Schooling inequality and the rise of research*

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Abstract

During the last twenty years the share of researchers in the workforce has been rising in OECD countries. The consistency of this pattern suggests that it is not a transitional phenomenon. This paper demonstrates that the rise of research can occur in the steady state when schooling inequality is declining. Comparative static analysis of a semi-endogenous growth model with a continuous distribution of skills shows that a reduction in skill inequality can have a variety of effects, which includes a rising share of researchers.

Additionally, the height of the growth rate of mean educational attainment is shown to have a positive effect on the proportion of researchers in the workforce, without causing it to grow.

Keywords: Schooling inequality; Economic growth
JEL classification: O40, I20, J24

1 Introduction

The number of researchers in the OECD has been growing at a higher rate than the OECD's workforce. Figure 1 displays summary statistics on the number of researchers measured in FTE relative to total employment for the last twenty years. A clear upward trend is visible in both the mean and median. Jones (2002) has argued that the upward trend is a reflection of transitional dynamics stemming from the rise in the years of schooling (figure 2). This paper proposes an alternative explanation for the rise of research, which builds on another empirical trend: the decline in schooling inequality. Figure 3 displays the evolution of the proportions of the population that have primary, secondary, and tertiary education as the highest completed level of education. Both the rise of tertiary education and the decline of primary education have contributed to a greater equality in educational attainment. The decline in schooling inequality has also been reported by

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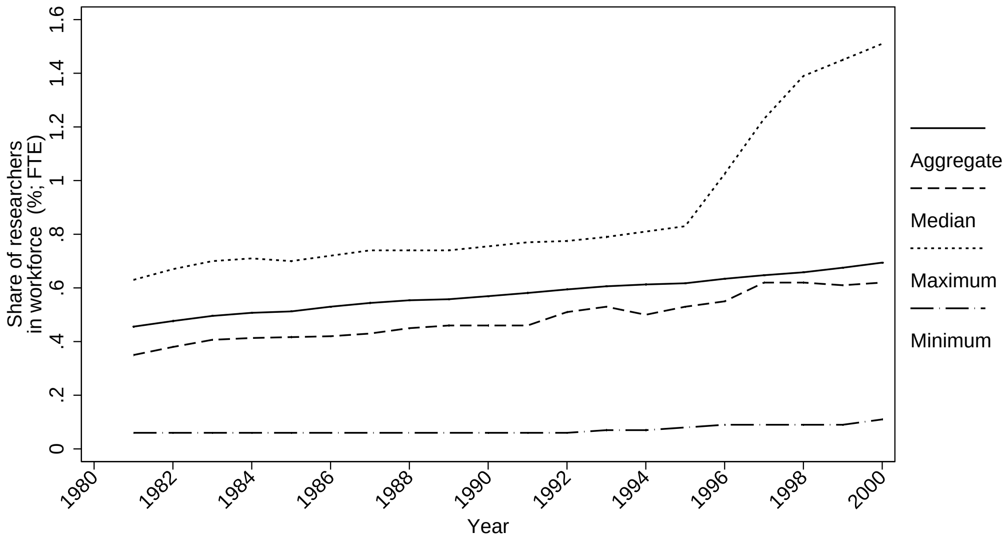

Figure 1. Number of researchers (FTE) in percentage of total employment; data have been interpolated at the country level
Sources: OECD (MSTI); World Bank (WDI)

Ram (1990), who finds that there exists an inverse relationship between schooling inequality and the average years of schooling if the average years of schooling in a country exceeds seven.

This paper addresses the consequences of the decline in schooling inequality on research and economic growth. A simple semi-endogenous growth model is presented in which workers differ in their education, or, more precisely, they differ in their skills. The model deviates from other models of economic growth in that it avoids the customary low-skill – high-skill dichotomy. Instead, skills are continuously distributed over workers. There are two kinds of jobs in the economy: jobs in the production sector and jobs in the research sector. In both types of jobs, workers with higher skills are more productive and receive a higher income. What distinguishes the two jobs from each other is that productivity is more sensitive to skills in research than it is in production. In other words, the 'superstar-effect' is stronger in research (Rosen 1981).

A change in the distribution of skills influences the proportion of researchers in the workforce through two channels. First, a change in the skill level of a worker changes the comparative advantage she has in her current job. A worker employed in the production sector, for example, might be more inclined to choose a job in research if her skills increase. I will refer to this as the 'skill' effect.

Second, a change in the distribution of skills will alter the amounts of aggregate human capital available for production and for research. In general this will lead to either over- or under-investment in research, such that the wage rates of both sectors will change. The change in wages induces a change in the proportion of researchers. I will refer to this as the 'wage' effect.

A mean-preserving change in the distribution of skills will trigger both 'skill'

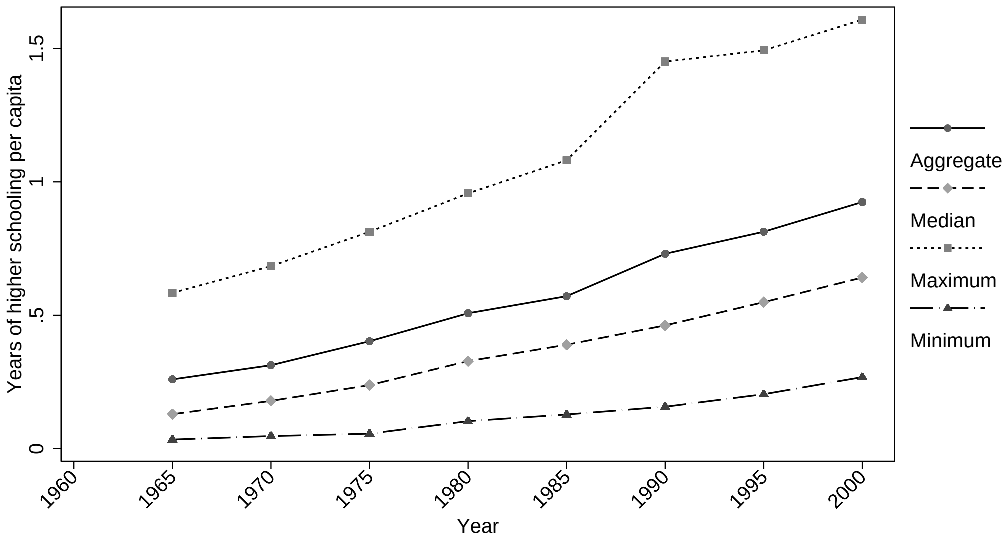

Figure 2. Years of higher schooling per capita (OECD)
Sources: Barro and Lee (2000); World Bank (WDI)

