

The Simplest Possible 2x2x2 CGE Diagraphomics:  
Revisiting the H-O, S-S Orthodoxies

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For Presentation at City University Hong Kong International Workshop on International Trade and Finance,

March 14, 2005

ABSTRACT

The simplest possible model of computable general equilibrium with trade is presented for purposes of diagraphomics pedagogica while probing/revisiting the orthodox Hecksher-Ohlin (HO) and Stolper-Samuelson (SS) theorems hopefully shedding some new light on each. Set forth against the HO orthodoxy is an H.O. heresy. The latter reverses the former outcomes, and turns the Leontief paradox no paradox accordingly. Re the SS theorem this paper challenge their policy recommendation to “bribe” the poor (or even the rich) adversely affected from free trade by pondering the meaning of *bribing* squarely under the light of the ‘ideal types’ model proposed herein. The paper also attempts to extend/generalize its static model by incorporating into it a growth model *a la* Jensen-Larsen (2004).

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## The Simplest Possible 2x2x2 CGE Diagrammatics: Revisiting the H-O, S-S Orthodoxies

### Introduction

In a similar workshop a year ago (2004) I presented my report with the following verbiage:

For the last several years I have been teaching my microeconomics courses even for freshmen what the Stolper-Samuelson theorem is all about. How do I do it? This note attempts a brief account of how I do it as an *alpha* part exercise for the Tokyo Workshop on the “Alpha and Omega Topics in International Trade and Finance”. The *alpha* here connotes something both elementary and substantial, while *omega* implying something highly advanced and comprehensive.

In what followed I assumed, as I do in class for an *alpha* exercise, two identical countries, identically endowed with homogeneous labor and capital (two factors). Pursuant to these basic assumptions I presented a simple (perhaps the simplest possible) diagrammatic account of the Stolper-Samuelson theory in a fully general equilibrium framework.

My class adventure reveals a **bitter maxim** of free trade. Underlying the proposed diagrammatics pedagogica, is a more formal system of mathematical equations also shown in my workshop paper. The paper’s Section 2 presented a fairly general EBD representation of optimal resource allocation and related income distribution. Section 3 in turn tried to interpret the Jensen-Larsen growth model within the confines of my simple diagrammatics, followed by Section 4 conclusions.

Among some intriguing comments offered by the then workshop participants I found *Kaku Furuya’s* the most intriguing. According to him I might as well set aside my critique of Stolper-Samuelson’s proposal on income redistribution to overcome the difficulty of free trade. He then suggested that my model be used to shed some new light on the orthodox H-O model and its basic predictions. Specifically starting with the same basic H-O model, what assumptions, say, on taste differentials may reverse their well-known conclusion of the H-O theorem. The purpose of my present piece is three-fold. First I will accept *Furuya’s* suggestion to treat the H-O model with my ideas on taste differentials in addition to H-O’s ideas on different endowments. My second task as an adamant old man’s is to get back to my old model to probe what Stolper-Samuelson had to say upon proposing to “bribe” the adversely affected class. And my third task is to revisit the Jensen-Larsen growth model using my 2x2x2 static model and related diagrammatics.

### Section 1 The Orthodox HO vs. Heretical H.O.

Consider two countries, labeled N and B, producing nuts and bananas. Assume that N is endowed with

more capital and less labor than B that is oppositely endowed to produce nuts and bananas, respectively. The two nationals share the same tastes for the produces under consideration. The production functions for each good ( $i=N, B$ ) are identical between the two countries and are linear homogeneous with variable factor proportions. Derived from these basic assumptions are the following autarky equilibria identified as  $E_N^A$  and  $E_B^A$  on the PPF of the Nutties and that of the Bananans.

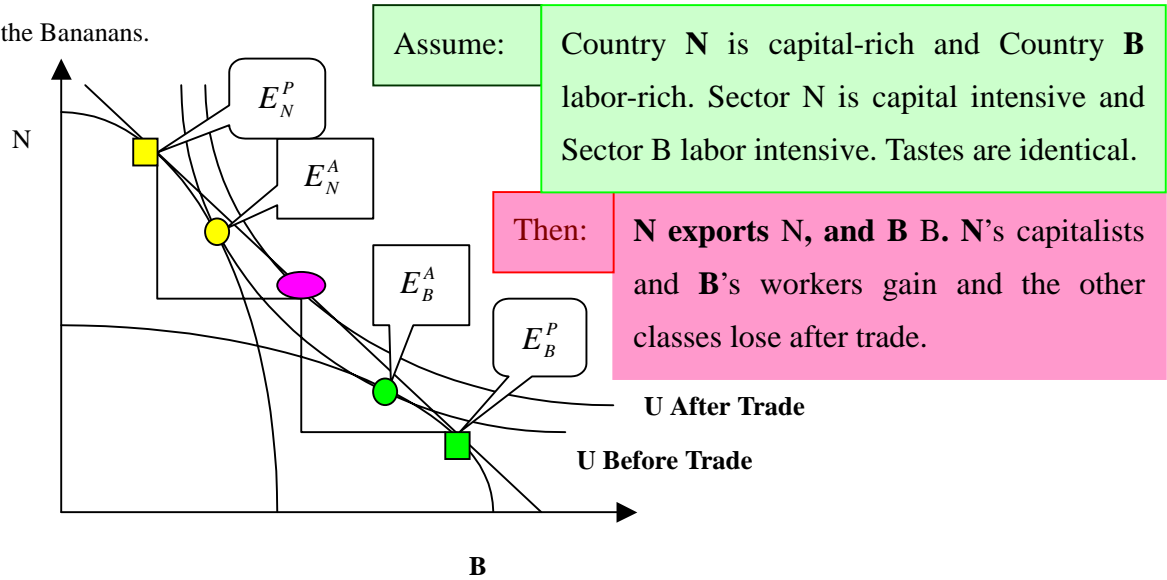


Figure 1 HO Orthodoxy

We now turn to H.O. heresy(?) which is predicated on the same assumptions as HO's but one: tastes are different between the two trading nations.

