}{s} \tag{A1.49}$$

It follows, that

$$\overline{e} = \frac{\overline{w}_R - w}{\overline{w} - w} \tag{A1.50}$$

$$\overline{W}_R = \frac{Y}{\overline{e}L} \frac{s-1}{s} \tag{A1.51}$$

Note that if condition

$$w < \overline{w}_R < \overline{w} \tag{A1.52}$$

holds, then $0 < \overline{e} < 1$.

Note that the stationary real reservation wage depends only on s whereas the stationary real wage rate depends on b and s and the stationary employment rate depends on w, b and s. We have

$$\frac{\partial \bar{e}}{\partial s} = \frac{b}{s}\frac{\left[w(1-b)+s(\bar{w}-w)\right]}{\bar{w}\left[s(\bar{w}-w)\right]^2} > 0 \qquad \text{(A1.53)}$$

$$\frac{\partial \bar{W}_R}{\partial s} = \frac{Y}{L(\bar{e}s)^2}\left(\bar{e} - s(s-1)\frac{\partial \bar{e}}{\partial s}\right) \qquad \frac{\partial \bar{w}}{\partial s} = \frac{1-b}{s^2} > 0 \qquad \frac{\partial \bar{w}_R}{\partial s} = \frac{1}{s^2} > 0 \quad \text{(A1.54)}$$

$$\text{with } \frac{\partial \bar{W}_R}{\partial s} \geq (<0) \quad \textit{for} \quad \frac{\partial \bar{e}}{\partial s} \leq (>) \frac{\bar{e}}{s(s-1)}$$

The impact of changes of b is given by

$$\frac{\partial \bar{e}}{\partial b} = -\frac{\bar{w}_R - w}{s(\bar{w}-w)^2} < 0 \qquad \text{(A1.55)}$$

$$\frac{\partial \bar{W}_R}{\partial b} = -\left(\frac{s-1}{s}\right)\frac{Y}{L\bar{e}^2}\frac{\partial \bar{e}}{\partial b} \qquad \frac{\partial \bar{w}}{\partial b} = \frac{1}{s} > 0 \qquad \text{(A1.56)}$$

The impact of changes of w is given by

$$\frac{\partial \bar{e}}{\partial w} = -\frac{b}{s}\frac{1}{(\bar{w}-w)^2} < 0 \qquad \text{(A1.57)}$$

$$\frac{\partial \bar{W}_R}{\partial w} = -\left(\frac{s-1}{s}\right)\frac{Y}{L\bar{e}^2}\frac{\partial \bar{e}}{\partial w} > 0 \qquad \text{(A1.58)}$$

A1.2 Bifurcation analysis

The Jacobian evaluated at the fixed point is

$$J_E = \begin{bmatrix} \left(\dfrac{s+b-1}{s-1}\right)\bar{e} & \left(\dfrac{s+b-1}{s-1}\right)\bar{W}_R - w\dfrac{Y}{L\bar{e}^2} \\ -\dfrac{s\,\bar{e}^2}{\bar{W}_R}\left(\dfrac{s+b-1}{s-1}\right) & 1-s-s\left(\dfrac{s+b-1}{s-1}\bar{e}-w\dfrac{Y}{L\bar{e}\bar{W}_R}\right) \end{bmatrix}$$

Determinant and Trace are:

$$\det J_E = -\left(s+b-1\right)\bar{e} \tag{A1.59}$$

$$\text{tr}\,J_E = 1-\left[(s+b-1)\bar{e}+s\left(\frac{\bar{w}_R-w}{\bar{w}_R}\right)\right] \tag{A1.60}$$

with $-\infty < \det J_E < 0$ and $-\infty < \text{tr}\,J_E < 1$.

The violation of one of the following conditions would involve instability for the system (in brackets the type of bifurcation involved):

(i) $1-\det J_E = 1+\left(s-1+b\right)\bar{e} > 0$ (Hopf bifurcation);

(ii) $1-\text{tr}\,J_E+\det J_E = \left(\dfrac{\bar{w}_R-w}{\bar{w}_R}\right)s > 0$ (saddle node bifurcation);

(iii) $1+\text{tr}\,J_E+\det J_E = 2\left[1-\left(s-1+b\right)\bar{e}\right]-\left(\dfrac{\bar{w}_R-w}{\bar{w}_R}\right) > 0$ (flip bifurcation).

