A Dynamic Growth Model for Flows of Foreign Direct Investment

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Abstract

In this work, we for the first time study the dynamic flows of the foreign direct investment (FDI) with a dynamic growth theory. We define the FDI flow as a process which transmits throughout a given social system by way of diverse communication channels. In model formulation, seven assumptions are thus proposed and the foreign capital policy of the host country is considered as an external influence; in addition, the investment policy of the investing country is modeled as an internal influence. Classification of influences is mainly according to the operational strategy as well as the consideration of economical/financial factors. The dynamic model of FDI flow is a differential equation which is solved numerically and verified with collected realistic data. Application of the developed model to explore, taking the electronics industry in Taiwan as an example, Taiwanese direct investment (TDI) in China (i.e. FDI flows from Taiwan to China) since 2001 is conducted. Our preliminary results successfully account for the dynamics of FDI flow for different amount of TDI outflows. It is found that the internal influence dominates the growth of TDI flow from Taiwan to China during 2001-2006.

Keywords: Foreign direct investment, dynamic flow theory, growth model, and numerical simulation.

1. Introduction

There are in general three kinds of international capital flows: foreign direct investment (FDI), portfolio capital and international bank lending. According to the data of International Monetary Fund, FDI has been growing faster than GDP, and is becoming a major component of foreign investment. FDI is defined as the flows of capital from a foreign to a host country to

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establish production or service facilities and conduct business activities. Research on FDI has attracted much attention of scholars in the field of international business and economics in recent years. Most literatures discussed FDI issue from the economic or financial point of view and based on the statistical methods [1-6]. Diverse approaches have their own advantages and disadvantages. However, studying the FDI flows could be from another point of view, such as a dynamic perspective [7]. Such approach benefits investigation of flow's forecasting, operation, planning and management, etc. We notice that the notion of growth (or called diffusion in market science [8-9]) on innovation was originated from 60’s. The growth theory has received much attention in market over the last decades [8]. Among approaches, the Bass model (1969) [9] is the most aggregated influence study. Robinson and Lakhani (1975) [10] were the first to include marketing variables in a growth model. Applications and extensions of the model have widely been written in different fields of innovation, such as high-tech or consumer product, technology, service and organization [11-14]. It will be an interesting way if the evolution of capital flows could be explored from dynamic growth point of view.

In this work, we for the first time study the dynamic flows of FDI with a dynamic growth theory. We define the FDI flows as a process which transmits throughout a given social system by way of diverse communication channels. In model formulation, seven restrict assumptions on growth process, potential amount, and influences are thus proposed and the foreign capital policy of the host country is considered as an external influence; in addition, the investment policy of the investing country is modeled as an internal influence. Classification of influences is mainly according to the operational strategy as well as the consideration of economical/financial factors. According to our assumptions and fundamental theory of dynamics, a dynamic model of FDI flows is thus formulated. The dynamic model is an ordinary differential equation (ODE), where the cumulative flow can be solved. Depending upon the models of external and internal influences, the resulting dynamic model may be a linear or nonlinear ODE. Therefore, in the solution of the developed model, numerical method is considered. It allows us to simulate the dynamics of FDI flow and verify the accuracy with the collected realistic data systematically. Application of the developed model to explore, taking the electronics industry in Taiwan as an example, Taiwanese direct investment (TDI) in China (i.e. FDI flows from Taiwan to China) since 2001 is conducted. Compared with the realistic data, our preliminary simulation results successfully account for the dynamics of FDI flow for different amount of TDI outflows. It is found that the internal influence dominates the growth of TDI flow from Taiwan to China during 2001-2006.

This paper is organized as follows. In Sec. 2, we state the growth theory for FDI flows. In Sec. 3, we show the application of model to TDI between Taiwan and China. Finally, we draw conclusions and suggest future work.
2. Assumption and Model Formulation of FDI Flows

2.1. Background of FDI
FDI could be categorized into the micro (industrial organization) theories and the macro (cost of capital) ones. The conventional theories that explain FDI in microeconomic terms focus on firm-specific advantage, location advantage or cost advantage [3, 15-20]. According to the perspectives, multinationals find it cheaper to expand directly in a foreign country rather than through trade in cases, where advantages associated with cost or product are based on internal, indivisible assets based on knowledge and technology. Studies on determinants of FDI flows could be summarized into three categories. The first group focuses on host country determinants [1-2, 21]. The second focuses on source country determinants [22]. The third one focuses on both source and host country determinants. They consider that core explanatory variables (such as GDP, language and distance) of host and source country in the traditional gravity models affect trade and FDI flows. Some emphasize the opacity (related accounting and regulations of FDI) policy of source and host country is an important factor of capital flows [1, 4, 6, 21]. Plenty literatures on the macroeconomic effects stemming from FDI flows are reported [6, 12, 14, 20, 23]. Studies examine the relationship between FDI and the economic system by employing the macro variables, such as domestic wealth or income effect, the financing cost or the real exchange rate. A capital-linked model based on marginal productivity of capital theory analyzed the mechanism of its capital linkage and tracked long-term change [24]. FDI-related issues in China are popular [2, 4, 17, 18, 25], but study on Taiwanese direct investment (TDI) in China is very limited.

