Integration, Wage Bargaining, and Growth with Creative Destruction

Tapio Palokangas*
Department of Economics, University of Helsinki
Discussion Paper No. 598:2004
ISBN 952-10-1528-4, ISSN 1459-3696

May 7, 2004

Abstract

We construct a Schumpeterian growth model of a common market with following properties. Households can stay as workers or become researchers at some cost. Workers are employed in production and researchers in *R&D*. Workers are unionized. A larger common market means a wider variety of products and more intensive goods market competition. The main findings are as follows. If the common market is able to carry out extensive labour market reforms, then it should accept new members as long as this increases consumption per capita. If no extensive reforms are feasible, then the common market should respond to excessive union power by accepting new members, which increases competition in the product market.

Journal of Economic Literature: $J50$, $O40$

Keywords: integration, wage bargaining, creative destruction

Corresponding author:

Tapio Palokangas, Department of Economics, P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland. Phone +358 9 191 28735, Fax +358 9 191 28736, Email: Tapio.Palokangas@helsinki.fi

*Financial support from the Yrjö Jahnsson Foundation is gratefully acknowledged.

1 Introduction

Economic integration is commonly known to intensify international competition and to increase the variety of products. Because it affects the position of labour unions in wage bargaining, it may have growth and welfare effects that so far have been neglected in the literature. To examine this, we construct a model of a common market with growth and wage bargaining.

Labour unions and employer federations have two roles which are often mixed in economic debates: (*i*) they bargain over wages and (*ii*) lobby the government for a number of issues (e.g. pension schemes, hiring and firing costs, restrictions on hours of work). To avoid confusion in this matter, this study concentrates wholly on role (*i*) and assumes that labour unions and employer federations try to increase their income through wage settlement. Political lobbying is ignored here and the author considers it elsewhere.[1]

The relationship of labour unions and economic growth may significantly depend on the production structure of the economy. Peretto (1998) examines the growth effects of union bargaining power by a product-variety model. His main result is that a fall in union power promotes *R&D* and growth through a higher profit margin. Peretto however assumes that (*i*) labour is employed only in production, (*ii*) final goods can be directly converted into *R&D*, (*iii*) labour unions completely ignore the effect of their wages on productivity through *R&D*, and (*iv*) union power is exogenous. We, on the contrary, assume that labour is used both in production and *R&D* and unions take into account also the effects through *R&D*. We also examine the case where union power can be altered through a labour market reform.

Grossman and Helpman (1991) (in ch. 4), Aghion and Howitt (1998), and Wälde (1999) examine economic growth from the viewpoint of creative destruction in which firms can step forward in the quality ladders of technology by investment on *R&D*. We take a similar 'Schumpeterian' approach, but instead of a competitive labour market we introduce wage bargaining in which unions and employers observe the effect of wages on both employment and investment.

Expensive labour may give rise to a higher growth rate. Cahuc and

[1] Using a common agency framework, Palokangas (2003) considers unions and employers as lobbies trying to influence the self-interested government.

Michel (1996) (using an OLG model) and Agell and Lommerud (1997) (using an extensive game framework) show that a minimum wage may create an incentive for workers to accumulate human capital. Palokangas (1996, 2000) introduces wage bargaining into Romer's (1990) product-variety model with skilled and unskilled workers. He shows that higher union bargaining power leads to higher wages for unskilled workers, higher unemployment for both skilled and unskilled workers in production, a lower wage for skilled workers. This decreases costs in $R\&D$ and promotes growth. All these models, however, ignore the uncertainty that is embodied in technological change. To eliminate this shortcoming, we use here a model of creative destruction.

The author uses a model of creative destruction for problems of growth and trade also in two other papers. Palokangas (2004a) examines the growth and welfare effects of union power in a model where research firms learn from each other. It shows that the international coordination of labour market policy raises the workers' wages and promotes growth and welfare. Palokangas (2004b) examines the growth and welfare effects of the expansion of common markets when the labour markets are perfect but economies differ in the productivity of $R\&D$. It shows that a small economy with low incentives to save do not growth at all, if left alone, but avoids stagnation if its $R\&D$ is productive enough to join a common market with a positive growth rate. In this study, we introduce labour unions into the production sector. We show that union power affects the integration strategy of the common market.

The remainder of this paper is organized as follows. Section 2 explains the institutional background of the model. The basic structures of the model are presented in section 3. Section 4 considers a household's consumption and saving as a problem of stochastic dynamic programming. It results in the savings and investment functions for the economies.[2] Section 5 examines wage bargaining and Section 6 the growth and welfare effects of integration.

[2]The study focuses entirely on the households' stationary equilibrium in which the allocation of resources is invariable across technologies, and ignores the behaviour of the system during the transitional period before the equilibrium is reached. In this study, the growth model is based on a Poisson process. This means that if the initial state is chosen outside a stationary equilibrium, then the model would most likely generate cycles, which are technically extremely difficult to cope with.

2 The setting

There is a common market with a given number J of member economies. Each economy contains a fixed amount of land and a fixed number of similar households.[3] Competitive firms in the common market produce the consumption good from land and intermediate goods of all member economies. Intermediate-good firms in the common market are subject to oligopolistic competition. They form expectations on each other's responses. Integration increases competition among the intermediate-good firms.

All households are modelled as dynastic families which are risk averters and share identical preferences. The members of such a family can be either *workers*, who are employed in production, or *researchers*, who are employed in *R&D*. Family-optimization considerations determine the evolution of consumption expenditure over time, the allocation of savings across shares in different firms, and the decision whether a family member becomes a researcher or enters the labour force as a worker. A single family takes prices, wages, profits, employment and aggregate labour supply as given. A single firm's technology is a random variable but the probability of its improvement in one unit of time depends on its *R&D*.

The structure of economy $j \in \{1, ..., J\}$ can be characterized as follows:

(i) One monopolist at a time produces the economy-specific intermediate good by workers. Several firms do *R&D* by using researchers and finance their expenditure by issuing shares. As soon as any of these completes a new innovation, it takes over the whole production of the intermediate good and drives the old producer out of the market.

(ii) The households decide on their labour supply before entering the labour market. They save in shares in research firms of their home economies.

(iii) The workers are unionized. The labour union can control the whole of the intermediate good industry, including potential entrants, so that the change of the incumbent producer does not affect the union's bargaining position. There is, however, a fixed number β_j of (employed

[3] The purpose of this admittedly strong assumption is to allow us to make welfare comparisons, which would be extremely problematic with heterogeneous households.

or unemployed) workers who shall not or cannot take part in strikes. In wage bargaining, the labour union maximizes the discounted value of the flow of the workers' wages and the employer federation the discounted value of the flow of the employers' profits.[4] Government regulations influence both the relative bargaining power of the parties (which we denote α_j) and the number β_j of non-striking workers.

(iv) Direct subsidy to $R\&D$ is commonly non-feasible.[5] Given this, the government regulates union power as a second-best policy.

