Investment in Schooling and the Marriage Market*

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Abstract

We produce a model with pre-marital schooling investment,endogenuos marital matching and spousal specialization in homework and market production Pre-marital investments generate two kinds of returns: a labor-market return due to the education premium and a marriage-market return because education can improve the intra-marital share of the surplus one can extract from marriage. When the returns to education are gender neutral, men and women educate in equal proportions and there is pure positive assortative matching in the marriage markets. But if the returns are not gender neutral, then there is mixing in equilibrium where some educated individuals marry uneducated spouses and those who educate less because their labor-market return is lower extract a relatively larger share of the marital surplus. Conditional on the choice of schooling, couples' career decisions affect the size of their marital surplus, but the existence of large and frictionless marriage markets can still produce efficient household specialization where the higher-wage spouse specializes in market production and the lower-wage spouse engages in homework. Even when cultural and social norms or the time requirements of homework dictate that wives devote relatively more time to homework, women can acquire more schooling than men if a gender wage gap exists but narrows with the level of education,.

1 Introduction

One of the pronounced trends in recent decades is the increased investment in education by women and the closing of the gap in schooling between men and women. In several developed countries, women now have more schooling than men. Beyond the

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important impact of schooling on the earning capacity of women and their economic independence, such trends influence the gains from marriage and marriage patterns. Couples sort according to schooling and, therefore, the educational gap within couples has declined too. At the same time we observe a decline in marriage and higher divorce rates, suggesting that the closing of the educational gaps had a larger influence on those who remain single than on those who marry. Women today receive lower wages and spend more time at home than men, although these gaps have narrowed over the years. One would think, therefore, that women should invest less in schooling, which appears to be less useful for them both at home and in the market. Yet, women have increased their schooling attainments sharply and now acquire *more* schooling than men.

Goldin et al. (2006) discuss the rise in investments in schooling of men and women and show that starting with the 1970 birth cohort, women have attained *higher* college graduation rates than men. Goldin (1997) provides a vivid and detailed description of the changes in the career and schooling choices of women in the last century. She compares female college graduates of several birth cohorts. Women who graduated from college during the early part of the twentieth century (1900 -1920) had sacrificed family to pursue a career; 50 percent of them had no children by age 35-44 and 30 percent of them never married. Women of later cohorts were able to mix family life with a career, but they altered the timing of their career and family-life choices. Those who graduated from college prior 1965 gradually raised their marriage and fertility rates and they typically had children before entering the labor market. In contrast, women who graduated from college after 1965 had lower marriage and fertility rates and they tended to start to work before having children.

These demographic changes were also associated with changes in the patterns of assortative mating by schooling. The degree of positive assortative matching by schooling has risen over time. But while college-educated women in the earlier birth cohorts married up and most of them had a college-educated husband, the rate of marrying up declined among educated-women of later cohorts because there were more educated women in those cohorts.

In contrast to other attributes such as race and ethnic background, schooling is an *acquired* trait and within some limits subject to choice. Presumably, agents who invest in schooling take into account the potential gains both in the labor market and within marriage. However, the gains from schooling within marriage strongly depend on the decisions of others to acquire schooling. Because much of schooling happens before marriage, partners cannot coordinate their investments. Rather, men and women make their choices separately, based on the anticipation of marrying a "suitable" educated spouse with whom the investments in schooling are expected to generate a higher return.

The purpose of this paper is to provide a simple general equilibrium framework for the *joint* determination of pre-marital schooling and choices couples make during their marriage. Hence, an important feature of the model is that the investment

choices of both men and women are established simultaneously. In our model, the returns to pre-marital investments can be decomposed into two parts: First, higher education raises ones wage rate and increases the payoff from time on the job (the *labor-market* return). Second, it can improve the intra-marital share of the surplus one can extract from marriage (the *marriage-market* return).[1]

The basic ingredients of our model are as follows. We consider a frictionless marriage market and assume that, conditional on the predetermined spousal schooling levels, the assignments are stable. That is, there are no men or women (married or single) who wish to form a new union and there are no men or women who are married but wish to be single. We then assume transferable utility between the spouses to characterize the stable assignment. We further assume that there men and women who can be divided into schooling classes (high and low) and the interactions between married spouses depend only on their education class. In particular, although men and women have idiosyncratic preferences for marriage and investment in schooling, they all have the same ranking over spouses of the opposite sex which depend only on their schooling. Thus, every educated man (woman) and every uneducated man (woman) has a perfect substitute. The absence of rents allows us to pin down the shares of the marital surplus of men and women in each schooling class. These shares, together with the known returns as singles, are sufficient to determine the investments in schooling of men and women.

Our main results are that, when the market return to education and household roles are gender neutral, men and women acquire education in equal proportions and, under the assumption that the schooling of the two spouses complement each other, a strictly positive assortative matching arises in the marriage markets. That is, educated men marry only educated women and uneducated men marry only uneducated women. But if the returns are not gender neutral, then there is mixing in equilibrium where some educated individuals marry uneducated spouses and individuals of the gender that marries down obtains a lower return from schooling within marriage. In particular, we show that a transition from an old regime in which women are required to work at home and expect lower market wages to a new regime where less work is required at home and the wages of educated women become more in line with that of educated men, women may *overtake* men in terms of schooling, despite their lower market wage rate and higher amount of housework compared with men. We hypothesize that the increase in the levels of schooling investment by women to and above the levels of men is a consequence of the higher return that women receive for schooling, reflecting lower labor market "discrimination" at higher levels of schooling.[2] The essence of the argument we make is that education can serve as a means

[1] Educational attainment could influence intra-marital spousal allocations directly (due to the fact that education raises household income) or indirectly (to the extent that educational attainment influences prospects for marriage and the determination of spousal roles within marriage.

[2] Mincer and Polachek (1974) and Weiss and Gronau (1981) provided explanations for the main patterns of the gender wage gap even in the absence of any discrimination based on lower investments

to escape discrimination.[3] Educated women are also more likely to marry in the new regime. However, when educated women become relatively abundant their share in the marital surplus declines.

2 Background

We begin with a brief description of the main facts that we wish to address. Figure 1 describes the time trends in level of completed schooling of men and women, aged 30 to 40, in the United States. As seen, the proportions of women with some college education, college education and advanced degrees (M.A., Ph.D.) have increased much faster than the corresponding proportions for men. By 2003, women had overtaken men in all of these three categories. Goldin et al. (2006) present trends for college graduation by gender and show that starting with the 1970 birth cohort, women have attained higher college graduation rates than men.

Figure 2 presents the time trends in the hourly wage differentials by schooling for men and women in the United States (for those who work at least 20 hours a week and adjusted for potential work experience).[4] Compared with high school, women receive a higher increase in wages than men when they acquire college or advanced degrees. These gender differences in the returns to schooling tend to widen with the level of schooling and narrow with time. They are positive and statistically significant for most years.

As is well known, gender wage differences are confounded by a variety of selection processes: selection into marriage, selection into schooling and selection into work. To disentangle all these effects is beyond the scope of this work (see Mulligan and Rubinstein, 2005). However, if we restrict the sample to "full time, full year" workers (who reported at least 35 hours a week and 51 weeks of work last year), the female advantage in the returns to schooling in the CPS is significant only for advanced degrees between the years 1968 and 1999, but is usually insignificant for other degrees. This suggests that additional schooling favors women mainly in terms access (or commitment) to full-time jobs.[5]

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on the job resulting from expected interruptions in participation. At that time, women also acquired less schooling. The current reversal in the schooling gender gap poses a challenge to this approach.

[3] Discrimination here simply means that conditioned on their level of schooling, women expect lower wages than men during their work career. This outcome can result from a variety of causes including self selection of women into part-time jobs with lower wages and weaker incentives for women to acquire, or for employers to provide, on the job training.

[4] The sample includes whites aged 25 to 60. Observations with hourly wages of less than 1 or more than 400 dollars were excluded. The presented coefficients are from year-by-year separate regressions for men and women with log hourly wages as the dependent variable and highest degree, experience and experience-squared as explanatory variables.