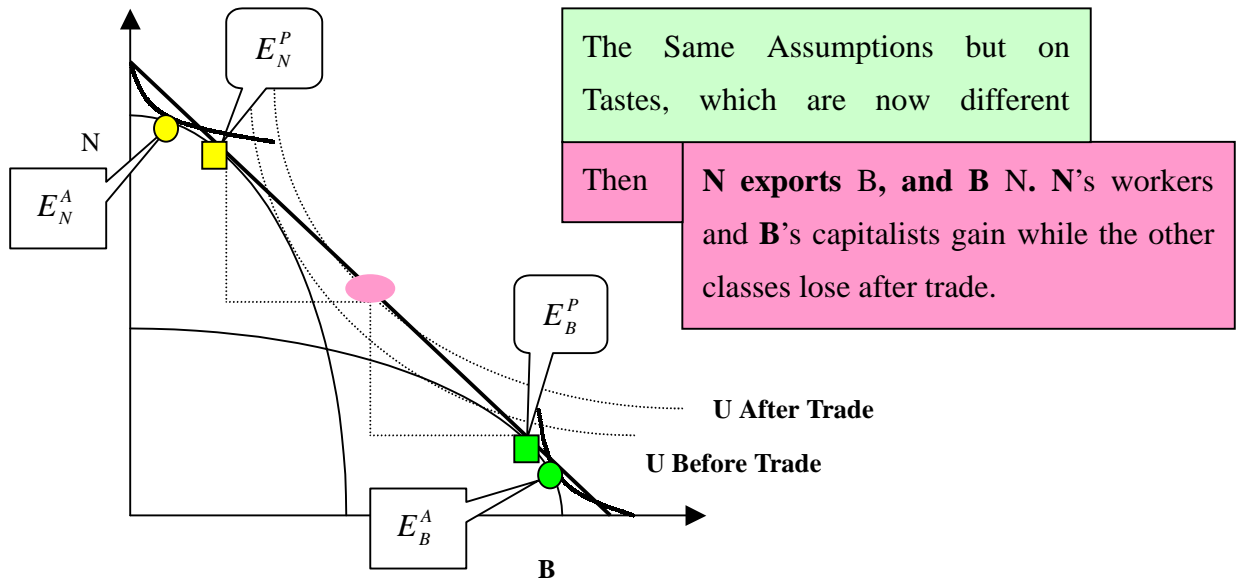


Figure 2 H.O. Heresy

Then it follows that the familiar HO theorem remains to be unaltered if only the taste differences between the

two nations remain small enough. But if and when tastes become sufficiently different for the two nationals as illustrated by Figure 2, then the HO theorem is reversed. It is its labor-intensive sector that is to be the capital-rich country's export sector, and capital-intensive goods N are to be imported. This may make the Leontief paradox is no more paradox than the H.O. heresy is, should it sound paradoxical.

**Section 2 SST Diagrammics**

After introducing the concept of the iso-quant map, followed by the concept of MRTS and the related law of diminishing MRTS, I draw a nonlinear (monotone) contract curve (as well as a linear one) over a square Edgeworth box of factor endowments. See Figure 3.

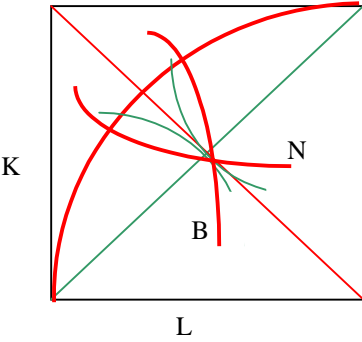


Figure 3 EBD of Endowments,  
Back-to-Back Iso-Quants and Contract

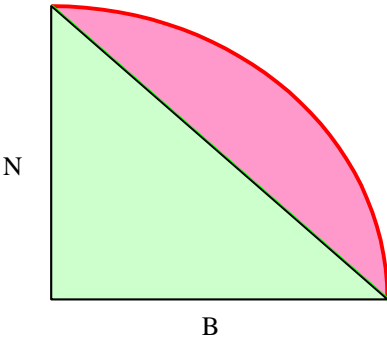


Figure 4 Production Possibilities  
Set and Frontier

**Two Laws of Importance of Being Different**

From this I either assert or demonstrate (geometrically) that a *concave* production possibility frontier can be derived, say, as Figure 4. Related to a red PPF is a red CC; blue PPF related to a blue CC. In this related vein I emphasize that being different in the methods of production in the two sectors has an important meaning as it enlarges the production possibilities set! I refer to this as the **first law** of importance (or beauty) of being different, followed in turn by the **second law** of importance (or beauty) of being different (in tastes).

Related to both laws of importance of being different is the concept of autarky, followed by that of trade. I identify arbitrarily two points of autarky equilibrium on the PPF. Prerequisite to this is the concept of the indifference map. Using two indifference curves to identify two distinct autarky equilibria along the PPF, I also explain the law of increasing marginal cost of either good in terms of the other good forgone, followed by the  $p = MC$  principle under conditions of perfect competition. You can produce either bananas or nuts by employing labor and capital optimally to maximize profit. Related to the two autarky points on the PPF are two corresponding points of optimal resource allocation identified along the contract curve.

Related to the contract curve is the concept of MRTS along an iso-quant curve. I explain how this ratio

varies along the contract curve, and why. In this connection I find it very helpful for the student to start with a reference point (circled in red, Figure 5) along the contract curve where  $MRTS = 1$  to intuitively appreciate its relation to factor prices. Assuming competitive equilibrium (in the factor market) requires relative factor prices to be equal to this MRTS, yields  $w/r = 1$ .

I then ask students what would happen to the factor intensities (say, capital/labor ratios) and factor prices as well if and when the productive optimum points and related factor combinations move along the contract curve, deviating from the reference point in Figure 5 circled in red. Compare it with a green circle point, the lower left corner, or a blue circle, the upper right corner of Figure 5. This is a heuristic method, which I find more effective than expected.

