SOFTWARE PRODUCTION AND ENDOGENOUS GROWTH: THEORETICAL ANALYSIS AND EMPIRICAL EVIDENCE FROM INDIA

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Abstract: Propelled by the rise of a vibrant software industry the Indian economy has demonstrated rapid growth since the 1990s. A novel three-sector endogenous growth model that encapsulates the salient features of an information technology oriented economy is developed. The dynamic optimization problem leads to a balanced growth path equilibrium characterized by output, physical capital, software assets, human capital and consumption growing at a uniform rate. Major implications of the model are reflected in empirical evidence from the growth trajectories of Indian states. The human capital production apparatus has a significant impact on economic growth. This has critical policy implications.

Keywords: endogenous growth, India, information technology, human capital, software.

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1. INTRODUCTION

Driven by the advent of a vibrant software services industry the Indian economy has displayed rapid growth since the 1990s. Though the information technology (IT) software sector contributed to less than 3% of Gross Domestic Product (GDP) in 2002-03, it accounted for over 28% of GDP growth between 2000 and 2002 (Athreye, 2005). This enormous impact of a relatively new industry on the Indian economy has engendered considerable interest. Understanding the dynamics of this process is crucial for framing policy initiatives directed at sustaining and accelerating India’s future economic growth. At the same time, lessons from this experience can be extended to address the issue of economic development in other countries. This paper analyses the process of economic growth in the context of software as a distinct production input.\(^1\) A theoretical model is developed to motivate the analysis. Major implications of the theoretical model are examined using empirical evidence from a panel of Indian states.

The literature on economic growth has not adequately examined the impact of software. Most of the existing research examines the role of IT in productivity of conventional sectors or the direct effect on economic growth. Jorgenson and Stiroh (2000) for instance, provide an in-depth analysis of the productivity and growth effects of IT investment in the U.S. Little has been done to identify the dynamics of the process. In particular, the existence of a separate production sector for software has not been conceptualized. Endogenous growth models can lend useful insights into the dynamics of software driven economic growth. Two-sector models of economic growth are quite prevalent since the work of Romer (1986) and Lucas (1988). The former deals with the

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\(^1\) Software refers to a collection of digital instructions that control the processing and flow of information in computers and other electronic devices.
innovation process while the latter models human capital. Three-sector growth models that incorporate three inputs in the final goods sector are rare. Rebelo (1991) uses a third non-reproducible factor in a Cobb-Douglas production function with two other reproducible inputs. The model developed in this paper uses three reproducible factors in a three-sector model of endogenous growth. This paper addresses a significant gap in the literature. It also deals with the practical aspect of explaining India’s rapid economic growth in an innovative and comprehensive manner.

The paper is structured as follows: Section 2 describes the theoretical model. The implications of the model are examined in Section 3. Section 4 discusses the empirical framework and estimates. Section 5 discusses the policy implications and concludes.

2. THE MODEL

This section develops a three-sector model of endogenous growth. The three inputs, physical capital, software and human capital are produced by three distinct sectors. For conceptual clarity a simple closed economy deterministic framework is used. Each sector is assumed to have a separate production function. Following the conventional assumption of most growth models, the goods sector output \( Y \) can be either consumed or installed as physical capital. This output is a homogenous good. All three factors depreciate at a common rate \( \delta \). There are no installation costs. The goods production function uses all three factor inputs, physical capital \( K \), software \( X \) and human capital \( H \). The production function is assumed to be Cobb-Douglas with constant returns to scale as follows:

\[
Y = AK^\alpha X^\beta (zH)^{1-\alpha-\beta}
\]  

(1)
where \( A \) is the technology level of the goods production sector, \( z \) is the proportion of human capital used for producing goods and the parameters of the production function are \( \alpha \) and \( \beta \).

Software is distinct from knowledge, human capital and physical capital. It has special characteristics, which are modeled in a simplified manner. It is assumed that software is non-rival but excludable.\(^2\) In contrast, knowledge is largely non-rival and non-excludable, while physical capital and human capital are rival and excludable. This assumption depicts an important feature of software in the real world. The software producer designs the asset and delivers it to the customer as a flow of services. The producer of the software continues to have access to the ‘blue print’ or ‘source code’ even after the asset has been installed. Therefore, the software firm can still utilize some proportion of the software asset that is available to the customer. Under the assumption that this proportion is a fixed parameter \( (b \leq 1) \), it is subsumed within the general technology parameter for the software sector \( B \) (where \( B=Fb^\psi \) and \( F \) is the original technology level of the software sector and \( \psi \) is the exponent of software input in the software production function described below). Software assets produced for the goods sector then have a correspondence with those available to the software sector. The ‘source code’ from the initial software output produced with human capital alone is instantly used for software production. It is assumed that software production does not require physical capital. Therefore, the software production function is highly intensive in human capital and software. It has the following form:

\[
Y_x = F(bX)^\psi (uH)^{1-\psi}
\]

\[ (2a) \]

