Product Market Competition, R&D Effort and Economic Growth*

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Abstract

Empirical evidence has recently pointed to the lack of any relationship between R&D intensity (variously defined and measured) and economic growth in the post-war period in the United States and other OECD countries. Using a framework that integrates human capital accumulation and purposive (horizontal) innovation activity, this paper looks at product market competition as a possible solution to this puzzle. Indeed, we find that changes in product market competition may well have no influence on human capital investment (the growth engine), while affecting R&D effort.

Key Words: *Endogenous Growth; R&D Investment; Human Capital Accumulation; Product Market Competition.*

JEL Classification: *D43; J24; L16; O31; O41*.

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1. Introduction

Empirical evidence suggests that Research and Development (R&D) activity and human capital accumulation are two of the most important determinants of technological progress and long-run growth.

As far as R&D is concerned, a recent study by OECD concludes: *"…a 0.1 percentage point increase in R&D*[1] *could boost output per capita growth by some 0.2 per cent"* (OECD, 2003, p.89). If correct, this estimate points to the existence of significant externalities from R&D capital.[2] Reflecting the importance of innovation activity in growth, over the last decades many industrialized countries have experienced large increases in R&D employment. For example, the number of scientists and engineers engaged in R&D in the United States was about 500.000 in 1965 and became about one million in 1989. For Japan these numbers are even more compelling: 117.000 in 1965 against about 461.000 in 1989 (see Segerstrom, 1998, Table 1, p. 1292). One problem that arises from these data is that, notwithstanding the relevant impact that R&D seems to have on output per capita growth and the huge amount of resources that most of OECD countries have devoted in the past to such activity, we do not observe any upward trend in growth rates in any of these countries over the long run (Jones, 1995a,b; 2002; 2004).[3] Thus, a still open issue in endogenous growth literature is to understand how we can explain theoretically the lack of any relation between R&D intensity and aggregate economic growth.

As for human capital, instead, the same OECD study mentioned above concludes: *"…the long-run effect on the level of GDP per capita of one additional year of education (corresponding to a rise in human capital by about 10 per cent) ranges between 4 and 7 per cent. These values contrast with many studies that found no or very limited effects of human capital on growth (see, for example, Benhabib and Spiegel, 1994…). …The magnitude of the impact of human capital on growth found in this analysis might be interpreted as suggesting…"* the existence of *"…links between education levels and advances in technology, through which human capital may not only affect the level of long-run output per capita, but may also have more persistent effects on growth"* (OECD, 2003, pp.76 and 78).

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[1] Business R&D in percentage of GDP.

[2] Measuring the social rates of return (spillovers) from R&D activity has proved to be not an easy task. After taking into account all the possible measurement problems, Griliches (1992) concludes that R&D spillovers are not only present, but their magnitude may also be quite large, with social rates of return being significantly above private rates. Nadiri (1993) supports this conclusion and suggests that the social rates of return to R&D average close to 50 per cent.

[3] According to Jones (2004, pp. 41-44): *"[…] A useful stylized fact that any growth model must come to terms with is the relative stability of growth rates in the United States over more than a century. […] This stylized fact represents an important benchmark that any growth model must match. Whatever the engine driving long-run growth, it must […] be able to produce relatively stable growth rates for a century or more. […] This stylized fact is even more problematic for the first-generation idea-based growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). These models predict that growth is an increasing function of research effort, but research effort has apparently grown tremendously over time. […] Between 1950 and 1993, […] research effort rose by more than a factor of eight. […] It also reflects a large increase in the fraction of employment devoted to research. A similar fact can be documented using just the data for the United States, or by looking at spending on R&D rather than employment. The bottom line is that resources devoted to research have exhibited a tremendous amount of growth in the post-war period, while growth rates in the United States have been relatively stable"*.

As a result of the empirical relevance of these links, we now have a number of models focusing on the relationship between R&D investment and human capital accumulation and their impact on economic growth. Notable examples of such models include Ziesemer (1991), Eicher (1996), Redding (1996), Arnold (1998), Blackburn *et al.* (2000), Sjögren (2000), Lloyd-Ellis and Roberts (2002). However, despite the fact that those articles are highly suggestive and represent important attempts to integrate skill accumulation and innovation activity within a unifying framework, our understanding of the possible reasons why R&D effort and per capita growth may appear uncorrelated in the data remains, at most, limited. The purpose of the present paper is to fill this gap in the literature.

In more detail, by combining in the simplest possible way the basic Lucas (1988) framework of human capital accumulation with (a version of) the R&D-based model of Grossman and Helpman (1991, ch.3) with imperfect competition in the product market, the objective of this work is to replicate, on theoretical grounds, the empirical evidence of a lacking link between R&D intensity (measured by the share of human capital devoted to research activity) and economic growth in the U.S. and other major industrialized countries in the second half of the twentieth century.

At this aim we consider an economy with three different productive sectors. An undifferentiated consumption good is produced using the services of a fixed-supply input (say, *land*) and intermediate goods. In order to produce intermediate goods, monopolistic firms employ only human capital. Through purposive R&D activity, technical progress expands the set of horizontally differentiated intermediates. Unlike the traditional R&D-based growth literature, we assume that the total supply of human capital may grow over time. In this respect we postulate the existence of a representative household that chooses plans not only for consumption, but also for skill acquisition. In the model there is no physical capital, and savings are used to finance innovative investments. Population is constant and skilled (each agent is endowed with a certain amount of skills that may grow over time through formal human capital investment). Human capital is a homogeneous input and is totally employed to produce intermediates, to perform R&D activity and to accumulate new human capital.

The main results we obtain are as follows. As in the basic Lucas (1988) model, growth is driven only by skill acquisition. Moreover, a change of the toughness of product market competition (PMC, henceforth), affects the amount of resources (human capital) devoted to research, but not economic growth. Indeed, in the model a decrease of competition increases the investment in R&D activity (this is the traditional *Schumpeterian effect* -Schumpeter, 1942- of less competition in the product market on innovation), but leaves human capital accumulation (the growth engine) unaffected, since agents' incentives to acquire skills are independent of PMC. It is in this specific sense that our model is able to account for the empirical evidence (mentioned earlier in this paragraph) of a rising amount of resources invested in R&D and a simultaneous approximate constancy of economic growth in the U.S. and other major industrialized countries in the second half of the twentieth century.[4]

The rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 presents the general equilibrium solution of it and Section 4 examines its properties along the balanced growth path equilibrium (BGPE, hereafter). In Section 5 we compute the equilibrium growth rate and sectoral distribution of human capital. The main result of the paper on the relationship between PMC, R&D effort and growth is presented in Section 6. Section 7 concludes.