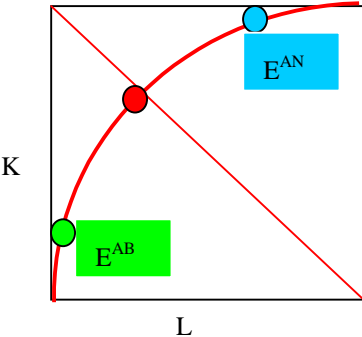


Figure 5 Trade and Factor price Equalization

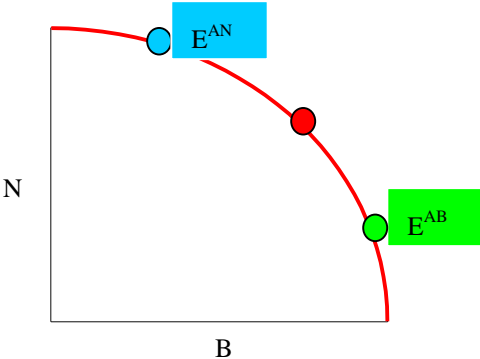


Figure 6 Productive Optimum before/after Trade

It goes without saying that production possibilities frontier PPF can readily be derived from or related to the contract curve of Figure 5. Figure 6 illustrates how points  $E^{AB}$ ,  $E^{AN}$  and a red point in-between along the PPF are related to those along the contract curve of Figure 5.

**The Third Law of ‘Importance of Being Unimportant’**

At this stage I also introduce a third law of importance, the concept of the “**importance of being unimportant**” in income distribution. The message stressed here is that the more minor a specific sector, say, capital-intensive sector, the higher the factor payment to its sector’s non-intensive factor, say, labor (not only in the minor sector itself, but also in the major sector as well). This is because the minor sector’s labor, which is scarce, becomes increasingly scarcer, thereby making the value of a marginal worker increasingly higher.

To cement this idea of ‘**importance of unimportance**’ suppose the capital-intensive sector shrinks. This requires the labor-intensive sector to expand. Then what happens to this labor-intensive sector’s capital intensity? It increases, too. Thus, the overall capital intensity tends to increase when labor-intensive sector expands while the capital-intensive (minor) sector shrinks to become even more minor. Both sectors becoming more capital-intensive means that labor is becoming relatively scarcer. This is the underlying reason why wage rate must be increasing as

both sectors become increasingly more capital intensive, or more pointedly as the (minor) capital-intensive sector diminishes by reallocating part of its scarce labor to the expanding (major) labor-intensive sector. Labor's value at the margin must increase as both sectors compete for the scarce labor that the minor capital-intensive sector has.

### **Factor Abundance Defined, or Redefined**

Now consider conditions before trade. Different tastes yield distinct autarky equilibria with different prices, both products and factor prices for two countries. Different prices induce trade, which tends to equalize not only product prices, but also factor prices. This does not require the Heckscher-Ohlin assumption of different factor endowments. Relative **factor scarcity** or abundance cannot be simply defined in terms of **different factor endowments**, but instead they ought to be defined in terms of **different factor prices**. If one country's wage rate is higher than the other country's wage rate, then that country must be defined to have scarcer endowment of labor.

It is then the differences, if any, in prices and related factor prices that induce international trade. However, trade tends to equalize product prices and factor prices as well *regardless of different or identical factor endowments*. And regardless of factor endowments the scarcity of any given factor can be defined in terms of factor price differentials, which tend to be equalized by trade. Note in this process of factor price equalization that the SS magnifying effects of changes in product prices upon an individual factor owner's welfare are **inescapable**.

### **Adverse Welfare Effects of Trade: The Stolper-Samuelson Magnifying Effects**

To appreciate the SS magnifying effect more fully, assume tastes are different such that everybody is equally paid *only after trade*. (This requires **symmetric taste differences** between the two nations assumed, identically endowed, however for simplicity, with  $K = L = 1$ , or, equivalently, equal numbers of workers and capitalists each endowed with one unit of labor and capital, respectively.) But remember output elasticities for a given factor are different in two sectors.) Then despite the **full equality** in income distribution attainable after trade, each country will find its richer class, either labor or capital, to lose from trade. Trade is therefore **no good news to the rich class in each country**. Unless the rich are promised a transfer in an amount that will keep them richer than the poor class after trade, they may have no incentives for free trade. The poor, by comparison, would only catch up with the rich in income distribution after trade. The poor may accordingly feel that the rich does not deserve any transfer from the poor.

Stolper and Samuelson may have wanted to show how the poor (or any other specific class adversely affected by trade) may deserve protection? However, the SST model reveals an intrinsic difficulty inherent in any such transfer proposed. Why? While free trade benefits all in terms of national income (welfare) increase, the adversely affected class, poor or rich, may naturally demand compensation. But if the rich were to be compensated *via* a transfer from the beneficiaries of free trade who remained unfairly poor (or rather *unjustly* poor) before trade, would the poor accept any such transfer from them to the rich? Would they deserve it? Theoretically they wouldn't. For any

transfer for a Pareto improvement implies a *de facto* deviation from the state of factor price equalization, keeping the poor before trade from becoming just equally rich as the former rich. There seems to be no justifiable excuse for the former rich to remain rich after trade via a transfer from the poor. Government intervention is therefore not warranted. The Stolper-Samuelson theorem 'theoretically' reveals the maxim of no government intervention.

A subtle distinction between 'justice' and 'fairness' may be warranted also. We are endowed with discriminatory amounts of talents and/or assets from our ancestors, or Heaven. That is unfair. But assuming that God is not to blame for this, and if market mechanism prevails, then factor payments will be made in accordance with one's value marginal contribution, given one's endowments. If such equilibrium were prevented by market imperfections including lack of free trade, it might be called *unjust*. Any regulation to prevent market from properly functioning may be called *unjust* rather than *unfair*.