Conditions (i) and (ii) always hold as long as $s > 1$ and $\overline{w}_R > w$. The flip bifurcation is the only possible. Condition (iii) corresponds to

$$\frac{\left[(3s-2)w+(4-3s)\overline{w}_R\right] b\overline{w}_R-(s-1)(\overline{w}_R-w)\left[(3s-4)\overline{w}_R-sw\right]}{\left[(s-1)(\overline{w}_R-w)+b\,\overline{w}_R\right]\overline{w}_R}>0 \qquad \text{(A1.61)}$$

In this condition for stability, the denominator is always positive. Therefore, the system is stable for

$$\left[(3s-2)w+(4-3s)\overline{w}_R\right] b\,\overline{w}_R>(s-1)(\overline{w}_R-w)\left[(3s-4)\overline{w}_R-sw\right]$$

We may distinguish three cases depending on s.

Case 1: $s<\frac{4}{3}$. The stability condition is satisfied for all other parameter values:

Case 2: $\frac{4}{3}<s<2$. The stability of the system depends upon the parameter values. There are three sub-cases that we have to take into account depending on w:

Case 2a: $w<\frac{3s-4}{3s-2}\overline{w}_R$. It follows $(3s-4)\overline{w}_R>(3s-2)w>2sw$. In this case, the stability condition is never satisfied.

Case 2b: $\frac{3s-4}{3s-2}\overline{w}_R<w<\frac{3s-4}{s}\overline{w}_R$. The system is stable for

$$b>b^{bif}=\frac{(s-1)(\overline{w}_R-w)[(3s-4)\overline{w}_R-sw]}{[(3s-2)w+(4-3s)\overline{w}_R]\overline{w}_R}>0 \qquad \text{(A1.62)}$$

If b falls below this value, the system looses stability through a Flip-bifurcation.

Case 2c: $\frac{3s-4}{3s-2}\overline{w}_R < \frac{3s-4}{s}\overline{w}_R < w < \overline{w}_R$. The system is stable for

$$b > 0 > b^{bif} = \frac{(s-1)(\overline{w}_R - w)[(3s-4)\overline{w}_R - sw]}{[(3s-2)w + (4-3s)\overline{w}_R]\overline{w}_R}$$

That is, the system is always stable.

Case 3: $2 < s$. As for Case 2, the stability of the system depends upon the parameter values. Depending on w we may identify two sub-cases:

Case 3a: $w < \frac{3s-4}{3s-2}\overline{w}_R$. As in case 2a, the stability condition is never satisfied.

Case 3b: $\frac{3s-4}{3s-2}\overline{w}_R < w < \overline{w}_R < \frac{3s-4}{s}\overline{w}_R$. As in case 2b, the system is stable for

$$b > b^{bif} = \frac{(s-1)(\overline{w}_R - w)[(3s-4)\overline{w}_R - sw]}{[(3s-2)w + (4-3s)\overline{w}_R]\overline{w}_R} > 0 \qquad \text{(A1.63)}$$

Appendix 2

In this appendix we examine some of the properties of the dynamic system related to the case in which the employment benefit is a proportion of the nominal wage rate earned in the lost job, $B_t = f W_{t-1}$. The dynamic system is

$$W_R^t = \left(\frac{s-1+b}{s-1}\right)[1-(1-f)(1-e_{t-1})]W_R^{t-1} \qquad \text{(A2.64)}$$

$$e_t = \left(\frac{Y}{L}\right)^s e_{t-1}^{1-s} \left\{\left(\frac{s}{s-1}\right) W_R^t\right\}^{-s} \quad \text{(A2.65)}$$

A2.1 Fixed point and comparative statics

From (42) and (43), we solve for the stationary employment rate and reservation wage

$$\bar{e} = \frac{(s-1)(1-f)-fb}{(s-1+b)(1-f)} \quad \text{(A2.66)}$$

$$\bar{W}_R = \frac{s-1}{s}\frac{Y}{\bar{e}L} \quad \text{(A2.67)}$$

Note that if condition

$$(s-1)(1-f)-fb>0 \quad \text{(A2.68)}$$

holds and $b > 0$, then $0<\bar{e}<1$.