2. The Model

The model economy is composed of a representative household and firms. The representative household consists of one infinitely lived agent being involved in four types of activities: consumption goods production, intermediate goods[5] manufacturing, human capital investment and R&D effort. Population is stationary and consumption goods are produced within a perfectly competitive market in which prices are taken as given and each input is compensated according to its own marginal product. In the intermediate goods sector monopolistic firms produce horizontally differentiated products entering the production function of the homogeneous consumption goods as an input. The household invests a fraction of its fixed time-endowment to acquire skills and at each point in time allocates portions of the available stock of human capital to produce intermediates, to invent new varieties of capital goods (research) and to accumulate new human capital.[6] Purposive R&D activity is the source of technological progress. In this economy technical progress happens through inventing new varieties of differentiated capital goods within a separate and competitive R&D sector. In order to produce new *ideas*, we assume that human capital and the existing stock of knowledge capital (approximated by the number of available capital good

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[4] Today there exists wide evidence in favor of the hypothesis that the relationship between PMC and productivity growth might be positive or, at most, inverse U-shaped at the firm or industry level (see, among others, Geroski, 1995; Nickell, 1996; Blundell *et al*., 1999; Aghion *et al*., 2002). For this reason the *Schumpeterian growth paradigm* has been recently extended along several lines and now we know that many alternative arguments can be put forward in order to explain theoretically why greater competition in the product market is likely to lead (at least up to a given threshold) to a better productivity performance (Aghion, Dewatripont and Rey, 1997 and 1999; Aghion and Howitt, 1996; Aghion, Harris and Vickers, 1997; Aghion *et al*., 2001. See also Aghion and Griffith, 2005 for a concise survey). It is outside the scope of the present article to build an endogenous growth model that reconciles the theory with the empirical evidence on the relationship between PMC and growth, our aim here being, instead, to provide an explanation to the lacking link between economic growth and (increasing) R&D intensity over the long run in many developed countries.

[5] In the remainder of the paper we shall often use such expressions as *intermediate goods*, *intermediate inputs*, *capital inputs*, *capital goods* or simply *intermediates* or *durables*. All these terms will be supposed to have the same meaning.

[6] As it is explained in Barro and Sala-I-Martin (1995, pp.172-173), one can think of the total stock of human capital (say, H) as the fixed size of the total labor force (population in our case, since in the economy under analysis there exists only one infinitely lived representative agent who performs several economic activities at the same time) multiplied by the average level of skill (quality) of the typical worker/member of population. Since H grows only because of improvements in the average skill level, in the rest of the paper we can use the term *skill* as a synonym of *human capital*. Therefore, and as an example, when we say *sectoral distribution of skills* we are, as a matter of fact, referring to the *sectoral distribution of human capital*.

varieties) are combined with constant returns to scale and postulate an R&D production function where technology spillovers, if positive, are incomplete.[7] We focus on this last peculiar hypothesis because it seems to accord well with most of the existing empirical literature[8] and because it allows us avoiding the implausible prediction (that we find in the first-generation R&D-based growth models) of exponentially increasing growth rates if human capital grows perpetually (*strong* scale effect). When a new blueprint is discovered, an intermediate goods producer acquires the perpetual patent over it and, hence, s/he can manufacture the new variety and practice monopoly pricing forever.

The economy under analysis presents two further peculiarities that are worth mentioning here. The first is that each sector employs human capital. More precisely, we assume that this factor input is employed (directly) in the intermediate and R&D sectors and (indirectly, through intermediate inputs) in the consumer goods industry. This is the same hypothesis on the sectoral distribution of (skilled) labor we may find in Grossman and Helpman (1991, ch.3). Furthermore, we hypothesize that the ratio of human to technological capital is constant in the very long run. This assumption, while being in line with the available evidence,[9] allows us to characterize a BGPE where these two forms of capital may grow over time at a common, steady and positive rate.

Besides assuming an R&D technology that displays incomplete knowledge spillovers, the main difference between our model and the path-breaking growth literature with horizontal product innovation (especially Grossman and Helpman, 1991, ch.3) is that in the economy we are going to analyze the supply of human capital (skilled labor) may grow over time. A deeper description of the model economy follows.

Consumer Goods

This sector is competitive and produces a homogeneous, traditional good through the following aggregate production function:

$$Y_t = l^{1-\alpha} \int_0^{N_t} \left(x_{jt}\right)^{\alpha}, \qquad \alpha \in (0;1). \tag{1}$$

[7] In R&D-based growth models technology is essentially envisaged as a *non-rival*, *partially excludable* good. As a non-rival good, it can be accumulated without bound on a per-capita basis, making it possible to generate inter-temporal spillovers. In the present framework, by incomplete (inter-temporal) technology spillovers we mean that, in the absence of another *reproducible* factor input (human capital in the paper), the production of new ideas (starting from the stock of already accumulated technical knowledge) comes ultimately to an end.

[8] See Keely (2001) and Keely and Quah (1998).

[9] See Goldin and Katz (1998). Recently, Caselli (1999) and Helpman and Rangel (1999) have emphasized the educational requirements of the new information technology, whereas Amable (2000) finds that education acts in complementarity with trade specialization in the sense that a sufficiently high level of education of the work force is required to benefit from specialization in electronics and other technologically advanced sectors.

According to this technology, at any time period *t* output (Y_t) is obtained by combining with constant returns to scale a fixed supply input (*e.g.* land, *l*), owned by the representative household, and N different varieties of intermediate inputs, each of which is employed in the quantity x_j. In the model α is a parameter that determines the elasticity of substitution (*e*) between any pair of intermediate inputs, equal to:

$$e = \frac{1}{1-\alpha}.$$

We assume that α is strictly between 0 and 1, which implies that intermediate inputs are imperfect substitutes in production.

Because this industry is populated by a large number of identical and atomistic firms engaging in perfect competition on the product market, in equilibrium each variety of intermediates receives its own marginal productivity:

$$p_{jt} = \alpha(x_{jt})^{\alpha-1}, \qquad \forall j \in (0; N_t). \tag{2}$$

In equation (2), p_{jt} is the inverse demand function faced at time *t* by the generic *j-th* intermediate producer, after normalizing the total amount of services of land (*l*) to one.

Intermediate Goods

The intermediate goods sector consists of monopolistically competitive firms, each producing a differentiated variety *j* with the same technology:

$$x_{jt} = h_{jt}, \qquad \forall j \in (0; N_t).$$

This production function is characterized by constant returns to scale in the only input employed (human capital) and, according to it, one unit of human capital is able to produce (at each time) one unit of whatever variety (*one-for-one* technology).[10] Therefore, for each producer of intermediate goods the marginal cost of production coincides with the wage rate accruing to one unit of human capital. Following Romer (1990) and Grossman and Helpman (1991, ch.3), we continue to assume that each intermediate input embodies a design created in the R&D sector and that there exists a patent law which prohibits any firm from manufacturing any intermediate good without the consent of the patent holder of the design.