It often is the rich who cries for their protection for the sake, or in the name, of the "poor", but in fact they are unjustly justifying their benefit to be protected as a vested interest, or vested authority. The Stolper Samuelson model reveals it.

What if the poor becomes poorer after trade? To answer this question note that the poor before trade *intra-nationally* is rich *internationally*. On what basis do they deserve protection from competition from the poor abroad who are equally capable as the domestic poor?

### **Edgeworth Box Diagram of a 2x2x2 GE Model**

Figure 7 below illustrates how both productive optimum and consumptive optimum may be obtained within a single Edgeworth box of production and consumption. In the box a concave production possibility curve PPC for nuts is shown to be tangent to another PPC for bananas. The back-to-back PPCs are shown to define general equilibrium of production at the point of tangency. Identified at this point of equilibrium  $E_p^*$  are both productive optimum and relative prices of bananas and nuts. Given the relative price of bananas (relative to nuts) as the slope of what I call **income line**, consumer optimum is also identified along the **income-line-turned budget line** at  $E_C^*$ . Here also note that two back-to-back community indifference curves are shown to be tangent to each other. Note further that these indifferent curves indicate respectively strictly higher community welfare than at  $E^{NA}$ , autarky equilibrium for the Nuts Country, or  $E^{BA}$ , autarky equilibrium for the Banana Country. The Nutties go bananas to consume more nuts than bananas; and the Bananans are nuts about consuming more bananas than nuts.

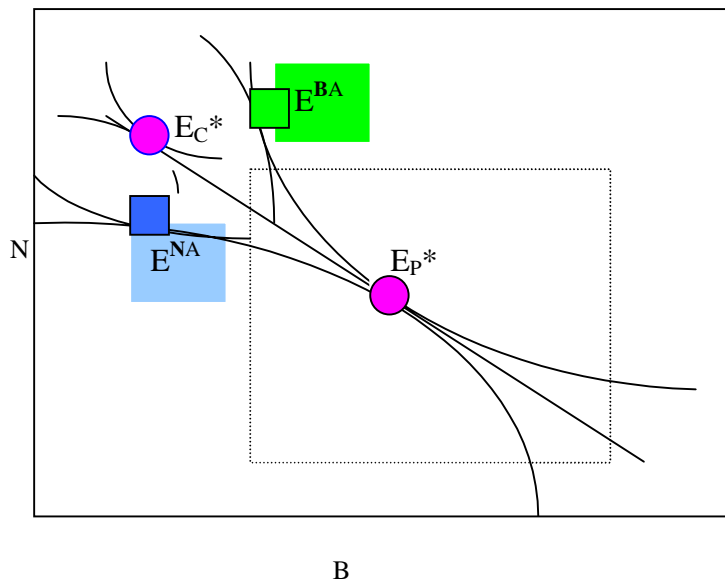


Figure 7 The EBD of General Equilibrium:  
Production and Exchange

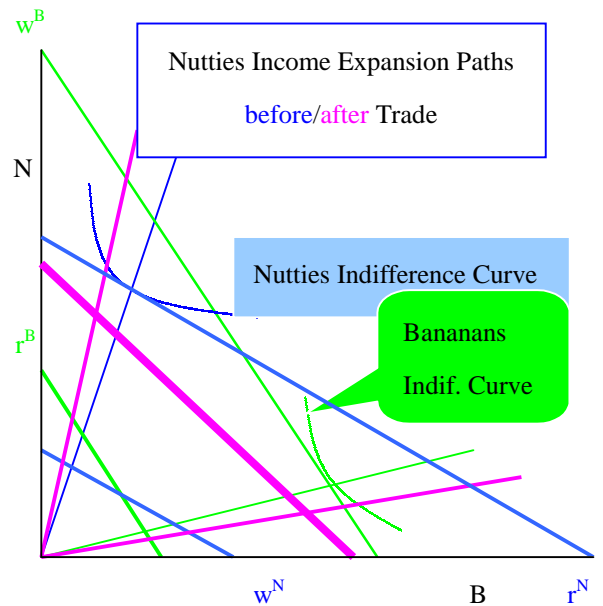


Figure 8 Budget Lines and Income  
Expansion Paths Before/After Trade

### Free Trade as a Difficult Proposition in Principle, besides in Practice

Figure 7 thus shows that trade helps to enlarge not only the size of the Edgeworth box (over and above a dotted box inscribed representing an EB before trade), but trade also increases each country's community welfare. Why is then free trade so adamantly resented? And resented by whom?

Figure 8 answers this question by individual budget lines and related income expansion lines in each country before trade and after trade. The two downward-sloping lines in blue are budget lines for the Nutties' rich capitalists and the poor worker before trade. Note they are parallel (and flatter than the green lines). Note also that the blue (or green) budget lines are intersected with a unique income expansion line identifying each income class' point of consumptive optimum. The two budget lines in green, by contrast, are the Bananans' rich workers' and poor capitalists' before trade with their income expansion line also in green identifying their consumptive optimum (at points intersected by the budget lines).

The budget lines in blue are flatter than those in green. This means that prices, or marginal costs, of bananas are lower for the Nutties than for the Bananans *before trade*.

Note finally that a single **solid** line in purple represents the unifying budget line for all income classes in both countries *after trade*. Everybody being endowed with either one unit of labor or capital will be equally paid after trade wherever he lives. Two distinctively different income expansion paths in purple reflect taste differences between two nations even after trade, however. In any case what happens to each income class' welfare after trade should be clear enough to confirm aforementioned intra-national as well as international conflicts that free trade provokes. The adversely affected income classes finding their enemies being either foreigners or own folks, but in



any case may feel that they deserve protection.

But do they? Aren't they simply whining? Stolper-Samuelson's concluding remarks that suggest to "bribe the poor" (or the rich adversely affected in our example) for market opening may call for a deeper scrutiny to determine on what basis bribing may be legitimate. I find none. This in a nutshell is my tentative conclusion of my *alpha* topic in economics in general and international trade in particular.

