Factor Replacement versus Factor Substitution, Mechanization and Asymptotic Harrod Neutrality

Danny Givon*

(March 2006)

Abstract:

This paper views technical change as a labor-saving, but capital-*using*, mechanization process, whereby capital *replaces* labor; though within any given technique, factors have a limited ability to substitute one another. This is formalized by reinterpreting the "distribution-parameters" of a low substitution CES aggregate production function as time-varying weights, such that technical change corresponds to a decrease in labor's weight, along with an increase in capital's. This "direction" of shift is considered a natural outcome of the fact that ideas are embedded within *capital*. As capital's weight tends to one, changes in it become increasingly negligible and balanced-growth is attained. Thus the proposed non-neutral mechanism is asymptotically equivalent to Harrod-neutrality. But during industrialization, when capital grows faster than output, its "*dis*-augmentation" is still significant; the result being constant factor-shares. This resolves a recent controversy regarding the measurement of TFP growth, specifically in East Asian NICs. The capital-using aspect of factors' replacement, along with the limited degree of factor substitution, also lead to time-ranked "appropriate-technologies", which are broadly consistent with under-development; despite the lack of non-convexities.

Keywords: Mechanization, Non-Neutral Technical Change, Dis-Augmentation, CES
JEL Classification: O33, O11, O14, E25

---

* E-mail: msdanig@mscc.huji.ac.il. I wish to thank: Buly Cardak, Mark Gradstein, Zvi Hercowitz, Charles Jones, Vikram Pathania, Dean Scrimgeour, David Wettstein, Joseph Zeira and seminar participants at Tel-Aviv and Ben-Gurion universities. An earlier version of section four has benefited from comments by: Philippe Aghion, Benny Bental, Benny Berdugo, Oded Galor, Moshe Hazan, Eran Manes, David Weil and Hosni Zoabi. I am grateful for the hospitality of U.C. Berkeley, where the paper was basically conceived.

1. **Introduction**

Long-run technical change can be described as a process by which physical capital undergoes significant qualitative changes due to new innovations, which enable its replacing of manual and/or cognitive services previously performed by labor. This view of a mechanization process is indeed unavoidable when thinking of initial industrialization.[1] Mechanization may also be a feature of more recent changes of technique; though these are clearly much less "dramatic" in featuring factor replacement.

As for actual modeling, the notions of mechanization or factor replacement are seldom formalized.[2] Rather, models wishing to comply with the stylized facts of industrialized economies, namely balanced growth with constant factor shares in income, are compelled to formalize strict "labor-augmentation", or Harrod neutrality.[3] Thus technical change is typically modeled not as an inherent change in the "shape" of the aggregate production function, as one might have supposed, but as an increment in a "technology index" ("A_t") multiplying the labor factor; simply interpreted as labor's "efficiency" or "productivity". Among other things, this means that producing the same level of output with a new technique is indeed "labor-saving", but leaves the capital requirement unchanged. Should one wish to model mechanization, whereby technical change is "capital-*using*" as well, the result would feature *non*-neutrality, such that labor is "augmented", but capital is "*dis*-augmented".

Through a refinement of the "factor efficiency augmentation" notion, the current paper is able to show that mechanization asymptotically tends to strict "labor augmentation", and thus there is

---

[1] Historical accounts of mechanization go at least as far back as Smith (1776), who notes that "every body must be sensible how much labour is facilitated and abridged by the application of proper machinery. It is unnecessary to give any example" (I.I.8). See more comprehensive accounts in Habakkuk (1962).

[2] A notable exception is Zeira (1998). Alternatively, high factor substitution variants of the "AK model" are occasionally considered as formalizing mechanization; albeit in somewhat extreme "reduced-form".

[3] See Harrod (1937), Robinson (1938), Uzawa (1961), and Barro & Sala-i-Martin (2004, ch 2).

no contradiction between these concepts. Technically, the proposed model reinterprets the CES "distribution parameters" of labor and capital as factors' respective "weights" or "impacts" on the production process; hence mechanization is formalized as a decrease in the weight or impact of labor, along with a simultaneous increase in the impact of capital. Since production techniques are characterized by a low elasticity, regardless of the technological level, capital accumulation brings about a rise in the valuation of labor, such that low factor substitution and a high degree of factor replacement balance each other.[4]

Despite the ongoing process of mechanization, the changes in capital's impact, or weight, become increasingly negligible as it asymptotically tends to one. Harrod neutrality and balanced growth are therefore attained in the limit. Thus, the idea of mechanization, formalized here by the shifting of factors' impacts or weights, provides a simple and intuitive explanation for the apparent "direction" of technical change.[5]

Applying insights gained by the "idea-based" new-growth literature of the 1990's, the current paper views mechanization as a result of the fact that production labor is the "basic" factor of production, empowered merely by the capital it operates.[6] The essence of the motor or cognitive services provided by labor is thus assumed to remain relatively unchanged through time, whereas capital undergoes persistent qualitative changes due to its enhancement by new innovations, or ideas, thereby increasing its impact on production while eroding labor's.

In centering on mechanization, the model is able to be consistent not only with the stylized facts of industrialized economies but of industrializing ones as well; two phases of growth which are difficult to reconcile. In particular, when assuming a low substitution aggregate production

[4] Careful econometric analyses have been consistently estimating low elasticities of substitution of the aggregate function for over 40 years. See David & van de Klundert (1965), and most recently Antràs (2004).
[5] The standard notion of "labor augmenting technical change" requires a theoretical analogue referring to capital, and consequently a theoretical need for explaining why technical changes are indeed *labor*, rather than *capital*, "augmenting". This theme is central to the "induced innovation" literature, initiated by Samuelson (1965) among others, which has been revived recently by Acemoglu (2003) and Jones (2005), within new-growth settings.
[6] As in many other idea-based models, human capital, though obviously a necessary ingredient, is absent here.

function, undergoing "factor-augmenting" technical change, then the rapid deepening of capital during industrialization ought to result in a decline of the capital share in income; a trend which does not seem to be empirically supported.[7]

Otherwise, technical change would have to be *non*-neutral, such that capital's "*dis*-augmentation" counteracts the fact that capital deepens at a faster rate than output growth, or the rate at which labor is apparently "augmented".[8] While this possibility has not gone unnoticed by the empirical literature, the deeply-rooted theoretical perception of "factor augmentation" has led scholars to reactions ranging from uneasiness to shear sarcasm whenever indeed confronted by capital "dis-augmentaion".[9] The contribution of the current model is therefore in theoretically grounding non-neutrality during industrialization, and thus reconciling low factor substitution with the relative constancy of factor shares at that stage. This helps resolve a recent controversy concerning growth accounting of the East Asian newly industrialized countries (NICs).

In addition to balanced growth and industrialization, the model is compatible with the experience of least developed countries (LDCs) as well. In particular, the inherent non-neutrality generates the result by which no technique dominates another, but rather the optimal technique is a function of the capital-labor ratio; thus an "appropriate technology". The "poverty-trap" literature indeed highlights the interaction of institutional arrangements, such as credit constraints, with appropriate technologies, usually induced by fixed-costs, in generating capital misallocation and under-development.[10] Though the current paper formulates fully convex

[7] A general lack of trend in factor shares, specifically during industrialization, is shown by Young (1995) and Gollin (2002), in longitudinal and cross-sectional contexts, respectively. Alternatively, see Abramovitz & David (1973), who document a consistent *rise* in the U.S. capital share during the19th century.

[8] Of course a unit elasticity delivers constant factor shares regardless of the capital-output or capital-labor ratios. Applying a sophisticated stochastic setup Jones (2005) indeed derives an inter-technique Cobb-Douglas form from low-substitution individual processes.

[9] Relaxing the Cobb-Douglas restriction, Caselli (2005) is among the first to accept non-neutrality, deducing from it that industrialized economies choose technologies which are *less* efficient in their use of physical capital. See also Young (1998).

[10] See Galor & Zeira (1993) and more recently Banerjee & Duflo (2004).

technologies, the intuition here is similar. Specifically, capital's replacement of labor can be seen as a dynamic equivalent of a fixed cost, thus an "*inter-temporal* non-convexity".

The paper is organized as follows. The next section introduces the basic model, which is a standard CES function, but with a "twist" whereby constants are made time dependant, and vice-versa. This is followed by a parameter transformation, showing the asymptotic equivalence to a Harrod neutral model. The third section emphasizes the non-neutrality aspects of the model; theoretically and empirically relevant for industrializing as well as under-developed countries. The fourth section elaborates by presenting an idea-based growth model. The fifth section concludes.

2. Basic Model

2.1 Replacement vs. Substitution

Before presenting the model, it seems worthwhile to elaborate on the fundamental distinction the current paper wishes to make between the notions of "factor replacement" and "factor substitution". "Substitution" is the standard producer-theory concept characterizing the curvature of the isoquants and indexed by the elasticity of substitution; thus a static property of any given technique. Alternatively, "replacement" (or "long-run substitution") refers to a dynamic process of mechanization, where the same level of output can be produced by using *less* labor but *more* capital, in terms of value. The notion of factor replacement is rarely incorporated in growth models, due to the fact that it is in stark contrast to neutrality, whether of the Hicks or the Harrod type.

