TEMPORARY STABILIZATION AND THE REAL OPTION OF WAITING WHEN CONSUMPTION CAN BE DELAYED: AN EXTREME VALUE APPROACH

Francisco Venegas-Martínez*

Finance Research Center, Tec de Monterrey, Mexico City Campus

Abstract

This paper develops, in a small open economy framework, a stochastic model of exchange-rate-based inflation stabilization that is expected to be temporary. Agents have expectations of devaluation driven by a mixed diffusion-jump process where the expected size of a possible devaluation is supposed to have an extreme value distribution of the Fréchet type; as the stylized facts from the Mexican's 1994 and Argentinean's 2001 cases have shown. Consumption and wealth equilibrium dynamics are examined when a stabilization plan is implemented. The case of a stochastic stabilization horizon guided by an exponential distribution is studied. Moreover, this paper also deals with pricing the real option of waiting for postponing consumption when a stabilization plan is about to be abandoned; a claim on a non-traded asset. We also assess the effects of exogenous shocks on consumption and economic welfare. Finally, we use the proposed model to carry out simulation experiments that reproduces the booms of private consumption in the Mexican case of 1989-1994 and the Argentinean case of 2001-2003, which resulted in extreme devaluations.

JEL Classification: F31, F41, G13

Keywords: Inflation stabilization, Contingent claims, Extreme values

1. Introduction

The recent Asian crisis, in 1998, had severe effects in countries managing a fixed exchange-rate, a fixed rate of devaluation, an exchange-rate adjustable band or a convertibility plan, like Brazil, Ecuador, Thailand, Korea, Indonesia, Russia, Bosnia, among others. The public in these countries anticipated that the stabilization plan was going to be temporary, resulting in a large expansion of the consumption of durable goods. According to the International Monetary Fund (IMF), more than 20% of the world countries have one of the above exchange-rate regimes; this proportion is similar to that in 1990.

* Mailing address: Calle del Puente 222, Col. Ejidos de Huipulco, Del. Tlalpan, 04200 México, D.F., E-mail: fvenegas@itesm.mx, Tel: +52 (55)54832254, Fax: +52 (55)54832234.

The Mexican experience in December 1994 brings the credibility of a stabilization plan, initiated at the beginning of 1989, to our attention once again. At the end of 1994 the public was skeptical about the government's commitment to defend a fixed rate of devaluation combined with an adjustable band, and the outcome was a consumption boom accompanied by an extreme devaluation. The case of Argentina's convertibility plan started in 1991, and abandoned in December 2001, deserves also some attention. Once again, in 2000, the Argentinean public anticipated the end of a stabilization plan, which led to a substantial increase in consumption and an extreme devaluation. In conclusion, one of the lessons that policymakers should be taken into account, from these experiences, when launching an exchange-rate-based inflation is that in most cases it result in a consumption boom and an extreme devaluation.[1].

Some of the studies in the literature on temporary stabilization that have considered a stochastic setting are, for instance, Drazen and Helpman (1988) examining stabilization with exchange-rate management under uncertainty, Calvo and Drazen (1997) contemplating uncertainty in the permanence of economic reforms[2], Mendoza and Uribe (1996) and (1998) modeling exogenous and endogenous probabilities of devaluation, respectively, Venegas-Martínez (2000a), (2000b), (2001), (2004a), (2004b), (2005a) and (2005b), (2005c) and Venegas-Martínez and González-Aréchiga (2000) studing exchange-rate-based stabilization with imperfect credibility[3]. While this literature has provided considerable theoretical advancement, there is a set of empirical regularities that still need to be explained as pointed out in Helpman and Razin (1987), Kiguel and Liviatan (1992), Végh (1992), and Rebelo and Végh (1995). First, the existing models have found difficulties in explaining the observed orders of magnitude of consumption booms. Secondly, in most

[1] The inflation stabilization plans, which took place in Argentina, Brazil, Chile, Uruguay, Israel, and Mexico between the 1970s and 1990s, have been widely documented, we direct the reader to the references contained in Calvo and Végh (1998)

[2] Though Calvo and Drazen (1997) focus on the duration of economic reforms, their results can be translated into an exchange-rate-based disinflation context. It is also important to point out that while Drazen and Helpman (1988), and Calvo and Drazen (1997) are mainly concerned with studying uncertainty about the timing of stabilization, we are interested in dealing with uncertainty about the exchange rate dynamics.

[3] A related research of pricing real options when prior information is available can be found in Venegas-Martínez (2005d), and Venegas-Martínez and Fundia Aizenstat (2005).

cases, stabilization inflation plans result in extreme devaluations. Third, in most models a plausible explanation of the lack of credibility is missing, even if there is no change in the parameters determining the expectations of devaluation. Fourth, most models forget that what makes a stabilization inflation plan temporary is uncertanty.

Real options have been recently attracting an increasing attention in macroeconomics, see for instance: Strobel (2005), and Henderson and Hobson (2002). The main issue associated with real options is how to value contingent claims on non-traded assets. In this paper, we are concerned with valuing the real option of waiting when consumption can be delayed for a representative consumer in a small open monetary economy. In this case, the underlying asset is the price of money in terms of goods. In the existing literature there is not a suitable analysis about the rationality to price the real option of waiting for postponing consumption when a stabilization plan is about to be abandoned; a non-traded claim. This paper provides an analytical solution for the price of such a real option.

This paper studies, in a small open economy setting, a stochastic model of exchange-rate-based stabilization recognizing the role of extreme movements in the dynamics of the expectations of devaluation. It is assumed that the expectations of devaluation follow a mixed diffusion-jump process where a Brownian motion drives the rate of devaluation and a Poisson process determines the number of posible devaluations. Additionally, the expected size of a posible devaluation is supposed to have an extreme value distribution of the Fréchet type. Using this framework, and assuming logarithmic utility, we examine the equilibrium dynamics of consumption and real wealth when a stabilization plan is implemented and the size of devaluation is expected to follow an extreme value Fréchet distribution. We also study the effects on consumption and economic welfare of changes in the parameters determining the expectations. The model is developed under the following two main asumptions: the revenue raised by seignorage is not rebated back to the agents and policy variables are stochastic. Our model derives tractable closed-forms solutions that make much easier the understanding of the key issues in the analysis of temporary stabilization when the expected size of a possible devaluation is supposed to have an extreme value distribution. Finally, several distinctive characteristic of our model are: 1) there is a lack of credibility even if parameter values determining the expectations of devaluation are not changed, 2) it deals with stochastic stabilization horizons, 3) it reproduces via Monte Carlo simulation, with suitable parameter values, the booms of private consumption in the Mexican case of 1989-1994 and the Argentinean case of 2001-2003, which resulted in an extreme devaluation, and 4) it prices the real option of waiting when consumption can be postponed.

The paper is organized as follows. In the next section, we work out a one-good, cash-in-advance, stochastic model where agents have expectations of devaluation driven by a mixed diffusion-jump process where the expected size of a posible devaluation is supposed to have an extreme value distribution. Through section 3, we solve the consumer's decision problem. In section 4, we undertake several policy experiments through comparative static exercises. In section 5, we examine the welfare implications. In section 6, we also study the dynamic behavior of wealth and consumption, and address a number of exchange-rate policy issues. In section 7, we carry out some Monte Carlo simulations of the response of consumption to permanent changes in the values of key parameters of the model when the expected size of a possible devaluation is supposed to have an extreme value distribution of the Fréchet type. In section 8, we deal with pricing the real option of waiting when consumption can be delayed. Finally, in section 9, we present conclusions, acknowledge limitations, and make suggestions for further research.

2. Structure of the model

Let us consider a small open economy populated by infinitely lived identical households in a world with a single consumption good internationally tradable.

2.1 Purchasing power parity and exchange rate dynamics

We assume that the good in the economy is freely traded, and its domestic price level, P_t, is determined by the purchasing power parity condition, namely

$$P_t = P_t^* e_t, \tag{1}$$

where P_t^* is the foreign-currency price of the good in the rest of world, and e_t is the nominal exchange rate. Throughout the paper, we will assume, for convenience, that P_t^* is equal to 1. We also suppose that the exchange-rate initial value, e_0, is known and equal to 1.