### Section 3 A 2x2x2 CGE Approach to the Stolper-Samuelson Theorem

Underlying the foregoing diaganomics is a more formal 2-sector, 2-factor, 2-country model of general equilibrium of production, trade, and consumption. A simple 2x2x2 CGE representation of the model is readily available subject to the basic theoretical requirements such as variable proportions and linear homogeneity. The system of equations is given as follows. (For the simplest possible CGE analysis with differentiated, rather than identical, tastes, both utility functions and production functions may be differentiated with the minimum possible parameters such as those underscoring Figures 9 and 10, pp. 10 - 11 below.)

#### 1. First-Order Conditions for Consumptive Optimum:

$$U^i = U^i(N^i, B^i), \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

where  $U^i$  is Country  $i$ 's utility as a function of nuts  $N^i$  and bananas  $B^i$  that  $i$ 's citizens consume. Citizens have different tastes internationally, but identical tastes intra-nationally (i.e., no taste differences within a country).

$$U_1^i(N^i, B^i) = U_2^i(N^i, B^i)/p_i \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

where  $U_1^i$  is Country  $i$ 's MU of the first good or nuts  $N^i$ ,  $U_2^i$  is Country  $i$ 's MU of the second good or bananas, and  $p$  is the relative price of bananas in terms of nuts assumed as the *numeraire*. Note also here that  $\mathbf{N}$  and  $\mathbf{B}$  in bold face letters stand for countries producing the goods  $N$  and  $B$ , respectively.

#### 2. Identical Production Functions:

$$N = f(K_N^i, L_N^i), \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

$$B = g(K_B^i, L_B^i), \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

where  $f$  and  $g$  stand for production functions for nuts and bananas, respectively, both being functions of capital  $K$  and labor  $L$ , but each function is identical internationally. That is,  $f$  and  $g$  are different intra-nationally, but each is identical internationally as a function.

#### 3. Factor Demand/Supply (Allocation) Equilibrium Conditions:

$$K_N^i + K_B^i = K(=1), \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

$$L_N^i + L_B^i = L(=1), \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

where both endowments are normalized, though not needed, to unity for simplification.

#### 4. First-Order Conditions for Productive Optimum (Before Trade):

$$w_i = f_2 = p_i g_2, \quad f_2 = f(k_N^i, 1) - k_N^i f_1(k_N^i, 1) = \phi(k_N^i), \quad g_2 = g(k_B^i, 1) - k_B^i g_1(k_B^i, 1) = \gamma(k_B^i), \quad i = \mathbf{N}, \mathbf{B}$$

$$r_i = f_1 = p_i g_1, \quad f_1 = f_1(k_N^i, 1) = \phi(k_N^i), \quad g_1 = g_1(k_B^i, 1) = \gamma(k_B^i), \quad i = \mathbf{N}, \mathbf{B}$$

5. Budget Constraints for Capitalists and Workers (Before Trade):

$$r_i = N_K^i + p_i B_K^i, \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

$$w_i = N_L^i + p_i B_L^i, \quad i = \mathbf{N}, \mathbf{B} \text{ (Countries)}$$

6. Law of Equi-Marginal Product of Labor per Dollar (After Trade):

$$f_2/w = p g_2/w$$

7. Law of Equi-Value Marginal Product of Capital (After Trade):

$$r = f_1 = p g_1$$

8. The Law of Equi-Value Marginal Product of Labor (After Trade):

$$w = f_2 = p g_2$$

9. Law of Equi-Value Marginal Product of Capital per Dollar (After Trade):

$$f_1/r = p g_1/r$$

10. Law of Equi-Marginal Rates of Technical Substitution (After Trade):

$$w/r = f_2/f_1 = g_2/g_1$$

11. Budget Constraint for Everybody (After Trade)

$$r = w = N_j^i + p B_j^i, \quad i = \mathbf{N}, \mathbf{B}; j = \mathbf{L}, \mathbf{K}$$

#### Section 4 EBD of 2x2x2 CGE: An Example

We have so far assumed that not only factor endowments but also production technologies are the same between the two countries N and B, but the production functions for the two sectors within each country are different. We now specify below the forms of the production functions in terms of output elasticity of production alone for simplicity. Moreover, we employ only one parameter  $\alpha$  while assuming the Cobb-Douglas type for both the production functions to differentiate the two sectors' methods of production, namely:

$$\text{The N Sector: } N^i = K_N^{i\alpha} L_N^{i(1-\alpha)}, \quad i = \mathbf{N}, \mathbf{B}$$

$$\text{The B Sector: } B^i = K_B^{i(1-\alpha)} L_B^{i\alpha}, \quad i = \mathbf{N}, \mathbf{B}$$

where the subscripts N and B refer to the nuts sector and the banana sector, and the superscript i (=N, B) stands for Country N (for nuts) or B (for bananas).

The following observations on the properties of the production functions above are important.

- 1) When  $\alpha = 1/2$ : The two production functions are identical.
- 2) When  $\alpha > 1/2$  ( $\alpha < 1/2$ ): The N sector is capital intensive, and the B sector is labor intensive. (The N sector is labor intensive, and the B sector is capital intensive.)
- 3) The larger the parameter  $\alpha$  (exceeding 1/2), the higher the capital intensity of the capital-intensive N sector is, and the higher the labor intensity of the B sector by comparison, and *vice versa*. [That is: The lower the  $\alpha$  (below 1/2), the higher the labor intensity of the labor-intensive N sector.]