[10] Grossman and Helpman (1991, ch.3) assume that intermediate local monopolists use a *one-for-one* technology in raw labor. A *one-for-one* technology in human capital for capital goods manufacturing is postulated by Arnold (1998), p. 85.

The generic *j-th* intermediate firm maximizes (with respect to x_{jt}) the instantaneous profit under the inverse demand constraint (equation 2). From the first order conditions, it is possible to obtain the wage rate accruing to one unit of human capital employed in the capital goods production (w_{jt}):

$$w_{jt} = \alpha^2 \left(x_{jt}\right)^{\alpha-1} . \qquad (3)$$

Since all intermediate good designs provide the same improvement in productivity, we can focus on a symmetric equilibrium where $x_{jt} = x_t$, $\forall j \in (0, N_t)$.[11] Accordingly, each local monopolist faces the same wage rate [$w_{jt} = w_t$, $\forall j \in (0, N_t)$]. Combining equations (2) and (3) yields:

$$p_{jt} = \frac{1}{\alpha} w_{jt} = \frac{1}{\alpha} w_t = p_t \quad , \qquad \forall j \in (0, N_t) . \qquad (4)$$

Hence, when firms producing capital goods are identical, each of them produces the same amount of output, faces the same wage rate accruing to intermediate human capital and fixes the same price for one unit of its own good. This price is equal to a constant *mark-up* ($1/\alpha$) over the marginal cost (w_t).

In the remainder of the paper we use α as a *proxy* for the degree of PMC in the uncompetitive intermediate sector. Indeed, the industrial organization literature (both empirical and theoretical) generally uses the so-called *Lerner index* to gauge the intensity of a firm's *monopoly power* within a market. Such an index equals the ratio of price (P) minus marginal costs (MC) over price. Given the definition of markup (price to marginal costs, *m*), the Lerner index can be written as:

Lerner Index = (P-MC)/P = 1-1/*m*, $\qquad m \equiv$ P/MC = $1/\alpha$.

From the last equation it is possible to conclude that:

(1-*Lerner Index*) = 1/*m* = α .

We see that (1-*Lerner Index*)[12] depends only on *m*: the lower the markup, the lower the monopoly power of a firm and the more competitive an industry. In turn, the markup is lower when the elasticity of substitution between each pair of intermediates is higher and in our model such elasticity depends solely (and positively) on α. Thus, α corresponds to standard measures of competition.

Since in this economy α represents also the share of total output going to capital (goods), looking at this parameter as a measure of competition has the implication that variations in the markup and

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[11] The hypothesis of symmetry is dictated by the way each variety of capital goods enters the final output technology and by the fact that all intermediate producers use the same (*one-for-one*) production function.
[12] This is the same measure of product market competition used in Aghion *et al*. (2002).

variations in the input income shares are strictly and univocally related to each other.[13] However, this is not a novelty in recent economic theory literature. Following Hall (1988) and Galì (1995), other papers that measure the aggregate markup as some function of the input shares in income in monopolistic competition models include Neiss (2001), Cavelaars (2003) and Przybyla and Roma (2005). Moreover, in the first-generation endogenous technical progress growth theory (*e.g.* Romer, 1990), monopolistic intermediate firms choose a markup that is exactly equal to the inverse of the capital share (see Jones and Williams, 2000, p. 68).

Defining by $H_{jt} \equiv \int_0^{N_t} h_{jt} dj$ the total amount of human capital employed in the intermediate sector and using the hypothesis of symmetry across intermediate firms, it is possible to obtain:

$$x_{jt} = \frac{H_{jt}}{N_t} = x_t, \qquad \forall j \in (0, N_t). \tag{5}$$

Given x_t, the instantaneous profit accruing to a generic *j-th* intermediate firm in the symmetric equilibrium is:

$$\pi_{jt} = \alpha(1-\alpha)\left(\frac{H_{jt}}{N_t}\right)^{\alpha} = \pi_t, \qquad \forall j \in (0, N_t). \tag{6}$$

As we would expect, equation (6) states that in the symmetric equilibrium, just as *p* and *x*, so too the instantaneous profit is equal for every variety of intermediates. Also notice that, since we are dealing with a monopolistic competition sector, the profit is decreasing in the number of intermediate producers (*N*).

R&D Activity

Producing the generic *j-th* variety of capital goods entails the purchase of a specific blueprint (the *j-th* one) from the competitive research sector, characterized by the following aggregate technology:

$$\dot{N}_t = bH_{Nt}^{1-\beta} N_t^{\beta}, \qquad b > 0, \qquad \beta \in [0;1), \tag{7}$$

where N_t denotes the number of capital good varieties existing at time *t*, H_N is the total amount of human capital employed in this sector and *b* is a positive productivity parameter. The production function of new ideas we employ here is a variant of the R&D technology used in Jones (1995a) and Arnold

[13] Recent empirical evidence (Galì, 1995, pp.58-60; Bentolila and Saint-Paul, 2003 and Jones, 2003b) points to the presence of substantial differences across countries and over time in the shares of factor inputs in income.

(1998). It displays constant returns to scale in H_{Nt} and N_t jointly considered and states that research human capital (H_N) is an indispensable input for the production of new ideas. The reason why we use this R&D technology is threefold.

First of all, and depending on the strength of knowledge spillovers in the innovation activity (measured by the parameter β), the technology reported in equation (7) allows us to keep two cases potentially distinct.[14] The first one ($\beta = 0$) is the case where there exists no knowledge spillover in the innovation activity and new ideas are obtained *linearly* from human capital input in research (H_N). The Jones (1995a) and Arnold's (1998) specification of the R&D process does not allow taking this particular case into account.[15] The second case is instead the one where $\beta \in (0;1)$. According to Keely and Quah (1998, pp.24-25) and Keely (2001), this is probably the most relevant (both theoretically and empirically), as in real life knowledge spillovers do occur, but are *incomplete* (either at the micro or the macro level). Indeed, when $\beta \neq 0$, equation (7) above highlights very well this idea of positive, but incomplete knowledge spillovers.