In what follows, we will suppose that the ongoing uncertainty in the dynamics of the expected rate of devaluation, and therefore in the inflation rate, is generated by a geometric

Brownian motion combined with Poisson jumps with random sizes driven by extreme value distributions of the Fréchet type:

$$\frac{\mathrm{d}e_t}{e_t} = \frac{\mathrm{d}P_t}{P_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t + Z \mathrm{d}N_t, \tag{2}$$

where $\mu, \sigma > 0$, $(W_t)_{t \geq 0}$ is a Brownian motion defined on a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P}_W)$, $\mathrm{d}N_t$ is a Poisson process with intensity parameter λ. The size of an upward jumps satisfies:

$$Z = \frac{1}{1 - X^{-\alpha}} - 1, \quad X > 0, \ \alpha > 0, \qquad \text{and} \qquad X = \frac{Y - \nu}{\kappa}, \quad \kappa, \ \nu > 0,$$

where Y is a Fréchet random variable with parameters α, ν and $\kappa > 0$. Clearly $Z > 0$. Notice that an increase in Y, leads to an increase in X, which, in turns, rises Z. The cumulative distribution function of Y is given by

$$F_Y(y) = \begin{cases} 0, & y < \nu, \\ \exp\left\{ - \left(\frac{y - \nu}{\kappa} \right)^{-\alpha} \right\}, & y \geq \nu. \end{cases} \tag{3}$$

The corresponding density of Y satisfies (see Figure 1 for a particular case of a Fréchet density):

$$f_Y(y) = \frac{\alpha}{\kappa} F_Y(y) \left(\frac{y - \nu}{\kappa} \right)^{-(1+\alpha)}, \qquad y \geq \nu \tag{4}$$

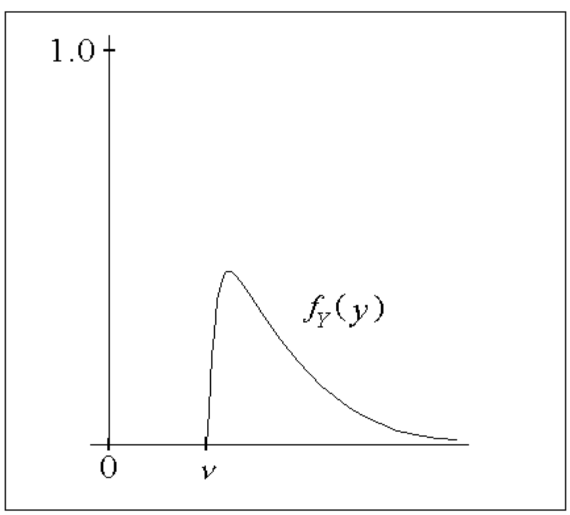

Figure 1. A Fréchet density for $\kappa = \alpha = 1$.

Notice that if $\alpha > 2$, then

$$\mathrm{E}\left[Y\right] = \nu + \kappa\Gamma\left(1 - \frac{1}{\alpha}\right),$$

$$\mathrm{E}\left[Y^2\right] = \kappa^2\Gamma\left(1 - \frac{2}{\alpha}\right) + 2\nu\kappa\Gamma\left(1 - \frac{1}{\alpha}\right) + \nu^2,$$

thus

$$\mathrm{Var}\left[Y\right] = \kappa^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \Gamma^2\left(1 - \frac{1}{\alpha}\right)\right],$$

On the other hand, since the number of expected devaluations (*i.e.,* upward jumps in the exchange rate) per unit of time follows a Poisson process $\mathrm{d}N_t$ with intensity λ, we have that

$$\mathbb{P}_N\{\text{one unit jump during } \mathrm{d}t\} = \mathbb{P}_N\{\mathrm{d}N_t = 1\} = \lambda\mathrm{d}t$$

and

$$\mathbb{P}_N\{\text{more than one unit jump during } \mathrm{d}t\} = \mathbb{P}_N\{\mathrm{d}N_t > 1\} = o(\mathrm{d}t),$$

so that

$$\mathbb{P}_N\{\text{no jump during } \mathrm{d}t\} = 1 - \lambda\mathrm{d}t + o(\mathrm{d}t),$$

where $o(\mathrm{d}t)/\mathrm{d}t \to 0$ as $\mathrm{d}t \to 0$. Observe that the expected time until the next jump is then $1/\lambda$. Finally, it is easy to show that $\mathrm{E}[\mathrm{d}N_t] = \mathrm{Var}[\mathrm{d}N_t] = \lambda\mathrm{d}t$.

2.2 Available assets in the economy

The representative consumer holds two real assets: real cash balances, $m_t = M_t/P_t$, where M_t is the nominal stock of money, and an international real bond, b_t. The bond pays a constant real interest rate r (*i.e.,* it pays r units of the consumption good per unit of time). Thus, the consumer's real wealth, a_t, is given by

$$a_t = m_t + b_t, \tag{5}$$

where a_0 is exogenously determined.

2.3 Financing consumption

The economy is equipped with a cash-in-advance constraint of the Clower type:

$$m_t = \int_t^{t+\psi} c_s \mathrm{d}s, \tag{6}$$

where c_t is consumption, and $\psi > 0$ is the time that money must be held to finance consumption. Condition (6) is critical in linking exchange-rate dynamics with consumption. Observe that

$$m_t = \int_t^{t+\psi} c_s \mathrm{d}s \approx \psi c_t + o(\psi).$$

In the sequel, we will assume that the error $o(\psi)$ is negligible and, for simplicity, $\psi = 1$. In this case, devaluation acts as a stochastic tax on real cash balances.

2.4 The return of real money balances

The stochastic rate of return of holding real cash balances, $\mathrm{d}R_m$, is simply the percentage change in the price of money in terms of goods. By applying Itô's lemma to the inverse of the price level, with (2) as the equation driven the underlying process, we get

$$\begin{aligned} \mathrm{d}\left(\frac{1}{P_t}\right) &= \left[-\left(\frac{1}{P_t^2}\right)\mu P_t + \tfrac{1}{2}\left(\frac{2}{P_t^3}\right)\sigma^2 P_t^2\right]\mathrm{d}t - \left(\frac{1}{P_t^2}\right)\sigma P_t \mathrm{d}W_t \\ &\quad + \left(\frac{-X^{-\alpha}+1}{P_t} - \frac{1}{P_t}\right)\mathrm{d}N_t \\ &= \frac{1}{P_t}\left[\left(-\mu+\sigma^2\right)\mathrm{d}t - \sigma \mathrm{d}W_t - X^{-\alpha}\mathrm{d}N_t\right]. \end{aligned} \tag{7}$$

The minus sign appearing in the term $-X^{-\alpha}$ is because forward jumps affect negatively money balances. Hence, the stochastic rate of return of holding real cash balances is given by

$$\mathrm{d}R_m = \left(-\mu+\sigma^2\right)\mathrm{d}t - \sigma \mathrm{d}W_t - X^{-\alpha}\mathrm{d}N_t. \tag{8}$$

3. The household's decision problem

The stochastic consumer's real wealth accumulation in terms of the portfolio shares, $w_t = m_t/a_t$, $1 - w_t = b_t/a_t$, and consumption, c_t, is given by

$$\mathrm{d}a_t = a_t w_t \mathrm{d}R_m + a_t(1-w_t)r\mathrm{d}t - c_t\mathrm{d}t,$$

with a_0 exogenously determined, which can be rewritten as

$$\mathrm{d}a_t = a_t\Big[(r-\rho w_t)\mathrm{d}t - w_t\sigma\mathrm{d}W_t - w_t X^{-\alpha}\mathrm{d}N_t\Big], \tag{9}$$

where $\rho = 1 + r + \mu - \sigma^2$.

The von Neumann-Morgenstern utility at time $t = 0$, v_0, of the competitive risk-averse consumer is assumed to have the time-separable form:

$$v_0 = \mathrm{E}_0 \left[\int_0^\infty \log(c_t)\ e^{-rt} \mathrm{d}t \right], \tag{10}$$

where E_0 is the conditional expectation on all available information at $t = 0$. To avoid unnecessary complex dynamics in consumption, we assume that the agent's subjective discount rate has been set equal to the constant real international rate of interest, r. This assumption leads to a steady state. We consider the logarithmic utility function in order to derive closed-form solutions and make the analysis tractable.

