A Small Open Economy Model with Currency Mismatches and a Financial Accelerator Mechanism

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Abstract

We develop a two-sectors small open economy model with imperfect competition, one-period nominal price rigidities and a financial accelerator mechanism. The latter assumes an asymmetric information problem between lenders and capital good producers (entrepreneurs). Studying the zero-inflation steady state, it is shown that the model with the financial accelerator mechanism nests a fairly standard RBC model; case in which entrepreneurs "disappear" as a differentiated sector from households. It is also explained that credit market imperfections essentially reduce the aggregate supply of capital relative to the RBC case. Turning to the dynamics, we study the effects of an unanticipated and permanent increase in the level of the money supply. In this context the exchange rate jumps immediately to its new steady state level without showing any overshooting process as in Dornbusch (1976). Analysing the case without credit market imperfections but with pre-set prices, it is demonstrated that money is not neutral in the long-run, that capital adds persistence to the initial shock, and that some traditional results of the Mundell-Fleming model still hold.

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1 Introduction

The classical Mundell-Fleming model shows that a currency depreciation has expansionary effects on output through expenditure-switching effects. This finding is also obtained in the well-known model developed by Obstfeld and Rogoff (1995). Although there is a long-standing debate on whether this effect holds for developing countries or not (see Agénor and Montiel 1999 for a survey), the recent experiences of contractionary depreciations have revitilised the discussion. To illustrate, Table 1 shows the negative association that existed between currency depreciations and real GDP growth rate for a number of selected countries. It is also noteworthy that the CPI inflation rate has been in all these cases below the WPI inflation rate[1].

Table 1. Selected macroeconomic indicators

Country	Year	Nominal depreciation	CPI inflation	WPI inflation	Real GDP growth rate
		(Dec-y_t/Dec-y_{t-1}) -in %-			
Argentina	2002	67.4	25.9	78.3	-10.9
Indonesia	1998	70.9	57.6	80.4	-13.1
Korea	1998	32.1	7.5	12.2	-6.9
Malaysia	1998	28.3	5.3	10.8	-7.4
Philippines	1998	27.9	9.7	11.7	-0.6
Thailand	1998	24.2	8.1	12.2	-10.5

Source: International Financial Statistics, IMF.

Calvo and Reinhart (1999) have pointed out that financial factors can be critical to understand contractionary depreciations. When liabilities are denominated in the foreign currency while assets are denominated in the domestic currency, the argument goes, an exchange rate depreciation increases the domestic value of liabilities. If the domestic value of assets does not increase *pari passu* with the exchange rate, indebted agents face negative net worth effects. This explanation is thus a reinterpretation of the debt-deflation mechanism stressed by Fisher (1933), but in the context of small open economies. Krugman (1999) has firstly formalised this argument in a highly stylised and static model. He shows that a combination of currency mismatches in the private sector, imperfections in credit markets and sudden changes in expectations could have explained what happened in the 1997-8 South East Asian crisis. To give an insight on the importance of private sector's foreign currency denominated liabilities, Figure 1 shows the total claims of foreign banks on the non-bank private sector for the same countries analysed in Table 1.

[1] This particular feature in the evolution of inflation rates is discussed later in this section.

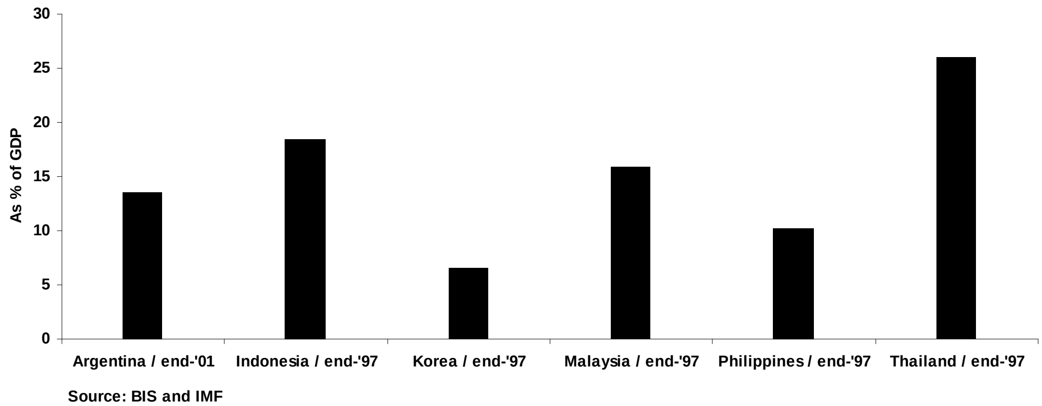

Figure 1. Claims of foreign banks on non-bank private sector

This rather imperfect measure of financial "dollarisation"[2],[3] puts forward the idea that foreign currency denominated debt has been an important element in the forefront of the currency depreciations in these countries.

Aghion et al (2000, 2001) follow the same line of reasoning as Krugman (1999), but provide a higher degree of formalisation. In particular, they assume the production of a single tradable good that faces one-period nominal price rigidities, in a context where the private sector has liabilities denominated in the foreign currency. An exchange rate depreciation thus generates negative net worth effects that reduce investment and output (i.e., balance-sheet effects).

Besides the lack of microfoundations present in their approach[4], the assumption that the tradable good sector faces nominal price rigidities seems to be an important drawback of their model. In this regard, Burstein et al (2005) show, analysing 5 recent episodes of large devaluations[5], that the main source of changes in the real exchange rate has been the slow adjustment in the prices of nontradable goods. This can provide an explanation of the relatively lower increase in the CPI inflation rate (since this index is highly influenced by nontradable goods) *vis a vis* the WPI inflation rate observed in Table 1.

The objective of this paper is thus to provide a rigorous though realistic framework in which to analyse why currency depreciations can be contractionary in the short-run. With

[2]Financial dollarisation is a widely used expression to indicate that the liabilities of certain sectors in a country are denominated in the foreign currency. Notice, however, that this foreign currency is not necessarily the US dollar.

[3]In particular, owing to lack of information we are not able to discriminate the currency of denomination of these liabilities. However, being the creditors foreign banks, it seems to be very likely that these loans were denominated in foreign currencies. Notice also that we are explicitly excluding currency mismatches in the public sector, which has been a critical feature specially in the Argentine crisis in 2001/2.

[4]For example, they directly postulate the existence and the form of credit constraints without deriving it from primitive assumptions.

[5]They consider the cases of Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994) and Thailand (1997).

this aim, we develop a well microfounded dynamic monetary general equilibrium model for a small open economy that considers a tradable and nontradable sector, imperfections in credit and goods markets, currency mismatches and one-period nominal price rigidities in the intermediate nontradable good.

Following Obstfeld and Rogoff (1996, Ch. 10.2) the output of the tradable sector will be assumed exogenous. Since we want to concentrate our attention in the nontradable sector, this assumption highly simplifies the analysis without affecting the main objectives of the paper. The nontradable sector is composed of a final producer firm, which is perfectly competitive, and a continuous number of intermediate firms that face monopolistic competition as in Blanchard and Kiyotaki (1987).

The production of capital is modelled as in Carlstrom and Fuerst (1997). This capital is afterwards utilised by intermediate firms. There is a continuum of entrepreneurs, each one producing capital with only one input, which is part of the final nontradable good. The production function of each entrepreneur has an idiosyncratic and stochastic element. To determine the amount of investment placed in production, entrepreneurs utilise their net worth in conjunction with external funding. This funding is, however, subject to frictions due to the presence of an asymmetric information problem between lenders and entrepreneurs. All the borrowing that entrepreneurs obtain is assumed to be denominated in the foreign currency.

Cespedes et al (2004) develop a similar model but with only one sector of production (tradable) and sticky wages. In the present paper we consider two sectors and fully flexible wages. Choi and Cook (2004), Cook (2004) and Devereux et al (2006) are probably the closest references. Essentially, all these models build on variants of the financial accelerator mechanism developed in Carlstrom and Fuerst (1997) and Bernanke et al (1999). However, besides some important differences in the specification of the models, they are only interested in numerical solutions to evaluate different exchange rate and monetary policies. Hence, there are relevant results and interactions that are hidden in the "black box" typically associated with calibration methods. In contrast, the present paper's objective is to work through the analytics of the model so as to provide, whenever possible, an analytical solution that highlights in a transparent way the mechanisms by which monetary and exchange rate policies affect the economy.

Although this paper is still work in progress, there are a number of intermediate results worth emphasizing. We firstly studied the properties of the model in the zero-inflation steady state. We show that those variables associated with the financial accelerator mechanism yield simple steady state solutions; depending only on the subjective discount factor, monitoring costs and the fraction of expected profits that entrepreneurs devote to consumption. Comparing the cases with and without credit market imperfections we show that the latter converges to a fairly standard RBC model in which entrepreneurs "disappear" as a differentiated class from households. It is also shown that credit market imperfections essentially reduce the supply of capital relative to the RBC case.

Turning to the dynamics, we study an unexpected and permanent increase in the level of the money supply under a floating exchange rate regime. It is shown that the nominal exchange rate immediately jumps to its new steady state level, therefore not showing any overshooting process as in Dornbusch (1976). As Fender and Rankin (2003) point out, this particular feature is a direct consequence of the household's logarithmic preferences assumed in the model. Without credit market imperfections and zero initial net foreign assets the monetary expansion with pre-set prices improves the short-run trade balance surplus, giving place to an accumulation of net-foreign assets. Owing to this effect money is not neutral in the long run. It is thus possible to show that the final nontradable output is positively affected in the short- and long-run. It is also explained that the long-run neutrality of money is recovered eliminating capital from the model.

The remainder of the paper is organised as follows. Section 2 develops the main elements of the model with the exception of the production of the capital good. Section 3 explains how capital is produced in the economy and develops the financial accelerator mechanism. Section 4 deals with aggregation and defines the equilibrium conditions of the model. Section 5 analyses the steady state. Section 6 deals with the dynamics of the model. Section 7 presents concluding remarks.

2 The model

We consider a small open economy model with two sectors: one tradable and one nontradable. The economy is composed of firms, households, the government and entrepreneurs that mutually interact within a monetary framework. The remainder of this section describes in detail the characteristics of each sector.

2.1 Firms

2.1.1 Tradable sector

There is a single homogeneous tradable good whose supply is constant and exogenously given each period t and is denoted by $Y_{T,t} = \overline{Y}_T$[6]. This output, in turn, becomes each period household's endowment.

2.1.2 Nontradable sector

The nontradable sector is composed of a continuum of intermediate firms that produce differentiated inputs and a perfectly competitive producer of the nontradable final good. There are a large number of firms indexed by i in the intermediate sector, where each one specialises in producing a particular input. Each firm, therefore, has some degree of monopoly power over its production. The imperfect competition in the production of nontradable inputs combined with nominal price rigidities in setting their prices (as explained below), provide an economic framework in which to rationalise that output could be demand-determined in the short-run.

The intermediate output of firm i at period t is produced by combining capital and labour services with a Cobb-Douglas production function as follows,

$$Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha},\ i \in [0,1],\ 0 < \alpha \leq 1, \tag{1}$$

where $Z_{i,t}$ indicates the production of input i, A_t is a technology parameter assumed to be common to all firms, $K_{i,t}$ is the stock of capital rented to entrepreneurs at the beginning of period t, $L_{i,t}$ indicates labour services obtained from households and α is the share of capital in the nontradable intermediate input (which is assumed to be the same for all firms).

The producer of the final nontradable good combines the inputs provided by intermediate firms and a tradable input with a Cobb-Douglas-type production function. This output is afterwards sold to domestic agents for consumption or to entrepreneurs for using it as an input in the production of the capital good. The production function of a representative firm is defined as follows,

$$Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma},\ \theta > 1,\ 0 < \gamma < 1, \tag{2}$$

where Y_t is the final nontradable good, θ is the elasticity of substitution between different nontradable inputs, γ is the share of nontradable components in the final nontradable good and $X_{T,t}$ is the tradable input that is used in producing the final good. This production

[6] A similar assumption is taken, for instance, in Obstfeld and Rogoff (1996, Ch. 10.2).

function intends to capture the fact that there are differentiated nontradable inputs required to produce a final good, such as transport services, retailing, etc. Each intermediate firm in the nontradable sector, therefore, faces the following downward sloping demand curve[7],

$$Z_{i,t} = Y_t \gamma^{\theta} [\frac{(1-\gamma)}{P_{T,t}}]^{\frac{(1-\gamma)(\theta-1)}{\gamma}} (P_{i,t})^{-\theta} (P_t)^{\frac{\theta-1+\gamma}{\gamma}}, \tag{3}$$

where $P_{T,t}$, $P_{i,t}$ and P_t are the prices of the tradable good, the intermediate good and the final good, respectively. It is worth noting that the marginal cost of the final producer firm is defined as $MC_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}(P_{T,t})^{1-\gamma}[\int_0^1 P_{i,t}^{1-\theta} di]^{\frac{\gamma}{1-\theta}}$[8].

Let us define $P_{N,t} = [\int_0^1 P_{i,t}^{1-\theta} di]^{\frac{1}{1-\theta}} = [P_t \gamma^{\gamma}(1-\gamma)^{(1-\gamma)}(P_{T,t})^{\gamma-1}]^{\frac{1}{\gamma}}$. We can therefore rewrite the demand curve that each intermediate firm faces as,

$$Z_{i,t} = Y_t (1-\gamma)^{\frac{\gamma-1}{\gamma}} (\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}} (\frac{P_{i,t}}{P_{N,t}})^{-\theta}. \tag{4}$$

It will be considered that the law of one price (LOOP) holds for tradable goods at all t, implying that,

$$P_{T,t} = S_t,$$

where S_t denotes the nominal exchange rate measured as the domestic price of foreign exchange. Note that the foreign price of the tradable good was normalised to one.

2.1.3 Demand for factors by intermediate firms

Intermediate firms determine their demand for factors by solving the following cost minimisation problem (taking the output level $Z_{i,t}$ as given):

$$\min_{\{K_{i,t}, L_{i,t}\}} R_t^k K_{i,t} + W_t L_{i,t} \text{ s.t. } Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \tag{5}$$

where R_t^k indicates the nominal rental price of capital and W_t denotes the nominal wage. It is worth highlighting that $K_{i,t}$ is a homogeneous capital good demanded by intermediate firms and supplied by a large number of entrepreneurs. This capital completely depreciates within the period. $L_{i,t}$, on the other hand, is a homogeneous type of labour demanded by intermediate firms and supplied by a large number of households. Since both inputs are homogeneous, supplied by a large number of agents and demanded by a large number of

[7]The final good producer solves the following cost minimisation problem:
$\min \int_0^1 Z_{i,t} P_{i,t} di + P_{T,t} X_{T,t}$ s.t. $Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma} \{X_{T,t}\}^{1-\gamma}$, giving the inverse demand function stated in Eq. 3.

[8]Notice that in equilibrium, the marginal cost of the final producer firm will be equal to the price of the final good, P_t.

firms, at the individual level each firm takes the nominal rental price of capital R_t^k and the nominal wage W_t as given.