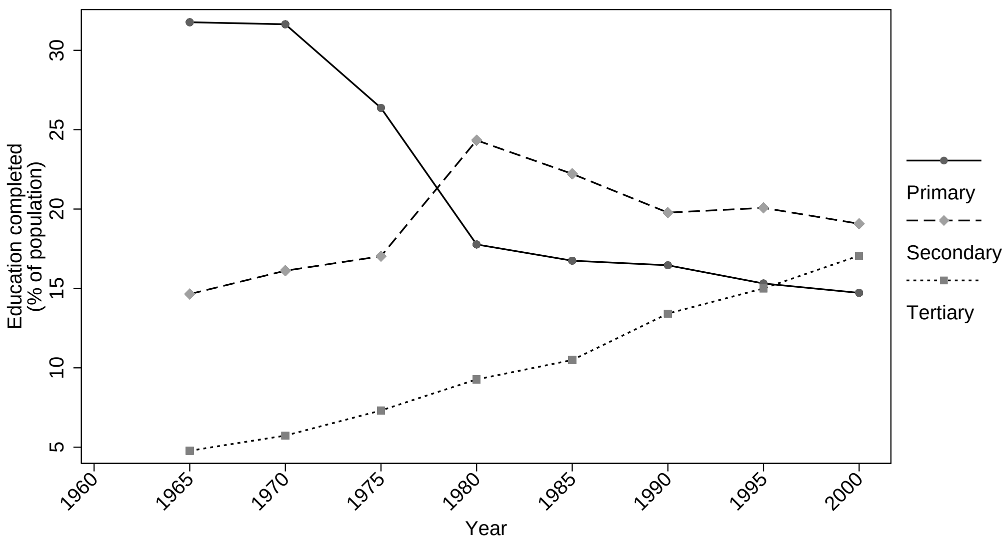

Figure 3. Proportion of population above 25 with primary, secondary, or tertiary eduction as the highest level of eduction completed (OECD)
Sources: Barro and Lee (2000); World Bank (WDI)

and 'wage' effects. The direction of the two effects depends on parameter settings. Furthermore, the effects may or may not work in the same directions, such that a change in schooling inequality can have a variety of steady state effects – amongst which a rise in the proportion of researchers.

When parameters are such that a reduction in inequality leads to a rise in the proportion of researchers, a higher level of welfare is not guaranteed. Although the reduction in inequality might induce more workers to become a researcher, it also has a negative effect on their productivity – provided that average skills are kept constant. The latter effect follows almost by definition: a mean-preserving decline in skill inequality can only be achieved by 'transferring' skills from high skilled workers to low skilled workers.

A second result is related to the growth in the average years of education. Advances in schooling induce economic growth, not only in the trivial way of raising productivity per worker, but also because they stimulate research in the same way as population growth does. A higher rate of growth in the average years of schooling leads to a higher – but constant – share of researchers in the workforce. Because of the latter effect, consumption per capita grows at a higher rate than the productivity of production workers does. This results has been established earlier by Arnold (1998).

The setup of the model presented below differs from that of Jones (2002) in several respects. First, the model allows for changes in the distribution of skills whereas all workers are equally capable in Jones' model. Second, Jones treats the process of education in some detail, while here educational attainment is exogenous.[1] Third, Jones does not provide a micro-foundation for his model. Finally, Jones takes into account knowledge spillovers.

With regard to the explanation for the rising share of research in employment, this paper deviates from Jones' paper in that a constant growth rate in average education does not cause the proportion of researchers to grow over time. Instead, the rising share of research is shown to be a possible consequence of a reduction in schooling inequality.

Besides Jones, several other authors have taken the rise in educational attainment as a starting point for explaining economic growth. The most well-known contributions in this direction are the endogenous growth models by Lucas (1988). After Lucas, the theoretical literature of the 1990s – with the exception of Barro, Mankiw, and Sala-i Martin (1995) – has largely ignored the effects of education on economic growth. Interest in the topic revived with the paper by Bils and Klenow (2000). In the model by Bils and Klenow the capability of teachers is larger when they are better educated themselves. In this way, young teachers will be more capable than old ones and, consequentially, human capital will accumulate and productivity will grow. The model by Jones builds on that of Bils and Klenow but is augmented with intertemporal knowledge spillovers.

As the current paper deals with the consequences of changes in the distribution of skills, it is also related to the literature on skill biased technological change and to the literature on job assignment and occupational choice. The first strand of

[1] Appendix A treats endogenous education.

literature has been surveyed by Acemoglu (2002). Although the model presented here does show that a rise in the 'college-premium' can be a consequence of a more equal distribution of skills, it does not really fit into this literature because it does not incorporate skill biased technological change. Laitner (2000) studies the relation between the distribution of 'natural abilities' and wage inequality using a model with unbiased – but exogenous – technological change.

The assignment of jobs to workers is very simple in the model presented below. There are just two kinds of jobs and skills are one-dimensional. The literature on assignment and occupational choice contains many more advanced configurations (Sattinger 1993). The approach followed by Teulings (1995) is particularly interesting in the context of the model presented here. In his model both the complexity of jobs and the skills of workers have a continuous distribution.

After the basic model has been presented in section 2, its steady state will be solved for in section 3. Particular attention is given to the effects of growth in the average skill level. Section 4 contains a discussion of how changes in the distribution of skills affect the job choice of individual workers as well as the economy as a whole. Section 5 summarizes the findings.

