Competition, Imitation and Growth with Non-Diversifiable Risk

Tapio Palokangas*
University of Helsinki and HECER
June 22, 2006

Abstract

This paper analyzes the growth and welfare effects of competition in an endogenously-growing economy with imitation and non-diversifiable risk. The main findings are as follows. There is no imitation without positive profits during innovation races. A larger proportion of competing industries leads to slower economic growth. When competitive profits are high or low, the economy grows faster than when they are of medium size. If the government subsidizes innovation and imitation optimally, then competitive profits are positively associated with welfare. With an optimal uniform subsidy to all R&D, there is an "inverted-U" relationship between competitive profits and welfare.

Journal of Economic Literature: $L11$, $L16$, $O31$, $O34$

Keywords: Imitation, competition, Schumpeterian growth

Corresponding author:

Tapio Palokangas, Department of Economics, P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland. Phone +358 9 191 28735, Fax +358 9 191 28736, Email: Tapio.Palokangas@helsinki.fi

*The author thanks Vesa Kanniainen, Juuso Välimäki, Otto Toivanen and Mikko Mustonen for helpful comments. Financial support from The Yrjö Jahnsson Foundation is gratefully acknowledged.

1 Introduction

This paper considers the growth and welfare effects of competition when households cannot wholly diversify their investment risk and economic growth is characterized by product cycles as follows. Through the development of new products, an innovator achieves a temporary advantage earning monopoly profits. This advantage ends when an imitator succeeds in copying the innovation, enters the market and starts competing with the innovator.

Product cycle models start from Segerstrom (1991), who assumes that (*i*) incumbents and outsiders have the same costs of innovation, and (*ii*) households eliminate investment risk wholly by diversification. Assumption (*i*) leads to leapfrogging: innovations will always be performed by outsiders and the current industry leaders will be wholly replaced. To eliminate this unrealistic outcome, Aghion et al. (1997, 2001) construct models where technological laggards must first catch up with the leading-edge technology before battling "neck-to-neck" for technological leadership in the future. They represent competition by the elasticity of substitution between firms' products and show that competition has in general a positive effect on economic growth. Mukoyama (2003) constructs a model in which only leaders can conduct next-round innovation, while outsiders can become leaders by imitation. He represents competition by the relative proportion of competing industries and shows that competition very commonly promotes economic growth.

The three papers above are based on the assumption (*ii*) of full diversification. Wälde (1999a, 1999b) shows that with non-diversifiable risk investment decisions are made by households rather than firms, and the equilibrium conditions differ substantially. To examine competition policy with non-diversifiable risk, I extend Wälde's one-industry growth model for an economy with many industries and incorporate Mukoyama's (2003) assumptions on imitation and cumulative technology into it. The model of this study is therefore characterized as follows:

(i) Labor is homogeneous and inelastically supplied. It is used in innovation, imitation or the production of the intermediate goods.

(ii) Competitive firms produce the consumption good from a great number of intermediate goods according to Cobb-Douglas technology.

(iii) Firms' products are imperfect substitutes. A successful innovator of a new technology crowds out all products made with old technology and becomes a monopolistic producer until its technology is imitated. A successful imitator starts producing a substitute for the innovator's product and establishes an innovation race with the incumbent producers. Imitation is necessary for an outsider to become an innovator.

(iv) R&D firms finance their expenditure by issuing shares. The households save only in these shares. Each R&D firm distributes its profit among those who had financed it in proportion to their investment in the firm. The subsidies to R&D are financed by lump-sum taxes.

The remainder of this paper is organized as follows. Sections 2 and 3 consider firms in production and R&D. Section 4 examines households deciding on saving. Section 5 examines general equilibrium and the effects of competition without government subsidies. Section 6 considers the effects of competition with government intervention.

2 Production

I assume a great number of intermediate-good industries that are placed over the limit $[0,1]$. Industry $j \in [0,1]$ contains intermediate-good firms $\kappa = 1, ..., a_j$. The representative consumption-good firm makes its output y from the products of all intermediate-good firms through technology

$$\log y = \int_0^1 \log[B_j x_j]dj, \quad x_j = \left[a_j^{-1/\varepsilon} \sum_{\kappa=1}^{a_j} x_{j\kappa}^{1-1/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \qquad \varepsilon > 1, \qquad (1)$$

where B_j is the productivity parameter in industry j, a_j the number of firms in industry j, x_j the quantity of intermediate good j, $x_{j\kappa}$ the output of firm κ in industry j, and ε the elasticity of substitution between the products in the same industry.[1] The consumption-good firm maximizes its profit

$$\Pi^c \doteq Py - \int_{j\in[0,1]} \sum_{\kappa=1}^{a_j} p_{j\kappa} x_{j\kappa} dj$$

[1] With the specification (1), the price p_j for the composite product of industry j will (in the symmetric equilibrium $p_{j\kappa} = p_{j1}$) be independent of the number of producers in that industry, a_j. Otherwise, the effect of a_j on p_j would excessively complicate the analysis.

by its inputs x_j, taking the output price P and the input prices $\{p_{j\kappa}\}$ as fixed. I normalize total consumption expenditure Py at unity. Because the consumption-good firm is subject to constant returns to scale, we then obtain

$$Py = 1, \quad \Pi^c = 0, \quad p_j x_j = 1 \text{ and } p_j = \left[\frac{1}{a_j}\sum_{\kappa=1}^{a_j} p_{j\kappa}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \text{ for all } j,$$
$$x_{j\kappa} = \frac{\partial p_j}{\partial p_{j\kappa}} x_j = \frac{1}{a_j}\left(\frac{p_j}{p_{j\kappa}}\right)^{\varepsilon} x_j = \frac{1}{a_j} p_j^{\varepsilon-1} p_{j\kappa}^{-\varepsilon} \text{ for all } j \text{ and } \kappa, \tag{2}$$

where p_j is the price of the composite product x_j.

I assume that all intermediate-good firms produce one unit of their output from one labor unit. Technological change is random. I assume that a successful innovator in industry j makes a perfect substitute for intermediate good j that is composed of the outputs all incumbent firms with older technology in industry j.[2] The innovator's profit is $\Pi_{j1} = (p_{j1} - w)x_{j1}$, where p_{j1} is its output price, x_{j1} its output (= labor input) and w is the wage.

The innovator's product provides exactly the constant $\mu > 1$ times as many services as the intermediate good of earlier generation. Firm κ of earlier generation earns the profit $\Pi^o_{j\kappa} = (p^o_{j\kappa} - w)x^o_{j\kappa}$, where $p^o_{j\kappa}$ is its output price and $x^o_{j\kappa}$ its output. The innovator pushes the old firms out of the market by choosing its price p_{j1} so that these earn no profit, $\Pi^o_{j\kappa} = 0$ and $p^o_{j\kappa} = w$. This and (2) yield $p_{j1}/\mu = p^o_j = p^o_{j\kappa} = w$, the mark-up rule $p_{j1} = \mu w$ and the innovator's output and profit as follows:

$$x_j = x_{j1} = 1/p_{j1} = 1/(\mu w) \text{ and}$$
$$\Pi_{j1} = (p_{j1} - w)x_{j1} = (1 - 1/\mu)p_{j1}x_{j1} = 1 - 1/\mu \doteq \Pi \text{ for } a_j = 1. \tag{3}$$

The innovator is the first leader (i.e. the first incumbent producer) in industry j. A successful imitator of the state-of-art good is able to make a close substitute for the product of the innovator. Thus with each imitation, the number of leaders and products increases by one. I assume that all leaders $1, ..., a_j$ in industry j behave in Bertrand manner, taking each other's prices as given. Given (1) and (2), leader κ in industry j maximizes its profit

$$\pi_{j\kappa} = p_{j\kappa}x_{j\kappa} - wx_{j\kappa} = (p_{j\kappa} - w)x_{j\kappa}, \tag{4}$$

[2]This assumption is in line with technology (1), because $x_j = x_{j1}$ for $a_j = 1$.

by its price $px_{j\kappa}$, assuming that the prices $p_{j\imath}$ for the other leaders $\imath \neq \kappa$ in industry j are kept constant. It therefore sets the wage w equal to the marginal product of labor. Noting (2), this leads to the first-order condition

$$\begin{aligned}\frac{\partial \pi_{j\kappa}}{\partial p_{j\kappa}} &= x_{j\kappa} + (p_{j\kappa} - w)\left[\frac{\partial x_{j\kappa}}{\partial p_{j\kappa}} + \frac{\partial x_{j\kappa}}{\partial p_j}\frac{\partial p_j}{\partial p_{j\kappa}}\right] \\ &= x_{j\kappa} + (p_{j\kappa} - w)\left[-\varepsilon\frac{x_{j\kappa}}{p_{j\kappa}} + (\varepsilon - 1)\frac{x_{j\kappa}}{p_j}\frac{1}{a_j}\left(\frac{p_j}{p_{j\kappa}}\right)^{\varepsilon}\right] \\ &= x_{j\kappa}\left\{1 + \left(1 - \frac{w}{p_{j\kappa}}\right)\left[-\varepsilon + \frac{\varepsilon - 1}{a_j}\left(\frac{p_j}{p_{j\kappa}}\right)^{\varepsilon-1}\right]\right\} = 0. \qquad (5)\end{aligned}$$