To grasp the inherent non-neutral nature of "replacement", consider the following simplistic example. Suppose a hydraulic excavator and its driver have the same output as 200 workers with shovels. If the excavator is worth more than 200 shovels (which is clearly an understatement) then capital "*dis*-augmentation" is induced. Such a (one shot) replacement of "workers" by "machines" is highlighted by Zeira (1998). More formally, and applying a Leontief function for simplicity, suppose the "shovel technology" is: $Y_v = \min[a_v L, d_v K]$, while the "excavator technology" is: $Y_x = \min[a_x L, d_x K]$. Mechanization, as defined above, requires: $a_x > a_v$ (labor saving/"augmenting") but also $d_v > d_x$ (capital using/"dis-augmenting"). See figure 1. Harrod neutrality, on the other hand, requires: $d_v = d_x$; displaying *strict* "labor augmentation" and a single relevant index for the technology level ("*a*").[11]

At least three things seem bothering in the above formulation. First, how does non-neutral technical change comply with the necessary requirement for strict "labor augmentation"? Second, instead of a single technology index, there are two: *a* and *d*, which seem to lack not only a conceptual mapping onto some "real-life" entity but also any connection between them. Third, it is not clear why and how do new innovation cause the specific non-neutral changes in these indices. While the attempt to answer the last question will be a dominant part of section four, the remainder of the current section will focus on the first two.

2.2 Dynamic CES, with a "Twist"

Consider the following CES production function of labor (*L*) and (physical) capital (*K*):

[11] Assume the output of an excavator or 200 shovels is normalized to one. Denote the prices of an excavator and a single shovel as p_x and p_v, respectively. Disregarding "qualitative" differences in labor (i.e. the knowledge of how to operate an excavator vs. the knowledge of operating a shovel), our numeric example assumes: $a_x = 1$ and $a_v = 1/200$, thus: $a_x = 200a_v > a_v$. As for capital, we have: $d_x = 1/p_x$ and $d_v = 1/200p_v$; thus non-neutrality requires: $p_x/p_v > 200$.

$$Y_t = \left[m_t(\lambda L_t)^{\rho} + (1-m_t)(\mu K_t)^{\rho}\right]^{\frac{1}{\rho}}. \tag{1}$$

While this formulation is similar to the standard one, introduced by Arrow, Chenery, Minhas & Solow (1961), three points should be stressed. First, the parameter ρ is restricted here to being strictly negative and finite, that is a low elasticity of substitution: $\sigma \equiv \frac{1}{1-\rho} \in (0,1)$.[12] Second, the strictly positive parameters λ and μ are *not* to be seen as time-varying "factor efficiency" indices, but rather as constant "unit adjustments".[13]

The third, and most important point concerns the time dependency of m ($m \in (0,1]$) and $1 - m$, originally termed by Arrow et al. (1961): the "distribution parameter(s)"; these determining the exact distribution of income only in the limiting Cobb-Douglas case ($\rho \to 0$). When $\rho \neq 0$, as in the current model, factor shares in income are also dependant on the capital-labor ratio, where capital deepening either raises or lowers the labor share, depending on whether the elasticity of substitution is low or high, respectively. Thus, with low factor substitution a rise in the capital-labor ratio and a reduction in m counteract each other concerning the factor shares.

Rather than "distribution parameters", the current paper interprets m and $1 - m$ as indicating the *weights*, or *impacts* of labor and capital on production, respectively. Furthermore, mechanization is viewed as a reduction in m, and thus an increase in $1 - m$. More specifically, we shall assume an exponential decline of m, given an initial m_0:

[12] Low elasticity reflects the limited ability to substitute factors within any given technique (as opposed to the high degree of factors' "replacement" induced by technical change). The model requires the Neo-Classical assumption of $\sigma > 0$ (i.e. *not* perfect complementarity), which is justified by "allowing" a certain degree of flexibility in the manner of applying the given technique; e.g. shift work, plant configuration, etc.

[13] Arrow et al. (1961) originally did not incorporate such "efficiency" indices, but rather a Hicks neutral one. But as we shall see below, λ is indeed of little importance, while "back of the envelope" calculations give: $\mu \cong 1$. Thus in the more elaborate model of section four these parameters shall be omitted for clarity.

$$m_t = m_0 e^{g_m t}\ ;\ g_m < 0\ , \dot{g}_m = 0\ \ \forall t, \tag{2}$$

where "g_z" denotes the exponential growth rate ($\frac{\dot{z}}{z}$) for any variable "z" used hereafter.

To help perceive the notion of "factors' impacts", consider both extreme cases, where labor's impact is either 1 or 0; output, as defined by (1), thus being either $Y = \lambda L$ or $Y = \mu K$, respectively. Such extreme differences in technique are an indication of the qualitative change in the "shape" of the production function induced by the current formulation of technical change. The case of $m = 1$ means that production is solely based on labor, thus a fully manual ("bare hands") technology, whereas $m = 0$ is a fully automated, "artificially intelligent" production process. Since even the most primitive conceivable techniques require some sort of capital, while even the most advanced ones require some sort of human intervention or presence, both polar cases are of course hypothetical. "Reality" is always somewhere in-between these extremes, such that technical change has an unambiguous direction, bringing techniques ever-closer to the "capital only" extreme; though never actually reaching it.

Somewhat equivalent to the question which motivated the "induced innovation" literature of the 1960's (e.g. Samuelson, 1965), namely why is labor "augmented" rather than capital, one may ponder regarding the assumption by which labor's impact is decreasing, rather than increasing. Section four provides an attempt to rationalize this "direction" of technical change based on more elaborate micro-foundations. It shall be argued that behind the notion of mechanization lies the fact that new knowledge is always embedded within capital, while the basic services provided by production labor remain unchanged; thus technical change increases capital's impact at the expense of labor's.

In the case of a low factor substitution not only do the marginal products tend to zero, but also (unlike the Cobb-Douglas case) the level of output is bounded when either of the two factors is being held constant; even as the input of the other factor increases. Though the point above, as well as the analyses below will be valid for any strictly negative value of ρ, it is helpful to keep in mind the simple example of $\rho = -1$ ($\sigma = 0.5$), in which case production takes the weighted harmonic mean form; m and $1 - m$ indeed serving as the weights:[14]

$$Y_t = \left(\frac{m_t}{\lambda L_t} + \frac{1 - m_t}{\mu K_t} \right)^{-1} . \tag{1'}$$

With this simple specification we clearly see how the constancy of either factor, in particular the non-accumulable labor factor, serves as a "drag" on the benefits from the accumulation of the other. It is thus natural to consider technical change as working through an alleviation of this limit, namely decreasing the impact, or weight of the hindering factor. This form also highlights the fact that when $\rho < 0$ the impacts, or weights, operate rather counter-intuitively. Specifically, a fall in m (disregarding the rise in $1 - m$), actually *increases* output.

Based on (1') we can give a somewhat alternative intuition for the growth experience. Given technical changes that decrease labor's impact and relax its drag on output, capital's accumulation soon follows, the result conceivably being balanced growth: a complementary process of technical change and capital deepening. Since a decrease in labor's impact, induced by technical change, makes labor more redundant, while capital deepening, given low substitution, makes labor scarcer, the constancy of factors shares is, again, a balance between these two forces.

---

[14] Econometric estimates of the elasticity of substitution typically narrow it to the 0.3 – 0.7 range. See David & van de Klundert (1965) and Antràs (2004). See also the partial review by Acemoglu (2003, footnote 3).

2.3 Asymptotic Equivalence to Harrod Neutrality

We now turn to applying a simple parameter transformation, which will show how the model is asymptotically equivalent to standard Harrod neutrality, or "labor augmenting" technical change. We shall define the following "synthetic" entities, which have no conceptual basis and are merely convenient transformations of the impact, or weight parameters:

$$B_t \equiv m_t^{\frac{1}{\rho}}; \tag{3}$$

$$Q_t \equiv (1-m_t)^{\frac{1}{\rho}}. \tag{4}$$

In the case of the harmonic mean ($\rho = -1$) B and Q are respectively the *reciprocals* of the labor and capital weights: $B \equiv \frac{1}{m}$ and $Q \equiv \frac{1}{1-m}$.

The revised production function and the capital share (θ^K) are thus, respectively:

$$Y_t = \left[(\lambda B_t L_t)^\rho + (\mu Q_t K_t)^\rho\right]^{\frac{1}{\rho}}; \tag{5}$$

$$\theta_t^K = \left[1 + \left(\frac{\lambda B_t L_t}{\mu Q_t K_t}\right)^\rho\right]^{-1} = \frac{(\mu Q_t K_t)^\rho}{(\lambda B_t L_t)^\rho + (\mu Q_t K_t)^\rho} = \left(\frac{\mu Q_t K_t}{Y_t}\right)^\rho. \tag{6}$$

The expression for the labor share ($1-\theta^K$) simply swaps the terms $\lambda B_t L_t$ and $\mu Q_t K_t$.