\( ^2 \)Software is assumed to be of the ‘closed source code’ type.
or

\[ Y_X = BX^\psi(uH)^{1-\psi} \quad (2b) \]

where \( B = Fb^\psi \) is the technology level of the software sector and \( u \) is the proportion of human capital devoted to this sector.

The human capital sector is assumed to use only human capital as an input in the manner of Lucas (1988). It has the following form:

\[ Y_H = D(1-u-z)H \quad (3) \]

where \( D \) is the technology level of the human capital sector. The economy has identical, everlasting households with no population growth. The households are assumed to own all the productive assets directly. They maximize a constant intertemporal elasticity of substitution utility function subject to the constraints of the equations of motion of the three factor inputs, the initial stocks of inputs \( K(0) = K_0 > 0, X(0) = X_0 > 0 \) and \( H(0) = H_0 > 0 \), and transversality conditions. The optimization problem can then be expressed as follows:

\[
\max U(0) = \int_0^\infty e^{-\theta t} U(C(t)).dt \quad \text{and} \quad U(C) = \frac{C^{1-\theta} - 1}{1-\theta} \quad (4)
\]

subject to

(a) \[ \dot{K} = AK^\alpha X^\beta (zH)^{1-\alpha-\beta} - C - \delta K \quad (5a) \]

(b) \[ \dot{X} = BX^\psi(uH)^{1-\psi} - \delta X \quad (5b) \]

(c) \[ \dot{H} = D(1-u-z)H - \delta H \quad (5c) \]

(d) \[ \lim_{t \to \infty} [Ke^{-r(t)}] \geq 0 \quad (5d) \]

(e) \[ \lim_{t \to \infty} [Xe^{-r(t)}] \geq 0 \quad (5e) \]
(f) \( \lim_{t \to \infty} [H \cdot e^{-r(t) t}] \geq 0 \) \hspace{1cm} (5f)

where \( C \) is consumption, \( \phi > 0 \) is the discount rate, the constant \( \theta > 0 \) has the interpretation of being the inverse of the elasticity of substitution and \( r(t) \) is an average interest rate between times 0 and \( t \).\(^3\)

This dynamic optimization problem has three control variables, consumption \( C \), the share of human capital in the goods sector \( z \) and the share of human capital in the software sector \( u \). The state variables are the stocks of the three factors, physical capital \( K \), software \( X \) and human capital \( H \). In the balanced growth path \( u \) and \( z \) are constant and \( C, K \) and \( X \) grow at constant but not necessarily equal rates.

The standard procedures described in Lucas (1988), Benhabib and Perli (1994), Xie (1994) and Barro and Sala-i-Martin (2004) are used to arrive at the balanced growth path (details in the Appendix). The balanced growth path is characterized by output, physical capital, software, human capital and consumption growing at a common constant rate:

\[
\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{X}}{X} = \frac{\dot{H}}{H} = \frac{\dot{C}}{C} = \frac{1}{\theta} (D - \delta - \phi)
\] \hspace{1cm} (6)

3. IMPLICATIONS AND EVIDENCE

Major implications of the model are reflected in empirical evidence from the growth trajectories of Indian states.\(^4\) The model has several major implications:

(i) Output, physical capital, software assets, human capital and consumption grow at a common rate along the balanced growth path.

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\(^3\) The dot over the letters has the conventional interpretation of the time derivative \( d/dt \).

\(^4\) India has 28 states and 7 union territories. In this paper all these political subdivisions are referred to as states.
(ii) Human capital is the key factor and the level of technology of this sector \( (D) \) determines the steady state rate of growth.

(iii) The model does not predict convergence. Rather it suggests that countries or regions would move along different balanced growth paths with perpetual growth rates determined by the level of educational (human capital) technology.