Because the conditions (2) and (5) hold for all $\kappa = 1, ..., a_j$, the symmetry $p_{j\kappa} = p_j$ holds throughout all κ. This, (1), (2), (4) and (5) yield

$$\begin{aligned}&p_{j\kappa}/w = \left\{1 - [\varepsilon + (1-\varepsilon)/a_j]^{-1}\right\}^{-1} \doteq \Phi(a_j), \quad \Phi' < 0, \quad a_j p_{j\kappa} x_{j\kappa} = 1, \\ &\pi_{j\kappa} = (p_{j\kappa} - w)x_{j\kappa} = \left[1 - \Phi(a_j)^{-1}\right]p_{j\kappa}x_{j\kappa} = \left[1 - \Phi(a_j)^{-1}\right]/a_j, \\ &x_j = a_j x_{j\kappa} = 1/p_{j\kappa} = 1/[\Phi(a_j)w]. \qquad (6)\end{aligned}$$

In order to make product market competition effective, I assume that the entry of the second leader decreases the first leader's mark-up:

$$\mu > \Phi(2). \qquad (7)$$

If anyone invests in imitative R&D to enter an industry with one leader, then his prospective profit is $\pi_{j\kappa}\big|_{a_j=2}$, but if he invests (with the same cost) in imitative R&D to enter an industry with more than two leaders, then his prospective profit is $\pi_{j\kappa}\big|_{a_j>2}$. Because, by (6), the profit is smaller with more than two leaders, $\pi_{j\kappa}\big|_{a_j=2} > \pi_{j\kappa}\big|_{a_j>2}$, investors invest in imitative R&D only to enter in one-leader industries. I summarize:

Proposition 1 *Each industry has one or two leaders. In one-leader industries the followers imitate and in two-leader industries the leaders innovate.*

I denote the set of one-leader industries by $\Theta \subset [0, 1]$, and the relative proportion of one-leader industries (two-leader industries), α (β) by

$$\alpha = \int_{j\in\Theta} dj, \quad \beta \doteq \int_{j\notin\Theta} dj = 1 - \alpha. \qquad (8)$$

Noting $a_j = 2$, (3), (6), (7) and (8), a firm's profit π (Π) and and total output x_α (x_β) in one-leader (two-leader) industry are given by

$$\Pi_j\Big|_{j\in\Theta} = \Pi,\ \ \Pi_{j\kappa}\Big|_{j\notin\Theta,\,a_j=2} \doteq [1-1/\Phi(2)]/2 \doteq \pi \in (0,\Pi/2),\ \ 1/\Phi(2) = 1-2\pi,$$
$$x_\beta = x_j\Big|_{j\notin\Theta,\,a_j=2} = \frac{1}{\Phi(2)w} = \frac{1-2\pi}{w} > x_\alpha = x_j\Big|_{j\in\Theta} = \frac{1}{\mu w} = \frac{1-\Pi}{w}. \tag{9}$$

The higher the elasticity of substitution between the products, ε, the closer $\Phi(2)$ to its lower limit 1 and the smaller π.[3] There are now two measures of competition: a competing firm's profit π and the relative proportion of the competing (two-leader) industries, β. The purpose of this paper is to examine the growth and welfare effects of these.

Noting (1), (3), (8) and (9), and summing up throughout all firms and industries, we obtain that the employment of labor in production, x, and total output y are determined as follows:

$$x \doteq \alpha x_\alpha + (1-\alpha)x_\beta = \frac{\varphi}{w},\ \ \varphi(\alpha,\pi) \doteq (1-\Pi)\alpha + (1-\alpha)(1-2\pi) < 1-2\pi,$$
$$\frac{\partial\varphi}{\partial\alpha} = 2\pi - \Pi < 0,\quad \frac{\partial\varphi}{\partial\pi} = 2(\alpha-1) < 0,\quad x_\alpha = (1-\Pi)\frac{x}{\varphi},\ \ \frac{\partial}{\partial\pi}\Big(\frac{x_\alpha}{x}\Big) > 0,$$
$$x_\beta = (1-2\pi)\frac{x}{\varphi} > x_\alpha,\ \ \frac{\partial}{\partial\pi}\Big(\frac{x_\beta}{x}\Big) = (2\pi-1)\frac{x}{\varphi^2}\frac{\partial\varphi}{\partial\pi} - 2\frac{x}{\varphi} = 2(\Pi-1)\frac{\alpha x}{\varphi^2} < 0;$$
$$y = Bx_\alpha^\alpha x_\beta^{1-\alpha} = \chi(\alpha,\pi)xB,\quad \chi(\alpha,\pi) \doteq (1-\Pi)^\alpha(1-2\pi)^{1-\alpha}/\varphi(\alpha,\pi),$$
$$\log B \doteq \int_0^1 \log B_j dj, \tag{10}$$

where x is employment, $\varphi = wx$ wage expenditure and B the average level of productivity in the production of intermediate goods $j \in [0,1]$. A decrease in a competing firm's profit π increases employment x and total wages in production, $\partial\varphi/\partial\pi < 0$. Because competing industries $j \notin \Theta$ employ more than monopoly industries $j \in \Theta$ (i.e. $x_\beta > x_\alpha$), a smaller proportion α of monopoly industries raises employment x and total wages φ in production.

Finally, given (10), we obtain

$$\frac{1}{\chi}\frac{\partial\chi}{\partial\pi} = \frac{\partial\log\chi}{\partial\pi} = \frac{2(\alpha-1)}{1-2\pi} - \frac{1}{\varphi}\frac{\partial\varphi}{\partial\pi} = \underbrace{2(\alpha-1)}_{-}\underbrace{\Big[\frac{1}{1-2\pi} - \frac{1}{\varphi}\Big]}_{-} > 0$$

and the following result:

[3]In papers that consider imitation in a framework with no growth, it is common to measure competition directly by the level of profit [Cf. Kanniainen and Stenbacka (2000)].

Proposition 2 *A higher competitive profit π is associated with a higher productivity χ of labor in production, $\partial\chi/\partial\pi > 0$.*

Total output $y = Bx_\alpha^\alpha x_\beta^{1-\alpha}$ must be maximized subject to the allocation of labor between one-leader and two-leader industries, $x = \alpha x_\alpha + (1-\alpha)x_\beta$, keeping total employment in production, x, constant. Output y is at the maximum, if all industries employ the same amount of labor, $x_\alpha = x_\beta$, and this holds true only if the two-leader industries collude and set monopoly prices, $\pi = \Pi/2$. A lower profit in the two-leader industries transfers labor into two-leader industries (i.e. x_α falls and x_β rises by (10)). The greater the difference $x_\beta - x_\alpha$, the lower y for given x.

3 Research

Given proposition 1, there are three types of R&D firms: the first leader (successful innovator), which I call firm 1, the second leader (successful imitator), which I call firm 2, and followers, which I call firm 0. In two-leader industry $j \notin \Theta$, firms 1 and 2 innovate and no firm imitates. The technological change of firm $\kappa \in \{1,2\}$ is characterized by a Poisson process $q_{j\kappa}$ in which the arrival rate of innovations, $\Lambda_{j\kappa}$, is in fixed proportion λ to the firm's own labor input $l_{j\kappa}$:

$$\Lambda_{j\kappa} = \lambda l_{j\kappa} \text{ for } j \notin \Theta \text{ and } \kappa \in \{1,2\}. \tag{11}$$

During a short time interval $d\nu$, there is an innovation $dq_{j\kappa} = 1$ in firm κ with probability $\Lambda_{j\kappa}d\nu$, and no innovation $dq_{j\kappa} = 0$ with probability $1 - \Lambda_{j\kappa}d\nu$.

In one-leader industry $j \in \Theta$, the representative follower (firm 0) imitates and no firm innovates. The technological change of firm 0 is characterized by a Poisson process Q_j in which the arrival rate of imitations is given by

$$\Gamma_j = \gamma l_{j0}^{1-\varsigma}\ell_\beta^\varsigma \text{ for } j \in \Theta, \tag{12}$$

where l_{j0} is the firm's own labor input, ℓ_β the average labor input to innovative R&D in the economy and $\gamma > 0$ and $\varsigma \in (0,1)$ are constants. The input ℓ_β characterizes the immediate spillover of knowledge from innovative to imitative R&D.[4] During a short time interval $d\nu$, there is an imitation $dQ_j = 1$ with probability $\Gamma_j d\nu$, and no imitation $dQ_j = 0$ with probability $1 - \Gamma_j d\nu$.

The invention of a new technology in industry j raises the number of technology in that industry, t_j, by one and the level of productivity, $B_j^{t_j}$, by $\mu > 1$. Given this and (10), the average productivity in the economy, B, is a function of the technologies of all industries, $\{t_k\}$, as follows:

$$\log B^{\{t_k\}} \doteq \int_0^1 \log B_j^{t_j} dj, \quad B^{t_j+1}/B_j^{t_j} = \mu, \tag{13}$$

where $\{t_k\}$ denotes a vector that consists of t_k for all k. The arrival rate of innovations in industry $j \notin \Theta$ is the sum of the arrival rates of both firms in the industry, $\Lambda_{j1} + \Lambda_{j2}$. The average growth rate of B_j due to technological change in industry j in the stationary state is then given by

$$E\big[\log B_j^{t_j+1} - \log B_j^{t_j}\big] = (\Lambda_{j1} + \Lambda_{j2}) \log \mu,$$

where E is the expectation operator.[5] Because only industries $j \notin \Theta$ innovate, then, noting (11), the average growth rate of the average productivity B in the stationary state is given by

$$\begin{aligned} g &\doteq \int_{j\notin\Theta} E\big[\log B_j^{t_j+1} - \log B_j^{t_j}\big] dj = (\log \mu) \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj \\ &= \lambda \int_{j\notin\Theta} (l_{j1} + l_{j2}) dj. \end{aligned} \tag{14}$$

Total employment in R&D is given by

$$l \doteq \int_{j\notin\Theta} (l_{j1} + l_{j2}) dj + \int_{j\in\Theta} l_j dj. \tag{15}$$

There exists a fixed number N of households, each supplying one labor unit. Total labor supply N is equal to inputs in production, x, and R&D, l:

$$N = x + l. \tag{16}$$

[4] In the case $\varsigma = 0$ investment in imitative R&D were subject to constant returns to scale and there were no equilibrium for a household (see section 4 and Appendix A, especially equations (53) and (54)). With the spillover effect $\varsigma > 0$, the average product of labor in innovative R&D, Γ_j / l_{j0}, falls with the increase in labor input l_{j0}. This property ensures that a household has an equilibrium.