The second reason, related to the previous one, why we use the R&D technology of equation (7) is that, when β is positive and lower than one, that equation continues to capture a *"crowding effect"* in research: increases in R&D human capital raise the number of inventions made in the time unit, but less than proportionally (the R&D technology is strictly concave in H_N). Many theoretical as well as empirical works have recently put this last feature of the innovative activity forward.[16]

Finally, as it will be clear in a moment, the R&D technology we use in this paper, together with the hypothesis that the ratio of human to knowledge capital is constant in the very long run and our assumption about the human capital accumulation technology (to be introduced shortly), allows us solving for a BGPE where the amount of human capital employed in each production sector (H_j and H_N) grows over time at a common, constant and positive rate (given by the growth rate of the aggregate stock of this factor input). In other words, equation (7) allows us to analyze the long run predictions of an endogenous

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[14] The Jones (1995a) and Arnold's (1998) R&D technology is of the form: $\dot{A} = \frac{A^{\chi} H_A^{\psi}}{a}$, with $\psi \in (0,1)$, $\chi \in [0,1)$, and where a is a positive constant, A denotes the number of intermediates producible at time *t* and H_A is the human capital input in research (see Arnold, 1998, p. 85, equation 3). According to Arnold (1998, p. 85, footnote 4): "…*It can be shown that if the R&D technology is homogeneous it must either have the Cobb Douglas form…or else reveal constant returns to scale…In order to avoid case distinctions, we, like Jones (1995), restrict attention to the Cobb Douglas case*". Contrary to these two very influential contributions, in this paper we want to focus our attention on a constant returns to scale R&D technology just because we are interested in maintaining the two cases discussed in the main text separate.

[15] When $\beta = 0$, the R&D technology of equation (7) coincides with the one used by Grossman and Helpman (1991, ch.3, pp. 43-57) and Funke and Strulik (2000, p. 494) in their respective endogenous growth models *without* knowledge spillovers. Since $\psi \in (0,1)$, this specific situation cannot be analyzed by Jones (1995a) and Arnold (1998).

[16] See, among others, Kremer (1993), Jones (1995a) and Stokey (1995).

growth model where $g_{H_j} = g_{H_N} = g_N = g_H \equiv g$ (with g_M denoting the growth rate of variable *M* and g being a positive constant, to be endogenously determined).[17] In turn, when this equality holds, then each economic sector that employs human capital receives a constant *share* of this factor input and it is exactly the aim of this paper to analyze the correlation between R&D effort (the share of human capital going to innovation activity) and economic growth in the long run (when both these two variables are supposed to be constant).

As a final comment, it is worth pointing out that using the R&D technology of equation (7) in a model where economic growth is sustained by innovative activity and the total amount of human capital is exogenously given may be rather problematic, since either when $\beta = 0$ or $\beta \in (0;1)$ it would imply the cessation of growth in the log run. Such an outcome cannot occur in this paper, since the engine of growth is human capital accumulation.

Given that the research sector is competitive, new firms will enter it until all profit opportunities are completely exhausted. Accordingly, the static zero profit condition amounts to setting:

$$\frac{1}{b}\left(\frac{H_{Nt}}{N_t}\right)^{\beta} w_{Nt} = V_{Nt} \tag{8}$$

$$V_{Nt} = \int_t^{\infty} \exp\left[-\int_t^{\tau} r(s)ds\right]\pi_\tau d\tau, \qquad \tau > t. \tag{9}$$

Symbols used in equations (8) and (9) have the following meaning: w_N is the wage rate accruing to one unit of human capital employed in research activity; the term $\exp\left[-\int_t^{\tau} r(s)ds\right]$ is a present value factor which converts a unit of profit at time τ into an equivalent unit of profit at time *t; r* is the real rate of return on the consumers' asset holdings (to e defined in a moment); π_j is the profit accruing to the *j-th* intermediate producer (once the *j-th* infinitely-lived patent has been attained) and V_N is the market value of one unit of research output (the generic *j-th* idea allowing to produce the *j-th* capital good variety). Notice that V_N is equal to the discounted value of the profit flow a local monopolist can potentially earn from *t* to infinity and coincides with the market value of the *j-th* intermediate firm (this must be so because in the model there exists a one-to-one relationship between number of patents and number of capital good producers).

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[17] It is easy to show that, with an R&D technology of the form: $\dot{N} = bN^{\chi}H_N^{\psi}$, $\chi \in [0,1)$ and $\psi \in (0,1)$, an equilibrium where $g_{H_N} = g_N \equiv g$ is constant does exist if and only if $\psi = (1-\chi)$. In sum, our model modifies the Jones (1995a) and Arnold's (1998) R&D technology so as to allow for such equilibrium to exist.

Households

We consider a closed economy where an undifferentiated final good can be consumed only. The economy under analysis is composed of a representative infinitely-lived household that owns assets in the form of ownership claims on firms and chooses plans for consumption (*c*), asset holdings (*a*) and human capital (*h*). The household has unit measure and there is no population growth.[18] This hypothesis implies that, at each time *t*, the household's own stock of human capital (*h*) equals the aggregate stock of this factor input (*H*). The household also owns the available amount of land (*l*), which is used just to produce the homogeneous final output and whose (fixed) supply was normalized to one. It sells the services of this input to the competitive consumption good firms and receives, as a price, its own marginal productivity. Following Lucas (1988), we also assume that the representative household is endowed with one unit of time and optimally allocates a fraction *u* of this time endowment to productive activities (research and capital inputs production) and the remaining fraction (1-*u*) to non-productive activities (education). Given the household's choice of the optimal *u*, the labor market clearing conditions determine the decentralized allocation of the productive human capital between manufacturing of intermediate goods and invention of new ideas (research).

With an instantaneous utility function $u(c_t) = \log(c_t)$, the decision problem of the household can be stated as follows:

$$\underset{\{c_t, u_t, a_t, h_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-\rho t} \log(c_t) dt \ , \qquad \rho > 0 \tag{10}$$

s.t.:

$$\dot{a}_t = r_t a_t + w_t u_t h_t + p_{lt} - c_t \tag{11}$$

$$\dot{h}_t = \delta(1-u_t)h_t - \phi h_t \ , \qquad \delta > 0 \ , \qquad \phi \in (0,1) \ , \qquad 0 \le u_t \le 1 \qquad \forall t \tag{12}$$

a_0, h_0, and p_{l0} are given.

The choice variables of this problem are c_t and u_t, whereas a_t and h_t are the state variables. Equation (10) is the household's inter-temporal utility function; equation (11) is its budget constraint and equation (12) represents the human capital supply function. The other symbols used in equations (10) through (12) are the following: ρ is the positive subjective discount rate; *r* is the real interest rate and *w* is the wage rate accruing to one unit of human capital; p_l is the price accruing to the household from selling the services of the fixed-supply input (land) to downstream firms; δ is a constant parameter reflecting the productivity of the education technology and ϕ denotes the constant human capital

[18] The introduction of exogenous population growth would not alter the main results of the model.

depreciation rate.[19] Since human capital is homogeneous (and, hence, accrues the same reward across sectors), in equation (11) we denoted the wage rate going to one unit of human capital at time *t* simply by w_t (without any subscript indicative of the sector where that unit of human capital is actually employed). Moreover, as many other models, in equation (12) we continue to assume that the education technology is linear in the available stock of human capital (*h*). While being aware of the so called *"linearity critique"* (see Stiglitz, 1990; Solow, 1994; Cannon, 2000; Jones, 2003a, 2004), we may easily justify this assumption on several grounds:[20]

> *"…In some cases this assumption is justified by reference to externality effects which convert diminishing returns at the individual level to constant returns at the aggregate level…. In others it is motivated by the inclusion of a broader set of inputs (aside from just time spent on education and training) in human capital production…. And in others, still, it is merited by appealing to an overlapping generations economy in which offspring inherit at least some fraction of the human capital of their parents…"* (Blackburn *et al.*, 2000, p. 195).