2.2 Assumption and Model
It is known that two origins of growth theory, one is from the British and German-Austrian schools of growth in anthropology and the other is the S-shaped growth curve proposed by French sociologist [8]. According to the dynamic theory, a growth function $y$ capturing the capital flow’s dynamic pattern, given the fact that the pattern is time-dependent, we denote a growth function by $y(t)$. The growth function is usually modeled as the solution of a differential equation $\frac{dy}{dt} = f(y, t)$ where the function $f$ determines the shape of the curve of dynamic growth. To explain the dynamics of FDI flows, we thus state a capital growth model based upon the theory of dynamic growth with the following assumptions [11, 26, 27].

Assumption 1: The growth process of FDI flows is binary. We assume that the FDI growth process is binary (two-stage); investors who enter the financial market either adopt or reject the outflow of capital. The model does not take into
account stages in the adoption process (such as awareness, knowledge, etc.). On the other hand, if firms remain flexible in their choices, they can profit from future developments by choosing options with higher profits on one side, and they can limit loss on the other, thus maximizing the value of investment projects. Typically, real options analysis focuses on the following stage of investment options: the option to abandon or to sell an investment project, to defer an investment, to expand or contract a project, or to switch project operations and to shut down and restart operations [28-30].

Assumption 2: Potential market ($M(t)$) of FDI flows remains constant over time. We assume that the size of the amount of capital is fixed, finite and known or can be estimated; $M(t)$ is constant. The potential market may be a function of exogenous and endogenous factors. Extension researches of $M(t)$ assumption in marketing field could be summarized to two fields: one is about products (durable consumer product and non-durable product) and the other is the decision variable of potential market (i.e. the form of $M(t)$). According to the gravity model, FDI flows depend on the GDPs of both economies and the distance between them. On the other hand, the business cycle, cost of capital, real wealth and real exchange rate could affect the amount of FDI flows [1, 2, 6].

Assumption 3: There are no supply restrictions. We assume that the supply side of capital is not considered. It does not capture the supply restriction. Ohnishi (1998) analyzed supply-side mechanism by a multi-country econometric model [24].

Assumption 4: The growth process of FDI flows is independent of all other kinds of capital flows. We assume that FDI flow is grown/diffused in isolation; complement, substitution or competition is not considered. As far as we know, there are three kinds of international capital flows: FDI, portfolio capital and international bank lending. In the presence of incomplete information, these flows are significantly different from one another. FDI flows do play an important role significantly to domestic investment in both the quantity and the quality dimensions. Outward FDI and local investment in the source countries are negatively co-related, indicating that these two forms of investment are substitutes [4, 5, 31].

Assumption 5: The growth process of FDI flows is not explicitly influenced by policies. We assume that the impact of policy of source and host country is...
implicitly captured by the model parameters. The explicit incorporation of policy variables not only makes the model realism but also helpful to improve investment performance by considering the possibility of altering the growth process through policy control. However, high opacity policy of host country may lead to a reduction in capital inflows in general. The crucial role government plays in outward FDI can be explained by the country’s current political and economic systems [5, 6].

**Assumption 6:** The rate of capital growth at any given point in time is directly proportional to the amount of remaining potential capital at that moment.

Mathematically, this can be represented as:

\[ N(t + \Delta t) - N(t) = gN(t)\Delta t, \quad (1) \]

where \( N(t) \) is the cumulative amount of capital at time \( t \), \( \Delta t \) is the increment of time and \( g(t) \) is the growth rate. Dividing by \( \Delta t \) to the both sides, we have

\[ \frac{N(t + \Delta t) - N(t)}{\Delta t} = gN(t), \quad (2) \]

Taking \( \Delta t \to 0 \) in Eq. (2), we have

\[ \lim_{\Delta t \to 0} \frac{N(t + \Delta t) - N(t)}{\Delta t} = \lim_{\Delta t \to 0} gN(t), \quad (3) \]

thus,

\[ \frac{dN(t)}{dt} = gN(t). \quad (4) \]

The Eq. (4) can be further modified by considering the effect of upper bound of the cumulative amount of capital on \( N(t) \),

\[ \frac{dN(t)}{dt} = g(t)(M - N(t)), \quad (5) \]

where \( M \) is the potential market (i.e., the total number of possible adopters in the social system). The difference \( M - N(t) \) mainly indicates the remaining amount of potential capital at time \( t \).

