

Sectors Expansion, Allocation of Talent and Adverse Selection in Development*

Esteban Jaimovich[†]

March 2006.

Abstract

This paper proposes a theory in which informational failures hindering an efficient operation of the economy are solved over the course development. Individuals are heterogeneous in terms of entrepreneurial talent, exhibiting different comparative advantages. Talent is subject to private information, giving rise to adverse selection problems. In this paper, adverse selection stems from sectors scarcity, which prevents some individuals from finding their "appropriate" sector. The availability of many sectors facilitates the allocation of individuals' unobservable talent. As a result, sectors expansion fosters growth because it helps to solve adverse selection problems. Successful long-run development is characterised by a continuous process of sectoral expansion, improved allocation of talent, and more efficient operation of financial institutions. Nevertheless, this model may also lead to poverty-traps; where economies are confined to a rudimentary situation with few sectors, poor allocation of talent, and underdeveloped financial institutions.

Key Words: Horizontal Innovation, Talent Allocation, Adverse Selection, Risk-Sharing.

JEL Codes: O10, O16, O31, D82

*I am very grateful to Nicola Pavoni and Andrew Newman for their advice, encouragement and very useful comments. I also thank Maitreesh Ghatak for helpful suggestions at an early stage of this work, as well as Wendy Carlin, Thomas Gall, Vincenzo Merella, and Matthias Messner for their comments. Special thanks to Jean Imbs for providing the author with the UNIDO data. All remaining errors are my fault.

[†]University College London, Department of Economics, Gordon Street, WC1H OAX, London, United Kingdom. Tel: +44-207-679-5227. E-mail: e.jaimovich@ucl.ac.uk.

1 Introduction

Over the course of development, the number of economic activities available to agents tends to increase, along with the aggregate stock of capital and output. In other words, while turning wealthier, societies not only accumulate larger amounts of capital, but capital accumulation itself is subject to changes, becoming progressively more differentiated. This dynamic pattern means that economic development partly manifests as a process of sectoral diversification (or *sectoral horizontal expansion*), a feature robustly documented by Imbs and Wacziarg (2003). We argue here that this process of sectoral diversification permits a less costly and more efficient allocation of individuals skills, and, as a consequence of this, an improved operation of a wide set of financial institutions.

This paper proposes a theory in which informational failures hindering the efficient allocation of individuals' talent are endogenously solved during the process of development. Individuals are heterogeneous in terms of talent or skills concerning different economic activities, characterised by distinct comparative advantages. However, skills are subject to asymmetric information, meaning that individuals possess better knowledge about their own capabilities than the rest of the economy. Given this sort of asymmetric information, adverse selection problems may naturally arise. In this model, adverse selection stems from sectors or activities scarcity, which obstructs an efficient allocation of unobservable talent. We suggest that some agents might be intrinsically gifted to specialise in activities or sectors that, for some reason, are not feasible or available. The unavailability of their "appropriate" sector or activity (i.e. the one for which they would enjoy comparative advantages), leaves those agents with no other choice but to specialise in activities for which they are not particularly talented. However, asymmetric information concerning intrinsic skills prevents (*ex-ante*) efficient screening of heterogeneous agents. As a result of this, not only those individuals whose "appropriate" sector is absent will suffer from this *sectors incompleteness*; but also those individuals whose "appropriate" sector actually exists will be affected, since they will have to cope with the adverse selection problem. In other words, those agents who are not really able to exploit their comparative advantages inflict a negative externality (through the adverse selection problem) on those who, in principle, would be able to fully exercise their skills.

Historically, variety of sectors has been considered *productivity-enhancing*, since it permits heterogeneous agents (in terms of skills) to obtain a better *individual-sector match*. Notice, however, this *matching-effect* is completely independent of the informational structure of the economy. Our theory claims that sectoral expansion brings about an additional positive effect on output and growth, because a larger number of activities helps to mitigate adverse selection problems associated to talent allocation.

In our model, adverse selection affects the operation of the economy because it hampers full risk-sharing. When individuals are *risk-averse*, investment in risky activities (*entrepreneurship*) will be discouraged by imperfect insurance provision. As a consequence of this, individuals' comparative advantages will not be fully exploited, and the economy will be prevented from reaching a *pareto-efficient equilibrium*. Agents' unwillingness to invest in risky (but highly productive) assets, due to inefficient risk-sharing, is the main

channel through which adverse selection will contaminate the functioning of the whole economy in our model.¹

The economy modelled in this paper is constituted by many (potential) sectors; each of them requiring some specific skill. At a particular period of time, some sectors are available to agents (i.e. some sectors actually *exist*), while some others are (still) absent. The creation of a sector is the result of a *successful innovation*. Innovation activities are undertaken by private agents who invest in research and development (R&D) and intend to maximise profits, in spirit of the *Endogenous Growth Theory* (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). The availability of many sectors facilitates the allocation of individuals talent, without interfering with other sectors. This fact reduces the severity of the adverse selection problem, enabling better insurance provision; which, in turn, encourages entrepreneurial activities. At last, vigorous entrepreneurship enhances the incentives to invest in R&D and innovate, since innovations are ultimately sold to entrepreneurs, who are the agents that ultimately put them in practise.

In this paper economic development is characterised by a continuous process of capital differentiation (sectoral expansion) and a more efficient allocation of skills. In addition to that, risk-sharing institutions become increasingly efficient over the path of development, as adverse selection problems tend to vanish away concomitantly with sectoral expansion. Nevertheless, our model may also generate a particular type of poverty-trap in which some economies are confined to an undesirable situation with few active sectors, poor allocation of individuals talent, and inefficient operation of risk-pooling institutions.

Ghatak, Morelli and Sjöström (2002) are the first authors to explicitly model misallocation of entrepreneurial talent as a consequence of an insufficiently attractive *outside option* available to the "wrong" types (i.e. those individuals who lack of entrepreneurial skills). In their model, the *outside option* available to agents who lack of entrepreneurial skills is the *market wage*. When the economy is able to provide high wages, low-quality entrepreneurs are better-off selling their work-force in the labour market. As a result, high wages help to "clean" the pool of credit applicants, reducing informational frictions and enabling better operation of the credit market.² Our paper, besides focusing on insurance instead of credit, looks at a different mechanism as the determinant of an efficient allocation of talent. We let innovation and the creation of new sectors/activities solve the adverse selection problem, by permitting individuals to specialise in the task they are best at. In that sense, one of the main insights of our model is that it concedes the innovation process a new role, very different from the one traditionally stressed in the growth literature. Innovation is not only desirable because it augments the productivity of inputs. It is also desirable because, by granting agents the chance to freely allocate their skills where they are most productive, it mitigates informational problems interfering with the

¹The claim that engaging in risky projects requires efficient risk-sharing institutions that protect from bad *states of nature* goes back to Arrow (1971). For some papers where that idea is incorporated into growth models, see Saint-Paul (1992) and Acemoglu and Zilibotti (1997).

²Grüner (2003) also provides a model where entrepreneurial skills are more efficiently allocated in richer, or less unequal, economies. However, in his model there is no interaction between different markets, and his results are driven by a pure "wealth-effect" that arises due to *limited liability issues*.

operation of the entire economy.

Acemoglu and Zilibotti (1999) also build a theory in which the quality of available information improves during the process of development. However, their motivation and object of study is completely different from ours. They are interested in how a society can provide correct incentives to agents, so that to solve moral hazard problems. They argue that, as an economy grows, each sector receives higher levels of capital, and this permits a more accurate use of *relative performance evaluation schemes* by repeating the same task. They do not study how the allocation of different skills evolves during development. Furthermore, they do not incorporate innovation decisions into their theory, which precludes the number of different economic activities from expanding along time.

In another paper, Acemoglu and Zilibotti (1997) construct a model where the degree of market incompleteness tends to disappear with capital accumulation, and sectoral differentiation enables better risk-sharing. Nonetheless, this model does not deal with the problem of skills allocation of heterogeneous agents and adverse selection. Insurance provision is enhanced with sectors horizontal expansion, simply because this allows better pooling of *sector-specific shocks*; while in ours it is the consequence of the amelioration of informational failures due to improvements in the skills allocation technology. In addition, the reason underlying their market incompleteness relates to *indivisibilities* (added to capital scarcity in poor nations). We do not need to rely on *production non-concavities*, deriving insurance under-provision from a standard asymmetric information problem.

The present paper is also related to the literature about credit market imperfections and poverty: Galor and Zeira (1993), Banerjee and Newman (1993), Piketty (1997), Aghion and Bolton (1997), and Lloyds-Ellis and Bernhardt (2000). These papers stress the importance of the initial-wealth distribution over the dynamic behaviour of the economy when agency-costs lead to credit rationing. As a general result, their models commonly lead to poverty-traps when the number of poor agents is large enough. Our theory contributes to this literature by different channels. We first provide a fully micro-founded explanation of why agency-costs may arise. Secondly, our paper is able to produce dynamics where these agency-costs are reduced as an economy develops. As a result, *rationing* is not just solved because people become rich enough (so that they can afford better credit or insurance contracts), but mainly because financial markets are able to function better.

1.1 Empirical Motivation and Some Stylised Facts

One of the fundamental observations that motivates our theory is the fact that economic development and the degree of sectoral diversification have historically displayed significant positive correlation. This correlation reflects our idea that the number of economic activities in which individuals can specialise grows simultaneously with the aggregate stock of capital and income.

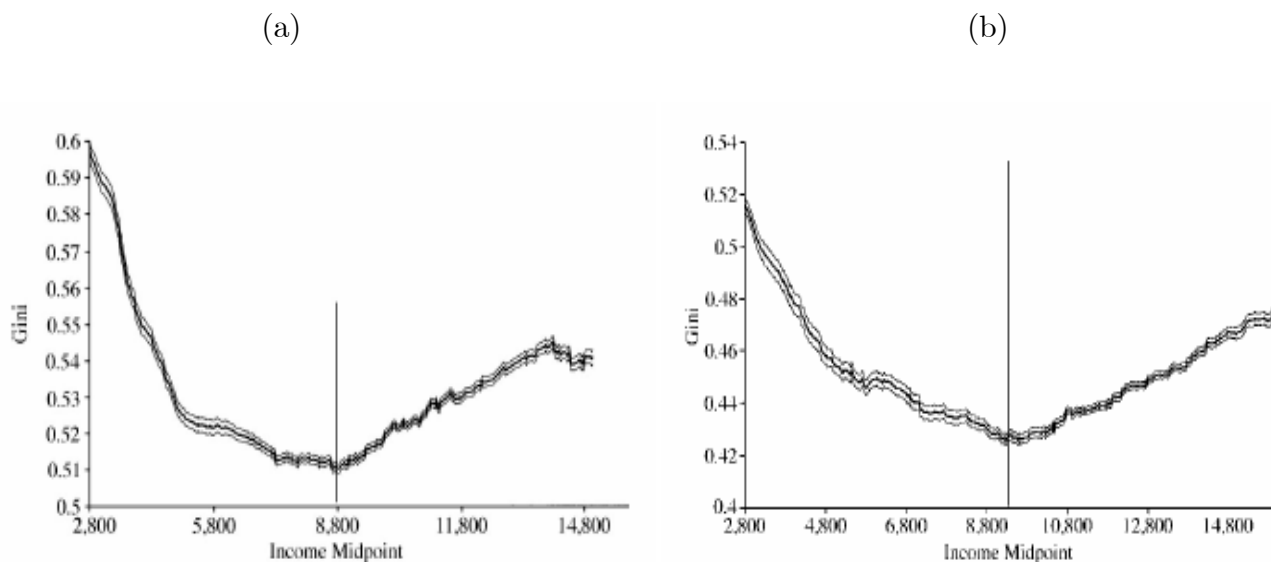
Imbs and Wacziarg (2003), making use of non-parametric techniques, provide strong and robust evidence that *sectoral concentration* (the opposite to *sectoral diversification*) follows a "U-shaped" pattern as the aggregate income level rises.³ They conclude that,

³Imbs and Wacziarg build five different concentration indices based on employment shares (these in-

over the path of development, poor countries initially experience a long process of sectoral differentiation, and this diversification pattern eventually reaches a maximum beyond which the process begins to revert. Given the implications of our paper, two observations need to be stressed here: (i) the "turning-point" in the sectoral differentiation process occurs at relatively high output levels (the authors argue that this point is located roughly at the real income level reached by Ireland in 1992), (ii) the eventual re-concentration process does only partly offset the effect of the initial diversification phase.

FIGURE 1 presents Figure 1 and 2 of Imbs and Wacziarg (2003). Panel (a) shows the non-parametric estimation when sectoral concentration is measured by the gini coefficient calculated using the 3-digit UNIDO employment data. Panel (b) displays the non-parametric results when 1-digit ILO employment data is used with the analogous purpose. Income is measured by income per-capita PPP in 1985 constant US dollars. Both figures show how sectoral concentration decreases at initial stages of development, eventually reaching a turning-point beyond which the initial diversification phase partially reverts.

FIGURE 1 : Estimated Association Between Sectoral Diversification and Income Level.



Our model also highlights the key role of financial development, through its positive effect on entrepreneurial investment and output growth. This relation between financial development and output/investment is long established and well documented in the literature: King and Levine (1993a, 1993b), Atje and Jovanovic (1993), and Benhabib

dices are: Gini coefficient, Herfindahl index, log-variance of sector shares, coefficient of variation, and the max-min spread). Their indices are constructed for three different datasets: 1-digit level from the International Labor Office (ILO), 3-digit level United Nations Industrial Development Organisation (UNIDO), and 2-digit level data from the OECD. For the UNIDO and OECD data, value added per sector are also available and utilised. All their results are strongly robust to the use of different indices and datasets.

and Spiegel (2000). Our theory particularly stresses the importance of efficient insurance schemes as a condition for entrepreneurial investment; this view is supported by Atje and Jovanovic (1993). They find evidence that the *ratio of annual value of all stock market trades to GDP* significantly increases the *average return on investment*, but the *ratio of credit extended by banks to GDP* does not seem to have any effect on that variable. Given that stock markets are better suited to diversifying idiosyncratic risks than lending institutions; their results may be interpreted as confirmation that investors do respond to the availability of improved insurance services, by shifting to riskier and more productive investment projects.