For our purposes, it is most straightforward to think in terms of the first alternative above. In other words, we consider the variant of the basic Lucas model in which the possible spillovers from education are internalized. In the present context this is definitely plausible since we are considering the case where there exists only one household (of unit measure) in the economy and population is stationary.

With λ_{1t} and λ_{2t} denoting respectively the shadow prices of the household's asset holdings and human capital stock, the first order conditions of the representative household's problem read as:

$$\frac{e^{-\rho t}}{c_t} = \lambda_{1t} \tag{13}$$

$$\lambda_{1t} = \lambda_{2t}\frac{\delta}{w_t} \tag{14}$$

$$\lambda_{1t} r_t = -\dot{\lambda}_{1t} \tag{15}$$

$$\lambda_{1t} w_t u_t + \lambda_{2t}\left[\delta(1-u_t) - \phi\right] = -\dot{\lambda}_{2t}\,. \tag{16}$$

Conditions (13) through (16) must satisfy the constraints (11) and (12), together with the two transversality conditions: $\lim_{t\to\infty}\lambda_{1t}a_t = 0$ and $\lim_{t\to\infty}\lambda_{2t}h_t = 0$.

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[19] According to equation (12) human capital may be accumulated devoting man-hours to formal education activities. Thus, the depreciation of the human capital stock in the schooling technology can be thought of as including the potential losses from skill deterioration (Barro and Sala-I-Martin, 1995, p. 173).

[20] A linear human capital accumulation technology is also present, among others, in Arnold (1998, p.85, equation 1), Blackburn *et al*. (2000, p. 196, equation 9) and Funke and Strulik (2000, p. 494, equation 5).

3. General Equilibrium Analysis

In order to solve for the general equilibrium of the model, we use the symmetry hypothesis - $x_{jt} = H_{jt} / N_t = x_t$, $\forall j \in (0, N_t)$ - and, for notation simplicity, drop the index *t* on the variables depending on time. Next, for given u^* (the optimal fraction of human capital that the household devotes to production activities[21]), the equilibrium allocation of human capital between capital inputs production (H_j) and research (H_N) is found by solving the following two-equations system:

$$H_j + H_N = u^* H \tag{17}$$

$$w_j = w_N . \tag{18}$$

Equation (17) is the market clearing condition for human capital, whereas mobility of this factor input across sectors implies the equalization of its wage rate (equation 18). In addition, as the total value of the household's assets must equal the total value of firms, the following condition must also be checked in a symmetric equilibrium:

$$a = NV_N , \tag{19}$$

where V_N is given by equation (9) and satisfies the following asset-pricing equation:

$$\dot{V}_N = rV_N - \pi_j \tag{19a}$$

with:

$$\pi_j = \alpha \frac{p_l}{N} , \tag{19b}$$

and

$$p_l = (1-\alpha) N \left(\frac{H_j}{N} \right)^{\alpha} , \qquad l \equiv 1 . \tag{19c}$$

In the model, one new *idea* allows a new intermediate firm to produce one new variety of capital goods. In other words, there exists a one-to-one relationship between number of *ideas*, number of capital good producers and number of intermediate input varieties. This explains why, in equation (19), the total value of the household's assets (*a*) is equal to the number of profit-making intermediate firms (*N*) times the market value (V_N) of each of them (equal, in turn, to the market value of the corresponding *idea*). On the other hand, equation (19a) suggests that the interest on the value of the *j-th* intermediate firm (rV_N) must be equal, in equilibrium, to the sum of two terms:

- the instantaneous monopoly profit (π_j) coming from the production of the *j-th* capital input;

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21 u^* will be endogenously determined in the next paragraph.

- the capital gain or loss matured on V_N during the time interval *dt* ($\dot{V}_N$).

Finally, it is worth noting that when H_j and *N* grow at the same constant rate (this happens in the BGPE, as we are going to show in a moment) equation (19c) incorporates the Malthusian idea that technological progress (in this model the continuous expansion of *N*) is the only force able to offset the law of diminishing returns in the use of the fixed-supply input (land, in our case). We can now move to a formal definition and characterization of the balanced growth path equilibrium of the model outlined in the previous sections.

4. The Long Run Balanced Growth Path Equilibrium (BGPE)

In this paragraph we restrict our attention to a perfect-foresight balanced growth path equilibrium where the growth rate of any variable depending on time is constant and the value of the ratio $R \equiv H_t / N_t$ remains invariant.

Continuing to define with $g_M \equiv \dot{M}/M$ the growth rate of variable *M* we note immediately that when g_H is constant, *u* is constant as well (see equation 12).[22] This means that the household will optimally decide to devote a constant fraction of its fixed time-endowment to work (u^*) and education (1-u^*) activities along the BGPE.

With *R*, u^* and g_N constant, equation (17) becomes the key one in the analysis. Indeed, under these conditions, H_j / N turns out to be also constant. Using this fact, it is possible to show that the following results do hold along the BGPE (mathematical derivation of such results can be obtained from the author upon request - see *Notes for the Referees not to be published* for details):

$$g_c = g_a = g_{pl} = g_N = g_H \equiv g = (\delta - \phi - \rho) \tag{20}$$

$$r = \delta - \phi \tag{21}$$

$$g_{V_N} = g_\pi = g_w = 0 \tag{22}$$

$$\frac{H_j}{N} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{b}\right)^{\frac{1}{1-\beta}} (\delta - \phi)(\delta - \phi - \rho)^{\beta/(1-\beta)}; \qquad \frac{H_N}{N} = \left(\frac{\delta - \phi - \rho}{b}\right)^{\frac{1}{1-\beta}} \tag{23}$$

[22] As already mentioned, our assumptions on the size of the representative household and the population growth rate imply that $H \equiv h$. Therefore, in the remainder of the paper we can use g_H instead of g_h.

$$u^* = \frac{\rho}{\delta}. \tag{24}$$

Equation (20) states that the balanced growth rate (*g*) is equal to the difference between the productivity of human capital at school (δ), the skill obsolescence rate in the education technology (ϕ) and the subjective discount rate (ρ). This is the common rate at which the household's asset holdings (*a*) and consumption (*c*), the price of the fixed supply input services (p_l), the number of capital good varieties (*N*) and the total stock of human capital (*H*) grow in the long run. According to equation (21), the real interest rate (*r*) is constant. Moreover, along the BGPE the market value of a generic idea (V_N), the profit (π) of the corresponding intermediate firm producing that idea and the wage accruing to one unit of human capital ($w_N = w_j \equiv w$) are also constant. This is written in equation (22). Equation (23) gives the equilibrium values of the constant H_j / N and H_N / N ratios, whereas equation (24) represents the optimal (and constant) fraction of the household's time endowment that it decides to devote to work (u^*) in equilibrium. Given the set of results (20) through (24), it is possible to note that for *g* to be positive the condition $\delta > \phi + \rho$ has to be checked. In turn, when *g*>0, and with $\rho > 0$, the real interest rate (*r*) is positive. Finally, when $\delta > \phi + \rho$ and with $\beta \in [0;1)$, $\alpha \in (0;1)$ and *b*>0, the two ratios H_j / N and H_N / N are both positive. Since $\phi > 0$, the condition $\delta > \phi + \rho$ also assures that $0 < u^* < 1$.