The stationary price is $\bar{p} = \frac{Y}{\bar{e}L}$ and the stationary nominal wage is $\bar{W} = \frac{s-1+b}{s-1}\bar{W}_R$. It follows that the stationary real wage and the stationary real reservation wage are:

$$\bar{w} = \frac{s-1+b}{s} \qquad \bar{w}_R = \frac{s-1}{s} \quad \text{(A2.69)}$$

The stationary real wage rate depends on b and s and the stationary employment rate depends on f, b and s. This is a consequence of our assumption according to which the reservation wage depends on the unemployment rate, on the nominal wage rate and the unemployment benefit. The comparative statics of the stationary state involves:

$$\frac{\partial \bar{e}}{\partial s} = \frac{b}{(s-1+b)^2(1-f)} > 0 \tag{A2.70}$$

$$\frac{\partial \bar{W}_R}{\partial s} = \frac{1}{(s\,\bar{e})^2}\frac{Y}{L}\left(e - s\,(s-1)\frac{\partial \bar{e}}{\partial s}\right) \qquad \frac{\partial \bar{w}}{\partial s} = \frac{1-b}{s^2} > 0 \qquad \frac{\partial \bar{w}_R}{\partial s} = \frac{1}{s^2} > 0 \tag{A2.71}$$

$$\text{with} \quad \frac{\partial \bar{W}_R}{\partial s} \geq (<) 0 \quad \textit{for} \quad \frac{\partial \bar{e}}{\partial s} \leq (>) \frac{\bar{e}}{s\,(s-1)}$$

The impact of changes of b is given by

$$\frac{\partial \bar{e}}{\partial b} = -\frac{s-1}{(s-1+b)^2(1-f)} < 0 \tag{A2.72}$$

$$\frac{\partial \bar{W}_R}{\partial b} = -\left(\frac{s-1}{s}\right)\frac{Y}{L\bar{e}^2}\frac{\partial \bar{e}}{\partial b} > 0 \qquad \frac{\partial \bar{w}}{\partial b} = \frac{1}{s} > 0 \tag{A2.73}$$

The impact of changes of f is given by

$$\frac{\partial \bar{e}}{\partial f} = -\frac{b}{(s-1+b)(1-f)^2} < 0 \tag{A2.74}$$

$$\frac{\partial \bar{W}_R}{\partial f} = -\left(\frac{s-1}{s}\right)\frac{Y}{L\bar{e}^2}\frac{\partial \bar{e}}{\partial f} > 0 \tag{A2.75}$$

A2.2 Bifurcation analysis

The Jacobian evaluated at the fixed point is

$$J_E = \begin{bmatrix} 1 & \dfrac{s - 1 + b}{s - 1}(1-f)\overline{W}_R \\ -s\dfrac{\overline{e}}{\overline{W}_R} & 1 - s - s\left(\dfrac{s - 1 + b}{s - 1}\right)(1-f)\overline{e} \end{bmatrix}$$

Determinant and Trace are:

$$\det J_E = 1 - s \qquad \text{(A2.76)}$$

$$\operatorname{tr} J_E = 2(1 - s) + \frac{s}{s - 1}\left(s - 1 + b\right)f \qquad \text{(A2.77)}$$

$$\text{with} \quad -\infty < \det J_E < 0 .$$

The violation of one of the following conditions would involve instability for the system (in brackets the type of bifurcation involved):

(iv) $1 - \det J_E = s > 0$ (Hopf bifurcation);

(v) $1 - \operatorname{tr} J_E + \det J_E = \dfrac{s}{s - 1}\left[(s - 1)(1-f) - fb\right] > 0$ (saddle node bifurcation);

(vi) $1 + \operatorname{tr} J_E + \det J_E = 1 + 3(1 - s) + \dfrac{s}{s - 1}\left(s - 1 + b\right)f > 0$ (flip bifurcation).

Conditions (i) and (ii) always hold as long as condition (A2.68) holds. The flip bifurcation is the only possible. Condition (iii) corresponds to

$$(3-f)s^2 - \left[7 - (1-b)f\right]s + 4 < 0 \qquad \text{(A2.78)}$$

If condition (A2.78) is not satisfied, the system looses stability through a Flip-bifurcation.