The first order conditions associated with this problem are,

$$K_{i,t}^* = (\frac{1-\alpha}{\alpha})^{\alpha-1}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{1-\alpha} \tag{6}$$

and

$$L_{i,t}^* = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_{i,t}}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}. \tag{7}$$

Note that the cost function evaluated at $K_{i,t}^*$ and $L_{i,t}^*$ takes the form,

$$C_{i,t}(Z_{i,t}, R_t^k, W_t) = C_{i,t}^* = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}Z_{i,t}W_t^{1-\alpha}(R_t^k)^{\alpha}. \tag{8}$$

2.1.4 Profit maximisation problem of intermediate firms

Intermediate firms determine the price level $P_{i,t}$ and output $Z_{i,t}$ that maximise profits subject to the cost function obtained in Eq. 8 and the inverse demand function stated in Eq. 4,

$$\max_{\{P_{i,t}\}} \pi_{i,t} = P_{i,t}Z_{i,t} - C_{i,t}^* \text{ s.t. } Z_{i,t} = Y_t(1-\gamma)^{\frac{\gamma-1}{\gamma}}(\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta}.$$

The solution of this problem gives the following price setting equation,

$$P_{i,t} = \frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R_t^k)^{\alpha}, \tag{9}$$

where $\frac{\theta}{\theta-1}$ is a markup over marginal costs[9].

This equation defines how intermediate firms optimally set the price level of their output $P_{i,t}$. It is worth highlighting that firms decide the price level that will prevail at period t at the end of period $t-1$. To be more precise, we can think of Eq. 9 as implicitly given by the following expression,

$$P_{i,t} = E_{i,t-1}\{\frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R^k)^{\alpha}\},$$

where $E_{i,t-1}$ indicates the expectation hold by agent i at the end of period $t-1$ given the information available at that time. This model assumes "perfect foresight". Therefore, the above expression will be identical to Eq. 9 for all periods but $t = 0$, when an unexpected shock hits the economy. During that period, the price $P_{i,0}$ differs from what firm i would

[9]Note that in the perfectly competitive case, when $\theta \to \infty$, the price of the intermediate firm is equal to the marginal cost.

have optimally chosen had it known the shock in advance. It is in this context that we can consider that the price level of the intermediate firm i is "given" at period $t = 0$.

2.2 Households

The representative household obtains utility from consumption of the final good C_t, real money balances $\frac{M_t}{P_t}$[10] and leisure (given by the disutility associated with working in the production of the nontradable input $-\frac{\kappa}{2}(L_t)^2$). Therefore, lifetime utility of the representative agent takes the form,

$$U_t = \sum_{t=0}^{\infty} \beta^t [\log C_t + \chi \log(\frac{M_t}{P_t}) - \frac{\kappa}{2}(L_t)^2]. \tag{10}$$

The budget constraint that the household faces when maximising utility, expressed in nominal terms, is defined by,

$$P_t C_t + M_t + S_t D_{t+1} = P_{T,t}\overline{Y}_T + W_t L_t + \pi_t + S_t R_t^* D_t + M_{t-1} + P_t T_t. \tag{11}$$

Household's sources of funding are given by the endowment of the tradable good $P_{T,t}\overline{Y}_T$, wage earnings for working in the nontradable intermediate sector $W_t L_t$, dividends from owning intermediate firms π_t, nominal gross return from previous-period foreign currency denominated deposits $S_t R_t^* D_t$[11], holdings of previous period nominal money balances M_{t-1} and lump-sum government transfers $P_t T_t$[12]. These resources are used to purchase consumption goods $P_t C_t$, to accumulate nominal money balances M_t or to acquire new interest-bearing deposits $S_t D_{t+1}$.

First order conditions associated to this problem are obtained by maximising Eq. 10 with respect to D_{t+1}, M_t, and L_t subject to the budget constraint stated in Eq. 11. We therefore have,

$$C_{t+1} = \beta R_{t+1}^* \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} C_t \tag{12}$$

$$\frac{M_t}{P_t} = \chi C_t \frac{R_{t+1}}{R_{t+1} - 1} \tag{13}$$

[10]The fact that households obtain utility from real balances is common in the Money-in-the-Utility function literature. It can be thought as money generating utility owing to the services that it provides in facilitating transactions (see Walsh 2003).

[11]Since one of the main objectives of the model is highlighting problems associated with currency mismatches, it is assumed that households only hold deposits denominated in the foreign currency.

[12]In facilitating the analysis it is assumed that government's transfers are only made in the final nontradable good.

$$\frac{1}{\kappa}\frac{1}{C_t}\frac{W_t}{P_t} = L_t. \tag{14}$$

Eq. 12 is a Euler equation indicating that the marginal rate of substitution of consumption in two subsequent periods must be equal to the real interest rate. Note that the UIP condition takes the form $R_{t+1} = R^*_{t+1}\frac{S_{t+1}}{S_t}$, where R_{t+1} and R^*_{t+1} indicate the gross nominal risk-free domestic and foreign interest rates, respectively.

The demand for real balances stated in Eq. 13 is positively associated with consumption and the weight in the utility function of having an extra-unit of real balances; and negatively related to the gross nominal interest rate. Finally, the labour supply equation shown in Eq. 14 increases in the real wage, while decreases in consumption and in the weight that the household gives to the disutility of working.

2.3 Government

It is assumed that government spending affects only the final nontradable good. In this simple setting, the only source of funding for the government's current spending and the lump-sum transfer that the government makes towards households is real seigniorage. Observe that the interpretation of T_t is twofold: whenever it takes a positive value it refers to a lump-sum transfer from the government to households, while if it takes a negative value it implies a lump-sum tax paid from households to the government. The government's budget constraint can therefore be expressed as,

$$G_t + T_t = \frac{(M_t - M_{t-1})}{P_t}, \tag{15}$$

where G_t indicates government's expenditure on the final good and $\frac{(M_t - M_{t-1})}{P_t}$ is the real seigniorage that the government is obtaining for issuing money between t and $t-1$. In facilitating the analysis, unless otherwise stated, it will be assumed that $G_t \equiv 0$ and therefore any revenue due to seigniorage is immediately rebated to households in a lump-sum way.

3 Entrepreneurs

Entrepreneurs will play a central role in the model. They will produce the capital good that is afterwards rented to firms. In producing the capital good, however, they must obtain external funding, which is denominated in foreign currency and subject to frictions. The present section provides a detailed analysis of the entrepreneurs' behaviour and their interactions with the credit market.

3.1 Partial equilibrium contracting problem

The analysis of the debt contracting problem under asymmetric information developed in this section closely follows Carlstrom and Fuerst (1997). It is assumed the existence of a continuous number of entrepreneurs indexed by j in the interval $[0,1]$ producing a homogeneous capital good. Each entrepreneur has the following stochastic linear technology,

$$K_{j,t+1} = \omega_{j,t} i_{j,t}, \tag{16}$$

where $K_{j,t+1}$ indicates the capital good produced by entrepreneur j at period $t,$ that will be incorporated in the production process of intermediate firms in period $t+1$; $i_{j,t}$ denotes the input utilised by entrepreneur j to produce the capital good, which is part of the final good produced in the economy; $\omega_{j,t}$ is a *iid* random variable with a common distribution across j, where the cumulative and density functions have positive supports and are denoted by $\Phi(\cdot)$ and $\phi(\cdot)$, respectively. To simplify the analysis it is assumed that $E(\omega) = 1$.

When the entrepreneur decides how much to invest at period t, he or she faces the following budget constraint,

$$S_t B^*_{j,t+1} = P_t(i_{j,t} - n_{j,t}), \tag{17}$$

where $S_t B^*_{j,t+1}$ indicates the domestic value of the foreign currency denominated debt[13] contracted at period t to be repaid at period $t+1$ and $n_{j,t}$ is the net worth of entrepreneur j at the beginning of period t. This constraint simply indicates that the entrepreneur can purchase inputs beyond his or her net worth only by contracting foreign currency denominated debt.

Following Townsend (1979) and Gale and Hellwig (1985) among others, the model assumes a costly state verification problem. In this context, the optimal contract between the borrower and the lender will take the form of a standard non-contingent debt contract. To simplify the model it will be assumed that there is enough anonymity in the credit market, so as to avoid issues related to how past records of interactions between entrepreneurs and lenders may affect the characteristics of the financial contract.

The contract specifies a fixed payment to the lender in all states where the project generates a nominal gross return above the fixed nominal value of the debt repayment. In contrast, when this condition is not satisfied, the entrepreneur defaults on the debt and the lender recoups as much as he or she can from the project, after paying a fixed monitoring cost.

[13] The fact that the entrepreneur can only obtain foreign currency denominated debt is taken as given in the model. It can be thought that the reason behind this situation is the so-called "original sin problem" (see Hausmann, 1999).

The random variable $\omega_{j,t}$, which can be thought of as a productivity parameter, is neither observed by the entrepreneur nor by the lender ex-ante. For the entrepreneur, however, it is costless to observe the ex-post value of $\omega_{j,t}$. The lender, in contrast, must incur in a monitoring cost to observe the true value of $\omega_{j,t}$.

The monitoring cost is given by the payment of $\mu i_{j,t}$ units of the final capital good, where $0 \leq \mu \leq 1$[14]. The payment to observe $\omega_{j,t}$, however, is only made in case the entrepreneur defaults on the debt. It is clear now where the costly state verification problem arises in the model: in order to observe the true realisation of $\omega_{j,t}$, the lender must incur in a deterministic pecuniary cost.

Let $\overline{\omega}_{j,t}$ denote the minimum value of $\omega_{j,t}$ at which default does not occur and let $R^{nd}_{j,t+1}$ indicate the non-default gross nominal interest rate charged on entrepreneur j when contracting the debt at period t. $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$ therefore satisfy,

$$R^k_{t+1}\overline{\omega}_{j,t}i_{j,t} = R^{nd}_{j,t+1}S_tB^*_{j,t+1} = R^{nd}_{j,t+1}P_t(i_{j,t} - n_{j,t}). \tag{18}$$

Eq. 18 indicates that entrepreneur j, with the associated value for the productivity parameter given by $\overline{\omega}_{j,t}$, produces $\overline{\omega}_{j,t}i_{j,t}$ units of the capital good that are afterwards rented to firms at the nominal rental price R^k_{t+1}. The term $R^k_{t+1}\overline{\omega}_{j,t}i_{j,t}$, therefore, represents the minimum nominal gross return of the produced capital required to repay the principal and interests on the debt, $R^{nd}_{j,t+1}S_tB^*_{j,t+1}$.

Note that Eq. 18 can be rewritten as follows,

$$R^{nd}_{j,t+1} = \frac{R^k_{t+1}\overline{\omega}_{j,t}}{P_t(1 - \frac{n_{j,t}}{i_{j,t}})}. \tag{19}$$

Eq. 19 gives a simple relation between $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$. It is worth highlighting that R^k_{t+1} is a market price, and as such it will be determined by the equilibrium conditions between aggregate supply and aggregate demand for capital. The general price index, P_t, is also a market price determined by equilibrium conditions in the market for goods. Therefore, from the entrepreneur's viewpoint, these variables are taken as given.

Also note that taking the net worth of entrepreneur j as given, the contractual problem between the lender and the entrepreneur is fully specified either in terms of $R^{nd}_{j,t+1}$ and $i_{j,t}$, or

[14]The assumption regarding the form of the monitoring cost implies that there is a fixed cost $\mu i_{j,t}$, known ex-ante by the lender, for observing the true realisation of the project. Note that this cost depends on the scale of the investment $i_{j,t}$, but it is independent of the ex-post realisation of $\omega_{j,t}$. A slightly different approach is taken in Bernanke et al (1999), where the monitoring cost is a fraction of the ex-post realisation of the project. It is worth observing, however, that the main results of the model remain the same, independently of the form in which monitoring costs are defined.

$\overline{\omega}_{j,t}$ and $i_{j,t}$ (see Eq. 18). Since the contract in terms $\overline{\omega}_{j,t}$ and $i_{j,t}$ is slightly easier to study, in the remainder of the section the optimal contractual problem is analysed only in terms of these two variables.

3.2 Expected profits

In determining the optimal contract it is assumed that both the entrepreneur and the lender are risk neutral. Recalling that capital fully depreciates within the period, the net expected profit of the entrepreneur in nominal terms can be expressed as follows,

$$R_{t+1}^{k}\int_{\overline{\omega}_{j,t}}^{\infty} i_{j,t}\omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]R_{j,t+1}^{nd}P_t(i_{j,t}-n_{j,t}),$$

where the first term indicates the expected gross income for producing the capital good whenever $\omega_{j,t} > \overline{\omega}_{j,t}$, while the second term shows the expected cost of the debt repayment in case the entrepreneur repays the debt as established in the contract (i.e., whenever $\omega_{j,t} > \overline{\omega}_{j,t}$). The term $[1-\Phi(\overline{\omega}_{j,t})]$ thus indicates the probability that the entrepreneur repays the debt. Observe that in case of default, or whenever $\omega_{j,t} < \overline{\omega}_{j,t}$, the entrepreneur receives nothing, and any remaining value of the project is completely seized by the lender.

Using Eq. 18 it is possible to rewrite the above expression as follows,

$$R_{t+1}^{k}i_{j,t}f(\overline{\omega}_{j,t}) = R_{t+1}^{k}i_{j,t}\{\int_{\overline{\omega}_{j,t}}^{\infty}\omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\}$$

where $f(\overline{\omega}_{j,t}) = \{\int_{\overline{\omega}_{j,t}}^{\infty}\omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\}$ indicates the expected share of the investment that the entrepreneur keeps when undertaking a successful project.

Following a similar way of reasoning, the net expected profit of the lender can be expressed as follows,

$$R_{t+1}^{k}\int_{0}^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega - R_{t+1}^{k}\mu i_{j,t}\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]R_{j,t+1}^{nd}P_t(i_{j,t}-n_{j,t}).$$

In this case $R_{t+1}^{k}\int_{0}^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega$ indicates the expected gross income generated by the project that is seized by the lender whenever $\omega_{j,t} < \overline{\omega}_{j,t}$ and $R_{t+1}^{k}\mu i_{j,t}\Phi(\overline{\omega}_{j,t})$ denotes the expected payment of the monitoring cost[15]. Note that $\Phi(\overline{\omega}_{j,t})$ indicates the probability that entrepreneur j defaults on the debt. In the case in which $\omega_{j,t} > \overline{\omega}_{j,t}$, on the other hand, the entrepreneur repays the loan as established in the contract, and thus the lender expects to receive $[1-\Phi(\overline{\omega}_{j,t})]R^{nd}_{j,t+1}P_t(i_{j,t}-n_{j,t})$.

Using Eq. 18 it is possible to define the expected profit for the lender as,

$$R^k_{t+1}i_{j,t}g(\overline{\omega}_{j,t}) = R^k_{t+1}i_{j,t}\{\int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\},$$

where $g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$ indicates the expected share of the investment that the lender keeps from the project.