The growth model is based on a Poisson process. We focus entirely on the households' stationary equilibrium in which the allocation of resources is invariable across technologies, and ignore the behaviour of the system during the transitional period before the equilibrium is reached. If the initial state is chosen outside a stationary equilibrium, then the model would most likely generate cycles, which are technically extremely difficult to cope with.

3 The model

(a) Consumption-good firms. Each economy possesses one unit of land. There is one consumption good in the common market and its price is normalized at unity. The representative consumption-good firm in the common market makes its output C from the quantity a_k of land and the quantity n_k

[4]Some papers assume that the expected wage outside the firm is the union's reference point, but this is not quite in line with the microfoundations of the alternating offers game. Binmore, Rubinstein and Wolinsky (1986) state (pp. 177, 185-6) that the the reference income should not be identified with the outside option point. Rather, despite the availability of these options, it remains appropriate to identify the reference income with the income streams accruing to the parties in the course of the dispute. For example, if the dispute involves a strike, these income streams are the employee's income from temporary work, union strike funds, and similar sources, while the employer's income might derive from temporary arrangements that keeps the business running.

[5]It is commonly suggested that in order to eliminate the externality due to $R\&D$, the government should directly subsidize $R\&D$. In reality, however, $R\&D$ is mostly carried out by research departments of companies that are also producing other goods, so that the government cannot completely distinguish between inputs being used in $R\&D$ and production. If $R\&D$ were subsidized, then it were in the interests of both employers and labour unions to hide costs of production under $R\&D$ expenditure and share the subsidy. For this discussion, see Palokangas (2000), chapter 8.

of the intermediate goods throughout all economies k by technology

$$C = \left\{ \sum_{k=1}^{J} B_k^{1-1/\theta} \big[n_k^{1-1/\theta} + \delta a_k^{1-1/\theta} \big] \right\}^{\theta/(\theta-1)}, \tag{1}$$

where B_k is the productivity parameter in economy k, $\theta > 1$ the constant elasticity of substitution and $\delta > 0$ a parameter. The firm maximizes its profit taking rents R_k and the prices of intermediate goods, p_k, throughout all economies k as given. This yields equilibrium conditions $a_k = 1$ and

$$p_j = \frac{\partial C}{\partial n_j} = B_j^{1-1/\theta} \Big(\frac{C}{n_j} \Big)^{1/\theta}, \quad \frac{R_j}{p_j} = \frac{\partial C}{\partial a_j} \Big/ \frac{\partial C}{\partial n_j} = \delta \Big(\frac{n_j}{a_j} \Big)^{1/\theta} = \delta n_j^{1/\theta}. \tag{2}$$

(b) Intermediate-good firms. There is one firm at a time as the incumbent producer of good j. It takes the productivity B_j as given and anticipates the reaction of the producers of the other goods $k \neq j$ by the function

$$C = \Phi(n_j, J) \text{ with } \phi(J) \doteq \frac{n_j}{\Phi} \frac{\partial \Phi}{\partial n_j} < 1 \text{ and } \phi' > 0,$$

and maximizes profit

$$\pi_j \doteq p_j n_j - w_j n_j \tag{3}$$

by its input n_j, given the demand function in (2). Without potential competition from new entrants, the profit-maximization condition is given by

$$w_j = p_j + n_j \frac{\partial p_j}{\partial n_j} = p_j + \frac{p_j}{\theta} \Big(\frac{n_j}{\Phi} \frac{\partial \Phi}{\partial n_j} - 1 \Big) = \Big[1 + \frac{\phi(J) - 1}{\theta} \Big] p_j.$$

This yields the monopoly price $p_j^m \doteq \theta w_j / [\theta + \phi(J) - 1]$. Each new generation of good j provides constant $\varepsilon > 1$ times as many services as the product of the generation before it. If the previous incumbent, whose productivity is $1/\varepsilon$ times the productivity of the current incumbent, makes a positive profit $\pi_j = (1/\varepsilon) p_j n_j - w_j n_j > 0$ for the monopoly price p_j^m, then the current incumbent sets $p_j = \varepsilon w_j$ to prevent the others from entering the market. Hence, the firm applies the mark-up rule $p_j^m = \epsilon(J) w_j$ with

$$\epsilon(J) \doteq \min \Big[\varepsilon, \frac{\theta}{\theta + \phi(J) - 1} \Big] > 1, \quad \epsilon' < 0 \text{ for } \epsilon < \varepsilon.$$

Noting this, (2) and (3), and assuming that the common market is large enough (i.e. J big enough), we obtain the equilibrium conditions

$$
\begin{aligned}
&p_j^m = \epsilon(J)w_j, \quad \epsilon' < 0, \quad w_j n_j = p_j n_j/\epsilon = C^{1/\theta} B_j^{1-1/\theta} n_j^{1-1/\theta}/\epsilon(J),\\
&\pi_j = [\epsilon(J)-1]w_j n_j, \quad R_j/(w_j n_j) = \epsilon R_j/(p_j n_j) = \delta\epsilon(J) n_j^{1/\theta-1}. \qquad (4)
\end{aligned}
$$

(c) Research firms. Because only researchers are used in *R&D*, investment expenditure in economy j is equal to labour cost $v_j l_j$, where l_j is the researchers' labour input and v_j their wage. When a research firm in economy j is successful, it uses its new technology to drive the old producer out and starts producing good j itself. Its profits are then distributed among those who had financed it. When *R&D* is not successful for a firm, there is no profit and the *ex post* value of a share of the firm is zero.

Economy j is subject to technological change which is characterized by a Poisson process q_j as follows. During a short time interval $d\tau$, there is an innovation $dq_j = 1$ with probability $\Lambda_j d\tau$, and no innovation $dq_j = 0$ with probability $1 - \Lambda_j d\tau$, where Λ_j is the arrival rate of innovations in the research process. We assume that the arrival rate Λ_j in in fixed proportion λ to research input in the economy j, l_j:

$$\Lambda_j = \lambda l_j. \qquad (5)$$

(d) Technological change. We denote the serial number of technology in economy k by t_k. The level of productivity in the production of intermediate good k, $B_k(t_k)$, is determined by the currently most advanced technology t_k. The invention of a new technology raises t_k by one and the level of productivity $B_k(t_k)$ by $\varepsilon > 1$. This implies

$$B_k(t_k) = B_k(0)\varepsilon^{t_k}. \qquad (6)$$

Because the average growth rate is in fixed proportion ($\log\varepsilon$) to the arrival rate $\Lambda_k = \lambda l_k$ and research input l_k,[6] we can use research input l_k as a proxy of the growth rate of economy k.

(e) Employment and labour supply. Because each family can change its members' occupation from a worker to an researcher at some cost and the abilities

[6] For this, see Aghion and Howitt (1998), p. 59.

of all individuals in economy j differ, there is a decreasing and convex transformation function between the supply of workers, N_j, and the supply of researchers, L_j, as:

$$N_j = N(L_j), \quad N' < 0, \quad N'' < 0. \tag{7}$$

More and more workers must be transformed in order to create one more research input. A worker's expected wage is equal to the wage w_j times the likelihood of employment, n_j/N_j:

$$w_j^e \doteq (n_j/N_j)w_j, \tag{8}$$

Because researchers are not unionized, they are always fully employed $l_j = L_j$ and their expected wage is equal to the wage v_j.