Define that value of s corresponding to the highest root that satisfies condition (A2.78) with an equality sign as

$$z(f,b)=\frac{7-(1-b)f+\sqrt{1+(1-b)^2f^2+2(1+7b)f}}{2(3-f)} \tag{A2.79}$$

The system is stable for

$$f>f_{bif}\equiv\frac{3s-4}{s-1+b}\left(\frac{s-1}{s}\right)$$

$$b>b_{bif}\equiv\frac{(3-f)s-4}{f}\left(\frac{s-1}{s}\right)$$

$$s<s_{bif}$$

with $0<f_{bif}<1$, iff $z(0,b)<s<z(1,b)$, $0<b_{bif}<1$ iff $z(f,0)<s<z(f,1)$ and $s_{bif}=z(f,b)$ for $0<b<1$ and $0<f<1$.

Note finally that

$$z(0,b)=\frac{4}{3}<s_{bif}<\frac{7}{4}+\frac{1}{4}\sqrt{17}=z(1,1) \tag{A2.80}$$

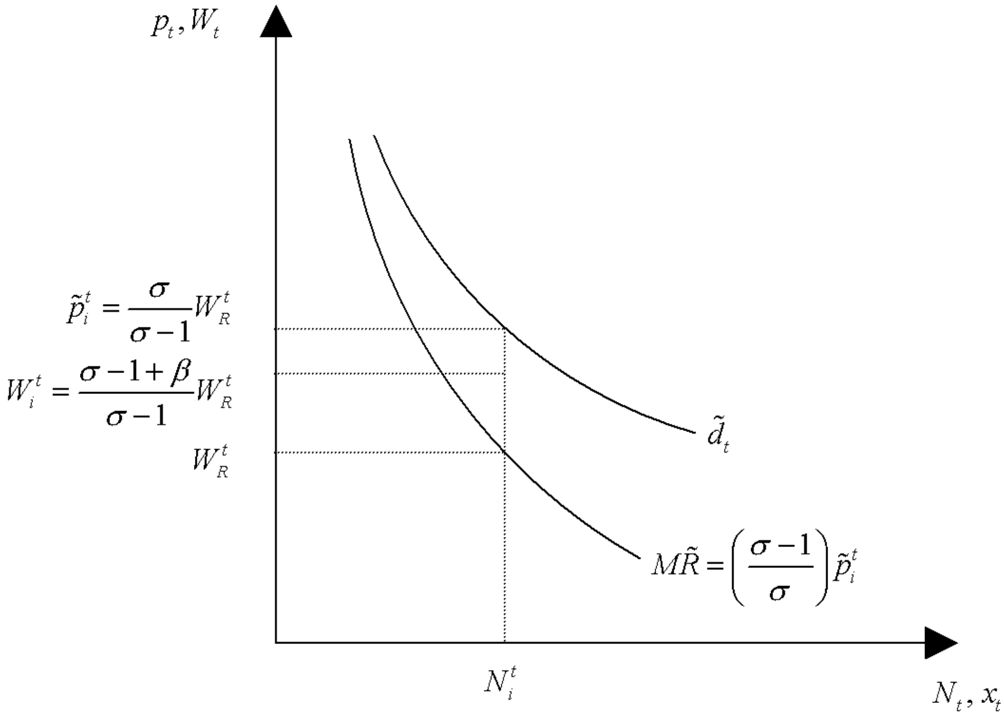