Considering the definitions of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ it is possible to show that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$[16]. This fact implies that a fraction $\mu\Phi(\overline{\omega}_{j,t})$ of the total investment made by entrepreneur j is expected to be lost owing to the presence of monitoring costs.

3.2.1 A note on the behaviour of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$

Let us consider again the fraction of the investment that the entrepreneur and the lender keep from the project $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$, respectively. In Appendix B it is shown that $f'(\overline{\omega}_{j,t}) = -[1-\Phi(\overline{\omega}_{j,t})]$ and that $f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t})$, implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$ (notice that monitoring costs, μ, do not affect $f(\overline{\omega}_{j,t})$) . In particular, note that $f'(\overline{\omega}_{j,t})$ will always be negative, unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $f'(\overline{\omega}_{j,t}) = 0$. Therefore, for a given level of investment $i_{j,t}$, entrepreneur's expected profits, $R^k_{t+1}i_{j,t}f(\overline{\omega}_{j,t})$, do not increase in $\overline{\omega}_{j,t}$.

Regarding $g(\overline{\omega}_{j,t})$, in this Appendix it is also shown that $g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]$ and that $g''(\overline{\omega}_{j,t}) = -[\mu\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]$. Note that, without monitoring costs (i.e., whenever $\mu = 0$), $g(\overline{\omega}_{j,t})$ is concave in $\overline{\omega}_{j,t}$ and $g'(\overline{\omega}_{j,t}) \geq 0$[17]. When monitoring costs are introduced in the model, there is an additional effect on $g(\overline{\omega}_{j,t})$. It can be shown that $g''(\overline{\omega}_{j,t}) < 0$ but, for

[15]Recall that when the entrepreneur defaults on the debt, the lender must pay $\mu i_{j,t}$ units of the capital good, which must therefore be priced at the rental price of capital R^k_{t+1}.

[16]To obtain this result, notice that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t})$ can be written as $\int_0^{\infty} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t})$. Recalling that $E(\omega) = \int_0^{\infty} \omega\phi(\omega)d\omega = 1$ gives $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1 - \mu\Phi(\overline{\omega}_{j,t})$.

[17]$g'(\overline{\omega}_{j,t})$ will always be positive unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $g'(\overline{\omega}_{j,t}) = 0$.

sufficiently high values of $\overline{\omega}_{j,t}$, $g'(\overline{\omega}_{j,t}) < 0$ (i.e., whenever $\mu\phi(\overline{\omega}_{j,t}) > [1-\Phi(\overline{\omega}_{j,t})]$). Therefore, with monitoring costs $g(\overline{\omega}_{j,t})$ becomes a hump shaped concave function, with a maximum at the value of $\overline{\omega}_{j,t}$ for which $\mu\phi(\overline{\omega}_{j,t}) = [1-\Phi(\overline{\omega}_{j,t})]$, call it $\overline{\omega}^*_{j,t}$.

Observe that for a given $i_{j,t}$, the behaviour of the lender's expected profits, $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})$, depends on the the behaviour of $g(\overline{\omega}_{j,t})$. In particular, whenever $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$ a small rise in $\overline{\omega}_{j,t}$ must increase lender's expected profits. To gain intuition on this result, note that a small rise in $\overline{\omega}_{j,t}$ has three effects on lender's expected profits: i. Increases the expected gross revenue of what the lender would recoup when the entrepreneur defaults on the debt, ii. Increases the expected monitoring costs and iii. Reduces the expected nominal value of the debt repayment. Therefore, it must be true that the first effect overcomes the second and third effects when $g'(\overline{\omega}_{j,t}) > 0$, so as to have that the lender's expected profits increase in $\overline{\omega}_{j,t}$ when $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$.

3.3 Determining the optimal contract

The optimal debt contract will be determined by a pair of values of $i_{j,t}$ and $\overline{\omega}_{j,t}$ that maximises the entrepreneur's expected profits, subject to the lender receiving at least the opportunity cost of the loan.

In what follows, it is assumed that the entrepreneur's participation constraint, given by $R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) > R_{t+1} P_t n_{j,t}$, holds. This condition indicates that the gross nominal rate of return that the entrepreneur expects to obtain for undertaking the project, $\frac{R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t})}{P_t n_{j,t}}$, must be greater than the gross nominal interest, R_{t+1}.

The lender's participation constraint, in turn, is given by $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) \geq R_{t+1} P_t (i_{j,t} - n_{j,t})$, indicating that the lender's expected gross nominal rate of return, $\frac{R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})}{P_t (i_{j,t} - n_{j,t})}$, must be at least the opportunity cost of the loan, R_{t+1}. Assuming that there are a large number of lenders in this economy, we can invoke arbitrage conditions so as to guarantee that the lender's participation constraint binds.

The optimisation problem that the entrepreneur faces, therefore, can be stated as follows,

$$\max_{\{i_{j,t}, \overline{\omega}_{j,t}\}} R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) \text{ s.t. } R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1} P_t (i_{j,t} - n_{j,t}).$$

From the first order conditions it is possible to obtain,[18]

$$R^k_{t+1} \{ g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})} g'(\overline{\omega}_{j,t}) \} = R_{t+1} P_t, \tag{20}$$

and

[18] See Appendix A for details.

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1}P_t} g(\overline{\omega}_{j,t})}. \tag{21}$$

It is worth observing that Eqs. 19, 20 and 21 constitute a system of three equations in three unknowns ($\overline{\omega}_{j,t}$, $R^{nd}_{j,t+1}$ and $i_{j,t}$), since $n_{j,t}$, P_t, R^k_{t+1}, and R_{t+1} are taken as given. In solving this system, notice that Eq. 20 gives an implicit function of the form,[19]

$$\overline{\omega}_{j,t} = F(\frac{R^k_{t+1}}{R_{t+1}P_t}) = \overline{\omega}_t, \text{ where } \frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}} = \frac{F'(\cdot)}{R_{t+1}P_t} > 0. \tag{22}$$

Eq. 22 implies that, in equilibrium, the value of $\overline{\omega}_{j,t}$ is the same for all entrepreneurs (and thus it is denoted by $\overline{\omega}_t$). Using Eqs. 21 and 22 it is possible to rewrite the demand function for the input $i_{j,t}$ as,

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1}P_t} g(\overline{\omega}_t)}. \tag{23}$$

Observe that this expression indicates that the demand function for the input $i_{j,t}$ linearly depends on the net worth of agent j, fact that facilitates aggregation. This result is a direct consequence of the linear production function of capital and the linear monitoring technology. Taking as given the net worth of entrepreneur j, moreover, Eq. 23 gives a positive relation between the rental price of capital, R^k_{t+1}, and the investment demand, $i_{j,t}$. Formally, differentiating this equation with respect to R^k_{t+1} it is possible to obtain,

$$\frac{\partial i_{j,t}}{\partial R^k_{t+1}} = \frac{1}{R_{t+1}P_t} \frac{i^2_{j,t}}{n_{j,t}} [g(\overline{\omega}_t) + R^k_{t+1} g'(\overline{\omega}_t) \frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}}] > 0^{20}, {}^{21}. \tag{24}$$

Also notice that Eq. 20 can be now rewritten as,

$$R^k_{t+1} = \{g(\overline{\omega}_t) - \frac{f(\overline{\omega}_t)}{f'(\overline{\omega}_t)} g'(\overline{\omega}_t)\}^{-1} R_{t+1}P_t. \tag{25}$$

[19]See Appendix B for details.

[20]A sufficient condition for having $\frac{\partial i_{j,t}}{\partial R^k_{t+1}} > 0$ is that $g'(\overline{\omega}_t) > 0$. In Appendix C it is shown that this condition must hold in order to satisfy the second order conditions of the entrepreneur's maximisation problem when ω is uniformly distributed in the interval $[0, 2]$. Moreover, the fact that in equilibrium $g'(\overline{\omega}_t) > 0$ can also be determined by analysing the maximand and the constraint of the entrepreneur's optimisation problem. To see this: let $g'(\overline{\omega}_t) < 0$. From the constraint, this fact implies that $\frac{\partial \overline{\omega}_t}{\partial i_t} < 0$. Using this result, we can see from the maximand that the entrepreneur can always increase the expected profit by increasing investment (since $f'(\overline{\omega}_t) \leq 0$), fact that must not be true in equilibrium. In equilibrium, therefore, $g'(\overline{\omega}_t) > 0$.

[21]It is worth emphasizing the close link between the entrepreneur's optimal contracting problem and the modern literature on credit rationing. In particular, the fact that in equilibrium it must be true that $g'(\overline{\omega}_t) > 0$, suggests that this model does not show "equilibrium credit rationing" in the sense of Stiglitz and Weiss (1981). Therefore, on the upward sloping part of $g(\overline{\omega}_t)$, the lender may provide any extra funding required by the entrepreneur at a higher interest rate R^{nd}_{t+1}, since lender's expected profits increase in that region. In contrast, whenever $g'(\overline{\omega}_t) \leq 0$, any further increase in the interest rate, which is associated with a higher probability of default of the entrepreneur, reduces lender's expected profits thus giving place to a situation where credit rationing holds.

Combining Eqs. 19, 22 and 23 we can compute the solution for $R^{nd}_{j,t+1}$,

$$R^{nd}_{j,t+1} = \frac{R^k_{t+1}\overline{\omega}_{j,t}}{P_t(1 - \frac{n_{j,t}}{i_{j,t}})} = R_{t+1}\overline{\omega}_t g(\overline{\omega}_t)^{-1} = R^{nd}_{t+1}. \tag{26}$$

This equation indicates that, in equilibrium, the non-default interest rate will be the same for all entrepreneurs since it does not depend on any variable of entrepreneur j.

4 Aggregation and equilibrium conditions

4.1 Aggregate net worth and aggregate investment

A key variable of the model is given by the entrepreneur's net worth. For simplicity, it will be assumed that entrepreneurs have an infinite horizon and that each period they devote a constant fraction $\upsilon \in (0, 1)$ of their aggregate net profits to the consumption of the final good. A slightly different approach is taken in Bernanke et al (1999), where entrepreneurs have a fixed probability of survival every period. As Carlstrom and Fuerst (2001) point out, it can be thought that those entrepreneurs who do not survive are "informed" at the beginning of the period and thus they consume their end-of-period profits just an instant before dying. These two alternative forms of modelling the evolution of entrepreneurs' consumption and thereby entrepreneurs' net worth are essentially equivalent (Carlstrom and Fuerst 2001, p. 7).

Recall that $R^k_{t+1}f(\overline{\omega}_{j,t})i_{j,t}$ denotes the expected net profits of entrepreneur j at period t. Using the fact that in equilibrium $\overline{\omega}_{j,t} = \overline{\omega}_t$ and summing over j, we can define the net expected profits of the entrepreneurial sector as $R^k_{t+1}f(\overline{\omega}_t)i_t$, where i_t denotes aggregate investment (defined below). Recalling that $f(\overline{\omega}_t) = 1 - \mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t)$, nominal aggregate net worth at the beginning of period $t + 1$ can be defined as,

$$P_{t+1}n_{t+1} = (1 - \upsilon)R^k_{t+1}(1 - \mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t))i_t.$$

Notice that the lender's constraint in the entrepreneur's maximisation problem can be written in the aggregate as $R^k_{t+1}i_t g(\overline{\omega}_t) = R_{t+1}P_t(i_t - n_t)$. Using this expression and the budget constraint of the entrepreneurial sector, $S_tB^*_{t+1} = P_t(i_t - n_t)$, we can rewrite the above equation as follows,

$$P_{t+1}n_{t+1} = (1 - \upsilon)\{R^k_{t+1}(1 - \mu\Phi(\overline{\omega}_t))i_t - R_{t+1}S_tB^*_{t+1}\}. \tag{27}$$

Aggregate consumption at period $t + 1$, C^e_{t+1}, is hence given by,

$$P_{t+1}C^e_{t+1} = \upsilon\{R^k_{t+1}(1 - \mu\Phi(\overline{\omega}_t))i_t - R_{t+1}S_tB^*_{t+1}\}. \tag{28}$$

Considering Eqs. 27 and 28 lagged one period, the budget constraint of the entrepreneurial sector at period t (*i.e.*, $S_tB^*_{t+1} = P_t(i_t - n_t)$) yields,

$$P_ti_t + P_tC^e_t + R_tS_{t-1}B^*_t = R^k_t(1 - \mu\Phi(\overline{\omega}_{t-1}))i_{t-1} + S_tB^*_{t+1}. \tag{29}$$

Each period t, entrepreneurs invest P_ti_t to produce the capital good, consume $P_tC^e_t$ of the final produced good and repays capital and interests of the debt contracted at period $t-1$, $R_tS_{t-1}B^*_t$[22]. These expenditures are financed with the aggregate income obtained for renting the produced capital good to firms, $R^k_t(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1}$[23], and by issuing new debt $S_tB^*_{t+1}$.

Aggregate investment can be obtained by summing over j the demand function for the input $i_{j,t}$ stated in Eq. 23, thus yielding,

$$i_t = \int_0^1 i_{j,t}dj = (1 - \frac{R^k_{t+1}}{R_{t+1}P_t}g(\overline{\omega}_t))^{-1}n_t = (1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})n_t, \tag{30}$$

where the last term in this expression is derived from Eq. 25. Eq. 30 shows that i_t linearly depends on n_t, the aggregate net worth available at the beginning of period t. It also indicates that, in equilibrium, aggregate investment at period t is determined by the aggregate net worth in the same period scaled by the factor $(1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})$, which can be thought of as a measure of the leverage ratio of the entrepreneurial sector as a whole.

Introducing Eqs. 25 and 30 into Eq. 27 it is possible to obtain,

$$P_{t+1}n_{t+1} = (1-\upsilon)R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_tn_t - S_tB^*_{t+1}\}. \tag{31}$$

Similarly, entrepreneurs' consumption can be expressed as,

$$P_{t+1}C^e_{t+1} = \upsilon R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_tn_t - S_tB^*_{t+1}\}. \tag{32}$$

Eq. 31 defines the evolution of aggregate net worth. It indicates that entrepreneurs obtain in the aggregate the gross nominal return $-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))R_{t+1}$ for investing their aggregate net worth P_tn_t to produce the capital good. They utilise this return to repay the amount $R_{t+1}S_tB^*_{t+1}$ for the debt contracted at period t. The difference between these two flows multiplied by $(1-\upsilon)$, the fraction of entrepreneurs' net profits not consumed, defines aggregate net worth at the beginning of period $t+1$.

[22]It is worth noting that, although each individual entrepreneur has to repay $R^{nd}_{j,t}S_{t-1}B^*_{j,t}$ to lenders (whenever the debt is repaid as established in the contract), at the aggregate level lenders receive the opportunity cost of the loan, $R_tS_{t-1}B^*_t$.

[23]The fact that $(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1}$ is equal to the supply of capital at period t is discussed in detail in the next subsection.