5. Economic Growth and the BGPE Distribution of Human Capital across Sectors

We now use the model developed in the previous sections to compute the output growth rate of this economy and to analyze the distribution of human capital across economic activities in the symmetric, balanced growth path equilibrium. At this aim, we first rewrite equation (1) as:

$$Y_t = l^{1-\alpha} N_t \left(\frac{H_{jt}}{N_t} \right)^{\alpha} = \Psi N_t, \qquad \Psi \equiv l^{1-\alpha} \left(\frac{H_{jt}}{N_t} \right)^{\alpha}.$$

Then, taking logs of both sides of this expression and totally differentiating with respect to time, we obtain:

$$\frac{\dot{Y}_t}{Y_t} \equiv g_Y = g_c = g_a = g_{p_l} = g_N = g_H \equiv g = (\delta - \phi - \rho). \tag{25}$$

Thus, as in the basic Lucas' model (1988), output growth depends exclusively on human capital accumulation. This result derives from our definition of the BGPE as an equilibrium where the ratio of human to technological capital is constant. To find out the equilibrium value of such a ratio, we plug equations (23) and (24) into (17) and obtain:

$$R_t \equiv \frac{H_t}{N_t} = \frac{\delta}{\rho}\left(\frac{\delta-\phi-\rho}{b}\right)^{\frac{1}{1-\beta}}\left[\frac{\delta-\phi-\rho(1-\alpha)}{(\delta-\phi-\rho)(1-\alpha)}\right] = R\,, \qquad \forall t\,. \tag{26}$$

In the expression above the human to technological capital ratio (*R*) has been obtained as a function of the productivity parameter of the human capital accumulation technology (δ), the constant obsolescence rate of skills (ϕ), the productivity parameter of the knowledge capital accumulation process (*b*), the subjective discount rate (ρ), the inverse of the mark-up charged over the marginal cost by each capital good producer (α) and β (which measures the strength of knowledge spillovers from technological capital in the innovation activity).

Summing-up, along the BGPE we see that:

- the growth rate of all variables depending on time is constant (in particular, we have $g_Y = g_c = g_a = g_N \equiv g = \delta - \phi - \rho$);
- the amount of human capital devoted respectively to intermediate inputs production (H_j) and to research (H_N) also grows at the common and constant rate $g \equiv g_{H_j} = g_{H_N} = g_H = g_N = \delta - \phi - \rho$;
- the rental price of the fixed supply input (p_l) grows at rate *g*. This happens because in the long run technical progress raises the productivity of such an input and, then, its market price;
- u^* and (1-u^*) are constant, meaning that the household optimally decides to devote a constant fraction of its fixed time-endowment to work and education;
- the real interest rate (*r*), the profit of the *j-th* intermediate firm (π), the market value of the *j-th* idea (V_N), and the wage going to one unit of human capital are also constant.

Given *R*, the shares of human capital devoted respectively to durables production (s_j), research (s_N) and skill acquisition (s_H) in the decentralized BGPE are easily obtained:[23]

$$s_j \equiv \frac{H_j}{H} = \frac{H_j}{N}\frac{N}{H} = \frac{H_j}{N}\frac{1}{\cdot R} = \frac{\alpha\rho(\delta-\phi)}{\delta[\delta-\phi-\rho(1-\alpha)]} \tag{27}$$

23 From equations (27) through (29), it is possible to check that, as we would expect, the following properties do hold in the presence of a positive growth rate (g>0): a) $s_j + s_N = u^* = \rho/\delta$; b) $s_j + s_N + s_H = 1$; c) $0 < s_j, s_N, s_H < 1$.

$$s_N \equiv \frac{H_N}{H} = \frac{H_N}{N}\frac{N}{H} = \frac{H_N}{N}\frac{1}{R} = \frac{\rho(\delta-\phi-\rho)(1-\alpha)}{\delta[\delta-\phi-\rho(1-\alpha)]} \tag{28}$$

$$s_H = 1-(s_j+s_N) = 1-u^* = \frac{\delta-\rho}{\delta}. \tag{29}$$

Looking at equations (27) through (29), we conclude that in the model the equilibrium distribution of human capital across sectors is, among other factors, also influenced by the degree of PMC in the intermediate sector (1-*Lerner Index* $\equiv \alpha$). However, this variable does not affect the output growth rate, g_Y. As a consequence, in this economy changes of α may well have in the long run a bearing on the economy-wide R&D effort (and, more generally, on the distribution of human capital across productive sectors, s_j and s_N), but not on economic growth. This is what we analyze in the next section in more depth.

6. Product Market Competition, R&D Effort and Economic Growth

The analysis of the last paragraph allowed us to detect a variable (PMC, α) able to affect R&D effort (s_N), but not economic growth. Since in this paper we want to explain why R&D intensity has increased so much in the last decades in many industrialized countries with per capita growth remaining simultaneously almost constant, the possible change over time of product market competition in the intermediate goods sector becomes a promising solution to this puzzle.

All the results stated up to now have been obtained under the assumption that δ is strictly greater than $(\phi+\rho)$. As already mentioned, this hypothesis guarantees that the balanced growth rate (*g*) is positive. In the present section, while continuing to keep this assumption, we study how the degree of PMC in the intermediate sector affects the shares of human capital devoted to each sector and the aggregate growth rate of output in our model economy. The results are summarized in the next table:

	α		g	s_N	s_j	s_H
$\forall \alpha \in (0,1)$	$\uparrow$		**0**	**-**	**+**	**0**

Table 1: *Comparative statics results*

The table above shows that an increase of PMC in the intermediate sector (an increase of α) has a positive impact on the share of human capital devoted to the production of capital goods (s_j) and a negative one on the share of human capital devoted to research (s_N). We also see that the same increase of PMC leaves unaffected both economic growth (g_Y) and the amount of resources going to the formation of new human capital (s_H). Hence, we can state the following:

PROPOSITION:

Within an integrated growth model of deterministic and horizontal R&D activity with incomplete knowledge spillovers and human capital accumulation where economic growth is sustained by a supply function of human capital à la Lucas (1988), an increase in the degree of product market power ($1/\alpha$) increases unambiguously R&D effort (s_N), while leaving aggregate economic growth (g) unchanged.