Closely related to issue to financial development and growth, we find the observation that income variability and output are negatively correlated. Section II in Acemoglu and Zilibotti (1997) thoroughly goes through several pieces of both recent and historical evidence of this fact. Ramey and Ramey (1995) show that “countries with higher output volatility have lower mean growth, even after controlling for other observables and for country-fixed effects”. Surprisingly, they show the negative effect of volatility on output does not seem to work through investment. This result quite contradicts the spirit of one of the main motivations of this paper; the fact that the incapacity to smooth out shocks may discourage entrepreneurial investment. However, Aizenman and Marion (1999) estimate the effect of volatility on investment, disaggregating between private and public investment for a sample of 43 developing countries. In opposition to Ramey and Ramey (who only utilise aggregate investment), they find a very strong and significant negative effect of volatility on private investment. On the other hand, when they look at public investment, it turns out that the effect volatility is positive in this case (which seems to explain the absence of any significant relation between volatility and total investment in Ramey and Ramey (1995)).

Finally, a fundamental prediction of our theory is that the degree of sectoral diversification has a positive effect on financial development. In particular, this paper argues that higher degree of sectoral diversification permits more efficient insurance provision, since it helps to solve adverse selection problems. This effect has neither been studied before, nor has it been incorporated into any theoretical model in the existing development literature. After we present and develop the model that illustrates our theory (in the following 4 sections), we will provide some evidence which is consistent with this prediction of the model. In any case, the reader can look at Tables 1 and 2 in Section 6, if he or she wishes to have an earlier look at these data.

The paper is organised as follows. Section 2 describes the basic set-up of the model. Section 3 presents the behaviour of the individuals/entrepreneurs, emphasizing how adverse selection problems affect their optimal portfolio allocations. Section 4 introduces the innovation activities into the model, which endogenises the number of sectors available in the economy. Section 5 proceeds to the dynamic study of this economy. Section 6 shows some additional descriptive evidence consistent with the model’s key predictions. Section 7 concludes with some further discussion.

2 Environment

We consider an economy in which goods production is organised by small and independent entrepreneurs. Life evolves during the time horizon $t = \{1, 2, \dots, \infty\}$. The economy enjoys full access to international credit markets, and is constituted by three different kinds of agents:

1. *Entrepreneurs*: These agents organise the final-goods production process. We will also refer to them simply as *individuals*.
2. *Insurance Companies*: They issue and sell insurance contracts that protect from *entrepreneurial failure*.
3. *Innovators*: They carry out R&D in order to transform basic knowledge into knowledge applicable to the production of goods. *Innovations* expand the number of sectors or activities available in the economy (*horizontal innovations*).

This economy is composed by a continuum of *sectors* indexed by the letter i along the interval $[0, 1]$. In addition to these *sectors*, there exists a continuum of *villages* also indexed by i along the interval $[0, 1]$. Each *village* $i \in [0, 1]$ is inhabited by an *Entrepreneur of Type- i* , to whom we will also refer simply as a *Type- i* . *Insurers* and *innovators* live outside these villages, in some piece of common-land located within the boundaries of the economy. Credit markets are perfect and subject to international capital mobility, so everybody can lend and borrow as much as desired at the given international interest rate $r = 0$.

At the beginning of $t = 1$ this economy "inherits" some degree of *market incompleteness* from the previous period $t = 0$.⁴ More precisely, during $t = 0$ only a fraction $n_0 \in (0, 1)$ of all *sectors* were able to enjoy the activity of productive industries. At the same time, the remaining fraction $(1 - n_0)$ lacked of any active industry whatsoever. The *degree of market incompleteness* in period t could be then measured by $1 - n_t$. We will denote the set of sectors with active industries in period t by $\mathcal{A}_t \subset [0, 1]$. We work under the assumption that the availability of productive industries is always the result of horizontal innovations (either during the past or in the present). Once a sector is created, it does never disappear (i.e. if *sector* $i \in \mathcal{A}_t$, then *sector* $i \in \mathcal{A}_{t+\Delta} \forall \Delta \geq 1$). Therefore, the set \mathcal{A}_t is determined by the "inherited" set \mathcal{A}_{t-1} (the *old sectors*), plus the *new sectors* resulting from horizontal innovations during period t (the set of *new sectors* is given by $\mathcal{A}_t \cap \mathcal{A}_{t-1}^c$). To ease notation, hereafter we skip the use of time-subscripts when creating no confusion. We will refer to those sectors belonging to \mathcal{A} as *active sectors*, while those sectors that do not pertain to \mathcal{A} will be denoted *inactive sectors*.

⁴We abuse a bit of the language here, and we use the term *market incompleteness* to refer to *sectors incompleteness* (or, which would be exactly the same in this model, *final-goods market incompleteness*).

2.1 Technology

A sector $i \in \mathcal{A}$ provides individuals the chance to invest in an entrepreneurial project called *Project- i* . The return of *Project- i* is random, and depends on the realisation of a purely idiosyncratic shock. For simplicity, we suppose there are only two states of nature: *good* (success) and *bad* (failure). Denote by K the amount of physical capital invested in the entrepreneurial project (imagine K as machinery, or any sort of physical capital measured in a common scale). If an individual invests K units of capital in *Project- i* , then, after the state of nature is revealed, this individual obtains:

$$\begin{aligned} \text{in good state of nature (success): } & F(K) + (1 - \delta)\mathcal{P}K \\ \text{in bad state of nature (failure): } & (1 - \delta)\mathcal{P}K \end{aligned}$$

Where $\delta \in (0, 1)$ represents the depreciation rate of K during one period, and \mathcal{P} is the price of each unit of K ; therefore, $(1 - \delta)\mathcal{P}K$ equals the *residual value* of the physical capital purchased at the beginning of the period. We suppose:

$$F(K) = \Gamma K^\alpha \quad \text{where: } \alpha \in (0, 1) \text{ and } \Gamma > 0$$

2.2 Agents

Entrepreneurs: All individuals live just for one period, let's say t , which lasts from t^- till t^+ . A new generation of individuals is born at the very end of period t . Each individual procreates 1 son, hence population remains constant across time. Individuals are *warm-glow* altruistic -see Andreoni (1989)- and, accordingly, leave positive bequests (b_{t+1}) to the next generation. Agents' decision timing is elicited below:

FIGURE 2: Individuals Decision Timing



Consumption (c_t) and bequests (b_{t+1}) occur at the end of period t (t^+), and they are subject to uncertainty due to the stochasticity of the agent's portfolio returns. In principle, any individual would be able to transform his initial endowments into resources for future consumption and bequests by two different means: 1) investing in a risky project (i.e. becoming an *entrepreneur*); or 2) storing wealth at the given international interest rate $r = 0$ (i.e. gross return is $1 + r = 1$). Diversification among entrepreneurial projects is not feasible; in other words, agents must *specialise* in one particular project.

All individuals share identical preferences: $E[u(\psi c_t^\rho b_{t+1}^{1-\rho})]$, where $\psi \equiv [\rho^\rho (1 - \rho)^{1-\rho}]^{-1}$ and $\rho \in (0, 1)$. The variable c_t represents "generation- t " agent consumption, and b_{t+1} stands for the amount bequeathed to the next generation (his offspring). In addition, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Given this formulation, by denoting *end-of-period-wealth* in t

by y_t , it can be shown that expected utility maximisation leads to $c_t^* = \rho y_t$ and $b_{t+1}^* = (1 - \rho)y_t$.⁵ As a result, we can simply re-formulate the individual's expected utility as follows: $E[u(\psi c_t^\rho b_{t+1}^{1-\rho})] = E[u(y_t)]$. For the rest of the paper, we work with a utility function displaying constant absolute risk aversion (CARA). That is:

$$u(\psi c_t^\rho b_{t+1}^{1-\rho}) = -\exp(-\psi c_t^\rho b_{t+1}^{1-\rho}) \quad \Rightarrow \quad u(y_t) = -\exp(-y_t)$$

Where, for simplicity, it has been assumed that the *coefficient of absolute risk aversion* is equal to 1. The use of CARA preferences is motivated by the fact that they do not exhibit "wealth-effects". As a consequence of that, income distribution will not matter for risky investment choices. As we will see later on, this fact will allow us to study the effect of the *number of active sectors* on portfolio allocation decisions, in complete isolation from income distribution issues.⁶

Recall that each single individual/entrepreneur belongs to a particular type i indexed along the interval $i \in [0, 1]$. Let us denote by $\phi_{i,s}$ the *failure probability* for entrepreneurs of *Type-s* $\in [0, 1]$ when investing in *Project-i*, where by *failure* is meant the *bad state of nature*. Types differ from each other with respect to their comparative skills, in particular:

$$\phi_{i,i} < \phi_{i,j}, \text{ for all } j \neq i.$$

In simple words, a *Type-i* is an individual with intrinsic comparative advantage in *Project-i* (of course, as long as this particular project actually exists!). For the sake of analytical simplicity, it will be supposed that $\phi_{i,j} = 1$ for all $j \neq i$, and $\phi_{i,i} = \phi \in (0, 1)$ for all $i \in [0, 1]$.

A key assumption in this paper is that individuals' types are private information. In other words, there is asymmetric information regarding the entrepreneurs' skills. In terms of our specific model, asymmetric information means the *place of residence* (or *citizenship*) of every single individual/entrepreneur is unobservable. In addition to that, we assume types are genetically uncorrelated, implying that parents' historical outcomes provide no information whatsoever about the type of a child.

Insurance Companies: Assume there exists an Arrow-Debreu commodity that protects from *entrepreneurial failure*. We suppose there are insurance companies who offer insurance contracts to entrepreneurs which (in principle) possess the following structure: 1) An *Type-j* must pay the amount $p^j \times q^j$ to buy q^j units of the Arrow-Debreu commodity that protect from entrepreneurial failure; 2) once the *state of nature* is revealed, if the project has failed (i.e. in the bad state of nature), each unit of Arrow-Debreu

⁵Notice *end-of-period wealth in t* (y_t) is endogenously determined by the *portfolio returns* of the individual (alive during period t).

⁶In another paper, Jaimovich (2005), I study how "wealth-effects" affect agents' optimal portfolio allocations when the economy is subject to similar adverse selection problems, utilising a utility function with *constant relative risk aversion* (which implies absolute risk aversion is decreasing in wealth). Succinctly, when absolute risk aversion is decreasing in wealth, poorer agents will suffer to a greater extent the negative consequences of adverse selection problems. Incorporating this effect would then just reinforce the results that will be shown in the present paper.

commodity entitles its owner to 1 unit of final output as indemnity. Thus, an *insurance contract* can be specified as a pair $(p^j, q^j) \in [0, \infty) \times [0, \infty)$, with its pay-off contingent on entrepreneurial success. It will be supposed that, at the moment of the insurance contract signing, insurers cannot observe the rest of the individual's portfolio allocation (i.e., they can neither observe how much an individual has invested in the risk-less asset, nor how much he has invested in the entrepreneurial project). This last assumption plays a very important role in the model, since observability of either investment in the risk-less asset or in the entrepreneurial project would provide insurance companies with valuable information when trying to infer someone's type from his actions.

The insurance industry is characterised by free-entry and absence of sunk or set-up costs. This entails insurers must make zero profits in equilibrium. Denote by $Q_t \subset [0, \infty) \times [0, \infty)$ the set of insurance contracts offered by insurance companies in period t .

Definition 1 (Equilibrium in the Insurance Market) *An equilibrium in the insurance market in period t consists of a set of contracts Q_t ; such that, given the entrepreneurs optimal behaviour: 1) no insurance company makes any losses; and 2) given the set Q_t , there exists no insurance contract \mathfrak{z} , such that $\mathfrak{z} \notin Q_t$ and $\mathfrak{z} \in [0, \infty) \times [0, \infty)$, which, if offered, would make non-negative expected profits.*

FIGURE 3: Innovators and Innovations Timing



Innovators: Finally, there exists a continuum of innovators indexed by $i \in [0, 1]$. Innovators also live just for one period. A new continuum $[0, 1]$ of innovators are spontaneously bred at the very beginning of each period. Innovators borrow capital from international credit markets in order to invest in R&D and produce *innovations*. Innovators "play" before insurers and entrepreneurs do (they "play" at some moment between t^- and *portfolio decisions* in FIGURE 2; see FIGURE 3 below). The rationale for this timing is straightforward: it is innovators' behaviour what ultimately determines the set \mathcal{A}_t ; and only when \mathcal{A}_t is actually determined can the entrepreneurs' optimisation problem in period t be fully specified.

Innovators are *risk-neutral* agents and, accordingly, seek to maximise expected profits. We rationalise this by supposing that innovators are agents whose assets ultimately belong to individuals. As a result of this, innovators will act as single-lived agents that maximise expected profits, and afterwards distribute their profits among individuals⁷ (according to their ownership shares).⁸

⁷The *risk-neutrality* result holds even if innovators face idiosyncratic shocks, since in that case (risk-averse) individuals would hold fully diversified portfolios of shares on innovators assets. However, it does necessarily require that there exist no aggregate shocks affecting innovators' outcomes. In that sense, the following model implicitly assumes no aggregate shocks of any sort affect our economy.

⁸Because the use of CARA preferences wipes out any sort of "wealth-effects", none of our model's main results will depend on how ownership on innovators assets is actually distributed among the population.

3 Entrepreneurs Optimisation Problem

In this section we present the optimisation problem faced by the (potential) entrepreneurs. As a benchmark, we start by looking at the hypothetical case in which types are publicly observable. Subsequently, we proceed to study the problem under asymmetric information, where adverse selection problems may distort the individuals optimal portfolio allocation. Hereafter, *full-information* will designate those cases in which types and portfolio allocations are publicly observable.

Recall that entrepreneurs choose their portfolio allocations after innovators have "played" in our model. This means that an entrepreneur born in period t receives the set \mathcal{A}_t as exogenously given. Furthermore, this also implies his budget constraint will include the share of innovators' profits over which he holds property rights. Let us define $w_t^j \equiv b_t^j + \pi_t^j$; where b_t^j is the bequest received by the Type- $j \in [0, 1]$ born in period t , and π_t^j represents the income derived from innovators profits received by this individual. Accordingly, the variable w_t^j will represent the total amount of resources (i.e. initial-wealth) that the Type- j born in period t will need to allocate in his portfolio allocation problem.