[5] For this, see Aghion and Howitt (1998), p. 59.

The government subsidizes R&D expenditures, but possibly at different rates in innovating and imitating industries. Given 9, we obtain total expenditures from these subsidies as follows:

$$R \doteq \tau_\alpha \int_{j\in\Theta} wl_{j0}dj + \tau_\beta \int_{j\notin\Theta} (wl_{j1} + wl_{j2})dj, \tag{17}$$

where wl_{j0} is expenditure on imitation in firm 0 industry $j \in \Theta$, $wl_{j\kappa}$ expenditure on innovation in firm $\kappa \in \{1,2\}$ in industry $j \notin \Theta$ and $\tau_\alpha \in (-\infty, 1)$ $\big(\tau_\beta \in (-\infty, 1)\big)$ is the subsidy to imitation (innovation). If the government cannot discriminate between innovation and imitation, then $\tau_\alpha = \tau_\beta$.

In industry $j \in \Theta$ firm 0 and in industry $j \notin \Theta$ firms 1 and 2 issue shares to finance their labor expenditure in R&D, net of government subsidies. Because the households invest in these shares, we obtain

$$\begin{aligned} &\sum_{\iota=1}^{N} S_{\iota j0} = (1-\tau_\alpha)wl_{j0} \ \text{ for } j \in \Theta, \\ &\sum_{\iota=1}^{N} S_{\iota j\kappa} = (1-\tau_\beta)wl_{j\kappa} \ \text{ for } \kappa \in \{1,2\} \text{ and } j \notin \Theta, \end{aligned} \tag{18}$$

where wl_{j0} is the imitative expenditure of firm 0 in industry $j \in \Theta$, τ_α the subsidy to it, $wl_{j\kappa}$ the innovative expenditure of firm $\kappa \in \{1,2\}$ in industry $j \notin \Theta$, τ_β subsidy to it, $S_{\iota j0}$ $(S_{\iota j\kappa})$ household ι's investment in firm 0 in industry $j \in \Theta$ (firm κ in industry $j \notin \Theta$), and $\sum_{\iota=1}^{N} S_{\iota j0}$ $\left(\sum_{\iota=1}^{N} S_{\iota j\kappa}\right)$ aggregate investment in firm 0 in industry $j \in \Theta$ (firm κ in industry $j \notin \Theta$). Household ι's relative investment shares in the firms are given by

$$i_{\iota j0} \doteq \frac{S_{\iota j0}}{(1-\tau_\alpha)wl_{j0}} \ \text{ for } j \in \Theta; \quad i_{\iota j\kappa} \doteq \frac{S_{\iota j\kappa}}{(1-\tau_\beta)wl_{j\kappa}} \ \text{ for } j \notin \Theta. \tag{19}$$

I denote household ι's income by A_ι. Total income throughout all households $\iota \in \{1, ..., N\}$ is then equal to income earned in the production of consumption goods, Py, plus income earned in R&D, wl, minus government expenditures R (= lump-sum taxes). Since $Py = 1$ by (2), this yields

$$\sum_{\iota=1}^{N} A_\iota = Py + wl - R = 1 + wl - R. \tag{20}$$

4 Households

The utility for risk-averting household $\iota \in \{1, ..., N\}$ from an infinite stream of consumption beginning at time T is given by

$$U(C_\iota, T) = E \int_T^\infty C_\iota^\sigma e^{-\rho(\nu - T)} d\nu \text{ with } 0 < \sigma < 1 \text{ and } \rho > 0, \tag{21}$$

where ν is time, E the expectation operator, C_ι the index of consumption, ρ the rate of time preference and $1/(1-\sigma)$ is the constant relative risk aversion.

Because investment in shares in R&D firms is the only form of saving in the model, the budget constraint of household ι is given by

$$A_\iota = PC_\iota + \int_{j \in \Theta} S_{\iota j0} dj + \int_{j \notin \Theta} (S_{\iota j1} + S_{\iota j2}) dj, \tag{22}$$

where A_ι is the household's total income, C_ι its consumption, P the consumption price, and $S_{\iota j0}$ ($S_{\iota j\kappa}$) the household's investment in firm 0 in industry $j \in \Theta$ (firm κ in industry $j \notin \Theta$). When household ι has financed a successful R&D firm, it acquires the right to the firm's profit in proportion to its relative investment share. Thus, I define:

$s_{\iota j\kappa}$ household ι's true profit from firm κ in industry j when the uncertainty in R&D is taken into account;

$i_{\iota j\kappa}$ household ι's investment share in firm κ in industry j [Cf. (19)];

$\Pi i_{\iota j\kappa}$ household ι's expected profit from firm $\kappa \in \{1, 2\}$ in industry $j \notin \Theta$ after innovation in firm κ have changed the two-leader industry j into a one-leader industry;

$\pi i_{\iota j0}$ household ι's expected profit from firm 0 in industry $j \in \Theta$ after imitation in firm 0 have changed the one-leader industry j into a two-leader industry.

The changes in the profits of firms in industry j are functions of the increments (dq_{j1}, dq_{j2}, dQ_j) of Poisson processes (q_{j1}, q_{j2}, Q_j) as follows:[6]

$$\begin{aligned} ds_{\iota j\kappa} &= (\Pi i_{\iota j\kappa} - s_{\iota j\kappa}) dq_{j\kappa} - s_{\iota j\kappa} dq_{j(\zeta \neq \kappa)} \text{ when } j \notin \Theta; \\ ds_{\iota j0} &= (\pi i_{\iota j0} - s_{\iota j0}) dQ_j \text{ when } j \in \Theta. \end{aligned} \tag{23}$$

[6]This extends the idea of Wälde (1999a, 1999b).

These functions can be explained as follows. If a household invests in leader κ in industry $j \notin \Theta$, then, in the advent of a success for that leader, $dq_{j\kappa} = 1$, the amount of its share holdings rises up to $\Pi i_{\iota j\kappa}$, $ds_{\iota j\kappa} = \Pi i_{\iota j\kappa} - s_{\iota j\kappa}$, but in the advent of success for the other leader $\zeta \neq \kappa$, its share holdings in leader κ fall down to zero, $ds_{\iota j\kappa} = -s_{\iota j\kappa}$. If a household invests in imitating firm 0 in industry $j \in \Theta$, then, in the advent of a success for the firm, $dQ_j = 1$, the amount of its share holdings rises up to $\pi i_{\iota j0}$, $ds_{\iota j0} = \pi i_{\iota j0} - s_{\iota j0}$.

Household ι's total income A_ι consists of its wage income w (the household supplies one labor unit), its profits $s_{\iota j1}$ from the single leader in each industry $j \in \Theta$, its profits $s_{\iota j1}$ and $s_{\iota j2}$ from the two leaders 1 and 2 in each industry $j \notin \Theta$, minus its share $1/N$ in the government's expenditures R (= the household's lump-sum tax). Given this and (9), we obtain

$$A_\iota = w + \int_{j\in\Theta} s_{\iota j1} dj + \int_{j\notin\Theta} (s_{\iota j1} + s_{\iota j2}) dj - \frac{R}{N}. \tag{24}$$

Household ι maximizes its utility (21) by its investment, $\{S_{\iota j0}\}$ for $j \in \Theta$ and $\{S_{\iota j1}, S_{\iota j2}\}$ for $j \notin \Theta$, subject to its budget constraint (22), the stochastic changes (23) in its profits, the composition of its income, (24), and the determination of its relative investment shares, (19), given the arrival rates $\{\Lambda_{j\kappa}, \Gamma_j\}$, the wage w, the consumption price P, the subsidies $(\tau_\alpha, \tau_\beta)$ and the government's expenditures R. In the households' stationary equilibrium in which the allocation of resources is invariable across technologies, this maximization yields the following results (see Appendix A):

$$\begin{gathered}
\begin{matrix} l_{j\kappa} = \ell_\beta \quad \text{for } j \notin \Theta, \\ l_{j0} = \ell_\alpha \quad \text{for } j \in \Theta, \end{matrix} \qquad \frac{\ell_\alpha}{\ell_\beta} = \psi(\pi, \tau_\alpha, \tau_\beta) \doteq \left[\frac{(1-\tau_\beta)\pi\gamma/2}{(1-\tau_\alpha)\Pi\lambda\mu^\sigma}\right]^{1/\varsigma}, \\
\frac{\partial\psi}{\partial\pi} = \frac{\psi}{\varsigma\pi} > 0, \qquad \frac{\partial\psi}{\partial\tau_\alpha} > 0, \qquad \frac{\partial\psi}{\partial\tau_\beta} < 0, \qquad \psi(\pi, \tau, \tau) = \psi(\pi, 1, 1),
\end{gathered} \tag{25}$$

$$w = \varphi(\alpha, \pi)/(N - l), \tag{26}$$

$$h = \frac{1}{1 + \{1 - [\tau_\alpha\alpha\psi + 2\tau_\beta(1-\alpha)][\alpha\psi + 2(1-\alpha)]^{-1}\}wl}, \tag{27}$$

$$g = \frac{(2\lambda \log\mu) l}{\alpha\psi/(1-\alpha) + 2}, \qquad \rho + \frac{1-\mu^\sigma}{\log\mu} g = \frac{\lambda h \mu^\sigma \Pi}{(1-\tau_\beta) w}. \tag{28}$$

Result (25) says that with a smaller subsidy τ_α to imitative R&D, a bigger subsidy τ_β to innovative R&D or with a lower profit π, investors spend relatively more in innovative than imitative R&D (i.e. a higher ℓ_β/l). With a uniform R&D subsidy $\tau_\alpha = \tau_\beta = \tau$, the relative investment in imitation is independent of the subsidy.