Finally, note that from the budget constraint of the entrepreneurial sector and Eq. 30 it is possible to obtain the aggregate demand for credit:

$$S_t B^*_{t+1} = -P_t n_t \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{f(\overline{\omega}_t) g'(\overline{\omega}_t)}. \tag{33}$$

4.2 Aggregate supply of the capital good

From the previous section we know that a fraction $\mu\Phi(\overline{\omega}_t)$ of the total investment made by entrepreneur j at period t is expected to be lost owing to the presence of monitoring costs. The expected aggregate supply of capital is hence defined as,

$$K^s_{t+1} = i_t(1 - \mu\Phi(\overline{\omega}_t)) = (1 - \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{f(\overline{\omega}_t) g'(\overline{\omega}_t)})(1 - \mu\Phi(\overline{\omega}_t)) n_t, \tag{34}$$

where the last equality is obtained from Eq. 30. The existence of asymmetric information problems between lenders and entrepreneurs implies that the aggregate supply of capital, K^s_{t+1}, is a fraction $(1 - \mu\Phi(\overline{\omega}_t))$ of what would be supplied under perfect information (i.e., whenever $\mu = 0$). Notice that, as R^k_{t+1} increases K^s_{t+1} is affected by two effects: i. Aggregate investment increases, positively affecting K^s_{t+1} and ii. Expected monitoring costs rises, negatively affecting K^s_{t+1}. The first effect, however, overcomes the second effect. This fact is formally assessed in the following remark:

Remark 1 *The model with monitoring costs provides an upward sloping supply curve of capital in the* (R^k_{t+1}, K_{t+1}) *space.*

Proof. Recalling that $f(\overline{\omega}_t) + g(\overline{\omega}_t) = 1 - \mu\Phi(\overline{\omega}_t)$, Eq. 34 can be rewritten as: $K^s_{t+1} = (1 - \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{f(\overline{\omega}_t) g'(\overline{\omega}_t)})(f(\overline{\omega}_t) + g(\overline{\omega}_t)) n_t$. Since in equilibrium $g'(\overline{\omega}_t) > 0$, we know that i_t is an increasing function of R^k_{t+1}. We also know that $\frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}} > 0$. To demonstrate that $\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}} > 0$, it is thus sufficient to show that $\frac{\partial}{\partial \overline{\omega}_t}(f(\overline{\omega}_t) - \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}) > 0$. Notice that $\frac{\partial}{\partial \overline{\omega}_t}(f(\overline{\omega}_t) - \frac{f'(\overline{\omega}_t) g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}) = \frac{g(\overline{\omega}_t)}{g'(\overline{\omega}_t)}\{-f''(\overline{\omega}_t) + \frac{f'(\overline{\omega}_t) g''(\overline{\omega}_t)}{g'(\overline{\omega}_t)}\}$; expression that must be positive to satisfy the second order condition of the entrepreneur's maximisation problem (see Appendix C). ■

4.3 Aggregate demand for the capital good

In this model only intermediate firms demand the capital produced by entrepreneurs. Using the fact that in a symmetric equilibrium each firm i sets the same price for the produced intermediate good (i.e., $P_{i,t} = P_{N,t}$), Eq. 4 thus implies that $Z_{i,t} = Z_t$ for all i. From Eq. 6, the aggregate demand for the capital good in period $t + 1$ takes the form,

$$K^d_{t+1} = (\frac{1-\alpha}{\alpha})^{\alpha-1}\frac{Z_{t+1}}{A_{t+1}}(\frac{W_{t+1}}{R^k_{t+1}})^{1-\alpha}. \quad (35)$$

4.4 Equilibrium conditions

To define the equilibrium of the model it is still necessary to specify: i. Money market equilibrium, ii. Goods market equilibrium, iii. Capital good market equilibrium, iv. Labour market equilibrium, v. Intertemporal balance of trade equilibrium and vi. Credit market equilibrium.

4.4.1 Money market equilibrium

Money market equilibrium is given by Eq. 13 under the assumption that aggregate supply equals aggregate demand for real money balances.

4.4.2 Goods market equilibrium

To determine the equilibrium conditions in the goods market it is worth recalling that there are two sectors in this economy: one tradable and one nontradable. Noting that the only source of absorption of tradable output is given by the demand for tradable inputs by the final producer firm, we can define the clearing market condition in the tradable sector as follows,

$$P_{T,t}(\overline{Y}_T - X_{T,t}) = TB_t, \quad (36)$$

where TB_t denotes the trade balance at period t measured in terms of tradable goods.

As previously pointed out, in a symmetric equilibrium we have that $P_{i,t} = P_{N,t}$ and that $Z_{i,t} = Z_t$ for all i. Therefore, the production function of the nontradable intermediate firm becomes $Z_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. Owing to the existence of imperfect competition in this sector, it must be true that the aggregate income of intermediate firms equates the payment of the two factors of production plus any remaining profits or: $P_{N,t}Z_t = R^k_t K_t + W_t L_t + \pi_t$.

Regarding the final producer firm, recalling that in equilibrium the marginal cost of the final producer firm equals the price level yields:

$$P_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)} {P_{T,t}}^{1-\gamma} P^{\gamma}_{N,t}, \quad (37)$$

where $P_{N,t}$ is given by,

$$P_{N,t} = \frac{\theta}{\theta - 1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R_t^k)^{\alpha}. \tag{38}$$

In equilibrium the production function of the final producer firm takes the form $Y_t = Z_t^{\gamma}X_{T,t}^{1-\gamma}$. Cost minimisation hence implies $P_tY_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$, and thus the demand functions for the nontradable and tradable inputs are given by $P_{N,t}Z_t = \gamma P_tY_t$ and $P_{T,t}X_{T,t} = (1-\gamma)P_tY_t$, respectively.

Finally, the clearing market condition for the final nontradable good implies,

$$Y_t = C_t + C_t^e + i_t. \tag{39}$$

4.4.3 Capital good market equilibrium

In equilibrium, the rental price of capital will adjust so as to equate aggregate supply and aggregate demand for the capital good: $K_t^s = K_t^d = K_t$. From Eqs. 34, 35 and 38 we thus have that,

$$R_t^k = \alpha\frac{\theta-1}{\theta}\frac{P_{N,t}Z_t}{K_t}, \tag{40}$$

where K_t is given by Eq. 34.

4.4.4 Labour market equilibrium

From Eq. 7, in a symmetric equilibrium the aggregate demand for labour is given by: $L_t^d = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_t}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}$. Using Eq. 38 and the equilibrium condition $L_t^s = L_t^d = L_t$ yields,

$$W_t = (1-\alpha)\frac{\theta-1}{\theta}\frac{P_{N,t}Z_t}{L_t}, \tag{41}$$

where L_t is given by Eq. 14.

4.4.5 Intertemporal balance of trade equilibrium

By adding the budget constraints of households, government and entrepreneurs we can obtain the budget constraint of the economy as a whole (i.e., the balance of payment):

$$\begin{aligned} & P_tC_t + P_tC_t^e + P_ti_t + S_tR_t^*B_t^* + S_tD_{t+1} \\ = \quad & P_{T,t}\overline{Y}_T + S_tB_{t+1}^* + R_t^k(1-\Phi(\overline{\omega}_{t-1}))i_{t-1} + W_tL_t + \pi_t + S_tR_t^*D_t. \end{aligned}$$

Using the fact that $R_t^k(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1} = R_t^k K_t$; $P_{N,t}Z_t = R_t^k K_t + W_t L_t + \pi_t$ and $P_t Y_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$ as well as the clearing market conditions for the tradable and nontradable sectors gives,

$$S_t(D_{t+1} - B_{t+1}^*) = TB_t + S_t R_t^*(D_t - B_t^*).$$

Let $F_t = D_t - B_t^*$ and $F_{t+1} = D_{t+1} - B_{t+1}^*$ denote the net foreign assets accumulation by the economy as a whole at period t and $t+1$, respectively, in foreign currency. The intertemporal national budget constraint in domestic currency can thus be written as,

$$S_t F_{t+1} = TB_t + S_t R_t^* F_t. \tag{42}$$

Iterating forward Eq. 42 yields:

$$-S_0 R_0^* F_0 = \sum_{t=0}^{\infty} TB_t(1.S_0(S_1R_1^*)^{-1}...S_{t-1}(S_tR_t^*)^{-1}) + \lim_{T\to\infty} S_0(-F_{T+1})(1.(R_1^*)^{-1}...(R_T^*)^{-1}).$$

Imposing the "no-Ponzi-game-condition", implying that $\lim_{T\to\infty} S_0(-F_{T+1})(1.(R_1^*)^{-1}...(R_T^*)^{-1}) = 0$, gives,

$$-S_0 R_0^* F_0 = \sum_{t=0}^{\infty} TB_t(1.S_0(S_1R_1^*)^{-1}...S_{t-1}(S_tR_t^*)^{-1}). \tag{43}$$

As usual, Eq. 43 simply states that any initial net foreign-currency indebtedness must be equal to the present value of future trade balance surpluses, guaranteeing that the economy is solvent from an intertemporal perspective.

4.4.6 Credit market equilibrium

By Walras' law, equilibrium in the credit market is guaranteed whenever the remaining markets of the economy are in equilibrium. In this context, from the intertemporal national budget constraint the credit market equilibrium condition becomes an identity given by:

$$S_t B_{t+1}^* \equiv S_t(D_{t+1} - F_{t+1}),$$

where the aggregate demand for credit (i.e., $S_t B_{t+1}^*$) is defined in Eq. 33; while the aggregate supply of credit is given by $D_{t+1} - F_{t+1}$. The overall amount of credit that is available in the economy is thus provided by domestic households in the form of deposits D_{t+1}, and by foreigners in the form of net foreign liabilities, $-F_{t+1}$.

5 Solving for the zero-inflation symmetric steady state

The steady state aggregate net worth can be derived from Eq. 31:

$$n[f(\overline{\omega})g'(\overline{\omega}) + (1-\upsilon)Rf'(\overline{\omega})(f(\overline{\omega}) + g(\overline{\omega}))] = -b(1-\upsilon)Rf(\overline{\omega})g'(\overline{\omega}), \tag{44}$$

where $b \equiv \frac{SB^*}{P}$. The aggregate demand for credit defined in Eq. 33 yields, in the steady state,

$$b = -\frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})}n. \tag{45}$$

From these two equations it is possible to pin-down the steady state value of $\overline{\omega}$. To see that, notice that Eq. 12 evaluated at the zero-inflation steady state gives,

$$R^* = R = r = \beta^{-1},^{24} \tag{46}$$

where r denotes the domestic risk-free gross real interest rate. Hence Eqs. 44 and 45 give the following expression,

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1}, \tag{47}$$

from which the steady state value of $\overline{\omega}$ is obtained. Entrepreneurial consumption therefore takes the form: $C^e = \frac{\upsilon}{1-\upsilon}n$. This expression indicates that in the steady state a share $\frac{\upsilon}{1-\upsilon}$ of entrepreneur's aggregate net worth is devoted to consumption. We can now study the steady state solution of the whole model. The analysis becomes simpler by defining all variables in real terms as follows: $r^k \equiv \frac{R^k}{P}$, $R^{nd} \equiv r^{nd}$, $p_N \equiv \frac{P_N}{P}$, $s = p_T \equiv \frac{S}{P}$, $w \equiv \frac{W}{P}$, $m \equiv \frac{M}{P}$, $b \equiv \frac{SB^*}{P}$, $tb \equiv \frac{TB}{P}$, and $f \equiv \frac{SF}{P}$. To facilitate the exposition the key endogenous variables of the model in the zero-inflation steady state are listed below:

$$C = m\frac{(1-\beta)}{\chi} \tag{48}$$

$$r^{nd} = \beta^{-1}\overline{\omega}g(\overline{\omega})^{-1} \tag{49}$$

$$r^k = \beta^{-1}(g(\overline{\omega}) - \frac{f(\overline{\omega})}{f'(\overline{\omega})}g'(\overline{\omega}))^{-1} \tag{50}$$

[24] It is important to highlight that we have also assumed that the risk-free gross nominal interest rate (R_t^*) is constant over time.

$$r^k = \alpha p_N \frac{\theta - 1}{\theta} \frac{Z}{K} \tag{51}$$

$$K = i(1 - \mu \Phi(\overline{\omega}_t)) \tag{52}$$

$$w = (1 - \alpha) p_N \frac{\theta - 1}{\theta} \frac{Z}{L} \tag{53}$$

$$L = \frac{1}{\kappa} \frac{1}{C} w \tag{54}$$

$$Z = A K^\alpha L^{1-\alpha} \tag{55}$$

$$Y = Z^\gamma X_T^{1-\gamma} \tag{56}$$

$$\gamma Y = p_N Z \tag{57}$$

$$(1 - \gamma) Y = s X_T \tag{58}$$

$$1 = \gamma^{-\gamma} (1 - \gamma)^{(\gamma - 1)} s^{1-\gamma} p_N^\gamma \tag{59}$$

$$p_N = \frac{\theta}{\theta - 1} \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1} A^{-1} w^{1-\alpha} (r^k)^\alpha \tag{60}$$

$$C^e = \frac{\upsilon}{1 - \upsilon} n \tag{61}$$

$$i = (1 - \frac{f'(\overline{\omega}) g(\overline{\omega})}{f(\overline{\omega}) g'(\overline{\omega})}) n \tag{62}$$

$$s(\overline{Y}_T - X_T) = tb \tag{63}$$

$$Y = C + C^e + i \tag{64}$$

$$b = -\frac{f'(\overline{\omega}) g(\overline{\omega})}{f(\overline{\omega}) g'(\overline{\omega})} n \tag{65}$$

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1} \tag{66}$$

$$-f\beta^{-1} = (1-\beta)^{-1}tb. \tag{67}$$

5.1 Analysis of $\overline{\omega}$, r^k and r^{nd}

Since ω is a random variable, solving the model analytically requires assuming a distribution function for it. To keep the analysis as simple as possible, we assume that ω is uniformly distributed in the interval $[0,2]$. Therefore, Eq. 66 yields (see Appendix D for details):

$$\overline{\omega} = 2 + \frac{\mu\beta}{1-(\beta+\upsilon)} \equiv \overline{\omega}^*. \tag{68}$$

Note that for $\overline{\omega}^*$ to be within the interval $[0,2]$ it is further required that $\beta+\upsilon > 1$ and that $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. From here onwards we will assume that these two conditions are satisfied.

Notice that the parameters affecting $\overline{\omega}^*$ are only μ, β and υ. Therefore, the steady state solution of $\overline{\omega}$ will not be affected by those parameters associated with production functions or preferences other than β. Also observe that to have $g'(\overline{\omega}^*) > 0$ it is required that $\upsilon < 1$, condition that is satisfied since $\upsilon \in (0,1)$[25].

It is possible to determine, therefore, that $\overline{\omega}^*$ decreases with monitoring costs μ (and thus, the probability of default decreases in μ), while it increases in the subjective discount factor β and in the fraction of expected profits devoted to entrepreneurs' consumption, υ.