The parameter B corresponds to the standard "labor efficiency" term, the source for balanced growth, whereas Q ("capital efficiency") is the source for non-neutrality and the reason technical change here is said to be asymptotically (or quasi) equivalent to "labor augmenting".[15]

From the definition of B and Q in (3) and (4) we can easily express their growth rates as:

$$g_B = \tfrac{1}{\rho} g_m ; \tag{7}$$

$$g_Q = -\tfrac{1}{\rho} g_m \left(\tfrac{m_t}{1-m_t}\right). \tag{8}$$

Now, recall our prior, as expressed in (2), being a constant exponential decrease in m. Keeping in mind that ρ is negative, the growth rate of the "labor augmenting" entity B, evident in (7), is thus indeed exponentially increasing, that is $g_B > 0$ with $\dot{g}_B = 0$, $\forall t$.

What is no less important to notice here, though, is that in parallel to the rise in B, not only does Q decrease, approaching one in the limit, but as clear from (8) so does its growth rate (in absolute value), which tends to zero. Thus we have a rapid decline of Q, reaching the vicinity of one "very quickly", while B perpetually increases at a constant rate.[16] When Q is "sufficiently" close to one, the economy approaches a balanced growth path, with a constant growth rate: g_B. In addition, as can be seen in (6), factor shares are constant as well.

It should be noted, however, that in the transitional stage too, when Q is still "significantly" larger than one (but decreasing), the factor shares may be constant. What is needed for this, as

[15] We can obtain the standard CES form: $Y_t = [\alpha(D_tK_t)^\rho + (1-\alpha)(A_tL_t)^\rho]^{1/\rho}$, by defining: $A_t \equiv \frac{\lambda B_t}{(1-\alpha)^{1/\rho}}$ and $D_t \equiv \frac{\mu Q_t}{\alpha^{1/\rho}}$ for some arbitrary constant $\alpha \in (0,1)$. But a major point of the paper, elaborated in sub-section 3.2, is that A and D (or rather B and Q) are inherently related to one another, and thus constitute but a *single* entity, or degree of freedom.

[16] The intuition can be traced back to changes in m vs. $1 - m$. Suppose m declines from 0.002 to 0.001. While negligible in percentage points, this is a significant 50% drop. Conversely, the corresponding change in $1 - m$: 0.998 to 0.999, is negligible in any sense. The same reasoning holds *a fortiori* for even smaller levels of m.

apparent from (6), is that the growth rate of per worker capital exceeds that of B; though converging to it as Q approaches one. Specifically, it must be that: $g_B \cong g_{K/L} + g_Q$; which, given (8), means: $g_B \cong (1-m)g_{K/L}$. This implies that output growth is less than capital growth, but converging to it, or in other words: the output-capital ratio is converging "from above" to a constant value. Such a phenomenon is indeed a stylized fact of growth, associated with the "transitional dynamics" of the neo-classic growth model. We will elaborate on this in sub-section 3.2, below.

2.3 "Quasi Steady State" in a Solow Growth Model

One can proceed to analyze the balanced growth path/steady-state of a standard neo-classic growth model. Define "per efficiency unit" variables: $\widetilde{y} \equiv \frac{Y}{BL}$ and $\widetilde{k} \equiv \frac{K}{BL}$, so that (5) becomes:

$$\widetilde{y}_t = \left[\lambda^\rho + \left(Q_t \mu \widetilde{k}_t\right)^\rho\right]^{\frac{1}{\rho}}. \tag{9}$$

With a constant saving rate s (as also in a Ramsey model on its balanced growth path) and the (slightly manipulated) law of motion: $\frac{1}{s}\dot{\widetilde{k}}_t = \widetilde{y}_t - E\widetilde{k}_t$, where $E \equiv \frac{g_B + g_L + \delta}{s}$ (δ is the depreciation rate), the steady-state value of $\widetilde{k}$ is easily obtained.[17] But here this will not be a true steady state since Q is time-dependant and thus the dynamical system can not be made fully autonomous. Therefore we shall refer to a "quasi steady-state" (QSS) and continue maintaining time

[17] Assuming $\rho < 0$ in (1), or (5), gives: $\lim_{K\to 0}\partial Y/\partial K = \mu Q < \infty$ (unless $m = 1$), violating the Inada condition for this limit, thus potentially leading to the "trivial" (zero) steady state (see Barro & Sala-i-Martin, 2005). A *sufficient* restriction in order to avoid this is: $\mu > (\delta + g_L)/s$.

subscripts for the QSS values, denoted by "hats". Solving the law of motion equation for the (instantaneous) steady-state "per efficiency unit" level of capital we get:

$$\hat{k}_t = \lambda\left[E^{\rho} - (Q_t\mu)^{\rho}\right]^{-\frac{1}{\rho}}; \tag{10}$$

and following the law of motion, we have: $\hat{y}_t = E\hat{k}_t$. Equation (10) clearly shows that as Q tends to one, the model approaches an autonomous state with a "true" balanced growth path, where "tilde", and thus "hat" variables are constant and time indices are redundant.

Substituting the steady-state output-capital ratio (i.e. E) in (6) we get a simple expression for the QSS capital share:

$$\hat{\theta}_t^K = \left(\frac{Q_t\mu}{E}\right)^{\rho} = \left(\frac{g_B + g_L + \delta}{sQ_t\mu}\right)^{-\rho}. \tag{11}$$

It should be noted that (11) is *not* the capital share during "transitional dynamics", since $\dot{\tilde{k}} > 0$ at that stage. Thus (11) is actually valid only for the "true" (i.e. limiting) steady-state, where we get: $\hat{\theta}_\infty^K \equiv \lim_{t\to\infty} \hat{\theta}_t^K = \left(\frac{g_Y+\delta}{s\mu}\right)^{-\rho}$, from which we can "back out" μ according to:

$$\mu = \left(\frac{\hat{g}_Y + \delta}{\hat{s}}\right)\left(\hat{\theta}_\infty^K\right)^{\frac{1}{\rho}}, \tag{12}$$

where $\hat{g}_y$ and $\hat{s}$ denote steady-state (balanced growth) levels of per worker output and the savings rate, respectively. Since the terms in both brackets are typically around one third, we have:

$\mu = \left(\frac{1}{3}\right)^{\frac{1+\rho}{\rho}}$ or $\mu^{\rho} = \left(\frac{1}{3}\right)^{1+\rho}$. Thus with σ = 0.5, or ρ = –1, we back out μ = 1. However, the estimation in (12) is somewhat sensitive to the chosen elasticity. For example: $\sigma = \frac{1}{3}$ (ρ = –2) sets $\mu = \frac{1}{\sqrt{3}} \cong 0.58$, while $\sigma = \frac{2}{3}$ (ρ = –0.5) sets μ = 3.

3. Non-Neutrality

3.1 The Rotating Production Function

The previous section has shown the tendency of the model for an asymptotic balanced growth path with constant factor shares. Nevertheless, we should bear in mind that that was a good approximation for an economy in which the level of technology (inversely related to labor's impact) and the capital-labor ratio are high. It is thus the initial phases of development, or the lack of it, which ought to be most interesting in the context of the current paper; these are the cases where the non-neutrality aspect of the model is most apparent.

The following proposition establishes the non-neutral, or "rotation" property of (1), distinguishing it from the standard case of neutral technical change.

PROPOSITION 3.1: *For any combination (L,K) satisfying $\frac{K}{L} = \frac{\lambda}{\mu}$, the level of output is the same for any level of technology (m). For any combination (L,K) satisfying $\frac{K}{L} > \frac{\lambda}{\mu}$, output is higher when m is lower (higher technologic level). For any combination (L,K) satisfying $\frac{K}{L} < \frac{\lambda}{\mu}$, output is lower when m is lower (higher technologic level)*

Proof: in the appendix.

Proposition 3.1 is depicted in figures 2 and 3. Figure 2 shows the production function (1) undergoing a discrete technical change in the isoquant plane. Unlike neutral changes, where isoquants are proportionally *compressed* along the dimension of the "augmented" factor, here they are *rotated*, such that same-level isoquants before and after the change intersect. The counterclockwise rotation in the (L,K) plane means that for capital-labor ratios above λ/μ the new technology indeed raises output for a given level of factors, or can maintain the level of output for a strict reduction in both factors. Furthermore, we know from section two that for high technology levels and capital-labor ratios the change will tend to resemble a neutral one. A corresponding rotation is shown in figure 3, where labor is normalized (say, to one) and per-worker output (y) is the "intensive form" function of per-worker capital (k).

However, as apparent in both figures, for capital-labor ratios which are below λ/μ, the new technology *reduces* output. This "adverse technical change" indeed seems rather strange at first glance and is certainly counter to the standard view of overall benefits from "better" technology, as implied by neutral technical change. But given the mechanization-induced replacement of labor by capital, featured in this low-substitution model, then the hypothetical adoption of a new technology without an appropriate increase in the level of capital may very well reduce output since labor will not be efficiently utilized. The notion of "appropriate technologies" will be further discussed in sub-section 3.3 below.

3.2 Non-Neutrality and Industrialization Accounting

Quantitative analysis of technical change is highly dependent on the assumptions regarding the "shape" of the production function, specifically whether or not it is characterized by a unit elasticity of factor substitution. The debate over the elasticity of substitution's significance has surfaced recently in the context of the East Asian newly industrialized countries (NICs). Nelson

& Pack (1999), Rodrik (1998) and Hsieh (2000) all make the claim by which standard growth accounting assuming unit elasticity overstates the contribution of factor deepening to growth, while understating the *direct* contribution of technical change.[18] Their alternative framework is a low substitution (high curvature) function undergoing "labor augmentation".