The equations (25) lead to the following result:

Proposition 3 *If there are no competitive profits, $\pi = 0$, then there is no imitation, $l_{j0} = \ell_\alpha = 0$ for $j \in \Theta$.*

With non-diversifiable risk, households hold the shares of all innovating firms in their portfolios. Given this, they have no incentives to invest in imitating R&D unless there are profits during the innovation race.

5 General equilibrium

When innovation occurs in an industry, this industry switches from the group of two-leader industries to that of one-leader industries, and when imitation occurs in an industry, this industry switches from one-leader industries to two-leader industries. In a steady-state equilibrium, every time a new superior-quality product is discovered in some industry, imitation must occur in some other industry.[7] Thus, the rate at which industries leave the group of two-leader industries, $\beta(\Lambda_{j1} + \Lambda_{j2})d\nu$, is equal to the rate at which industries leave the group of one-leader industries, $\alpha\Gamma_j d\nu$. This, (8), (11), (12) and (25) yield $\beta(\Lambda_{j1} + \Lambda_{j2}) = \alpha\Gamma_j$ and

$$\frac{\alpha}{1-\alpha} = \frac{\alpha}{\beta} = \frac{\Lambda_{j1} + \Lambda_{j2}}{\Gamma_j} = \frac{\lambda(l_{j1}^{1-\varsigma} + l_{j2}^{1-\varsigma})}{\gamma l_{j0}^{1-\varsigma}} = \frac{2\lambda\ell_\beta^{1-\varsigma}}{\gamma\ell_\alpha^{1-\varsigma}} = \frac{2\lambda}{\gamma}\psi^{\varsigma-1}.$$

Given this equation, one solves for the proportion of one-leader industries as:

$$\alpha(\pi, \tau_\alpha, \tau_\beta) = \Psi(\psi) \doteq \frac{2\lambda}{2\lambda + \gamma\psi^{1-\varsigma}}, \quad \Psi' = \frac{d\Psi}{d\psi} = (\varsigma - 1)(1-\alpha)\frac{\alpha}{\psi} < 0,$$
$$\partial\alpha/\partial\pi < 0, \ \partial\alpha/\partial\tau_\alpha < 0, \ \partial\alpha/\partial\tau_\beta > 0, \quad \alpha(\pi, \tau, \tau) = \alpha(\pi, 1, 1). \qquad (29)$$

[7] Cf. Segerstrom (1991), p. 817.

Finally, given (10), (25) and (29), one obtains that wage expenditure in production, $wx = \varphi$, depends on the profit π as follows:

$$
\begin{aligned}
&\frac{d\varphi}{d\pi} = \frac{\partial\varphi}{\partial\alpha}\frac{\partial\alpha}{\partial\psi}\frac{\partial\psi}{\partial\pi} + \frac{\partial\varphi}{\partial\pi} = (2\pi - \Pi)\Psi'\frac{\partial\psi}{\partial\pi} + 2(\alpha - 1) \\
&= (1-\alpha)\left[(1-\varsigma)(\Pi - 2\pi)\frac{\alpha}{\psi}\frac{\partial\psi}{\partial\pi} - 2\right] = (1-\alpha)\left[\Big(\frac{1}{\varsigma} - 1\Big)\Big(\frac{\Pi}{\pi} - 2\Big)\alpha - 1\right], \\
&d\varphi/d\pi < 0 \ \text{ for } \pi > \pi_0, \quad d\varphi/d\pi > 0 \ \text{ for } \pi < \pi_0, \\
&\lim_{\pi\to 0}\frac{d\varphi}{d\pi} = \frac{1-\alpha}{\pi}\lim_{\pi\to 0,\,\alpha\to 1}\left[\Big(\frac{1}{\varsigma} - 1\Big)\Big(\Pi - 2\pi\Big)\alpha - \pi\right] = \frac{1-\alpha}{\pi}\Big(\frac{1}{\varsigma} - 1\Big)\Pi > 0, \\
&\lim_{\pi\to\Pi/2}\frac{d\varphi}{d\pi} = \alpha - 1 < 0, \qquad (30)
\end{aligned}
$$

where the constant $\pi_0 \in (1, \mu)$ is defined by the equation

$$(1/\varsigma - 1)(\Pi/\pi_0 - 2)\alpha(\pi_0, \tau_\alpha, \tau_\beta) = 1.$$

Inserting (29) into equations (28) and noting (8), (10), (25), (26), (27) and (30), one obtains

$$
\begin{aligned}
&l(\psi, g) = \left(\frac{1}{\gamma}\psi^\varsigma + \frac{1}{\lambda}\right)\frac{g}{\log\mu}, \quad \frac{\partial l}{\partial\psi} > 0, \quad \frac{\partial l}{\partial g} = \frac{l}{g} > 0, \qquad (31) \\
&\rho + \frac{1-\mu^\sigma}{\log\mu}g = \nabla(l, \tau_\alpha, \tau_\beta, \pi) \doteq \Delta(l, \alpha, \tau_\alpha, \tau_\beta, \pi) \doteq \\
&\frac{\lambda\mu^\sigma\Pi(N-l)/(1-\tau_\beta)}{\varphi(\alpha,\pi) + \big\{1 - \big[\tau_\alpha\alpha\psi + 2\tau_\beta(1-\alpha)\big]\big[\alpha\psi + 2(1-\alpha)\big]^{-1}\big\}\varphi(\alpha,\pi)^2 l/(N-l)}, \\
&\left.\frac{\partial\Delta}{\partial\beta}\right|_{\tau_\alpha=\tau_\beta=0} = -\left.\frac{\partial\Delta}{\partial\alpha}\right|_{\tau_\alpha=\tau_\beta=0} < 0, \quad \left.\frac{\partial\nabla}{\partial\tau_\beta}\right|_{\tau_\alpha=\tau_\beta=0} > 0, \quad \left.\frac{\partial\nabla}{\partial\tau}\right|_{\tau=\tau_\alpha=\tau_\beta} > 0, \\
&\frac{\partial\nabla}{\partial l} < 0, \quad \left.\frac{\partial\nabla}{\partial\pi}\right|_{\tau_\alpha=\tau_\beta=0,\,\pi>\pi_0} > 0, \quad \left.\frac{\partial\nabla}{\partial\pi}\right|_{\tau_\alpha=\tau_\beta=0,\,\pi<\pi_0} < 0. \qquad (32)
\end{aligned}
$$

The equation (31) says that the demand for labor devoted to R&D, l, is in fixed proportion to the growth rate g, and the equation (32) that a household's subjective discount factor $\rho + \frac{1-\mu^\sigma}{\log\mu}g$ is equal to the rate of return to savings, ∇. These two equations form a system of two unknown variables l and g. The equilibrium is in the intersection Q of these. By the comparative statics of this system, one obtains

$$g=\hat{g}(\pi,\tau_\alpha,\tau_\beta)=\tilde{g}(\alpha,\pi,\tau_\alpha,\tau_\beta),\ \left.\frac{\partial\tilde{g}}{\partial\beta}\right|_{\tau_\alpha=\tau_\beta=0}<0\ \Leftrightarrow\ \left.\frac{\partial\hat{g}}{\partial\pi}\right|_{\tau_\alpha=\tau_\beta=0,\,\pi>\pi_0}>0$$
$$\Leftrightarrow\ \left.\frac{\partial\hat{g}}{\partial\pi}\right|_{\tau_\alpha=\tau_\beta=0,\,\pi<\pi_0}<0\ \Leftrightarrow\ \left.\frac{\partial\hat{g}}{\partial\tau}\right|_{\tau_\alpha=\tau_\beta=\tau}>0\ \Leftrightarrow\ \left.\frac{\partial\hat{g}}{\partial\tau_\beta}\right|_{\tau_\alpha=\tau_\beta=0}>0$$
$$\Leftrightarrow\ \underbrace{\left(\frac{1}{\gamma}\psi^\varsigma+\frac{1}{\lambda}\right)}_{+}\underbrace{\frac{\partial\nabla}{\partial l}}_{-}<\underbrace{1-\mu^\sigma}_{-}. \tag{33}$$

Unfortunately, these results are ambiguous, because an increase in the growth rate g lowers both a household's subjective discount factor $\rho+\frac{1-\mu^\sigma}{\log\mu}g$ and the rate of return to savings, ∇, through lower employment l in R&D. Empirically, one can assume that a small targeted subsidy τ_β to innovative R&D is growth enhancing. In such a case, the effect through the rate of return to savings outweighs that through the subjective discount factor. The results (33) can then be rephrased as follows:

Proposition 4 *A higher proportion β of competing industries in the economy decreases the growth rate. A uniform subsidy τ to all R&D is growth enhancing. When the competitive profit π is initially lower (higher) than the constant π_0, an increase in it is growth-hampering (growth-enhancing).*

A higher proportion of competing industries raises the demand for labor in production. This decreases labor devoted to R&D and the growth rate. A higher subsidy to all R&D increases investment in R&D and the growth rate. A decrease in the profit is in general growth enhancing because of cost escaping effect, except that at high initial profit margins (i.e. $\pi>\pi_0$) it is outweighed by the "wage effect" as follows. With lower profits, firms charge lower prices, produce more and employ more labor in production.