[5] A familiar conceptual issue is which variables should be held fixed when one considers the impact of schooling. It seems that for the analysis of schooling investment, variables such as hours

Positive assortative matching by schooling is prevalent and about half of the married couples have the same levels of schooling. This proportion remained stable over time. However, the proportion of couples in which the husband has a higher degree than his wife declined with time and in the most recent cohort it is the wife that is more educated than her husband among young couples for whom the husband is 30 to 40 years old (see Figure 3). For married couples in the decades 1970-1979 and 1990-1999, Figure 4 shows the distribution of the spouse's education for husbands and wives with different level of schooling. At low levels of schooling, each gender mainly marries similarly-educated individuals of the opposite sex. This pattern became more prevalent over time as the distribution of education among women and men became more similar. In particular, we see a large increase in marriages where the husband and the wife have some college education. Because the number of women with some college has risen sharply relative to men, we see that husbands with some college attainment have substituted wives with high school with wives with some college education, while wives with some college education replaced men with college education and higher degrees with men with some college attainment. However, at higher levels of schooling (such as B.A. and more) and among whom women are still relatively scarce, we see that more highly-educated men still marry down while college-educated women marry up.

3 The Basic Model

We begin with a benchmark model in which men and women are completely symmetric in their preferences and opportunities. However, by investing in schooling, agents can influence their marriage prospects and labor market opportunities. Competition over mates determines the assignment (i.e., who marries whom) and the shares in the marital surplus of men and women with different levels of schooling, depending on the aggregate number of women and men that acquire schooling. In turn, these shares together with the known market wages guide the individual decisions to invest in schooling and to marry. We investigate the rational-expectations equilibrium that arises under such circumstances.

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of work and job characteristics, and perhaps even experience, should be allowed to vary. Mulligan and Rubinstein (2005) show that, in the CPS, the gender wage gap declines with schooling if one compares men and women who work full time but does not control for experience. Dougherty (2005) and O'Neill and O'Neill (2006) show, using NLSY data, that the gender differences in the impact of schooling are eliminated when detailed employment and occupational characteristics are added. Gronau (1998) shows, using PSID data, that education strongly affects access to on the job training opportunities, but the difference between men and women in this regard is not significant.

3.1 Assumptions

There are two equally large populations of men and women to be matched. Individuals live for two periods. Each person can choose whether to acquire schooling or not and whether and whom to marry. Investment takes place in the first period of life and marriage in the second period. Investment in schooling is lumpy and takes one period so that a person who invests in schooling works only in the second period, while a person who does not invest works in both periods. To simplify, we assume no credit markets.[6] All individuals with the same schooling earn the same wage rate and we denote the wage of an educated person by w_2 and the wage of an uneducated person by w_1 where $w_2 > w_1$. Market wages are taken as exogenous and we do not attempt to analyze here the feedbacks from the marriage market and investments in schooling to the labor market. We shall discuss, however, different wage structures.

We denote a particular man by i and a particular woman by j. The schooling level (class) of man i by $I(i)$ where $I(i) = 1$ if i is uneducated and $I(i) = 2$ if he is educated. Similarly, we denote the class of woman j by $J(j)$ where $J(j) = 1$ if j is uneducated and $J(j) = 2$ if she is educated. The surplus generated by a marriage of man i and woman j is

$$s_{ij} = z_{I(i)J(j)} + \theta_i + \theta_j, \tag{1}$$

where θ_i and θ_j represent the non-economic gains of man i and woman j from their marriage and $z_{I(i)J(j)}$ is the material surplus that the marriage generates. Married partners can divide their material surplus and their utilities are linear in the shares (transferable utility).

We assume that the schooling levels of married partners complement each other so that

$$z_{11} + z_{22} > z_{12} + z_{21}. \tag{2}$$

When men and women are viewed symmetrically, we also have $z_{12} = z_{21}$.

The per-period material utilities of man i and woman j as singles are denoted by $z_{I(i)0}$ and $z_{0J(i)}$ and are assumed to increase in $I(i)$ and $J(j)$. Thus, a more educated person has a higher utility as a single. Men and women who acquire no schooling and never marry have life time utilities of $2z_{10}$ and $2z_{01}$, respectively. A person that invests in schooling must give up the first period utility and, if he\she remains single, the life time utilities are z_{20} for men and z_{02} for women. Thus, the (absolute) return from schooling for never married men and women are $R^m = z_{20} - 2z_{10}$ and $R^w = z_{02} - 2z_{01}$, respectively.[7] The return to schooling of never-married individuals depends only on their own market wages and we shall refer to it as the *labor-market* return. A married person has an additional return from schooling investment because

[6] Allowing borrowing and lending raises issues such as whether or not one can borrow based on the income of the future spouse and enter marriage in debt (see Browning et al., in progress, ch. 7).

[7] Because we assume away the credit market, the rate of return from investment in schooling depends on consumption decisions and is in utility terms. Also note that, with inelastic labor supply and linear utility, $z_{10} = z_{01} = w_1$ and $z_{20} = z_{02} = w_2$.

it can influence whom one marries and the shares each spouse receives from the marital surplus. In addition to the returns in the market or marriage, investment in schooling is associated with idiosyncratic costs (benefits) denoted by μ_i for men and μ_j for women.

The idiosyncratic preference parameters are assumed to be independent of each other and across individuals. We denote the distributions of θ and μ by $F(\theta)$ and $G(\mu)$ and assume that these distributions are symmetric around their zero means. This specification is rather restrictive because one might expect some correlations between the taste parameters and the observable attributes. For instance, individuals that have a low cost of schooling may also have a high earning capacity and individuals may derive different benefits from marriage depending on the observed quality of their spouses. One may also expect a correlation between the emotional valuations of the marriage by the two spouses. Thus, the model is very basic and intended mainly as an illustration of the possible feedbacks between the marriage market and investment in schooling.

3.2 The Marriage Market

Any *stable* assignment of men to women must maximize the *aggregate surplus* over all possible assignments (Shapley and Shubik, 1972). The dual of this linear programming problem posits the existence of non-negative shadow prices associated with the constraints of the primal that each person can be either single or married to one spouse. We denote the shadow price of woman j by u_j and the shadow price of man i by v_i. The complementarity slackness conditions require that

$$z_{I(i)J(j)} + \theta_i + \theta_j \leq v_i + u_j, \tag{3}$$

with equality if i and j are married and inequality otherwise.

Condition (3) is equivalent to

$$\begin{aligned} v_i &= Max\{\underset{j}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - u_j], 0\} \\ u_j &= Max\{\underset{i}{Max}[z_{I(i)J(j)} + \theta_i + \theta_j - v_i], 0\}, \end{aligned} \tag{4}$$

which means that the assignment problem can be *decentralized*. That is, given the shadow prices u_j and v_i, each agent marries a spouse that yields the highest "profit." Alternatively, we can view the shadow prices u_j and v_i as the reservation utility levels that woman j and man i require to participate in *any marriage*.

Our specification imposes a restrictive but convenient structure in which the interactions between agents depend on their group affiliation only, i.e., their level of

schooling. Therefore, the endogenously-determined shadow prices of married man i in I and married woman j in J can be written in the form,

$$v_i = V_{I(i)} + \theta_i \qquad \text{and} \qquad u_j = U_{J(j)} + \theta_j, \tag{5}$$

where

$$V_I = \underset{J}{Max}[z_{IJ} - U_J] \qquad \text{and} \qquad U_J = \underset{I}{Max}[z_{IJ} - V_I] \tag{6}$$

are the *shares* that the partners receive from the "material" surplus of the marriage (not accounting for the idiosyncratic effects θ_i and θ_j). All agents of a given type receive the *same* share of the material surplus z_{IJ} no matter whom they marry, because all the agents on the other side rank them in the same manner. Any man (woman) of a given type who asks for a higher share than the "going rate" cannot obtain it because he or she can be replaced by an equivalent alternative.[8]

Although we assume equal numbers of men and women, it is possible that the number of educated men and women will differ. We shall assume throughout that the variances in θ and μ are large enough to ensure that there will always be some uneducated men who marry uneducated women and some educated men who marry educated women. This means that the equilibrium shares must satisfy

$$U_2 + V_2 = z_{22}, \tag{7}$$

$$U_1 + V_1 = z_{11}. \tag{8}$$

We can then classify the possible matching patterns as follows. Under *strict* positive assortative mating, educated men marry only educated women and uneducated men marry only uneducated women. Then,

$$U_1 + V_2 \geq z_{21}, \tag{9}$$

$$U_2 + V_1 \geq z_{12}. \tag{10}$$

If there are more educated men than women among the married, some educated men will marry uneducated women and condition (9) also will hold as an equality. If there are more educated women than men among the married, equation (10) will

[8] Our model features transferable utility and marriage markets that operate frictionlessly. As such, the couples' outside prospects–be it their utility as singles or in another potential marriage–play an independent role in whether or not household allocations are made efficiently. Thus, no committment mechanism exists between the potential spouses and the ability to remarry acts as a substitute for commitment. This result holds even when there are some search frictions in spousal matching. For more details on the role of commitment in intra-household allocations, see Lundberg and Pollak (1993).