Once the solution of $\overline{\omega}$ is obtained r^k is also pinned down. From Eqs. 50 and 68 it is possible to obtain,

$$r^k = (\beta g(\overline{\omega}^*) + (1-\upsilon)f(\overline{\omega}^*))^{-1} = \beta^{-1}(1-\mu-\frac{\mu^2}{4}\frac{\beta}{1-(\beta+\upsilon)})^{-1} \equiv r^{k*}. \tag{69}$$

Eq. 69 implies that r^{k*} increases in μ if $\beta > \frac{2(1-\upsilon)}{2-\mu}$. It is also possible to show that r^{k*} decreases in β if $\frac{r^{k*}\beta^2\mu^2}{4} < \frac{[1-(\beta+\upsilon)]^2}{1-\upsilon}$. On the other hand, r^{k*} will always increase in υ.

We can also obtain the solution of the non-default real interest rate r^{nd} from Eq. 68:

$$r^{nd} = \overline{\omega}^*(\beta g(\overline{\omega}^*))^{-1} = \beta^{-1}(\frac{1}{2}-\frac{1}{2}\mu-\frac{1}{4}\mu\frac{\beta}{1-(\beta+\upsilon)})^{-1} \equiv r^{nd*}. \tag{70}$$

[25] This can be easily seen by noting that $g'(\overline{\omega}^*) = \frac{\mu}{2}\frac{1-\upsilon}{\beta+\upsilon-1}$ when ω is uniformly distributed in the interval $[0,2]$ (see Appendix D).

Notice the similarity between Eq. 69 and Eq. 70. Their levels, however, will differ depending on the degree of credit market imperfections (i.e., the value of μ). It is possible to verify that r^{nd*} decreases in μ if $\beta < 2(1-\upsilon)$ and that it increases in β if $\frac{r^{nd*}\beta^2\mu}{4} > \frac{[1-(\beta+\upsilon)]^2}{1-\upsilon}$. As it happened in the case of the rental rate of capital, r^{nd*} unequivocally rises in υ. Table 2, below, summarises these results.

Table 2. Summary of comparative static analysis

Effect on	Change in		
	μ	β	υ
$\overline{\omega}^*$	(-)	(+)	(+)
r^{k*}	(+) if $\beta > \frac{2(1-\upsilon)}{2-\mu}$	(-) if $r^k < \frac{4}{\beta^2\mu^2}\frac{(1-(\beta+\upsilon))^2}{1-\upsilon}$	(+)
r^{nd*}	(-) if $\beta < 2(1-\upsilon)$	(+) if $r^{nd} > \frac{4}{\beta^2\mu^2}\frac{(1-(\beta+\upsilon))^2}{1-\upsilon}$	(+)

From this table it is possible to observe that the effects of changes in μ and β on r^{k*} and r^{nd*} depend on the values of the parameters β, μ and υ. To give an illustration of this comparative static exercise, therefore, a small scale calibration-type analysis is introduced.

We have to determine the baseline values of μ, υ and β. Regarding μ, following Carlstrom and Fuerst (1997) it will be considered that monitoring costs are equivalent to bankruptcy costs. In case of default, hence, the lender will have to incur in direct costs (e.g., legal costs) but also indirect costs to seize the entrepreneur's project. The latter costs can be associated with those in which the lender must incur for having the entrepreneur's assets idle while these are being liquidated. Since we will explore the effects of raising μ we initially start with a relatively low value: $\mu = 0.12$. This value is the same as in Bernanke et al (1999) but is below of that considered in Carlstrom and Fuerst ($\mu = 0.25$). Regarding the subjective discount factor, we initially set $\beta = 0.98$.

Observe that having defined μ and β, the value of υ is restricted. In fact, υ must be determined so as to satisfy: $\beta + \upsilon > 1$ and $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. Notice that the second restriction implies that $\upsilon \geq 1 + \beta(\frac{1}{2}\mu - 1)$. Since the share of expected profits consumed by entrepreneurs is likely to be relatively low we set $\upsilon = 0.12$. This value implies that, in the steady state, entrepreneurs consume 12% of their net expected profits. Table 3 illustrates the values of $\overline{\omega}$, $\Phi(\overline{\omega})$ (i.e., the steady state probability of default), r, r^k, r^{nd} and the spreads $r^k - r$ and $r^{nd} - r$ for the baseline parameters.

Table 3. The benchmark case

$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$
0.82	0.41	1.02	1.11	1.39	0.09	0.37

From Table 3 we can observe that $\overline{\omega}$ is below the mean value of ω ($\equiv 1$) in the benchmark case. Therefore, we should expect a relatively low failure rate in this scenario. In fact, this is the case since $\Phi(\overline{\omega}) = 0.41$. It is worth observing that the credit spread (or risk premium), $r^{nd} - r$ ($= 0.37$), is markedly above the excess return for investing in producing capital, $r^k - r$ ($= 0.09$). This fact is a direct consequence of the asymmetric information problem assumed in the model. As it will be explained below, $\mu = 0.12$ implies that the agency problem is relatively low and therefore the supply of capital is relatively high in this case. As a consequence of this, the real rental rate of capital, r^k, is pushed at a level close to the risk-free real interest rate, r. To understand this feature in more detail, we can study the behaviour of the model when μ rises from 0.12 to 0.15 (see Table 4).

Table 4. A change in μ : from $\mu = 0.12$ to $\mu = 0.15$

In levels							As % change wrt benchmark case							
$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$	μ	$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$
0.53	0.27	1.02	1.13	1.29	0.11	0.27	25	-35.7	-35.7	0	1.1	-7.4	13.2	-27.8

From Table 4 we can observe that the rise in μ raises r^k. A rise in monitoring costs hence implies a higher steady state return on capital, r^k. To understand this, note that there is a direct effect of μ on the total supply of capital. Taking the level of aggregate investment i and of $\overline{\omega}$ as given, Eq. 52 shows that the aggregate supply of capital must fall after the shock. A shortage in the supply of capital is thus associated with a higher rental price r^k (see Eq. 51). Everything else equal, an increase in r^k reduces the probability that the entrepreneur defaults on the debt $\Phi(\overline{\omega})$, and therefore $\overline{\omega}$ and r^{nd} decreases. Notice that, in this model, economies with a more developed financial system are associated with a lower value of μ, and thus a higher steady state supply of capital. Therefore, the excess return $r^k - r$ brings a direct measure of the degree of imperfections in financial markets.

As a second exercise, we consider an increase in β from 0.98 to 0.99. As a direct consequence of this shock there is a reduction in the steady state level of the risk-free gross real interest rate (from 1.02 to 1.01). Everything else equal, there is a direct effect on r^k from Eq. 50, thus implying that r^k must fall. The reduction in r^k is thus associated with a higher probability of default of the entrepreneur and therefore $\overline{\omega}$ and r^{nd} must rise (see Table 5).

Table 5. A change in β : from $\beta = 0.98$ to $\beta = 0.99$

In levels							As % change wrt benchmark case							
$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$	β	$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$
0.92	0.46	1.01	1.11	1.42	0.1	0.41	1	11.7	11.7	-1	-0.7	2.3	2.7	11.6

The final exercise stresses the effect of an increase in υ from 0.12 to 0.15. This shock implies that entrepreneurs devote a higher fraction of their expected profits to consumption. Everything else equal, there is a direct negative effect on the aggregate steady state net worth. The reduction in net worth implies that entrepreneurs are riskier in the steady state and therefore $\Phi(\overline{\omega})$, $\overline{\omega}$ and r^{nd} must be at a higher level. Notice that the fall in net worth has an indirect negative effect on the aggregate investment and thus on the aggregate capital supply. This reduction in the steady state aggregate capital supply thus implies that the rental price of capital, r^k, must be located at a higher level (see Table 6). This latter effect diminishes although does not offset completely the initial increase in $\Phi(\overline{\omega})$.

Table 6. A change in υ : from $\upsilon = 0.12$ to $\upsilon = 0.15$

In levels							As % change wrt benchmark case							
$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$	υ	$\overline{\omega}$	$\Phi(\overline{\omega})$	r	r^k	r^{nd}	$r^k - r$	$r^{nd} - r$
1.10	0.55	1.02	1.12	1.53	0.1	0.51	25	32.9	32.9	0	0.9	10.2	10.6	38.3

5.2 Analysis of the remaining steady state variables of the model

In this subsection the solutions of the remaining variables for the steady state are obtained analytically. Notice that unless we assume zero steady state net foreign assets it is not possible to find closed-form solutions. For brevity, we discuss the steady state solutions of the model under the assumption that $f = 0$. This assumption of often considered in the new open economy macroeconomics literature (see, for instance, Obstfeld and Rogoff, 1996). From Eqs. 58, 63 and 67 we thus have that $X_T = \overline{Y}_T$ and thus $s = Y(1-\gamma)\overline{Y}_T^{-1}$. Notice that from the steady state solution of $\overline{\omega}$ ($\equiv \overline{\omega}^*$) we can derive the associated steady state values of $f(\overline{\omega}^*)$ and $g(\overline{\omega}^*)$. In Appendix E we show that output, consumption and net worth are given by:

$$Y = \{1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}\}^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}}\{A^{-1}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}(\frac{r^{k*}}{\alpha})^{\alpha}(\gamma\frac{\theta-1}{\theta})^{-\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\}^{\frac{\gamma}{\alpha\gamma-1}} \tag{71}$$

$$C = Y\{1 - \alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}\}, \tag{72}$$

and

$$n = Y\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)(1-\upsilon)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}. \tag{73}$$

From the above solutions it is possible to undertake some exercises of comparative statics. It is interesting, in particular, to notice that a rise in $\overline{Y}_T$ raises Y, C and n but has a negative effect on the steady state real exchange rate, s. At the steady state level, we can thus observe some sort of "Dutch disease" phenomenon: a sudden expansion in the tradable sector is spread in the economy hence expanding nontradable output and consumption, but also gives place to a more appreciated real exchange rate in the long-run.

Table 7, below, shows a subset of the key endogenous variables of the model relative to K[26]. In order to illustrate the differences between the case with and without credit market imperfections, we also compute the steady state solutions when $\mu = 0$.

Table 7. Steady state solutions of the main real variables of the model

Variable	Credit markets imperfections case: $\mu \in (0, 1]$	RBC case: $\mu = 0$
$\frac{Y}{K}$	$r^{k*} \frac{1}{\alpha\gamma} \frac{\theta}{\theta-1}$	$\frac{1}{\beta} \frac{1}{\alpha\gamma} \frac{\theta}{\theta-1}$
$\frac{i}{K}$	$\frac{1}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{b}{K}$	$r^{k*} \frac{\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	1
$\frac{n}{K}$	$r^{k*} \frac{(1-\upsilon) f(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$	0

Without asymmetric information problems (i.e., when $\mu = 0$), it can be easily seen that $f(\overline{\omega}^*) = 0$ and that $g(\overline{\omega}^*) = 1$[27]. This case becomes a fairly standard RBC model, similar to that developed, for instance, in McCallum (1989)[28],[29]. Since we have assumed that capital fully depreciates within the period, when $\mu = 0$ investment i and the capital stock K coincide, as we would expect: $i/K = 1$. With asymmetric information, since a fraction of investment is lost owing to the presence of agency costs, we have that $i/K > 1$.

Notice also that, although the steady state level of output is higher in the RBC case, the ratio Y/K is lower compared to that obtained with agency costs (see Table 7). This fact is a direct consequence of the relatively higher increase in K, compared to Y, that is obtained when monitoring costs are eliminated.

An interesting feature of the RBC case is the fact that as $\mu \to 0$, $n \to 0$. From Eq. 61 we can also observe that $C^e \to 0$, implying that entrepreneurs' consumption must also be zero.

[26]Notice that Table 7 will be exactly the same whether we assume that $f = 0$ or not.

[27]Notice that when $\mu = 0$, $\overline{\omega}^* = 2$. From Appendix D, this fact implies that $f(\overline{\omega}^*) = 0$ and therefore $g(\overline{\omega}^*) = 1$.

[28]For the case in which McCallum assumes full capital depreciation, a Cobb-Douglas production function and logarithmic preferences.

[29]It should be noted, however, that other relevant differences between these models remain. McCallum considers a closed-economy model, with flexible prices and without money. However, the assumption previously made that $f \equiv 0$ and that the nominal price rigidity only lasts one period imply that the steady state solutions of the two models do not differ for these reasons. Both solutions differ, nevertheless, since McCallum assumes perfectly competitive sectors. In the present paper we are considering that intermediate nontradable firms face monopolistic competition. This fact indeed yields slightly different steady state solutions.

Although this observation is pointed out in Carlstrom and Fuerst (2001, p. 12), they do not discuss the issue in depth.

Remark 2 *Without asymmetric information problems entrepreneurs disappear as a differentiated class from households. Without credit constraints there is no need for accumulation of net worth. The supply of capital in the economy is thus directly provided by households in the form of savings for future consumption; and thus can be interpreted as a one to one linear transformation from savings:* $b \equiv K = i = Y - C$.

Since the real amount of debt b and the stock of capital K coincide in the RBC case, we have $b/K = 1$ and $\frac{n}{K} = 0$ as shown in Table 7.

6 Dynamics

To study the dynamics of the model it is useful to take advantage of the dichotomy which exists between the monetary and the real side of the economy, due to the assumed households' logarithmic preferences. A similar approach for solving their respective models is taken, for instance, in Benassy (1995) and in Fender and Rankin (2003).

Since this model is highly non-linear, however, we will analyse its dynamic properties undertaking a linear approximation of it about the zero-inflation steady state. With a number a few exceptions, a lower-case variable denotes a percentage deviation of the original variable with respect to the initial steady state. For instance, for any variable X_t we define $x_t \equiv \frac{X_t - X^{ss}}{X^{ss}}$[30] $\left(\approx \log \frac{X_t}{X^{ss}}\right)$, where X^{ss} is the initial steady state value of X_t[31]. We now start analysing the linear approximation of those variables associated with the monetary side of the model.

$$x_t \equiv m_t - p_t - c_t \tag{74}$$

$$h_t \equiv m_t - m_{t+1} \tag{75}$$

$$x_t = -\frac{\beta}{1-\beta} r_{t+1} \tag{76}$$

[30] Since we previously used lower-case notations for $\overline{\omega}_t$, n_t and i_t their linear approximations are defined as: $\widehat{\omega}_t \equiv \frac{\overline{\omega}_t - \overline{\omega}^*}{\overline{\omega}^*}$, $\widehat{n}_t \equiv \frac{n_t - n^{ss}}{n^{ss}}$ and $\widehat{i}_t \equiv \frac{i_t - i^{ss}}{i^{ss}}$, respectively.