The software sector in India is at a nascent stage of development. As a result, it is unlikely that the balanced growth path conditions would hold completely at this point of time. Nevertheless, in a transition to this steady state, the growth of output, physical capital, consumption, software assets and human capital can be expected to show some consonance. It is evident from the Indian growth experience that software driven growth shows certain patterns (Arora and Athreye, 2002; D’Costa, 2003; Arora and Gambardella, 2004; World Bank, 2004). The growth of high technology services has been accompanied by increased overall output, urban consumption growth and demand for technical education, at the national and regional level. Regions that are endowed with better educational facilities have generally grown faster. Growth in urban consumption at the regional levels shows a clear correlation with the growth of high technology industries. The growth of business services and physical capital investment also shows correlation with the rise of high technology service industries. While exports were the initial impetus for high technology industries in India, domestic demand is now growing steadily. These patterns appear to reflect the general conclusions of the model.

Growth of the software industry has been accompanied by increased per capita incomes, manufacturing output growth and demand for technical education. Table 1 depicts the correlation between the aforementioned variables. Per capita income growth
has a stronger correlation with software services growth (0.306) than manufacturing sector growth (0.071). The number of engineering colleges per million population and number of engineering students per million population can be considered as proxies of the human capital production technology and human capital output respectively. Both these variables have a strong positive correlation with per capita income growth (0.523 and 0.591). There is a distinct correlation between manufacturing output growth and software service growth (0.211).\(^5\) Software services growth is positively correlated with engineering colleges (0.254) and engineering students (0.291). It appears that the software sector, the manufacturing sector and the technical education sector are propelling each other in a virtuous endogenous growth cycle.

**Table 1. Correlation Matrix of State Per Capita Income Growth, Manufacturing Output Growth, Software Services Growth, Number of Engineering Colleges and Number of Engineering Students.**

<table>
<thead>
<tr>
<th></th>
<th>Per capita income growth</th>
<th>Manufacturing growth</th>
<th>Software services growth</th>
<th>No. of engineering colleges per million population</th>
<th>No. of engineering students per million population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita income growth</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing growth</td>
<td>0.071</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Software services growth</td>
<td>0.306</td>
<td>0.211</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of engineering colleges per million population</td>
<td>0.523</td>
<td>-0.111</td>
<td>0.254</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>No. of engineering students per million population</td>
<td>0.591</td>
<td>-0.021</td>
<td>0.291</td>
<td>0.866</td>
<td>1.000</td>
</tr>
</tbody>
</table>


\(^5\) The weak negative correlation between engineering colleges and the manufacturing sector has a simple explanation. States with larger software industries attract the majority of engineering colleges given the attractive job opportunities due to the presence of the software sector. States with more manufacturing output are not attractive destinations for technical education institutions due to the comparatively unattractive remuneration.
Indian states have not shown convergence tendencies. The southern region, which is abundant in human capital, initially attracted software sector investment and subsequently displayed rapid growth. The northern region where the process of software sector growth started later is now showing rapid growth in the demand and supply of technical education (Arora and Gambardella, 2004). These regional aspects are reflected in Figure 1.

![Figure 1. Relation between regional software growth and technical education. Notes: Growth in software output: Sales growth of major IT companies (author’s calculation based on Centre for Monitoring Indian Economy (2005) data set. Number of engineering colleges: Private engineering colleges based on Arora and Gambardella (2004).](image)

The figure shows that for the period 1995-96, the South and the West had a large number of technical education institutes and this was correlated with the high growth of
software output. In contrast, the North and the East lagged behind in both technical education and software output growth. For the period 2002-03 the South and the West maintained their lead while the North and the East were beginning to catch up in terms of technical education. The North also showed some growth in software output but the East was yet to come on par. This probably indicates that human capital creation precedes increases in software output.

Figure 2. Absence of convergence among Indian states.
Note: Data from Reserve Bank of India (2007).

The lack of convergence among Indian states is clearly reflected in Figure 2. The figure shows that there is a positive correlation between the initial state per capita income

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6 The North comprises Jammu and Kashmir, Himachal Pradesh, Punjab, Haryana, Uttar Pradesh, Uttarakhand, Rajasthan, Bihar, Delhi and Chandigarh. East comprises West Bengal, Orissa, Chhattisgarh, Assam, Sikkim, Arunachal Pradesh, Tripura, Manipur, Meghalaya, Mizoram and Nagaland. West is composed of Gujarat, Maharashtra, Madhya Pradesh, Dadra and Nagar Haveli, Daman and Diu. South is composed of Karnataka, Kerala, Tamil Nadu, Andhra Pradesh, Goa and Pondicherry.
(1993-94) and the growth rate achieved during the period 1993-94 to 2003-04. This is contrary to predictions of the exogenous Solow-Swan growth model but adheres to the implications of the endogenous growth model described in this research.