Figure 1: Bargaining outcome Week t

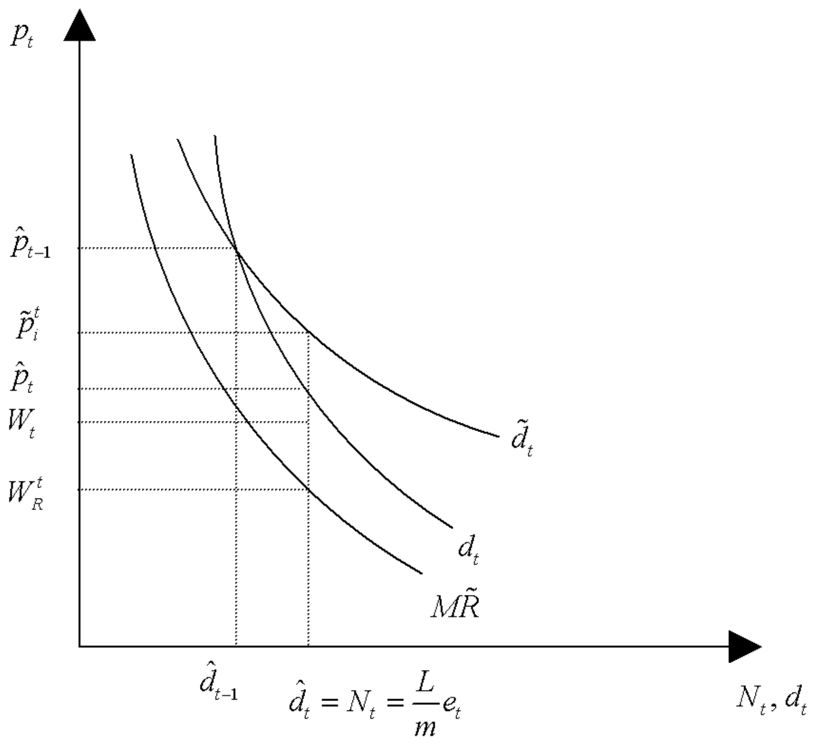

Figure 2: Short run general equilibrium Week t

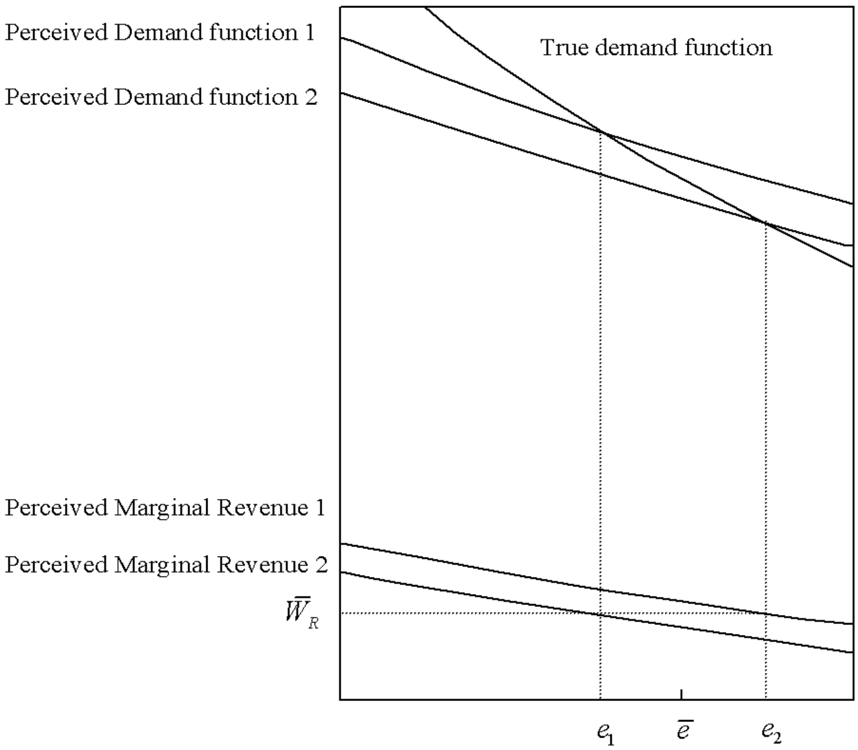

Figure 3: Period-two Cycle

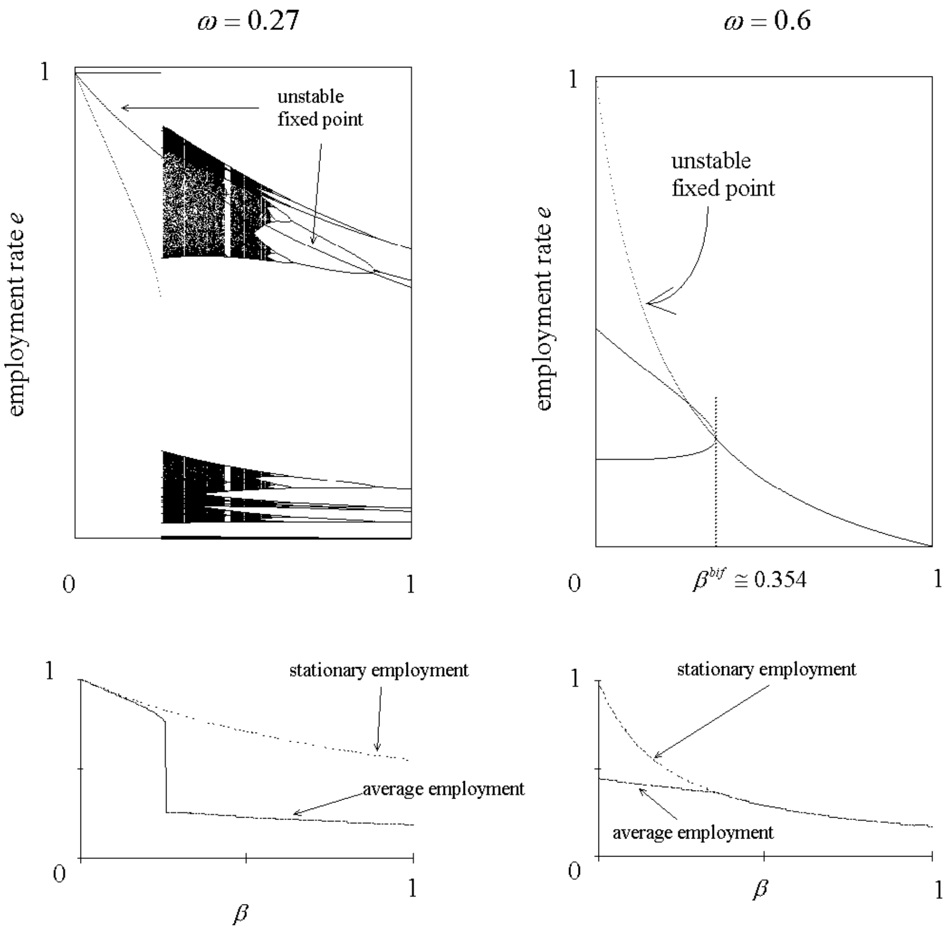

Figure 4: Bifurcation with respect to the bargaining power of trade unions