2 The model

Let $\mathcal{L}$ be the set of all workers and order the workers according to their skills. Skills, denoted by k, are one-dimensional, implying that they reflect some general notion of intelligence or capability. The workers are indexed from 0 to L, where L is both the worker with highest skills and the total amount of labor. The skills of worker i depend on his relative ranking, i/L, and on the parameters s and σ ($s, \sigma > 0$).

$$k(i) = (\sigma + 1)\, s\, (i/L)^{\sigma} \tag{1}$$

Integration of $k(i)$ over $\mathcal{L}$ shows that s is simply the average level of skills.

$$\frac{1}{L}\int_0^L (\sigma + 1)\, s\, (i/L)^{\sigma}\, \mathrm{d}i = \left[s\frac{i^{\sigma+1}}{L^{1+\sigma}} \right]_0^L = s \tag{2}$$

The chosen specification of k has the advantage that changes in σ do not affect average skills. The shape of the distribution of k can be altered by varying σ without having to worry about changes in the mean of the distribution. This distribution can be derived in a straightforward manner by solving equation 1 for i/L, which yields the cumulative distribution, and differentiating the result with respect to k.[2]

$$\frac{i}{L} = \frac{1}{[(\sigma+1)\, s]^{\frac{1}{\sigma}}} k^{\frac{1}{\sigma}} \tag{3}$$

$$f(k) = \frac{1}{\sigma\,[(\sigma+1)\, s]^{\frac{1}{\sigma}}} k^{\frac{1-\sigma}{\sigma}} \tag{4}$$

[2]The way in which the distribution of skills is modelled resembles the specification of the distribution of jobs in Dupuy and Marey (2004).

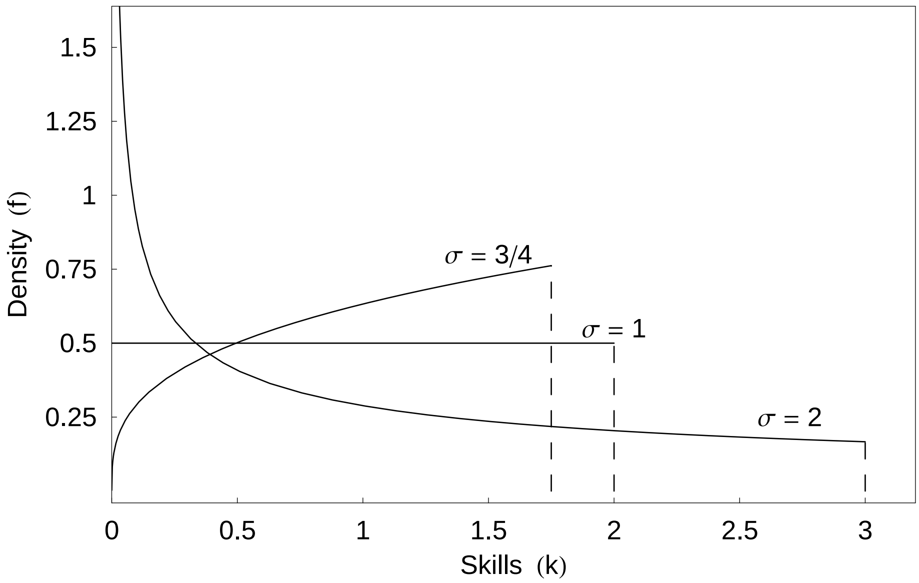

Figure 4. Density plots for skill distributions

In this last expression, f is the density function. Figure 4 shows the distributions of k for three values of σ, having $s = 1$. The figure illustrates that although the function in 1 is quite simple, it still allows for a reasonable flexibility in the distribution of k.

A larger σ causes the maximum skill level to increase, even though s is fixed. A rise in σ therefore widens the gap between the workers with minimal skills and the workers with maximal skills. This follows directly from $k(L) = (\sigma + 1)s$. Besides affecting the domain of k, σ also affects its variance.

$$\operatorname{var}(k) = \int_0^{(\sigma+1)s} k^2 f(k)\,\mathrm{d}k - s^2 = \frac{\sigma s^2}{2 + \sigma^{-1}} \tag{5}$$

The variance of k is clearly increasing in σ. A rise in σ not only widens the skill-gap between workers 0 and L, it also raises the variance of the skill distribution. These two properties of the skill distribution make σ a reasonably appropriate measure of inequality and whenever I mention 'inequality' below, I will implicitly refer to σ.

Consumers maximize the discounted stream of instantaneous utility using the subjective discount rate ρ.

$$\max_{\{c_t\}_0^\infty} \left\{ \int_0^\infty \ln(c_t) \exp[-\rho t]\,\mathrm{d}t \right\} \tag{6}$$

Here, instantaneous utility is assumed to equal the log of a consumption index c. Consumers have the usual CES preferences over n symmetric goods. The quantity of each good consumed by household j is $x(j)$, such that household j's consumption equals $c(j) = n^{\frac{1}{\gamma}} x(j)$, where γ determines the elasticity of substitution. Because consumption is homogenous of degree one in quantities, we can write the aggregate consumption index, C, as a function of the total quantity that is produced of each type, x.

$$C = n^{\frac{1}{\gamma}} x \tag{7}$$

The production of x requires an amount of human capital equivalent to H_x/n (human capital will be defined later on). Aggregate consumption is therefore a function of n and H_x.

$$C = n^{\frac{1-\gamma}{\gamma}} H_x \tag{8}$$

The flow of new goods depends on the amount of human capital available for research, H_n.

$$\dot{n} = H_n \tag{9}$$

Entry into the research sector is free, meaning that the value of an invention, v, equals the wage rate per unit of human capital, w_n. Research is funded through the savings of consumers, who in return get a share of the profits, π, that an invention generates. The rate of return on investing in research is π/v. The aggregate Ramsey rule that follows from utility maximization is therefore given by

$$\hat{C} = g_L + \frac{\pi}{v_n} - \rho. \tag{10}$$

Here, $\hat{C}$ is the growth rate of consumption and g_L is the (exogenous) growth rate of the workforce (a hat denotes a growth rate; g is reserved for fixed growth rates).

There are two types of jobs in the economy: research jobs and production jobs. Workers may freely choose which type of job they take, but are assumed to choose the job that gives them the highest income. Although skills are both valuable in research and production, they are appreciated differently. The skills of person i allow either for a production of $h_x(i)$ consumption goods or for the invention of $h_n(i)$ new product designs. Their exact specifications are

$$h_x(i) = ak(i)^{\alpha} = a\tilde{s}^{\alpha}(i/L)^{\sigma\alpha} \tag{11}$$
$$h_n(i) = bk(i)^{\beta} = b\tilde{s}^{\beta}(i/L)^{\sigma\beta} \tag{12}$$
$$\tilde{s} \equiv (\sigma+1)s,$$

where $a, b > 0$ and $\beta > \alpha \geq 0$. The latter condition ensures that the marginal importance of skills is higher for researchers than for production workers. With this setup, the people with relatively high skills will end up in research.[3] What remains to be determined – and this amounts to solving the model – is what level of skills marks the border between production workers and researchers.