The model allows us to test one critical aspect of the software driven economy. This is the hypothesis that software is a distinct input. It is this aspect that is tested within an empirical framework in the subsequent section.

4. EMPIRICAL MODEL AND ESTIMATES

4.1 Empirical specification

In this section the impact of software assets \((X)\) on aggregate output \((Y)\) is examined in an empirical framework. The aggregate production function is conceived to have software as an input in addition to physical capital \((K)\) and human capital \((H)\). The aggregate production function is represented as:

\[
Y=f(K,H,X)
\]

This production function is estimated for a panel of Indian states. If the production function is assumed to be Cobb-Douglas, it can be estimated using the following log-linear empirical specification:

\[
\ln Y_{it}=\ln A_{it}+\alpha \ln K_{it}+\beta \ln H_{it}+\gamma \ln X_{it}+\varepsilon_{it}
\]

where \(Y_{it}\) is the aggregate output of the \(i\)th state in time period \(t\). \(A_{it}\) is the technology level, \(K_{it}\) is physical capital, \(H_{it}\) is human capital and \(X_{it}\) represents software assets of a particular state at a specific time period. The general error term is denoted by \(\varepsilon_{it}\). This equation can be estimated using standard panel data estimation techniques. To establish
the robustness of the results Ordinary Least Squares (OLS), Fixed Effects (FE) and Random Effects (RE) estimators are used.

4.2. Data description

The data used for the empirical analysis draws upon, statistical reports published by India’s premier government statistical bureau, the Central Statistical Organisation. The main variables are derived from accounts of the various states and data from the Annual Survey of Industries. The balanced sample has data for 29 states for six years (Financial years 1998-99 to 2003-2004). This leads to 174 observations. The variables are described below:

(i) Aggregate output ($Y_{it}$): Gross State Domestic Product at factor cost is used to represent aggregate state output (Central Statistical Organisation, 2007a).

(ii) Physical capital ($K_{it}$): Fixed capital reported by the Annual Survey of Industries is used to represent physical capital (Central Statistical Organisation, 2007b).

(iii) Human capital ($H_{it}$): Number of workers reported by the Annual Survey of Industries is used to represent human capital (Central Statistical Organisation, 2007b).

(iv) Software assets ($X_{it}$): In the Indian system of statistical accounting the output of software services is represented by the category ‘other services’(Central Statistical Organisation, 2007a). However, this just gives the flow of high-technology services and not the corresponding assets. To convert the service flows to represent assets, the delivered services are capitalized to develop a proxy for software assets. This method follows research undertaken with respect to the accounting of high-technology services in

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7 The Indian financial year commences on the 1st of April of a calendar year and ends on the 31st of March of the following calendar year.

8 The term $H$ is used to denote skilled labor as opposed to unskilled labor, which has the general designation $L$. In the Indian context industrial workers can be considered skilled in comparison with unskilled agricultural and minimum wage day laborers.
the U.S. (Nakamura, 1999; Corrado, Hulten and Sichel, 2005). The flows are capitalized at a rate of 20% with a 10% depreciation rate using the perpetual inventory method. The descriptive statistics for the above variables is given in Table 2.

Table 2. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Aggregate Output</th>
<th>Physical Capital</th>
<th>Human Capital</th>
<th>Software Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3966832</td>
<td>1433323</td>
<td>210255.7</td>
<td>272858.2</td>
</tr>
<tr>
<td>Median</td>
<td>2517674</td>
<td>648638.5</td>
<td>92907.5</td>
<td>172541.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>19219046</td>
<td>8707855</td>
<td>947590</td>
<td>1550777</td>
</tr>
<tr>
<td>Minimum</td>
<td>59815</td>
<td>377</td>
<td>201</td>
<td>4787.728</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4115728</td>
<td>2003543</td>
<td>261085.5</td>
<td>287110.7</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.265408</td>
<td>1.989732</td>
<td>1.424707</td>
<td>1.483404</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.349215</td>
<td>6.518108</td>
<td>3.952829</td>
<td>5.555777</td>
</tr>
</tbody>
</table>

4.3. Empirical estimation results

The empirical estimation results are reported in Table 3. The OLS results are reported first followed by FE and RE estimates.