**Optimal Resource Allocation Requires the Law of Equi-MRTS of L for K:**

Focusing on one country for the time being, and omitting the superscript  $i$  accordingly, the following optimization conditions are to be observed along a contract curve, which is concave (when  $\alpha > 1/2$ ) or convex (when  $\alpha < 1/2$ ).

$$-\frac{dK_N}{dL_N} = \frac{K_N}{L_N} \frac{\alpha}{1-\alpha} = -\frac{dK_B}{dL_B} = \frac{K_B}{L_B} \frac{1-\alpha}{\alpha}$$

This combined with the conditions of a given endowment of L and K, which are assumed both to be unity, yields a unique relation between  $L_N$  and  $K_N$  (or  $L_B$  and  $K_B$ ).

$$\frac{K_N}{L_N} = \frac{(1-\alpha)^2}{\alpha^2} \frac{K(=1) - K_N}{L(=1) - L_N}$$

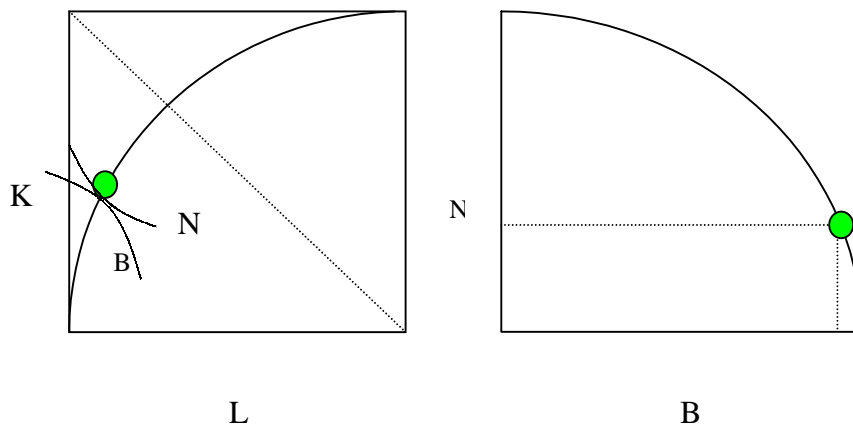
Assuming  $\alpha = .9$  for simulative purposes yields:

$$K_N = \left(\frac{1}{9}\right)^2 \frac{KL_N}{L-L_N} - \left(\frac{1}{9}\right)^2 \frac{1}{L-L_N} K_N L_N; \left(1 + \left(\frac{1}{9}\right)^2 L_N (L-L_N)^{-1}\right) K_N = \left(\frac{1}{9}\right)^2 K \frac{L_N}{L-L_N}$$

$$K_N = \frac{\left(\frac{1}{9}\right)^2 K}{L-L_N + \left(\frac{1}{9}\right)^2 L_N} L_N = \frac{\left(\frac{1}{9}\right)^2 K}{L + \left(\left(\frac{1}{9}\right)^2 - 1\right) L_N} L_N = \frac{\left(\frac{1}{9}\right)^2}{\frac{1}{L_N} + \left(\left(\frac{1}{9}\right)^2 - 1\right)}, \text{ assuming } K=L=1$$

And more generally,

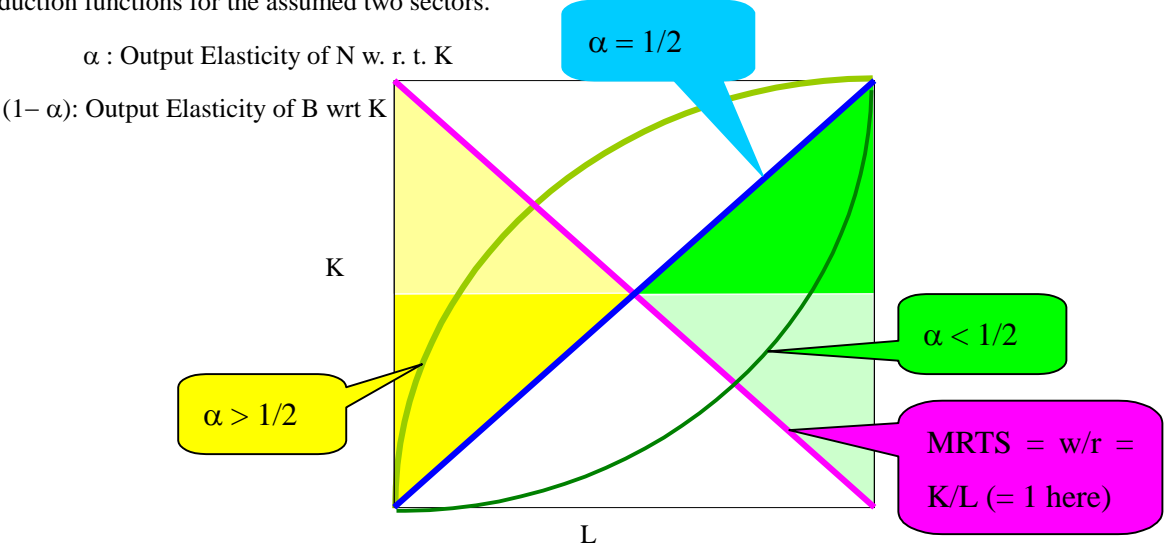
$$K_N = \frac{\left(\frac{1-\alpha}{\alpha}\right)^2}{\frac{1}{L_N} + \left(\left(\frac{1-\alpha}{\alpha}\right)^2 - 1\right)} \quad (K=L=1)$$



**Figure 9 Given  $\alpha$ , Arbitrary  $L_N$  Determines  $K_N$ , Hence  $(L_N, K_N)$  as a One-to-One Correspondence with  $(B, N)$ , and Equilibrium Factor Price Ratio  $w/r (=MRTS)$**

**Factor Allocation and Related Income Distribution: General EBD Representation**

For a more general CGE analysis let us probe an Edgeworth box of endowment for concrete characterization of factor allocation and related income distribution under fairly general conditions of well-defined production functions for the assumed two sectors.



**Figure 10 The EB Partitioned to Represent Factor Allocation and Related Income Distribution**

In the Figure above the darker areas imply factor combinations that yield the higher wage rates and shares relative to rent on capital.