Notation - For the rest of this paper *superscripts* will always indicate the entrepreneur's type, S_t will represent the amount of initial wealth allocated to the safe-asset yielding fixed net return $r = 0$, $K_{i,t}$ will stand for the amount of capital invested in *Project- i* , and q_t will designate the quantity of Arrow-Debreu commodity insuring against entrepreneurial failure that is purchased.

Definition 2 (Entrepreneurs Problem) *Given (i) the set of active sectors \mathcal{A}_t , (ii) the set of offered insurance contracts Q_t , (iii) the set of prices $\{P_i\}_{i \in \mathcal{A}_t}$, and (iv) the level of initial-wealth w^j . In equilibrium, the entrepreneur of type- $j \in [0, 1]$ selects the portfolio allocation $\left[S_t^j(w^j), q_t^j(w^j), \{K_{i,t}^j(w^j)\}_{i \in \mathcal{A}_t} \right]$ that maximises his expected utility function, subject to his budget constraint.*

3.1 Full-Information

3.1.1 Optimal allocation by a type whose "appropriate" sector is *active*:

Assume *sector $i \in \mathcal{A}_t$* , and study the optimisation problem of the Type- i entrepreneur. Denote by p^j the price of the Arrow-Debreu commodity which protects from entrepreneurial failure charged upon individuals of Type- $j \in [0, 1]$. Then, under *full-information*, a Type- i will be confronted to $p^i = \phi$ in case he specialises in *Project- i* , and to $p^i = 1$ if he instead specialises in some *Project- $h \neq i$* , such that $h \in \mathcal{A}_t$.⁹ Given those prices, it must be straightforward that a Type- i will always *specialise* in *Project- i* . Thus, his

⁹This stems from free-entry, plus the absence of any kind of fixed/set-up costs in the insurance industry.

maximisation problem reads as follows:

$$\begin{aligned} \underset{\{S_t^i, q_t^i, K_{i,t}^i\}}{MAX} : E(U_t^i) = & - \{ \phi \exp [-(S_t^i + q_t^i + (1 - \delta)\mathcal{P}_{i,t} K_{i,t}^i)] \\ & + (1 - \phi) \exp [-(S_t^i + F(K_{i,t}^i) + (1 - \delta)\mathcal{P}_{i,t} K_{i,t}^i)] \} \end{aligned} \quad (I)$$

$$s.t. : w_t^i = S_t^i + \mathcal{P}_{i,t} K_{i,t}^i + \phi q_t^i \quad \text{and} \quad K_{i,t}^i, q_t^i \geq 0$$

Recall $F(K) = \Gamma K^\alpha$. To ease the algebra, assume henceforth that $\Gamma = [\alpha(1 - \phi)]^{-1}$. In addition to that, in order to avoid muddling the notation of subsequent equations, denote $\zeta_i \equiv (\delta\mathcal{P}_i)^{-1/(1-\alpha)}$ (hereafter we drop the time-subscripts when creating no confusion). Note that ζ_i is actually a decreasing function of \mathcal{P}_i ; however, from the entrepreneurs' perspective \mathcal{P}_i is exogenously determined, hence we can safely take ζ_i as an given parameter throughout Section 3.¹⁰ From *Problem (I)* we may get the FOC: $F'(K_i^i) = \delta\mathcal{P}_i(1 - \phi)^{-1}$; which leads to the following optimal portfolio allocation:

$$K_i^i = \zeta_i \quad (1)$$

$$q^i = \frac{1}{\alpha(1 - \phi)} \zeta_i^\alpha \quad (2)$$

$$S^i = w^i - \mathcal{P}_i \zeta_i - \frac{\phi}{\alpha(1 - \phi)} \zeta_i^\alpha. \quad (3)$$

From Eq.(1)-(3) it can be noticed that individuals are able to completely smooth out consumption across states of nature (*end-of-period-wealth* is given by $y^i = w^i + (1 - \alpha)/\alpha$, regardless the state of nature).¹¹

3.1.2 Optimal allocation by a type whose "appropriate" sector is *inactive*:

Let's study now the situation faced by an individual of type $h \in [0, 1]$, assuming that *sector* $h \notin \mathcal{A}$. In principle, a Type- h would be able to invest in any *Project- i* , such that $i \in \mathcal{A}$. At the same time, he could also buy insurance protecting from entrepreneurial failure. However, given full-information, a Type- h should pay $p^h = 1$ for each unit of Arrow-Debreu commodity he buys. Since $\phi_{i,h} = 1$ and $\delta < 1$, then this Type- h will never invest any amount of capital in *Project- i* . Clearly, by investing in the safe-asset, he is able to obtain higher gross return ($1 + r = 1$), bearing no risks. As a result, straightforwardly, the Type- h optimisation problem -analogous to *Problem (I)*- delivers: $S^h = w^h$, $q^h = 0$, and $K_i^h = 0$ for all $i \in \mathcal{A}$. Lastly, notice this Type- h obtains as (certain) maximum utility level: $-e^{-w}$.¹²

¹⁰In Section 4, once we introduce the innovators decision problem, the equilibrium value of \mathcal{P}_i will be determined within the model (and the exact value of ζ_i will be accordingly pinned down as well)

¹¹Eq.(1)-(3) will be in fact the solution of *Problem (I)* for any utility function $u(\cdot)$, such that: $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

¹²Rigorously speaking, given that $p^h = \phi_{i,h} = 1$, there exist an infinite number of equilibria for this particular Type- h entrepreneur. More precisely, any allocation in which: $S^h = w^h - q^h$, $K_i^h = 0$ and

3.2 Asymmetric Information

We proceed now to study the entrepreneurs' optimisation problem when types are private information; this situation will naturally give rise to adverse selection problems in the insurance market. In particular, any individual would be able to hide his *type*, and try to take advantage of possible mismatches between the insurance *market price* and his particular *actuarially fair price*.

Suppose *sector* $h \notin \mathcal{A}$, *sector* $i \in \mathcal{A}$, and *sector* $j \in \mathcal{A}$. Adverse selection problems may emerge in two distinct modes. First, a Type- $h \notin \mathcal{A}$ may wish to replicate the Type- $i \in \mathcal{A}$ behaviour in the market of the Arrow-Debreu commodity. Second, a Type- $i \in \mathcal{A}$ may wish to deviate from *Project- i* , and specialise in *Project- j* ; trying to take advantage of the fact that $\phi_{j,i} = 1$. Notice that, in the first case, the *outside option* available to a Type- $h \notin \mathcal{A}$ is just $S^h = w^h$; whereas, in the second case, a Type- $i \in \mathcal{A}$ has access to a better *outside option*, since he could also succeed in *Project- i* . As result of this, the adverse selection problem generated by a Type- $h \notin \mathcal{A}$ will bring about worse consequences than the adverse selection problem created by a Type- $i \in \mathcal{A}$. This is the key insight that will be exploited in this paper; i.e. the fact that increasing the number of active sectors reduces the severity of the adverse selection problem, by improving the outside option available to some individuals.

For the rest of the paper we restrict the analysis to *symmetric equilibria*; where by *symmetric* it is meant what follows: if *sector* $i \in \mathcal{A}$ and *sector* $j \in \mathcal{A}$, then both the *type- i entrepreneur* and the *type- j entrepreneur* must purchase the *same* amount of insurance in equilibrium. This requires setting $\mathcal{P}_i = \mathcal{P}$, $\forall i \in \mathcal{A}$ (i.e. all sorts of physical capital, used for different entrepreneurial projects, are sold at the same price \mathcal{P}).¹³ In any case, as it will become clear later on in Section 4, any equilibrium in this model will necessarily be symmetric.

In principle, given asymmetric information and the set of active sectors $\mathcal{A} \subset [0, 1]$, one may expect two different kinds of equilibria to arise in the insurance market: 1) a *pooling equilibrium*, in which all types who buy the same amount of Arrow-Debreu commodity and pay a unique price; 2) a *separating equilibrium*, in which different types buy different amounts of contingent commodity, and pay the actuarially fair price according to their particular type (this would represent a Rothschild and Stiglitz (1976) equilibrium). In the remainder of this section we show that any equilibrium in the insurance market must necessarily entail some sort of *pooling-contracts*. Subsequently, we fully characterise the equilibrium pooling-contracts and the optimal portfolio allocations under asymmetric information.

$q^h > 0$, is also an optimum in this problem. We will disregard all these trivial results. Note that, because $p^h = \phi_{i,h} = 1$, an Arrow-Debreu commodity sold to a Type- $h \notin \mathcal{A}$ is (under full information) indistinguishable from the safe-asset, since they both ultimately display exactly the same pay-off function.

¹³Note that if $\mathcal{P}_i = \mathcal{P}$ for all $i \in \mathcal{A}$, then $\zeta_i = \zeta \equiv (\delta\mathcal{P})^{-1/(1-\alpha)}$ for all $i \in \mathcal{A}$.

3.2.1 Separating Insurance Contracts

Recall that K_i^h and S^h are both unobservable to insurance companies (at least at the moment when signing up the insurance contract). Given that $\phi_{i,h} = 1$ for all $h \neq i$ and $\delta < 1$, a Type- h will always optimally choose $K_i^h = 0$ (he would never invest any capital in an entrepreneurial project for which he lacks the required skills, as he could never succeed in that project). As a consequence of this, given the budget constraint, it must be the case that $S^h = w^h - p^h q^h$. *Proposition 1* states that no separating equilibria, where insurers screen agents according to their type, may possibly exist.

Proposition 1 *Assume the set of inactive sectors is non-empty (i.e. $\mathcal{A}^c \neq \emptyset$). Take any sector $i \in \mathcal{A}$ and any sector $h \notin \mathcal{A}$. Then, there can never exist an equilibrium in the insurance markets in which: $q^i \neq q^h$ and $q^i > 0$.*

Proof. *Given free-entry in the insurance industry, in an equilibrium with $q^i \neq q^h$, where $i \in \mathcal{A}$ and $h \notin \mathcal{A}$, failure probabilities are truthfully revealed; then, $p^i = \phi$ and $p^h = 1$ (so that insurance companies make zero profits). Take some Type- $h \notin \mathcal{A}$; he would optimally choose q^h at unit-price $p^h = 1$, instead of $q^i > 0$ at unit-price $p^i = \phi$, only if the contract ($p^h = 1, q^h$) is incentive-compatible.*

Denote by \tilde{S}^h the amount invested in the safe-asset by the Type- h when "mimicking" the Type- i behaviour in the insurance market. Since $\phi_{i,h} = 1$, the incentive compatibility constraint for a Type- h with initial-wealth w^h is given by:¹⁴

$$-\exp(-w^h) \geq -\exp\left[-(\tilde{S}^h + q^i)\right] \quad (4)$$

Given $\phi_{i,h} = 1$ and $\delta < 1$, it is always the case that in an optimum $K_i^h = 0$. Thus, by the budget constraint, we get: $\tilde{S}^h = w^h - \mathcal{P}_i K_i^h - p^i q^i = w^h - \phi q^i$. Then, plugging $\tilde{S}^h = w^h - \phi q^i$ into Eq.(4) and cancelling out repeated expressions, we can re-write it as:

$$w^h \geq (1 - \phi)q^i + w^h \quad (5)$$

Finally, from Eq.(5) it can be straightforwardly observed that incentive-compatibility would necessarily require $q^i = 0$. ■

3.2.2 Pooling Insurance Contracts

In a pooling equilibrium a certain amount of Arrow-Debreu commodity (say q) is sold at a unique unit-price (say p) to any Type- $l \in [0, 1]$ who wishes to buy it. Thus, a pooling insurance contract displays, in general, the following structure: $(p, q) \in [0, \infty) \times (0, \infty)$, with pay-off contingent on entrepreneurial success.

Keep assuming sector $i \in \mathcal{A}$ and sector $h \notin \mathcal{A}$. Notice that a Type- h will always wish to buy a pooling contract $(p, q) \in (\phi, 1) \times (0, \infty)$, as this contract renders him strictly

¹⁴Eq.(4) states that the utility obtained by a Type- h when he replicates the Type- i behaviour in the insurance market should not exceed the utility this Type- h gets from his outside option ($y^h = w^h$).

positive pay-off because his failure probability equals 1 in all *active sectors* (additionally, he is indifferent between buying, or not, any insurance contract sold at $p = 1$). Therefore, given free-entry, any pooling equilibrium in the insurance market must verify the following two properties. First, the *equilibrium price* must range $p \in (\phi, 1]$; this is because the Type- h always buys the pooling contract. Second, the equilibrium insurance contract must maximise the expected utility of any Type- i , such that sector $i \in \mathcal{A}$; otherwise insurance companies could offer a different contract such that it makes non-negative profits and, at the same time, makes a Type- i better-off. Bearing all this in mind, in order to derive the equilibrium pooling contract, as a first step, we can start by solving the following problem for any Type- i , such that sector $i \in \mathcal{A}$:

$$\begin{aligned} \underset{\{S^i, q^i, K_i^i\}}{\text{MAX}} : & E(U^i) = - \left\{ \phi \exp \left[-(S^i + q^i + (1 - \delta) \mathcal{P}_i K_i^i) \right] \right. \\ & \left. + (1 - \phi) \exp \left[-(S^i + F(K_i^i) + (1 - \delta) \mathcal{P}_i K_i^i) \right] \right\} \end{aligned} \quad (\text{II})$$

$$\text{s.t. : } w_t^i = S^i + \mathcal{P}_i K_i^i + p q^i \quad , \quad K_i^i ; q^i \geq 0 \quad \text{and} \quad p \in (\phi, 1]$$

The solution of *Problem (II)* is characterised in *Lemma 1*.