More specifically, Nelson & Pack (1999), Rodrik (1998) and Hsieh (2000) use the formula:

$$g_{\theta^K} = \left(1-\theta^K\right)\rho\left[\left(g_K - g_L\right) - \left(g_A - g_D\right)\right], \tag{13}$$

where *A* and *D* are pure "labor efficiency" and "capital efficiency" terms, respectively, similar to *B* and *Q* in (5) above (see footnote 13), and $\rho = \frac{\sigma-1}{\sigma} < 0$. These authors calculate TFP growth as the difference between *actual* output growth between two periods and the growth which *would have* occurred in the absence of technical change ($g_A = g_D = 0$); in which case the capital share *would have* fallen, due to the effect of capital deepening when factor substitution is low.

Young (1998) implies at least two problems with such an analysis. The first concerns the somewhat casual treatment of the "capital efficiency" term. Indeed, Nelson & Pack, Rodrik and Hsieh are right in claiming that "labor augmentation" offsets capital deepening in preserving the constancy of factor shares. See equation (13). But as clearly seen in (6), this means that *strict* "labor augmentation" will hold only during balanced growth (i.e. when $g_Y = g_K$). Otherwise (i.e. $g_Y < g_K$) there has to be negative "capital efficiency" growth. As Young clearly explains: "In order for the Nelson-Pack-Rodrik-Hsieh framework to explain the facts of East Asian growth, it is not only necessary that factor augmenting technical change offset the growth of the capital-labor ratio, it is also necessary (given the relatively slow growth of output relative to

[18] Of-course one can claim technical change has also the *indirect* effect of leading to the mere capital deepening.

factor accumulation) that the production function *rotate*" (1998, pp. 4-5, italics in the original text).

The second, related, point concerns the "path dependency" of TFP growth estimates; as rigorously shown by Hsieh (2000). The implication is that if one rejects Cobb-Douglas, or more generally Hicks-neutrality, then the measuring of TFP growth is highly problematic. As Young (1998) points out, there is indeed no reason to prefer one "path", say "capital deepening first and then technical change" over another, say "technical change first". Young (1998) refers to TFP measures along these two paths as resembling "Paasche" and "Laspeyres" indices, respectively. Moreover, as implied by Young, the standard (Cobb-Douglas) method in some sense provides an "average" of the TFP growth estimates obtained from both these "extreme-case" paths; as can be seen in table 1 here, which is Young's (1998) table 2.

TFP measure	Hong Kong	Singapore[19]	South Korea	Taiwan
"Paasche" ($\sigma = 0.3$)	3.4	1.8	3.3	3.5
Standard ($\sigma = 1$)	2.4	0.1	1.6	2.1
"Laspeyres" ($\sigma = 0.3$)	0.9	–1.6	–1.4	–1.1

Table 1: TFG growth rates (%) in NICs, under different methods (1966-90). Source: Young (1998), table 2.

In three out of the four NICs we see that the "Laspeyres" estimates show up as negative, implying highly dominant "capital dis-augmentation". These results are explained graphically by Young (1998) and superimposed here in figure 3. Suppose the two points of actual estimation are: *I* and *J*. The so-called "Paasche" index of TFP growth corresponds to the vertical distance between *J* and the hypothetical point $\tilde{J}$ obtained by applying formula (13) under the assumption of no technical change. The "Laspeyres" index corresponds to the vertical distance from the

[19] Subsequent work has revealed potential problems with Singaporian data.

point I to the hypothetical point $\widetilde{I}$ obtained too by applying (13) without technical change; but "in reverse". Figure 3 shows the *hitherto* puzzling case of a negative "Laspeyres" index.[20]

Nelson & Pack (1999), Rodrik (1998) and Hsieh (2000) rely on a long interval (extreme case) path; specifically the "capital accumulation first" ("Paasche") path. But one may ponder whether the claim by which TFP growth depends on the elasticity of substitution applies for the more realistic infinitesimal, or yearly, conjoined path.

Recall that changes in TFP correspond to "shifts in the production function", such that: $g_Y = g_{TFP} + \theta^K g_K + \left(1-\theta^K\right)g_L$, or: $g_y = g_{TFP} + \theta^K g_k$, under the standard assumptions. It is straightforward to show that given "factor augmenting" technical change, where output is a constant returns to scale function of "efficiency units" of capital (D_tK_t) and labor (A_tL_t), we obtain:

$$g_{TFP} = \theta^K g_D + \left(1-\theta^K\right)g_A. \tag{14}$$

If one does not consider "capital (dis-)augmentation" then of-course (14) simplifies to:

$$\widetilde{g}_{TFP} = \left(1-\theta^K\right)g_A; \tag{14'}$$

where the elasticity of substitution, or curvature, does not seem to affect the parsing of growth into technical change and capital accumulation; due to two possible reasons. The first, which does not seem to be the case, is that factor shares are not constant. Thus overlooking their change in (14') may conceal the actual dependence on the elasticity of substitution.

[20] A negative "Laspeyres" index is indicative of significant mechanization, and need not always be the case. It is indeed not surprising that Hong Kong, being initially the most developed of the four NICs, has all indices positive.

But the second, and most likely reason for the elasticity of substitution's lack of appearance in (14') is that it is a biased formula, as it does not consider "capital (dis-)augmentation"; which in-fact it *should*, given (6) and the relative constancy of factor shares. The model here easily corrects this bias, and **without adding a degree of freedom** in the form of a separate "capital efficiency/productivity" entity. Indeed, rewriting (14) in terms of the synthetic parameters B and Q, defined in (3) and (4), we get, given (7) and (8):

$$g_{TFP} = \left(1 - \tfrac{\theta^K}{1-m}\right) g_B \,. \tag{15}$$

With constant factor shares we can estimate: $g_B = g_y$. Since $g_B > 0$ ($g_m < 0$), it is clear that (15) is consistent with either a positive or a negative growth rate of TFP, depending on whether the technological level is high (m closer to 0) or low (m closer to 1), respectively. Applying (6) and (4), then (15) can be rewritten as:[21]

$$g_{TFP} = \left[1 - \left(\tfrac{\mu K}{Y}\right)^{\rho}\right] g_B \,. \tag{15'}$$

The claim by which the elasticity of factor substitution ought to negatively affect TFP growth (i.e. low substitution means high TFP growth) is supported by (15'); but only if $\mu K > Y$, which holds only for relatively technologically advanced stages. If $\mu K < Y$ then the lower is the elasticity the smaller is TFP growth; which may even be negative, due to the dominance of "capital dis-augmentation". Such a form of TFP growth would be interpreted as an indication of intense mechanization (i.e. "capital *using*").

---

[21] If, for example, $\rho = -1$ and thus $\mu \cong 1$, we have $g_{TFP} = (1 - \tfrac{Y}{K})g_B$.

3.3 Non-Neutrality and Low Development

In a long-anticipated attempt to bridge growth theory and development economics, Banerjee & Duflo (2004) have skillfully synthesized the idea of "appropriate technologies" with micro-level studies, inferring the occurrence of "traps"; though on an intra- rather than inter-country level. Relying on fixed costs and decreasing returns to scale, these authors show how imperfections, mainly credit constraints, generate a misallocation of capital and lack of sufficient technology adoption. More specifically, if there is heterogeneity in the credit given to various agents and if a "better" technology requires a greater fixed cost then there will be heterogeneity in the level of technology. Conversely, if credit rationing is relatively low, as in developed economies, then fixed costs hardly matter and the latest technologies will most often be adopted.

Rather than assuming fixed-costs at a static level (i.e. within a given technique), the non-neutrality induced by mechanization, which means a requirement for less labor but more capital in order to produce the same level of output, can be thought of as a dynamic equivalent; or an "inter-temporal non-convexity". Thus the current model too, despite its convexity, generates the result by which no technique dominates another. Rather, there is an "ideal" technique which is a function of the capital-labor ratio: an "appropriate technology".[22] Indeed, the sign of the "technology index derivative" will vary here across the production function's domain.[23] A "classic" reference to appropriate technologies is Atkinson & Stiglitz (1969), who criticize the idea of neutral technical change, due to its dubious implication by which a new innovation boosts production levels for *all* capital-labor ratios.[24]

---

[22] The basic model introduced in section 2 and discussed so far is of an "all or none" type appropriateness, but the more elaborate model introduced in the next section has an infinite number of appropriate technologies.

[23] But as shown in the next section, a marginally higher technological level will indeed increase output *on the balanced growth path*.

[24] Basu & Weil (1998) are renowned for reviving the idea of appropriate technologies, though the interpretation in the current paper is more in line with Zeira (1998), whose model indeed implies a non-neutral (albeit discrete) technical change. Caselli (2005) and Jones (2005) too link non-neutrality and appropriate technologies, though in

A glance at figure 3 reveals that in similar to the approach undertaken by Banerjee & Duflo (2004), and in stark contrast to standard (neutral) formulations, for low levels of investment a newer technology is inferior in terms of its output per worker, as well as in its marginal product of capital per worker. Formally, finding the expression for the cross-derivative of the intensive form of (1): $\frac{\partial^2 y}{\partial k \partial m}$, which is non-monotone in k, gives the unique value of k: $\left(1-\frac{\rho}{1-m}\right)^{1/\rho}\frac{\lambda}{\mu}$, which is smaller than $\frac{\lambda}{\mu}$, below which a rise (fall) in m raises (lowers) the marginal product of capital per worker, and above which a rise (fall) in m lowers (raises) it.