That noted, when spouses need to make career choices and investment in children, some commitment to stay married may be required to implement the efficient household division of labor if spouses need to make career choices and time investment in children. We shall return to this topic in subsection 4.4.

hold as an equality. It is impossible that all four conditions will hold as equalities because this would imply

$$z_{22} + z_{11} = z_{12} + z_{21}, \tag{11}$$

which violates assumption (2) that the education levels of the spouses are complements. Thus, either educated men marry uneducated women or educated women marry uneducated men but not both.

When types mix and there are more educated men than educated women among the married, conditions (7) to (9) imply

$$\begin{aligned} U_2 - U_1 &= z_{22} - z_{21}, \\ V_2 - V_1 &= z_{21} - z_{11}. \end{aligned} \tag{12}$$

If there are more educated women then men among the married then conditions (7), (8) and (10) imply

$$\begin{aligned} V_2 - V_1 &= z_{22} - z_{12}, \\ U_2 - U_1 &= z_{12} - z_{11}. \end{aligned} \tag{13}$$

One may interpret the differences $U_2 - U_1$ and $V_2 - V_1$ as the *return to schooling in marriage* for women and men, respectively. The quantity $z_{22} - z_{21}$ which reflects the contribution of an educated woman to the material surplus of a marriage with an educated man provides an *upper bound* on the return that a woman can obtain through marriage, while her contribution to a marriage with an uneducated man, $z_{12} - z_{11}$, provides a *lower bound*. When there are more educated women than men, analogous bounds apply to men. When types mix in the marriage market equilibrium, we see that the side that is in short supply receives the marginal contribution to a marriage with an educated spouse, while the side in excess supply receives the marginal contribution to a marriage with an uneducated spouse.

One issue of concern is whether the "material shares" defined above are non-negative. In practice, if the only means to transfer utility within couples is via the transfer of consumption goods, which are bounded from below at zero, then the non-negativity constraints on consumption bind, utility is no longer transferable and it becomes difficult to determine the stable assignments and the marital shares that support these assignments. However, we may assume that the partners can also exchange "signs of endearment" in which case the marital shares are no longer material and can be negative or positive. In the subsequent analysis, we shall provide examples in which the material shares are positive in equilibrium.

3.3 Investment Decisions

We assume rational expectations so that, in equilibrium, individuals know V_I and U_J, which are sufficient statistics for investment decisions. Given these shares and knowledge of their own idiosyncratic preferences for marriage, θ, and costs of schooling, μ, agents know for sure whether or not they will marry in the second period, conditional on their choice of schooling.

Man i chooses to invest in schooling if

$$z_{20} - \mu_i + Max(V_2 + \theta_i\ ,0) > 2z_{10} + Max(V_1 + \theta_i\ ,0). \tag{14}$$

Similarly, woman j chooses to invest in schooling if

$$z_{02} - \mu_j + Max(U_2 + \theta_j\ ,0) > 2z_{01} + Max(U_1 + \theta_j\ ,0). \tag{15}$$

Figure 5 describes the choices made by different men. Men for whom $\theta < -V_2$ do not marry and invest in schooling if and only if $\mu < R^m \equiv z_{20} - 2z_{10}$. Men for whom $\theta > -V_1$ always marry and they invest in schooling if and only if $\mu < R^m + V_2 - V_1$. Finally, men for whom $-V_2 < \theta < -V_1$ marry if they acquire education and do not marry if they do not invest in schooling. These individuals will acquire education if $\mu < R^m + V_2 + \theta$. In this range, there are two motivations for schooling: to raise future earning capacity and to enhance marriage.

The proportion of men who invest in schooling is

$$G(R^m)F(-V_2) + [1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta, \tag{16}$$

the proportion of men who marry is

$$[1 - F(-V_1)] + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta, \tag{17}$$

and the proportion of men who invest and marry is

$$[1 - F(-V_1)]G(R^m + V_2 - V_1) + \int_{-V_2}^{-V_1} G(R^m + V_2 + \theta)f(\theta)d\theta. \tag{18}$$

The higher are the returns from schooling in the labor market, R^m, and in marriage, $V_2 - V_1$, the higher is proportion of men who acquire schooling. A common increase in the levels V_2 and V_1 also raises investment because it makes marriage more attractive and schooling obtains an *extra* return within marriage. For the same reason, an increase in the market return R^m raises the proportion of men that marry. Analogous expressions hold for women.

3.4 Equilibrium

In the marriage market equilibrium, the number of men and women who marry must be the same. Using equation (17) and applying symmetry, we can write this condition as

$$F(V_1) + \int_{V_1}^{V_2} G(R^m + V_2 + \theta) f(\theta) d\theta = F(U_1) + \int_{U_1}^{U_2} G(R^w + U_2 + \theta) f(\theta) d\theta. \tag{19}$$

Under strictly positive assortative mating, the number of men and women in each education group is equal. Given that we impose condition (19), it is necessary and sufficient to require that the number of men and women who marry but do *not* invest in schooling is the same. Using condition (18) and symmetry, we can write this condition as

$$F(V_1)G(-R^m + V_1 - V_2) = F(U_1)G(-R^w + U_1 - U_2). \tag{20}$$

Together with conditions (7) and (8), conditions (19) and (20) yield a system of four equations in four unknowns that are, in principle, solvable.

If there is some mixing of types, equation (20) is replaced by an inequality and the shares are determined by the boundary conditions on the returns to schooling within marriage for either men or women, whichever is applicable. If there are more educated men than women among the married,

$$F(V_1)G(-R^m + V_1 - V_2) < F(U_1)G(-R^w + U_1 - U_2) \tag{20a}$$

and educated women receive their maximal return from marriage while men receive their minimal return so that condition (12) holds. Conversely, if there are more educated women than men among the married we have

$$F(V_1)G(-R^m + V_1 - V_2) > F(U_1)G(-R^w + U_1 - U_2) \tag{20b}$$

and educated men receive their maximal return from marriage while educated women receive their minimal return so that condition (13) holds. Together with condition (7) and (8), we have four equations in four unknowns that are in principle solvable.[9]

The two types of solutions are described in Figures 6 and 7, where we depict the equilibrium conditions in terms of V_1 and V_2 after we eliminate U_1 and U_2, using (7) and (8). The two positively sloped green lines in these figures describe the boundaries on the returns to schooling of men within marriage. The negatively sloped red line describes the combinations of V_1 and V_2 that maintain equality in the number of men

[9]Note the system of equations consisting of (7), (8) and (11) and the system consisting of (7), (8) and (12) impose only *three* independent requirements.

and women who wish to marry. The positively sloped blue line describes the combinations of V_1 and V_2 that maintain equality in the number of men and women with each level of schooling among the married. The slopes of these lines are determined by the following considerations: An increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, induces more men and fewer women to prefer marriage. An increase in V_2 holding V_1 has a similar effect. Thus, V_1 and V_2 are substitutes in terms of their impact on the incentives of men to marry and U_1 and U_2 are substitutes in terms of their impact on the incentives of women to marry. Therefore, equality in the number of men and women who wish to marry can be maintained only if V_2 declines when V_1 rises.[10] At the same time, an increase in V_1 (and a reduction in U_1), keeping V_2 and U_2 constant, reduces the return to education that men receive within marriage and raises the return to education that women receive within marriage. Hence, fewer educated men and more educated women will wish to marry. Therefore, equality in the number of educated men and women who wish to marry can be maintained only if V_2 rises when V_1 rises so that the rates of return to education within marriage are restored.[11]

As long as the model is completely symmetric, that is $R^m = R^w$ and $z_{12} = z_{21}$, the equilibrium is characterized by equal sharing: $V_2 = U_2 = z_{22}/2$ and $U_1 = V_1 = z_{11}/2$. With these shares, men and women have identical investment incentives. Hence, the number of educated (uneducated) men equals the number of educated (uneducated) women, both among the singles and the married. Such a solution is described by point e in Figure 6, where the lines satisfying conditions (19) and (20) intersect. There is a unique symmetric equilibrium. However, with asymmetry, when either $R^m \neq R^w$ or $z_{12} \neq z_{21}$, there may be a mixed equilibrium where the line representing

[10] Differentiating (19),

$$\begin{aligned} 0 &= \{f(V_1)[1-G(R^m+V_2-V_1)] + f(z_{11}-V_1)[1-G(R^w+z_{22}-z_{11}-(V_2-V_1)]\}dV_1 \\ &\quad +\{G(R^m)f(V_2)+G(R^w)f(z_{22}-V_2)] \\ &\quad +[\int_{V_1}^{V_2} g(R^m+V_2-\theta)f(\theta)d\theta + \int_{U_1}^{U_2} g(R^w+U_2-\theta)f(\theta)d\theta]\}dV_2 \end{aligned}$$

implying that

$$\frac{dV_2}{dV_1} < 0.$$

[11] The slope line satisfying condition (20) must exceed 1 because

$$\begin{aligned} & f(V_1)G(R^m-(V_1-V_2)) + f(z_{11}-V_1)G(R^m-(z_{22}-z_{11})+(V_1-V_2)]dV_1 \\ = \; & F(V_1)g(R^m-(V_1-V_2)+F(z_{11}-V_1)g(R^w-(z_{22}-z_{11})+(V_1-V_2)]d(V_2-V_1) \end{aligned}$$

and therefore

$$\frac{d(V_2-V_1)}{dV_1} = \frac{dV_2}{dV_1} - 1 > 0.$$

condition (19) intersect either the lower or upper bound on $V_2 - V_1$ so that condition (20) holds as an inequality. Such a case is illustrated by the point $e\prime$ in Figure 7. In this equilibrium, educated men obtain the lower bound on their return to education within marriage, $z_{21} - z_{11}$. The equilibrium point $e\prime$ is on the lower bound and above the blue line satisfying condition (20), indicating excess supply of educated men.