Proof:

From equations (25) and (28), we have $\frac{\partial g}{\partial \alpha} = 0$ and $\frac{\partial s_N}{\partial \alpha} < 0$.

In our paper human capital may be accumulated over time through devoting a fraction of the household's fixed time-endowment to education investment and R&D activity requires (together with the existing stock of knowledge capital) only human capital to run. Consequently, and unlike the contributions by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), we can regard the *share* (instead of the stock) of human capital that the household allocates to innovation at each point in time along the BGPE as a *proxy* for the economy-wide research effort. In this respect our model suggests that, *ceteris paribus*,[24] in the post-war period (1950-1993) there might have been in the United States (and, more generally, in the G5 countries)[25] a decrease of the degree of PMC in the intermediate sector that in those countries led to a rise in R&D incentives (the share of human capital resources allocated to innovation activity) without any concomitant increase in the growth rate of income (driven only by private schooling investment decisions). The empirical test of a similar hypothesis is left to future research.

7. Concluding Remarks

In the second half of the last century the amount of resources devoted to R&D activity has risen considerably in the US and many other industrialized countries, without any simultaneous and proportional increase of the growth rate of output that, instead, in most cases has remained relatively

[24] Namely, for a given productivity of education technology (δ), human capital depreciation rate (ϕ) and time preference rate (ρ).

[25] The evidence of a rising investment in R&D and a simultaneous relative constancy of economic growth is similar also for France, Germany and Japan. See Jones (1995b), pp.516-519, Figures IV and V.

constant (Jones, 1995a,b; 2002; 2004). By considering an endogenous growth model that integrates purposive and horizontal R&D activity with human capital accumulation, this paper provided a possible theoretical answer to such empirical puzzle. Indeed, within a theoretical framework where innovation takes place through an R&D technology that displays constant returns to human capital and the existing stock of knowledge, and where individuals may increase their own level of skills without employing knowledge capital, we found that skill accumulation is the only force driving long term economic growth. Moreover, and under the hypothesis that the ratio of human to knowledge capital is constant along the BGPE, we showed that the degree of competition among intermediate firms plays no role on economic growth, but influences the allocation of the reproducible factor input (skills) across productive sectors (research and intermediate inputs production). In more detail, an increase of the monopoly power enjoyed by uncompetitive producers increases unambiguously the share of human capital resources devoted to R&D without affecting the equilibrium output growth rate. Accordingly, the model predicts that changes in the level of product market competition in the intermediate sector may have represented an element of paramount importance in the explanation of how the rising investment in R&D can be reconciled with the approximate constancy of income growth rates.

However, behind these results two important questions still remain open in the future research agenda. We believe that further empirical research (especially at the macro level) needs to be done in order to shed light on the impact the degree of product market competition (differently defined and measured) may exert on growth, R&D effort and, more generally, on the sectoral distribution of skills in the presence of human capital accumulation. Furthermore, and awaiting for this empirical test, one would analyze how the theoretical findings of the present paper might change in the presence of richer hypotheses on the human capital accumulation process and its interaction with disembodied technological progress.

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Notes for the Referees
(<u>NOT TO BE PUBLISHED</u>)

In these notes we derive the set of results (20) through (24) and equation (26) in the main text. In what follows, we continue to denote by g_M the growth rate of variable *M*. Moreover, our assumptions on the size of the representative household and the population growth rate imply that $H \equiv h$. Hence, we can use interchangeably g_H or g_h.

From equation (12), when g_h is constant *u* turns out to be constant, too. This means that along the balanced growth path equilibrium (BGPE)[26] the household will devote a constant fraction of its own time-endowment to work (*u*) and educational (*1-u*) activities. Consequently, the optimal *u* (which we denote by u^*) will be constant and endogenously determined through the solution to the household decision problem. From equation (17), and with u^*, $g_N \equiv \frac{\dot{N}_t}{N_t} = b\left(\frac{H_{Nt}}{N_t}\right)^{1-\beta}$ and $R \equiv \frac{H_t}{N_t}$ time-invariant, H_{jt}/N_t is constant in equilibrium. In turn, this implies that x is also constant along the balanced growth path (see equation 5 in the main text).

Consider now the representative consumer's problem (equations 10 through 12 in the main text), whose first order conditions are stated in equations 13 through 16 and that we rewrite below for convenience, together with the consumer's constraints and the transversality conditions:

(13) $\quad \frac{e^{-\rho t}}{c_t} = \lambda_{1t}, \qquad \rho > 0$

(14) $\quad \lambda_{1t} = \lambda_{2t}\frac{\delta}{w_t}, \qquad \delta > 0$

(15) $\quad \lambda_{1t} r_t = -\dot{\lambda}_{1t}$

(16) $\quad \lambda_{1t} w_t u_t + \lambda_{2t}[\delta(1-u_t) - \phi] = -\dot{\lambda}_{2t}, \qquad \phi \in (0;1)$

(11) $\quad \dot{a}_t = r_t a_t + w_t u_t h_t + p_{lt} - c_t$

(12) $\quad \dot{h}_t = \delta(1-u_t)h_t - \phi h_t$

a_0, h_0, and p_{l0} are given,

$\lim_{t\to\infty} \lambda_{1t} a_t = 0 \quad$ and $\quad \lim_{t\to\infty} \lambda_{2t} h_t = 0$.

[26] As mentioned in Section 4 of the main text, the BGPE is defined as an equilibrium where the growth rate of any variable depending on time is constant, as well as the ratio of human to technological capital (*R*).

From now on we omit the index *t* near the time-dependant variables. Combining equations 14 and 16 we get:

A) $\frac{\dot{\lambda}_2}{\lambda_2} = -(\delta - \phi)$,

whereas, from (15):

B) $\frac{\dot{\lambda}_1}{\lambda_1} = -r$.

Equation 14 implies:

C) $\frac{\dot{\lambda}_1}{\lambda_1} = \frac{\dot{\lambda}_2}{\lambda_2} - g_w$, or:

D) $r = (\delta - \phi) + g_w$.

Along the BGPE the wage accruing to human capital[27] is constant ($g_w = 0$, see later on in these notes). Accordingly, the real interest rate (*r*) is also constant. With *r* and H_j / N being constant, and using (6) in the main text, equation 9 becomes:

E) $V_N = \alpha(1-\alpha)\left(\frac{H_j}{N}\right)^{\alpha} \int_t^{\infty} e^{-r(\tau - t)} d\tau$, $\tau > t$, $\alpha \in (0;1)$.