Lemma 1 *Assume $\mathcal{P}_i = \mathcal{P}$ for all $i \in \mathcal{A}$, and denote by \bar{p} the value of x that solves*

$$\frac{1}{\alpha(1 - \phi)} \zeta^\alpha \left(\frac{1 - x}{1 - \phi} \right)^{\frac{\alpha}{1 - \alpha}} + \ln \left(\frac{\phi}{1 - \phi} \frac{1 - x}{x} \right) = 0,$$

and note that $\bar{p} \in (\phi, 1)$. Then:

1) *For all $p < \bar{p}$, Problem (II) yields:*

$$q^i(p) = \frac{1}{\alpha(1 - \phi)} \zeta^\alpha \left(\frac{1 - p}{1 - \phi} \right)^{\frac{\alpha}{1 - \alpha}} + \ln \left(\frac{\phi}{1 - \phi} \frac{1 - p}{p} \right) \quad (6)$$

$$K_i^i(p) = \zeta \left(\frac{1 - p}{1 - \phi} \right)^{\frac{1}{1 - \alpha}} \quad (7)$$

$$S^i(p) = w - \mathcal{P} K_i^i(p) - p q^i(p) \quad (8)$$

2) *For all $p \geq \bar{p}$, Problem (II) delivers:*

$$q^i = 0 \quad (9)$$

$$K_i^i = \bar{K} \equiv \zeta \left(\frac{1 - \bar{p}}{1 - \phi} \right)^{\frac{1}{1 - \alpha}} \quad (10)$$

$$S^i = w - \mathcal{P} \bar{K} \quad (11)$$

Lemma 1 states that when the price of the Arrow-Debreu commodity rises above \bar{p} , then the optimal demand of this commodity falls down to zero. As a consequence, only

when a pooling equilibrium can be supported at a price $p \in (\phi, \bar{p})$, will this equilibrium entail positive insurance provision. From *Lemma 1* we can also observe that, for all $p > \phi$, the amount invested in *Project-i* by the Type- i will be strictly below ζ . Recalling from Eq.(1) that $K_i^i = \zeta$ represents its *full-information* solution; this means the market failure in the insurance market also distorts entrepreneurs' investment decisions, discouraging risk-taking.

The Incentive Compatibility Constraint: Solving *Problem (II)* does not suffice to fully characterise the pooling equilibrium in the market of the Arrow-Debreu commodity. Because $\phi_{i,h} = 1$, when *sector* $h \notin \mathcal{A}$, a Type- h will always find profitable to buy an insurance contract $(p_i, q_i) \in (\phi, 1) \times (0, \infty)$. However, we haven't yet studied whether a Type- i , given that *sector* $i \in \mathcal{A}$, indeed desires to behave according to what *Lemma 1* stipulates. In fact, since a Type- i can also hide his type, he might well wish to deviate from choosing *sector* i , and pretend to be, for instance, a Type- j by specialising in *sector* $j \in \mathcal{A}$. As a consequence of this, we also need to check the incentive compatibility constraint (IC) for a type whose "appropriate" sector is *active*, so that to offer only incentive-compatible insurance contracts.

Consider the situation in which both sector i and j belong to \mathcal{A} . The IC for an entrepreneur of type- i appears below in Eq.(12), where $q^j(p)$ represents the amount of Arrow-Debreu commodity purchased by Types- j at price p (notice we have already incorporated into Eq.(12) the fact that, as a general property of CARA preferences, the value of w will only affect the amount invested in the safe-asset; being q^i , q^j and K_i^i always independent of w):¹⁵

$$(IC) \quad - \left\{ \phi \exp \left[- \left(S^i(w^i, p) + q^i(p) + (1 - \delta) \mathcal{P} K_i^i(p) \right) \right] + (1 - \phi) \exp \left[- \left(S^i(w^i, p) + F(K_i^i(p) + (1 - \delta) \mathcal{P} K_i^i(p)) \right) \right] \right\} \geq - \exp \left\{ - \left[(1 - p)q^j(p) + w^i \right] \right\} \quad (12)$$

The *LHS* of Eq.(12) amounts the expected utility achieved by a Type- i when his portfolio allocation is given by $[S^i(w^i, p), q^i(p), K_i^i(p)]$. The *RHS* equals the utility obtained by this Type- i when he replicates the behaviour Type- j in the insurance market, and, at the same time, invests the remainder of w^i in the safe-asset (which then equals $w^i - pq^j(p)$). Thus, Eq.(12) simply states that a Type- i should find no incentive to deviate from the portfolio allocation $[S^i(w^i, p), q^i(p), K_i^i(p)]$, by pretending to be an individual of Type- j .

In a symmetric equilibrium, by definition: $q^i(p) = q^j(p) = q(p)$, must hold $\forall i, j \in \mathcal{A}$. Replacing $q^i(p) = q^j(p) = q(p)$ into Eq.(12), we can easily provide another interpretation to the incentive-compatibility constraint. Assume that a Type- i will need to exert some *effort* in order to succeed in *Project-i* with probability $1 - \phi$; but suppose effort is cost-less (to avoid mixing adverse selection with moral hazard problems). In that case, Eq.(12) could also be understood as an IC on the *effort level* exerted by this Type- i when running his own *Project-i*. In that sense that, on the one hand, if he plans to *shirk* and let *Project-i*

¹⁵N.B: $S^i(w^i, p)$, $q^i(p)$ and $K_i^i(p)$, are not necessarily given by *Problem (II)* solution, represented by Eq.(8) or (11), Eq.(6) or (9), and Eq.(7) or (10); respectively. As just mentioned in the previous paragraph, those solutions are not necessarily incentive-compatible.

fail with probability equal to 1, his optimal allocation would be given by: $K_i^i = 0$, $q^i = q$ and $S^i = w - pq$; leading to the utility level shown in *RHS* of Eq.(12). On the other hand, in the case the Type- i chooses to put the needed effort in *Project- i* , the *LHS* of Eq.(12) amounts the expected utility he would then obtain. As long as Eq.(12) holds, this Type- i will indeed exert the required effort level in order to succeed in *Project- i* with probability $1 - \phi$.

Denote by p^* the equilibrium price in the insurance market. Then, it follows:

Lemma 2 *Take any i and $j \in \mathcal{A}$. Then, there exists a unique real value $\hat{p} \in (\phi, \bar{p})$, such that: $i) \forall p^* > \hat{p}$, the IC stipulated in Eq.(12) does not bind in equilibrium; and $ii) \forall p^* \leq \hat{p}$, the IC stated in Eq.(12) binds in equilibrium.*

The result enunciated in *Lemma 2* is key in order to find our model's equilibrium solution. Succinctly, if the equilibrium price p^* exceeds the value \hat{p} , where $\hat{p} \in (\phi, \bar{p})$; then, the optimal solutions stated in *Lemma 1* (i.e. Eq.(7)-(8), or Eq.(10)-(11)) will indeed correspond to the pooling-equilibrium of our model. Intuitively, whenever $p^* > \hat{p}$, *Problem (II)* solution is also incentive-compatible for any Type- i , such that $i \in \mathcal{A}$; therefore, it can be safely implemented under asymmetric information. In addition to that, since any individual of type- i , such that $i \in \mathcal{A}$, is maximising his expected utility, there exists no other feasible contract which yields non-negative pay-off and would still be purchased by those individuals.

What happens when $p^* \leq \hat{p}$? According to *Lemma 2*, the IC will be binding in equilibrium. Denote by $\hat{q}(p^*)$ the incentive-compatible level of q at price level $p = p^*$ derived from Eq.(12). When the IC binds, the pooling equilibrium contract can be found by solving *Problem (II)* including the additional constraint $q^i \leq \hat{q}(p^*)$. This leads to a Lagrangian function

$$\begin{aligned} \mathcal{L} = & - \left\{ \phi \exp \left[- (S^i + q^i + (1 - \delta) \mathcal{P} K_i^i) \right] + (1 - \phi) \exp \left[- (S^i + F(K_i^i) + (1 - \delta) \mathcal{P} K_i^i) \right] \right\} \\ & + \lambda (w - S^i - \mathcal{P} K_i^i - p^* q^i) + \mu (\hat{q}(p^*) - q^i) \end{aligned}$$

Since the IC binds, it must be the case that $q^i = \hat{q}(p^*)$. On the other hand, *inada conditions* imply that, in an optimum, $K_i^i > 0$ must always hold. Hence, the FOC $\partial \mathcal{L} / \partial S^i = \partial \mathcal{L} / \partial K_i^i = \lambda$ lead to:

$$\hat{q}(p^*) = F(K_i^i) + \ln \left(\frac{\phi}{1 - \phi} \frac{\delta \mathcal{P}}{F'(K_i^i) - \delta \mathcal{P}} \right) \quad (13)$$

Eq.(13) implicitly delivers a function $K_i^i = \hat{K}(\hat{q}(p^*))$, strictly increasing in $\hat{q}(p^*)$, which describes the optimal choice of K_i^i when $q^i = \hat{q}(p^*)$. In addition to this, using the budget constraint, $S^i = \hat{S}(\hat{q}(p^*)) = w - \mathcal{P} \hat{K}(\hat{q}(p^*)) - p^* \hat{q}(p^*)$. We lastly need to pin down the exact functional form of $\hat{q}(p^*)$. To do this, we can replace $q^i = \hat{q}(p^*)$, $K_i^i = \hat{K}(\hat{q}(p^*))$ and $S^i = \hat{S}(\hat{q}(p^*))$ into Eq.(12), to obtain

$$e^{\delta \mathcal{P} \hat{K}(\hat{q}(p^*))} \left[\phi e^{-\hat{q}(p^*)} + (1 - \phi) e^{-F(\hat{K}(\hat{q}(p^*)))} \right] = e^{-\hat{q}(p^*)}$$

which using the condition $F(\widehat{K}(\widehat{q}(p^*))) = \widehat{q}(p^*) + \ln\left(\frac{1-\phi}{\phi}(\delta\mathcal{P})^{-1}[F'(\widehat{K}(\widehat{q}(p^*))) - \delta\mathcal{P}]\right)$, leads to:

$$\phi e^{\delta\mathcal{P}\widehat{K}(\widehat{q}(p^*))} \left[\frac{F'(\widehat{K}(\widehat{q}(p^*)))}{F'(\widehat{K}(\widehat{q}(p^*))) - \delta\mathcal{P}} \right] = 1 \quad (14)$$

Lemma 3 *The value of \widehat{q} that solves Eq.(14) does not depend on p^* ; i.e., $\widehat{q}(p^*) = \widehat{q}$.*

We can now use *Lemma 2* to exactly pin down the value of \widehat{q} . More precisely, notice that, by the definition of \widehat{p} in *Lemma 2*, \widehat{q} must be given by the value taken by q^i in Eq.(6) when $p = \widehat{p}$; that is,

$$\widehat{q} \equiv \frac{1}{\alpha(1-\phi)} \zeta^\alpha \left(\frac{1-\widehat{p}}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-\widehat{p}}{\widehat{p}} \right) \quad (15)$$

Where, it must be clear that, since $\widehat{p} < \bar{p}$, then $\widehat{q} > 0$. Furthermore, Eq.(15), incidentally, leads to:

$$\widehat{K} = \zeta \left(\frac{1-\widehat{p}}{1-\phi} \right)^{\frac{1}{1-\alpha}} \quad (16)$$

Where it be observed that, since $\phi < \widehat{p} < \bar{p}$; then, the value of \widehat{K} must be strictly smaller than ζ (which represents the full-information solution according to Eq.(1)) and, at the same time, \widehat{K} will be strictly bigger than the value of K_i^i delivered by the *RHS* of Eq.(10).

The pooling equilibrium in the insurance market, given the equilibrium price p^* , can now be fully characterised.

Proposition 2 *Assume $\mathcal{A} \subset [0, 1]$ and $\mathcal{A} \neq \emptyset$; and take some sector $i \in \mathcal{A}$. Then:*

- i) A pooling equilibrium insurance contract with $q^* > 0$ exists if, and only if, $p^* < \bar{p}$.*
- ii) Whenever $p^* \in [\bar{p}, 1)$, in equilibrium: $q^* = 0$. In addition to that,*

$$K_i^i = \bar{K} \equiv \zeta \left(\frac{1-\bar{p}}{1-\phi} \right)^{\frac{1}{1-\alpha}} ;$$

and the IC constraint stipulated in Eq.(12) does not bind in equilibrium.

- iii) Whenever $p^* \in (\widehat{p}, \bar{p})$, the equilibrium insurance contract encompasses:*

$$q^* = \frac{1}{\alpha(1-\phi)} \zeta^\alpha \left(\frac{1-p}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-p}{p} \right),$$

and the IC constraint stipulated in Eq.(12) does not bind. In addition to that, in such an equilibrium, the amount invested in Project- i by Type- i is given by:

$$K_i^i = \zeta \left(\frac{1-p^*}{1-\phi} \right)^{\frac{1}{1-\alpha}} .$$

iv) Whenever $p^* \in (\phi, \widehat{p}]$, the equilibrium insurance contract encompasses:

$$q^* = \widehat{q} \equiv \frac{1}{\alpha(1-\phi)} \zeta^\alpha \left(\frac{1-\widehat{p}}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-\widehat{p}}{\widehat{p}} \right),$$

and the IC constraint stipulated in Eq.(12) is binding. In addition to that, in such an equilibrium:

$$K_i^i = \widehat{K} \equiv \zeta \left(\frac{1-\widehat{p}}{1-\phi} \right)^{\frac{1}{1-\alpha}}.$$

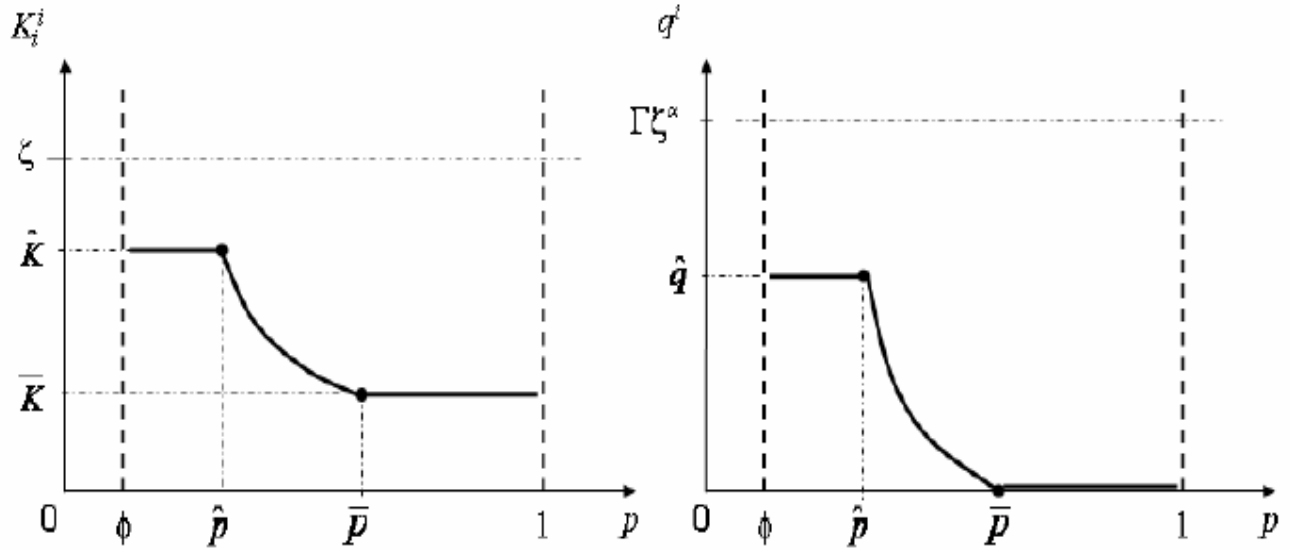


FIGURE 4: Equilibrium Levels of K_i^i and q_i^i .