4 Gender Differences in the Incentive to Invest

In this section, we discuss differences between women and men that can cause them to invest at different rates. We discuss two possible sources of asymmetry:

- Women may receive different market wages, which may lead to lower returns for single women.
- Women may receive a lower return from schooling within marriage because they must take care of children.

Either of the above causes can induce women to invest less in schooling. Therefore, the lower incentives of women to invest can create equilibria with mixing, where educated men are in excess supply and some of them marry less-educated women.

4.1 The Household Production Technology

We use a rudimentary structural model to trace the impact of different wages and household roles of men and women on the marital output and surplus. We assume that, irrespective of the differences in wages or household roles, men and women have the same preferences given by

$$u = cq + \theta, \tag{21}$$

where c is a private good, q is a public good that can be shared if two people marry but is private if they remain single and θ is the emotional gain from being married (relative to remaining single). The public good is produced within the household according to a household production function

$$q = e + \gamma t, \tag{22}$$

where e denotes purchased market goods, t is time spent working at home and γ is an efficiency parameter.

This specification implies transferable utility between spouses and allows us to trace the impact of different market wages or household roles on the decisions to invest and marry. Time worked at home is particularly important for parents with children. To simplify, we assume that all couples have children and that rearing children requires

a specified amount of time $t = \tau$, where τ is a constant such that $0 < \tau < 1$. Moreover, *all* the time provided at home is supplied by the *mother*. Hence, women spend τ amount of their time on childrearing and $1 - \tau$ of it on market work.

If man i of class I with wage $w^m_{I(i)}$ marries woman j of class J with wage $w^w_{J(j)}$, their joint income is $w^m_{I(i)}+(1-\tau)w^w_{J(j)}$. Any efficient allocation of the family resources maximizes the partners' sum of utilities given by $[w^m_{I(i)}+(1-\tau)w^w_{J(j)}-e](e+\tau\gamma)+\theta_i+\theta_j$, which implies that the partners agree to spend half of the family income on the public good. The *marital output* is, therefore,

$$o_{ij} = \frac{[w^m_{I(i)} + \tau\gamma + (1-\tau)w^w_{J(j)}]^2}{4} + \theta_i + \theta_j. \tag{23}$$

Note that the wages of the husband and wife complement each other in generating marital output, which is a consequence of sharing the child(ren)–the public good of their marriage.

Since an unmarried man i solves

$$\underset{e_i,c_i}{Max} \;\; c_i e_i \tag{24}$$

subject to

$$c_i + e_i = w^m_{I(i)}, \tag{25}$$

his optimal behavior generates a utility level of $(w^m_{I(i)}/2)^2$. A single woman j solves an analogous problem and obtains $(w^w_{I(i)}/2)^2$. Therefore, the total material *surplus* generated by the marriage in the second period is

$$s_{ij} \;\; = \;\; \frac{[w_{I(i)} + \tau\gamma + (1-\tau)w_{J(j)}]^2 - w^2_{I(i)} - w^2_{J(j)}}{4} + \theta_i + \theta_j \;\; \equiv z_{I(i)J(j)} + \theta_i + \theta_j \; . \tag{26}$$

The mother's effective wage is now a weighted average of her market wage and productivity at home. We assume that $w_2 > \gamma > w_1$ so that having children is costly for educated women but not for uneducated women.[12] The surplus function (26) maintains complementarity between the wages of the husband and wife, which is a consequence of sharing a public good.[13] However, the assumed asymmetry in

[12] We also implicitly assume that the prerequisite for having offspring is getting married. Thus, while we do not model cohabitation, the implications of the model could apply to that as well.

[13] The expression in (26) applies only if, at the optimum, the family spends money on the public good, $e > 0$. Otherwise the surplus becomes

$$\tau\gamma(w_{I(i)} + (1-\tau)w_{J(j)}) - \frac{w^2_{I(i)} + w^2_{J(j)}}{4} + \theta_i + \theta,$$

in which case the surplus becomes additive. A sufficient condition for a positive e is $w_1+(1-\tau)w_1 > \tau\gamma$.

household roles between men and women implies that a higher husband's wage always raises the surplus but a higher mother's wage can reduce the surplus. In other words, it may be costly for a high-wage woman to marry because she must spend time on the child, while if the mother does not marry her utility as a single remains $w_{J(j)}^2/4$. In addition, it is no longer true that $z_{21} = z_{12}$ as in the benchmark case. In fact,

$$z_{21} - z_{12} = \frac{\tau(w_2 - w_1)}{2}[(1-\tau)\frac{w_2 + w_1}{2} + \tau\gamma] > 0, \tag{27}$$

which means that the marriage of an educated man with an uneducated woman creates a larger marital surplus than the marriage of an educated woman with an uneducated man.

4.2 Gender Roles I: The Impact of the Wage Gap

We are now ready to examine the implications of the gender wage differences. Let w_2 and w_1 denote the wage that educated and uneducated men obtain, respectively, and let $d_2 w_2$ and $d_1 w_1$ be the corresponding wages of women. The gender difference in wages can be an outcome of discrimination associated, for instance, with lower options for investment on the job.[14] Such discrimination can reduce or increase the incentives of women to invest, depending on whether discrimination is stronger at the low or high levels of schooling.

We shall begin our analysis assuming that discrimination is uniform across schooling levels so that $d_1 = d_2 \equiv \delta < 1$. In this case, women have lower market return to schooling investment than men.[15] We shall later discuss a case in which discrimination against educated women is weaker so that $d_1 < 1$ and $d_2 = 1$.

The returns to investment in schooling for singles are

$$R^m = z_{20}^m - 2z_{10}^m = (\frac{w_2^m}{2})^2 - 2(\frac{w_1^m}{2})^2 \tag{28}$$

and

$$R^w = z_{02}^w - 2z_{01}^w = (\frac{w_2^w}{2})^2 - 2(\frac{w_1^w}{2})^2 = (\frac{\delta w_2^m}{2})^2 - 2(\frac{\delta w_1^m}{2})^2 \tag{29}$$

for men and women, respectively. Thus, we have $R^w = \delta^2 R^m < R^m$.

Starting from an initial situation of equality described by the point e in Figure 6, consider an increase in the wages of educated men, w_2^m, combined with a reduction

[14] For related papers that emphasize the same dual-feedback mechanism between the intensity homework and labor market wage rates we discuss here, see Albanesi and Olivetti (2005, 2006) and Chichilnisky (2005).

[15] In standard human capital models in which the only costs of investment are forgone earnings and the only returns are higher future earnings, uniform discrimination has *no* impact on investment. In this model, however, the absolute market returns are added to the returns within marriage and together determine investment decisions (see equations (14) and (15)) Therefore absolute market returns matter.

in the wages of educated women, w_2^w, holding the wages of uneducated men and women at their benchmark value w_1. To isolate the role of market returns, we assume that the increase in the wage of educated men exactly compensates the reduction in the wage of educated women so that marital output is unaffected and symmetry is maintained.[16] In other words, the change in wages affect directly only the returns as singles, R^m and R^w.

The higher market return from schooling of men encourages their investment in schooling and also strengthens their incentives to marry, because schooling obtains an additional return within marriage. In contrast, the lower return to schooling for women reduces their incentives to invest and marry. These changes create excess supply of men who wish to invest and marry. Consequently, to restore equilibrium, the rates of returns that men receive within marriage must decline implying that for any V_1, the value of V_2 that satisfies conditions (19) and (20) must decline. These shifts in the equilibrium lines are represented by the broken blue and red lines in Figure 8.