6 Optimal public policy

The symmetry across the households $\iota=1,...,n$ yields $C_\iota=y/N$. Noting $C_\iota=y/N$, (10), (16), (29) and (31), a single household's consumption relative to the level of productivity, c, can be written as follows:

$$c(g,\alpha,\pi) \doteq \frac{C_\iota}{B^{\{t_k\}}} = \frac{y/N}{B^{\{t_k\}}} = \frac{x}{N}\chi = \left[1 - \frac{l(\psi,g)}{N}\right]\chi$$
$$= \chi(\alpha,\pi)\left[1 - \frac{1}{N} l\left(\Psi^{-1}(\alpha), g\right)\right], \quad \frac{\partial c}{\partial g} = -\frac{\chi}{N}\frac{\partial l}{\partial g} = -\frac{cl}{xg} < 0, \tag{34}$$

where Ψ^{-1} is the inverse function of Ψ. Given this, a single household's utility function (21) takes the form

$$U(C_\iota, T) = E\int_T^\infty c(g,\alpha,\pi)^\sigma \left(B^{\{t_k\}}\right)^\sigma e^{-\rho(\nu - T)} d\nu. \tag{35}$$

On the assumption that the government is benevolent, it maximizes the representative household's welfare (35). I consider two cases:

(a) *First-best policy.* The government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$. Because there is one-to-one correspondence from $(\tau_\alpha, \tau_\beta)$ to (g, α) through (25), (29) and (33), the government can control the growth rate g and the proportion of imitating industries, α, by the subsidies $(\tau_\alpha, \tau_\beta)$. It maximizes social welfare (35) by the growth rate g and the proportion of imitating industries, α.

(b) *Second-best policy.* The government cannot discriminate between innovation and imitation, $\tau_\alpha = \tau_\beta = \tau$. Given (25), (29) and (33), the proportion of imitating industries, α, is wholly exogenous and the growth rate g can be controlled by the uniform subsidy τ. The government then maximizes social welfare (35) by g.

I denote by $\Upsilon(\{t_k\})$ the value of each industry k using current technology t_k, and by $\Upsilon\left(t_j + 1, \{t_{k\neq j}\}\right)$ the value of industry j using technology $t_j + 1$, when other industries $k \neq j$ use current technology t_k. The maximization problems in both the first-best (a) and second-best (b) cases above lead to the Bellman equation

$$\rho\Upsilon(t) = \begin{cases} \max_{g,\alpha}\mathcal{F} & \text{in the case of first-best policy (a)}, \\ \max_g \mathcal{F} & \text{in the case of second-best policy (b)}, \end{cases} \quad \text{where}$$

$$\mathcal{F} \doteq c(g,\alpha,\pi)^\sigma \left(B^{\{t_k\}}\right)^\sigma + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2})\left[\Upsilon\left(t_j+1, \{t_{k\neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right] dj$$
$$= \frac{c(g,\alpha,\pi)^\sigma}{\left(B^{\{t_k\}}\right)^{-\sigma}} + \frac{g}{(1-\alpha)\log\mu}\int_{j\notin\Theta}\left[\Upsilon\left(t_j+1, \{t_{k\neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right] dj. \tag{36}$$

(a) First-best policy. The socially optimal levels for the growth rate g and the proportion of imitating industries, α, are given by [see Appendix B]

$$g^* = \frac{\rho\sigma \log \mu}{(\mu^\sigma - 1)(\sigma + x/l)}, \quad \alpha^* = \frac{\eta}{\eta + l/x}, \tag{37}$$

where

$$\eta(g, \alpha, \pi) \doteq -\frac{\alpha}{c}\frac{\partial c}{\partial \alpha} \tag{38}$$

is the elasticity of consumption with respect to the proportion of imitating industries. Inserting $g = g^*$ from (37) into (28) yields the following result:

Proposition 5 *The welfare-maximizing subsidy to innovative R&D is*

$$\tau_\beta^* = 1 - \frac{hz}{\rho + \frac{1-\mu^\sigma}{\log \mu} g^*} = 1 - \left(\sigma\frac{l}{x} + 1\right)\frac{hz}{\rho}. \tag{39}$$

If the government cannot discriminate between innovative and imitative R&D, then this is also the welfare-maximizing uniform subsidy to all R&D.

In explaining proposition 5, the starting point is that τ_α^* determines the welfare-maximizing levels for both subsidies $(\tau_\alpha, \tau_\beta)$. The next proposition considers how much τ_α and τ_β should be differentiated. The lower the propensity to consume, h, the average rate of return to investment in imitative R&D, z, or the relative proportion of workers in R&D, l/x, the more R&D should be subsidized. The promotion of R&D by subsidies speeds up growth and increases future consumption and welfare. On the other hand, it crowds out the production of consumption goods through higher wages and decreases welfare. The subsidies to R&D should be increased as long as the former growth effect dominates over the latter current-consumption effect. The lower the propensity to consume, h, the weaker the current-consumption effect and the higher the optimal subsidy. The lower the "private" rate of return z to imitative R&D, the higher subsidy is needed to cover the gap between it and the social rate of return to imitative R&D. Finally, the lower the relative proportion of workers in R&D, l/x, the less a proportional increase in R&D crowds out current consumption and the higher the optimal subsidy.

Inserting (37) into (25), we obtain [see Appendix C]:

Proposition 6 *If the government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$, then the welfare-maximizing subsidy to imitative R&D, τ_α^*, is determined by*

$$\frac{1-\tau_\beta}{1-\tau_\alpha^*} = \left[\frac{\lambda}{\gamma} + \frac{\left(\frac{\lambda}{\gamma}+1\right)\xi}{\frac{\gamma}{2}\left(1+\frac{1}{\eta}\frac{l}{x}\right)-\xi}\right]\mu^\sigma \frac{\Pi}{\pi}.$$

The bigger the relative profit in the two-leader industries, π/Π, or the less workers there are in R&D (i.e. the smaller l/x), the more the government should prefer innovation to imitation (i.e. the higher τ_β^* relative to τ_α and the lower the ratio $(1-\tau_\beta^*)/(1-\tau_\alpha)$). The profit in the two-leader industries, π, and the subsidy to imitative R&D, τ_α, are strategic substitutes, for they both increase the incentives to imitate. Therefore, at the optimum, the increase in π relative to Π should lead to the decrease in τ_α relative to τ_β.

Noting (34), (36) and proposition 2, we obtain $\partial\chi/\partial\pi > 0$, $\partial c/\partial\pi > 0$, $\partial\mathcal{F}/\partial\pi > 0$ and the following result:

Proposition 7 *In the first-best case $\tau_\alpha \neq \tau_\beta$, an increase in the competitive profit π is welfare-enhancing (i.e. $\mathcal{F}$ rises).*

(b) Second-best policy. In this case, the rule (39) determines the uniform subsidy $\tau = \tau_\alpha = \tau_\beta$ and the welfare-maximizing level α^* of α is given by (37). Because α is an decreasing function of π through ψ [cf. (25) and (29)], there is a welfare-maximizing level π^* for the mark-up factor π in the two-leader industries as well. This result can be rephrased as follows:

Proposition 8 *If the government cannot discriminate between innovation and imitation but uses the uniform subsidy $\tau = \tau_\alpha = \tau_\beta$ optimally, then there is an "inverted-U" relationship between the competitive profit π and welfare.*

A decrease in the profit has two opposing effects. It decreases the consumption price and thereby increases current consumption and welfare. On the other hand, it transfers labor from R&D to the production of goods. This decreases the growth rate, future consumption and welfare. These opposing effects are balanced for $\pi = \pi^*$ and $\alpha = \alpha^*$.

7 Conclusions

This paper examines a multi-industry economy in which growth is generated by creative destruction. In each industry, a firm creating the newest technology by a successful innovative R&D project crowds out the other firms with older technologies from the market and becomes the first leader of the industry. A firm creating a copy of the newest technology starts producing a close substitute for the innovator's product and establishes an innovation race with the first leader. There is systematic investment risk that the households cannot eliminate by diversification. The government subsidizes R&D, possibly discriminating between innovative and imitative R&D, and affects the competing firms' mark up rate through competition policy.