Because households must choose their combination (L_j, N_j) of labour supply before entering the labour market, this choice is based on the transformation function (7) and the expected wages (v_j, w_j^e) which the household takes as given. This equilibrium is found by maximizing expected income $v_j L_j + w_j^e N_j = v_j L_j + w_j^e N(L_j)$ by L_j, which yields the first order condition $v_j/w_j^e = -N'(L_j)$. This condition, $l_j = L_j$, (7) and (8) yield

$$-\frac{N'(l_j)}{N(l_j)} = -\frac{N'(L_j)}{N(L_j)} = \frac{v_j}{w_j^e N_j} = \frac{v_j}{w_j n_j}. \tag{9}$$

4 Consumption and saving

Economy j contains a fixed number κ of similar households which consist of both workers and researchers.[7] The utility for household $\ell \in \{1, ..., \kappa\}$ in economy j from an infinite stream of consumption beginning at time T is

$$U_j(C_{j\ell}, T) = E \int_T^\infty C_{j\ell}^\sigma e^{-\rho(\tau - T)} d\tau \text{ with } 0 < \sigma < 1 \text{ and } \rho > 0, \tag{10}$$

where τ is time, E the expectation operator, $C_{j\ell}$ consumption, ρ the rate of time preference and $1/(1-\sigma)$ is the constant rate of relative risk aversion.

When household ℓ has financed a successful *R&D* project, it acquires the right to a certain share of profits the successful firm earns in the production

[7]See footnote 3.

of final goods. Since the old producer is driven out of the market, all shares held in it lose their value. Let $s_{j\ell}$ be the true profit share of household ℓ when the uncertainty of the outcome of the projects are taken into account. Following Wälde (1999), we assume that the change in this share, $ds_{j\ell}$, is a function of the increment dq_j of a Poisson process q_j as follows:

$$ds_{j\ell} = (i_{j\ell} - s_{j\ell})dq_j \text{ with } i_{j\ell} \doteq S_{j\ell}/(v_j l_j), \tag{11}$$

where $S_{j\ell}$ is saving by household ℓ in economy j. When a household does not invest in the upcoming vintage, its share holdings are reduced to zero in the case of research success $dq_j = 1$. If it invests, then the amount of share holdings depends on its relative investment in the vintage.

Labour income in economy j is equal to wages paid in production and $R\&D$, $w_j n_j + v_j l_j$. The total income of household $\ell \in \{1, ..., \kappa\}$ in economy j, $A_{j\ell}$, consists of an equal share $1/\kappa$ of labour income and rents $w_j n_j + v_j l_j + R_j$ and the share $s_{j\ell}$ of the profits of the intermediate-good firm, π_j:[8]

$$A_{j\ell} \doteq (v_j l_j + w_j n_j + R_j)/\kappa + s_{j\ell}\pi_j. \tag{12}$$

Because the price of the consumption good is normalized at unity, the budget constraint of household ℓ in economy j is given by

$$A_{j\ell} = C_{j\ell} + S_{j\ell}, \tag{13}$$

where $C_{j\ell}$ is consumption and $S_{j\ell}$ saving.

We denote the value of receiving a share $s_{j\ell}$ of the profits of the monopolists using current technology t_j by $\Omega(s_{j\ell}, t_j)$, and the value of receiving a share $i_{j\ell}$ of the profits of the monopolists of the next generation by $\Omega(i_{j\ell}, t_j + 1)$. Household ℓ maximizes its utility (10) subject to stochastic process (11) and the budget constraint (13) by its saving $S_{j\ell}$, given wages (w_j, v_j), profits π_j, employment (n_j, l_j) and prices (p_j, R_j). This maximization leads to the Bellman equation[9]

$$\rho\Omega(s_{j\ell}, t_j) = \max_{S_{j\ell}} \big\{ C_{j\ell}^{\sigma} + \Lambda_j [\Omega(i_{j\ell}, t_j + 1) - \Omega(s_{j\ell}, t_j)] \big\}, \tag{14}$$

[8]Because the consumption-good firms are subject to technology (1) with constant returns to scale, in equilibrium they have no profits to be distributed to households.

[9]Cf. Dixit and Pindyck (1994).

where $C_{j\ell} = A_{j\ell} - S_{j\ell}$ and $\Lambda_j = \lambda l_j$. The first order condition associated with the Bellman equation (14) is given by

$$\lambda l_j \frac{d}{dS_{j\ell}}[\Omega(i_{j\ell}, t_j + 1) - \Omega(s_{j\ell}, t_j)] = \sigma C_{j\ell}^{\sigma-1}. \tag{15}$$

We try the solution that consumption expenditure $C_{j\ell}$ is a share $0 \leq c_{j\ell} \leq 1$ out of income $A_{j\ell}$, and the value function is of the form $\Omega = (c_{j\ell}A_{j\ell})^\sigma / r_{j\ell}$, where the consumption-income ratio $c_{j\ell}$ and the (subjective) interest rate $r_{j\ell}$ are independent of income $A_{j\ell}$. Inserting that solution into (14) and (15), we obtain the following results for economy j (Appendix A). First, every innovation that replaces technology t_j by $t_j + 1$ raises consumption C_j and domestic output y_j in economy j as follows:

$$C^{t_j+1}/C^{t_j} = y_j^{t_j+1}/y_j^{t_j} = \varepsilon^{1-1/\theta} > 1. \tag{16}$$

Second, workers' employment n_j is determined by the function

$$n_j = n(l_j, J), \quad \frac{\partial n}{\partial J} < 0, \quad \frac{\partial^2 n}{\partial J \partial l_j} = \frac{1 + (2 - 1/\theta)\delta n^{1-1/\theta}}{n + \delta n^{2-1/\theta}} \frac{\partial n}{\partial l_j} \frac{\partial n}{\partial J}. \tag{17}$$

This result can be rephrased as follows:

Proposition 1 *When research input l_j is kept constant, economic integration (i.e. a bigger J) decreases workers' employment n_j.*

Greater competition due to integration decreases the price for good j and the employment of workers in the production of good j.