[31] A convenient way for obtaining log-deviations is proposed in Uhlig (1999). For any variable X_t we can define: $X_t = X^{ss} e^{x_t}$. A first order Taylor approximation about the point $x_t = 0$ gives, after rearranging, $x_t \equiv \frac{X_t - X^{ss}}{X^{ss}}$.

$$x_{t+1} = \beta^{-1} x_t - h_t \tag{77}$$

$$s_{t+1} - s_t = r_{t+1}. \tag{78}$$

Defining the demand for real money balances per unit of consumption as X_t ($\equiv \frac{M_t}{P_t C_t}$) and the inverse of the (gross) growth rate of money supply between $t+1$ and t as H_t ($\equiv \frac{M_t}{M_{t+1}}$), it is easy to see that Eqs. 74 and 75 are log-linear versions of these two equations. Substituting the Euler equation for consumption (i.e., Eq. 12), and the UIP condition in the demand for money equation derived in Eq. 13 and linearising about the zero-inflation steady state gives Eq. 76. From the Euler equation, the UIP condition and Eqs. 75-76 it is possible to obtain Eq. 77. It is easy to see that the log-linear version of the UIP condition yields Eq. 78[32].

The solution of the model becomes easier by firstly solving Eq. 77, which is a first order linear difference equation in the forward-looking variable x_t. Since $\beta < 1$, this difference equation is unstable in its forward dynamics. Assuming that h_t is constant over time (i.e., $h_t = h \; \forall \; t$), saddle point stability thus requires that x_t immediately jumps to the steady state value $\frac{h}{\beta^{-1}-1}$.

Since the economic policy exercise in which we are interested is a permanent and unanticipated change in the log-deviation of the money supply at time t, we further have that $h = 0$ (i.e., $m_t = m_{t+1} \equiv \overline{m}$)[33]. In this case, therefore, x_t jumps immediately after the shock to its new steady state value ($= 0$).

Notice that Eq. 74 implies that $c_t = \overline{m} - p_t$, and thus consumption and real money balances move together over time. From Eq. 76 we also have that $r_{t+1} = 0$ and hence Eq. 78 gives $s_t = s_{t+1} \equiv \overline{s}$. This is an important implication of the model since it embeds the fact that the exchange rate immediately jumps to its new steady state value after the shock. This model does not show, therefore, non-trivial exchange rate dynamics as in the case of the well-known overshooting model of Dornbusch (1976).

It is worth highlighting that this dichotomy between the monetary and the real side of the model is not complete since $\overline{s}$ is an endogenous variable and we still have to solve for it. To do that, it will be necessary to consider the real side of the model, the direction in which we are now moving.

[32] Recall the assumption that R_t^* is constant over time.

[33] Observe that $\overline{m}$ can be thought as the percentage deviation of the new steady state level of the money supply ($=M^{ss\prime}$) with respect to the pre-shock steady state level of the money supply ($=M^{ss}$): $\overline{m} \equiv \frac{M^{ss\prime} - M^{ss}}{M^{ss}}$.

To facilitate the exposition, below is presented a list with the key variables of the model, where we have made use of the results previously obtained for the monetary sector.

$$c_{t+1} = p_t - p_{t+1} + c_t \tag{79}$$

$$c_t = \overline{m} - p_t \tag{80}$$

$$z_t = a_t + \alpha k_t + (1 - \alpha) l_t \tag{81}$$

$$y_t = \gamma z_t + (1 - \gamma) x_{T,t} \tag{82}$$

$$y_{T,t} = 0 \tag{83}$$

$$x_{T,t} = y_t - (\overline{s} - p_t) \tag{84}$$

$$z_t = y_t - (p_{N,t} - p_t) \tag{85}$$

$$p_t = (1 - \gamma)\overline{s} + \gamma p_{N,t} \tag{86}$$

$$p_{N,t} = -a_t + (1 - \alpha) w_t + \alpha r_t^k \tag{87}$$

$$\widehat{i}_t = \sigma_1 \widehat{n}_t + \sigma_2 b_{t+1} \tag{88}$$

$$\widehat{\omega}_t = \sigma_6 \widehat{n}_t - \sigma_6 b_{t+1} \tag{89}$$

$$r_{t+1}^k - p_t = \sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t \tag{90}$$

$$\widehat{n}_{t+1} = c_{t+1}^e = b_{t+1} - (p_{t+1} - p_t) + \sigma_4 \widehat{\omega}_t \tag{91}$$

$$k_{t+1} = -\sigma_5 \widehat{\omega}_t + \sigma_1 \widehat{n}_t + \sigma_2 b_{t+1} \tag{92}$$

$$r_t^k - p_{N,t} = z_t - k_t \tag{93}$$

$$w_t - p_{N,t} = z_t - l_t \tag{94}$$

$$l_t = w_t - p_t - c_t \tag{95}$$

$$y_t = \sigma_7 c_t + \sigma_8 c_t^e + \sigma_9 \widehat{i}_t \tag{96}$$

$$-x_{T,t} = \tau_t \tag{97}$$

$$-\beta^{-1} f_{-1} = \sum_{t=0}^{\infty} \beta^t \tau_t \tag{98}$$

where:

$\sigma_1 \equiv (1 + \frac{\beta g(\overline{\omega}^*)}{(1-\upsilon) f(\overline{\omega}^*)})^{-1}$; $\sigma_{1/\mu=0} \equiv 0$
$\sigma_2 \equiv (1 + \frac{(1-\upsilon) f(\overline{\omega}^*)}{\beta g(\overline{\omega}^*)})^{-1}$; $\sigma_{2/\mu=0} \equiv 1$
$\sigma_3 \equiv \frac{1}{4} \frac{(\overline{\omega}^*)^2}{g(\overline{\omega}^*)} - 1$; $\sigma_{3/\mu=0} \equiv 0$
$\sigma_4 \equiv \frac{1}{4} (\overline{\omega}^*)^2 (g(\overline{\omega}^*)^{-1} + f(\overline{\omega}^*)^{-1}) - f(\overline{\omega}^*)^{-1}$; $\sigma_{4/\mu=0} \to -\infty$
$\sigma_5 \equiv (f(\overline{\omega}^*) + g(\overline{\omega}^*))^{-1} - 1$; $\sigma_{5/\mu=0} \equiv 0$
$\sigma_6 \equiv \frac{(\overline{\omega}^* - 2)(\mu + \overline{\omega}^* - 2)}{2\overline{\omega}^{*2} + \mu\overline{\omega}^* - 4\overline{\omega}^* + (2\overline{\omega}^{*2} + 2\mu\overline{\omega}^* - 4\overline{\omega}^*) \frac{(1-\upsilon)}{\beta} \frac{f(\overline{\omega}^*)}{g(\overline{\omega}^*)}}$; $\sigma_{6/\mu=0} \equiv 0$
$\sigma_7 \equiv \frac{C}{Y} = 1 - \alpha\gamma \frac{\theta-1}{\theta} \frac{f(\overline{\omega}^*) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}$; $\sigma_{7/\mu=0} \equiv 1 - \alpha\gamma\beta \frac{\theta-1}{\theta}$
$\sigma_8 \equiv \frac{C^e}{Y} = \alpha\upsilon\gamma \frac{\theta-1}{\theta} (1 + \frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}$; $\sigma_{8/\mu=0} \equiv 0$
$\sigma_9 \equiv \frac{i}{Y} = \alpha\gamma \frac{\theta-1}{\theta} \frac{f(\overline{\omega}^*)(1-\upsilon) + \beta g(\overline{\omega}^*)}{f(\overline{\omega}^*) + g(\overline{\omega}^*)}$; $\sigma_{9/\mu=0} \equiv \alpha\gamma\beta \frac{\theta-1}{\theta}$.

Eq. 79 is derived from the Euler equation for consumption, where we have considered the fact that $r_{t+1} = 0$. Eq. 74 is rewritten in Eq. 80 for the case in which $x_t = 0$. The supply side of the model is described in Eqs. 81-83. The log-linear production function of intermediate firms is defined in Eq. 81, while that of the final producer firm is given by Eq. 82. The log-linear expression for the tradable output, which is equal to zero owing to the assumption that its supply is exogenous and constant over time, is given by Eq. 83.

Eqs. 84 and 85 are log-linear versions of the input demand functions of the final producer firm. We have used the fact that the LOOP holds in this model and thus $p_{T,t} = \overline{s}$. The linearised price index of the economy is represented by Eq. 86; while that of nontradable goods is given by Eq. 87.

The presence of credit market imperfections is essentially reflected in Eqs. 88-92. The linear approximation of aggregate investment (i.e., Eq. 30), is given by Eq. 88[34], where we have used the following definition $b_{t+1} \equiv b^*_{t+1} + \overline{s} - p_t$. Similarly, Eq. 89 is the linear approximation of the aggregate demand for credit equation defined in Eq. 33. Eqs. 90-92 are linear approximations of Eqs. 25, 31, 32 and 34, respectively, considering Eq. 33. It is worth observing that since entrepreneur's net worth and consumption are constant fractions of expected profits ($(1-\upsilon)$ and υ, respectively), the log-deviation of these variables is the same. Eqs. 93-95 are log-linear versions of Eqs. 40, 41 and 14, respectively. Eqs. 96 and 97 are linear approximations of the clearing market conditions for the nontradable and tradable sector.

To facilitate studying the analytics of the model, we approximate the trade balance about an initial steady state in which the trade balance is zero. In this steady state, therefore, net foreign assets are also zero. We thus define τ_t ($\equiv \frac{TB_t}{S\overline{Y}_T}$) as the absolute deviation of the trade balance at period t deflated by the value of tradable output evaluated at that initial steady state (i.e., before the change in the money supply). The linear approximation of the intertemporal national budget constraint defined in Eq. 43 is given by Eq. 98. For further reference, we also define $f_{-1}\{\equiv \frac{S_0 F_0}{P_0}/(\frac{S}{P}\overline{Y}_T)\}$ as the absolute deviation of (inherited) net foreign assets in real terms deflated by the real value of domestic output evaluated at the initial steady state. Observe that F_0 is given from the previous history of the model. In contrast, the real exchange rate $\frac{S_0}{P_0}$ can be affected by current shocks. When setting $F_0 = 0$ Eq. 98 takes the form: $0 = \sum_{t=0}^{\infty} \beta^t \tau_t$. The parameters σ_i, $i = 1, ..., 9$, are complicated functions of the structural parameters of the model. For future comparisons we also include the values for the case in which asymmetric information problems are absent (i.e., $\mu = 0$), which we denote as $\sigma_{i/\mu=0}$, $i = 1, ..., 9$. Notice that $\sigma_{4/\mu=0} \to -\infty$, implying that net worth is not well defined in such case. This is a direct consequence of the fact that the steady state level of net worth is zero when $\mu = 0$.

6.1 Dynamics without credit market imperfections

Without loss of generality it is assumed that $a_t = 0 \; \forall \; t$. Let $\overline{e} \equiv \overline{s} - \overline{m}$ denote the difference between the steady state level of the exchange rate and the money supply, respectively. From Appendix F it can be seen that Eqs. 79-96 can be reduced to the following system of linear

[34]In obtaining Eq. 88 we also considered Eq. 33: $\frac{S_t B^*_{t+1}}{P_t} = -n_t \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}$.

difference equations in the case in which credit market imperfections are absent (i.e., $\mu = 0$),

$$\begin{bmatrix} k_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{2}{1+\alpha}\frac{1}{\gamma\beta}\frac{\theta}{\theta-1} & 1-\frac{2}{1+\alpha}\frac{1}{\gamma^2}\frac{1}{\alpha\beta}\frac{\theta}{\theta-1} \\ -\frac{1-\alpha}{1+\alpha}\frac{1}{\beta}\frac{\theta}{\theta-1} & \alpha\gamma+\frac{1-\alpha}{1+\alpha}\frac{1}{\gamma\alpha\beta}\frac{\theta}{\theta-1} \end{bmatrix} \begin{bmatrix} k_t \\ c_t \end{bmatrix} + \begin{bmatrix} \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma^2}\frac{1}{\alpha\beta}\frac{\theta}{\theta-1} \\ (\gamma-1)(1-\frac{1-\alpha}{1+\alpha}\frac{1}{\alpha\beta\gamma}\frac{\theta}{\theta-1}) \end{bmatrix} \overline{e}. \quad (99)$$

This representation takes the value of the endogenous variable $\overline{s}$ as given. This variable, however, is not affecting the matrix of coefficients in the above system and thus will not affect the speed of convergence to the new steady state after the shock. Once the solutions of k_t and c_t are obtained, we can solve for $\overline{s}$ considering Eqs. 97 and 98. Notice that at each period t the stock of capital k_t is a predetermined variable, while c_t is a non-predetermined or jump variable. In particular, at $t = 0$, k_0 is also given by the previous history of the model.

To satisfy the saddle point property of the model it is required that the matrix of coefficients of Eq. 99 has one root inside and one root outside the unit circle. In the present case the two roots are given by $\lambda_1 = \alpha\gamma < 1$ and $\lambda_2 = \frac{\theta}{\theta-1}\frac{1}{\alpha\beta\gamma} > 1$; and hence the saddle point property is satisfied. The final solution can be written as,

$$\begin{bmatrix} k_t \\ c_t \end{bmatrix} = \kappa_1 \begin{bmatrix} 1 \\ \alpha\gamma \end{bmatrix} (\alpha\gamma)^t + \frac{1-\gamma}{\alpha\gamma-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \overline{e}. \quad (100)$$

As usual, the constant associated with the unstable root is set to zero so as to eliminate any explosive path. To determine κ_1 we consider the fact that k_t is a predetermined variable, thus yielding: $\kappa_1 = (k_0 - \frac{1-\gamma}{\alpha\gamma-1}\overline{e})$. The system described in Eq. 100 hence gives the solutions of k_t and c_t conditional on $\overline{s}$. It is now possible to solve for this remaining variable. From Appendix F we can write $l_t = -\frac{1}{1+\alpha}\frac{1}{\gamma}c_t + \frac{\alpha}{1+\alpha}k_t + \frac{1}{1+\alpha}\frac{\gamma-1}{\gamma}\overline{e}$, which is equal to zero after substituting for the solutions of k_t and c_t. Therefore, with $\mu = 0$ labour is unaffected by the change in the money supply under flexible prices[35]. The linearised version of the demand for the tradable input can therefore be written as: $x_{T,t} = \alpha k_t + -\gamma^{-1}(\overline{e} + c_t)$ (see Appendix F). Substituting for k_t and c_t we finally arrive at:

$$x_{T,t} = -\overline{e} = -(\overline{s} - \overline{m}).$$

Without nominal price rigidities and assuming zero initial net foreign assets (i.e., $F_0 = 0$), Eqs. 97 and 98 give $0 = (1-\beta)^{-1}\tau_t$. This implies that $\tau_t = 0$, and therefore $\overline{s} = \overline{m}$. In this case, the increase in the money supply does not have current account effects, and therefore

[35] When considering one-period nominal price rigidities at period $t = 0$, however, labour will be affected by the change in the money supply during that period. From $t = 1$ onwards, when prices are fully flexible, the fact that $l_t = 0 \; \forall t \geq 1$ is again recovered.

money is neutral in the long-run. This result is modified when introducing nominal price rigidities, as shown below.