As stressed by Banerjee & Duflo (2004), a Cobb-Douglas production function (or, as should be added, any function in which technical change is neutral) can not easily reconcile differences between rich and poor countries in their output-capital ratios, as well as their ratios of rates of return. The reason is that in standard models the required level of the technology gap induces a lower than observed ratio of returns when fitting the observed output ratio. Alternatively, one will obtain a lower than observed output ratio when fitting the observed rate of return ratio. The underlying reason is that in these models both the rate of return and the output level are increasing with the level of technology.

The implication of time-ranked non-neutrality, as featured here, is that economic agents facing high interest rates will in-fact prefer investing in old technologies, even if newer ones are fully at their disposal.[25] The picture depicted here (e.g. figure 3) and formalized above, is of older technologies being characterized by initially soaring marginal productivity of capital, which levels-of rather fast. In accordance with the "poverty trap" literature, the implication is that

their view, these are alternatives along a concurrent, specifically the frontier, "technology menu", rather than a time ranking of techniques.

[25] According to Banergee & Duflo (2004), for example, the median textile "firm" in India is a tailor using primitive technology, despite the existence of more modern (capital intensive) firms; perhaps due to the median entrepreneur's limited access to capital. Though these authors stress internal reasons for high interest rates, a contributing factor may be the interest rate relevant for foreign loans, which may be high due to various risk related factors. A high interest rate as a possible cause for deficient capital flows is also highlighted by Lucas (1990).

neither differences in technology, nor "institutional arrangements" alone can provide explanations for under-development. Rather, there has to be some sort of interaction between these two elements.[26]

4. Elaborate Model

4.1 Production Technology

Somewhat inspired by the Classical approach to production, where labor is the "basic" factor of production, the formulation below assumes that production is the result of a low-substitution process by which capital "empowers", or "intensifies" workers' vital motor or cognitive services. Abstracting from important issues concerning human capital acquisition, the model implicitly assumes that workers are always knowledgeable as to the means by which to operate the contemporary technology, embedded within capital.

More specifically, it is assumed that capital empowers labor through a sequential process of n succeeding tasks, each with its own amount of "capital input". A higher number of tasks corresponds to a more complex type of capital, or equipment, embodying a higher technological level. These tasks seemingly bear resemblance to the intermediate capital-good variants of the expanding-variety growth model, originally due to Romer (1990). There are several main differences, though, which shall be highlighted below; one of which is that the model here assumes a low, rather than high elasticity of substitution among the different tasks.

[26] The dependence on the interaction between technology *and* credit constraints is dominant in much of the poverty traps literature. See Galor & Zeira (1993).

Suppose each i-th task ($2 \leq i \leq n$) can augment the value of a given amount of the previous task's output (x_{i-1}), by using it as an input, to be combined with its own amount of capital (k_i), to produce intermediate output (x_i). The first task, x_1, augments the "basic input" of the production process, which is the labor input. Thus, in contrast to the product variety model, here labor is inherently embedded within the complexity of the production process. Omitting time subscripts for clarity, suppose the series of intermediate output $\{x_i\}_{i=1}^{n}$ in the process described above is recursively defined by the following CRS-CES function per each task, with $x_0 \equiv L$:

$$x_i = \left[\gamma x_{i-1}^{\rho} + (1-\gamma)k_i^{\rho}\right]^{\frac{1}{\rho}}, \tag{16}$$

where in similar to section two: $\rho \in (-\infty, 0)$, or in terms of the elasticity of substitution: $\sigma \equiv \frac{1}{1-\rho} \in (0,1)$; but the so-called "distribution parameter" $\gamma \in (0,1)$ is now constant.

Assume further that k_i is a "composite capital", comprised of κ capital types, defined by yet another CES function (per each production task):

$$k_i = \left[\sum_{j=1}^{\kappa} \beta^j \left(k_i^j\right)^{\varepsilon}\right]^{\frac{1}{\varepsilon}}, \tag{17}$$

where the superscripts denote capital type, the β-s are respective weights ($\Sigma\beta^j = 1$, $\beta^j \geq 0$ $\forall j$), and $\varepsilon \leq 1$ determines the elasticity of substitution ($\frac{1}{1-\varepsilon}$) between them. A strong simplifying assumption is, of course, that ε and the β-s are identical across tasks.

Recursive substitution in (16), and defining the final task's output (the final output) as Y, yields the following $n + 1$ inputs CES production function:

$$Y = \left[\gamma^n L^\rho + (1-\gamma) \sum_{i=1}^{n} \gamma^{n-i} k_i^{\ \rho} \right]^{\frac{1}{\rho}}. \tag{18}$$

One can already observe how the assumption regarding labor as basic input will bring about its decreasing impact when the task number increases as technology advances.

Besides the inherent inclusion of labor, as well as low substitution among tasks, two additional differences between (18) and the standard expanding-variety growth model are apparent. First, notice the weight per each specific task, which is dependant on n. This shall manifest itself in the existence of an "appropriate" number of tasks per a given capital-labor ratio (an "appropriate technology"), as opposed to an unambiguous global optimality of a higher technology index in a variety setting; or in any less micro-founded model of neutral technical change. The additional difference is a non-symmetry property, evident in the appearance of i in the weighs; in contrast to the symmetric, or commutative, feature of the variety model. Thus capital in different tasks, though (in principle) homogeneous, is non-symmetric in its impact on final output.

Let us assume the usual simplifying assumption whereby the various k-s are homogeneous in that we can specify: $\Sigma k^j = K^j$, where K^j is "raw (j-type) capital", that is "forgone consumption" measured in units of output. Now regardless of whether it is done by the final goods' producers themselves or by intermediate goods' producers, there is a question of how should a given stock of "raw capital", be allocated among the various n tasks.

PROPOSITION 4.1: *If maximizing the output of (18), given a constraint on total amounts of capital utilized, then for all j capital types and any n,* $\{k_i^j\}_{i=1}^n$ *progresses geometrically, with a quotient:*

$$q \equiv \left(\frac{1}{\gamma}\right)^{\sigma} > 1 . \tag{19}$$

Proof: in the appendix.

From (19) we can write the exact definition of the optimal allocation values as:

$$k_i^j = q^{i-1}\left(\frac{q-1}{q^n - 1}\right)K^j , \tag{20}$$

where $K^j \equiv \sum_{i=1}^n k_i^j$ is the total amount of the *j*-th type capital. The increasing result should come as no surprise given the assumptions of this section, namely that the earlier tasks (as well as the basic labor input) undergo further enhancement by later ones, which thus diminish the formers' impact; whereas the impact of later tasks on final output is more salient. Qualitatively, this increasing pattern is similar to the results obtained in other serial-production models, such as those by Locay (1990) and Kremer (1993).

Substituting (20) back into (18) yields the aggregative specification:

$$Y = \left[\left(\gamma^n\right) L^{\rho} + \left(1-\gamma^n\right)\left(\frac{G}{G_n}K\right)^{\rho} \right]^{\frac{1}{\rho}}, \tag{21}$$

where: $G \equiv \left[\frac{1-\gamma^{\sigma}}{(1-\gamma)^{\sigma}}\right]^{\frac{1}{1-\sigma}}$, $G_n \equiv \left[\frac{1-\gamma^{n\sigma}}{(1-\gamma^{n})^{\sigma}}\right]^{\frac{1}{1-\sigma}}$ and based on (17): $K \equiv \left[\sum_{j=1}^{\kappa} \beta^{j}\left(K^{j}\right)^{\varepsilon}\right]^{1/\varepsilon}$ is the "aggregate composite capital". If we define labor's impact: $m_t \equiv \gamma^{n_t}$, then (21) bears resemblance to (1) and thus, following (3), we can define the (synthetic) "labor efficiency/productivity" term:

$$B_t \equiv \left(\gamma^{\frac{1}{\rho}}\right)^{n_t} = e^{\frac{\ln\gamma}{\rho} n_t}; \tag{22}$$

where $\frac{\ln\gamma}{\rho} > 0$ is important for converting "ideas" into "productivity", as shown below.

Notice that G is constant, while G_n tends to one as n tends to infinity. Thus, following the analysis of section 2, the additional term G_n reinforces the "capital efficiency" term, here given (4): $Q_t \equiv \left(1-\gamma^{n_t}\right)^{1/\rho}$, as both tend to one and changes in them become increasingly negligible. In terms of an accounting exercise, with respect to the growth rate of "labor productivity", B, we get an expression which is slightly different than that in (14); though still larger than the estimate of the standard exercise.[27]

As opposed to the formulations of section 2, the inclusion of $\frac{G}{G_n}$ in (21) allows for an infinite number of appropriate technologies, as seen in figure 4, depicting (21) with various levels of n. Formally, optimizing (21) with respect to n gives the (continuous) "ideal" or "appropriate" technology as an increasing function of the capital-labor ratio:[28]

[27] Given (21) and (22), the equivalent of (15) is: $g_{TFP} = (1 - \frac{\theta^{K}}{1-m^{\sigma}})g_B$.