Proposition 2 is illustrated in FIGURE 4.¹⁶ Whenever $\mathcal{A} \subset [0, 1]$ and $\mathcal{A} \neq \emptyset$, in equilibrium, p^* must necessarily lie within the interval $(\phi, 1)$. In FIGURE 4: (i) when $p^* \in (\overline{p}, 1)$ the non-negativity constraint on q^i binds, while the IC in Eq.(12) is not binding; as a consequence, $q^i = q^* = 0$ and $K_i^i = \overline{K}$; (ii) $\forall p^* \in [\widehat{p}, \overline{p}]$ neither the IC, nor the $q^i \geq 0$ binds; accordingly, in that range K_i^i is given by Eq.(7) and q^i is described by Eq.(6), with $p = p^*$; (iii) $\forall p^* \in (\phi, \widehat{p})$ the IC is binding (while $q^i \geq 0$ does not bind); thus, over that interval $K_i^i = \widehat{K}$ and $q^i = \widehat{q}$. Finally, notice that, for all $p^* \in (\phi, 1)$, investment in *Project-i* remains below ζ and the demand of the Arrow-Debreu commodity stays below $\Gamma \zeta^\alpha$; which represent their *full-information* solutions, respectively - see Eq.(1) and Eq.(2), after plugging in $\mathcal{P}_i = \mathcal{P}$.

¹⁶The functions $K_i^i(p)$ and $q_i(p)$ in Figure 2 are plotted for a given value of \mathcal{P} (which, in turn, implies a given value of ζ).

3.2.3 The Equilibrium Price in Pooling Contracts

Proposition 2 characterises the entrepreneurs' optimal portfolio allocation, given the pooling-equilibrium insurance price p^* . Hence, in order to find the exact portfolio allocation that holds in period t , it only remains to pin down the value taken by p^* in that period.

Insurance companies make zero profits in equilibrium. This implies the value of p^* should just reflect the *average failure probability* of the *pool* of individuals who buy the insurance contract. Let's denote the *average failure probability* in period t by f_t . Then, $p_t^* = f_t$. How is the value of f_t determined?

Note firstly that, since insurance contracts offered in equilibrium comply with the IC written down in Eq.(12); then, any individual of type- $i \in [0, 1]$, such that $i \in \mathcal{A}_t$, will always specialise and invest in *Project- i* . This implies that a fraction n_t of the population in the economy displays *failure probability* equal to ϕ .

Secondly, take any individual of type- $h \in [0, 1]$, such that *sector* $h \notin \mathcal{A}_t$. This set of individuals represents a fraction $1 - n_t$ of the whole population. Additionally, since $\phi_{i,h} = 1$ for all $i \neq h$, they all exhibit *failure probability* equal to 1, no matter which sector $i \in \mathcal{A}_t$ they choose to specialise in.

Proposition 3 $p_t^* = p_t^*(n_t) = 1 - (1 - \phi)n_t$.

Proof. The proof is straightforward from the fact that $f_t = \phi \times n_t + 1 \times (1 - n_t)$, and the fact that $p_t^* = f_t$. ■

Proposition 3 reflects one of the key insights of this paper. Increasing the number of *active sectors* induces more efficient operation of risk-sharing institutions; this is because a higher value of n_t permits better allocation of talents, ameliorating the adverse selection problem affecting this economy. In order to close the model, we must then proceed to finally make explicit how the value of n_t is determined in equilibrium; this requires incorporating innovators' optimisation problem into the model.

4 Innovators, Innovations and Sectors Expansion

We model the appearance of new sectors as the result of innovations. These innovations are the consequence of deliberate R&D policies undertaken by private agents to which we refer to as *innovators*. We focus only on horizontal innovations, since these are the kind of innovations that will lead to improvements in the allocation of individuals' talent; the key mechanism at work in this paper.

Recall there is a continuum of innovators indexed by i along the interval $[0, 1]$. Except for their particular index $i \in [0, 1]$, all innovators are *ex-ante* identical; displaying risk-neutrality, living for one period only, and starting off their lives with zero initial-wealth. These conditions are identically reproduced in every single period t . We assume an innovator $i \in [0, 1]$ can only possibly innovate for *sector* $i \in [0, 1]$. Since we disregard vertical innovations, (by construction) the subset of innovators who innovate for *sectors* which

were already *active* at the end of period $t - 1$ will not play any relevant role in period t (in other words, if *sector* $i \in \mathcal{A}_{t-1}$, then innovator i remains "inactive" in period t).

An innovation in period t for sector $h \notin \mathcal{A}_{t-1}$ materialises as the chance to invest in *Project-h* (in other words, it creates the technology needed to undertake *Project-h*). This, in turn, implies that sector $h \in \mathcal{A}_t$. We suppose innovations arise as *embodied technical change*. This means the innovation that brings *Project-h* into life is "contained" within each unit of physical capital K_h (and, accordingly, putting this innovation into action requires buying K_h). It is assumed that each unit of capital good K_h is produced by its corresponding innovator $h \in [0, 1]$ at a constant unit-cost equal to 1. As a result, the innovation activity entails two distinct types of costs: first, innovators must invest in R&D in order to be able to *design* an innovation; second, they must also incur in a unit-cost equal to 1 in order to physically *produce* each unit of capital good K_h "containing" the innovation. At last, an innovator $h \in [0, 1]$, such that $h \in \mathcal{A}_t$, must choose the optimal price \mathcal{P}_h at which to sell each unit of K_h .¹⁷

Innovators seek to maximise expected profits. Their optimisation problem consists in choosing two variables: 1) how much to *ex-ante* invest in R&D (henceforth denoted by ι); 2) the price \mathcal{P}_h at which to sell their (embodied) innovations (of course, as long as $h \in \mathcal{A}_t$).¹⁸ A higher value of ι will be naturally associated with a higher probability of producing an innovation. Given an amount ι invested in R&D, the probability of designing an innovation is given by the non-decreasing function $\beta(\iota)$; for simplicity and without any loss of generality, suppose:

$$\beta(\iota) = 0 \text{ if } \iota \in [0, 1), \quad \text{and} \quad \beta(\iota) = \gamma \in (0, 1) \text{ if } \iota \geq 1$$

4.1 Innovators Problem under Full Information

By assumption, vertical innovations are precluded. Therefore, if sector $i \in \mathcal{A}_{t-1}$, then innovator $i \in [0, 1]$ will trivially choose $\iota_{i,t}^* = 0$. Because of that, only an innovator $h \in [0, 1]$, such that sector $h \notin \mathcal{A}_{t-1}$, will turn out "active" during period t in our model.

Assume sector $h \notin \mathcal{A}_{t-1}$, and take some entrepreneur of type- h born in period t . Under full information, this Type- h would invest $K_{h,t}^h = \zeta_{h,t} = (\delta \mathcal{P}_{h,t})^{-1/(1-\alpha)}$, were *Project-h* to become available in period t (i.e. in case sector $h \in \mathcal{A}_t$). On the other hand, a Type- $j \neq h$ would always choose $K_{h,t}^j = 0$. As a result, the individual of type- $h \in [0, 1]$ represents the whole prospective market for innovator $h \in [0, 1]$, and the optimisation problem this innovator solves reads as follows:

$$\begin{aligned} \underset{\iota_{h,t} \geq 0, \mathcal{P}_{h,t} \geq 0}{MAX} & : & E [\Pi_h(\iota_{h,t}, \mathcal{P}_{h,t})] &= \beta(\iota_{h,t}) (\mathcal{P}_{h,t} - 1) K(\mathcal{P}_{h,t}) - \iota_{h,t} & \text{(III)} \\ \text{s.t.} & : & K(\mathcal{P}_{h,t}) &= (\delta \mathcal{P}_{h,t})^{-1/(1-\alpha)} \end{aligned}$$

¹⁷Notice that, if sector $i \in \mathcal{A}_{t-1}$, although (by assumption) innovator i will not need to invest in R&D for this sector in period t , he will still need to produce the capital-goods K_i at unit-cost equal to 1, and also select the price \mathcal{P}_i at which to sell each unit of K_i .

¹⁸It is implicitly assumed that innovators cannot charge contingent-prices on their innovations. If that were possible, risk-neutral innovators could offer some type of insurance by charging a lower price when the entrepreneurial project fails.

Leaving aside "knife-edge" situations, due to its linearity, *Problem (III)* will yield corner solutions; i.e. either $\iota_{h,t}^* = 0$ or $\iota_{h,t}^* = 1$.

Lemma 4 *Take any innovator $h \notin \mathcal{A}_{t-1}$ and assume $\iota_{h,t}^* = 1$. If this innovator h succeeds in designing an innovation for sector h in period t (i.e. if $h \in \mathcal{A}_t$), then he will optimally charge a price $\mathcal{P}_{h,t}^* = \alpha^{-1}$.*

Notice from *Lemma 4* that all successful innovators charge an identical price on each unit of K_h (which is also constant across t); namely, $\mathcal{P}_{h,t}^* = \mathcal{P}^* = \alpha^{-1}$. Moreover, *Lemma 4* implies that, if $\iota_{h,t}^* = 1$, then expected profits of any innovator h , such that $h \notin \mathcal{A}_{t-1}$, are given by:

$$\Pi_{FI}^* \equiv \gamma(1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}}\delta^{-\frac{1}{1-\alpha}} - 1$$

We are interested in those cases where economies would display positive innovation under full information. Accordingly, suppose parameters are such that $\Pi_{FI}^* > 0$, so that *Problem (III)* indeed delivers $\iota_{h,t}^* = 1$ for all $h \notin \mathcal{A}_{t-1}$. As final remark, notice that, so far, we haven't said anything at all about the value of n_t (i.e. the fraction *active sectors*). In other words, under full-information, the degree of *market incompleteness* becomes utterly irrelevant, since (by definition) no adverse selection problems may possibly arise.

4.2 Innovators Behaviour under Asymmetric Information

When there is asymmetric information about skills and adverse selection problems accordingly arise, the fraction of *active sectors* (n_t) becomes relevant. Intuitively, the degree of market incompleteness, measured by $(1 - n_t)$, will affect the severity of the adverse selection problem in the insurance market. This, in turn, influences the optimal entrepreneurial investment level, affecting innovators' expected profits.

Denote by $\bar{\iota}_t$ the value of ι_t chosen by all innovators $k \neq h \in [0, 1]$, such that *sector $k \notin \mathcal{A}_{t-1}$* .¹⁹ The maximisation problem for innovator h in period t , reads now:

$$\underset{\iota_{h,t} \geq 0, \mathcal{P}_{h,t} \geq 0}{MAX} : E[\Pi_h(\iota_{h,t}, \mathcal{P}_{h,t}, \mathcal{P}_t^*)] = \beta(\iota_{h,t}) (\mathcal{P}_{h,t} - 1) K(\mathcal{P}_t^*, \mathcal{P}_{h,t}) - \iota_{h,t} \quad (IV)$$

¹⁹More precisely, $\bar{\iota}_t$ represents the *rational expectation* formed by innovator h about the value of ι_t chosen by all the other "active" innovators in period t .

subject to:

$$p_t^* = 1 - (1 - \phi)n_t \quad (17)$$

$$n_t = n_{t-1} + \beta(\bar{\iota}_t)(1 - n_{t-1}) \quad (18)$$

$$K(p_t^*, \mathcal{P}_{h,t}) = \begin{cases} \left(\frac{1}{\delta\mathcal{P}_{h,t}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\widehat{p}}{1-\phi}\right)^{\frac{1}{1-\alpha}} & \text{if } p_t^* \in (\phi, \widehat{p}) \\ \left(\frac{1}{\delta\mathcal{P}_{h,t}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-p_t^*}{1-\phi}\right)^{\frac{1}{1-\alpha}} & \text{if } p_t^* \in [\widehat{p}, \bar{p}] \\ \left(\frac{1}{\delta\mathcal{P}_{h,t}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi}\right)^{\frac{1}{1-\alpha}} & \text{if } p_t^* \in (\bar{p}, 1) \end{cases} \quad (19)$$

$$\frac{1}{\alpha(1-\phi)} \left(\frac{1}{\delta\mathcal{P}_{h,t}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi}\right)^{\frac{\alpha}{1-\alpha}} + \ln\left(\frac{\phi}{1-\phi} \frac{1-\bar{p}}{\bar{p}}\right) = 0 \quad (20)$$

$$\left(\frac{1}{\delta\mathcal{P}_{h,t}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\widehat{p}}{1-\phi}\right)^{\frac{1}{1-\alpha}} + \ln\left(\frac{\phi}{\widehat{p}}\right) = 0 \quad (21)$$

Eq.(17) reflects *Proposition 3*. Eq.(18) stems from the assumption that, once a sector becomes *active* it does never revert to *inactive* in the future; added to the fact that a fraction $\beta(\bar{\iota})$ of all sectors which were *inactive* during period $t-1$ become *active* in period t as a result of innovations perpetrated by innovators $k \neq h$ (such that $k \notin \mathcal{A}_{t-1}$). Eq.(19) is taken from *Proposition 2*. Eq.(20) implicitly pins down the value taken by \bar{p} ; notice that $\bar{p} = \bar{p}(\mathcal{P}_{h,t})$, and $\bar{p}'(\mathcal{P}_{h,t}) < 0$. Finally, Eq.(21) implicitly yields the value taken by \widehat{p} ; again, note that $\widehat{p} = \widehat{p}(\mathcal{P}_{h,t})$, and $\widehat{p}'(\mathcal{P}_{h,t}) < 0$.