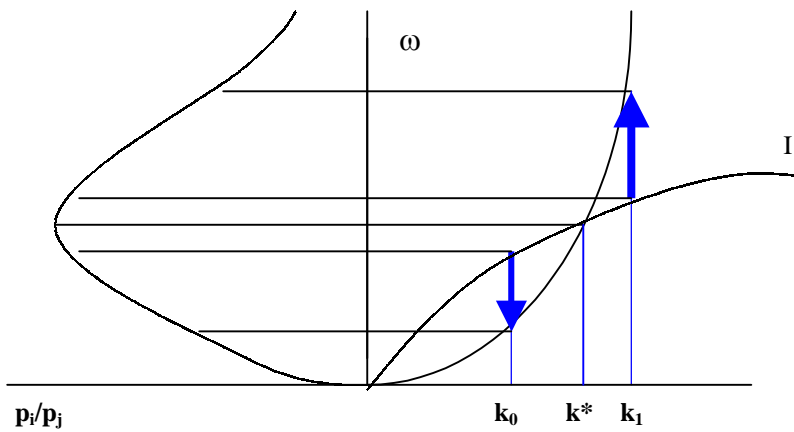
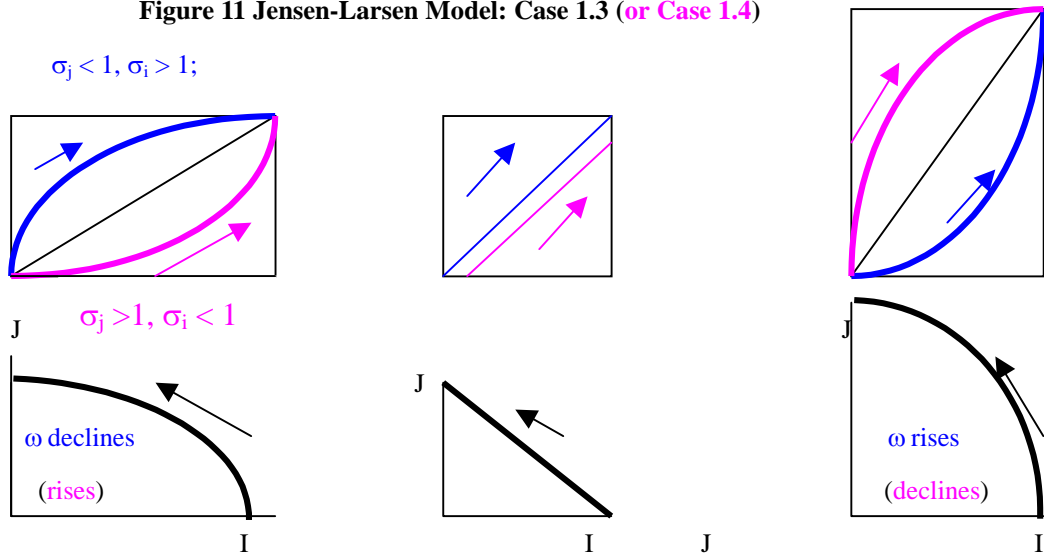
**Section 5 Jensen-Larsen Model Simplified under the Light of H.O. Diagrammatics**

This section briefly reviews a recent contribution by Jensen and Larsen (2004) on general equilibrium model of growth in terms of commodity price CP, factor price FC, and output mix on the production possibilities frontier over consumption goods J and investment goods I. We confine our attention on their multi-sector model to the one with two sectors only following along our analysis above. Moreover, abstracting from trade, we deal with a 2x2 model of general equilibrium while generalizing the production functions assumed to the extent that elasticities of substitution are no longer limited to unity.

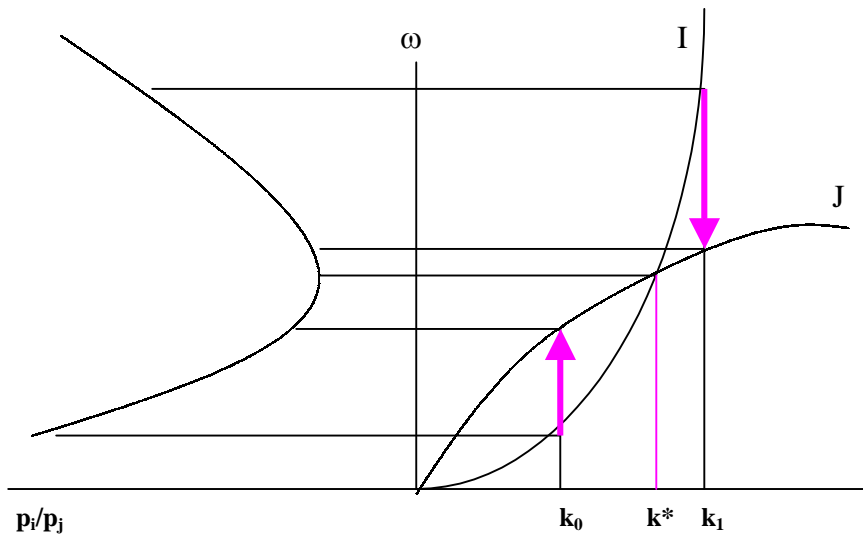
In what follows we will show how different elasticity of substitution parameters  $\sigma$  in different sectors of production are related to the convexity/concavity of the contract curve in the factor endowment space, capital intensity k, relative factor price  $\omega (=w/r)$ , and commodity price p ( $=p_i/p_j$ ). Simple Edgeworth box diagrams and related back-to-back diagrams can be used to do the job. We also probe the impact of growth upon the form of contract curves and related production possibilities set. An interesting example case is when one sector's  $\sigma$  is large ( $> 1$ ) and the other small ( $< 1$ ). As capital accumulation proceeds with a rise in capital intensity k, we can see, as Figure 11 below illustrates, how price in the elastic sector tends to rise initially, but

followed by a decline eventually.