**Assumption 7:** The growth rate of the probability of capital depends on time through a linear function of \( N(t) \) [32]. It measures the rate at which the amount of capital outflows and, the response rate of the non-outflows. The growth rate depending on the internal and external influences is given by,

\[ g(t) = (a + bN(t)), \quad (6) \]

where the parameters \( a \) and \( b \) show the degree of intensity of resource for the external and
internal influences, respectively. Substituting Eq. (6) into Eq. (5), and based on above assumptions, the dynamic change rate of the capital amounts is proportional to the current amount. Thus, the dynamics of FDI flows is expressed as:

\[
\frac{dN(t)}{dt} = (a + bN(t))(M - N(t)).
\]  

(7)

We notice that the amount of capital at time \( t \) can be calculated by

\[
n(t) = \frac{dN(t)}{dt}.
\]  

(8)

The difference \( M - N(t) \) could be referred to as the effective potential market. When \( N(t) \) approaches \( M \), the growth rate decreases. In the model formulation, the foreign capital policy of the host country is considered as an external influence, whereas the investment policy of the investing country is modeled as an internal influence. We notice that the model is the most parsimonious one which aggregates the mixed influences, and follows the form of the growth model suggested in the marketing literature [8-9]. In general, communication channels are an important element in the growth process of any innovation. They constitute the means through which the innovation expands among the members of a social system. The external communication include through investment advertising or explanation from financial operators, while the external communication include interpersonal communication, interaction among members of a social system. Both of them are complementary and they can play different roles during investment process. Depending on the importance of each source of influence, three versions can be derived from the fundamental growth model. When \( b = 0 \), the model only consider the effect of external influences on \( N(t) \); when \( a = 0 \), the model only consider internal influence. When \( a \neq 0 \) and \( b \neq 0 \), the model is called a mixed influence growth model.

As far as we know, there are two kinds of methods to solve the ODE (7). One is analytical method and the other is computer simulation. If parameters in the developed FDI model are assumed to be constants, we can use analytical method and get the explicit solution. However, the unknown \( N(t) \) and different influence may be time-dependent nonlinear parameters which complicate analytical solution of the ODE. In contrast to the analytical approach, computer simulation is considered due to its generality. A computational procedure is thus applied to solve and calibrate the simulation result with the realistic data. First of all we set initial parameters of \( (a, b, M) \) and then solve Eq. (7). To solve the ODE numerically, a 4th-order Runge-Kutta algorithm is implemented in our solution scheme. Once the computed solution is obtained, we calibrate it with the realistic data for the accuracy of simulation. If result is acceptable, the simulation will be terminated. Otherwise, a least square optimization technique will enable us to update the newer parameters \( a \) and \( b \) for the next simulation.
3. Application to Study FDI Flows from Taiwan to China

3.1 Problem Description
The growth of FDI in China has been dramatic since the beginning of the economic reforms in 1978. China is the first largest recipient of foreign capital in the world. After the two sides across the Taiwan Strait started to exchange visits in the 1980s, direct investment by Taiwan businessmen began to rise rapidly. China has an advantage in attracting FDI from Taiwan since two economies share a very close cultural background in spite of different economic and political systems. We in this section will apply the developed dynamic model to study the dynamics of TDI flow from Taiwan to China; in particular, for the electronics industry.

Annual data of TDI amount approved by government is summarized in Table I. According to the information published by the Investment Commission of Ministry of Economic Affairs (MOEA) of Taiwan, China has also played the largest share of TDI flows in the global recently, as shown in Table I. Taiwanese electronics industry (including both the design and fabrication sub-industries) has been approved to outflow since 2001. The collected realistic data used in our study includes 63 integrated circuit (IC) initial public offerings (IPO) firms of Taiwan. The investigated time period is from 2001 to 2006, the amount unit is thousand NT dollars. Data resource is from The Investment Commission of MOEA of Taiwan.