6.1.1 Analysis including short-run dynamics

The case in which intermediate nontradable firms do not adjust prices at period $t = 0$ adds interesting dynamics into the analysis. In what follows, the subindex 0 indicates the short-run value of any given variable (i.e., while prices of intermediate nontradable goods are unchanged). In this case we have that $p_0 = (1-\gamma)\overline{s}$, implying that $c_0 = \overline{m} - (1-\gamma)\overline{s}$. It can be seen that the demand for the tradable input takes the form: $x_{T,0} = y_0 - \gamma\overline{s}$. Let $\sigma \equiv \alpha\gamma\beta\frac{\theta-1}{\theta}$ (< 1). The clearing market condition for nontradable goods can thus be written as: $y_0 = (1-\sigma)\{\overline{m} - (1-\gamma)\overline{s}\} + \sigma k_1$ (since $\widehat{i}_0 = k_1$ under the assumption of full capital depreciation). Immediately arises the fact that the evolution of the capital stock gives persistence to the initial shock. From Eq. 100, we have that $k_1 = \{k_0 - \frac{1-\gamma}{\alpha\gamma-1}(\overline{s} - \overline{m})\}\alpha\gamma + \frac{1-\gamma}{\alpha\gamma-1}(\overline{s} - \overline{m})$. After some manipulations the demand for tradable inputs at period $t = 0$ takes the form:

$$x_{T,0} = \overline{m}(1 - \sigma\gamma) - \overline{s} + \sigma\alpha\gamma k_0 = -\tau_0. \tag{101}$$

Since prices are fully flexible $\forall t \geq 1$ we also have that,

$$x_{T,t} = -\overline{e} = -(\overline{s} - \overline{m}) = -\tau_t, \forall t \geq 1.$$

The trade balance is hence divided in two differentiated phases. There is a short-run trade balance that lasts only one period, when prices of intermediate nontradable goods are unchanged. The second phase is characterised by price flexibility and holds $\forall t \in 1...\infty$. The national intertemporal budget constraint stated in Eq. 98 has to be analysed accordingly. Assuming that the system is in the steady state before the shock, we have that $k_0 = 0$. Eq. 98 gives,

$$\overline{s} = \overline{m} - \{f_{-1}\frac{1-\beta}{\beta} + (1-\beta)\sigma\gamma\overline{m}\}. \tag{102}$$

With the solution of $\overline{s}$ at hand, we can now recover the solutions of the other variables of the model. Notice that that $\overline{e} = -\{f_{-1}\frac{1-\beta}{\beta} + (1-\beta)\sigma\gamma\overline{m}\}$. For comparability with the model developed in Obstfeld and Rogoff (1996, Ch. 10.2) we further assume a zero initial net assets position (i.e., $f_{-1} = 0$). It is thus possible to show that the trade balance surplus in the short- and long-run is given by:

$$\tau_0 = \sigma\beta\gamma\overline{m} = -x_{T,0},$$

and

$$\tau_t = -(1-\beta)\sigma\gamma\overline{m} = -x_{T,t},\ \forall\ t \geq 1.$$

Therefore, the increase in the money supply improves the trade balance surplus on impact. Recalling that tradable output is exogenous and constant over time, it is easy to see then that the improvement in the trade balance surplus is driven by the reduction in the demand for tradable inputs. This fact is a direct consequence of the reduction in the relative price of nontradable inputs. There is therefore a boom in that sector, whose output is given by: $z_0 = \overline{m}(1-\sigma\gamma)$. Notice also that since the stock of capital is zero at $t = 0$, the rise in the nontradable output is only possible if labour increases at that time. It can be easily seen that $l_0 = \frac{\overline{m}(1-\sigma\gamma)}{1-\alpha}$[36].

The steady state trade balance is reached at $t = 1$. Notice that from this period onwards the positive wealth effect generated on impact (i.e., there is an accumulation of net foreign assets) allows the country to reduce future trade balance surpluses. Therefore, in this framework, we have the result that money is not neutral in the long-run. Setting $\alpha = 0$ (i.e., the model without capital), recovers the long-run neutrality of money. In this case there are no current account imbalances and $\tau_0 = \tau_t = 0\ \forall\ t \geq 1$, bringing similar results as those obtained in Obstfeld and Rogoff (1996, Ch. 10.2); where labour is the only input utilised in production.

Remark 3 *The introduction of capital in the model in combination with one-period nominal price rigidities bring the result that money is not neutral in the long-run. The unexpected and permanent increase in the money supply affects the current account on impact, giving place to an accumulation of net foreign assets. This wealth effect is thus the source of the long-run non-neutrality of money.*

The accumulation of net foreign assets also affects the new steady state level of the remaining endogenous variables. To explain this, observe that the short- and long-run solutions of the capital stock are given by,

$$k_0 = 0,$$

and

$$k_t = \frac{1-\gamma}{1-\alpha\gamma}(1-(\alpha\gamma)^t)(1-\beta)\sigma\gamma\overline{m}, \forall t \geq 1.$$

[36]Interestingly, it can be shown that neither z_0 nor l_0 are affected by f_{-1} in the more general case in which $f_{-1} \neq 0$.

Although $k_0 = 0$, the monetary shock has effects on the evolution of capital from period $t = 1$ onwards. The supply of capital increases over time until the new steady state is reached. A similar result is also observed in the case of the final nontradable good, whose solutions for the short- and long-run are stated below:

$$y_0 = \gamma \overline{m} \{(1 - \sigma) + (1 - \gamma)(1 - \beta)\sigma\},$$

and

$$y_t = (1 - \gamma) \frac{1 - (\alpha\gamma)^{t+1}}{1 - \alpha\gamma} (1 - \beta)\sigma\gamma\overline{m}, \ \forall\, t \geq 1.$$

We can observe that there is a short- and long-run positive impact on output. As previously mentioned, the production of nontradable intermediate inputs is demand-determined in $t = 0$, boosted by the positive effect of $\overline{m}$ on y_0. Although labour increases at period $t = 0$, for $t \geq 1$ we have shown that $l_t = 0$. This fact will negatively affect y_1 compared to y_0. However, the increase in the supply of capital from $t = 1$ pushes up output. Consumption, not discussed here, is also positively affected by the monetary shock. Finally, it is worth highlighting that although the nominal exchange rate immediately jumps to its new steady state value, the real exchange rate shows interesting non-trivial dynamics. Owing to the assumption that prices of nontradable goods do not adjust at $t = 0$, there is a sort of short-run overshooting in the real exchange rate. Simple computations show that at period $t = 0$ this is given by,

$$\overline{s} - p_0 = \gamma\overline{s} = \gamma\overline{m}\{1 - (1 - \beta)\sigma\gamma\}.$$

Hence, the real exchange rate depreciates in the short-run. For $t \geq 1$ we have,

$$\overline{s} - p_t = -\left(\frac{\gamma(1 - \alpha) + (1 - \gamma)(\alpha\gamma)^{t+1}}{(1 - \alpha\gamma)}\right)(1 - \beta)\sigma\gamma\overline{m}.$$

Clearly, the real exchange rate becomes more appreciated in the new steady state after the shock. This effect is consistent with the accumulation of net foreign assets at period $t = 0$. Notice that the increase in the domestic price level at period $t = 1$, due to the rise in prices of nontradable goods, is one of the main source of this real appreciation. The positive wealth affect given by the accumulation of net foreign assets also increases aggregate demand, thus pushing up the price level. This further contributes to appreciate the real exchange rate. For illustrative purposes, Figure 2 sketches the impulse-response functions of the variables discussed previously.

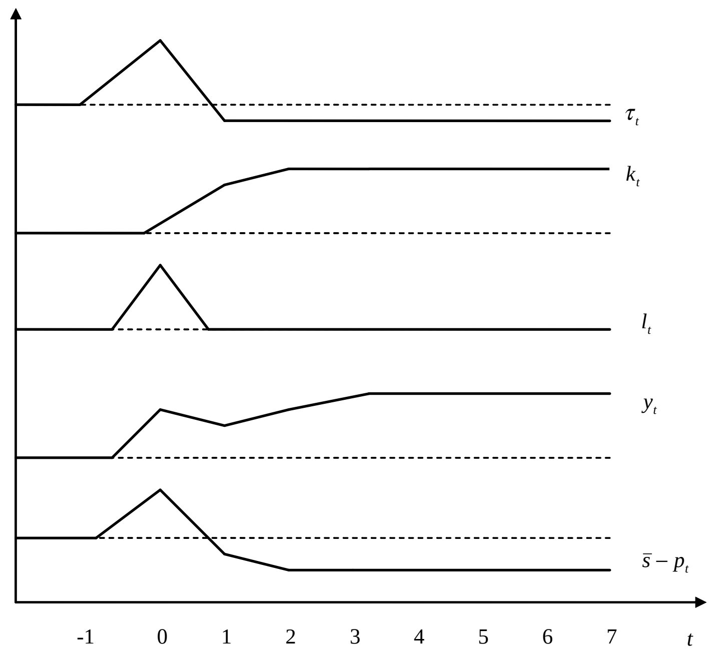

Figure 2. Impulse-response functions.

6.2 Dynamics with credit market imperfections (under current research)

From Appendix F it can also be shown that Eqs. 79-96 give a linear system in the endogenous variables k_{t+1}, $\widehat{n}_{t+1}$ and c_{t+1}. It is worth noting that when credit market imperfections are introduced into the model the stock of capital k_t and net worth $\widehat{n}_t$ become the predetermined variables at each period t. In particular, at $t = 0$, k_0 and $\widehat{n}_0$ are given. Consumption c_t, again, is the non-predetermined or jump variable. The introduction of credit market imperfections therefore adds an additional state variable into the analysis, expanding the dynamic behaviour of the model. Since the coefficients of the system are highly nonlinear, verifying that the saddle point property is satisfied requires some sort of calibration-type exercise. Preliminary results show that this is the case[37]. Moreover, it can be also demonstrated that l_t is now affected even with flexible prices for all $t \in 0, ...\infty$. As a consequence of this, the trade balance does not reach the steady state at period $t = 1$ (as it was the case without credit market imperfections).

[37]Setting $\mu = 0.12$, $\beta = 0.98$, $\alpha = 0.33$, $\gamma = 0.7$, $\theta = 10$ and $\upsilon = 0.12$ we have been able to prove that the matrix of coefficients of the system of difference equations has the following roots: 0.23, 0.94, and 5.01. Therefore, there are two roots inside and one root outside the unit circle; and hence the saddle point property is satisfied.

7 Concluding Remarks

This paper develops a fully microfounded two sectors small open economy model with imperfect competition in the intermediate nontradable sector, infinitely lived agents, currency mismatches in the denomination of assets and liabilities and imperfect credit markets.

The economic policy exercise considered here is an unexpected and permanent increase in the level of the money supply under a floating exchange rate regime. It was shown that the nominal exchange rate does not show non-trivial dynamics as in the Dornbusch (1976) well known overshooting model.

When studying the dynamic properties of the model without credit market imperfections, it was demonstrated that the long-run neutrality of money is not satisfied due to the accumulation of net foreign assets in the period of the shock. It was also shown that the currency depreciation is expansionary as in the Mundell-Fleming textbook model. When considering credit market imperfections entrepreneurs' net worth is included as an additional state variable into the model, thus affecting the dynamic behaviour of the system. It remains to be answered, therefore, whether credit market imperfections can explain contractionary depreciations or not. Another important aspect of the model that could modify these results is the assumption of zero initial net foreign assets. If the currency depreciation takes place in an economy with initial net foreign liabilities, it might be the case that the real value of these liabilities rises with the exchange rate; thus generating negative wealth effects that might overcome the positive expenditure-switching effects of the currency depreciation on output. A natural extension of the paper will be to study the model under a fixed exchange rate regime. Welfare comparisons can also be introduced in a future stage of this research.

Appendix A

The maximisation problem of the entrepreneur can be written in terms of the following Lagrangean,

$$\max_{\{i_{j,t},\overline{\omega}_{j,t},\lambda\}} L = R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t})], \tag{AA1}$$

The associated first order conditions are:

$$\frac{\partial L}{\partial i_{j,t}} = R^k_{t+1} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t] = 0, \tag{AA2}$$

$$\frac{\partial L}{\partial \overline{\omega}_{j,t}} = R^k_{t+1} i_{j,t} f'(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t})] = 0, \tag{AA3}$$

and

$$\frac{\partial L}{\partial \lambda} = R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t}) = 0. \tag{AA4}$$

Note that Eq. AA3 implies $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$. Replacing this expression in Eq. AA2 and rearranging gives Eq. 20. Solving Eq. AA4 for $i_{j,t}$ gives Eq. 21.

Appendix B

From the main text we have: $f(\overline{\omega}_{j,t}) = \int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Observe that $f(\overline{\omega}_{j,t})$ can be written as $f(\overline{\omega}_{j,t}) = \int_0^{\infty} \omega\phi(\omega)d\omega - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Recalling that $E(\omega) = \int_0^{\infty} \omega\phi(\omega)d\omega = 1$, we can obtain,

$$f(\overline{\omega}_{j,t}) = 1 - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$f'(\overline{\omega}_{j,t}) = -[1 - \Phi(\overline{\omega}_{j,t})], \tag{AB1}$$

and

$$f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t}), \tag{AB2}$$

implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$.

Similarly, from the main text we have,

$$g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1 - \Phi(\overline{\omega}_{j,t})], \tag{AB3}$$

and

$$g''(\overline{\omega}_{j,t}) = -[\mu\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]. \tag{AB4}$$

Let us now consider the first order condition stated in Eq. 20. After rearranging terms, this equation takes the form,

$$\frac{R^k_{t+1}}{R_{t+1}P_t} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}.$$

Let us define $G(\overline{\omega}_{j,t}) = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}$. Therefore,

$$\frac{\partial G(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-2}\frac{f(\overline{\omega}_{j,t})g'(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})^2}\{\frac{g''(\overline{\omega}_{j,t})f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} - f''(\overline{\omega}_{j,t})\} > 0,$$

considering the second order conditions of the entrepreneur's maximisation problem (see Appendix C) and the fact that in equilibrium $g'(\overline{\omega}_{j,t}) > 0$. This implies that $\frac{\partial\overline{\omega}_{j,t}}{\partial R^k_{t+1}} > 0$, as explained in the main text.