$\sigma = 3,\ m = 100,\ Y = 1000,\ L = 100$

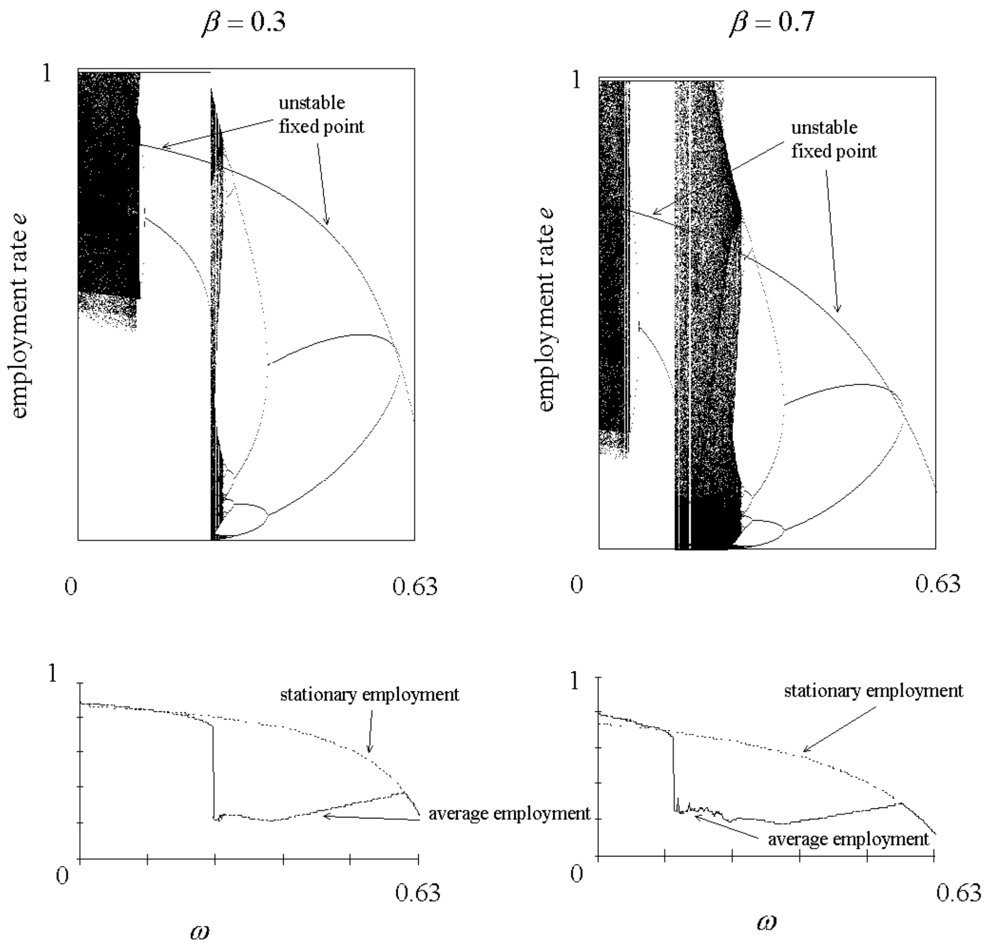

Figure 5: Bifurcation with respect to the real unemployment benefit

$\sigma = 3,\ m = 100,\ Y = 1000,\ L = 100$

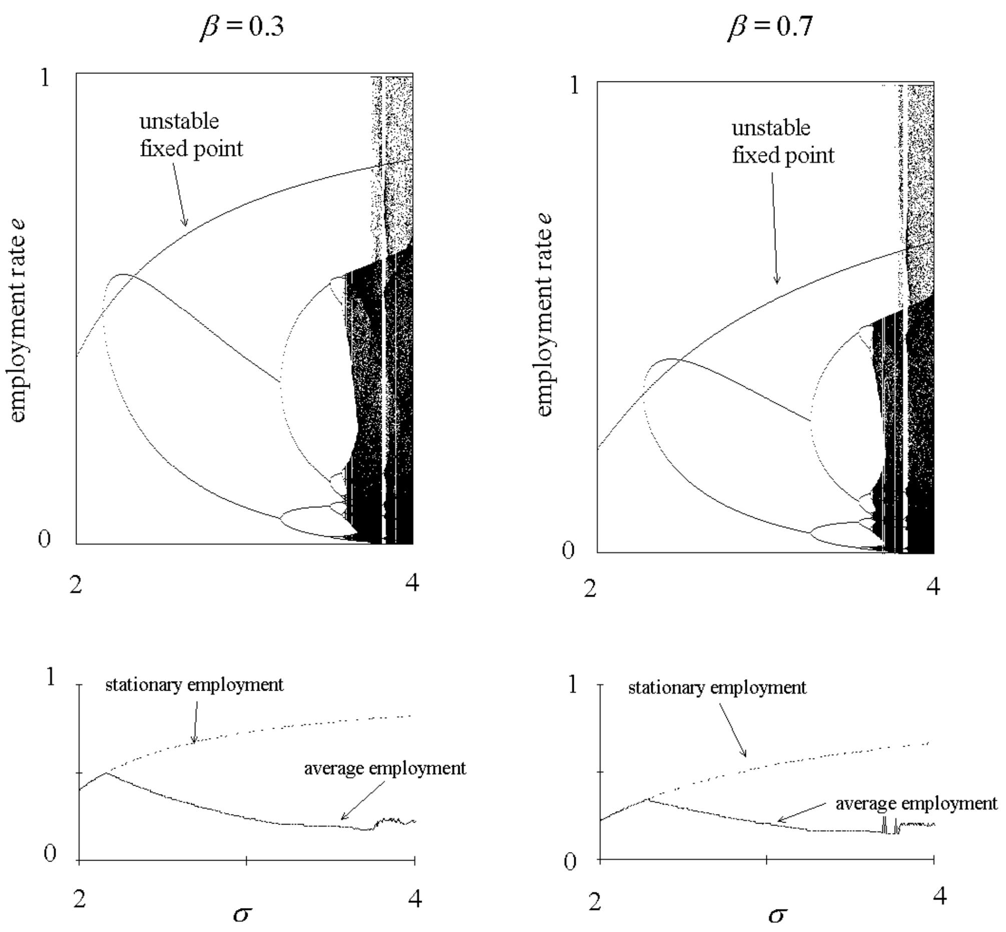

Figure 6: Bifurcation with respect to the mark-up