Mukoyama (2003) assumes that firms' products are perfect substitutes and shows that firms' are ready to imitate in order to be able to participate in the innovation race, although during the race there are no profits. After the assumption of fully diversifiable risk is relaxed, this is no more possible. With non-diversifiable risk, households hold the shares of all innovating firms in their portfolios. Given this, they have no incentives to invest in imitating R&D unless there are profits during the innovation race.

In the literature, the degree of product market competition has been represented by either the elasticity of the substitution of firms' products [Cf. Aghion et al. (1997, 2001)] or the proportion of competing industries in the economy [Cf. Mukoyama (2003)]. This paper shows that with non-diversifiable risk these two representations are qualitatively different. The elasticity of product substitution is otherwise growth enhancing, except that at high elasticities it is outweighed by the "wage effect" as follows. With more intense PMC firms charge lower prices, produce more and employ more labor in production. The proportion of competing industries is negatively associated with the growth rate.

The other findings of this paper are as follows. In the first-best case where the government is able to set different subsidies to innovation and imitation, a higher elasticity of product substitution diminishes welfare. It transfers labor from the one-leader industries, which contain a recent innovator and a number of imitating followers, into the two-leader industries, in which an innovator and a recent imitator are in an innovation race. Because the decrease in output in the one-leader industries outweighs the increase in output in the two-leader industries in terms of consumption, consumption and welfare must fall.

In the second-best case where the government cannot discriminate between innovation and imitation but uses a uniform subsidy to all R&D, there is an "inverted-U" relationship between the elasticity of product substitution and social welfare. A higher elasticity has two opposing effects. It decreases the consumption price and thereby increases current consumption and welfare. On the other hand, it transfers labor from R&D to the production of goods and thereby decreases the growth rate, future consumption and welfare. The elasticity of product substitution is at its welfare-maximizing level when these two opposing effects are balanced. When it is below (above) the welfare-maximizing level, it should be increased (decreased).

Appendix

A. Results (25)-(33)

I denote:

$\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)$ the value of receiving profits $s_{\iota k\upsilon}$ from all firms υ in all industries k using current technology t_k.

$\Omega\big(\Pi i_{\iota j\kappa},0,\{s_{\iota(k\neq j)\upsilon}\},t_j+1,\{t_{k\neq j}\}\big)$ the value of receiving the profit $\Pi i_{\iota j\kappa}$ from firm κ in industry $j\notin\Theta$ using technology t_j+1, but receiving no profits from the other firm which was a leader in that industry when technology t_j was used, and receiving profits $s_{\iota(k\neq j)\upsilon}$ from all firms υ in other industries $k\neq j$ with current technology t_k.

$\Omega\big(\pi i_{\iota j1},\pi i_{\iota j2},\{s_{\iota(k\neq j)\upsilon}\},\{t_k\}\big)$ the value of receiving profits $\pi i_{\iota j\kappa}$ from firms $\kappa\in\{1,2\}$ in industry $j\in\Theta$, but receiving profits $s_{\iota(k\neq j)\upsilon}$ from all firms υ in the other industries $k\neq j$ with current technology t_k.

The Bellman equation associated with the household's maximization is[8]

$$\rho\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)=\max_{S_{\iota j}\geq 0\text{ for all }j}\Xi_\iota, \tag{40}$$

[8] Cf. Dixit and Pindyck (1994).

where

$$\Xi_\iota \doteq C_\iota^\sigma + \int_{j\in\Theta} \Gamma_j \Big[\Omega\big(\pi i_{\iota j1}, \pi i_{\iota j1}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)\Big] dj$$
$$+ \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \Big[\Omega\big(\Pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)\Big] dj. \tag{41}$$

Because $\partial C_\iota / \partial S_{\iota j\kappa} = -1/P$ by (22), the first-order conditions are given by

$$\Lambda_{j\kappa} \frac{d}{dS_{\iota j\kappa}} \Big[\Omega\big(\Pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)\Big] = \frac{\sigma}{P} C_\iota^{\sigma-1}$$
$$\text{for } j \notin \Theta \text{ and } \kappa \in \{1, 2\}, \tag{42}$$

$$\Gamma_j \frac{d}{dS_{\iota j0}} \Big[\Omega\big(\pi i_{\iota j1}, \pi i_{\iota j2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) - \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)\Big] = \frac{\sigma}{P} C_\iota^{\sigma-1}$$
$$\text{for } j \in \Theta. \tag{43}$$

I try the solution that for each household ι the propensity to consume, h_ι, and the subjective interest rate r_ι are independent of income A_ι, i.e. $PC_\iota = h_\iota A_\iota$ and $\Omega = C_\iota^\sigma / r_\iota$. Let us denote variables depending on technology t_k by superscript t_k. Since according to (24) income $A_\iota^{\{t_k\}}$ depends directly on variables $\{s_{\iota k}^{t_k}\}$, I denote $A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\})$. Assuming that h_ι is invariant across technologies yields

$$P^{\{t_k\}} C_\iota^{\{t_k\}} = h_\iota A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\}). \tag{44}$$

The share in the next innovator t_j+1 is determined by investment under the present technology t_j, $s_{\iota j\kappa}^{t_j+1} = \Pi i_{\iota j\kappa}^{t_j}$ for $j \notin \Theta$. The share in the next imitator is determined by investment under the same technology t_j, $s_{\iota j\kappa}^{t_j} = \pi i_{\iota j\kappa}^{t_j}$ for $j \in \Theta$. The value functions are then given by

$$\Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) = \Omega\big(\pi i_{\iota j1}, \pi i_{\iota j2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big) = \frac{1}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma,$$
$$\Omega\big(\Pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) = \frac{1}{r_\iota}\big(C_\iota^{t_j+1, \{t_{k\neq j}\}}\big)^\sigma. \tag{45}$$

Given this, we obtain

$$\frac{\partial \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big)}{\partial S_{\iota j}^{t_j}} = 0. \tag{46}$$

From (19), (24), (44), (45), $s^{t_j+1}_{\iota j\kappa} = \Pi i^{\cdot t_j}_{\iota j\kappa}$ for $j \notin \Theta$, and $s^{t_j}_{\iota j\kappa} = \pi i^{\cdot t_j}_{\iota j\kappa}$ for $j \in \Theta$ it follows that

$$\frac{\partial s^{t_j+1}_{\iota j\kappa}}{\partial i^{\cdot t_j}_{\iota j\kappa}} = \Pi \text{ for } j \notin \Theta, \quad \frac{\partial s^{t_j}_{\iota j0}}{\partial i^{\cdot t_j}_{\iota j0}} = \pi \text{ for } j \in \Theta, \quad \frac{\partial A^{t_j+1,\{t_{k\neq j}\}}_{\iota}}{\partial s^{t_j+1}_{\iota j\kappa}} = \frac{\partial A^{\{t_k\}}_{\iota}}{\partial s^{t_j}_{\iota j\kappa}} = 1,$$

$$\frac{\partial i^{\cdot t_j}_{\iota j0}}{\partial S^{t_j}_{\iota j0}} = \frac{1}{(1-\tau_\alpha) w^{\{t_k\}} l^{\{t_k\}}_{j0}} \text{ for } j \in \Theta, \quad \frac{\partial i^{\cdot t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}} = \frac{1}{(1-\tau_\beta) w^{\{t_k\}} l^{\{t_k\}}_{j\kappa}} \text{ for } j \notin \Theta,$$

$$\frac{\partial \Omega\big(\Pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big)}{\partial S^{t_j}_{\iota j\kappa}}$$

$$= \frac{\sigma}{r_\iota}\big(C^{t_j+1,\{t_{k\neq j}\}}_{\iota}\big)^{\sigma-1} \underbrace{\frac{\partial C^{t_j+1,\{t_{k\neq j}\}}_{\iota}}{\partial A^{t_j+1,\{t_{k\neq j}\}}_{\iota}}}_{h_\iota / P^{t_j+1,\{t_{k\neq j}\}}} \underbrace{\frac{\partial A^{t_j+1,\{t_{k\neq j}\}}_{\iota}}{\partial s^{t_j+1}_{\iota j\kappa}}}_{=1} \underbrace{\frac{\partial s^{t_j+1}_{\iota j\kappa}}{\partial i^{\cdot t_j}_{\iota j\kappa}}}_{=\pi} \frac{\partial i^{\cdot t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}}$$

$$= \frac{\Pi\sigma h_\iota \big(C^{t_j+1,\{t_{k\neq j}\}}_{\iota}\big)^{\sigma-1}}{r_\iota P^{t_j+1,\{t_{k\neq j}\}}} \frac{\partial i^{\cdot t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}} = \frac{\Pi h_\iota \sigma \big(C^{t_j+1,\{t_{k\neq j}\}}_{\iota}\big)^{\sigma-1}}{(1-\tau_\beta) r_\iota w^{\{t_k\}} P^{t_j+1,\{t_{k\neq j}\}} l^{\{t_k\}}_{j\kappa}} \quad \text{for } j \notin \Theta, \tag{47}$$

$$\frac{\partial \Omega\big(\pi i_{\iota j1}, \pi i_{\iota j2}, \{s_{\iota(k\neq j)\upsilon}\}, \{t_k\}\big)}{\partial S^{t_j}_{\iota j0}} = \frac{\sigma}{r_\iota}\big(C^{\{t_k\}}_{\iota}\big)^{\sigma-1} \underbrace{\frac{\partial C^{\{t_k\}}_{\iota}}{\partial A^{\{t_k\}}_{\iota}}}_{=h_\iota/P^{\{t_k\}}} \underbrace{\frac{\partial A^{\{t_k\}}_{\iota}}{s^{t_j}_{\iota j0}}}_{=1} \underbrace{\frac{s^{t_j}_{\iota j0}}{\partial i^{t}_{\iota j0}}}_{=\pi} \frac{\partial i^{t}_{\iota j0}}{\partial S^{t}_{\iota j0}}$$

$$= \frac{\pi\sigma h_\iota}{r_\iota P^{\{t_k\}}}\big(C^{\{t_k\}}_{\iota}\big)^{\sigma-1} \frac{\partial i^{t}_{\iota j0}}{\partial S^{t}_{\iota j0}} = \frac{\pi h_\iota \sigma \big(C^{\{t_k\}}_{\iota}\big)^{\sigma-1}}{(1-\tau_\alpha) r_\iota w^{\{t_k\}} P^{\{t_k\}} l^{\{t_k\}}_{j0}} \quad \text{for } j \in \Theta. \tag{48}$$