Table I. The Approved TDI During 2001~2006

<table>
<thead>
<tr>
<th>Year</th>
<th>(1) American area</th>
<th>(2) European area</th>
<th>(3) Asia area</th>
<th>(4) China</th>
<th>(5) Total</th>
<th>China/Total = (4) / (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>3,460,901</td>
<td>46,870</td>
<td>814,981</td>
<td>2,784,147</td>
<td>11,567,455</td>
<td>24.07%</td>
</tr>
<tr>
<td>2002</td>
<td>2,475,574</td>
<td>154,416</td>
<td>530,055</td>
<td>3,858,757</td>
<td>10,598,849</td>
<td>36.41%</td>
</tr>
<tr>
<td>2003</td>
<td>2,731,271</td>
<td>76,724</td>
<td>1,063,915</td>
<td>4,594,985</td>
<td>12,532,161</td>
<td>36.67%</td>
</tr>
<tr>
<td>2004</td>
<td>1,881,380</td>
<td>61,913</td>
<td>1,275,089</td>
<td>6,940,663</td>
<td>13,704,707</td>
<td>50.64%</td>
</tr>
<tr>
<td>2005</td>
<td>1,618,228</td>
<td>299,314</td>
<td>430,573</td>
<td>6,006,953</td>
<td>10,901,851</td>
<td>55.10%</td>
</tr>
<tr>
<td>2006</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7,642,328</td>
<td>16,273,178</td>
<td>46.96%</td>
</tr>
</tbody>
</table>

Data Resource: Investment Commission, MOEA, Taiwan

3.2 Results and Discussion
Realistic model of permitted amount of FDI outflows from Taiwan to China is complicated. To verify the accuracy of theoretical work, we assume two different cases of the amount of the permitted FDI flows (Mr) in the dynamic simulation of FDI flows. While in the following description, we denote that Mr is the permitted amount (potential market) of TDI outflows, Nr
is the realistic cumulative amount of capital of TDI outflows, $M_s$ is the assumed potential market of TDI outflows, and $N_s$ is the simulated cumulative amount of capital of TDI outflows.

Case 1: Fixed potential market of TDI outflows (i.e., constant $M_s$ is assumed in Eq. (7)). Three different sizes of the fixed amount of capital are analyzed in this case. First of all, $M_{s1-1}$ means the minimum of $M_r$ (i.e., $M_{s1-1} =$ the low bound of $M_r$); the second, $M_{s1-2}$ is $M_r/2$ (i.e., $M_{s1-2} =$ the average of $M_r$); and the third one $M_{s1-3}$ is the maximum of $M_r$ (i.e., $M_{s1-3} =$ the upper bound of $M_r$). Our simulation implies that there is only the result of the case of $M_{s1-1}$ is accurate, as shown in Fig. 1. The simulated $N_s$ is basically traced the behavior of realistic $N_r$, where the longitudinal axis is the cumulative amount of capital and the transverse is time scale. Compared with the realistic data $M_r$, if we set $M_s$ to be the low bound of $M_r$, the simulation result is more accurate than other two settings.

![Figure 1](image-url)

Figure 1. The collected realistic $M_r$ (the solid line), the assumed $M_s$ (the short-dashed line), the collected realistic $N_r$ (the long-dashed line), and the simulated $N_s$ (the dotted lines) of the electronics industry of TDI in China. $M_r$ is the permitted amount of TDI, $N_r$ is the realistic cumulative amount of capital of TDI, $M_s$ is the assumed amount of TDI, and $N_s$ is the simulated cumulative amount of capital of TDI.

Besides, according to Eq. (8), the derivatives of $N_s$ and $N_r$ (denoted by $dN_r$ and $dN_s$, i.e., the collected realistic and simulated amounts of capital at time $t$) are compared and have
shown the similar tendency at time $t$, as shown in Fig. 2. There is time-lag effect during 2003 and 2004, it means there is time-lag between the permitted amounts of capital outflows in the resource country and the amounts of capital inflows to the host country.

It is found when $M_s$ is assumed to be constant in our simulation, the relationship of $a < b$ is held always. It suggests that the external influence is smaller than the internal influence. Thus, the capital policy of the source country dominates the dynamics of FDI flow. However, the case of $M_{s1-1}$ is undervalued and unreasonable; also, the amount of potential market may not be constant. We thus continuously discuss a time-dependent $M_s$ and simultaneously optimized the parameters $a$ and $b$ in the next case.