5 Wage bargaining

In each economy j, the workers' wage w_j is determined by bargaining between a union representing workers in economy j and a federation representing employers of these workers. We assume, for simplicity, that these both are risk neutral and have the same rate of time preference $\varrho > 0$. The union controls the whole of the intermediate-good industry inclusive of the possible entrants, but there is a fixed number β_j of workers who cannot go on strike or who are willing to work even during strikes. This means that the reference income is zero for the union and $\pi_j|_{n_j=\beta_j}$ for the federation.[10] The union attempts then to maximize the expected value $\mathcal{U}_j$ of the stream of workers' wages $w_j n_j$, while the federation attempts to maximize the expected value $\mathcal{F}_j$ of the stream of employers' profits over the reference income, $\pi_j - \pi_j|_{n_j=\beta_j}$. Given the result (16) and the stochastic technological progress (see part (c) in section 3), these targets take the form:[11]

$$\mathcal{U}_j(l_j, C) \doteq E\int_0^\infty e^{-\varrho\tau} w_j n_j \, d\tau = \frac{B_j(0)^{1-1/\theta} w_j n_j}{B_j^{1-1/\theta}[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j]},$$
$$\mathcal{F}_j(l_j, C) \doteq E\int_0^\infty e^{-\varrho\tau}[\pi_j - \pi_j|_{n_j=\beta_j}] d\tau = \frac{B_j(0)^{1-1/\theta}[\pi_j - \pi_j|_{n_j]=\beta_j}}{B_j^{1-1/\theta}[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j]}. \quad (18)$$

The union (federation) maximizes its welfare $\mathcal{U}_j$ ($\mathcal{F}_j$) by workers' wage w_j, taking world consumption C as given. Because there is one-to-one correspondence from w_j to l_j through (17), w_j can be replaced by l_j as the instrument of bargaining. The outcome of bargaining is then obtained through maximizing the Generalized Nash Product $\mathcal{U}_j^{\alpha_j}\mathcal{F}_j^{1-\alpha_j}$, where constant $\alpha_j \in (0,1)$ is relative union bargaining power, by l_j, keeping C constant. Given this maximization, research input is determined as (Appendix B):

$$l_j = l(\alpha_j, \beta_j, J) \text{ with } \partial n/\partial l_j < 0, \ \lim_{\alpha_j \to 1} l_j = \lim_{\beta_j \to 0} l_j, \ \partial l/\partial \alpha_j > 0,$$
$$\frac{\partial l}{\partial \beta_j} < 0, \ \frac{\partial l}{\partial J} \begin{cases} > 0 & \text{if } \alpha_j \text{ is large enough or } \beta_j \text{ small enough,} \\ < 0 & \text{otherwise.} \end{cases} \quad (19)$$

These results can be rephrased as follows:

Proposition 2 *The increase in union power (i.e. a bigger α_j or a smaller β_j) promotes R&D l_j and growth in economy j. When the common market takes new members (i.e. J increases) but its unions are strong enough, R&D and growth will be at a higher level for all its members.*

With higher union power, workers' wage w_j increases, but their employment l_j and expected wage w_j^e falls. With a lower relative expected wage for a worker, more households choose to become researchers rather than workers.

[10] See footnote 4.

[11] For this, see e.g. Aghion and Howitt (1998), p. 61.

A higher number of researchers promotes $R\&D$ and economic growth. Integration shift the demand for workers' labour to the left (see proposition 1). This (i) makes labour demand more inelastic, which increases the wage, but on the other hand it (ii) decreases employment the more, the stronger the union is. If the union is strong enough, then effect (ii) outweighs (i), workers' expected wage w_j^e falls, more households choose to become researchers rather than workers, and $R\&D$ increases. If the union is weak, then effect (i) outweighs (ii), workers' expected wage w_j^e increases, more households choose to become workers rather than researchers, and $R\&D$ decreases.

6 Social welfare

We define the level of productivity in the consumption-good sector as

$$B \doteq \left[\frac{1}{J}\sum_{k=1}^{J} B_k^{1-1/\theta}\right]^{\theta/(\theta-1)}. \tag{20}$$

Because there is symmetry throughout economies $j = 1, .., J$, there exists an equilibrium with $\alpha_j = \alpha$, $\beta_j = \beta$, $l_j = l$ and $B_j = B$. In that equilibrium, the average growth rate of consumption (= the arrival rate of jumps $\varepsilon > 1$ in the level of productivity in the consumption-good sector) is given by

$$\Lambda \doteq \frac{\partial B}{\partial B_k}\Lambda_k\bigg|_{B_k=B} = \frac{1}{J}\sum_{k=1}^{J}\left(\frac{B}{B_k}\right)^{1/\theta}\bigg|_{B_k=B}\Lambda_k = \frac{1}{J}\sum_{k=1}^{J}\Lambda_k.$$

We denote the serial number of consumption technology by t. Choosing $B(0) = 1$, we then obtain $B = \varepsilon^t$. Noting this, $a_k = 1$, (1), (17), (19) and (20), we can define consumption per capita in the common market as follows:

$$\begin{aligned}
&\frac{C}{J\kappa} = \psi(l,J)\left[\frac{1}{J}\sum_{k=1}^{J} B_k^{1-1/\theta}\right]^{\theta/(\theta-1)} = \psi(l,J)B = \varepsilon^t\psi(l,J),\\
&\psi(l,J) \doteq J^{1/(\theta-1)}\left[n(l,J)^{1-1/\theta}+\delta\right]^{\theta/(\theta-1)}/\kappa, \quad \partial\psi/\partial l < 0,\\
&\frac{1}{\psi}\frac{\partial\psi}{\partial J} = \frac{1/J}{\theta-1} + \frac{1}{n+\delta n^{1/\theta}}\frac{\partial n}{\partial J}.
\end{aligned} \tag{21}$$

The utility (10) of a single consumer in the common market is then given by

$$U_j = E\int_T^\infty \left(\frac{C}{J\kappa}\right)e^{-\rho(\tau-T)}d\tau = E\int_T^\infty \psi(l,J)^\sigma\varepsilon^{\sigma t}e^{-\rho(\tau-T)}d\tau. \tag{22}$$

Consider first the case where the common market is able to reform the labour market. The social planner in the common market then maximizes social welfare (22) by union power (α or β) and the size of the common market, J. Because l depends on these parameters through (see (19)), the number of researchers $l_j = l$ can be used as the instrument of maximization. Denoting the value of the state of technology t for this government by $\Upsilon(t)$, and noting (5) and (19), we obtain the Bellman equation for this policy as:

$$\rho\Upsilon(t, J) = \max_{J,l} \mathcal{Q}(l, J, t), \text{ where } \mathcal{Q} \doteq \psi(l, J)^{\sigma}\varepsilon^{\sigma t} + \lambda l[\Upsilon(t+1) - \Upsilon(t)]. \tag{23}$$

We define $(J^*, l^*) = \arg\max_{J,l} \mathcal{Q}(l, J, t)$. This, (21) and (23) yield

$$J^* = \arg\max_{J} \mathcal{Q}(l^*, J, t) = \arg\max_{J} \psi(l^*, J) = \arg\max_{J} [C/(J\kappa)]_{l=l^*}.$$

This result can be rephrased as follows:

Proposition 3 *If the common market is free to make a labour market reform of any size, then the expansion of the common market (i.e. the increase in J) is welfare enhancing as long as it increases consumption per capita, $C/(J\kappa)$.*

With the possibility of extensive labour market reforms, the common market can control its growth rate through union power. In such a case, the integration of new members can be wholly determined by the maximization of current consumption with no concern of economic growth.