Remark 1 *Since all innovators $h \in [0, 1]$, such that sector $h \notin \mathcal{A}_{t-1}$, (i.e. all "active" innovators in t) are ex-ante identical, Problem (IV) will deliver the same optimal solution $(\iota_{h,t}^*, \mathcal{P}_{h,t}^*) = (\iota_t^*, \mathcal{P}_t^*) \forall h$. Additionally, note that all innovators $i \in [0, 1]$, such that sector $i \in \mathcal{A}_{t-1}$, (i.e. all "inactive" innovators in t), solve:*

$$\text{MAX}_{\mathcal{P}_{i,t}} : \widetilde{\Pi}_i(\mathcal{P}_{i,t}, p_i^*) = (\mathcal{P}_{i,t} - 1)K(p_i^*, \mathcal{P}_{i,t}),$$

subject to: Eq.(17)-(21). It must be straightforward to observe that this last optimisation problem and Problem (IV) must necessarily deliver an identical value $\mathcal{P}_{h,t}^ = \mathcal{P}_{i,t}^* = \mathcal{P}_t^*$, $\forall i, h$. This finally entails that the equilibrium in the insurance market must necessarily be symmetric, as originally supposed in Section 3.2.*

From *Problem (IV)*, we can derive the following condition for the optimal choice of ι_t :

$$\iota_t^* = 1 \iff E[\Pi(\iota_t = 1, \mathcal{P}_t^*, \bar{\iota}_t, n_{t-1})] \geq 0, \quad \text{otherwise: } \iota_t^* = 0.$$

Problem (IV) cannot be, however, analytically solved; mainly for two different reasons. First, the function $K(p_t^*, \mathcal{P}_{h,t})$ displays "kinks" at \bar{p} and \widehat{p} , and it is accordingly non-differentiable at those points (Figure 2). Second, and more importantly, it is not possible

to provide a closed-form solution for the relations $\bar{p}(\mathcal{P}_{h,t})$ and $\hat{p}(\mathcal{P}_{h,t})$; though we can prove these relations are indeed *continuous* and *strictly decreasing* functions. Despite the lack of analytical solution, some important general results for *Problem (IV)* can be proved without much trouble. Denote by Π_{FI}^* the *maximal* of the function $E[\Pi(\cdot)]$ under *full information*, and by Π_{AI}^* the *maximal* of the function $E[\Pi(\cdot)]$ under *asymmetric information*; *Proposition 4* provides the main results derived from Section 4.

Proposition 4 (i) $\Pi_{FI}^* > \Pi_{AI}^*$, (ii) Π_{AI}^* is non-decreasing in n_{t-1} , (iii) Π_{AI}^* is non-decreasing in $\bar{\iota}_t$.

Firstly, the intuition for $\Pi_{FI}^* > \Pi_{AI}^*$ lies on the fact that entrepreneurial investment under full information is always higher than under asymmetric information, as can be observed from comparing Eq.(1) to results in *Proposition 2*. Secondly, Π_{AI}^* is non-decreasing in n_{t-1} , because n_{t-1} and n_t exhibit positive serial correlation, and a higher n_t is associated with less severe adverse selection problems in period t . Finally, Π_{AI}^* is non-decreasing in $\bar{\iota}_t$, as a larger value of $\bar{\iota}_t$ helps to reduce adverse selection problems during period t by expanding the number of *active sectors* in t due to innovations by other innovators; this fact implies there exist positive externalities across innovators.

4.3 A Numerical Solution for *Problem (IV)*

In this sub-section we present a numerical solution for the innovators' optimisation problem under asymmetric information. The intention of this sub-section is just to further illustrate the ideas developed so far in this paper and, additionally, to provide a clear numerical (and graphical) exposition of the results stated in *Proposition 4*. We set the parameters' values as follows: $\phi = 0.1$, $\alpha = 0.3$, $\delta = 0.3$, and $\gamma = 0.6$. The maximisation problem was solved with MATLAB. Notice those parameters imply $\Pi_{FI}^* = 0.4$.²⁰

FIGURE 5 depicts the value of $\Pi_{AI}^*(\iota_t = 1)$, defined as the *maximum level of expected profits under asymmetric information if we fixed $\iota_t = 1$* , for different values of n_{t-1} and $\bar{\iota}_t$. The thicker line shows $\Pi_{AI}^*(\iota_t = 1, n_{t-1})$ when $\bar{\iota}_t = 1$, while the thinner line portrays $\Pi_{AI}^*(\iota_t = 1, n_{t-1})$ when $\bar{\iota}_t = 0$. Both functions are (non-strictly) increasing in n_{t-1} , in concordance with point (ii) of *Proposition 4*. In addition to that, $\Pi_{AI}^*(\iota_t = 1, n_{t-1}, \bar{\iota}_t = 1) \geq \Pi_{AI}^*(\iota_t = 1, n_{t-1}, \bar{\iota}_t = 0)$, for all $n_{t-1} \in [0, 1]$, reflecting point (iii) of *Proposition 4*. Notice, as well, that $\Pi_{AI}^*(\iota_t = 1, n_{t-1})$ always remains below $\Pi_{FI}^* = 0.4$, both for $\bar{\iota}_t = 1$ and $\bar{\iota}_t = 0$; in agreement with point (i) of *Proposition 4*. Finally, observe that for values of n_{t-1} sufficiently low, both functions eventually display negative expected profits. This implies that for those (sufficiently low) values of n_{t-1} , $\iota_t = 1$ cannot be an optimal choice, and innovators will instead set $\iota_t^* = 0$ (so that to obtain $\Pi_{AI}^* = 0$).

²⁰The MATLAB codes are available from the author upon request. In Appendix B the solution to this problem is presented in further detail. Additionally, examples with different parameters settings are available upon request, in case the reader wishes to check the robustness of the model.

$$\Pi_{AI}^*(\iota_t=1, n_{t-1})$$

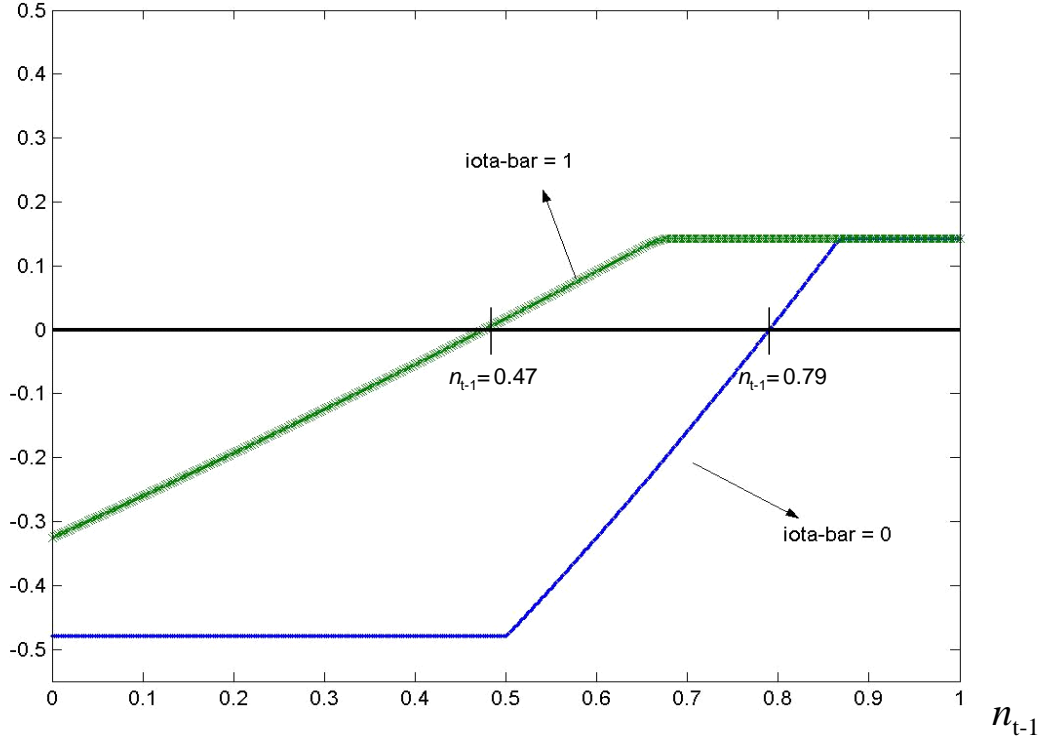


FIGURE 5: Innovators' expected profits when $\iota_t = 1$, as a function of n_{t-1} and $\bar{\iota}_t$.

No Innovation Due to Coordination Failures: Notice that, for a given n_{t-1} , expected profits are (weakly) increasing in $\bar{\iota}_t$. As a result of this, in general, ι_t^* will be a non-decreasing function of $\bar{\iota}_t$; which reflects the fact that innovators' behaviour displays *strategic complementarity*, a property that may bring forth *coordination failures* (Cooper and John (1988)). In this model, coordination failures arise when everybody optimally responds with $\iota_t^* = 0$, if everybody expects $\bar{\iota}_t = 0$; but, the optimal choice would had been $\iota_t^* = 1$, had everyone instead coordinated their expectations on $\bar{\iota}_t = 1$. Looking at FIGURE 5, we can observe that coordination failures may arise for values of $n_{t-1} \in [0.47, 0.79]$. Over that range, if $\bar{\iota}_t = 0$ holds, then $\iota_t^* = 0$; on the other hand, if $\bar{\iota}_t = 1$, then $\iota_t^* = 1$ will be verified.

No Innovation as Unique Optimal Solution (no coordination failures): Under some circumstances, no investment in R&D might too be the optimal innovators' action even if expectations were fully optimistic. That is to say, $\iota_t^* = 0$ may sometimes be optimal even under $\bar{\iota}_t = 1$. Going back to FIGURE 5, whenever $n_{t-1} \leq 0.47$, then $\Pi_{AI}^*(\iota_t = 1, n_{t-1}, \bar{\iota}_t = 1) < 0$. As a consequence of this, for all $n_{t-1} < 0.47$, the unique optimal solution to the innovators' maximisation problem is given by $\iota_t^* = 0$ (no matter what the value of $\bar{\iota}_t$ is).

Positive Innovation as Unique Optimal Solution: This last case arises for all values of $n_{t-1} > 0.79$. In those cases, all innovators would optimally select $\iota_t^* = 1$, regardless

whether $\bar{l}_t = 0$ or $\bar{l}_t = 1$. This is the case because, for all $n_{t-1} > 0.79$, both $\Pi_{AI}^*(l_t = 1, n_{t-1}, \bar{l}_t = 0)$ and $\Pi_{AI}^*(l_t = 1, n_{t-1}, \bar{l}_t = 1)$, are strictly positive.

5 Aggregate Dynamic Analysis

The analysis of this economy has remained until now within a static framework, in the sense that initial conditions have been taken as exogenously set, and we have focused on agents' optimal behaviour given these initial conditions. In particular, the set \mathcal{A}_{t-1} (and, hence, the variable n_{t-1}) has been so far taken as exogenously given. The use of CARA preferences implies wealth distribution loses all relevance concerning risk-taking choices; leaving the fraction of active sectors (i.e. n_t) as the sole variable whose behaviour we need to study, in order to keep track of this economy's dynamics. That is to say, n_t remains as the only relevant state-variable in our model. We provide now the definitions of both *static equilibrium* and *dynamic equilibrium*. Subsequently, we describe the distinct characteristics of three different cases in terms of its dynamic paths, which correspond to dissimilar initial conditions. These cases are referred to as: *prosperity and development*, *secular stagnation*, and *multiple equilibria (history vs. expectations)*.²¹

Definition 3 (Static Equilibrium) *Given: (i) the size of the set \mathcal{A}_{t-1} measured by the variable n_{t-1} , and (ii) the set of bequests $\{b_i^i\}_{i \in [0,1]}$, describing the amount of bequest received by each Type- i at the beginning of period t (b_t^i). An equilibrium in period t is a situation in which:*

- a) *The insurance market is in equilibrium, according to Definition 1.*
- b) *The entrepreneurs portfolio allocations are optimal for all $i \in [0, 1]$, according to Definition 2.*
- c) *The pair $(\iota_{h,t}^*, \mathcal{P}_{h,t}^*)$ maximises the expected profits function of innovator $h \in [0, 1]$; given the set $\{(\iota_{k,t}^*, \mathcal{P}_{k,t}^*)\}_{\forall k \neq h \in [0,1]}$, and the fact that innovators' expectations are formed rationally.*

Section 4 has shown that the pair $(\iota_{h,t}^*, \mathcal{P}_{h,t}^*) = (\iota_t^*, \mathcal{P}_t^*)$ for all $h \in \mathcal{A}_{t-1}$. We can, thus, provide a our definition of a *dynamic equilibrium* in a very concise way.

Definition 4 (Dynamic Equilibrium) *A dynamic equilibrium is a sequence of static equilibria, linked together across time by the "law of motion" of n_t specified in Eq.(22)*

$$n_t = n_{t-1} + \beta(\iota_t^*)(1 - n_{t-1}). \quad (22)$$

5.1 Secular Stagnation

Take an economy for which the value of n_0 is such that $n_0 < 0.47$. For this economy, the equilibrium in period $t = 1$ is unique, and it is characterised by $\iota_1^* = 0$. In addition to zero

²¹For expositional clarity we will describe those three cases by referring to the numerical example presented in Section 4.3.

investment in R&D and absence of innovation; this economy will exhibit very inefficient insurance services provision (underdeveloped risk-sharing institutions), and low levels of entrepreneurial investment. Inefficient insurance provision is the consequence of severe adverse selection problems, which stem from the high degree of sectors incompleteness. On the other hand, repressed entrepreneurship is the result of both lack of opportunities (due to sectors incompleteness) and discouraged risk-taking behaviour due to inefficient insurance provision.