[3]The model can be extended with other types of jobs. The inclusion of, for example, managerial or professional jobs avoids the implication of the model that all the smart people end up doing research. This modification is unlikely to lead to qualitatively different outcomes.

The worker that is indifferent between a production job and a research job is indexed L_x, such that the workers 0 through L_x produce consumption goods and the workers L_x through L invent new products. The worker that is indifferent between production and research, must earn the same income with both kinds of jobs.

$$w_x h_x (L_x) = w_n h_n (L_x) \tag{13}$$

Here, w_x is wage rate per unit of human capital in production, and w_n is the wage rate per unit of human capital in research. After substitution for h_x and h_n, the ratio of the wage rates can be seen to be related to the allocation of labor.

$$\frac{w_x}{w_n} = \frac{b\tilde{s}^{\beta} (L_x/L)^{\sigma\beta}}{a\tilde{s}^{\alpha} (L_x/L)^{\sigma\alpha}} = \frac{b}{a}\tilde{s}^{\beta-\alpha} \left(\frac{L_x}{L}\right)^{\sigma(\beta-\alpha)} \tag{14}$$

The aggregate amounts of human capital can be found by integration over the appropriate range of the labor force.

$$H_x = \int_0^{L_x} a\tilde{s}^{\alpha} (i/L)^{\sigma\alpha} \, \mathrm{d}i = \frac{a\tilde{s}^{\alpha}}{\sigma\alpha+1} \left(\frac{L_x}{L}\right)^{\sigma\alpha+1} L \tag{15}$$

$$H_n = \int_{L_x}^{L} b\tilde{s}^{\beta} (i/L)^{\sigma\beta} \, \mathrm{d}i = \frac{b\tilde{s}^{\beta}}{\sigma\beta+1} \left(1 - \left(\frac{L_x}{L}\right)^{\sigma\beta+1}\right) L \tag{16}$$

(The constants of integration have been set to zero for convenience.)

The profit value ratio, π/v, follows from the zero profit condition in research, $v = w_n$, and the part of consumption that is being paid out as dividends.

$$\frac{\pi}{v} = \frac{(1-\gamma) C}{n w_n} = \frac{1-\gamma}{\gamma} \frac{w_x}{w_n} \frac{H_x}{n} \tag{17}$$

After substitution for the ratio of wage rates and human capital employed in production, the profit value ratio becomes

$$\frac{\pi}{v} = \frac{(1-\gamma) b\tilde{s}^{\beta}}{\gamma (\sigma\alpha+1)} \left(\frac{L_x}{L}\right)^{\sigma\beta+1} \frac{L}{n}. \tag{18}$$

Using this last expression the Ramsey rule can be formulated in terms of L_x/L and L/n.

$$\hat{C} = \frac{(1-\gamma) b\tilde{s}^{\beta}}{\gamma (\sigma\alpha+1)} \left(\frac{L_x}{L}\right)^{\sigma\beta+1} \frac{L}{n} + g_L - \rho \tag{19}$$

Equation 8 yields another expression for the growth rate of consumption.

$$\hat{C} = \frac{1-\gamma}{\gamma} \hat{n} + \hat{H}_x \tag{20}$$

The growth rates of $\hat{H}_x$ and n can be obtained from 15 and 9 together with 16.

$$\hat{H}_x = \alpha g_s + (\sigma\alpha + 1)\,\hat{L}_x - \sigma\alpha g_L \tag{21}$$
$$\hat{n} = \frac{H_n}{n} = \frac{b\tilde{s}^{\beta}}{\sigma\beta+1}\left(1 - \left(\frac{L_x}{L}\right)^{\sigma\beta+1}\right)\frac{L}{n} \tag{22}$$

In the first expression, g_s is the exogenous growth rate of average skills (endogenous skill growth is discussed in the appendix). Substitute for $\hat{n}$ and $\hat{H}_x$ to get the growth rate of consumption in terms of L_x/L and L/n.

$$\hat{C} = \frac{(1-\gamma)\,b\tilde{s}^{\beta}}{\gamma\,(\sigma\beta+1)}\left(1 - \left(\frac{L_x}{L}\right)^{\sigma\beta+1}\right)\frac{L}{n} + \alpha g_s + (\sigma\alpha+1)\,\hat{L}_x - \sigma\alpha g_L \tag{23}$$

Together, equations 19, 22, and 23 provide sufficient information to study the dynamic behavior of the model.

3 Steady state

Before we proceed with the analysis of the dynamic properties of the model, let us first rephrase the condensed model formed by equations 19, 22, and 23 in order to reduce its complexity. Define $\Lambda \equiv L_x/L$ and $\lambda \equiv s^{\beta}L/n$. It turns out to be that the steady state of the model coincides with constant values for Λ and λ.

$$\hat{C} = \frac{(1-\gamma)\,b\,(\sigma+1)^{\beta}}{\gamma\,(\sigma\alpha+1)}\Lambda^{\sigma\beta+1}\lambda + g_L - \rho \tag{24}$$
$$\beta g_s + g_L - \hat{\lambda} = \frac{b\,(\sigma+1)^{\beta}}{\sigma\beta+1}\left(1 - \Lambda^{\sigma\beta+1}\right)\lambda \tag{25}$$
$$\hat{C} = \frac{(1-\gamma)\,b\,(\sigma+1)^{\beta}}{\gamma\,(\sigma\beta+1)}\left(1 - \Lambda^{\sigma\beta+1}\right)\lambda + \alpha g_s + (\sigma\alpha+1)\,\hat{\Lambda} + g_L \tag{26}$$

After substituting out $\hat{C}$ and solving for $\hat{\Lambda}$, we obtain a system of two equations in Λ and λ.

$$\hat{\Lambda} = \frac{(1-\gamma)\,b\,(\sigma+1)^{\beta}}{\gamma\,(\sigma\alpha+1)}\left(\left(\frac{1}{\sigma\alpha+1} + \frac{1}{\sigma\beta+1}\right)\Lambda^{\sigma\beta+1} - \frac{1}{\sigma\beta+1}\right)\lambda - \frac{\alpha g_s + \rho}{\sigma\alpha+1} \tag{27}$$
$$\hat{\lambda} = \beta g_s + g_L - \frac{b\,(\sigma+1)^{\beta}}{\sigma\beta+1}\left(1 - \Lambda^{\sigma\beta+1}\right)\lambda \tag{28}$$