[28] Not surprisingly, the function is homogeneous of degree zero in L and K, meaning that the appropriate technology is not dependant on scale; consistent with the CRS property of the production function (16).

$$n^* = \log_q\left(G\frac{K}{L}+1\right). \tag{23}$$

This expression can be substituted in (21), generating: $Y^*(L,K) \equiv \max_n\{Y(L,K;n)\} = GK + L$, the upper-envelope function seen in figure 4. The "appropriate technology" specified in (23) will, at least along the (asymptotic) balanced growth path, *not* be the "actual" one employed, or at all in existence, as we shall see below.

4.2 Technical Change along the (Asymptotic) Balanced Growth Path

Rather than complicating the current framework with the product-variety settings central to the, by-now classic, models of Romer (1990), Grossman & Helpman (1991) and Aghion & Howitt (1992), we shall adopt a more reduced-form analysis.[29] Following Jones (1995), define the following "idea flow" function of existing knowledge and researcher input:

$$\dot{n} = \xi\phi^n L_R^\eta, \tag{24}$$

with all parameters (ξ, ϕ and η) strictly positive. This differs from Jones (1995) in that existing knowledge enters through an exponential, rather than a power function. But as shall be discussed below, the formulas are in-fact fully equivalent; whereas the difference is conceptual.

Following (22), then on a balanced growth path it must be that n is a linear (or rather, an affine) function of time; namely: $\ddot{n} = 0$. Therefore, similar to Jones' (1995) methodology, we ought to differentiate (24) and equate to zero, giving the solution:

---

[29] Such a complication would require, for example, assuming final output is produced by highly substitutable parallel processes as described by (16) – (18). As discussed below, the model can be intuitively viewed as combining features of both the expanding-variety (Romer) and the quality-ladder (static variety) models.

$$\dot{n} = \frac{\eta}{\ln\left(\phi^{-1}\right)} g_{L_R} . \tag{25}$$

Thus, from (22), given (25), we get the economy's (asymptotic) balanced growth rate:

$$g_B = \frac{\ln\gamma}{\rho} \frac{\eta}{\ln\left(\phi^{-1}\right)} g_{L_R} . \tag{26}$$

In a fully specified Ramsey model, equation (26) can be substituted in the Euler equation, solving for the interest rate and thus also the allocation of labor to production and R&D. As we are following Jones' (1995) critique, whose implication is that the number of researchers only affects the *level* of the balanced growth path but not the growth rate itself, we do not need to simultaneously solve for the growth rate *and* interest rate, as in models exhibiting a "scale effect" (in growth rates). Therefore (26) is determined regardless of preference-related parameters.[30]

Recall that the current model implies the existence of appropriate technologies, following (23), which means that given prevailing levels of labor and capital an increase in the technological level, as defined by n, or B, may either increase or decrease output. But the following proposition shows that along the (asymptotic) balanced growth path the "actual" technological level is always smaller than the "appropriate" level. In other words, despite the potential for an "overly optimal" technology, the latest technologies are always dominated by at least a sub-set of those

[30] The fertility rate could be endogenized in a Beckerian type model. But it seems a significant gap still exists in the literature, as the growth rate of R&D-engaged labor in G7 countries, shown by Jones (1995), Kortum (1997) and Segerstrom (1998), has been greater than the growth rate of the labor force, which itself has been greater than the fertility rate (e.g. due to women's increasing labor force participation). Thus the growth rate of researchers can not be purely due to population growth *per se*.

not yet invented. Thus: $\frac{\partial Y}{\partial n} > 0$, or $\frac{\partial Y}{\partial B} > 0$, like in the standard (neutral) model of technical change.

PROPOSITION 4.2: *Along the (asymptotic) balanced growth path:* $n_t < n^*_t$

Proof: in the appendix

4.3 Implications Concerning Knowledge Accumulation

As apparent from (25) or (26), it must be that: $0 < \phi < 1$, in order to avoid "explosive" growth. Thus, as is clear from (24), the model implies negative inter-temporal knowledge spillovers ("fishing out" of ideas). While this restriction diminishes the generality of the current analysis, it should be noted that the elaborately micro-founded models of technical change due to Kortum (1997) and Segerstrom (1998) generate an equivalent implication. A main empirical justification of Kortum (1997) and Segerstrom (1998) for the negative inter-temporal knowledge spillover is that in parallel to the exponential increase of R&D-engaged workers in industrialized countries, not only have productivity growth rates remained relatively constant, but so has the flow of new patents. The current model too is consistent with these stylized facts, though there is a conceptual difference here between the stock of ideas (n), and "productivity" (B), a reciprocal of labor's impact. These facts necessitate, as in Kortum (1997) and Segerstrom (1998), that new patents, or ideas, become increasingly more valuable with time.

The fact that ideas in the current model are accumulated linearly along a balanced growth path is derived from the formalization *assumed* in (24). As was stated above, (24) differs from Jones' (1995) formulation by relating the flow of new ideas to the existing stock of ideas through an exponential, rather than a power function. But it should be stressed that there is no contradiction between the models. In-fact, given (22) we can transform (24), expressed in terms of ideas, into a "productivity-level terms" specification, as follows:

$$\dot{B} = \Xi B^{\Phi} L_R^{\eta}, \qquad (27)$$

where $\Xi = \xi \frac{\ln \gamma}{\rho} > 0$ and $\Phi = \frac{\rho \ln \phi}{\ln \gamma} + 1$. This expression is equivalent to the one proposed by Jones (1995), where the result required $\Phi < 1$; indeed so if (in the current model) $0 < \phi < 1$. Notice that for the range $0 < \Phi < 1$, rather than $\Phi < 0$, (27) can appear as if exhibiting slight *positive* past-research spillovers. What is actually happening in this range, as clear from (24), is that if ϕ is relatively close to one then an increase of n only slightly decreases $\dot{n}$ (*ceteris paribus*, i.e. with no change in L_R), whereas B is an exponential function of n. Thus since $\dot{n}$ is still positive, $\dot{B}$ (but not $\frac{\dot{B}}{B}$) may still (slightly) increase.

Once acknowledging the difference between "productivity levels" and "ideas", we can also say that there is no contradiction with the models due to Kortum (1997) and Segerstrom (1998). The former, for example, shows balanced growth as a possible outcome of productivity levels being drawn from a "thick-tailed" Pareto stationary search distribution; such as: $F(B) = 1 - \left(\frac{B}{B_0}\right)^{-\Omega}$, with $\Omega > 0$ (or $\Omega > 1$ if one wishes a finite mean). Given the relationship (22), it can easily be shown that this is equivalent here to assuming that the ideas themselves (i.e. n) are being drawn from an *exponential* stationary search distribution, with the CDF: $F(n) = 1 - e^{-\omega(n - n_0)}$, where $\omega = \frac{\ln \gamma}{\rho}\Omega > 0$ and n_0 relates to B_0 according to (22).[31]

[31] In Kortum's (1997) model, where researchers directly sample the *productivity levels*, an exponential (rather than a Pareto) stationary search distribution delivers counterfactual linear (rather than exponential) growth.

On a more conceptual level, the model presented here combines basic intuitions from both the expanding-variety as well as the quality-ladder models. Equation (18), incorporating an additive separable term with a varying number of components, clearly resembles the former, while the inherent non-symmetry of the model and the fact that new ideas diminish the significance of previous ones resembles a Schumpeterian "creative destruction"-type process. In a sense, the current model can thus be seen as merging the orthogonal, but by no means contradictory, views of knowledge implicit in both.

The expanding-variety model highlights the fact that the technology index, corresponding to the "stock of knowledge", measures the cumulation of ideas or designs (which are modeled as symmetric for reasons of tractability). As such and as explicitly stated by Romer (1990, p. 79): "each new unit of knowledge corresponds to a design for a new good, so there is no conceptual problem measuring [the technology index]. It is a count of the number of designs". The quality ladder approach highlights the dominance of new ideas; though, as in the original neo-classic growth model, refraining from an attempt to map the quality or technology index onto some well-defined "real-life" accumulable stock (such as: "the number of ideas").

A combination of the two approaches requires a slight refinement of the notion of an "idea", freeing it of the commutativity, or symmetry feature inherent in the expanding-variety model. An idea is both an individual "sub-design" and an integrated "meta-design", which includes previous ideas as well. Knowledge is thus conceived here as a *hierarchical* cumulation of the set of ideas existing at a point in time.[32] This conceptualization fits well with the fundamental "standing on the shoulders of giants" property of knowledge.

[32] This can be explained with the help of the following simplistic example. A hybrid car incorporates many different ideas, most of which were added on along a time line. These would include: wheel, axel, chassis, shaft, piston, *internal* combustion, power charge, planetary gear set and power split device. On one hand, the knowledge incorporated in such cars is a cumulation of *all* these ideas, and in that sense resembles the expanding variety view (albeit with *low* substitution among the ideas). But despite their vitality, these ideas are not symmetric in their impact, in that the introduction of a new one creates a *superior* "meta-design" driving-out the previous one.