From Eq.(22), since $\iota_1^* = 0$, then $n_1 = n_0$; and, accordingly, $\iota_2^* = 0$ will hold as unique equilibrium in $t = 2$. Furthermore, in the absence of any substantial exogenous technological shock, this sort of equilibrium will perpetuate itself for all $t = \{3, 4, \dots, \infty\}$; constituting a "poverty-trap" in this model, and characterised by complete stagnation.

5.2 Prosperity and Development

Let's look now at an economy for which n_0 is large enough; in particular, $n_0 > 0.79$. The equilibrium in period $t = 1$ is unique, and displays $\iota_1^* = 1$. Intuitively, the initial degree of market incompleteness is quite low, which implies adverse selection problems do not substantially distort the operation of this economy. Given Eq.(22), since $\iota_1^* = 1$, thus $n_1 > n_0$. As a consequence of this, the equilibrium in period $t = 2$ will also exhibit $\iota_2^* = 1$. Moreover, this prosperous sequence will be perpetuated *ad infinitum*. *Proposition 5* formally states the prosperous dynamics of an economy that starts off with a sufficiently high level of n_0 .

Proposition 5 *Denote by \tilde{n} the value of n_t , such that it (implicitly) solves the following equation: $\gamma(\mathcal{P}^*(\tilde{n}) - 1)K^*(1 - (1 - \phi)\tilde{n}, \mathcal{P}^*(\tilde{n})) = 1$; where $K^*(\cdot)$ is given by Eq.(19). Then, if $n_0 > \tilde{n}$, this economy will monotonically (and asymptotically) converge to an equilibrium in which $n_\infty = 1$.*

Proposition 5 argues that, when the initial degree of market incompleteness is sufficiently low (i.e. n_0 is sufficiently large), this economy will eventually reach an equilibrium characterised by *complete markets* ($n_\infty = 1$). During the transition period, the economy experiences development and growth, which manifests as a continuous process of sectors expansion (capital differentiation) and a more efficient allocation of skills. Simultaneously, the operation of risk-pooling institutions concomitantly improves, as a consequence of less severe adverse selection problems due to lower degree of market incompleteness.

5.3 Multiple Equilibria (History vs. Expectations)

When the value of $n_0 \in [0.47, 0.79]$, this economy will be subject to multiple equilibria. Equilibrium multiplicity will be driven by innovators' expectations. In particular, if innovators' expectations coordinate in $\bar{\iota}_1 = 0$, then $\iota_1^* = 0$; whereas, if they instead coordinate in $\bar{\iota}_1 = 1$, then $\iota_1^* = 1$. More importantly, from a dynamic perspective, whether expectations in $t = 1$ lead to $\bar{\iota}_1 = 0$ or $\bar{\iota}_1 = 1$, may carry dramatic future consequences. By Eq.(22), $\iota_1^* = 0$ entails that $n_1 = n_0$, so initial conditions (in terms of n_{t-1}) for $t = 2$ would

identically replicate those faced in $t = 1$. On the other hand, $\iota_1^* = 1$ means $n_1 > n_0$; as a result, this might shoot up n_1 above 0.79, and ignite a process of continuous prosperity and development, as the one described in Section 5.2. For an economy with $n_{t-1} \in [0.47, 0.79]$, the bigger n_{t-1} is, the higher the chances that $n_t > 0.79$ will hold if $\iota_{t-1}^* = 1$, and that the economy is accordingly put on safe track for long-run prosperity from $t + 1$ onwards.

When the economy is located within $[0.47, 0.79]$, then both *history* and *expectations* matter, in the sense of Krugman (1991). Since, in each single period, a new generation of innovators is born; there is no reason why to expect that $\bar{\iota}_t$ should have any effect on any $\bar{\iota}_{t+\Delta}$, where $\Delta \in \{1, 2, \dots, \infty\}$ (that is to say, there is no reason why expectations should exhibit any sort of serial correlation, concerning different static equilibria). Therefore, even if expectations in period t were, for any reason, coordinated in $\bar{\iota}_t = 1$ (leading to $\iota_t^* = 1$); we may well still commence period $t + 1$ again within the interval $[0.47, 0.79]$, at the risk of experiencing coordination failures. This could lead to cyclical dynamics, driven by expectations, with periods of growth and technical change, followed by periods of stagnation.

6 A Quick Look at Some Data

A key prediction of our theory is that the degree of sectoral diversification positively affects the operation of financial institutions. In particular, this paper argues that higher degree of sectoral diversification permits an improved operation of risk-sharing institutions, since it helps to ameliorate adverse selection problems. We present in this section some empirical observations which are consistent with this prediction of the model.

Table 1 provides some evidence consistent with this fact. To measure sectoral concentration, we utilise the same *Gini* and *Herfindahl* indices for employment shares based on the UNIDO data used by Imbs and Wacziarg (2003).²² The dependent variable in all regressions is the *logarithm of the stock market capitalisation to GDP*, whose data is taken from Beck *et al* (1999).²³ Data on *GDP per capita* is taken from Heston, Summers and Aten (2002).

The first two columns in Table 1 show that both measures of sectoral concentration are negatively (and significantly) correlated with the level of stock market development. Given that these estimates may be simply capturing the fact that GDP per-capita is correlated with both the independent and dependent variables, we include GDP per-capita in regressions (3) and (4); the regression coefficients associated to the Gini and Herfindahl are reduced in magnitude, but they still remain significant at 1% level. Furthermore, all these results are also robust to the inclusion of country fixed-effects, as shown in the last two columns of Table 1

²²The author is indebted to Jean Imbs for kindly providing him with this data.

²³Similar regressions were also run for other financial development indicators taken from Beck *et al* (1999); namely: stock market total value traded to GDP, liquid liabilities to GDP, and non-life insurance premiums as a share of GDP. All regressions led to similar results as those shown in Table 1, and are available from the author upon request.

TABLE 1: Contemporaneous Regression

Independent Variable	OLS (1)	OLS (2)	OLS (3)	OLS (4)	fixed-effects (1)	fixed-effects (2)
Gini	-5.59 (0.835)		-2.87 (0.831)		-8.76 (1.592)	
Herfindahl		-11.03 (1.028)		-7.88 (1.057)		-8.54 (1.503)
GDP per-capita			0.113 (0.013)	0.093 (0.012)	0.396 (0.025)	0.355 (0.023)
R squared	0.087	0.197	0.220	0.285	0.220	0.251

Dependent variable: logarithm of stock market capitalisation to GDP (1963-1992)

Standard errors in parentheses. All regressions include intercept. Observations: 471

All estimates are significant at 1% level.

Reverse causation might be an issue leading to overestimation of the regressors associated to the concentration indices in Table 1. In particular, increasing stock markets development might encourage agents to specialise in different sectors (for which they maybe possess comparative advantages), since sector-specific and idiosyncratic shocks can be then more efficiently insured. Table 2 intends to address this matter by regressing the *average log stock market capitalisation to GDP* over the period 1981-1992, on the level of sectoral concentration in 1980 (measured again by the Gini and Herfindahl indices). Both regressions indicate that higher degree of (past) sectoral diversification significantly predicts more developed stock markets.

TABLE 2: Stock Market Development and Initial Sectoral Concentration

Independent Variable	OLS (5)	OLS (6)
Gini in 1980	-4.93* (2.881)	
Herfindahl in 1980		-8.04** (3.279)
R squared	0.070	0.134
Observations	41	41

Dependent variable: avg. log of stock market cap. to GDP (1981-1992)

Standard errors in parentheses. All regressions include intercept.

* significant at 10% level, ** significant at 5% level.

7 Concluding Discussion

Credit motives vs. insurance motives: Throughout this paper we have completely abstracted from credit-constraints issues, by assuming that all agents have perfect access to credit markets at the given international interest rate. In principle, our adverse selection argument stemming from unobservability of individuals' skills could be also applied to credit markets, in a similar way as done by Ghatak, Morelli and Sjöström (2002). The credit-channel would work analogously as the insurance-channel presented here; the availability of many sectors permits better allocation of individual talent, ameliorating adverse selection problems which, in turn, enables better operation of credit markets. Yet, we have decided to close this additional channel, making use only of the inefficiency in the insurance market. The reason for this choice is motivated by our intention to isolate the effect of the number of active sectors; this requires wiping out any sort of "wealth-effect" which may possibly arise in our model. When working with imperfect credit markets, limited-liability issues immediately generate positive wealth-effects (in the sense that richer agents enjoy better access to credit markets, because they are further away from their limited-liability constraint), which inevitably incapacitates the model to isolate the "sectors-effect".

Insurance companies vs. stock markets: This paper has synthesized any sort of risk-sharing institution by assuming that insurance services are provided by private agents called *insurance companies*. The insurance companies artifact is just posed for modelling simplicity. In reality, agents share idiosyncratic risks by different means, and one of the most important institutions to deal with pooling risks are stock markets. Our model could be reformulated in a way such that entrepreneurs will insure themselves by utilising stock markets. More precisely, we could suppose that individuals can issue and sell a continuum of *securities* on their own projects. These *securities* would then represent shares on the project's return, whose pay-off would be contingent on the *state of nature*. Since project risks are purely idiosyncratic, individuals can diversify away all risks by holding fully diversified portfolios. Furthermore, in equilibrium, competition on securities will drive its price down to its expected return. Expected returns are negatively affected by adverse selection, because more severe adverse selection problems means higher average failure probability; this implies the securities equilibrium price will be smaller, the higher the level of market incompleteness. This last result is exactly analogous to *Proposition 3* expressing that the pooling-equilibrium insurance price increases when adverse selection problems worsen.

Poverty alleviation programmes: This paper predicts that some economies might get stuck in a poverty-trap; this is the result of a "deep-rooted" organisational failure, affecting several different markets. Underdevelopment is characterised by few sectors in which individuals can specialise, underdeveloped financial institutions, and scant innovation activities. The market failure contaminating the operation of the economy in our paper (i.e. the adverse selection problem) derives from the incapacity of some individuals to find the activity or sector for which they display comparative skills. In most of theories of

poverty-traps, economies can be easily rescued from long-run poverty simply by receiving a sufficiently large wealth-transfer. Instead, our theory claims that foreign-aid should presumably also include important transfers of technology, as standard wealth-transfers alone might not be enough to solve the adverse selection problem this economy suffers from (at least in a reasonably short time frame).

Testing the theory: In Section 1.1, Tables 1 and 2 have intended to provide some evidence consistent with the theory presented in this paper. Nevertheless, these observations by no means prove our theory key predictions. In order to do so, we would ideally need to count with some important exogenous shock that suddenly shrinks (or increases) the degree of market incompleteness that individuals face. The European Union last enlargement in May 2004, could presumably be exploited as such a shock. The last EU enlargement has incorporated 10 new countries whose income and development level was significantly lower than for the average previous EU-15 countries. From the new 10 economies perspective, the incorporation to the EU will expectably mean a drastic expansion in terms of both product markets and labour markets.²⁴ According to our model, this sudden expansion in the set of economic activities or sectors available to individuals should permit a smoother allocation of talent and, consequently, improve the operation of financial institutions within the newly incorporated 10 countries. We let this issue pending for future research.

²⁴At this moment, the expansion in labour markets has been somewhat limited; as only the only EU-15 countries that have permitted unrestricted access to their labour markets have been: Ireland, United Kingdom and Sweden.

Appendix

Proof of Lemma 1. The Lagrangian function of *Problem (II)* is (after setting $\mathcal{P}_i = \mathcal{P}$):

$$\mathcal{L} = - \left\{ \phi e^{-[S^i + q^i + (1-\delta)\mathcal{P}K_i^i]} + (1-\phi)e^{-[S^i + F(K_i^i) + (1-\delta)\mathcal{P}K_i^i]} \right\} + \lambda(w - S^i - \mathcal{P}K_i^i - pq^i)$$

Bearing in mind the non-negativity constraints $K_i^i \geq 0$ and $q^i \geq 0$, we obtain the following first order conditions (FOC):

$$\phi e^{-q^i} + (1-\phi)e^{-F(K_i^i)} = \widehat{\lambda} \quad (\text{L.1.1})$$

$$\frac{\phi}{p}e^{-q^i} \leq \widehat{\lambda} \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial q^i} q^i = 0 \quad (\text{L.1.2})$$

$$\phi(1-\delta)e^{-q^i} + (1-\phi) \left[\frac{F'(K_i^i)}{\mathcal{P}} + (1-\delta) \right] e^{-F(K_i^i)} \leq \widehat{\lambda} \quad \text{and} \quad \frac{\partial \mathcal{L}}{\partial K_i^i} K_i^i = 0 \quad (\text{L.1.3})$$

$$S^i + \mathcal{P}K_i^i + pq^i = w^i \quad (\text{L.1.4})$$

Where $\widehat{\lambda} \equiv \lambda e^{[S^i + (1-\delta)\mathcal{P}K_i^i]}$. Finally, in addition to these equations, $p \in (\phi, 1]$.

First of all, notice that given the *inada conditions* on $F(K)$, $K_i^i > 0$ must always hold in the optimum; hence, Eq.(L.1.3) will always hold with strict equality. Secondly, in order to find the solution of *Problem (II)*, it is easier start solving the problem under the (temporary) assumption that $q^i > 0$ verifies in the optimum, so that Eq.(L.1.2) must also hold with strict equality. Given this, from Eq.(L.1.1) and (L.1.2) we obtain:

$$q^i = F(K_i^i) + \ln \left(\frac{\phi}{1-\phi} \frac{1-p}{p} \right) \quad (\text{L.1.5})$$

At the same time, from Eq.(L.1.1) and (L.1.3) we get:

$$q^i = F(K_i^i) + \ln \left(\frac{\phi}{1-\phi} \frac{\delta\mathcal{P}}{F'(K_i^i) - \delta\mathcal{P}} \right) \quad (\text{L.1.6})$$

Consequently, equating Eq.(L.1.5) and (L.1.6), in the optimum it must be verified: $F'(K_i^i) = \delta\mathcal{P}(1-p)^{-1}$. This last expression can be explicitly solved for K_i^i , leading to:

$$K_i^i = \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-p}{1-\phi} \right)^{\frac{1}{1-\alpha}} \quad (\text{L.1.7})$$

Finally, replacing Eq.(L.1.7) into Eq.(L.1.6), the expression for the optimal q^i is obtained.

$$q^i = \frac{1}{\alpha(1-\phi)} \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-p}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-p}{p} \right) \quad (\text{L.1.8})$$

Claim A.1.1 There exists a unique value $\bar{p} \in (\phi, 1)$, such that: *i*) when $p = \bar{p}$, Eq.(L.1.8) yields $q^i = 0$, *ii*) $\forall p < \bar{p}$, (L.1.8) delivers $q^i > 0$, *iii*) $\forall p > \bar{p}$, (L.1.8) gives $q^i < 0$.