For moderate changes in wages, strictly positive assortative mating continues to hold. However, the equilibrium value of V_2 declines and educated men receive a *lower* share than educated women when they marry each other. When the gap between R^m and R^w becomes large, the equilibrium shifts to a mixed equilibrium, where some educated men marry uneducated women. That is, because of their higher tendency to invest some educated men must "marry down." This equilibrium is represented by the point $e\prime$ in Figure 8, where the broken red line representing equality in the number of men and women that wish to marry (condition (19)) intersects the green line representing the lower bound on the share that educated men obtain in the marital surplus, $z_{21} - z_{11}$. As seen, *both* V_1 and V_2 are lower in the new equilibrium so that *all* men (women), educated and uneducated, receive lower (higher) shares of the marital surplus when men have stronger market incentives to invest in schooling than women.

These results on the shares of married men and women in the *marital surplus* must be distinguished from the impact of the shares in the *marital output.* If men get a higher return from schooling as singles, their share in the marital output can increase even though they receive a lower share of the surplus.

4.3 Gender Roles II: The Impact of Children

Recall that we assume marriage is always associated with having children. To investigate what impact children can have on pre-marital investments and household

[16] When wages change $z_{I(i)J(j)}$ usually changes. Also, when wages differ by gender, we generally do *not* maintain symmetry in the contribution of men and women to marriage so that $z_{12} \neq z_{21}$. It is only in the special case in which the product $w^m_{I(i)} w^w_{J(j)}$ remains invariant under discrimination that the marital surplus generated by all marriages is intact. The qualitative results for shares are not affected by this simplification.

roles, ceteris paribus, we assume equal market wages for men and women so that $\delta = 1$. Starting with the symmetric case with $\tau = 0$, consider an increase in τ. Such an increase reduces the contribution educated women make to marital output and raises the contribution of uneducated women. That is, z_{11} and z_{21} rise because uneducated women are more productive at home, $\gamma > w_1$, while z_{12} and z_{22} decline because educated women are less productive at home, $\gamma < w_2$. Consequently, both equilibrium lines corresponding to conditions (19) and (20) shift down so the V_2 is lower for any V_1. At the same time, the boundaries on the rate of returns from schooling that men can obtain within marriage shift as $z_{21} - z_{11}$ rises and $z_{22} - z_{12}$ declines. These changes are depicted in Figure 9.

For moderate changes in τ, strictly assortative mating with equal sharing continues to hold. As long as a symmetric equilibrium is maintained, the returns to schooling that men and women receive within marriage, $V_2 - V_1$ and $U_2 - U_1$, are equal. Hence, men and women have the same incentives to invest. But because the surplus shares (and consequently utilities within marriage) of educated men and women $z_{22}/2$ decline with τ, while those of uneducated men and women $z_{11}/2$ rise, both men and women will reduce their investment in schooling by the same degree.

As τ rises further, the difference in the contributions of men and women to marriage can rise to the extent that an educated man contributes to a marriage with uneducated woman more than an educated woman contributes to a marriage with an educated man.[17] That is,

$$z_{21} - z_{11} > z_{22} - z_{21}. \tag{30}$$

Condition (30) implies that the *lower* bound on the return to schooling that men receive within marriage exceeds the *upper* bound on the return to schooling that woman receive within marriage. In this event, the symmetric equilibrium in Figure 9 is eliminated and instead there is a mixed equilibrium with some educated men marrying uneducated women (point $e\prime$ in Figure 9). This outcome reflects the lower incentive of educated women to enter marriage and the stronger incentive of men to

[17] Consider the expression

$$\begin{aligned} h(w_1 \quad, w_2, \tau) \equiv\ & 2z_{21} - z_{11} - z_{22} = 2[w_2 + \tau\gamma + (1-\tau)w_1]^2 \\ & -[w_1 + \tau\gamma + (1-\tau)w_1]^2 - [w_2 + \tau\gamma + (1-\tau)w_2]^2 \end{aligned}$$

as a function of w_1 and w_2 and τ. For $w_1 = w_2 = \gamma$, $h(\gamma, \gamma, \tau) = 0$ and

$$\begin{aligned} h_1(\gamma, \gamma, \tau) &= -4\gamma\tau, \\ h_2(\gamma, \gamma, \tau) &= 4\gamma\tau. \end{aligned}$$

Therefore, for a positive τ, w_1 slightly below γ and w_2 slightly above γ, $h(w_1, w_2, \tau) > 0$. Also

$$h_3(w_1, w_2, \tau) = (w_2 - w_1)[w_2(4 - 2\tau) + 2\tau(2\gamma - w_1)] > 0$$

and for all $w_2 > \gamma > w_1$, $h(w_1, w_2, 0) < 0$ and $h(w_1, w_2, 1) > 0$. Therefore, the larger is τ the broader will be the range in which $h(w_1, w_2, 0) > 0$.

invest because their return from schooling within marriage, $V_2 - V_1 = z_{21} - z_{11}$, exceeds the return to schooling that women can obtain within marriage. Consequently, some educated men must marry "down" with uneducated women.

4.4 Division of Labor and Career Choice

We can further refine the family decision problem by letting the partners decide whether to have children and who shall take care of them. Reinterpreting τ as a temporal choice, imagine that one of the partners must first spend τ units of time during marriage on the child(ren) and later enter the labor market and work for the remainder of the period (length $1 - \tau$).

An important idea of Becker (1991, ch. 2) is that wage differences among identical spouses can be created endogenously and *voluntarily* because of learning by doing and increasing returns. Thus, it may be *optimal* for the household for *one* of the spouses to take care of the child(ren) and the other to enter the labor market immediately, thereby generating a higher wage in the remainder of the period. Because we assume transferable utility between spouses, household roles will be determined efficiently by each married couple.[18] Also, because human capital is to some extent self productive, the spouse with more education will specialize in market work. Thus, by choosing schooling ahead of marriage one can influence his\her household role within marriage.

As we stated above, the ability to remarry acts as a substitute for commitment in models of frictionless matching and this result holds even when there are some search frictions in spousal matching. However, the existence of children and, in particular, their custody assignment in divorce can produce asymmetric costs of committing to having children or childrearing time (especially if doing so affects one's labor market wages subsequently). Thus, to implement the efficient outcome, there is need for some commitment. We can assume that at the time of marriage, each couple agrees on the efficient household division of labor and to divide the resulting household surplus according to the anticipated, market-determined marital shares. In addition, each spouse commits not to divorce. Then, when marital output is realized, there is

[18] The required level of efficiency is a conditional one. In particular, the household choices need to be efficient taking as given the pre-marital decisions made by each spouse (and how those decisions manifest themselves in spousal inputs to the marriage). In fact, Iyigun (2005) and Iyigun-Walsh (2005) demonstrate that large marriage and frictionless markets produce unconditionally efficient outcomes in which both pre-marital investments and intra-household allocations are Pareto efficient. For more detail on the efficiency of investments and spousal allocations, also see Browning et al. (in progress, ch. 7).

In addition, Chichilnisky (2005) shows that efficient household assignment need not be efficient from the economy's point of view, because the effects of household work on market productivity are not fully internalized by the household. Specifically, a wife who works more at home may be assigned in the labor market to a man who works less at home and their total output would be higher if they would have been equally productive.

no mutual gain from renegotiation and therefore the ex-ante agreement will not be renegotiated so that expectations are in fact realized.[19]

Suppose now that men and women are ex-ante identical and there are no predetermined household roles. If an educated woman (man) marries a less-educated man (woman) then the more educated spouse will work in the market and the other will stay at home. Anticipating that, each gender has the same incentives to educate and therefore a strictly assortative matching pattern arises with equal number of men and women in each education class. In particular, note that equation (26) applies and attains the same value for all couples. As a result, we have $z_{12} = z_{21}$. Moreover, since men and women are ex-ante identical, $R^m = R^w$. The equilibrium in this case is unique, symmetric and characterized by equal sharing $V_2 = U_2 = z_{22}/2$ and $U_1 = V_1 = z_{11}/2$. With these shares, men and women have identical investment incentives. Hence, the number of educated (uneducated) men equals the number of educated (uneducated) women, both among the singles and the married. In this equilibrium, it is immaterial who works at home and who works in the market and the two spouses obtain equal shares of the maximized marital output.