3.1 First order conditions for an interior solution

The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing utility, $\log(c_t) = \log(a_t w_t)$, and assuming that Y is stochasticaly independent of $\mathrm{d}N_t$ satisfies

$$\begin{aligned} \max_w H(w_t; a_t, t) \equiv \max_w \Big\{ & \log(a_t w_t) e^{-rt} + I_a(a_t, t) a_t (r - \rho w_t) + I_t(a_t, t) \\ & + \tfrac{1}{2} I_{aa}(a_t, t) a_t^2 w_t^2 \sigma^2 - \lambda \mathrm{E}\Big[I\Big(a_t \left(w_t X^{-\alpha} + 1 \right), t \Big) - I(a_t, t) \Big] \Big\} = 0. \end{aligned} \tag{11}$$

The first-order condition for an interior solution is $H_{w_t} = 0$. Given the exponential time discounting in (10), we postulate $I(a_t, t)$ in a time-separable form as

$$I(a_t, t) = e^{-rt} [\beta_1 \log(a_t) + \beta_0], \tag{12}$$

where β_0 and β_1 are to be determined from (11). In this case, condition (11) becomes

$$\begin{aligned} \max_w H(w_t; a_t, t) \equiv \max_w \Bigg\{ & \log(a_t w_t) + \beta_1 (r - \rho w_t) - r[\beta_1 \log(a_t) + \beta_0] \\ & - \tfrac{1}{2} \beta_1 w_t^2 \sigma^2 + \beta_1 L(w_t) \Bigg\} = 0, \end{aligned} \tag{13}$$

where

$$L(w_t) = -\lambda \mathrm{E} \left[\log \left(w_t X^{-\alpha} + 1 \right) \right]. \tag{14}$$

Notice that the argument in the above logarithmic function remains positive. In order to compute the first-order condition, we need first to find

$$\frac{\partial}{\partial w_t} L(w_t) = \mathrm{E} \left[\frac{\partial}{\partial w} \log \left(w_t X^{-\alpha} + 1 \right) \right] = \mathrm{E} \left[\frac{X^{-\alpha}}{w_t X^{-\alpha} + 1} \right].$$

From the above result, we may conclude that optimal w_t is time-invariant, $w_t \equiv w$, and it satisfies

$$\frac{1}{\beta_1 w} - \frac{\lambda}{w}\mathrm{E}\left[\frac{X^{-\alpha}}{X^{-\alpha}+w^{-1}}\right] = \rho + w\sigma^2. \tag{15}$$

By defining the change of variable

$$\zeta = \left(\frac{y-\nu}{\kappa}\right)^{-\alpha},$$

the above mathematical expectation can be written as

$$\begin{aligned}\mathrm{E}\left[\frac{X^{-\alpha}}{X^{-\alpha}+w^{-1}}\right] &= \int_0^\infty \frac{[(y-\nu)/\kappa]^{-\alpha}}{[(y-\nu)/\kappa]^{-\alpha}+w^{-1}} f_Y(y)\mathrm{d}y \\ &= \int_0^\infty \frac{\zeta}{\zeta+w^{-1}} e^{-\zeta}\mathrm{d}\zeta \\ &= \frac{1}{w}e^{1/w}\Gamma(-1,1/w).\end{aligned} \tag{16}$$

where $\Gamma(-1,1/w) = -\Gamma(0,1/w) + e^{-1/w}w$. We also have the following formula:

$$\begin{aligned}\Gamma(0,1/w) &= \int_{1/w}^\infty \frac{e^{-u}}{u}\mathrm{d}u \\ &= we^{-1/w}\left(1-w+o(w)\right),\end{aligned} \tag{17}$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function. Figure 2 shows the graph of the funtion $\Gamma(0,1/w)$. Observe that $\Gamma(0,0)=0$, $\Gamma(0,\infty)=0$, and $\Gamma(0,1)\approx e^{-1}$. Hence, from (16) and (17), the first order condition can be written as

$$\frac{1}{\beta_1 w} - \frac{\lambda}{w}\left(1 - \frac{1}{w}e^{1/w}\Gamma(0,1/w)\right) = \rho + w\sigma^2. \tag{18}$$

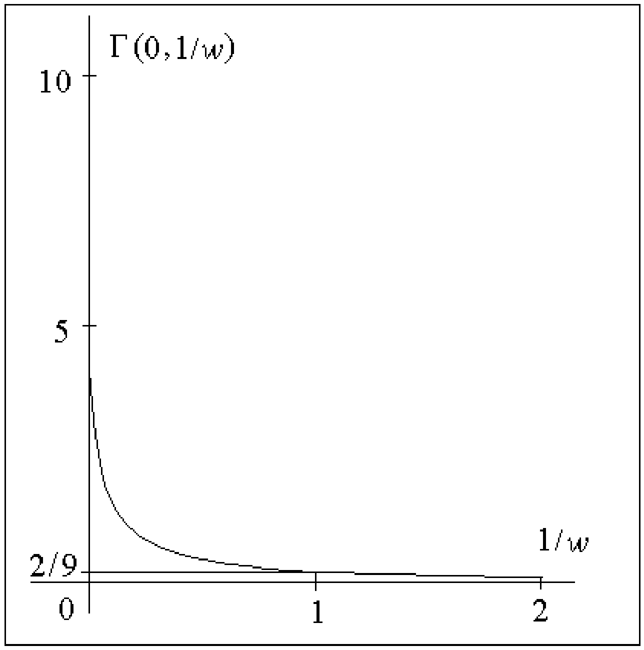

Figure 2. Graph of the funtion $\Gamma(0, 1/w)$.

If we assume that $0 < w < 1$, then the approximation $\Gamma(0, 1/w) \approx we^{\frac{1}{w}}(1-w)$ performs well, and we may write the first order condition as

$$\frac{1}{\beta_1 w} - \lambda = \rho + w\sigma^2. \tag{19}$$

Once an optimal w is chosen, coefficients β_0 and β_1 are determined from (11) as follows:

$$(1 - r\beta_1)\log(a_t) - r\beta_0 + r\beta_1 + \log(w) - \beta_1\left[\rho w + \tfrac{1}{2}w^2\sigma^2 - L(w)\right] = 0 \tag{20}$$

which leads to

$$\beta_1 = r^{-1} \tag{21}$$

and

$$\beta_0 = \frac{1}{r}[1 + \log(w)] - \frac{1}{r^2}\left[\rho w + \tfrac{1}{2}w^2\sigma^2 - L(w)\right].$$

Thus, the first order condition becomes

$$\frac{r}{w} = \rho + \lambda + w\sigma^2,$$

which is a quadratic equation with real solutions and only one positve given by

$$w = \frac{-(\rho+\lambda) + \sqrt{(\rho+\lambda)^2 + 4\sigma^2 r}}{2\sigma^2} > 0 \tag{22}$$

Notice that always $0 < 1 + \mu + \lambda$, and we can readily show that $0 < 1 + \mu + \lambda$ iff

$$\sqrt{(\rho+\lambda)^2 + 4\sigma^2 r} < 2\sigma^2 + (\rho+\lambda)$$

iff $0 < w < 1$. Figure 3 shows optimal w as a function of $\rho + \lambda$ and σ^2.

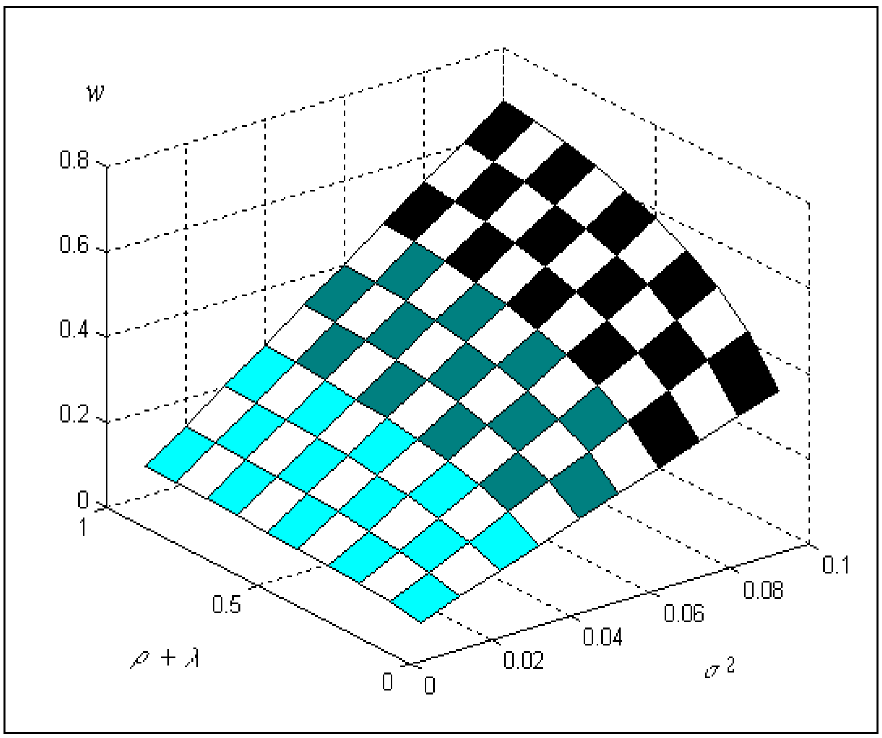

Figure 3. Optimal w as a function of $\rho + \lambda$ and σ^2.