Appendix C

To obtain the second order conditions we also need the following partial derivatives of the Lagrangean analysed in Appendix A:

$$\frac{\partial^2 L}{\partial i^2_{j,t}} = 0,$$

$$\frac{\partial^2 L}{\partial i_{j,t}\partial\overline{\omega}_{j,t}} = \frac{\partial^2 L}{\partial\overline{\omega}_{j,t}\partial i_{j,t}} = R^k_{t+1}[f'(\overline{\omega}_{j,t}) + \lambda g'(\overline{\omega}_{j,t})] = 0^{38},$$

$$\frac{\partial^2 L}{\partial\overline{\omega}^2_{j,t}} = R^k_{t+1}i_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})]$$

[38] To obtain this result, it is worth recalling that $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$.

$$\frac{\partial}{\partial i_{j,t}}(\frac{\partial L}{\partial \lambda}) = R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t,$$

and

$$\frac{\partial}{\partial \overline{\omega}_{j,t}}(\frac{\partial L}{\partial \lambda}) = R^k_{t+1}i_{j,t}g'(\overline{\omega}_{j,t}).$$

We can now form the bordered Hessian,

$$H = \begin{bmatrix} 0 & R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t & R^k_{t+1}i_{j,t}g'(\overline{\omega}_{j,t}) \\ R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t & 0 & 0 \\ R^k_{t+1}i_{j,t}g'(\overline{\omega}_{j,t}) & 0 & R^k_{t+1}i_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})] \end{bmatrix}.$$

To satisfy the associated second order condition for a maximum, we need the determinant of the matrix H to be greater or equal to zero (see, for instance, Simon and Blume 1994, p. 461). This determinant takes the form,

$$|H| = -R^k_{t+1}i_{j,t}[R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t]^2[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})].$$

Since $[R^k_{t+1}g(\overline{\omega}_{j,t}) - R_{t+1}P_t]^2 \geq 0$, to satisfy the second order condition it is needed that $[f''(\overline{\omega}_{j,t}) - \frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}g''(\overline{\omega}_{j,t})] \leq 0$. Whenever ω is uniformly distributed in the interval $[0, 2]$, the second order condition can be written as $1 \leq \frac{(1-\frac{1}{2}\overline{\omega})}{1-\frac{1}{2}(\overline{\omega}+\mu)}$ (see Appendix D). For this inequality to be satisfied, we further require that $g'(\overline{\omega}) = 1-\frac{1}{2}(\overline{\omega}+\mu) > 0$ (or that $\mu < 2-\overline{\omega}$). Moreover, providing that $\mu > 0$ the second order condition is satisfied as an strict inequality.

Appendix D

In this appendix we consider the particular case in which ω is uniformly distributed in the interval $[0, 2]$; therefore the mean of ω is 1, as stated in the main text. The following results immediately follows: $\phi(\overline{\omega}) = \frac{1}{2}$, $\frac{\partial \phi(\overline{\omega})}{\partial \overline{\omega}} = 0$, $\Phi(\overline{\omega}) = \frac{1}{2}\overline{\omega}$ and $1-\ \Phi(\overline{\omega}) = 1 - \frac{1}{2}\overline{\omega}$. Using the results obtained in Appendix B, we can easily obtain:

$$g(\overline{\omega}) = -\frac{1}{4}\overline{\omega}^2 + \overline{\omega}(1 - \frac{\mu}{2}),$$

$$g'(\overline{\omega}) = 1 - \frac{1}{2}(\mu + \overline{\omega}),$$

$$g''(\overline{\omega}) = -\frac{1}{2},$$

$$f(\overline{\omega}) = \frac{1}{4}\overline{\omega}^2 - \overline{\omega} + 1,$$

$$f'(\overline{\omega}) = -(1 - \frac{1}{2}\overline{\omega}),$$

and

$$f''(\overline{\omega}) = \frac{1}{2}.$$

Appendix E

In this appendix it is shown how to obtain the steady state solutions of the main endogenous variables of the model. Recalling that $s = Y(1-\gamma)\overline{Y}_T^{-1}$, from Eq. 59 it is possible to obtain,

$$Y = \overline{Y}_T(\frac{p_N}{\gamma})^{\frac{\gamma}{\gamma-1}}.$$

This equation, however, does not pin-down Y since p_N is an endogenous variable. To solve for p_N note that the equilibrium condition in the labour market, described by Eqs. 53 and 54, in conjunction with Eq. 57 bring the relation,

$$w = \{\gamma(1-\alpha)\frac{\theta-1}{\theta}\kappa Y C\}^{\frac{1}{2}}. \tag{AE1}$$

Hence, the price index for the nontradable good given in Eq. 60 can be expressed as,

$$p_N = A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}\{\frac{\gamma\kappa}{(1-\alpha)}YC\}^{\frac{1-\alpha}{2}}.$$

Combining the above equation with the expression for Y obtained previously and solving for C gives,

$$C = Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}\}^{\frac{2}{\alpha-1}}.$$

This equation gives a relation between two endogenous variables: C and Y. The clearing market condition for the nontradable good, Eq. 64, in combination with Eqs. 61, 62 and 66 also give,

$$n = (1-\upsilon)(1+\beta\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}\{Y - Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}\}^{\frac{2}{\alpha-1}}\}, \tag{AE2}$$

after substituting for C. Observe that Eq. AE2 relates two endogenous variables: n and Y.

We can obtain a second expression in these two variables as follows. The production function of intermediate firms, $Z = AK^{\alpha}L^{1-\alpha}$, and Eq. 57 give,

$$Y = A\gamma^{-1}K^{\alpha}L^{1-\alpha}p_N.$$

Notice that $L^{1-\alpha}p_N = A^{-1}(\frac{\theta}{\theta-1}\frac{r^{k*}}{\alpha})^{\alpha}(\gamma Y)^{1-\alpha}$ (from Eqs. 54, 60 and AE1). Also observe that $r^{k*}K = (1-\upsilon)^{-1}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})n$ (from Eqs. 50, 52, 62 and 66).

Therefore, it is possible to obtain,

$$n = Y\alpha\gamma\frac{\theta-1}{\theta}(1-\upsilon)(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}. \tag{AE3}$$

Substituting Eq. AE3 into Eq. AE2 and rearranging gives the solution for Y,

$$Y = \{1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}\}^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}}\{A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{1}{\gamma}\frac{\theta}{\theta-1})^{\frac{1+\alpha}{2}}(\frac{\kappa}{1-\alpha})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}\}^{\frac{\gamma}{\alpha\gamma-1}}.$$

Having solved for Y we can now obtain the steady state solutions of all the remaining endogenous variables of the model.

Appendix F

This appendix explains how to derive the minimum state-space system described in Eq. 99. Without loss of generality it is assumed that $a_t = 0\,\forall\, t$. Let $\overline{e} \equiv \overline{s} - \overline{m}$ denote the difference between the steady state level of the exchange rate and the money supply, respectively. To express the system in terms of the endogenous variables k_{t+1}, $\widehat{n}_{t+1}$ and c_{t+1}, as a first order linear system of difference equations, it is necessary to firstly obtain the following intermediate results: (i) $p_{t+1} - p_t$ as a function of k_{t+1}, c_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (ii) c_{t+1} as a function of k_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (iii) $\widehat{n}_{t+1}$ as a function of k_{t+1}, c_{t+1}, c_t, $\widehat{n}_t$ and $\overline{e}$; (iv) k_{t+1} as a function of k_t, $\widehat{n}_t$, c_t, $\overline{e}$ and (v) c_{t+1} and $\widehat{n}_{t+1}$ as a function of k_t, $\widehat{n}_t$, c_t, $\overline{e}$. In what follows, we explain in detail how to obtain each of these results.

(i) From Eqs. 80, 81, 82 and 84 it is possible to write $x_{T,t}$ as:

$$x_{T,t} = \alpha k_t + (1-\alpha)l_t - \gamma^{-1}(\overline{e} + c_t). \tag{AF1}$$

Substituting Eqs. 81, 82 and AF1 into Eq. 85 gives:

$$p_{N,t+1} = p_{t+1} + \frac{\gamma-1}{\gamma}\overline{e} + \frac{\gamma-1}{\gamma}c_{t+1}. \tag{AF2}$$

Notice that using Eqs. 81, 94 and 95 it is possible to obtain the following expression for w_{t+1},

$$w_{t+1} = \frac{\alpha}{1+\alpha}(\overline{m} + k_{t+1}) + \frac{1}{1+\alpha} p_{N,t+1}. \tag{AF3}$$

Eqs. 87, 90 and AF3, in turn, bring:

$$p_{N,t+1} - p_t = \frac{1-\alpha}{2}(c_t + k_{t+1}) + \frac{1+\alpha}{2}(\sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t). \tag{AF4}$$

Substituting Eq. AF2 into Eq. AF4 yields,

$$p_{t+1} - p_t = \frac{1-\gamma}{\gamma} c_{t+1} + \frac{1-\gamma}{\gamma}\overline{e} + \frac{1-\alpha}{2}(c_t + k_{t+1}) + \frac{1+\alpha}{2}(\sigma_1 b_{t+1} - \sigma_1 \widehat{n}_t + \sigma_3 \widehat{\omega}_t). \tag{AF5}$$

Notice that from Eqs. 89 and 92 b_{t+1} and $\widehat{\omega}_t$ can be expressed as:

$$\widehat{\omega}_t = \frac{\sigma_6(\sigma_1+\sigma_2)}{\sigma_2+\sigma_5\sigma_6}\widehat{n}_t - \frac{\sigma_6}{\sigma_2+\sigma_5\sigma_6}k_{t+1}, \tag{AF6}$$

$$b_{t+1} = \frac{\sigma_5\sigma_6 - \sigma_1}{\sigma_2+\sigma_5\sigma_6}\widehat{n}_t + \frac{1}{\sigma_2+\sigma_5\sigma_6}k_{t+1}. \tag{AF7}$$

Therefore, $p_{t+1} - p_t$ is given by,

$$\begin{aligned} p_{t+1} - p_t \;&=\; \frac{1-\gamma}{\gamma}c_{t+1} + \frac{1}{2}\{1-\alpha+(1+\alpha)\frac{(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}\}k_{t+1} + \frac{1-\alpha}{2}c_t \\ &\quad -\frac{1+\alpha}{2}\frac{(\sigma_1+\sigma_2)(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}\widehat{n}_t + \frac{1-\gamma}{\gamma}\overline{e}. \end{aligned} \tag{AF8}$$

(ii) From the linearised Euler equation for consumption given in Eq. 79 and using Eq. AF8 it is possible to obtain:

$$\begin{aligned} c_{t+1} \;&=\; -\frac{1}{2}\gamma\{1-\alpha+(1+\alpha)\frac{(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}\}k_{t+1} + \frac{(1+\alpha)}{2}\gamma c_t \\ &\quad +\frac{1}{2}\gamma(1+\alpha)\frac{(\sigma_1+\sigma_2)(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}\widehat{n}_t + (\gamma-1)\overline{e}. \end{aligned} \tag{AF9}$$

(iii) Substituting Eqs. AF5, AF6 and AF7 into Eq. 91 yields,

$$\begin{aligned} \widehat{n}_{t+1} \;&=\; \{\frac{1-\sigma_6\sigma_4}{\sigma_2+\sigma_5\sigma_6} - \frac{1}{2}(1-\alpha+\frac{1+\alpha}{\sigma_2+\sigma_5\sigma_6}(\sigma_1-\sigma_3\sigma_6))\}k_{t+1} + \frac{\gamma-1}{\gamma}c_{t+1} \\ &\quad +\frac{\{(\sigma_1+\sigma_2)(\sigma_6\sigma_4+\frac{1+\alpha}{2}(\sigma_1-\sigma_3\sigma_6))+\sigma_5\sigma_6-\sigma_1\}}{\sigma_2+\sigma_5\sigma_6}n_t - \frac{1-\alpha}{2}c_t + \frac{\gamma-1}{\gamma}\overline{e}. \end{aligned} \tag{AF10}$$

(iv) Observe that Eqs. 80, 95, AF2 and AF3 give:

$$l_t = -\frac{1}{1+\alpha}\frac{1}{\gamma}c_t + \frac{\alpha}{1+\alpha}k_t + \frac{1}{1+\alpha}\frac{\gamma-1}{\gamma}\overline{e}.$$

Substituting this expression into Eq. 82 and using Eqs. 81 and AF1:

$$y_t = \frac{2\alpha}{1+\alpha}k_t + \frac{1}{\gamma}((\gamma-1) - \frac{1-\alpha}{1+\alpha})c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\overline{e}.$$

Finally, substituting the above equation in the clearing market condition stated in Eq. 96 and using Eqs. 88, 91 and AF7 it is possible to obtain:

$$k_{t+1} = 2\frac{\alpha}{1+\alpha}\Delta_1 k_t - \Delta_1\Delta_2\widehat{n}_t + \Delta_1\Delta_3 c_t + \frac{2}{1+\alpha}\frac{\gamma-1}{\gamma}\Delta_1\overline{e}, \tag{AF11}$$

where:

$\Delta_1 \equiv \frac{\sigma_2+\sigma_5\sigma_6}{\sigma_9\sigma_2}$

$\Delta_2 \equiv \sigma_8 + \frac{\sigma_9\sigma_5\sigma_6(\sigma_1+\sigma_2)}{\sigma_2+\sigma_5\sigma_6}$

$\Delta_3 \equiv \frac{\gamma-1}{\gamma} - \frac{1}{\gamma}\frac{1-\alpha}{1+\alpha} - \sigma_7$ (v) Replacing Eq. AF11 into Eq. AF9 gives, after some manipulation:

$$c_{t+1} = -\frac{\alpha\gamma}{1+\alpha}\Delta_4 k_t + \frac{\gamma}{2}\{(1+\alpha)\Delta_6 + \Delta_2\Delta_4\}\widehat{n}_t + \frac{\gamma}{2}\{1+\alpha-\Delta_3\Delta_4\}c_t + (\gamma-1)(1-\frac{\Delta_4}{1+\alpha})\overline{e}. \tag{AF12}$$

Introducing Eqs. AF11 and AF12 into Eq. AF10 yields,

$$\begin{aligned}\widehat{n}_{t+1} \;=\; & \frac{\alpha}{1+\alpha}\{2\Delta_5\Delta_1 - \gamma\Delta_4\}k_t + \{\gamma\frac{(1+\alpha)}{2}\Delta_6 + 1 - \Delta_2\Delta_5\Delta_1 + \frac{\gamma}{2}\Delta_2\Delta_4 - (\sigma_1+\sigma_2)\Delta_5\}\widehat{n}_t \\ & \{\Delta_3\Delta_5\Delta_1 - \frac{1}{2}(1-\gamma)(1+\alpha) - \frac{\gamma}{2}\Delta_3\Delta_4 - \frac{1-\alpha}{2}\}c_t + \frac{\gamma-1}{\gamma}\{\frac{2}{1+\alpha}\Delta_5\Delta_1 + \gamma - \frac{\gamma}{1+\alpha}\Delta_4\}\overline{e},\end{aligned} \tag{AF13}$$

where

$\Delta_4 \equiv \frac{(1-\alpha)(\sigma_2+\sigma_5\sigma_6)}{\sigma_9\sigma_2} + \frac{(1+\alpha)(\sigma_1-\sigma_3\sigma_6)}{\sigma_9\sigma_2}$

$\Delta_5 \equiv \frac{1-\sigma_4\sigma_6}{\sigma_2+\sigma_5\sigma_6}$

$\Delta_6 \equiv \frac{(\sigma_1+\sigma_2)(\sigma_1-\sigma_3\sigma_6)}{\sigma_2+\sigma_5\sigma_6}$.

Finally, since prices are fully flexible in the steady state it must be true that $\overline{e} = 0$.

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