I focus on a stationary equilibrium where the growth rate g and the allocation of labor, $(l_{j\kappa}, x)$, are invariant across technologies. Given (2), (10), (13) and (16), this implies

$$l^{\{t_k\}}_{j\kappa} = l_{j\kappa}, \quad x^{\{t_k\}} = x = N - l, \quad w^{\{t_k\}} = w = x/\varphi,$$
$$\frac{P^{\{t_k\}}}{P^{t_j+1,\{t_{k\neq j}\}}} = \frac{C^{t_j+1,\{t_{k\neq j}\}}_{\iota}}{C^{\{t_k\}}_{\iota}} = \frac{A^{t_j+1,\{t_{k\neq j}\}}_{\iota}}{A^{\{t_k\}}_{\iota}} = \frac{y^{t_j+1,\{t_{k\neq j}\}}}{y^{\{t_k\}}} = \frac{B^{t_j+1,\{t_{k\neq j}\}}}{B^{\{t_k\}}} = \mu. \tag{49}$$

Inserting (14), (41), (44), (45), (49) and $g \doteq \int_{j\notin\Theta} l_j dj$ into (40) yields

$$0 = \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj + \int_{j\in\Theta} \Gamma_j dj\Big] \Omega\big(\{s_{\iota k\upsilon}\}, \{t_k\}\big) - \big(C^{\{t_k\}}_{\iota}\big)^{\sigma}$$
$$- \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \Omega\big(\Pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)\upsilon}\}, t_j+1, \{t_{k\neq j}\}\big) dj$$

$$
\begin{aligned}
&\quad - \int_{j\in\Theta} \Gamma_j \Omega\big(\pi i_{\iota j1}, \pi i_{\iota j2}, \{s_{\iota(k\neq j)v}\}, \{t_k\}\big) dj \\
&= \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj\Big] \frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma \\
&\quad - \int_{j\notin\Theta} \sum_{\kappa=1,2} \frac{\Lambda_{j\kappa}}{r_\iota} \big(C_\iota^{\{t_j+1\},\{t_{k\neq j}\}}\big)^\sigma dj \\
&= \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj\Big] \frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma - \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \frac{\mu^\sigma}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma dj \\
&= \frac{1}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma \Big[\rho + (1-\mu^\sigma) \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj - r_\iota\Big] \\
&= \frac{1}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma \Big[\rho - r_\iota + \frac{1-\mu^\sigma}{\log\,\mu} g\Big].
\end{aligned}
$$

This equation is equivalent to

$$
r_\iota = \rho + \frac{1-\mu^\sigma}{\log\,\mu} g. \tag{50}
$$

Because there is symmetry throughout all households ι, their propensity to consume is equal, $h_\iota = h$. This, (17), (18), (20), (22), (24) and (44) yield

$$
\begin{aligned}
wl - R &= w \int_{j\in\Theta} l_{j0} dj + w \int_{j\notin\Theta} (l_{j1} + l_{j2}) dj - R \\
&= (1-\tau_\alpha) w \int_{j\in\Theta} l_{j0} dj + (1-\tau_\beta) w \int_{j\notin\Theta} (l_{j1} + l_{j2}) dj \\
&= \sum_{\iota=1}^N \Big[\int_{j\in\Theta} S_{\iota j0} dj + \int_{j\notin\Theta} (S_{\iota j1} + S_{\iota j2}) dj\Big] = \sum_{\iota=1}^N (A_\iota - PC_\iota) \\
&= (1-h) \sum_{\iota=1}^N A_\iota = (1-h)(1 + wl - R).
\end{aligned}
$$

Solving for the propensity to consume, we obtain

$$
h_\iota = h = (1 + wl - R)^{-1}. \tag{51}
$$

Given (10) and (16), we obtain the wage

$$
w = \varphi/x = \varphi(\alpha, \pi)/(N - l). \tag{52}
$$

I define the rate of return to imitative R&D by $z \doteq \pi\Gamma_j/(wl_{j0})$. Inserting this, (11), (12), (46), (47), (48) and (9) into (42) and (43), we obtain

$$\begin{aligned}
&\frac{\Pi h\sigma\mu^\sigma\left(C_\iota^{\{t_k\}}\right)^{\sigma-1}\lambda}{(1-\tau_\beta)\left(\rho+\frac{1-\mu^\sigma}{\log\mu}g\right)wP^{\{t_k\}}} = \frac{\sigma\Pi h_\iota\mu^\sigma\Lambda_{j\kappa}\left(C_\iota^{\{t_k\}}\right)^{\sigma-1}}{(1-\tau_\beta)r_\iota wl_{j\kappa}P^{\{t_k\}}} \\
&= \frac{\sigma\Pi h_\iota\Lambda_{j\kappa}\left(C_\iota^{t_j+1,\{t_{k\neq j}\}}\right)^{\sigma-1}}{(1-\tau_\beta)r_\iota wl_{j\kappa}P^{t_{t_j+1,\{t_{k\neq j}\}}}} = \Lambda_{j\kappa}\frac{d}{dS_{\iota j\kappa}}\Omega\left(\Pi i_{\iota j},\{s_{\iota(k\neq j)}\},t_j+1,\{t_{k\neq j}\}\right) \\
&= \frac{\sigma}{P^{\{t_k\}}}\left(C_\iota^{\{t_k\}}\right)^{\sigma-1} \text{ for } j\notin\Theta \text{ and } \kappa\in\{1,2\}, \qquad (53)\\
&\frac{\pi h\sigma\left(C_\iota^{\{t_k\}}\right)^{\sigma-1}\gamma l_{j0}^{-\varsigma}\ell_\beta^\varsigma}{(1-\tau_\alpha)\left(\rho+\frac{1-\mu^\sigma}{\log\mu}g\right)wP^{\{t_k\}}} = \frac{\sigma h\left(C_\iota^{\{t_k\}}\right)^{\sigma-1}z}{(1-\tau_\alpha)r_\iota wl_{j0}P^{\{t_k\}}} = \frac{\sigma\pi h_\iota\Gamma_j\left(C_\iota^{\{t_k\}}\right)^{\sigma-1}}{(1-\tau_\alpha)r_\iota wl_{j0}P^{\{t_k\}}} \\
&= \Gamma_j\frac{d}{dS_{\iota j0}}\Omega\left(\{\pi i_{\iota j1},\pi i_{\iota j2},\{s_{\iota m(k\neq j)}\},\{t_k\}\right) = \frac{\sigma}{P^{\{t_k\}}}\left(C_\iota^{\{t_k\}}\right)^{\sigma-1} \text{ for } j\in\Theta. \qquad (54)
\end{aligned}$$

Given equations (53) and (54) and (9), we obtain

$$\begin{aligned}
&\begin{matrix} l_{j\kappa}=\ell_\beta & \text{for } j\notin\Theta, \\ l_{j0}=\ell_\alpha & \text{for } j\in\Theta, \end{matrix} \quad \frac{\ell_\alpha}{\ell_\beta} = \psi(\pi,\tau_\alpha,\tau_\beta) \doteq \left[\frac{(1-\tau_\beta)\pi\gamma/2}{(1-\tau_\alpha)\Pi\lambda\mu^\sigma}\right]^{1/\varsigma}, \\
&\partial\psi/\partial\pi>0, \quad \partial\psi/\partial\tau_\alpha>0, \quad \partial\psi/\partial\tau_\beta<0, \quad [\partial\psi/\partial\tau]_{\tau_\alpha=\tau_\beta=\tau}=0. \qquad (55)
\end{aligned}$$