5. Concluding Remarks

This paper has linked the well-known empirically induced requirement for Harrod neutral technical change with the intuitively appealing, albeit *non*-neutral notion of mechanization, or factor replacement. The proposed model is capable of complying not only with the stylized-facts of industrialized economies, but of industrializing and under-developed ones as well.

While it has by no means been the intention of the paper to downplay the importance of “labor augmentation”, it does highlight two insights which are masked if one interprets this concept *literally*. First, the seemingly orthogonal “capital-” and “labor efficiency augmentation” facets of technology are shown to be two sides of the same coin. This implies that there is but a single “technological entity” and a single “direction” for technical change; not just in practice, but in potential as well. Thus a simple explanation for the seeming “labor-bias” of technical change in offered, along which inventions need not be classified as either “capital-” or “labor augmenting”; the former either ignored or altogether precluded in equilibrium.[33]

The elaborate version of the model also highlights the fact that capital (equipment), rather than labor, is the factor which embeds new ideas or designs. The upshot here is that technical change refers to qualitative changes in *capital*, which is indeed what basic common sense would lead us to think; although the intuition is not conveyed by standard models, such as the expanding variety types, unless they are restricted to multiplicative (Cobb-Douglas) forms.

[33] Samuelson (1965, p. 355) puts it most bluntly (all punctuation and styling as in the original text):

> For the most part, labor-saving innovation has a spurious attractiveness to economists because of a fortuitous verbal muddle. When writers list inventions, they find it easy to list labor-saving ones and exceedingly difficult to list capital-saving ones. (Cannan is much quoted for his brilliance in being able to think up wire*less* as a capital-saving invention, the syllable “less” apparently being a guarantee that it does save capital!). That this is all fallacious becomes apparent when one examines a mathematical production function and tries to decide in advance whether a particular described invention changes the partial-derivatives of marginal productivity imputations one way or another.

Relaxing the Cobb-Douglas assumption can enable the deterministic and technology-based analysis of what seem to be pronounced "medium run" trends in factor shares, discussed by Blanchard (1997) and Bentolila & Saint Paul (2003), among others; though such analyses typically require some sort of "friction". These trends in factor shares perhaps indicate a complex innovation-accumulation interrelationship, inducing "technological cycles", by which a few years may elapse before capital deepening and technical change are actually aligned. More specifically, from the rather hump-shaped pattern of labor's share during the 2nd half of the 20th century one might speculate for instance that the rate of capital deepening initially overtook the rate of technical change; after which a reversal of trends occurred.

This dynamic innovation-accumulation relationship could be extended in order to analyze issues concerning skilled versus unskilled labor. For example, "skilled-biased" technical change may refer to episodes when capital equipment providing cognitive type services is accumulated, relative to being innovated, at a faster rate than the capital equipment providing physical or motor type services. Thus, the well-documented seemingly high substitution between skilled and unskilled labor, can perhaps be more elaborately described as factors' "replacement".

A further, possibly related, extension concerns durable investment goods' declining price, as notably emphasized by Greenwood, Hercowitz & Krusell (1997), which some may interpret as capital "saving" or "augmenting" technical change. Alternatively, following Whelan (2003), the decline in the price of durables can be analyzed by a two sector model, distinguishing between equipment, on one hand, and non-durables and structures, on the other. Technical change, interpreted as the decrease in labor's impact, may (for reasons which are yet to be fully understood) be more rapid in the former sector.

Appendix: Proofs of Propositions

PROPOSITION 3.1:

We seek combinations of factors (L,K) which produce the same output under two different technologies: m_1 and m_2. Thus: $\left[m_1(\lambda L)^\rho + (1-m_1)(\mu K)^\rho\right]^{\frac{1}{\rho}} = \left[m_2(\lambda L)^\rho + (1-m_2)(\mu K)^\rho\right]^{\frac{1}{\rho}}$, which with a bit of algebra becomes: $(m_1 - m_2)(\lambda L)^\rho = (m_1 - m_2)(\mu K)^\rho$. Thus we have: $\frac{K}{L} = \frac{\lambda}{\mu}$, which is independent of the technologies, and is therefore valid for any m_1 and m_2.

Without loss of generality, assume: $m_2 < m_1$ (i.e. $B_2 > B_1$), thus "2" is the higher technological level. We now seek combinations of (L,K) where $Y_2 = Y(L,K;m_2)$ is greater (smaller) than $Y_1 = Y(L,K;m_1)$. Applying the algebra as in the previous paragraph but with an inequality < (>), gives: $\frac{K}{L} > \frac{\lambda}{\mu}$ ($\frac{K}{L} < \frac{\lambda}{\mu}$).

PROPOSITION 4.1:

Differentiating (18) by k_i^j and k_{i-1}^j (for any factor type j) given a constrained total amount of it, and dividing the constrained problem's first order conditions gives (with a little algebra):

$$\left(\frac{1}{\gamma}\right)^{\frac{1}{\varepsilon-1}} \left[\frac{\sum_{f=1}^{\kappa} \beta^j \left(k_i^f\right)^\varepsilon}{\sum_{f=1}^{\kappa} \beta^j \left(k_{i-1}^f\right)^\varepsilon}\right]^{\frac{\rho-\varepsilon}{\varepsilon(\varepsilon-1)}} \frac{k_i^j}{k_{i-1}^j} = 1 \tag{A1}$$

Differentiating (18) with respect to any other (constrained total amount) k'-th type factor gives the same expression, except for the third LHS term, which is a ratio of $k^{j'}$-s. Therefore:

$\frac{k_i^j}{k_{i-1}^j} = \frac{k_i^{jw}}{k_{i-1}^{j'}} \equiv q_i$ for any two factor types: *j, j'*. Substituting k_i^j for $q_i k_{i-1}^j$ in (A1) collapses the second LHS term to $q_i^{(\rho-\varepsilon)/(\varepsilon-1)}$. The same applies when substituting $k_i^{j'}$ for $q_i k_{i-1}^{j'}$. Additional algebra shows that the subscript *i* is no longer needed, eliminates ε, and yields the proposition.

PROPOSITION 4.2:

Along a balanced growth path the growth rate of "labor efficiency/productivity" equals the growth rate of the capital-labor ratio *k*, thus given (22): $g_k = g_B = \frac{\ln\gamma}{\rho}\dot{n}$. Differentiating (23) by time yields: $\dot{n}* = \frac{1}{\ln q}\frac{G\dot{k}}{Gk+1}$. Given (19) and multiplying by $\frac{k}{k}$ we get: $\dot{n}* = \frac{1}{-\sigma\ln\gamma}\left(\frac{Gk}{Gk+1}\right)g_k$. Noting that: $\lim_{t\to\infty}\frac{Gk}{Gk+1} = 1$, and given the above expression for g_k, we have: $\lim_{t\to\infty}\dot{n}* = \frac{1}{1-\sigma}\dot{n}$. This implies that the growth of *n* is always smaller than the growth of *n**. Thus even if $n*_0 < n_0$, then *n** must surpass *n* at some finite time (specifically at $t = -\rho\frac{n_0 - n*_0}{\dot{n}}$, where $\dot{n}$ is given by equation 25); as formally stated in the proposition.

References:

Abramovitz, Moses & Paul A. David (1973): "Reinterpreting Economic Growth: Parables and Realities", *American Economic Review (Papers and Proceedings)*, 63(2): 428-439.

Acemoglu, Daron (2003): "Labor- and Capital Augmenting Technical Change", *Journal of European Economic Association*, 1: 1-37.

Aghion, Philippe & Peter Howitt (1992): "A Model of Growth through Creative Destruction", *Econometrica*, 60(2): 323-351.

Antràs, Pol (2004): "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution", *Contributions to Macroeconomics*, 4(1): article 4.

Arrow, Kenneth J., Hollis B. Chenery, Bagicha S. Minhas & Robert M. Solow (1961): "Capital-Labor Substitution and Economic Efficiency", *Review of Economics and Statistics*, 43(3): 225-250.

Atkinson, Anthony B. & Joseph E. Stiglitz (1969): "A New View of Technological Change", *Economic Journal*, 79(315): 573-578.

Banerjee, Abhijit & Esther Duflo (2004): "Growth Theory through the Lens of Development Economics", MIT Working Paper, December; forthcoming in Philippe Aghion & Steven Durlauf (eds.): *Handbook of Economic Growth*.

Barro, Robert & Xavier Sala-i-Martin (2004): *Economic Growth*, Cambridge MA: MIT Press; 2nd edition.

Basu, Susanto & David N. Weil (1998): "Appropriate Technology and Growth", *Quarterly Journal of Economics*, 113(4): 1025-1054.

Bentolila, Samuel & Gilles Saint-Paul (2003): "Explaining Movements in the Labor Share", *Contributions to Macroeconomics*, 3(1): article 9.

Blanchard, Olivier (1997): "The Medium Run", *Brookings Papers on Economic Activity*, 1997(2): 89-141.

Caselli, Francesco (2005): "Accounting for Cross-Country Income Differences", CEP Discussion Paper 667, January; forthcoming in Philippe Aghion & Steven Durlauf (eds.): *Handbook of Economic Growth*.

David, Paul & Theo van de Klundert (1965): "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960", *American Economic Review*, 55: 357-393.

Galor, Oded & Joseph Zeira (1993): "Income Distribution and Macroeconomics", *Review of Economic Studies*, 60(1): 35-52.