4. Policy experiments and comparative statics

In this section we carry out some comparative static experiments regarding the optimal share w. We will see the effects of changes in the mean expected rate of inflation μ, the instantaneous volatility of inflation and the intensitity parameter λ on optimal w, $0 < w < 1$. By differentiating the first order condition, we get

$$\mathrm{d}\mu + (w-1)\mathrm{d}\sigma^2 - \mathrm{d}\lambda + A(w)\mathrm{d}w = 0, \tag{23}$$

where

$$A(w) = \sigma^2 + \frac{r}{w^2}.$$

We are now in a position to derive our first result: a once-and-for-all increase in the rate of devaluation, which results in an increase in the future opportunity cost of purchasing goods, leads to a permanent decrease in the proportion of wealth devoted to future consumption. To see this, it is enough to use (23) to find that

$$\frac{\partial w}{\partial \mu} = -\frac{1}{A(w)} < 0. \tag{24}$$

Notice also that a once-and-for-all increase in the variance of the diffusion component, will produce a contrary effect to that of μ on w since

$$\frac{\partial w}{\partial \sigma^2} = \frac{1-w}{A(w)} > 0 \tag{25}$$

In other words, the consumer sets aside a larger proportion of wealth to maintain real monetary balances to finance consumption in order to deal with a higher variance in consumption prices.

Another result is the response of the equilibrium share of real monetary balances, w, to once-and-for-all changes in the intensity parameter, λ. A once-and-for-all increase in the expected number of extreme devaluations per unit of time causes an increase in the future opportunity cost of purchasing goods. This, in turn, permanently decreases the proportion of wealth set aside for future consumption. From (11), we get

$$\frac{\partial w}{\partial \lambda} = -\frac{1}{A(w)} < 0. \tag{26}$$

5. Impact on economic welfare

We will now assess the effects of exogenous shocks on economic welfare. As usual, the welfare criterion, W, of the representative individual is the maximized utility starting from the initial real wealth, a_0. Therefore, welfare is given by

$$\begin{aligned} W(\mu, \sigma^2; a_0) \equiv I(a_0, 0) =& \frac{1}{r}[1 + \log(a_0) + \log(\alpha^{-1} w)] \\ & - \frac{1}{r^2}\Big[\rho w + \tfrac{1}{2} w^2 \sigma^2 - L(w)\Big)\Big]. \end{aligned} \tag{27}$$

Table 1 shows the impacts on welfare of once-and-for-all changes in the mean expected rate of devaluation, the inverse of volatility, the probability of devaluation, and the expected size of a devaluation. Table 1 also shows that under the assumption of logarithmic utility, it is welfare-reducing to increase the stochastic tax coming from devaluation.

In order to compute economic welfare W, we need to find, explicitely, $L(w)$. To do this, we use the same change of variable as in (16), so

$$\begin{aligned} \mathrm{E}\left[\log\left(wX^{-\alpha}+1\right)\right] =& \int_0^\infty \log(w^2\zeta+1)e^{-\zeta}\mathrm{d}\zeta \\ =& e^{1/w}\Gamma(0,1/w). \end{aligned} \tag{28}$$

It is important to point out that (16) can be also obtained by differentiating (28)

$$\begin{aligned} \frac{\partial}{\partial w}e^{1/w}\Gamma(0,1/w) &= -\frac{1}{(w)^2}e^{1/w}\Gamma(0,1/w) + e^{1/w}\left(\frac{\partial\Gamma(0,1/w)}{\partial(1/w)}\right)\left(\frac{\partial(1/w)}{\partial w}\right) \\ &= -\frac{1}{(w)^2}e^{1/w}\Gamma(0,1/w) + e^{1/w}\left(-we^{-1/w}\right)\left(-\frac{1}{(w)^2}\right) \\ &= -\frac{1}{(w)^2}e^{1/w}\Gamma(0,1/w) + \frac{1}{w}. \end{aligned}$$

Notice also that since there is a differentiation process on $L(w)$ in the first order condition, we may use now the aproximation $\Gamma(0,1/w) = we^{-1/w}(1+O(w))$, thus

$$L(w) = -\lambda w = \int_0^w \lambda \mathrm{d}x.$$

increase in	effect on welfare
μ	$-\frac{w}{r^2} < 0$
σ^2	$-\frac{(w)^2}{r^2} < 0$
λ	$-\frac{w}{r^2} < 0$

Table 1. Effects of policy changes on economic welfare.

6. Wealth, consumption and dynamic implications

We now derive the stochastic process that generates wealth when the optimal share is applied. After substituting the optimal share w into (9), we get

$$\mathrm{d}a_t = a_t\Big[(\lambda + w^2\sigma^2)\mathrm{d}t - w\sigma\mathrm{d}W_t - wX^{-\alpha}\mathrm{d}N_t\Big], \tag{29}$$

The solution to the above stochastic differential equation, conditional on a_0, is

$$a_t = a_0 e^{\delta_t}, \tag{30}$$

where

$$\delta_t = \eta_t - \gamma_t, \quad \eta_t \sim \mathcal{N}[F(w)t, G(w)t], \quad \gamma_t = H(w,X)N_t,$$
$$F(w) = \lambda + \frac{w^2\sigma^2}{2}, \quad G(w) = w^2\sigma^2, \qquad \text{and} \qquad H(w,X) = \log(wX^{-\alpha}+1), \tag{31}$$

Notice that

$$\begin{aligned}\mathrm{E}[\delta_t] &= [F(w) - \lambda\mathrm{E}[H(w,X)]]\,t \\ &= [F(w) - \lambda w(1 - w + o(w))]\,t\end{aligned} \tag{32}$$

In virtue of (6), the stochastic process for consumption, in (30), can be written as

$$c_t = wa_0e^{\delta_t}. \tag{33}$$

This indicates that, in the absence of contingent-claims markets, the devaluation risk has an effect on wealth via the uncertainty in δ_t, that is, uncertainty changes the opportunity set faced by the consumer. On the other hand, the devaluation risk also affects the composition of portfolio shares via its effects on w. Thus, a policy change will be accompanied by both wealth and substitution effects. We cannot determine the level of consumption in our stochastic framework. We can only compute the probability that, at a given time, a certain level of consumption occurs. Notice, however, that by Jensen's inequality, mean consumption satisfies:

$$\mathrm{E}[c_t] \geq a_0we^{\mathrm{E}[\delta_t]}. \tag{34}$$

In contrast with our stochastic setting, c_t shows a dynamic behavior, even if the rate of devaluation is expected to remain fixed forever. This is because δ_t is a time-varying, state-contingent variable. We may conclude that uncertainty is the clue to rationalize richer consumption dynamics that could not be obtained from deterministic models.