Second, consider the case where the common market cannot make a labour market reform that is large enough to attain the unconstrained social optimum (J^*, l^*). We can then assume that the parameter of union power, α and β, are exogenous. The Bellman equation for this policy is as follows:

$$\rho\Upsilon(t, J) = \max_{J} \mathcal{Q}(l(\alpha, \beta, J), J, t). \tag{24}$$

We restrict ourselves to small departures from the social optimum $\partial\mathcal{Q}/\partial l = 0$, for simplicity. Large departures would involve ambiguous results.

In the neighbourhood of $\partial\mathcal{Q}/\partial l = 0$, given (17), (19), and (23), we obtain

$$\frac{\partial\mathcal{Q}}{\partial J} = \sigma\frac{\mathcal{Q}}{\psi}\frac{\partial\psi}{\partial J} + \frac{\partial\mathcal{Q}}{\partial l}\frac{\partial l}{\partial J} = \sigma\mathcal{Q}\left[\frac{\theta/J}{\theta-1} + \frac{1}{n+\delta n^{1/\theta}}\frac{\partial n}{\partial J}\right] \text{ for } \frac{\partial\mathcal{Q}}{\partial l} \approx 0,$$

$$
\begin{aligned}
\frac{\partial^2 \mathcal{Q}}{\partial J \partial l} &= \frac{1}{\mathcal{B}} \frac{\partial \mathcal{Q}}{\partial J} \frac{\partial \mathcal{Q}}{\partial l} + \sigma \mathcal{Q} \left[\frac{1}{n + \delta n^{1/\theta}} \frac{\partial^2 n}{\partial J \partial l} - \frac{1 + \delta n^{1/\theta - 1}/\theta}{(n + \delta n^{1/\theta})^2} \frac{\partial n}{\partial J} \frac{\partial n}{\partial l} \right] \\
&= \frac{1}{\mathcal{B}} \frac{\partial \mathcal{Q}}{\partial J} \frac{\partial \mathcal{Q}}{\partial l} + \frac{\sigma \mathcal{Q}}{n + \delta n^{1/\theta}} \left[\frac{1 + (2 - 1/\theta)\delta n^{1-1/\theta}}{n + \delta n^{2-1/\theta}} - \frac{1 + \delta n^{1/\theta - 1}/\theta}{n + \delta n^{1/\theta}} \right] \frac{\partial n}{\partial J} \frac{\partial n}{\partial l} \\
&> \frac{1}{\mathcal{B}} \frac{\partial \mathcal{Q}}{\partial J} \frac{\partial \mathcal{Q}}{\partial l} + \frac{\sigma \mathcal{Q}}{n + \delta n^{1/\theta}} \left[\frac{1}{n} - \frac{1 + \delta n^{1/\theta - 1}/\theta}{n + \delta n^{1/\theta}} \right] \frac{\partial n}{\partial J} \frac{\partial n}{\partial l} > 0 \text{ for } \frac{\partial \mathcal{Q}}{\partial l} \approx 0, \\
\frac{\partial}{\partial J} \Big(\frac{\partial \mathcal{Q}}{\partial l} \frac{\partial l}{\partial \alpha} \Big) &= \frac{\partial^2 \mathcal{Q}}{\partial l \partial J} \frac{\partial l}{\partial \alpha} + \frac{\partial \mathcal{Q}}{\partial l} \frac{\partial^2 l}{\partial \alpha \partial J} > 0 \text{ for } \frac{\partial \mathcal{Q}}{\partial l} \approx 0, \\
\frac{\partial}{\partial J} \Big(\frac{\partial \mathcal{Q}}{\partial l} \frac{\partial l}{\partial \beta} \Big) &= \frac{\partial^2 \mathcal{Q}}{\partial l \partial J} \frac{\partial l}{\partial \alpha} + \frac{\partial \mathcal{Q}}{\partial l} \frac{\partial^2 l}{\partial \alpha \partial J} < 0 \text{ for } \frac{\partial \mathcal{Q}}{\partial l} \approx 0.
\end{aligned} \tag{25}
$$

The first-order and second-order conditions for J are then given by $\partial \mathcal{Q}/\partial J = 0$ and $\partial^2 \mathcal{Q}/\partial J^2 < 0$. Differentiating $\partial \mathcal{Q}/\partial J = 0$ totally, and noting (25) and $\partial^2 \mathcal{Q}/\partial J^2 < 0$, we obtain

$$
\frac{dJ}{d\alpha} = -\frac{\partial}{\partial J} \Big(\frac{\partial \mathcal{Q}}{\partial l} \frac{\partial l}{\partial \alpha} \Big) \Big/ \frac{\partial^2 \mathcal{Q}}{\partial J^2} > 0 \text{ and } \frac{dJ}{d\beta} = -\frac{\partial}{\partial J} \Big(\frac{\partial \mathcal{Q}}{\partial l} \frac{\partial l}{\partial \beta} \Big) \Big/ \frac{\partial^2 \mathcal{Q}}{\partial J^2} < 0
$$

for $\partial \mathcal{Q}/\partial l \approx 0$. This result can be rephrased as follows:

Proposition 4 *If the common market cannot make large enough labour market reform, then its optimal size is the bigger, the stronger its labour unions are (i.e. the bigger α or the smaller β). In other words, the common market should outweigh excessive union power through economic integration, which increases competition in the goods market.*

7 Conclusions

This paper examines a common market with the following properties. First, The expansion of the common market increases the variety of products and the intensity of competition in the goods market. Second, growth is generated by creative destruction. A firm creating the latest technology through a successful *R&D* project crowds the other firms with older technologies out of the market so that they lose their value. Third, households save by buying shares in *R&D* projects. Fourth, households decide whether their members are researchers who are used in *R&D*, or workers who are employed in production. A change of occupation involves a cost. Fifth, direct subsidy to *R&D*

is commonly non-feasible. Sixth, wages are determined by union-employer bargaining. The main findings are as follows.

Union power has a positive impact on the growth rate, but a negative impact on current income. With higher union power, workers' wages increase, but their employment and expected wage falls, and more households choose to become researchers rather than workers. With lower employment for workers, current output and income is smaller. On the other hand, with a larger number of researchers, there will be more innovations and a higher growth rate in future. The welfare effect of union power is positive (negative) if the latter effect through growth dominates (is dominated by) the former effect through employment. Union power and the growth rate are socially optimal when the growth and employment effects exactly outweigh each other.

When the common market takes new members, $R\&D$ and the growth rate increase (decrease) if unions are strong (weak). The integration of new members decreases the prices for goods and the demand for workers' labour. This (i) makes labour demand more inelastic, which increases the wage, but on the other hand it (ii) decreases employment the more, the stronger the union is. If unions are strong enough, then effect (ii) outweighs (i), workers' expected wage falls, more households choose to become researchers rather than workers, and $R\&D$ increases. If unions are weak, then effect (i) outweighs (ii), workers' expected wage w_j^e increases, more households choose to become workers rather than researchers, and $R\&D$ decreases.