Equations (2), (8), (11), (14), (15), (17), (51), (54) and (55) yield

$$\begin{aligned}
l &= \int_{j\notin\Theta}(l_{j1}+l_{j2})dj + \int_{j\in\Theta}l_jdj = \ell_\beta\int_{j\notin\Theta}dj + \ell_\alpha\int_{j\in\Theta}dj \\
&= \alpha\ell_\alpha + 2(1-\alpha)\ell_\beta = [\alpha\psi+2(1-\alpha)]\ell_\beta, \\
\ell_\beta &= [\alpha\psi+2(1-\alpha)]^{-1}l, \quad \ell_\alpha = [\alpha\psi+2(1-\alpha)]^{-1}\psi l, \\
R &= \tau_\alpha\int_{j\in\Theta}wl_{j0}dj + \tau_\beta\int_{j\notin\Theta}(wl_{j1}+wl_{j2})dj \\
&= \tau_\alpha w\ell_\alpha\int_{j\in\Theta}dj + 2\tau_\beta w\ell_\beta\int_{j\notin\Theta}dj = \tau_\alpha w\ell_\alpha\alpha + 2\tau_\beta w\ell_\beta(1-\alpha) \\
&= \left[\tau_\alpha\alpha\psi+2\tau_\beta(1-\alpha)\right]wl[\alpha\psi+2(1-\alpha)]^{-1}, \\
h &= \frac{Py}{\sum_\iota A_\iota} = \frac{1}{1+wl-R} \\
&= \frac{1}{1+\left\{1-\left[\tau_\alpha\alpha\psi+2\tau_\beta(1-\alpha)\right][\alpha\psi+2(1-\alpha)]^{-1}\right\}wl}, \qquad (56)\\
\Lambda_{j\kappa} &= \lambda\ell_\beta = \lambda[\alpha\psi+2(1-\alpha)]^{-1}l \text{ for } j\notin\Theta \text{ and } \kappa\in\{1,2\},
\end{aligned}$$

$$g = (\log \mu) \int_{j \notin \Theta} (\Lambda_{j1} + \Lambda_{j2}) dj = (2 \log \mu)(1-\alpha)\Lambda_{j\kappa}$$
$$= \frac{(2\lambda \log \mu)(1-\alpha) l}{\alpha\psi + 2(1-\alpha)} = \frac{(2\lambda \log \mu) l}{\alpha\psi/(1-\alpha) + 2}, \tag{57}$$
$$\rho + \frac{1-\mu^\sigma}{\log\ \mu} g = \frac{h\mu^\sigma \Pi \Lambda_{j\kappa}}{(1-\tau_\beta) w l_{j\kappa}} = \frac{\lambda h \mu^\sigma \Pi}{(1-\tau_\beta) w}. \tag{58}$$

Equations (52), (55), (56), (57) and (58) define (25)-(28).

B. Results (37)

Noting (34), the first-order conditions for g and α in the government's maximization are given by

$$\frac{\partial \mathcal{F}}{\partial g} = \sigma c^{\sigma-1} \left(B^{\{t_k\}}\right)^\sigma \frac{\partial c}{\partial g} + \frac{1}{(1-\alpha)\log \mu} \int_{j \notin \Theta} \left[\Upsilon\left(t_j + 1, \{t_{k \neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right] dj$$
$$= 0, \tag{59}$$
$$\frac{\partial \mathcal{F}}{\partial \alpha} = \sigma c^{\sigma-1} \left(B^{\{t_k\}}\right)^\sigma \frac{\partial c}{\partial \alpha} + \frac{g}{(1-\alpha)^2 \log \mu} \int_{j \notin \Theta} \left[\Upsilon\left(t_j + 1, \{t_{k \neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right] dj$$
$$= 0. \tag{60}$$

I try the solution

$$\Upsilon\left(\{t_k\}\right) \doteq \vartheta c^\sigma \left(B^{\{t_k\}}\right)^\sigma, \tag{61}$$

where ϑ is independent of the endogenous variables of the system. Noting (13) and (61), we then obtain

$$\Upsilon\left(t_j + 1, \{t_{k \neq j}\}\right) = \vartheta c^\sigma \left(B^{t_j+1, \{t_k\}}\right)^\sigma = \vartheta \mu^\sigma c^\sigma \left(B^{\{t_k\}}\right)^\sigma = \mu^\sigma \Upsilon\left(\{t_k\}\right). \tag{62}$$

Inserting (61) and (62) into the Bellman equation (36), we obtain

$$0 = c^\sigma \left(B^{\{t_k\}}\right)^\sigma + \frac{g/(1-\alpha)}{\log \mu} \int_{j \notin \Theta} \left[\Upsilon\left(t_j + 1, \{t_{k \neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right] dj - \rho \Upsilon\left(\{t_k\}\right)$$
$$= \Upsilon\left(\{t_k\}\right)[1/\vartheta - \rho + (\mu^\sigma - 1) g/(\log \mu)]$$

and

$$1/\vartheta = \rho - (\mu^\sigma - 1) g/(\log \mu) < \rho. \tag{63}$$

Given (34), (38)-(61), (62) and (63), we obtain

$$\frac{\partial \mathcal{F}}{\partial g} = \sigma c^{\sigma-1} \left(B^{\{t_k\}}\right)^\sigma \frac{\partial c}{\partial g} + \frac{\mu^\sigma - 1}{\log \mu} \Upsilon\left(\{t_k\}\right) = \left(\frac{\sigma}{\vartheta c} \frac{\partial c}{\partial g} + \frac{\mu^\sigma - 1}{\log \mu}\right) \Upsilon\left(\{t_k\}\right)$$

$$
\begin{aligned}
&= \Big(\frac{\mu^\sigma - 1}{\sigma \log \mu} - \frac{l}{\vartheta x g}\Big)\sigma \Upsilon\big(\{t_k\}\big) = \Big[\frac{\mu^\sigma - 1}{\sigma \log \mu} - \Big(\rho - \frac{\mu^\sigma - 1}{\log \mu} g\Big)\frac{l}{xg}\Big]\sigma \Upsilon\big(\{t_k\}\big) \\
&= 0, \qquad (64)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{F}}{\partial \alpha} &= \sigma c^{\sigma-1}\big(B^{\{t_k\}}\big)^\sigma \frac{\partial c}{\partial \alpha} + \frac{(\mu^\sigma - 1)g}{(1-\alpha)\log \mu}\Upsilon\big(\{t_k\}\big) \\
&= \Big(\frac{\sigma}{c\vartheta}\frac{\partial c}{\partial \alpha} + \frac{\mu^\sigma - 1}{\log \mu}\frac{g}{1-\alpha}\Big)\Upsilon\big(\{t_k\}\big) = \Big(\frac{1}{c}\frac{\partial c}{\partial \alpha} + \frac{\mu^\sigma - 1}{\sigma \log \mu}\frac{\vartheta g}{1-\alpha}\Big)\frac{\sigma}{\vartheta}\Upsilon\big(\{t_k\}\big) \\
&= \Big[-\frac{\eta}{\alpha} + \frac{l}{(1-\alpha)x}\Big]\frac{\sigma}{\vartheta}\Upsilon\big(\{t_k\}\big) = 0. \qquad (65)
\end{aligned}
$$

Noting (64), we obtain

$$
g = \frac{\rho\sigma \log \mu}{(\mu^\sigma - 1)(\sigma + x/l)}.
$$

Given (38) and (65), $\partial c/\partial \alpha < 0$, $\eta > 0$ and $\alpha \doteq \eta/(\eta + l/x)$ hold.

C. Proposition 6

Inserting $\alpha = \alpha^*$ and (37) into (29) and noting yields

$$
\frac{\gamma/2}{(\lambda+\gamma)\psi + \xi} = \alpha = \alpha^* = \frac{\eta}{\eta + l/x}.
$$

From this and (25) it follows that

$$
\xi\Big[\frac{(1-\tau_\beta)\pi\gamma}{(1-\tau_\alpha)\Pi\mu^\sigma} - \lambda\Big]^{-1} = \psi = \frac{1}{\lambda+\gamma}\Big[\frac{\gamma}{2}\Big(1 + \frac{1}{\eta}\frac{l}{x}\Big) - \xi\Big].
$$

Solving for the ratio $(1-\tau_\beta^*)/(1-\tau_\alpha)$ and noting (16), we obtain

$$
\frac{1-\tau_\beta^*}{1-\tau_\alpha} = \Big\{\frac{\lambda}{\gamma} + \Big(\frac{\lambda}{\gamma} + 1\Big)\xi\Big[\frac{\gamma}{2}\Big(1 + \frac{1}{\eta}\frac{l}{x}\Big) - \xi\Big]^{-1}\Big\}\mu^\sigma\frac{\Pi}{\pi}.
$$

References:

Aghion, P., Harris, C. and Vickers, J. (1997). Competition and growth with step-by-step innovation: an example. *European Economic Review 41*, 771-782.

Aghion, P., Harris, C., Howitt, P. and Vickers, J. (2001). Competition, Imitation and growth with step-by-step innovation. *Review of Economic Studies 68*, 467-492.

Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory.* Cambridge (Mass.): MIT Press.

Barro, R.J. and Sala-i-Martin, X. (1995). Economic Growth. New York: MacGraw-Hill.

Cheng, L.K. and Tao, Z. (1999). The impact of public policies on innovation and imitation: the role of R&D technology in growth models. *International Economic Review 40*, 187-207.

Dixit, A. and Pindyck, K. (1994). *Investment under Uncertainty.* Princeton: Princeton University Press.

Kanniainen, V. and Stenbacka, R. (2000). Endogenous imitation and implications for technology policy. *Journal of Institutional and Theoretical Economics 156*, 360-381.

Mukoyama, T. (2003). Innovation, imitation, and growth with cumulative technology. *Journal of Monetary Economics 50*, 361-380.

Segerstrom, P.S. (1998). Innovation, imitation, and economic growth. *Journal of Political Economy 99*, 807-827.

Segerstrom, P.S. (1991). Endogenous growth without scale effects. *The American Economic Review 88*, 1290-1310.

Wälde, K. (1999a). A model of creative destruction with undiversifiable risk and optimizing households. *The Economic Journal 109*, C156-C171.

Wälde, K., (1999b). Optimal saving under Poisson uncertainty. *Journal of Economic Theory 87*, 194–217.