If, however, women receive lower market wages as we discussed in subsection 4.2, then we have

$$s_{ij} = \frac{[w_{I(i)} + \tau\gamma + (1-\tau)\delta w_{J(j)}]^2 - w_{I(i)}^2 - \delta^2 w_{J(j)}^2}{4} + \theta_i + \theta_j \equiv z_{I(i)J(j)} + \theta_i + \theta_j \ , \tag{31}$$

if husband i works in the market and wife j stays at home, and

$$s_{ij} = \frac{[\delta w_{J(j)} + \tau\gamma + (1-\tau) w_{I(i)}]^2 - w_{I(i)}^2 - \delta^2 w_{J(j)}^2}{4} + \theta_i + \theta_j \equiv z_{I(i)J(j)} + \theta_i + \theta_j \ , \tag{32}$$

if husband i stays home and wife j works in the market. Thus, if the partners jointly maximize the surplus, the spouse that will take care of the children is the one with the lower market wage.

Given that, for all matches between man i and woman j (31) exceeds (32), each couple will assign the wife to stay at home, which will erode her future market wage

[19] Lundberg and Pollak (1993, pp. 1002-1006) discuss in elaborate detail the role of commitment in intra-household allocations and spousal transfers. Their primary focus is on the mode of household decision-making (i.e., cooperative versus non-cooperative Nash bargaining) and how the existence of transfers between the spouses can alter the allocation of resources under the non-cooperative form. Lundberg and Pollak conclude that either commitment to a transfer scheme within couples prior to the formation of their marriage, or the ability to null a marriage and remarry costlessly and frictionlessly both produce efficient outcomes (due to spousal competition in large marriage markets).

and reinforce the unequal division of labor. Similarly, if there are predetermined household roles such that women must take care of the child(ren) then women will end up with lower market wages. Thus, inequality at home and the market are interrelated. Models of statistical discrimination tie household roles and market wages through employers beliefs about female participation. Typically, such models generate multiple equilibria and inefficiency (Hadfield, 1999, Lommerud and Vagstad, 2002). Here, we do not require employers' beliefs to be correct. Instead, we think of household roles and discrimination as processes that evolve slowly and can be taken as exogenous in the medium run.

4.5 Why Women May Acquire More Schooling than Men

We have examined two possible reasons why women may invest less than men in schooling. The first reason is that women may receive lower return from investment in the market because of discrimination. The second reason is that women may receive a lower return to schooling in marriage because of the need to care for children (due to social and cultural norms or the biological time requirements of childcare), in which case the contribution of schooling to marital output is lower.

Over time, fertility has declined and female wages have risen in industrialized countries, a pattern being replicated in many developing countries too, which is consistent with the increased investment in education of women. The fact that women are now slightly more educated than men, on average, appears surprising given the fact that women still earn substantially less than men. However, in dealing with investments in education, the crucial issue is whether the gender wage gap rises or declines with schooling, or equivalently, whether women obtain a higher rate of return from schooling. There is some evidence that this is indeed the case and that the gender gap declines with schooling (Dougherty, 2005).

Now consider a comparison of the following two situations. An "old" regime in which married women must spend a relatively large fraction of their time at home and a "new" regime in which, because of reductions in fertility and improved technology in home production, married women spend less time at home and work more in the market (Greenwood, Seshadri and Yorukoglu, 2005). Assume further that women suffer from statistical discrimination because employers still expect them to invest less on the job. However, this discrimination is weaker against educated women that are expected to stay in the labor market. Then, it is possible that in the new regime women will invest in schooling *more* than men. The presence of discrimination *raises* the return of women relative to men because schooling serves as an instrument for women to escape discrimination. The fact that women are still tied up in homework lowers their return from schooling relative to men because women obtain lower returns from schooling within marriage. However, as women raise their participation, this second effect weakens and the impact of discrimination can dominate.

In Figure 10, we display the transition between the two regimes. We assume that $d_2 > d_1$ so that discrimination against women is lower at higher level of schooling. This feature generates stronger incentives for women than for men to invest in schooling. However, the fact that women must spend time working at home reduces has the opposite effect. We then reduce the amount of time that the mother has to spend at home and raise d_2, which strengthens the incentives of women to invest in schooling and to marry. Therefore, an increase in V_2 relative to V_1 is required to maintain equality between the number of men who wish to invest and marry and the number of women who wish to invest and marry. This effect is represented by the upwards shifts to the broken red and blue lines in Figure 10. The impact is assumed to be large enough to generate an equilibrium in which the two equilibrium requirements–equality of the number of women and men that wish to acquire schooling and marry (the broken blue line) and equality of the total number of men and women that wish to marry (the broken red line)–yield an intersection above the upper bound on the returns from schooling that men can receive within marriage. Therefore, strictly positive assortative mating cannot be sustained as an equilibrium and the outcome is a mixed equilibrium in which there are more educated women than men among the married and some educated women marry uneducated men. This new mixed equilibrium is indicated by the point e'' in Figure 10.

The result that there are more educated women among the married does not, by itself, imply that women invest in schooling more than men. But, in this example, we assume that women have higher return from schooling as singles so that there are also more educated women among the singles. The basic logic is that the gender with higher market returns invests more in schooling and as, a consequence, has higher incentives to marry, which can drive the returns to schooling within marriage of that gender to the lower bound, where an excess supply of the educated members of that gender among the married as well as among the singles is created.

4.5.1 A numerical example

Suppose that μ and θ are uniformly distributed on the interval $[-3, 3]$ and that wages of men and women are are given by

	Uneducated	Educated
men	$w_1^m = 2$	$w_2^m = 3$
women	$w_1^w = 1$	$w_2^w = 2.1$

Thus women earn less than men, $d_1 = .5$ and $d_2 = .7$. However, the absolute and relative wage gains from schooling are larger for women. We set $\gamma = 2$ so that an educated woman is more productive in the market and uneducated women are more

productive at home, . Note that educated women can earn more than uneducated men. The assumed wages imply the utility levels of single men and women given below

	Uneducated	Educated	
Men	$z_{10} = 1.00$	$z_{20} = 2.25$	$R^m = z_{20} - 2z_{10} = .25$
Women	$z_{01} = .25$	$z_{02} = 1.10$	$R^w = z_{02} - 2z_{01} = .60$

Thus, women enjoy higher returns from schooling as singles (in utility terms), reflecting the reduction in the gender wage gap as schooling rises.

Now consider a change in regimes such that the wife has to spend less time at home (τ declines from .6 to .25) and wages of educated women rise (from $w_2^m = 2.1$ to $w_2^m = 2.4$) holding all other wages constant. A direct effect of this change is an increase in the returns to schooling of single women from $R^w = .60$ to $R^w = .94$. The marriage market implications of these changes are summarized in tables 1-3 below.

Table 1: Impact of parameter changes on marital surplus

Old regime: $\tau = .6$, $\gamma = 2$, $d_1 = .5$, $d_2 = .7$			New regime: $\tau = .25$, $\gamma = 2$, $d_1 = .5$, $d_2 = .8$		
	Uned. wife	Educ. wife		Uned. wife	Educ. wife
Uned. husband	$z_{11} = 1.99$	$z_{12} = 2.10$	Uned. husband	$z_{11} = 1.39$	$z_{12} = 2.40$
Educ.. husband	$z_{21} = 2.79$	$z_{22} = 3.00$	Educ. husband	$z_{21} = 2.02$	$z_{22} = 3.33$

A decrease in the amount of time worked at home, raises the contribution of and educated women to the marital surplus and lowers the contribution of uneducated women. In the old regime, with $\tau = .6$, the marital surplus *declines* with the education of the wife when the husband is uneducated, while in the new regime with $\tau = .25$ it rises. This happens because educated women are more productive in the market than uneducated women but, by assumption, equally productive at home. Even if the wife has the higher wage, which happens when the husband is uneducated, it is still costly to have an educated wife, because the opportunity cost is the wage of an uneducated woman that could replace the husband in home work him if he would marry her.