The decision on accepting new members should depend on the rigidity of labour market institutions. With the possibility of extensive labour market reforms, the common market can control its growth rate through union power. In such a case, the integration of new members can be wholly determined by the maximization of current consumption with no concern of economic growth. In the absence of extensive reforms, the common market should outweigh excessive union power through economic integration, which increases competition in the goods market. This means that common markets with strong labour unions should have a lower threshold of taking in new members than those with weak unions.

Appendix A

Let us denote variables depending on technology t_j by superscript t_j. Since according to (12) income $A_{j\ell}^{t_j}$ depends directly on the share $s_{j\ell}^{t_j}$, we denote $A_{j\ell}^{t_j}(s_{j\ell}^{t_j})$. Guessing that $c_{j\ell}$ is invariant across technologies, we obtain

$$C_{j\ell}^{t_j} = c_{j\ell} A_{j\ell}^{t_j}(s_{j\ell}^{t_j}), \quad S_{j\ell}^{t_j} = (1 - c_{j\ell}) A_{j\ell}^{t_j}(s_{j\ell}^{t_j}). \tag{26}$$

The share in the next producer $t_j + 1$ is determined by investment under technology t_j, $s_{j\ell}^{t_j+1} = i_{j\ell}^{t_j}$. The value functions are then given by

$$\Omega(s_{j\ell}^{t_j}, t_j) = (C_{j\ell}^{t_j})^\sigma / r_{j\ell}, \quad \Omega(i_{j\ell}^{t_j}, t_j + 1) = (C_{j\ell}^{t_j+1})^\sigma / r_{j\ell}. \tag{27}$$

Given this, we obtain

$$\partial\Omega(s_{j\ell}^{t_j}, t_j)/\partial S_{j\ell}^{t_j} = 0. \tag{28}$$

From (11), (12), (26) and (27) it follows that

$$\begin{gathered}
\frac{\partial i_{j\ell}^{t_j}}{\partial S_{j\ell}^{t_j}} = \frac{1}{v_j^{t_j} l_j^{t_j}}, \quad \frac{\partial[A_{j\ell}^{t_j+1}(i_{j\ell}^{t_j})]}{\partial i_{j\ell}^{t_j}} = \frac{\partial[A_{j\ell}^{t_j+1}(s_{j\ell}^{t_j+1})]}{\partial s_{j\ell}^{t_j+1}} = \pi_j^{t_j+1}, \\
\frac{\partial\Omega(i_{j\ell}^{t_j}, t_j + 1)}{\partial S_{j\ell}^{t_j}} = \frac{\sigma}{r_{j\ell}} (C_{j\ell}^{t_j+1})^{\sigma-1} \frac{\partial C_{j\ell}^{t_j+1}}{\partial A_{j\ell}^{t_j+1}} \frac{\partial A_{j\ell}^{t_j+1}}{\partial i_{j\ell}^{t_j}} \frac{\partial i_{j\ell}^{t_j}}{\partial S_{j\ell}^{t_j}} = \sigma \frac{c_{j\ell}(C_{j\ell}^{t_j+1})^{\sigma-1} \pi_j^{t_j+1}}{r_{j\ell} v_j^{t_j} l_j^{t_j}}.
\end{gathered} \tag{29}$$

We focus on a stationary equilibrium where the allocation of labour, $(l_j^{t_j}, n_j^{t_j})$, is invariant across technologies. Given (7), this implies

$$l_j^{t_j} = l_j, \quad n_j^{t_j} = n_j, \quad N_j = N(L_j) = N(l_j). \tag{30}$$

From (4), (6), (12), (26) and (30) it then follows that

$$\begin{aligned}
C_{j\ell}^{t_j+1}/C_{j\ell}^{t_j} &= A_{j\ell}^{t_j+1}/A_{j\ell}^{t_j} = v_j^{t_j+1}/v_j^{t_j} = \pi_j^{t_j+1}/\pi_j^{t_j} \\
&= w_j^{t_j+1}/w_j^{t_j} = (B_j^{t_j+1}/B_j^{t_j})^{1-1/\theta} = \varepsilon^{1-1/\theta} > 1.
\end{aligned} \tag{31}$$

Inserting (5), (27) and (31) into equation (14), we obtain

$$\begin{aligned}
0 &= (\rho + \Lambda_j)\Omega(s_{j\ell}^{t_j}, t_j) - (C_{j\ell}^{t_j})^\sigma - \Lambda_j \Omega(i_{j\ell}^{t_j}, t_j + 1) \\
&= (\rho + \Lambda_j)(C_{j\ell}^{t_j})^\sigma / r_{j\ell} - (C_{j\ell}^{t_j})^\sigma - \Lambda_j (C_{j\ell}^{t_j+1})^\sigma / r_j \\
&= (C_{j\ell}^{t_j})^\sigma [\rho + \Lambda_j - r_{j\ell} - \varepsilon^{(1-1/\theta)\sigma} \Lambda_j] / r_{j\ell} \\
&= (C_{j\ell}^{t_j})^\sigma \big\{\rho - r_{j\ell} + [1 - \varepsilon^{(1-1/\theta)\sigma}] \lambda l_j \big\} / r_{j\ell}.
\end{aligned}$$

This leads to the function

$$r_j = r_{j\ell} = \rho + [1 - \varepsilon^{(1-1/\theta)\sigma}]\lambda l_j. \tag{32}$$

From (4), (9), (17) and (34) it follows that

$$v_j^{t_j}/(w_j^{t_j} n_j) = -N'(l_j)/N(l_j). \tag{33}$$

Inserting (4) and (28)-(33) into (15) yields

$$0 = \lambda l_j \frac{\partial \Omega(i_{j\ell}^{t_j}, t_j + 1)}{\partial S_{j\ell}^{t_j}} - \sigma(C_{j\ell}^{t_j})^{\sigma-1} = \lambda l_j \sigma c_{j\ell} \frac{(C_{j\ell}^{t_j+1})^{\sigma-1} \pi_j^{t_j}}{r_j v_j^{t_j} l_j} - \sigma(C_{j\ell}^{t_j})^{\sigma-1}$$
$$= \sigma(C_{j\ell}^{t_j})^{\sigma-1} \Big[\lambda c_{j\ell} \varepsilon^{(1-1/\theta)(\sigma-1)} \frac{\pi_j^{t_j}}{r_j v_j^{t_j}} - 1\Big]$$

and

$$c_{j\ell} = \varepsilon^{(1/\theta-1)(\sigma-1)} \frac{r_j v_j^{t_j}}{\lambda \pi_j^{t_j}} = \varepsilon^{(1/\theta-1)(\sigma-1)} \frac{\rho + [1 - \varepsilon^{(1-1/\theta)\sigma}]\lambda l_j}{(\epsilon - 1)\lambda N(l_j)/N'(l_j)} \doteq c(l_j), \tag{34}$$

where the sign of $c(l_j)$ is ambiguous. Given $\sum_{\ell=1}^{\kappa} s_{j\ell}^{t_j} = 1$, (4), (12), (26), (33) and (34), we obtain $v_j l_j = \sum_{\ell=1}^{\kappa} S_{j\ell}$ and