The steady state of this system is characterized by a constant share of production workers in the labor force, Λ, and a constant λ. Setting $\hat{\lambda} = 0$ in 28 and $\hat{\Lambda} = 0$ in 27 yields the steady state value of λ as functions of Λ^*, the steady state value of Λ (steady state levels carry a star).

$$\lambda^* = \frac{(\sigma\beta + 1)(\beta g_s + g_L)}{b(\sigma + 1)^\beta} \left(1 - \Lambda^{*\sigma\beta+1}\right)^{-1} \tag{29}$$

$$\lambda^* = \frac{[(1-\gamma) g_L + \gamma\rho + (\gamma\alpha + (1-\gamma)\beta) g_s](\sigma\alpha + 1)}{(1-\gamma) b(\sigma+1)^\beta} \Lambda^{*-\sigma\beta-1} \tag{30}$$

The first expression has been used to simplify the second expression. Equate both expressions for λ^* to get a solution for Λ^* and, after substitution of Λ^*, a solution for λ^* as well.

$$\Lambda^* = \left(\frac{\Theta}{1+\Theta}\right)^{\frac{1}{\sigma\beta+1}} \tag{31}$$

$$\lambda^* = \frac{(\sigma\beta + 1)(\beta g_s + g_L)}{b(\sigma+1)^\beta}(1+\Theta) \tag{32}$$

$$\Theta \equiv \frac{\sigma\alpha + 1}{\sigma\beta + 1}\left(1 + \frac{\gamma(\alpha g_s + \rho)}{(1-\gamma)(\beta g_s + g_L)}\right) \tag{33}$$

The steady state growth rate of consumption can be retrieved either by substituting for λ^* in equation 24 using 30 or by substituting for λ^* in equation 26 using 29.

$$g_C = \frac{1}{\gamma} g_L + \frac{\gamma\alpha + (1-\gamma)\beta}{\gamma} g_s \tag{34}$$

As was to be expected of a semi-endogenous growth model, the growth rate of consumption in the steady state depends on the growth rate of the population. A novel feature here is that consumption growth also depends on the growth rate of average skills. A substantial difference with the standard semi-endogenous growth model is that steady state economic growth is also feasible in the absence of population growth. This is also reflected by the fact that Λ^* is smaller than one if population growth is zero but skill growth is positive (see equation 31). Even when the population is fixed, researchers are employed and new products are introduced to the market. This is why the steady state growth rate of consumption is higher than the rate of productivity growth in the production sector as long as $g_s > 0$ (remember $\beta > \alpha$). However, as both population growth and skill growth are exogenous, the label 'semi-endogenous' is still appropriate. A similar result has been found by Arnold (1998).

The solution for g_C in 34 could also have been found using a shortcut. The steady state growth rates of $\hat{H}_x$ and $\hat{n}$ are given by

$$g_{H_x} = \alpha g_s + g_L \tag{35}$$

$$g_n = \beta g_s + g_L. \tag{36}$$

Applying these growth rates to equation 20 immediately yields the steady state growth rate of consumption. Above expressions clearly illustrate that growth in average skills raises both the productivity of production workers and researchers. By doing so, advances in education affect economic growth in much of the same way as population growth does.

4 Schooling inequality

We have seen that a change in the parameter σ alters the shape of the skill distribution without affecting its mean. The change in the shape of the distribution is such that a rise in σ always widens the gap between the minimum and the maximum skill level, while simultaneously increasing the variance of the skill distribution. A reduction in σ has the reverse effect. This property of σ makes it a suitable measure of schooling inequality – at least within the context of the model.

The steady state values of λ and Λ have been derived in the previous section. Equations 31, 32, and 33 show that the steady state values are dependent on σ: schooling inequality matters for the amount of research being done as well as the number of product types available for consumption. The fact that σ occurs several times in each of these equations indicates that the impact of a change in σ is quite complex. Below we will analyze the effects of a change in σ on the steady state in three steps.

In the first step it will be shown how σ affects the kind of job – production or research – that is preferred by worker L_x, while keeping the wage rates constant. This effect of σ on the labor market is labelled the 'skill' effect. With the second step it is shown how the wage rates will adjust after the 'skill' effect has taken place. The adjustment of the wage rates naturally causes workers to reconsider their job choice. This second effect is labelled the 'wage' effect. The third and last step involves the effect of σ on product variety given that both the 'skill' and the 'wage' effect have taken place and the proportion of production workers has reached the steady state.

The three steps do not reflect the transitional dynamics of the model and are only used to make the comparative static effects of a change in σ more insightful.

The first step starts by analyzing how the skills of an individual worker are affected by inequality, after this the effects on his income are discussed. A change in the shape of the skill distribution may have a positive or a negative effect on the skills of person i, depending on his ranking. The skills of person i will increase in response to a rise in σ if the following condition holds:

$$i/L > \exp\left[\frac{-1}{\sigma+1}\right] \tag{37}$$

This condition is obtained by differentiating equation 1 with respect to σ. A graphical representation is given in figure 5.

A change in skills affects the amount of human capital a worker can supply to production or research. A worker will be more inclined to do research if $w_x \mathrm{d}h_x < w_n \mathrm{d}h_n$ and he will be more inclined to take a production job if $w_x \mathrm{d}h_x > w_n \mathrm{d}h_n$ (we keep wages fixed for the moment). The change relative attractiveness of the jobs can be found by differentiating equations 11 and 12 with respect to $k(i)$. The condition below marks the skill level at which $w_x \mathrm{d}h_x = w_n \mathrm{d}h_n$.

$$k(i) = \left(\frac{\alpha a}{\beta b}\frac{w_x}{w_n}\right)^{\frac{1}{\beta-\alpha}} \tag{38}$$

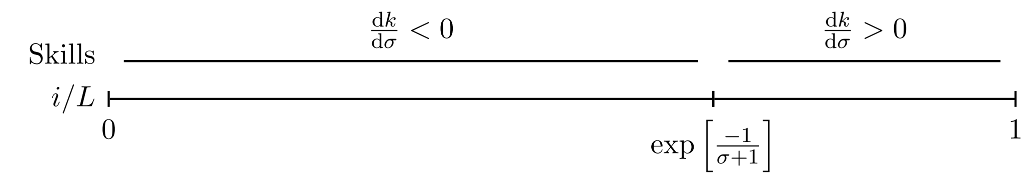

Figure 5. Change in the skills of worker i due to a change in inequality

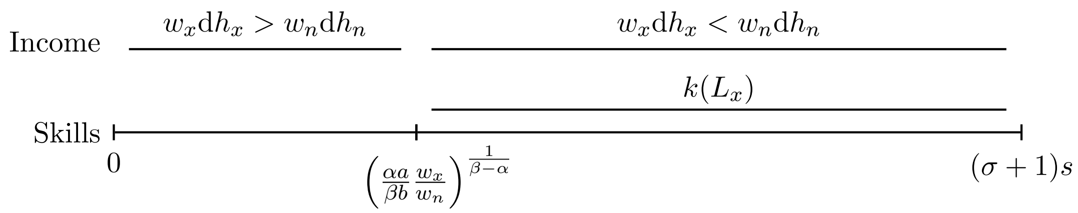

Figure 6. Effect of skill change on income (constant wages) and skill domain of worker L_x