Table 3. Empirical estimates (Dependant variable is ln $Y_a$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS</th>
<th>FE</th>
<th>RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.12 (20.21)***</td>
<td>Various</td>
<td>5.38 (19.40)***</td>
</tr>
<tr>
<td>ln $K_a$</td>
<td>0.05 (2.84)***</td>
<td>0.003 (0.27)</td>
<td>0.05 (2.92)***</td>
</tr>
<tr>
<td>ln $H_a$</td>
<td>0.16 (5.28)***</td>
<td>0.06 (3.48)***</td>
<td>0.19 (9.58)***</td>
</tr>
<tr>
<td>ln $X_a$</td>
<td>0.75 (28.94)***</td>
<td>0.44 (21.85)***</td>
<td>0.54 (21.91)***</td>
</tr>
<tr>
<td>Adjusted R-sq.</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>F-statistic</td>
<td>2916.91</td>
<td>4238.44</td>
<td>--</td>
</tr>
<tr>
<td>Durbin Watson</td>
<td>0.14</td>
<td>1.28</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Notes: Eviews 4.1 used for estimation. T-statistics in parenthesis. *** denotes significance at 1% level.
The OLS estimates show that the model has a good fit. All the variables are statistically significant. The adjusted R-square is around 0.98. Given the log-linear specification of the model the coefficients represent the elasticity of output with respect to the particular input. The elasticity of output to physical capital is 0.05 approximately. Human capital has a larger effect of about 0.16 and the impact of software is even larger at around 0.75. However, the Durbin-Watson statistic indicates the presence of serial correlation. This is probably due to the presence of cross-section specific effects. As a result, the FE and RE estimates are likely to be more reliable.

The FE estimates show that the effect of physical capital is small and not statistically significant. Human capital is statistically significant and has a coefficient of about 0.06. Software also has a significant impact and a coefficient of around 0.44. The RE estimates are likely to be robust owing to the use of Generalized Least Squares (GLS). These estimates show all the variables to be statistically significant. The elasticity of output with respect to physical capital is around 0.05. For human capital the effect is larger at 0.19. Software assets have the largest impact. A unit increase in software assets leads to an output increase of about 0.54.

The results clearly demonstrate that software assets and human capital have a large and significant effect on aggregate output. This indicates that large variations in output and income levels across countries and regions may be attributable to the use of software and availability of human capital.
5. CONCLUSIONS AND POLICY IMPLICATIONS

This research developed a three-sector endogenous growth model to explain the phenomenon of software driven growth. The model implied that economies would grow in proportion to their educational technology and would not show convergence tendencies. The major implications of the model are supported by data from Indian states. An empirical model is used to test the importance of various inputs including software assets on aggregate output. Panel data evidence from Indian states show that human capital and software assets have a major impact on output.

The theoretical and empirical results of this research have momentous policy implications. With particular regard to the Indian economy it shows that software production and quality technical education have a large impact on state level output and prosperity. It explains the disparities in income and growth among Indian states to a large extent. Policy makers need to heed these warning signs. The process of economic growth in India is clearly leading to significant disparities. This study provides a simple solution to the problem. Greater stress on improvement of educational standards in laggard states together with encouragement of software investment in those regions is likely to counteract rising inequality.

The results of this research can be generalized and extended to other countries and regions. The crucial conclusion of this research is that the digital divide and the educational divide are interlinked. Improving access to both quality education and better information technology assets is a viable formula for addressing the issue of long-term economic growth.
APPENDIX: MATHEMATICAL DERIVATION OF THE BALANCED GROWTH PATH.

The Hamiltonian for the optimization problem is:

\[ J = e^{-\psi}U(C) + v(AK^\alpha X^\beta (zH)^{1-\alpha-\beta} - C - \delta K) + \lambda(BX^\psi (uH)^{1-\psi} - \delta X) \]

\[ + \mu(D(1-u-z)H - \delta H) \]  

(A1)

The first order conditions lead to:

\[ C^{-\theta} = \nu e^{\psi} \]  

(A2)

\[ \frac{\mu}{\lambda} = \frac{B}{D}(1-\psi)u^{-\psi}\varphi^\psi \]  

(A3)

\[ \frac{\mu}{\nu} = \frac{A}{D}(1-\alpha-\beta)z^{-\alpha-\beta}\omega^\alpha\varphi^\beta \]  

(A4)

\[ \cdot \]

\[ \frac{v}{\nu} = -A\alpha z^{-\alpha-\beta}\omega^{\alpha-1}\varphi^\beta + \delta \]  

(A5)

\[ \cdot \]

\[ \frac{\mu}{\mu} = -D + \delta \]  