$$\frac{v_j l_j}{1 - c(l_j)} = \frac{1}{1 - c(l_j)} \sum_{\ell=1}^{\kappa} S_{j\ell} = \sum_{\ell=1}^{\kappa} \frac{S_{j\ell}}{1 - c_{j\ell}} = \sum_{\ell=1}^{\kappa} A_{j\ell} = v_j l_j + w_j n_j + R_j + \pi_j$$
$$= v_j l_j + \epsilon w_j n_j + R_j,$$
$$v_j l_j = [1/c(l_j) - 1](\epsilon w_j n_j + R_j) = \epsilon[1/c(l_j) - 1]\big(1 + \delta n_j^{1/\theta-1}\big) w_j n_j,$$
$$\frac{l_j}{1 - 1/c(l_j)} \frac{N'(l_j)}{N(l_j)} = \frac{1}{1/c(l_j) - 1} \frac{v_j l_j}{w_j n_j} = \epsilon(J)\big[1 + \delta n_j^{1/\theta-1}\big]. \tag{35}$$

Equation (35) defines the function $n_j = n(l_j, J)$. Keeping l_j constant, we obtain $\partial n/\partial b$ by differentiating the logarithm of the term $\epsilon(J)\big[1 + \delta n_j^{1/\theta-1}\big]$ totally. This, $\epsilon' < 0$ and $\theta > 1$ yield

$$\frac{\partial n}{\partial J}(l_j, J) = \frac{\theta}{\theta - 1}\Big[\delta n(l_j, J)^{2-1/\theta} + n(l_j, J)\Big] \frac{\epsilon'(J)}{\epsilon(J)} < 0.$$

Differentiating the logarithm of this equation with respect to l_j yields

$$\frac{\partial^2 n}{\partial J \partial l_j} \Big/ \frac{\partial n}{\partial J} = \frac{1 + (2 - 1/\theta)\delta n^{1-1/\theta}}{n + \delta n^{2-1/\theta}} \frac{\partial n}{\partial l_j}.$$

Appendix B

Given (4), (17) and (18), the logarithm of the Generalized Nash product $\mathcal{U}_j^{\alpha}\mathcal{F}_j^{1-\alpha}$ takes the form

$$
\begin{aligned}
&\Gamma_j(l_j, C, \alpha_j, \beta_j, \theta) \doteq \alpha_j \log \mathcal{U}_j + (1-\alpha_j)\log \mathcal{F}_j \\
&= \alpha_j \log\big[w_j n_j B_j^{1/\theta-1}\big] + (1-\alpha_j)\log\big[\big(\pi_j - \pi_j|_{n_j=\beta_j}\big)B_j^{1/\theta-1}\big] \\
&\quad + (1-1/\theta)\log B_j(0) - \log[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j] \\
&= \alpha_j \log\big[w_j n_j B_j^{1/\theta-1}\big] + (1-\alpha_j)\big[\log\big(1 - \pi_j|_{n_j=\beta_j}/\pi_j\big) + \log \pi_j B_j^{1/\theta-1}\big] \\
&\quad + (1-1/\theta)\log B_j(0) - \log[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j] \\
&= \log\big[w_j n_j B_j^{1/\theta-1}\big] + (1-\alpha_j)\log\big[1 - (\beta_j/n_j)^{1-1/\theta}\big] + (1-\alpha_j)\log(\epsilon-1) \\
&\quad + (1-1/\theta)\log B_j(0) - \log[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j] \\
&= (1-1/\theta)\log\, n_j + (1-\alpha_j)\log\big[1 - \beta_j^{1-1/\theta} n_j^{1/\theta-1}\big] + (1-\alpha_j)\log(\epsilon-1) \\
&\quad + (1-1/\theta)\log B_j(0) - \log[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j] - \log(\epsilon J) + \log[\gamma\mu(J)] \\
&\quad + (1/\theta)\log C \\
&= (1-1/\theta)\alpha_j \log\, n_j + (1-\alpha_j)\log\big[n_j^{1-1/\theta} - \beta_j^{1-1/\theta}\big] + (1-\alpha_j)\log(\epsilon-1) \\
&\quad + (1-1/\theta)\log B_j(0) - \log[\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j] - \log(\epsilon J) + \log[\gamma\mu(J)] \\
&\quad + (1/\theta)\log C \;\text{ with }\; \varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j > 0 \;\text{ and }\; n_j = n(l_j, J).
\end{aligned} \tag{36}
$$

Because a logarithm is an increasing transformation, the outcome of bargaining is obtained through maximizing the function (36) by l_j, taking C as given. This leads to the first-order condition

$$
\begin{aligned}
\frac{\partial \Gamma_j}{\partial l_j} &= \overbrace{\left(1 - \frac{1}{\theta}\right)}^{+}\overbrace{\left[\frac{\alpha_j}{n(l_j,J)} + \frac{(1-\alpha_j)n(l_j,J)^{-1/\theta}}{n(l_j,J)^{1-1/\theta} - \beta_j^{1-1/\theta}}\right]\frac{\partial n}{\partial l_j}}^{+} + \overbrace{\frac{(\varepsilon^{1-1/\theta}-1)\lambda}{\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j}}^{+} \\
&= \left(1 - \frac{1}{\theta}\right)\frac{n(l_j,J)^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n(l_j,J)[n(l_j,J)^{1-1/\theta} - \beta_j^{1-1/\theta}]}\frac{\partial n}{\partial l_j} + \frac{(\varepsilon^{1-1/\theta}-1)\lambda}{\varrho + (1-\varepsilon^{1-1/\theta})\lambda l_j} = 0.
\end{aligned} \tag{37}
$$

Note that if all workers are controlled by the union and able to strike, $\beta_j \to 0$, then the outcome is the same as with a monopoly union $\alpha \to 1$.

In equilibrium, there must be $\pi_j > \pi_j|_{n_j=\beta_j}$ and $n_j > \beta_j$. Noting $\alpha_j < 1$,

$\theta > 1$ and $n_j > \beta_j$, we show first that

$$
\begin{aligned}
& n_j \frac{\partial}{\partial n_j}\left[\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}\right] \Big/ \left[\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}\right] \\
& = n_j \frac{\partial}{\partial n_j} \log\left[\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}\right] \\
& = n_j \frac{\partial}{\partial n_j}\left[\log\bigl(n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}\bigr) - \log\bigl(n_j^{1-1/\theta} - \beta_j^{1-1/\theta}\bigr) - \log\, n_j\right] \\
& = \Bigl(1 - \frac{1}{\theta}\Bigr) n_j^{1-1/\theta}\left[\frac{1}{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}} - \frac{1}{n_j^{1-1/\theta} - \beta_j^{1-1/\theta}}\right] - 1 \\
& = \frac{(1-1/\theta)(\alpha_j - 1)\beta_j^{1-1/\theta} n_j^{1-1/\theta}}{\bigl(n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}\bigr)\bigl(n_j^{1-1/\theta} - \beta_j^{1-1/\theta}\bigr)} - 1. \qquad (38)
\end{aligned}
$$