Table 2: Impact of parameter changes on the equilibrium shares

Old regime: $\tau = .6$, $\gamma = 2$, $d_1 = .5$, $d_2 = .7$			New regime: $\tau = .25$, $\gamma = 2$, $d_1 = .5$, $d_2 = .8$		
	Uneducated	Educated		Uneducated	Educated
Men	$V_1 = .82$	$V_2 = 1.62$	Men	$V_1 = .78$	$V_2 = 1.71$
Women	$U_1 = 1.17$	$U_2 = 1.38$	Women	$U_1 = .60$	$U_2 = 1.62$

The implied returns from schooling within marriage in the old regime are

$$\begin{aligned} U_2 - U_1 &= 1.38 - 1.17 = z_{22} - z_{21} = 3.00 - 2.79 = .21\ , \\ V_2 - V_1 &= 1.62 - .82 = z_{21} - z_{11} = 2.79 - 1.99 = .80\ . \end{aligned}$$

That is men receive the *lower* bound on their return from schooling within marriage while women receive the *upper* bound on their return from schooling. This pattern is reversed in the new regime

$$\begin{aligned} U_2 - U_1 &= 1.62 - .61 = z_{12} - z_{11} = 2.40 - 1.40 = 1.0 \\ V_2 - V_1 &= 1.71 - .78 = z_{22} - z_{12} = 3.33 - 2.40 = .93, \end{aligned}$$

where women receive their lower bound and men receive their upper bound. Both men and women receive higher returns from schooling within marriage in the new regime because the effective wage of educated women $\tau\gamma + (1-\tau)w_2^m$ has risen and the wages of the two spouses complement each other.

Table 3: Impact of parameter changes on the equilibrium investment and marriage rates*

Old regime: $\tau = .6$, $\gamma = 2$, $d_1 = .5$, $d_2 = .7$				New regime: $\tau = .25$, $\gamma = 2$, $d_1 = .5$, $d_2 = .8$			
	Married	Unmarried	All		Married	Unmarried	All
Educ.	.510, .463	.125, .162	.635, .625	Educ.	.535, .621	.116, .151	.651, .772
Uned.	.207, .253	.158, .121	.365, .383	Uned.	.191, .105	.157, .123	.349, .228
All	.717, .717	.284, .284	1	All	.726, .726	.284, .284	1

$*$ The first entry in each cell refers to men and the second entry refers to women

In the old regime, more men invest in schooling than women and some educated men marry down with uneducated women. This pattern is reversed in the new regime and women invest in schooling more than men and some educated women marry down with uneducated men. Educated women are less likely to marry than educated men, because their return for schooling in marriage is lower, anticipating that they will do the home work if they marry educated men. This effect is stronger in the old regime where the time requirements at home are relatively large.

So far, we assumed that couples choose household efficiently and if the wife more educated than her husband she goes to work full time and the husband takes care of the children. It is interesting to examine the impact of social norms such that the wife is always responsible for child care, irrespective of who has the higher wage. An interesting feature of this example is that such norms have only a small effect on the outcomes (compare tables 3 and 4). Note, first, that in the old regime such norms have no effect whatsoever, as long as a mixed equilibrium is maintained such that there are more educated men than women among the married and some men

marry uneducated women. In such an equilibrium, there are *no* couples in which the wife is more educated than her husband, and couples that do marry satisfy the norm because the wage of an educated wife is lower than that of and educated husband, hence it is efficient that the wife takes care of the children.

In the new regime, where the mixing is reversed and there are married couples in which the wife is more educated, the presence of norms reduces diminishes the gains from mixed marriages as z_{12} drops from 2.4 to 2.18 and the consequences for the marriage market are displayed in tables 4 and 5.

The marriage rate, investment rate and the proportion of educated women who wish to marry decline when educated women are forced to stay at home. Men, however, increase their investment in schooling and uneducated men are less inclined to marry, due to the loss of efficiency in mixed marriages with a wife which is more educated than the husband.

Table 4: Impact of norms on equilibrium investment and marriage rates in the new regime*

Efficient allocation				Wife always works at home			
	Married	Unmarried	All		Married	Unmarried	All
Educ.	.535, .621	.116, .151	.651, .772	Educ.	.564, .592	.111, .157	.675, .746
Uned.	.191 ,.105	.157, .123	.349, .228	Uned.	.161, .133	.160, .119	.325, .351
All	.726, .726	.284, .284	1	All	.725, .725	.285, .285	1

$*$ The first entry in each cell refers to men and the second entry refers to women

Changes in the equilibrium shares are associated with the changes in marriage and investment patterns described above. Because educated women and uneducated men generate together a lower surplus shares they both lose surplus shares in all marriages (including marriages of educated women with educated men). Educated men gain from the reduced share of their wives, while uneducated women who are married to uneducated men obtain also obtain a larger surplus share. ,All these changes reflect the weekend competitiveness of educated women. Thus the return for schooling that men receive within marriage rise while the return for schooling that women receive decline, supporting the rise in the proportion of men who acquire schooling and the reduction in the proportion of women that acquire schooling.

Table 4: Impact of norms on the equilibrium shares in the new regime

Efficient allocation			Wife always works at home		
	Uneducated	Educated		Uneducated	Educated
Men	$V_1 = .78$	$V_2 = 1.71$	Men	$V_1 = .61$	$V_2 = 1.76$
Women	$U_1 = .60$	$U_2 = 1.62$	Women	$U_1 = .77$	$U_2 = 1.56$

5 Conclusions

In standard models of human capital, individuals invest in schooling with the anticipation of being employed at a higher future wage that would compensate them for the current foregone earnings. In this paper, we add another consideration: the anticipation of being matched with another agent with whom one can share consumption and coordinate work activities. Schooling has an added value in this context because of complementarity between agents, whereby the contribution of the agent schooling to marital output rises with the schooling of his\her spouse. In the frictionless marriage market considered here, the matching pattern is fully predictable and supported by a unique distribution of marital gains between partners. Distribution is governed by competition, because for each agent there exist a perfect substitute that can replace him\her in marriage. There is thus no scope for bargaining and, therefore, premarital investments are efficient. This simple framework allows us to jointly determine investment and marriage patterns as well as the welfare of men and women under a variety of circumstances.

One of our main findings is that, when the market return to education and household roles are gender neutral, men and women acquire education in equal proportions and, under the assumption that the schooling of the two spouses complement each other, a strictly positive assortative matching arises in the marriage markets. That is, educated men marry only educated women and uneducated men marry only uneducated women. But if the returns are not gender neutral, then there is mixing in equilibrium where some educated individuals marry uneducated spouses and individuals of the gender that marries down obtains a lower return from schooling within marriage. In particular, we show that a transition from an old regime in which women are required to work at home and expect lower market wages to a new regime where less work is required at home and the wages of educated women become more in line with that of educated men, women may *overtake* men in terms of schooling, despite their lower market wage rate and higher amount of housework compared with men.

From the perspective of family economics, gender differences in investment in schooling are of particular interest because assortative mating based on schooling is a common feature of marriage patterns in modern societies. However, schooling is an acquired trait that responds to economic incentives. We mentioned two interrelated causes that may diminish the incentives of women to invest in schooling: lower market wages and larger amount of household work. Although we did not fully specify the sources of discrimination against women in the market, we noted that such discrimination tends to decline with schooling and therefore increases the incentive to invest. This is a possible explanation for the slightly higher investment in schooling by women that we observe today. We do not view this outcome as a permanent phenomenon but rather as a part of an adjustment process, whereby women who now enter the labor market in increasing numbers, following technological changes at home and in the market that favor women, must be "armed" with additional schooling to

overcome norms and beliefs that originate in the past.

We should add that there are other possible reasons for why women may invest in schooling more than men. One reason is that there are more women than man in the marriage market at the relatively young ages at which schooling is chosen, because woman marry younger. Iyigun and Walsh (2005) have shown, using a similar model to the one discussed here, that in such a case women will be induced to invest more than men in competition for the scarce males. Another reason is that divorce and divorce is more harmful to women, because men are more likely to initiate divorce when the quality of match is revealed to be low. This asymmetry is due to the higher income of men and the usual custody arrangements (see Chiappori and Weiss, 2005). In such a case, women may use schooling as an insurance device that mitigates their costs from unwanted divorce.