Proof. Recall that $\zeta \equiv (\delta\mathcal{P})^{-1/1-\alpha}$ and define:

$$A(p) \equiv \frac{1}{\alpha} \zeta^\alpha \left(\frac{1-p}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad B(p) \equiv (1-\phi) \ln \left(\frac{1-\phi}{\phi} \frac{p}{1-p} \right)$$

Thus, $q^i \geq 0 \iff A \geq B$. Remember that $p \in (\phi, 1]$, and calculate the values of A and B when p approaches its *lower-bound* (ϕ) and its *upper-bound* (1).

$$\begin{aligned} A(p \rightarrow \phi) &= \frac{1}{\alpha} \zeta^\alpha > 0 & A(p \rightarrow 1) &= 0 \\ B(p \rightarrow \phi) &= 0 & B(p \rightarrow 1) &= \infty \end{aligned}$$

Now, bearing in mind that both A and B are *continuous* and *differentiable* all over the interval $(\phi, 1)$, let's calculate the derivatives of A and B with respect to p .

$$\begin{aligned} \frac{dA}{dp} &= -\frac{1}{1-\alpha} \frac{1}{1-\phi} \zeta^\alpha \left(\frac{1-p}{1-\phi} \right)^{\frac{2\alpha-1}{1-\alpha}} < 0 \\ \frac{dB}{dp} &= \frac{1-\phi}{p(1-p)} > 0 \end{aligned}$$

Therefore, as $A(p \rightarrow \phi) > B(p \rightarrow \phi)$ and $A(p \rightarrow 1) < B(p \rightarrow 1)$, being A strictly decreasing in p_i and B strictly increasing in p over the interval $(\phi, 1)$; A and B must necessarily cross each other once (and only once) at some value of $p \in (\phi, 1)$. Let's denote this *crossing-point* as \bar{p} ; so that, $A(p = \bar{p}) = B(p = \bar{p})$. Hence, for all $p \in (\phi, \bar{p})$, $A > B$ must hold; and for all $p \in (\bar{p}, 1)$, $B > A$ must be verified. \blacklozenge

Claim A.1.1 implies that, for all $p \leq \bar{p}$, Eq.(L.1.7) and Eq.(L.1.8) indeed yield the optimal values of K_i^i and q^i , respectively. However, according to *Claim A.1.1*, for all $p > \bar{p}$ the non-negativity constraint $q^i \geq 0$ will be binding; consequently, we still need to find the optimal values of K_i^i and q^i when $p > \bar{p}$.

Assume for the moment that $q^i = 0$ is optimal $\forall p \in (\bar{p}, 1)$. Then, from the FOC and the fact that Eq.(L.1.3) must always hold with strict equality, it must be the case that:

$$\phi + (1-\phi)e^{-F(K_i^i)} = \phi(1-\delta) + \frac{(1-\phi)}{\mathcal{P}} [F'(K_i^i) + (1-\delta)\mathcal{P}] e^{-F(K_i^i)} \geq \frac{\phi}{p} \quad (\text{L.1.9})$$

for all $p \in (\bar{p}, 1)$. From the first two members in Eq.(L.1.9), we get:

$$F(K_i^i) + \ln \left(\frac{\phi}{1-\phi} \frac{\delta\mathcal{P}}{F'(K_i^i) - \delta\mathcal{P}} \right) = 0$$

which (uniquely) yields:

$$K_i^i = \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi} \right)^{\frac{1}{1-\alpha}} \equiv \bar{K} \quad (\text{L.1.10})$$

To finally complete the proof, we still need to show that Eq.(L.1.9) holds when we plug Eq.(L.1.10) into it. Take the last two members of Eq.(L.1.9); apply logarithms on both sides, replace K_i^i by the *RHS* of Eq.(L.1.10), and re-arrange terms; to obtain:

$$\frac{1}{\alpha(1-\phi)} \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-\bar{p}}{p} \frac{1-(1-\delta)p}{1-(1-\delta)\bar{p}} \right) \leq 0 \quad (\text{L.1.11})$$

Notice that, by definition of \bar{p} :

$$\frac{1}{\alpha(1-\phi)} \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-\bar{p}}{\bar{p}} \right) = 0 \quad (\text{L.1.12})$$

and, since the *LHS* of Eq.(L.1.11) is strictly decreasing in p , it must necessarily be the case that:

$$\frac{1}{\alpha(1-\phi)} \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\bar{p}}{1-\phi} \right)^{\frac{\alpha}{1-\alpha}} + \ln \left(\frac{\phi}{1-\phi} \frac{1-\bar{p}}{p} \frac{1-(1-\delta)p}{1-(1-\delta)\bar{p}} \right) < 0 \quad \forall p \in (\bar{p}, 1)$$

Hence, Eq.(L.1.11) holds for all $p \in (\bar{p}, 1)$, which completes the proof. \blacksquare

Proof of Lemma 2. First, take any $p \in [\bar{p}, 1)$. Then, according to *Lemma 1*: $q^i = 0$, $K_i^i = \bar{K} > 0$, and $S^i = w^i - \mathcal{P}\bar{K}$. If this Type- i decided to mimic the behaviour of the Type- j in the insurance market, then he would choose: $q^i = q^j = 0$, $K_i^i = 0$, and $S^i = w^i$. Notice now that the vector $[q^i = 0, K_i^i = 0, S^i = w^i]$ is actually part of the feasible set in *Problem(II)*; furthermore, it is never an optimal choice for any $p \in [\bar{p}, 1)$. Therefore, it must be the case that the IC stipulated in Eq.(12) does not bind for values of $p \in [\bar{p}, 1)$.

Given than in a symmetric equilibrium $q^i = q^j = q$, the IC in Eq.(12) can be simplified to:

$$e^{\delta\mathcal{P}K_i^i} \left[\phi e^{-q} + (1-\phi)e^{-F(K_i^i)} \right] \leq e^{-q}. \quad (\text{L.2.1})$$

Now, take any $p \in (\phi, \bar{p})$; FOC of *Problem (II)* necessarily lead to the following relation between q^i and K_i^i :

$$q^i = F(K_i^i) + \ln \left(\frac{\phi}{1-\phi} \frac{\delta\mathcal{P}}{F'(K_i^i) - \delta\mathcal{P}} \right) \quad (\text{L.2.2})$$

Plugging Eq.(L.2.2) into Eq.(L.2.1), we can obtain (after some simple algebra):

$$\phi e^{\delta\mathcal{P}K_i^i} \frac{F'(K_i^i)}{F'(K_i^i) - \delta\mathcal{P}} \leq 1 \quad (\text{L.2.3})$$

If the IC does not bind in equilibrium, then in the optimum it must be verified the following condition: $F'(K_i^i) = \delta\mathcal{P} (1-p)^{-1}$. Replacing this condition in Eq.(L.2.3), we get:

$$K_i^i \leq \frac{1}{\delta\mathcal{P}} \ln \left(\frac{p}{\phi} \right) \quad (\text{L.2.4})$$

Finally, from *Lemma 1*, if the IC does not bind in equilibrium, then Eq.(7) must hold for any $p \in (\phi, \bar{p})$. Replacing Eq.(7) into Eq.(L.2.4), leads to the following condition:

$$J(p) \equiv \left(\frac{1}{\delta\mathcal{P}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-p}{1-\phi}\right)^{\frac{1}{1-\alpha}} + \ln\left(\frac{\phi}{p}\right) \leq 0. \quad (\text{L.2.5})$$

Notice that $J(p)$ is *continuous* and *differentiable* all over the interval $[\phi, \bar{p}]$. Set $p = \phi$, which leads to $J(\phi) = \zeta^\alpha > 0$; therefore, the IC must bind in the vicinity of ϕ . Finally, note that $\partial J/\partial p < 0$ for any $p \in (\phi, \bar{p})$, and that the IC does not bind at $p = \bar{p}$, which implies that $J(\bar{p}) < 0$ must hold. As a result, there must necessarily exist some *cut-off* value $\hat{p} \in (\phi, \bar{p})$, such that when $p = \hat{p} : J(\hat{p}) = 0$; and $\forall p \in (\phi, \hat{p}) : J(p) > 0$, and $\forall p \in (\hat{p}, \bar{p}) : J(p) < 0$. ■

Proof of Lemma 3. Take some value $p^* = p_0$, and denote: $\hat{q}_0 \equiv \hat{q}(p_0)$ and $\hat{K}_0 \equiv \hat{K}(q_0)$. Applying logarithms to Eq.(14),

$$\ln(\phi) + \delta\mathcal{P} \hat{K}_0 + \ln\left(\frac{F'(\hat{K}_0)}{F'(\hat{K}_0) - \delta\mathcal{P}}\right) = 0 \quad (\text{L.3.1})$$

Differentiating both sides of Eq.(L.3.1), we obtain:

$$\delta\mathcal{P} \left[1 - \frac{F''(\hat{K}_0)}{F'(\hat{K}_0)(F'(\hat{K}_0) - \delta\mathcal{P})}\right] \frac{d\hat{K}}{d\hat{q}} \Bigg|_{\hat{q}_0} \frac{d\hat{q}}{dp} \Bigg|_{p_0} = 0 \quad (\text{L.3.2})$$

Eq.(13) implies $\frac{d\hat{K}}{d\hat{q}} > 0$. Moreover, $\frac{F''(\hat{K}_0)}{F'(\hat{K}_0)(F'(\hat{K}_0) - \delta\mathcal{P})} < 0$, because $F''(K) < 0$ and $F'(\hat{K}_0) \geq \delta\mathcal{P}(1 - \phi)^{-1} > \delta\mathcal{P}$. As a consequence, for Eq.(L.3.2) to hold, $\frac{d\hat{q}}{dp} = 0$ must necessarily be true for any value of $p^* = p_0$. ■

Proof of Proposition 2. Omitted.

Proof of Lemma 4. Take some site $h \notin \mathcal{A}_{t-1}$, and assume in period t innovator h chooses $\iota_{h,t} = 1$ and manages to design an innovation for sector h . Then, the optimal price \mathcal{P}_h^* must solve:

$$MAX_{\mathcal{P}_h \geq 0} : \left\{ \Upsilon \equiv (\mathcal{P}_h - 1) K_h^h(\mathcal{P}_h) = (\mathcal{P}_h - 1) \left(\frac{1}{\delta\mathcal{P}_h}\right)^{\frac{1}{1-\alpha}} \right\}$$

This leads to the following FOC:

$$\left(\frac{1}{\delta\mathcal{P}_h}\right)^{\frac{1}{1-\alpha}} \frac{1 - \alpha\mathcal{P}_h}{(1 - \alpha)\mathcal{P}_h} = 0$$

Which yields $\mathcal{P}_h^* = \alpha^{-1}$. Finally, $\mathcal{P}_h^* = \alpha^{-1}$ represents a *global maximum* of Υ , since $\Upsilon''(\mathcal{P}_h) < 0$ whenever $\Upsilon'(\mathcal{P}_h) \geq 0$. ■

Proof of Proposition 4.

(i) First, notice that, by assumption in Section 4.1, $\Pi_{FI}^* > 0$. Then, if $\iota^* = 0$ in *Problem (IV)*, $\Pi_{FI}^* > \Pi_{AI}^* = 0$.

Now, assume *Problem (IV)* delivers $\iota^* = 1$. Thus, if

$$\gamma(\mathcal{P} - 1) \left(\frac{1}{\delta\mathcal{P}} \right)^{\frac{1}{1-\alpha}} - 1 > \gamma(\mathcal{P} - 1) K(p^*, \mathcal{P}) - 1 \quad \forall \mathcal{P} > 1, \quad (\text{P.4.1})$$

where $K(p^*, \mathcal{P})$ is given by Eq.(19), then: $\Pi_{FI}^* > \Pi_{AI}^*$. Eq.(P.4.1) leads to:

$$1 > \frac{1 - \hat{p}(\mathcal{P})}{1 - \phi} \quad (\text{P.4.2})$$

And Eq.(P.4.2) always holds, since $\hat{p}(\mathcal{P}) > \phi$ for all $\mathcal{P} > 1$.²⁵ ◆

(ii) **PROOF INCOMPLETE**

Proof of Proposition 5. Given $\gamma(\mathcal{P}^*(\tilde{n}) - 1) K^*(1 - (1 - \phi)\tilde{n}, \mathcal{P}^*(\tilde{n})) = 1$, if $n_0 > \tilde{n}$; then, equilibrium in $t = 1$ is unique, and verifies $\iota_1^* = 1$. As a result Eq.(22) implies $n_1 > n_0 > \tilde{n}$; which, in turn, means that $\iota_2^* = 1$. Repeating this reasoning *ad infinitum*, we can deduce that for any economy such that $n_0 > \tilde{n}$, the sequence $\{\iota_1^* = 1, \iota_2^* = 1, \dots, \iota_\infty^* = 1\}$ always holds in a dynamic equilibrium. This means Eq.(22) turns into:

$$n_t = (1 - \gamma)n_{t-1} + \gamma \quad (\text{P.5.1})$$

Finally, Eq.(P.5.1) provides a difference equation that converges monotonically to $n_\infty = 1$. ■

²⁵More precisely, $\hat{p}(\mathcal{P})$ is strictly decreasing in \mathcal{P} and $\lim_{\mathcal{P} \rightarrow \infty} \hat{p}(\mathcal{P}) = \phi$.

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