6.1 Consumption booms

In this section, we will analyze a policy of the form:

$$\mu_t = \begin{cases} \mu_1 & \text{for} \quad 0 \leq t \leq T, \\ \mu_2 & \text{for} \quad t > T, \end{cases}$$

where T is exogenously determined, and $\mu_1 < \mu_2$. Notice that in our stochastic setting, there is a lack of credibility even if we do not change the four parameters since agents always assign some probability to the event of currency devaluation. Let us examine the response of consumption to the above policy. From (33), we may write

$$\frac{c_{T+\Delta}}{c_T} = \frac{w_2}{w_1} h_{_T}(\Delta; \mu_1, \mu_2),$$

where $h_{_T}(\Delta; \mu_1, \mu_2) \equiv \exp\{-(\delta_T(\mu_1) - \delta_{T+\Delta}(\mu_2))\}$ tends to 1 as $\Delta \to 0$ a.s.(almost surely). The limit means that although the stationary components of the parameters of η_t and γ_t are different before and after time T, such a difference becomes negligible when $\Delta \to 0$. Consequently,

$$\lim_{\Delta \to 0} c_{T+\Delta} = c_T \frac{w_2}{w_1} \quad \text{a.s.} \tag{35}$$

We also notice that $w_2/w_1 < 1$, together with (35), imply $c_T > \lim_{\Delta \to 0} c_{T+\Delta}$ a.s., indicating a jump (boom) in consumption at time T.

If μ were to be constant forever, *i.e.*, if $\mu_t = \mu_2$ for all $t \geq 0$, then we would have

$$c_{t+\Delta} = c_t h_{_t}(\Delta; \mu_2, \mu_2). \tag{36}$$

On the right-hand side of (36), the factor $h_{_t}(\Delta; \mu_2, \mu_2) \to 1$ as $\Delta \to 0$ a.s. Hence, consumption would be continuous a.s. for all t. If the plan is expected to be temporary, then $c_T > \lim_{\Delta \to 0} c_{T+\Delta}$ a.s., indicating a jump in consumption at T, as we have shown above. The above analysis can be entirely applied to any of the remaining parameters determining the expectations of devaluation, namely μ or λ.

6.2 Stochastic stabilization horizons

Let T be an exponentially distributed random variable with parameter $\overline{T}^{-1}$. In such a case, a higher parameter value will increase the probability of shortening the horizon of the stabilization plan. Suppose that T is independent of $\mathrm{d}W_t$, $\mathrm{d}N_t$ and Y. Leaving out $o(w)$ from (32) and writing $c_0 = a_0 w$, we have that the growth rate of consumption satisfies

$$\mathrm{E}[\log(c_T/c_0)] = \mathrm{E}[\mathrm{E}[\log(c_T/c_0)|T]] = [F(w) - \lambda w(1-w)]\,\overline{T}.$$

Moreover, from the application of Jensen's inequality in (34) the mean consumption has a lower bound given by $LB(c_t) = c_0 e^{\mathrm{E}[\delta_t]}$. Figure 4 shows such a lower bound as a function of c_0 and $\overline{T}$. Units of c_0 are scaled in $\times 10^{11}$ pesos of 1993. It can be seen that as c_0 and $\overline{T}$ increase, $LB(c_t)$ rises.

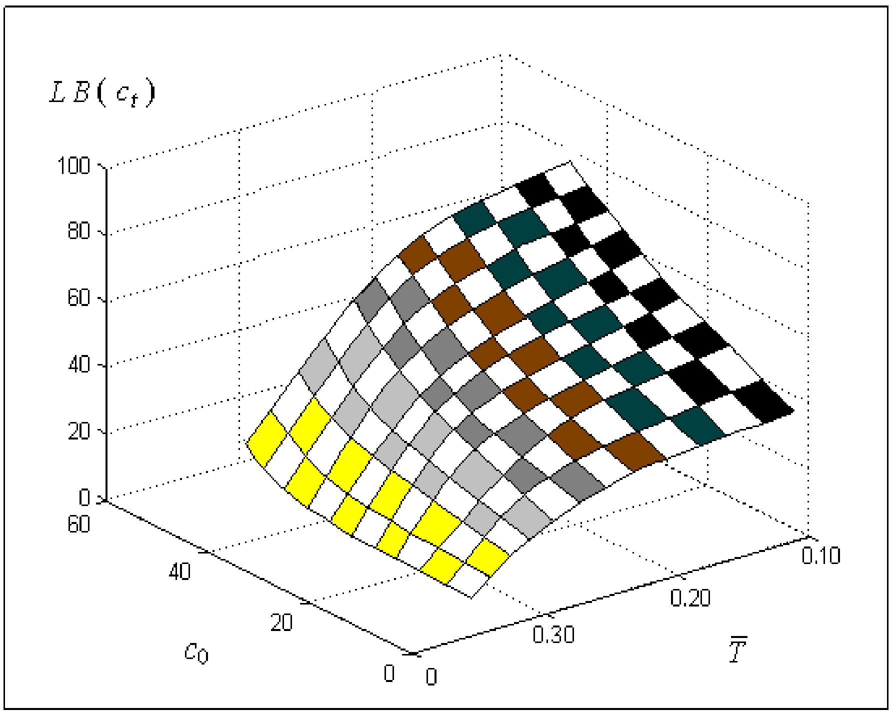

Figure 4. Consumption as a function of c_0 and $\overline{T}$,

7. Simulation exercise

The following experiment is intended to simulate through Monte Carlo methods the response of consumption to permanent changes in the values of the parameters that determine the expectations of devaluation. Table 1 presents a couple of vectors of parameter values, $(\mu_j, \sigma_j^2, \lambda_j)$, $j = 1, 2$, that reproduce the observed consumption trends and observed in Mexico between 1989 and 1994[4]. It is also supposed that $r = 0.085000$ and $a_0 = 1.849 \times 10^{12}$ (pesos of 1993). It is also assumed that $\kappa = 1$ and $\nu = 0$.

[4] We tried about 800 combinations of parameter values.

$w_1 = 0.455$	$w_2 = 0.444$
$\mu_1 = 0.192$	$\mu_2 = 0.298$
$\sigma_1^2 = 0.499$	$\sigma_2^2 = 0.409$
$\lambda_1 = 0.010$	$\lambda_2 = 0.100$

Table 1. Consumption shares and parameter values (Mexican case).

In Figure 5 (a), starting from below there are simulated paths of the Brownian motion, the geometric Brownian motion, and the diffusion-jump-extreme process[5]. In figure 5 (b), several simulated paths are shown before and after the first quarter of 1993. In Figure 6 (a), the dashed line corresponds to observed consumption. In Figure 6 (b) the light solid line represents the simulated trend of consumption with the diffusion-jump component. Figure 6 (c) shows a heavy solid line representing the simulated trend of consumption with the diffusion-jump component with a jump size guided by a Fréchet distribution. Finally, Figure 6 (d) compares the trends in (b) and (c)[6]. Notice that, with the above parameter values, the stochastic simulation considering an extreme value distribution mimics the order of magnitude of the consumption jump observed in the first quarter of 1993; a jump about 60 thousands of millions of pesos of 1993. Table 2 and Figure 7 repeat the same exercise for the Argentinean case 2001-2003[7].

[5] We used the statistical software "Xtremes" (Reiss and Thomas, 2001) and Ripley's methodology (1987) for Monte Carlo Simulations.

[6] Data Source: Instituto Nacional de Estadística Geografía e Informática, México.

[7] Source: Instituto Nacional de Estadística y Censos de la República de Argentina.

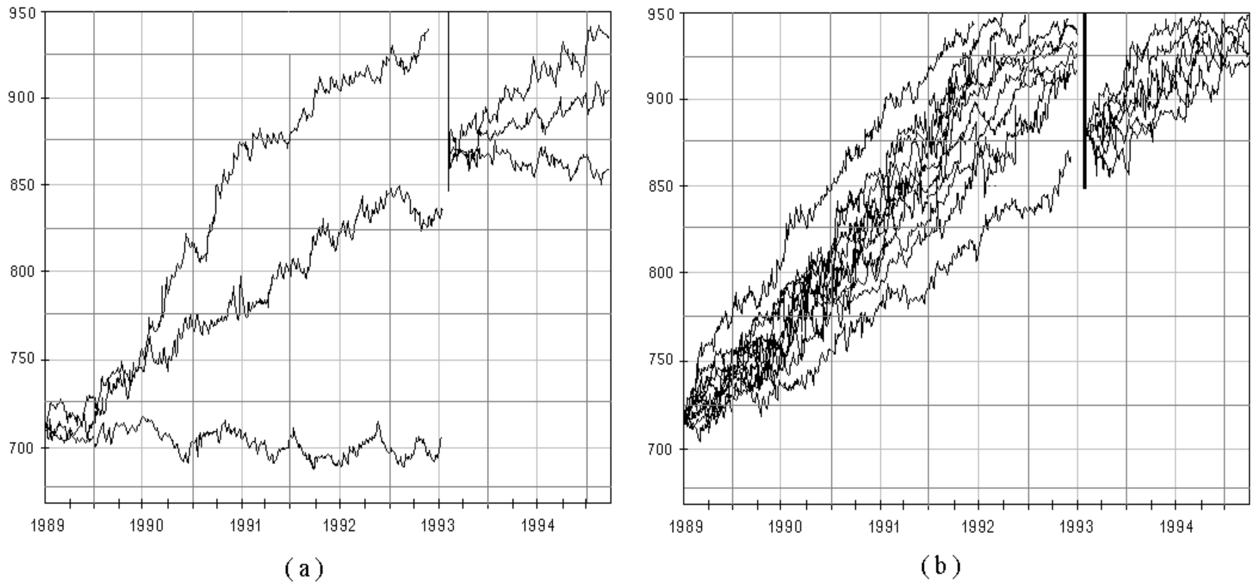

Figure 5. (a) simulated paths of the Brownian motion, geometric Brownian motion, and the diffusion-jump-extreme process, (b) several simulated paths of consumption before and after the first quarter of 1993.
(thousands of millions of pesos of 1993)