$\omega = 0.4, \ m = 100, \ Y = 1000, \ L = 100$

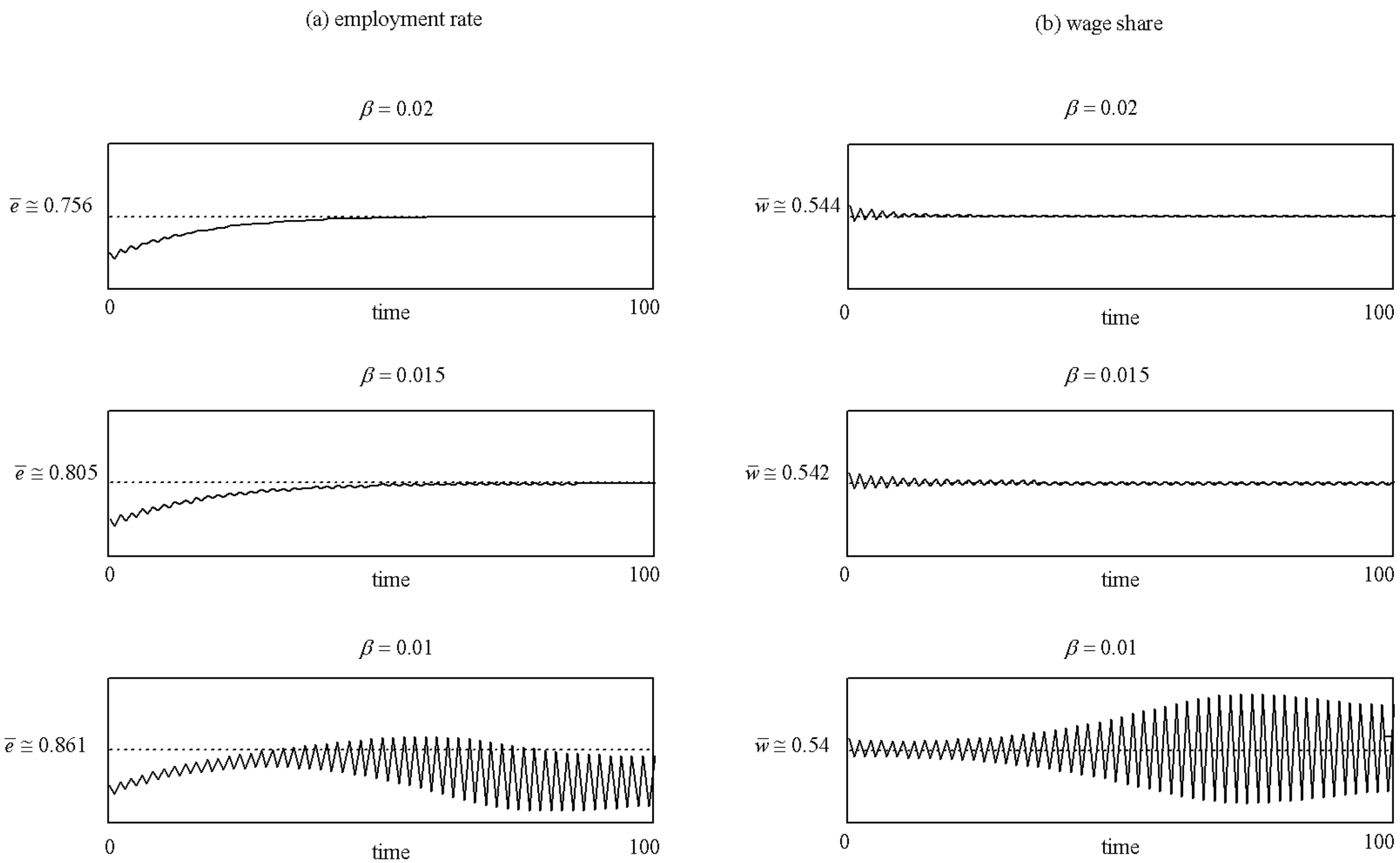
(a) employment rate
(b) wage share
$\beta = 0.02$
$\bar{e} \cong 0.756$
$\bar{w} \cong 0.544$
$\beta = 0.015$
$\bar{e} \cong 0.805$
$\bar{w} \cong 0.542$
$\beta = 0.01$
$\bar{e} \cong 0.861$
$\bar{w} \cong 0.54$
0
time
100

Figure 7