A skill level that exceeds this value will encourage workers to do research. If the skills of a worker are lower than this value, a marginal increase in skills will raise the attractiveness of a production job. Low-skilled people benefit from a higher skill level because it makes them more productive in their current occupation. Their productivity as a researcher remains very low, causing their wage gap between production and research to widen and not to reduce. The reverse applies to high skilled production workers. A rise in their skills will reduce the difference between their current income and the income that they would earn in research. The first line in figure 6 shows how the attractiveness of a job depends on the skills of the worker.

In general, a change skills can either raise or lower the attractiveness of a job in research, depending on the skill level of the worker. However, there is only one worker that might actually switch jobs: worker L_x. Can we be more specific about the incentives faced by L_x? Fortunately, we can. Use equation 14 to solve for $k(L_x)$ as a function of the ratio of wages and compare the outcome with the skill level for which $w_x \mathrm{d}h_x = w_n \mathrm{d}h_n$.

$$k(L_x) = \left(\frac{a}{b}\frac{w_x}{w_n}\right)^{\frac{1}{\beta-\alpha}} > \left(\frac{\alpha a}{\beta b}\frac{w_x}{w_n}\right)^{\frac{1}{\beta-\alpha}} \tag{39}$$

This leaves us with the clean result that if the skills of worker L_x increase, then he will choose to be a researcher; if they decrease, he will choose a job in production. This is the 'skill' effect: after a change in σ, worker L_x can improve his income by switching jobs because his skill level has changed. Figure 7 contains a graphical representation of this result. The domain labeled 'Production' is where worker L_x chooses a production job; the domain labeled 'Research' is where he chooses to become a researcher.

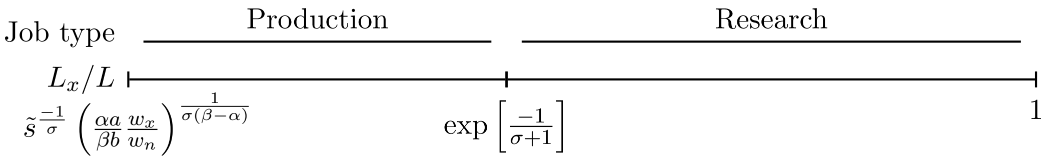

Figure 7. Type of job chosen by worker L_x in response to a rise in inequality (constant wages)

So far, we have analyzed the effects of a change in inequality keeping wage rates constant. However, a change in inequality is unlikely to leave wage rates unaffected. The underlying reason is that a change in inequality will affect both types of human capital. First, there is a direct effect through the presence of σ in equations 11 and 12. Second, σ is important for the job choice of worker L_x. If H_x and H_n change, then – in general – there will be over- or under-investment in research. When this happens a change in wages is required to bring the economy back to the steady state.

The complexity of the model makes it impractical to discuss the impact of a change in σ on the wages through its effect on H_x and H_n. In stead, I will discuss the change in the wage rates using the ratio of wage bills as this is mathematically more convenient. An expression for the ratio of the wage bills can be derived using the labor market skill equation (14) in combination with the definitions of human capital (11, 12).

$$\frac{w_x H_x}{w_n H_n} = \frac{\sigma\beta+1}{\sigma\alpha+1}\frac{1}{\left(\left(\frac{L_x}{L}\right)^{-\sigma\beta-1}-1\right)} \tag{40}$$

The steady state value of the wage bill ratio follows from substituting L_x/L with Λ^*, which is given by 31.

$$\left(\frac{w_x H_x}{w_n H_n}\right)^* = 1 + \frac{\gamma\left(\alpha g_s+\rho\right)}{\left(1-\gamma\right)\left(\beta g_s+g_L\right)} \tag{41}$$

The convenient property of the wage bill ratio is that it is independent of σ in the steady state. A change in σ will therefore only have temporary effects on the wage bill ratio.

If we differentiate the log of the wage bill ratio in equation 40 with respect to σ while keeping L_x/L constant, we find that the sign of this derivative depends on L_x/L.

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln\left(\frac{w_x H_x}{w_n H_n}\right) = \frac{\beta}{\sigma\beta+1} - \frac{\alpha}{\sigma\alpha+1} + \frac{\beta\ln\left[\frac{L_x}{L}\right]}{\left(\frac{L_x}{L}\right)^{-\sigma\beta-1}-1} \tag{42}$$

The precise value of L_x/L for which $\frac{\mathrm{d}}{\mathrm{d}\sigma}\ln\left(\frac{w_x H_x}{w_n H_n}\right) = 0$ can be found by numerically

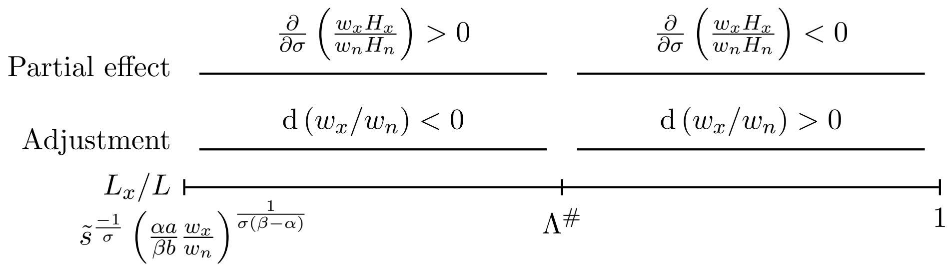

Figure 8. Adjustment of wage ratio to steady state

solving the following equation:

$$\frac{\beta}{\sigma\beta+1} - \frac{\alpha}{\sigma\alpha+1} = \frac{-\beta \ln\left[\frac{L_x}{L}\right]}{\left(\frac{L_x}{L}\right)^{-\sigma\beta-1} - 1}. \tag{43}$$

I will label the solution to this equality $\Lambda^{\#}$. There will be at most one solution as both $\beta \ln\left[\frac{L_x}{L}\right]$ and $\left(\left(\frac{L_x}{L}\right)^{-\sigma\beta-1} - 1\right)^{-1}$ are monotonically increasing in L_x/L. A higher σ causes the wage bill ratio to rise above its steady state value if the proportion of production workers is lower than $\Lambda^{\#}$; the wage bill ratio will decline if $L_x/L > \Lambda^{\#}$. The first line in figure 8 refers to this partial effect of a change in σ on the wage bill ratio.