(A6)

\[ \frac{\lambda}{\lambda} = \left[ -\frac{(1-\psi)}{(1-\alpha-\beta)}\beta z^{-\psi u} \right] Bu^{-\psi}\varphi^{\psi-1} + \delta \]  

(A7)

If the ratios \( \frac{C}{K} = \chi \), \( \frac{K}{H} = \omega \) and \( \frac{X}{H} = \varphi \) are used, the first order conditions lead to the growth rate equations:

\[ \cdot \]

\[ \frac{\dot{C}}{C} = \frac{1}{\theta} \left( A\alpha z^{-\alpha-\beta}\omega^{\alpha-1}\varphi^\beta - \delta - \phi \right) \]  

(A8)

\[ \cdot \]

\[ \frac{\dot{K}}{K} = Az^{-\alpha-\beta}\omega^{\alpha-1}\varphi^\beta - \chi - \delta \]  

(A9)

\[ \cdot \]

\[ \frac{\dot{X}}{X} = Bu^{-\psi}\varphi^{\psi-1} - \delta \]  

(A10)

\[ \cdot \]

\[ \frac{\dot{H}}{H} = D(1-u-z) - \delta \]  

(A11)
This leads to the following system of differential equations:

\begin{align}
\frac{\omega}{\omega} &= A z^{1-\alpha-\beta} \omega^{-1} \varphi^{\beta} - \chi - D(1-u-z) \tag{A12} \\
\frac{\phi}{\varphi} &= B u^{1-\psi} \varphi^{-1} - D(1-u-z) \tag{A13} \\
\frac{\chi}{\chi} &= \frac{\alpha - \theta}{\theta} A z^{1-\alpha-\beta} \omega^{\alpha-1} \varphi^{\beta} + \chi - \frac{1}{\theta} [\delta(1-\theta) + \phi] \tag{A14} \\
\frac{u}{u} &= \frac{D}{\psi} u + \frac{D}{\psi} z^{1-\psi} \beta \varphi^{\psi-1} - \frac{(1-\psi) \beta z}{\psi(1-\alpha-\beta)} \tag{A15} \\
\frac{z}{z} &= 1 - \frac{\alpha}{\psi} \chi - (\alpha + \beta)(1-u-z) + \frac{\beta}{D} B u^{1-\psi} \varphi^{\psi-1} \tag{A16}
\end{align}

To derive the steady state, the time derivatives are set to zero. Solving the five simultaneous equations in five variable leads to the steady state values of \( \chi, \varphi, \omega, z \) and \( u \):

\begin{align}
\chi^* &= D \left( \frac{1}{\alpha} - \frac{1}{\theta} + \pi \right) \quad \text{where} \quad \pi = \frac{1}{D \theta} [\delta(1-\theta) + \phi] \tag{A17} \\
z^* &= \frac{(1-\alpha-\beta)}{(1-\psi)} \left[ \frac{\theta}{1-\pi \theta} - 1 \right] \tag{A18} \\
u^* &= 1 + \pi - \frac{1}{\theta} \left[ \frac{(1-\alpha-\beta)}{(1-\psi)} \left( \frac{\theta}{1-\pi \theta} - 1 \right) \right] \tag{A19} \\
\varphi^* &= \left[ 1 + \pi - \frac{1}{\theta} \left( \frac{(1-\alpha-\beta)}{(1-\psi)} \left( \frac{\theta}{1-\pi \theta} - 1 \right) \right) \right] \left[ \frac{D(\pi + 1/\theta)}{B} \right]^{1/\varphi^{-1}} \tag{A20} \\
\omega^* &= \frac{D \left( \frac{1}{\alpha} \right)}{A \left[ \frac{(1-\alpha-\beta)}{(1-\psi)} \left( \frac{\theta}{1-\pi \theta} - 1 \right) \right]^{1-\alpha} \left[ \left( \frac{1-\pi \theta (1-\psi)}{(\theta (1-\alpha-\beta) - 1)} \right) \left( \frac{D(\pi + 1/\theta)}{B} \right) \right]^{1/\varphi^{-1}}} \tag{A21}
\end{align}
The results in equation (6) can then be arrived at by substituting these steady state values in the growth rate equations (A8-A11) and using the following representation of the growth rate of output which is derived from equation (1):

\[
\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{X}}{X} + (1 - \alpha - \beta) \left( \frac{\dot{z}}{z} + \frac{\dot{H}}{H} \right)
\]  

(A22)

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