Noting this, $\varepsilon > 1$, (4), (17), (19), (37) and (38), we obtain $\partial n/\partial l_j < 0$ and

$$
\frac{\partial^2 \Gamma_j}{\partial l_j \partial \alpha_j} = \underbrace{\Bigl(\frac{1}{\theta} - 1\Bigr)}_{-} \underbrace{\frac{\beta_j^{1-1/\theta}}{n(l_j, J)[n(l_j, J)^{1-1/\theta} - \beta_j^{1-1/\theta}]}}_{+} \underbrace{\frac{\partial n}{\partial l_j}}_{-} > 0,
$$

$$
\frac{\partial^2 \Gamma_j}{\partial l_j \partial \beta_j} = \underbrace{\Bigl(1 - \frac{1}{\theta}\Bigr)}_{+} \underbrace{(1 - \alpha_j)}_{+} \underbrace{\frac{\partial n}{\partial l_j}}_{-} \underbrace{\frac{\partial}{\partial \beta_j}\left[\frac{n(l_j, J)^{-1/\theta}}{n(l_j, J)^{1-1/\theta} - \beta_j^{1-1/\theta}}\right]}_{+} < 0,
$$

$$
\begin{aligned}
\frac{\partial^2 \Gamma_j}{\partial l_j \partial J} &= \Bigl(1 - \frac{1}{\theta}\Bigr)\left\{\frac{\partial n}{\partial l_j} \frac{\partial}{\partial n_j}\left[\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}\right] \frac{\partial n}{\partial J}\right. \\
&\qquad \left. + \frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})} \frac{\partial^2 n}{\partial l_j \partial J}\right\} \\
&= \Bigl(1 - \frac{1}{\theta}\Bigr) \frac{\partial n}{\partial l_j} \frac{\partial n}{\partial J}\left\{\frac{\partial}{\partial n_j}\left[\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}\right]\right. \\
&\qquad \left. + \frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})} \frac{1 + (2-1/\theta)\delta n_j^{1-1/\theta}}{n_j + \delta n_j^{2-1/\theta}}\right\} \\
&= \Bigl(1 - \frac{1}{\theta}\Bigr) \frac{\partial n}{\partial l_j} \frac{\partial n}{\partial J} \frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j^2(n_j^{1-1/\theta} - \beta_j^{1-1/\theta})} \\
&\qquad \left\{\frac{(1-1/\theta)(\alpha_j - 1)\beta_j^{1-1/\theta} n_j^{1-1/\theta}}{\bigl(n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}\bigr)\bigl(n_j^{1-1/\theta} - \beta_j^{1-1/\theta}\bigr)} - 1 + \frac{1 + (2-1/\theta)\delta n_j^{1-1/\theta}}{1 + \delta n_j^{1-1/\theta}}\right\}
\end{aligned}
$$

$$
= \overbrace{\left(1 - \frac{1}{\theta}\right)^2}^{+} \overbrace{\frac{\partial n}{\partial l_j}}^{-} \overbrace{\frac{\partial n}{\partial J}}^{-} \overbrace{\frac{n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}}{n_j^2 (n_j^{1-1/\theta} - \beta_j^{1-1/\theta})}}^{+} \overbrace{n_j^{1-1/\theta}}^{+}
$$

$$
\left\{ \frac{(\alpha_j - 1)\beta_j^{1-1/\theta}}{\left(n_j^{1-1/\theta} - \alpha_j \beta_j^{1-1/\theta}\right)\left(n_j^{1-1/\theta} - \beta_j^{1-1/\theta}\right)} + \frac{1}{1 + \delta n_j^{1-1/\theta}} \right\}
$$

$$
\begin{cases} > 0 & \text{if } \alpha_j \text{ is large enough or } \beta_j \text{ small enough,} \\ < 0 & \text{otherwise.} \end{cases}
$$

Given these inequalities, (17), $\partial n / \partial l_j < 0$ and the second-order condition $\partial^2 \Gamma_j / \partial l_j^2 < 0$, the comparative statics of the equation (37) produces the function $l_j = l(\alpha_j, \beta_j, J)$ with the properties

$$
\frac{\partial l_j}{\partial \alpha_j} = -\frac{\partial^2 \Gamma_j}{\partial l_j \partial \alpha_j} \bigg/ \frac{\partial^2 \Gamma_j}{\partial l_j^2} > 0, \quad \frac{\partial l_j}{\partial \beta_j} = -\frac{\partial^2 \Gamma_j}{\partial l_j \partial \beta_j} \bigg/ \frac{\partial^2 \Gamma_j}{\partial l_j^2} < 0,
$$

$$
\frac{\partial l_j}{\partial J} = -\frac{\partial^2 \Gamma_j}{\partial l_j \partial J} \bigg/ \frac{\partial^2 \Gamma_j}{\partial l_j^2} \begin{cases} > 0 & \text{if } \alpha_j \text{ is large enough or } \beta_j \text{ small enough,} \\ < 0 & \text{otherwise.} \end{cases}
$$

References:

Agell, J. and Lommerud, K.J. (1997). Minimum wages and the incentives for skill formation. *Journal of Public Economics 64*, 25-40.

Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory.* Cambridge (Mass.): MIT Press.

Binmore, K., Rubinstein, A. and Wolinsky, A. (1986). The Nash Bargaining Solution in Economic Modelling. *Rand Journal of Economics 17*, 176-188.

Cahuc, P. and Michel, P. (1996). Minimum wage unemployment and growth. *European Economic Review 40*, 1463-1482.

Dixit, A. and Pindyck, K. (1994). *Investment under Uncertainty.* Princeton: Princeton University Press.

Palokangas, T. (1996). Endogenous growth and collective bargaining. *Journal of Economic Dynamics and Control 20*, 925-944.

Palokangas, T. (2000). *Labour Unions, Public Policy and Economic Growth.* Cambridge (U.K.): Cambridge University Press.

Palokangas, T. (2003). The political economy of collective bargaining. *Labour Economics 10*, 253-264.

Palokangas, T. (2004a). International Labour Union Policy and Growth with Creative Destruction. *Discussion Paper No. 591 (February 25, 2004), ISBN 952-10-1519-5.* Department of Economics, University of Helsinki. Downloadable: http://ethesis.helsinki.fi/valkandis.html

Palokangas, T. (2004b). Common Markets, Economic growth, and Creative Destruction. *Discussion Paper No. 594 (March, 2004), ISBN 952-10-1519-5,* Department of Economics, University of Helsinki. Downloadable: http://ethesis.helsinki.fi/valkandis.html

Peretto, P.F. (1998). Market Power, Growth and Unemployment. *Duke Economics Working Paper 98-16.*

Romer, P.M. (1990). Endogenous technological change. *Journal of Political Economy 98*, S71-S102.

Wälde, K. (1999). A model of creative destruction with undiversifiable risk and optimizing households. *The Economic Journal 109*, C156-C171.