When the wage bill ratio deviates from its steady state value, an adjustment on the labor market needs to take place to reach the steady state again. Suppose a rise in inequality leads to an increase in the wage bill ratio, then a return to the steady state requires a decrease in the wage bill for production relative to that of research. This can only be accomplished by a drop in w_x relative to w_n. Alternatively, if $L/L_x > \Lambda^{\#}$, then the wage rate for researchers is too high relative to wage rate for production workers. The second line in figure 8 shows how wages adjust to a change in σ.

Above we have first established the effect of a change in σ on job choice keeping wage rates constant. Second, we have established the effect of a change in σ on the wage rates assuming that workers choose their jobs optimally. Combining the two effects allows us to analyze the overall comparative static effects of a rise in σ. As we rely on equation 14 for the analysis of wage adjustment, we will have to start with analyzing the 'skill' effect before we can turn to the 'wage' effect. The overall comparative static effects are summarized in figure 9. The 'skill' effect is shown on the first line, which is identical to figure 7. The second line shows the 'wage' effect assuming that the 'skill' effect has already taken place.

The three dashed arrows represent three scenario's for arriving at a new steady state when inequality increases. The leftmost arrow shows the response of worker L_x if a large part of the workforce is employed in research. First, worker L_x finds out that his skill level is lower, which induces him to take a production job. Second, the new worker L_x is confronted with a decline in w_x/w_n causing him to

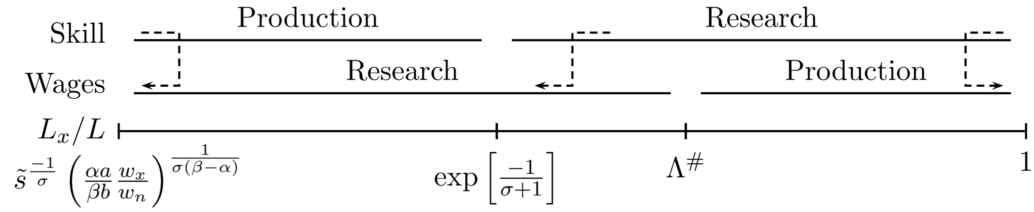

Figure 9. Job choice by worker L_x in response to a rise in inequality: skill and wage effects ($\exp\left[\frac{-1}{\sigma+1}\right] < \Lambda^{\#}$)

become a researcher. Which of the two effects is dominant depends on the precise parameter values. The rightmost arrow describes the opposite situation. Worker L_x experiences a rise in skills and decides to do research. The second worker L_x sees a rise in w_x/w_n and takes a production job.

The middle arrow shows a situation in which the 'skill' and the 'wage' effects work in the same direction. If $\exp\left[\frac{-1}{\sigma+1}\right] < L_x/L < \Lambda^{\#}$ like in figure 9, higher inequality will cause the proportion of researchers in the workforce to increase. If $\Lambda^{\#} < L_x/L < \exp\left[\frac{-1}{\sigma+1}\right]$, then the shift will be towards production.

As discussed in the introduction, figures 1 and 3 show that the rise in the proportion of researchers has coincided with a decline in schooling inequality. In the model a more equal distribution of educational attainment can be simulated by lowering σ. Figure 10 summarizes the effects of a decline in σ on the proportion of researchers. Not surprisingly, the figure is a 'mirror image' of figure 9 as all effects work exactly in the opposite direction. Assuming it is appropriate to consider the proportion of researchers in OECD countries to be 'small' (meaning $L_x/L > \Lambda^{\#}$), the model is able to explain how a reduction in schooling inequality can lead to a rising proportion of researchers.[4] In particular, the comparative static analysis demonstrates that a continuously rising proportion of researchers can be a property of an economy that is in the steady state. A growing proportion of researchers need not be a transitional effect if schooling inequality is also continuously declining. Of course, this can only be true if the 'skill' effect is smaller than the 'wage' effect.

On the rightmost part of the domain sketched in figure 10 the model yields the prediction that a more equal skill distribution raises the wage rate for researcher relative to that of production workers. This result comes very close to what Acemoglu (1998) calls the 'strong induced-bias hypothesis': "... directed technical change can make the long-run relative demand curve [for skilled labor] slope up." (p. 783). Of course, the model presented here differs in one important respect from Acemoglu's approach: it does not involve skill-biased technical change!

[4] The precise condition for $\mathrm{d}\ln\Lambda^*/\mathrm{d}\sigma < 0$ is

$$\left(1 - \frac{\alpha\,(\sigma\beta+1)}{\beta\,(\sigma\alpha+1)}\right)\frac{\Theta^2}{1+\Theta} < -\ln\left(\frac{\Theta}{1+\Theta}\right).$$

This follows from differentiating 31.

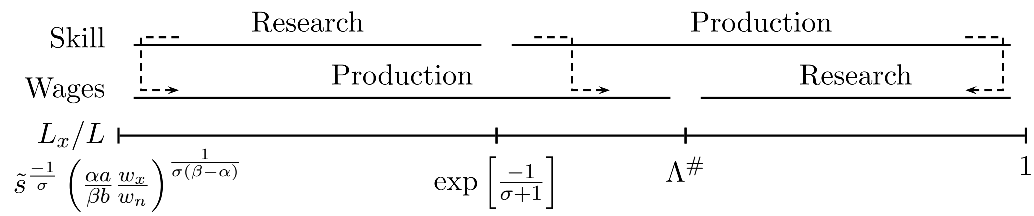

Figure 10. Job choice by worker L_x in response to a decline in inequality: skill and wage effects ($\exp\left[\frac{-1}{\sigma+1}\right] < \Lambda^{\#}$)