Figure 11 Jensen-Larsen Model: Case 1.3 (or Case 1.4)



Relations of Sectoral  $k_i$ 's to CFP, Given  $\sigma_j < 1, \sigma_i > 1$ , and  $k$  at  $k = k_0, k^*, k_1$



Relations of Sectoral  $k_i$ 's to CFP, Given  $\sigma_j > 1, \sigma_i < 1$ , and  $k$  at  $k = k_0, k^*, k_1$

Figure 11 illustrates how relative commodity price CP is related to relative factor price FP, and FP to sectoral capital intensities  $k_i$ 's as well as aggregate capital intensity  $k$ . The last two panels above show how according as either ' $\sigma_j$  low and  $\sigma_i$  high' or ' $\sigma_j$  high and  $\sigma_i$  low'  $\omega$  tends to either rise or decline as relative price changes along the PPF. Let us probe deeper below.

- 1) Given different elasticities of substitution, say,  $\sigma_j < 1$  and  $\sigma_i > 1$ , economic growth in terms of increasing capital endowments and related increases in capital intensity (from, say,  $k_0$  to  $k^*$  to  $k_1$ ) must expand the *more* capital-intensive sector *more than* does the *less* capital-intensive sector, be it the capital goods sector  $j$  or the consumption goods sector  $i$ .
- 2) Any particular sector, say, capital goods sector, may be capital intensive at an earlier stage of growth. But a continued growth must eventually reverse each sector's factor intensity insofar as the one elasticity of substitution is assumed as small and the other large.
- 3) The relative wage rate  $\omega$  tends to increase monotonically with the aggregate capital intensity, but the product prices, say, in the consumption sector  $i$ , that *may* also increase with  $\omega$  initially (as does under the present parameter combinations) must peak out eventually while  $\omega$  keeps increasing.
- 4) Opposite relations must hold for the other sector's prices. Prices, say, in the capital goods sector must decline initially as  $\omega$  increases, but the decreasing prices must bottom out eventually and start rising as  $\omega$  keeps increasing with capital accumulation.

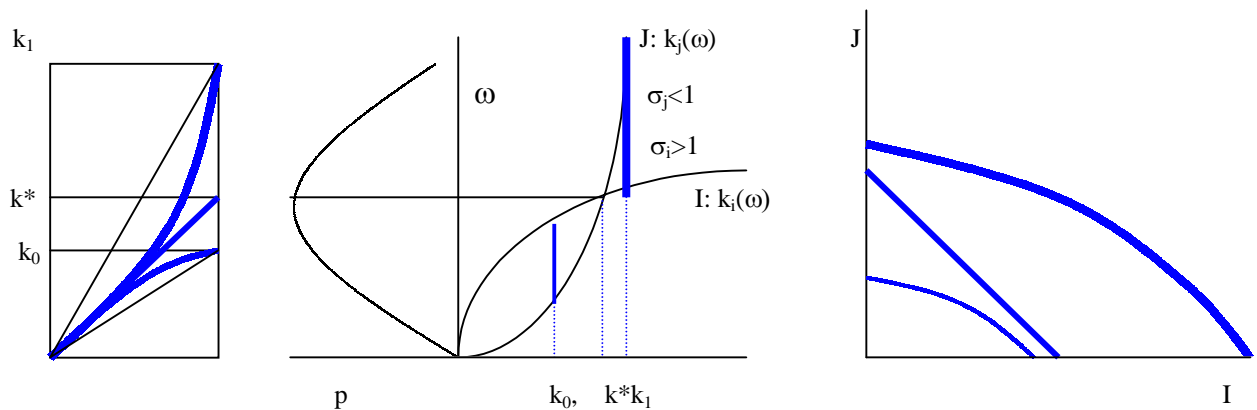


Figure 12 **Impact of Growth upon Contract Curves, and PP Sets**

#### 5) Short-Run (Given $k$ ) Results: Summary Proposition A

Wages may either rise or fall if and when any structural changes take place along a given production possibilities frontier according as either the one sector's elasticity is small and the other large or oppositely large and small. Related to this is either a concave or convex contract curve reflecting relevant differences in the method of production in two sectors.

#### 6) Short-Run (Given k) Results: Summary Proposition B

When  $k$  is small enough a rise in real wages can be accompanied by a fall in a given sector's output, say, in the capital good sector if its elasticity of substitution is small enough relative to the other sector, and *vice versa*. (The importance of being unimportant for workers' welfare requires elasticity of substitution for the minor (capital good) sector to be small enough relative to the other sector.)

#### 7) Short-Run (Given k) Results: Summary Proposition C

When  $k$  is large enough wages increase with the small elasticity of substitution sector.

#### 8) Long-Run Results A: Early Stage

Growth at an early stage requires lower real wages and higher capital good prices relative to consumption goods in order to reallocate resources to promote the capital good sector.

#### 9) Long-Run Results B: Intermediate Stage

Growth in terms of increasing  $k$  tends to raise consumer prices (*a la* Balassa-Samuelson effects?) along with wage rates. The prices that may have been decreased in the short run must start to rise as the PPF shifts out with a particular skew, strictly steeper (and linear) than ever.

#### 10) Long-Run Results C: Mature Stage

However, consumer prices will eventually peak out as growth continues with rising wages, consumer prices must start falling thereafter.

### 6. Conclusion

Probing the classic SST within the confines of a simple diaganomics has led to some interesting discoveries or rediscoveries of fundamental principles of economics revealing the importance of being different not only in technology (human capability as well), but also in tastes. The importance of being unimportant in income distribution is another discovery under well-defined conditions. Our inquiry also revealed why free trade is such a difficult proposition *not only in practice*, but also more disturbingly *in theory*. We suggested that the S-S suggestion to "bribe the poor" or the rich might not be a good idea. We also briefly examined the Jensen-Larsen models of economic growth and trade. We find that there is nothing that keeps wages from rising **unless** checked by free trade with the rest of the world. But the rest of the world being endowed with abundant labor in the scale of billions perhaps, the rich working class in the highly developed countries could be one of the stumbling blocks to free trade. However, the factor price equalization won't materialize without free trade and/or mobile factor movement, which may be either resented by vested interest groups in both developed and developing countries or else simply averted in considerations of various risk and uncertainty related to health, security, mores, and more.

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