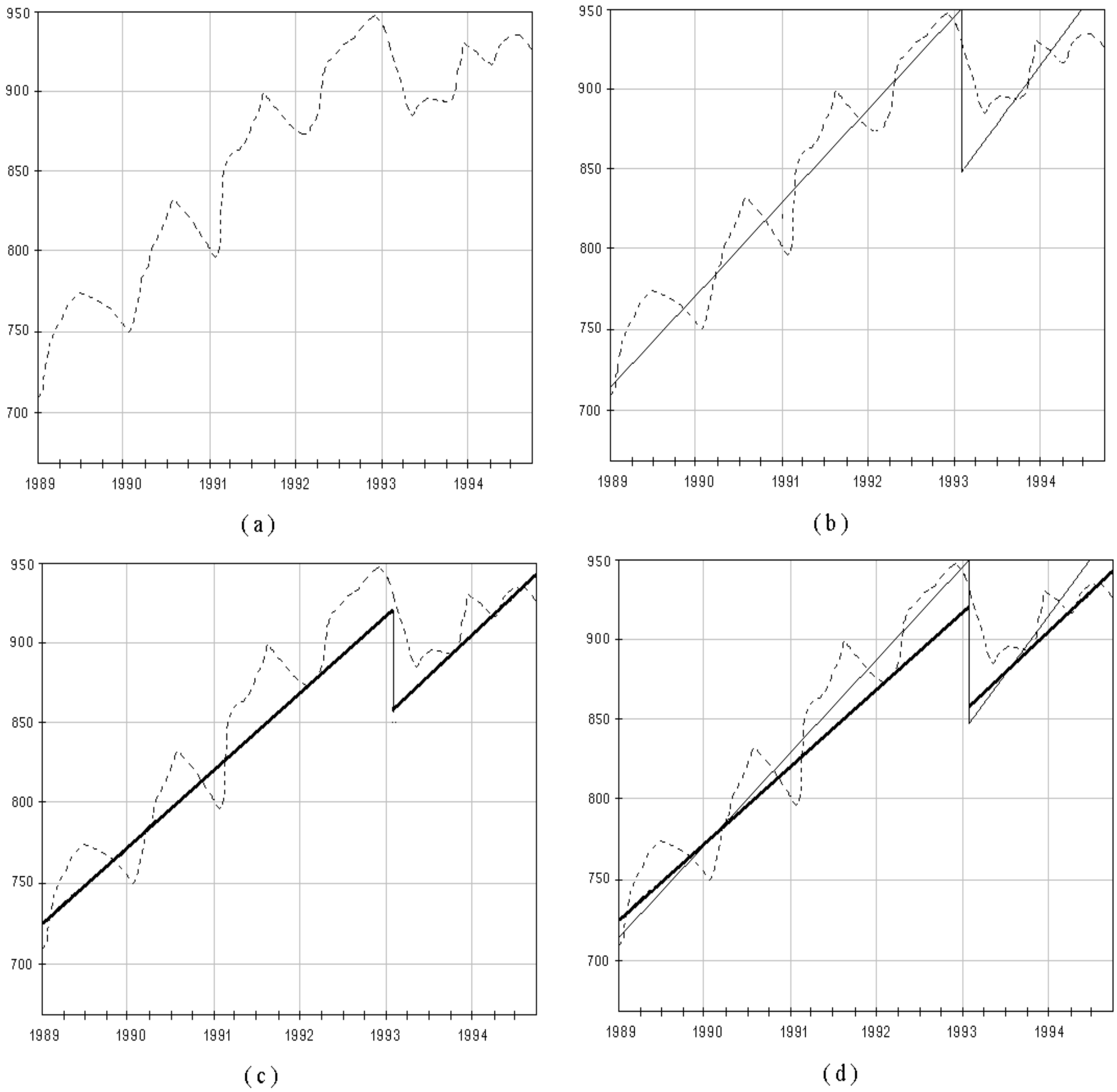

Figure 6. Simulated trends of consumption: (a) observed consumption, (b) the simulated trend of consumption with the diffusion-jump component, (c) the simulated trend of consumption with the diffusion-jump-extreme component, (d) compares the trends in (b) and (c) (thousands of millions of pesos of 1993)

$w_1 = 0.532$	$w_2 = 0.477$
$\mu_1 = 0.112$	$\mu_2 = 0.154$
$\sigma_1^2 = 0.390$	$\sigma_2^2 = 0.416$
$\lambda_1 = 0.011$	$\lambda_2 = 0.053$

Table 2. Consumption shares and parameter values (Argentinean case).

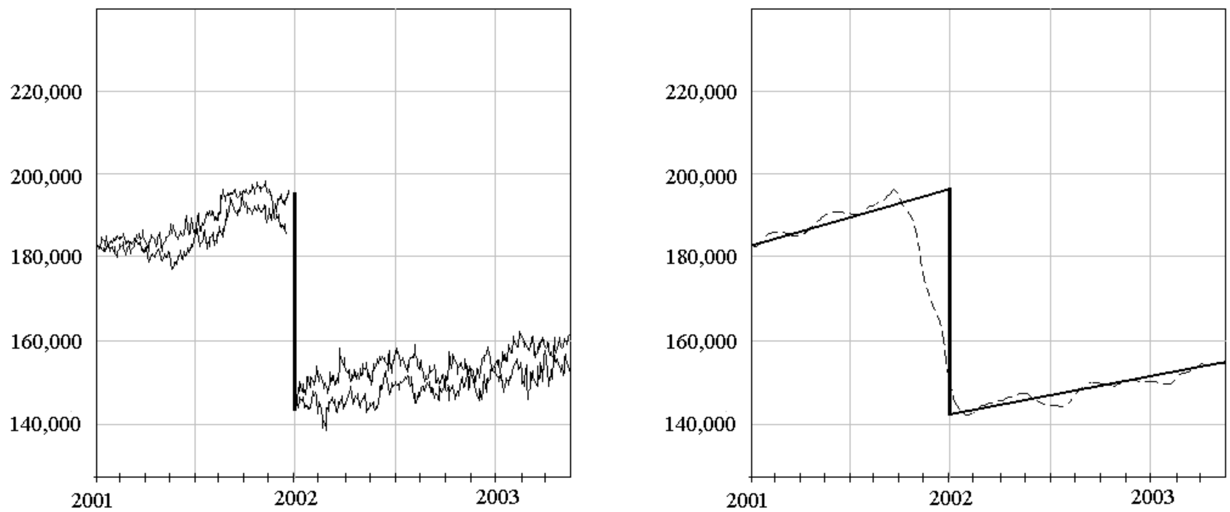

Figure 7. (a) simulated paths of consumption, (b) the simulated trends (Dashed line corresponds to observed consumption in millions of pesos Argentinean pesos of 1993).