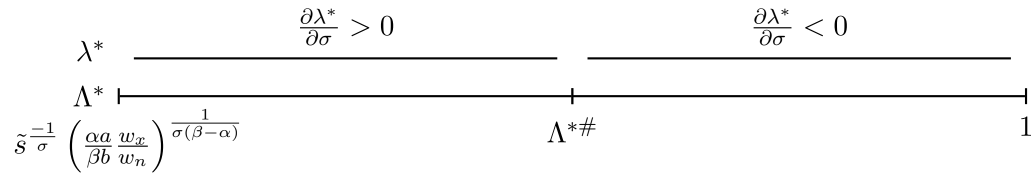

Figure 11. Product variety in the steady state

Having analyzed the effects of schooling inequality on the proportion of production workers, what remains to be done is the analysis of the effects on product variety. Equation 29 shows that λ^* depends on σ both directly and indirectly through Λ^*. The direct effect is due to a change in the average productivity of researchers and is given by the partial derivative of λ^* with respect to σ. Unfortunately, the sign of the partial derivative cannot be found analytically.

$$\frac{\partial \ln \lambda^*}{\partial \sigma} = \left(\frac{\beta}{\sigma\beta+1} - \frac{\beta}{\sigma+1} + \frac{\beta \ln[\Lambda^*]}{\Lambda^{*-\sigma\beta-1}-1}\right)\lambda^* \tag{44}$$

The solution for $\partial \ln(\lambda^*)/\partial\sigma = 0$ is labeled $\Lambda^{*\#}$ and can be found by solving the following equation numerically:[5]

$$\frac{\beta}{\sigma\beta+1} - \frac{\beta}{\sigma+1} = \frac{-\beta \ln[\Lambda^*]}{\Lambda^{*-\sigma\beta-1}-1} \tag{45}$$

Figure 11 shows the domain of Λ^* for which a rise in σ has a positive direct effect on product variety per capita ($\partial\lambda^*/\partial\sigma < 0$) and the domain for which the direct effect is negative ($\partial\lambda^*/\partial\sigma > 0$).

The indirect effect depends on how Λ^* is affected by σ. We have seen above that this relationship is quite complex. Once we know the change in Λ^*, however, remaining part of the analysis is straightforward as $\partial\lambda^*/\partial\Lambda^*$ is always positive. This follows directly from the fact that a larger number of researchers invent a greater number of new products as equation 29 is just a reformulation of $\dot{n} = H_n$.

[5] As equation 45 is similar to equation 43, it is possible to find the condition under which $\Lambda^{\#} < \Lambda^{*\#}$. This condition turns out to be $\sigma < \frac{\beta-\alpha}{\alpha(1-\beta)}$.

We have noted before that schooling inequality has declined in the OECD, while the proportion of researchers has risen. Given these facts, the model predicts that the total effect of the decline in schooling inequality on product variety per capita has probably been positive, provided that the direct effect on λ^* is not too strong if positive.

5 Concluding remarks

It has been demonstrated how economic growth can be affected by changes in the distribution of skills and by growth in the level of education. Although the specification of the model outlined above is still fairly simple, it has proved to be difficult to draw some general conclusions about the effects of a change in the shape of the skill distribution. A closed form solution has been found for the steady state, but whether the influence of a change in schooling inequality on welfare and the share of researchers in the workforce is positive or negative, depends entirely on parameter settings. For certain parameter settings, the model does provide an explanation for some empirical trends. The model shows that a reduction in schooling inequality might raise the proportion of researchers in the workforce, while the wage rate for researchers rises relative to that for production workers.

Advances in schooling induce economic growth, not only in the trivial way of raising a worker's productivity, but also because it stimulates research in the same way as population growth does. Because of the latter effect, consumption per capita grows at a higher rate than the productivity of production workers does.

A Endogenous skill growth

The assumption that the average level of skills grows at an exogenous and constant rate, $\hat{s} = g_s > 0$, has been made for analytical convenience. However, in real life education is not free and therefore growth in skills requires growth in resources devoted to education. This appendix discusses two cases for which constant growth in average skills is feasible in the steady state.

In order to avoid notational changes in the model assume that the population, P, consists of the normal workforce, L, and the part of the population being a teacher or student, P_s ($P = P_s + L$). Furthermore, suppose that the change in average skills is affected by the amount of human capital per capita that is available for education, H_s/P, and by a discount factor, δ. In particular, the change in average skills is given by $\dot{s} = H_s/P - \delta s$. Human capital depends on the people involved in education activities and on their average education, which is assumed to equal that of the population: $H_s = s^{\varepsilon} P_s$. (Better educated teachers will teach more effectively; better educated students will learn quicker.) Substituting for H_s and dividing by s gives an expression for $\hat{s}$.

$$\hat{s} = s^{\varepsilon - 1} \frac{P_s}{P} - \delta \tag{46}$$

Define $\Lambda_s \equiv P_s/P$ and take the growth rate of $(\hat{s}+\delta)$ to get

$$\mathrm{d}\ln(\hat{s}+\delta)/\mathrm{d}t = (\varepsilon-1)\,\hat{s} + \hat{\Lambda}_s. \tag{47}$$

This last expression implies that there can be two specifications that allow for a constant and positive growth rate of skills in the steady state. First, $\varepsilon = 1$ in combination with $\hat{P}_s = \hat{P}$ yields $g_s = \Lambda_s^* - \delta$. With this specification, skill growth stems entirely from the positive effect of skills as an input on skills as an output, while the proportion of people involved in education remains constant. This specification has been proposed by Lucas (1988) and Rosen (1976).

Second, if $\varepsilon < 1$, $\mathrm{d}\ln(\hat{s}+\delta)/\mathrm{d}t$ will go to zero as time proceeds. Setting $\mathrm{d}\ln(\hat{s}+\delta)/\mathrm{d}t = 0$ yields $\hat{s} = \frac{1}{1-\varepsilon}\hat{\Lambda}_s$. Steady state skill growth can only be positive if the proportion of the population active in education is growing, but for this proportion to grow at a constant rate, the population should grow at a different rate than the workforce. If both $\hat{\Lambda}_s$ and $\hat{L}$ are to be constant, the population should grow according to $\hat{P} = g_{\Lambda_s}\frac{P_s}{L} + g_L$.

The results presented above demonstrate that the growth rate of skills can be positive and constant in the steady state, but only under very restrictive assumptions. A more detailed and general treatment of the effects of schooling on economic growth is given by Bils and Klenow (2000).

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