Gollin, Douglas (2002): "Getting Income Shares Right", *Journal of Political Economy*, 110(2): 458-474.

Greenwood, Jeremy, Zvi Hercowitz & Per Krusell (1997): "Long-Run Implications of Investment-Specific Technological Change", *American Economic Review*, 87(3): 342-362.

Grossman, Gene M. & Elhanan Helpman (1991): *Innovation and Growth in the Global Economy*, Cambridge MA: MIT Press.

Habakkuk, John H. (1962): *American and British Technology in the Nineteenth Century: The Search for Labor Saving Technical Change*, Cambridge: Cambridge University Press.

Harrod, Roy F. (1937): Review of "Essays in the Theory of Employment" by Joan Robinson, *Economic Journal*, 47(186): 326-330.

Hsieh, Chang-Tai (2000): "Measuring Biased Technology", Princeton University Working Paper, October.

Jones, Charles I. (1995): "R&D Based Models of Economic Growth", *Journal of Political Economy*, 103(4): 759-784.

——— (2005): “The Shape of the Production Functions and the Direction of Technical Change”, *Quarterly Journal of Economics*, 120(2): 517-549.

Kortum, Samuel S. (1997): “Research, Patenting, and Technological Change”, *Econometrica*, 65(6): 1389-1419.

Kremer, Michael (1993): “The O-Ring Theory of Economic Development”, *Quarterly Journal of Economics*, 108(3): 551-575.

Locay, Luis (1990): “Economic Development and the Division of Production between Households and Markets”, *Journal of Political Economy*. 98(5, pt. 1): 965-82.

Lucas, Robert E. Jr. (1990): “Why Doesn’t Capital Flow from Rich to Poor Countries?”, *American Economic Review (Papers and Proceedings)*, 80(2): 92-96.

Nelson, Richard R. & Howard Pack (1999): “The Asian Miracle and Modern Growth Theory”, *Economic Journal*, 109(457): 416-436.

Robinson, Joan (1938): “The Classification of Inventions”, *Review of Economic Studies*, 5(2): 139-142.

Rodrik, Dani (1998): “TFPG Controversies, Institutions and Economic Performance in East Asia”, in Yujiro Hayami & Masahiko Aoki (eds.): *The Institutional Foundation of Economic Development in East Asia*, London: Macmillan.

Romer, Paul M. (1990): “Endogenous Technical Change”, *Journal of Political Economy*, 98(5 pt. 2): S71-S102.

Samuelson, Paul A. (1965): “A Theory of Induced Innovation along Kennedy-Weisäcker Lines”, *Review of Economics and Statistics*, 47(4): 343-356.

Segerstrom, Paul S. (1998): “Endogenous Growth without Scale Effects”, *American Economic Review*, 88(5): 1290-1310.

Smith, Adam (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*. http://www.econlib.org/LIBRARY/Smith/smWN.html.

Uzawa, Hirofumi (1961): "Neutral Inventions and the Stability of Growth Equilibrium", *Review of Economic Studies*, 28(2): 117-124.

Whelan, Karl (2003): "A Two-Sector Approach to Modeling U.S. NIPA Data", *Journal of Money, Credit, and Banking*, 35(4): 627-656.

Young, Alwyn (1995): "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience", *Quarterly Journal of Economics*, 110(3): 641-680.

——— (1998): "Paasche vs. Laspeyres: The Elasticity of Substitution and Bias in Measures of TFP Growth", NBER Working Paper 6663.

Zeira, Joseph (1998): "Workers, Machines and Economic Growth", *Quarterly Journal of Economics*, 113(4): 1091-1113.

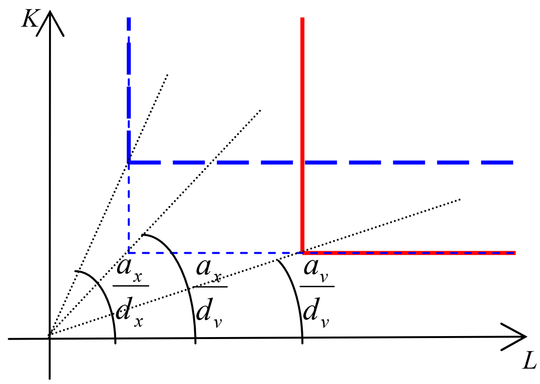

Figure 1: same level isoquants: “shovel technology”, “excavator technology” (thick-dashed) and excavator technology under Harrod neutrality (thin-dashed)

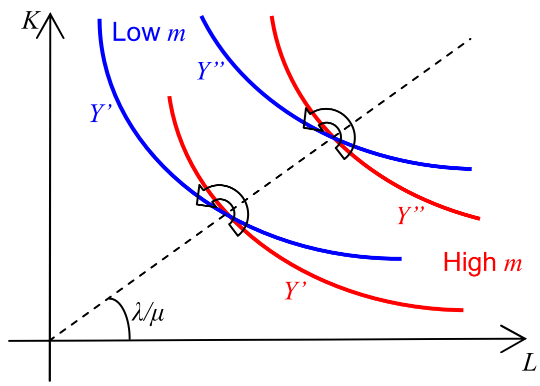

Figure 2: technical change (“high *m*” to “low *m*”) in the isoquant plane

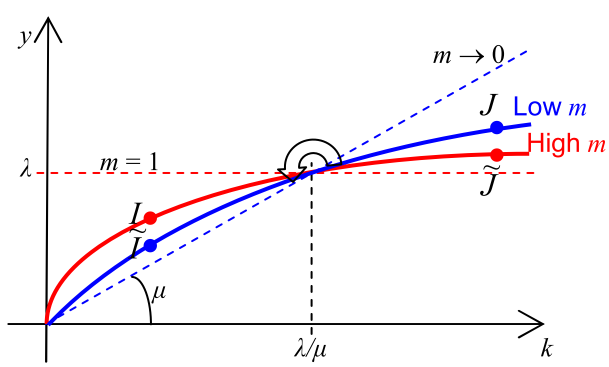

Figure 3: technical change (“high *m*” to “low *m*”) in per-worker terms (“intensive form”)

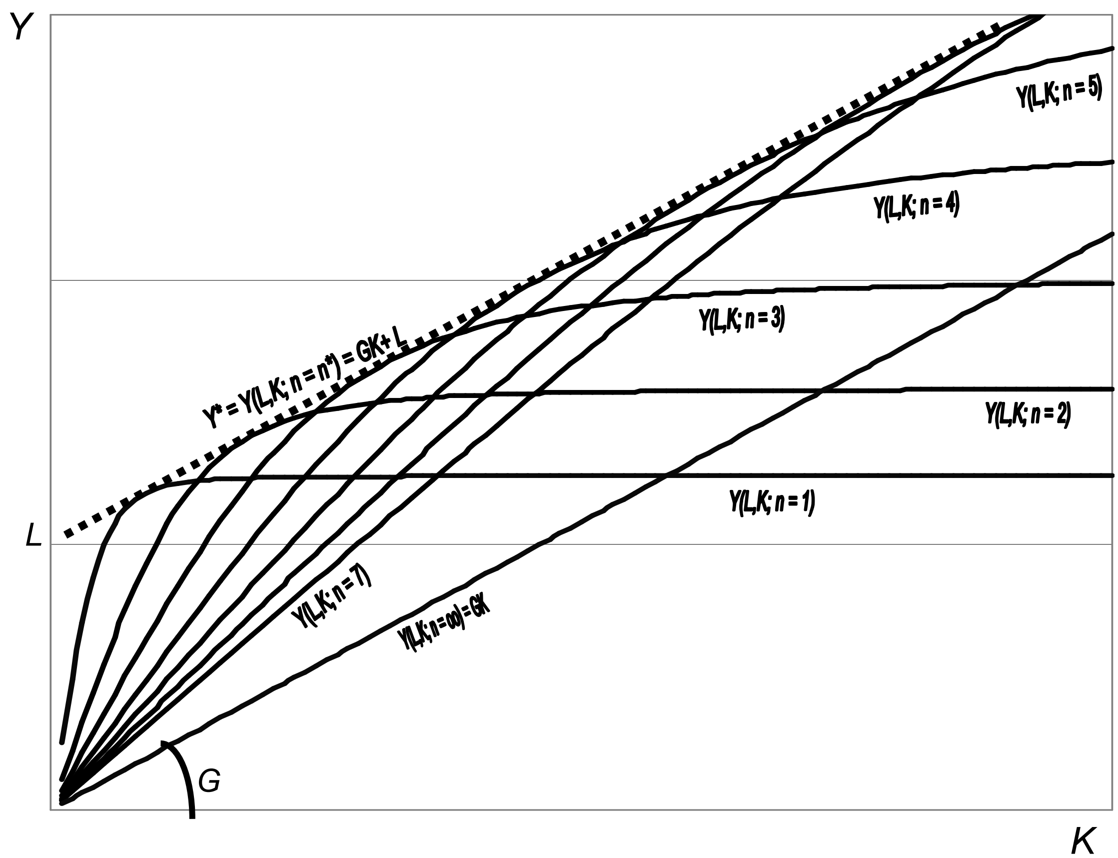

Figure 4: output as a function of capital, as in (21), given various technologies (levels of n)