8. The real option of waiting when consumption can be delayed

In this section we caracterize the price of the real option of waiting when consumption can be delayed as the solution of a differential-integral equation. Moreover, we find the solution of such a differential-integral equation. Observe first that equation (7) can be written as

$$\mathrm{d}S_t = \phi S_t \mathrm{d}t + \sigma S_t \mathrm{d}W_t + \xi S_t \mathrm{d}N_t, \tag{37}$$

where

$$S_t = 1/P_t, \qquad \phi = -\mu + \sigma^2, \qquad \text{and} \qquad \xi = -X^{-\alpha}.$$

Notice that $S_t = 1/P_t$ is the price of money in terms of goods; a non-traded asset. Obviously, from the definition of Z given in (2), we have

$$\xi = \frac{1}{1+Z} - 1.$$

Thus, from (37) the stochastic rate of return of holding real cash balances, $\mathrm{d}R_m = \mathrm{d}S_t/S_t$, can be written as

$$\mathrm{d}R_m = \phi \mathrm{d}t + \sigma \mathrm{d}W_t + \xi \mathrm{d}N_t. \tag{38}$$

If $V(S_t, t)$ denotes the value of the real option of waiting when consumption can be delayed, then Itô's lemma leads to

$$\mathrm{d}V = \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S_t}\phi S_t + \tfrac{1}{2}\frac{\partial^2 V}{\partial S_t^2}\sigma^2 S_t^2 \right) \mathrm{d}t + \frac{\partial V}{\partial S_t}\sigma S_t \mathrm{d}W_t + \left[V(S_t(\xi+1), t) - V(S_t, t)\right] \mathrm{d}N_t.$$

or

$$\mathrm{d}V = \phi_V V \mathrm{d}t + \sigma_V V \mathrm{d}W_t + \xi_V V \mathrm{d}N_t, \tag{39}$$

with

$$\phi_V = \frac{1}{V}\left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S_t}\phi S_t + \tfrac{1}{2}\frac{\partial^2 V}{\partial S_t^2}\sigma^2 S_t^2 \right),$$

$$\sigma_V = \frac{1}{V}\frac{\partial V}{\partial S_t}\sigma S_t$$

and

$$\xi_V = \frac{1}{V}\left[V(S_t(\xi+1), t) - V(S_t, t)\right].$$

In this case, the stochastic consumer's real wealth accumulation in terms of the portfolio shares, $w_{1t} = m_t/a_t$, $w_{2t} = V_t/a_t$, $1 - w_{1t} - w_{2t} = b_t/a_t$, and consumption, c_t, becomes

$$\mathrm{d}a_t = a_t w_{1t} \mathrm{d}R_m + a_t w_{2t} \mathrm{d}R_V + a_t(1 - w_{1t} - w_{2t}) r \mathrm{d}t - c_t \mathrm{d}t,$$

with a_0 exogenously determined. Thus, the above budget constraint can be written as

$$\mathrm{d}a_t = a_t\Big[(r + (\gamma - r)w_{1t} + (\phi_V - r)w_{2t})\mathrm{d}t + (w_{1t}\sigma + w_{2t}\sigma_V)\mathrm{d}W_t + (w_{1t}\xi + w_{2t}\xi_V)\mathrm{d}N_t\Big], \tag{40}$$

where $\gamma = \sigma^2 - \mu - 1 = \phi - 1$. The Hamilton-Jacobi-Bellman equation for the stochastic optimal control problem of maximizing utility, with $\log(c_t) = \log(a_t w_t)$, is given by

$$\max_{w_{t1}, w_{2t}} H(w_t; a_t, t) \equiv \max_{w_{t1}, w_{2t}} \Bigg\{ \log(a_t w_t)e^{-rt} + I_a(a_t, t)a_t[r + (\gamma - r)w_{1t} + (\phi_V - r)w_{2t}] + I_t(a_t, t) + \tfrac{1}{2} I_{aa}(a_t, t)a_t^2(w_{1t}\sigma + w_{2t}\sigma_V)^2 + \lambda \mathrm{E}_\xi\Big[I\Big(a_t(w_{1t}\xi + w_{2t}\xi_V + 1), t\Big) - I(a_t, t)\Big]\Bigg\} = 0. \tag{41}$$

The first-order conditions for an interior solution are:

$$H_{w_{1t}} = 0 \qquad \text{and} \qquad H_{w_{2t}} = 0.$$

We postulate $I(a_t, t)$ in a time-separable form as

$$I(a_t, t) = e^{-rt}[\beta_1 \log(a_t) + \beta_0],$$

where β_0 and β_1 are to be determined from (41). Hence, we obtain

$$\max_{w_{t1}, w_{2t}} H(w_{1t}, w_{2t}; a_t, t) \equiv \max_{w_{t1}, w_{2t}} \Bigg\{ \log(a_t w_t) + \beta_1[r + (\gamma - r)w_{1t} + (\phi_V - r)w_{2t}]$$
$$- r[\beta_1 \log(a_t) + \beta_0]$$
$$- \tfrac{1}{2}\beta_1(w_{1t}\sigma + w_{2t}\sigma_V)^2$$
$$+ \lambda\beta_1 \mathrm{E}_\xi\left[\log(w_{1t}\xi + w_{2t}\xi_V + 1)\right] \Bigg\} = 0$$

If we now compute the first-order conditions, we find that

$$\frac{1}{\beta_1 w_{1t}} + \mathrm{E}_\xi\left[\frac{\lambda\xi}{w_{1t}\xi + w_{2t}\xi_V + 1}\right] + \gamma - r = (w_{1t}\sigma + w_{2t}\sigma_V)\sigma$$

and

$$\mathrm{E}_\xi\left[\frac{\lambda\xi_V}{w_{1t}\xi + w_{2t}\xi_V + 1}\right] + \phi_V - r = (w_{1t}\sigma + w_{2t}\sigma_V)\sigma_V.$$

So far we have not made any assumption on the parameter values. From now on, we assume that $\gamma = \phi - r$. If we suppose a corner solution, $w_{1t} = 1$ and $w_{2t} = 0$, then

$$\frac{1}{\beta_1} + \lambda\mathrm{E}_\xi\left[\frac{\xi}{\xi + 1}\right] + \gamma - r = \sigma^2 \tag{42}$$

and

$$\lambda\mathrm{E}_\xi\left[\frac{\xi_V}{\xi + 1}\right] + \phi_V - r = \sigma\sigma_V \tag{43}$$

It can be shown that $\beta_1 = r^{-1}$. After some simple computations, we have that conditions (42) and (43) become

$$\phi = r + \sigma^2 - \lambda\mathrm{E}_\xi\left[\frac{\xi}{\xi + 1}\right], \tag{44}$$

and

$$\lambda\mathrm{E}_\xi\left[\frac{\xi_V}{\xi + 1}\right] + \phi_V - r = \sigma\sigma_V, \tag{45}$$

where. From (45) it follows

$$\lambda \mathrm{E}_\xi \left[\frac{V(S_t(\xi+1),t) - V(S_t,t)}{\xi+1} \right] + \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial S_t}\phi S_t + \tfrac{1}{2}\frac{\partial^2 V}{\partial S_t^2}\sigma^2 S_t^2 \right) - rV = \frac{\partial V}{\partial S_t}\sigma^2 S_t.$$

If we use now (44), we get

$$\lambda \mathrm{E}_\xi \left[\frac{V(S_t(\xi+1),t) - V(S_t,t) - \xi S_t \frac{\partial V}{\partial S_t}}{\xi+1} \right] + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial S_t} r S_t + \tfrac{1}{2}\frac{\partial^2 V}{\partial S_t^2}\sigma^2 S_t^2 - rV = 0. \quad (46)$$