Solving the integral above yields:

E') $V_N = \alpha \frac{(1-\alpha)}{r}\left(\frac{H_j}{N}\right)^{\alpha}$.

Such result was obtained under the hypothesis that $r > 0$. In a moment we shall show that this hypothesis is always checked along the BGPE. Equation (E') says that in equilibrium the market value of an *idea* (V_N) is constant.

Given V_N and making use of equation (8) in the main text, w_N (the wage rate accruing to research human capital) is equal to:

F) $w_N = bV_N\left(\frac{H_N}{N}\right)^{-\beta} = b\frac{\alpha(1-\alpha)}{r}\left(\frac{H_j}{N}\right)^{\alpha}\left(\frac{N}{H_N}\right)^{\beta}$.

From equation (3) in the main text we know that in a symmetric equilibrium w_j (the wage rate accruing to human capital employed in the intermediate sector) is:

[27] In equilibrium the wage accruing to the human capital input employed in the intermediate and research sectors is the same.

G) $w_j = \alpha^2 \left(\frac{H_j}{N} \right)^{\alpha - 1}$.

Equating w_N and w_j (see equation 18 in the main text), one can determine the equilibrium constant value of H_j / N :

H) $\frac{H_j}{N} = r \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{1}{b} \right)^{1/(1-\beta)} (g_N)^{\beta/(1-\beta)}$.

Combining equations 13, 15 and (B) in these notes we are able to obtain the usual Euler equation, giving the optimal household's consumption path:

I) $\frac{\dot{c}}{c} \equiv g_c = r - \rho$.

From the equation above we clearly see that r must be greater than ρ (and, then, positive) in order for g_c to be positive.

In the symmetric case (and with $l \equiv 1$), from the final output production function the price (p_l) of the services of the fixed supply input - land - at time *t* can be written as:

(L) $p_l = (1-\alpha) N \left(\frac{H_j}{N} \right)^{\alpha}$ (See also (19c) in the main text).

This implies that:

(L') $g_{p_l} = g_N$.

From equation (19) in the main text and using (E') in these notes:

(M) $g_a = g_N + g_{V_N} = g_N$.

Combining (L') and (M) above, we obtain:

(N) $g_{p_l} = g_a = g_N$.

Using equations (11) and (B) in these Notes, we have:

O) $\frac{\dot{\lambda}_1}{\lambda_1} = -g_a + wu \frac{h}{a} + \frac{p_l}{a} - \frac{c}{a}$.

Instead, from equations (12) and (A) we obtain:

P) $\frac{\dot{\lambda}_2}{\lambda_2} = -g_h - u\delta$.

Equations (F) and (G) together also imply that:

Q) $g_{w_N} = g_{w_j} \equiv g_w = 0$,

whereas employing equations (C), (O), (P) and (Q) above yields:

S) $\frac{c}{a} = wu\frac{h}{a} + \frac{p_l}{a} + \delta u$.

In obtaining this result we also used the fact that $g_a = g_N$ (see equation M above) and that $g_h = g_N$ along the BGPE (where *H* and *N* grow at the same constant rate).

Using equations (Q) and (N), and knowing that: 1) *u* is constant in equilibrium; 2) $g_h = g_N$; 3) a_0, h_0 and p_{l_0} are given constants, (S) leads to the conclusion that *c/a* is constant. In other words:

T) $g_c = g_a = g_{p_l} = g_N$.

Putting equations (T) and (I) together it is possible to obtain:

D') $r = g_N + \rho$,

whereas equating (D') and (D) yields:

Q') $g_w = g_N + \rho - (\delta - \phi)$.

At this point, equating (Q') and (Q), we are able to compute the growth rate of *N* (g_N) along the BGPE:

U) $g_N = g_H \equiv g = (\delta - \phi - \rho)$. (See equation 20 in the main text)

Given g_N , it is now possible to calculate:

T'') $g_c = g_a = g_{p_l} = g_N \equiv g = (\delta - \phi - \rho)$; (See equation 20 in the main text)

D'') $r = \delta - \phi$; (See equation 21 in the main text)

Q'') $g_{V_N} = g_\pi = g_w = 0$; (See equation 22 in the main text)

H') $\frac{H_j}{N} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{b}\right)^{\frac{1}{1-\beta}}(\delta - \phi)(\delta - \phi - \rho)^{\beta/(1-\beta)}$; (See equation 23 in the main text)

$$\frac{H_N}{N} = \left(\frac{\delta - \phi - \rho}{b}\right)^{\frac{1}{1-\beta}} .$$

Notice that for *g* to be positive the condition $\delta > \phi + \rho$ has to be checked. When this condition is met, the real interest rate (*r*) is positive (since ρ >0). In turn, this implies that the market value of one unit of research output (V_N) is positive for each $N > 0$ and $H_j > 0$ along the BGPE (see equation E' above).

To find out the optimal *u* (and denoted by u^*), we combine equations (A) and (P), recalling that $g_h \equiv g_H = g_N$, obtaining:

V) $u^* = \frac{\rho}{\delta}$. (See equation 24 in the main text)

When $g > 0$, and with $\phi \in (0,1)$, δ is strictly greater than ρ, which implies $0 < u^* < 1$. Also note that, under equations (A), (B), (U), (T') and (D") and with $\rho > 0$, the two transversality conditions are trivially checked, since:

$$\lim_{t\to\infty} \lambda_{1t} a_t = \lambda_{10} a_0 \lim_{t\to\infty} e^{-\rho t} = 0;$$

$$\lim_{t\to\infty} \lambda_{2t} h_t = \lambda_{20} h_0 \lim_{t\to\infty} e^{-\rho t} = 0,$$

where λ_{10} and λ_{20} are respectively the given shadow prices of the household's asset holdings and human capital stock at the initial time (t=0).

Finally, using equation 17 in the main text, and with:

- $u^* = \dfrac{\rho}{\delta}$;
- $\dfrac{H_j}{N} = \left(\dfrac{\alpha}{1-\alpha}\right)\left(\dfrac{1}{b}\right)^{\frac{1}{1-\beta}} \cdot (\delta - \phi)(\delta - \phi - \rho)^{\frac{\beta}{1-\beta}}$, and
- $g_N = b\left(\dfrac{H_N}{N}\right)^{1-\beta} \equiv g = \delta - \phi - \rho$,

it is straightforward to obtain (see equation 26 in the main text):

Z) $R_t \equiv \dfrac{H_t}{N_t} = \dfrac{\delta}{\rho}\left(\dfrac{\delta - \phi - \rho}{b}\right)^{\frac{1}{1-\beta}} \left[\dfrac{\delta - \phi - \rho(1-\alpha)}{(\delta - \phi - \rho)(1-\alpha)}\right] = R, \quad \forall t$.