![Figure 2. The collected realistic $N_r$ (the long-dashed line), the simulated $N_s$ (the dotted lines), $dN_r$ (the dish-dotted line) and $dN_s$ (the dotted line) of the electronics industry of TDI in China ($M_s$ is constant). Where $dN_r$ is the derivatives of $N_r$, $dN_s$ is the derivatives of $N_s$. $N_s$ and $N_r$, $dN_r$ and $dN_s$ have shown the similar tendency at time $t$.](image)

**Case 2:** *Time-dependent potential market of TDI outflows (i.e., a linear $M_s$ is assumed in Eq. (7)).* $M_s$ is formulated as a time-dependent function to approximate the $M_r$. In contrast to the case 1, four different settings on the external and internal influences (i.e., $a$ and $b$) are analyzed in this case. $M_{s2-1}$ means $a$ and $b$ both are fixed. $M_{s2-2}$ means $a$ is fixed and $b$ varies with time. $M_{s2-3}$ means $a$ varies with time and $b$ is fixed. $M_{s2-4}$ means $a$ and $b$ both vary with time. Our simulation implies that the results of the cases of $M_{s2-2}$ and $M_{s2-4}$ are accurate, as shown in*
Figs. 3(a), 3(b), 4(a) and 4(b). Compared with the realistic data Mr, if we set $M_{S2-4}$ as a time-dependent function and consider the external and internal influences ($a$, $b$) as time-varying parameters, shown in Fig. 4(b), the simulation result is further accurate, compared with the result of $M_{S2-2}$, as shown in Fig. 3(b).

![Figure 3](image_url)

Figure 3 (a) Results of $M_{S2-2}$ ($M_s$ is time-dependent, parameter $a$ is fixed and $b$ varies with time). Comparison among Mr, Ms, Nr, and Ns of the electronics industry of TDI in China. (b). Using the Nr and Ns, the dNr and dNs are further computed for the electronics industry of TDI in China.

Besides, in case $M_{S2-2}$, when we compare $dNr$ with $dNs$, they have shown the similar tendency at time $t$, as shown in Fig. 3(b). It is found when $M_s$ is assumed as time-dependent function, parameter $b$ varies with time in our simulation, and the result is more accurate than case $M_{S2-1}$ and $M_{S2-3}$. However, the fixed setting of parameter $a$ is not reasonable; the external influence could vary with time. So, in case $M_{S2-4}$, we formulate $a$ and $b$ both vary with time. The simulation result is further accurate than the result of $M_{S2-2}$ and the formulation in consistent with the fact. Insignificant time-lag effect is observed during 2002-2004. We also find the result $a < b$. This suggests that the internal influence is the key factor of FDI flows. Thus, the capital policy of the source country dominates the dynamics of FDI outflows.

According to the simulation result, we get the following economic findings. When $M_s$ is constant setting, we get the result: $a < b$. This suggests that the foreign capital policy of China is smaller than the capital or investment-related policy of Taiwan; i. e., the capital or investment-related policy of Taiwan dominates. When $M_s$ is formulated as a time-dependent function and if parameter $b$ varies with time, the results ($M_{S2-2}$ and $M_{S2-4}$) are more accurate. It suggests that the capital or investment-related policy of Taiwan is the key factor of TDI outflows to China.

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So far, we have investigated the relaxation of Assumption 2: Market potential \( M(t) \) of FDI flows remains constant over time. However, the simulation result may be improved by adjusting the form of potential market size \( M(t) \). More extensions of growth model could be investigated. For example, the form of \( M(t) \) may be nonlinear, such as \( M \) is quadratic or exponential functions; internal influence and external influence may be functional of time, the cumulative amount of capital, the potential market, there are other parameters required to model the growth patterns, etc. For example, the influence factor (parameter \( a \) and \( b \)) could be divided into four fields as follow. (A) Resource/cost incentives: natural resources, cost of production endowment (rent, labor, capital), culture and customs, language, distance, etc. (B) Industry-specified incentives: industry cluster, technology transfer or reutilized, extend the life cycle of product, close to market, etc. (C) Policy of the host country: tax and tariff concessions, cash grants, special subsidies, infrastructure, political stability, etc. (D) Policy of the resource country: open policy of capital outflow, political stability, general business climate, legal system, tax concessions or subsidies, etc. Calibration of parameters using statistical hypothesis is under investigated. Besides, the heterogeneous property among capital flows and non-symmetric growth patterns could be considered in further research.

4. Conclusions

In this work, we have proposed a capital growth model to analyze the dynamic growth behaviors of FDI flows. In the model formulation, the foreign capital policy of the host country
has been considered as an external influence, whereas the investment policy of the parent country was modeled as an internal influence. Under seven assumptions, the classification of influences is mainly according to the operational strategy as well as the consideration of economical/financial factors. The resulting nonlinear dynamic model has been solved numerically and successfully applied to explore the electronics industry of TDI in Mainland China during 2001-2006. When $M_s$ is modeled as a time-dependent function and $b$ varies with time, we have obtained more accurate results. It implies that the internal influence dominates the growth of FDI. Capital policy of the source country is the key factors of FDI flows. More applications and simulation studies on different industry and different country, relaxations of the assumptions are currently under investigated.

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