We impose the boundary conditions $V(0,t) = 0$ and $V(S_T,t) = \max(S_T - K, 0)$ where K is the exercise price of the real option. Notice that the above expected value

$$\mathrm{E}_\xi \left[\frac{V(S_t(1+\xi),t) - \lambda V(S_t,t)}{\xi+1} \right] = \int_{-\infty}^{\infty} \frac{V(S_t(1+\xi),t) - \lambda V(S_t,t)}{\xi+1} f_\xi(\xi)\mathrm{d}\xi,$$

where $f_\xi(\cdot)$ is the density function of ξ produces in (46) a differential-integral equation. Notice that if ξ is constant in (46), then by redefining λ as $\lambda/(\xi+1)$ we obtain Merton's (1973) equation. Finally, observe that when $\xi = 0$ or $\lambda = 0$, equation (46) reduces to the Black-Scholes' (1973) second order parabolic partial differential equation. Notice now that if we write

$$\zeta = \left(\frac{y-\nu}{\kappa} \right)^{-\alpha},$$

then

$$\begin{aligned} \mathrm{E}\left[\frac{\xi}{\xi+1} \right] =& \mathrm{E}\left[\frac{X^{-\alpha}}{X^{-\alpha}-1} \right] \\ =& \int_0^\infty \frac{[(y-\nu)/\kappa]^{-\alpha}}{[(y-\nu)/\kappa]^{-\alpha}-1} f_Y(y)\mathrm{d}y \\ =& \int_0^\infty \frac{\zeta}{\zeta-1} e^{-\zeta}\mathrm{d}\zeta \\ =& -e\Gamma(-1,1), \end{aligned}$$

where $\Gamma(-1,1) = -\Gamma(0,1) + e^{-1}$. It can be shown that $\Gamma(0,1) \approx 2/9$ (in fact, $\Gamma(0,1) = 0.219383934...$). Therefore, equation (46) is transformed into

$$\lambda \mathrm{E}_\xi \left[\frac{V(S_t(1+\xi),t) - V(S_t,t)}{\xi+1} \right] + \frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2} + \left[r + \lambda(\tfrac{2}{9} - 1) \right] S_t \frac{\partial V}{\partial S_t} - rV = 0.$$

A possibility to determine $V(S_t,t)$ consists in defining a sequence of random variables Y_n with the distribution of the product of n independent and identically distributed random variables $\xi + 1$ with $Y_0 = 1$. In other words, if $\{\xi_n\}_{n\in\mathbb{N}}$ is a sequence of independent and identically distributed random variables. We define

$$\begin{aligned}
Y_0 &= 1\\
Y_1 &= \xi_1 + 1\\
Y_2 &= (\xi_1 + 1)(\xi_2 + 1)\\
&\vdots\\
Y_n &= \prod_{k=1}^{n}(\xi_k + 1)\\
&\vdots
\end{aligned}$$

In this case, the solution of equation (46) with the boundary conditions

$$V(0,t) = 0, \qquad \text{and} \qquad V(S_T, 0) = \text{máx}(S_T - K, 0),$$

is given by

$$V(S_t,t) = \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n}\left[\frac{e^{-\lambda(T-t)/(\xi+1)}[\lambda(T-t)/(\xi+1)]^n}{n!} V_{\text{BS}}(S_t Y_n e^{-\lambda \mathrm{E}_\xi[\xi/(\xi+1)](T-t)}, t)\right], \tag{47}$$

where ξ is independent of $\{\xi_n\}_{n\in\mathbb{N}}$ and $V_{\text{BS}}(\cdot,\cdot)$ is the basic Black-Scholes solution. Indeed, consider

$$V(S_t,t) = \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n}[P_{n,t}\ V_{\text{BS}}^{(n)}\ ], \tag{48}$$

where

$$P_{n,t} = \frac{e^{-\lambda(T-t)/(\xi+1)}[\lambda(T-t)/(\xi+1)]^n}{n!},$$

$$U_{n,t} = Y_n e^{-\lambda \mathrm{E}_\xi[\xi/(\xi+1)](T-t)}$$

and

$$V_{\text{BS}}^{(n)} = V_{\text{BS}}(S_t U_{n,t}, t).$$

In what follows, it it will convenient to introduce the notation

$$Q_{n,t} = S_t U_{n,t}.$$

In such a case,

$$\frac{\partial V}{\partial S_t} = \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ U_{n,t} \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}} \right], \tag{49}$$

$$\frac{\partial^2 V}{\partial S_t^2} = \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ U_{n,t}^2 \frac{\partial^2 V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}^2} \right] \tag{50}$$

and

$$\begin{aligned} \frac{\partial V}{\partial t} =& \lambda \mathrm{E}_\xi[\xi/(\xi+1)] \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ Q_{n,t} \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}} \right] \\ &+ \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial t} \right] \\ &+ \lambda \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[\frac{P_{n,t}\ V_{\mathrm{BS}}^{(n)}}{\xi+1} \right] \\ &- \lambda \sum_{n=1}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[\frac{e^{-\lambda(T-t)/(\xi+1)} [\lambda(T-t)/(\xi+1)]^{n-1}}{(n-1)!} \left(\frac{V_{\mathrm{BS}}^{(n)}}{\xi+1} \right) \right]. \end{aligned} \tag{51}$$

Hence, by virtue of (49), (50) and (51), we get

$$\begin{aligned} \frac{\partial V}{\partial t} =& \lambda \mathrm{E}_\xi[\xi/(\xi+1)] S_t \frac{\partial V}{\partial S_t} + \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial t} \right] + \lambda \mathrm{E}_\xi \left[\frac{V(S_t,t)}{\xi+1} \right] \\ &- \lambda \sum_{m=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_{m+1}} \left[\frac{e^{-\lambda(T-t)/(\xi+1)} [\lambda(T-t)/(\xi+1)]^{m}}{m!} \left(\frac{V_{\mathrm{BS}}^{(m+1)}}{\xi+1} \right) \right]. \end{aligned} \tag{52}$$

Observe that the last term in the above equation can be written as

$$\begin{aligned} \mathrm{E}_\xi \left[\frac{V((\xi+1)S_t,t)}{\xi+1} \right] &= \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ \frac{V_{\mathrm{BS}}^{(n)}(Q_{n,t}(1+\xi),t)}{\xi+1} \right] \\ &= \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_{n+1}} \left[P_{n,t}\ \frac{V_{\mathrm{BS}}^{(n+1)}(Q_{n+1,t},t)}{\xi+1} \right], \end{aligned} \tag{53}$$

since $Q_{n+1,t}$ y $Q_{n,t}(1+\xi)$ are independent and identically distributed random variables. Therefore, equation (52) is transformed into

$$\frac{\partial V}{\partial t} = \sum_{n=0}^{\infty} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[P_{n,t}\ \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial t} \right] - \lambda \mathrm{E}_\xi \left[\frac{V(S_t(\xi+1),t) - V(S_t,t) - \xi S_t \frac{\partial V}{\partial S_t}}{\xi+1} \right]. \tag{54}$$

From (49), (50) and (54), it follows that

$$\begin{aligned}
&\frac{\partial V}{\partial t} + \tfrac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2} + r S_t \frac{\partial V}{\partial S_t} - rV \\
&= \sum_{n=0}^{\infty} P_{n,t} \mathrm{E}_\xi \mathrm{E}_{Y_n} \left[\frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial t} + \tfrac{1}{2}\sigma^2 Q_{n,t}^2 \frac{\partial^2 V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}^2} + r Q_{n,t} \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}} - r V_{\mathrm{BS}}^{(n)} \right] \\
&\quad - \lambda \mathrm{E}_\xi \left[\frac{V(S_t(\xi+1), t) - V(S_t, t) - \xi S_t \frac{\partial V}{\partial S_t}}{\xi + 1} \right].
\end{aligned} \tag{55}$$

Since

$$\frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial t} + \tfrac{1}{2}\sigma^2 Q_{n,t}^2 \frac{\partial^2 V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}^2} + r Q_{n,t} \frac{\partial V_{\mathrm{BS}}^{(n)}}{\partial Q_{n,t}} - r V_{\mathrm{BS}}^{(n)} = 0$$

holds for all $n \in \mathbb{N} \cup \{0\}$, we deduce, immediately, that (47) is solution of (46).

9. Conclusions

We have developed a stochastic model of exchange-rate-based stabilization with imperfect credibility. An important feature of our formulation is that there is a lack of credibility even if parameter values determining the expectations of devaluation are not changed and that extreme devaluation can take place. By using a logarithmic utility, we have derived closed-form solutions to examine the dynamic implications of extreme devaluations. These explicit solutions have made much easier the understanding of the key issues of inflation stabilization plans that are expected to be temporary by the public.

Our stochastic framework, in which a Brownian motion and a Poisson process with the jump size driven by a extreme value distribution, provides new elements to carry out simulation experiments and empirical research on some observed stylized facts. We have also obtained the price of the real option of waiting when consumption can be delayed as the solution of a differential-integral equation.

It is worthwhile mentioning that the derived results depend on the assumption of logarithmic utility, which is a limit case of the family of constant relative risk aversion utility functions. Needles to say, both nontradable and durable goods, which provide more realistic assumptions, should be considered in extending the model. These extensions will lead to more complex transitional dynamics, but results will certainly be richer.

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