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Figure 1: Proportion of some college, college and advance education by sex, 30-40 years old.US 1968-2003 (CPS)

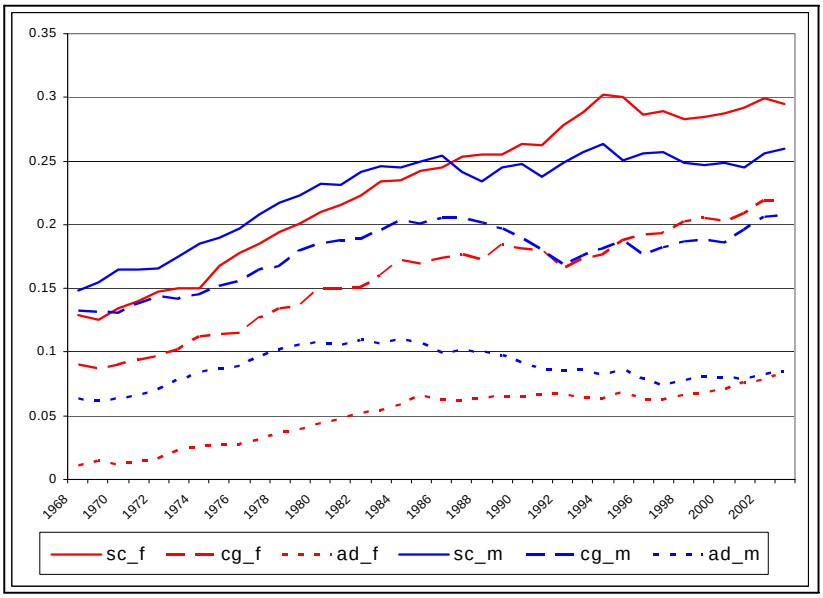

Figure 2: Impacts of higher degrees (relative to high school) on log-wages, adjusted for (potential) experience by sex, US 1988-2003 (CPS)

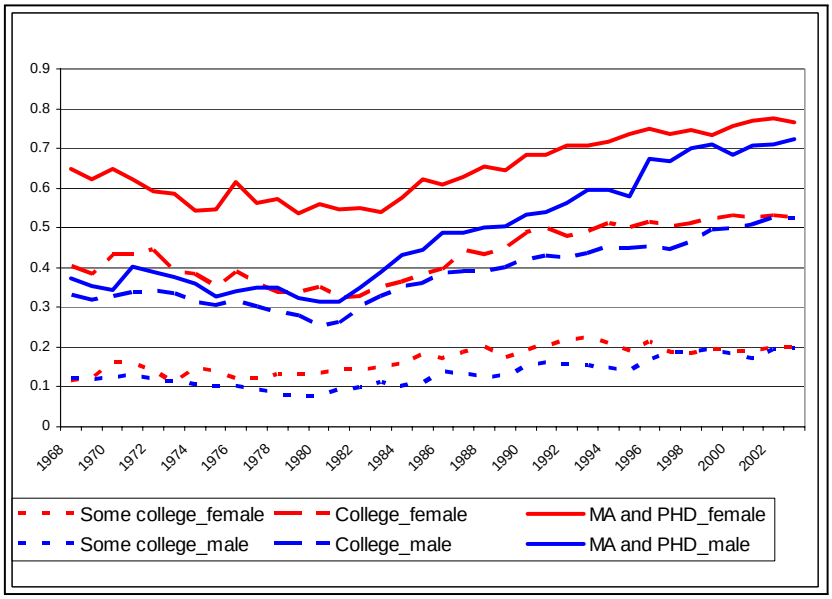

Figure 3: Educational Attainment of Spouses

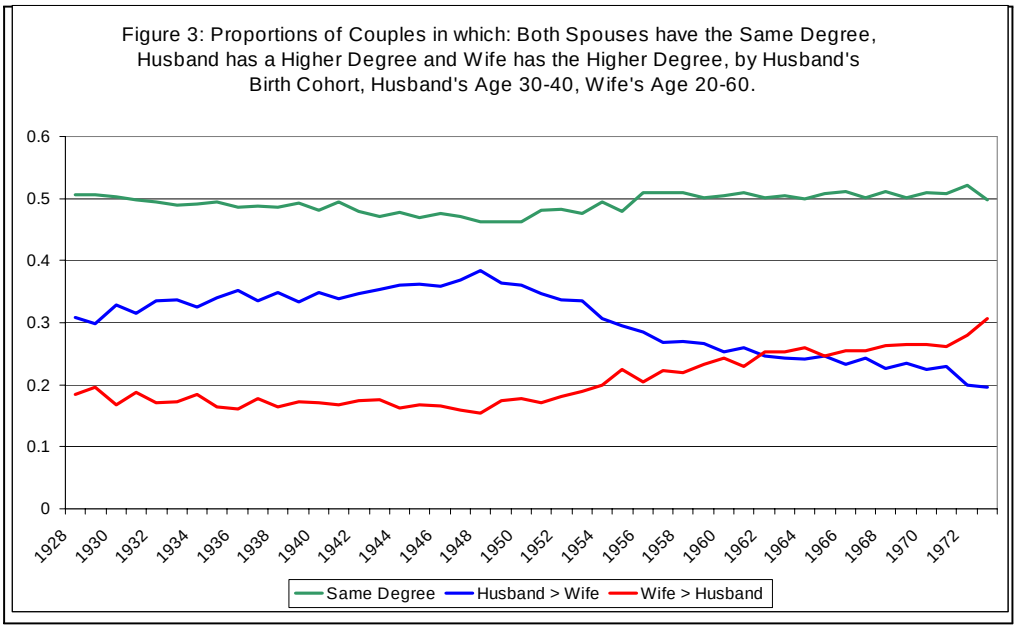

Figure 4: Distribution of Spouse's Education by Own Level of Schooling

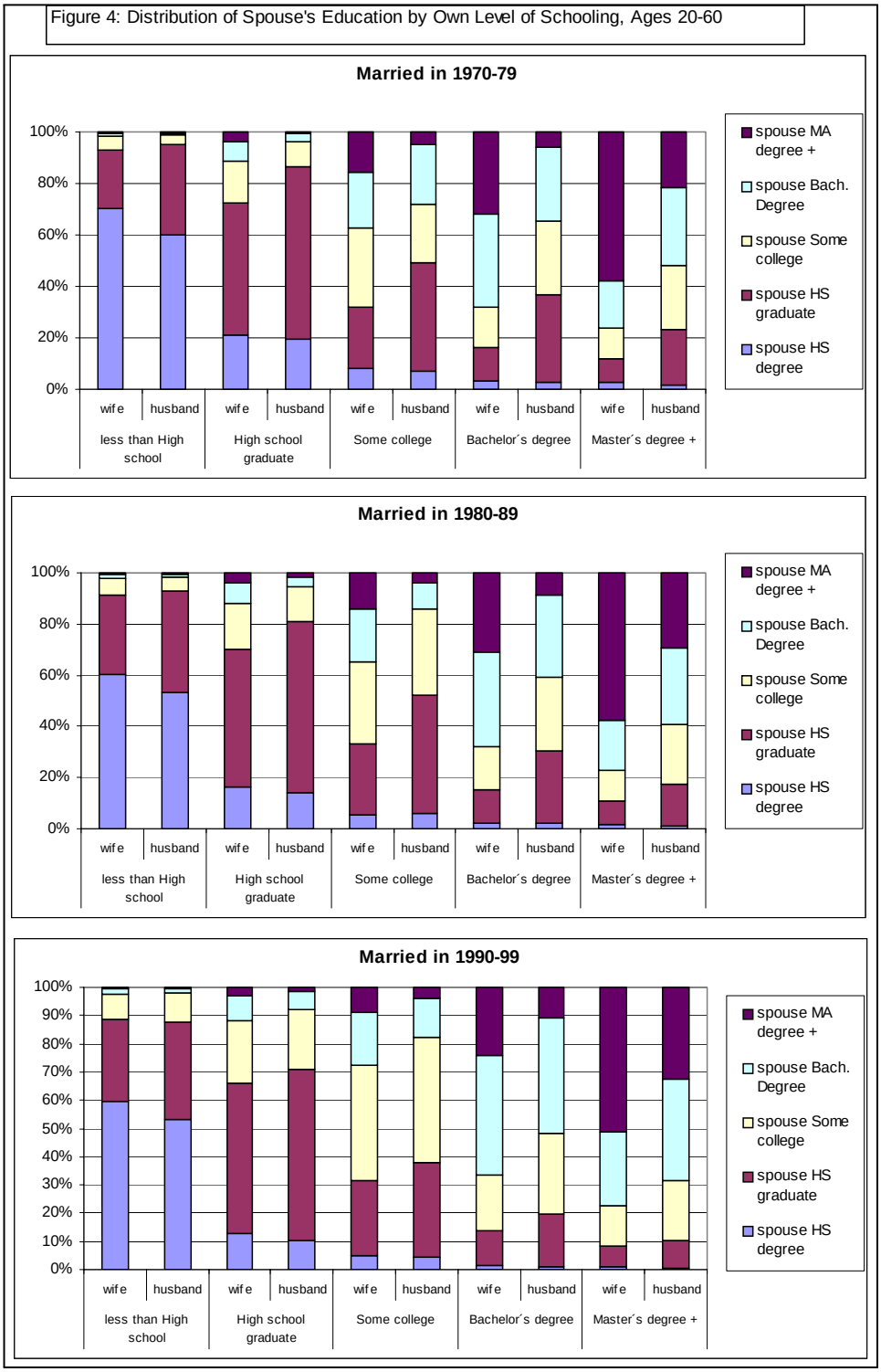

Figure 5: Regions for Marriage and Investment

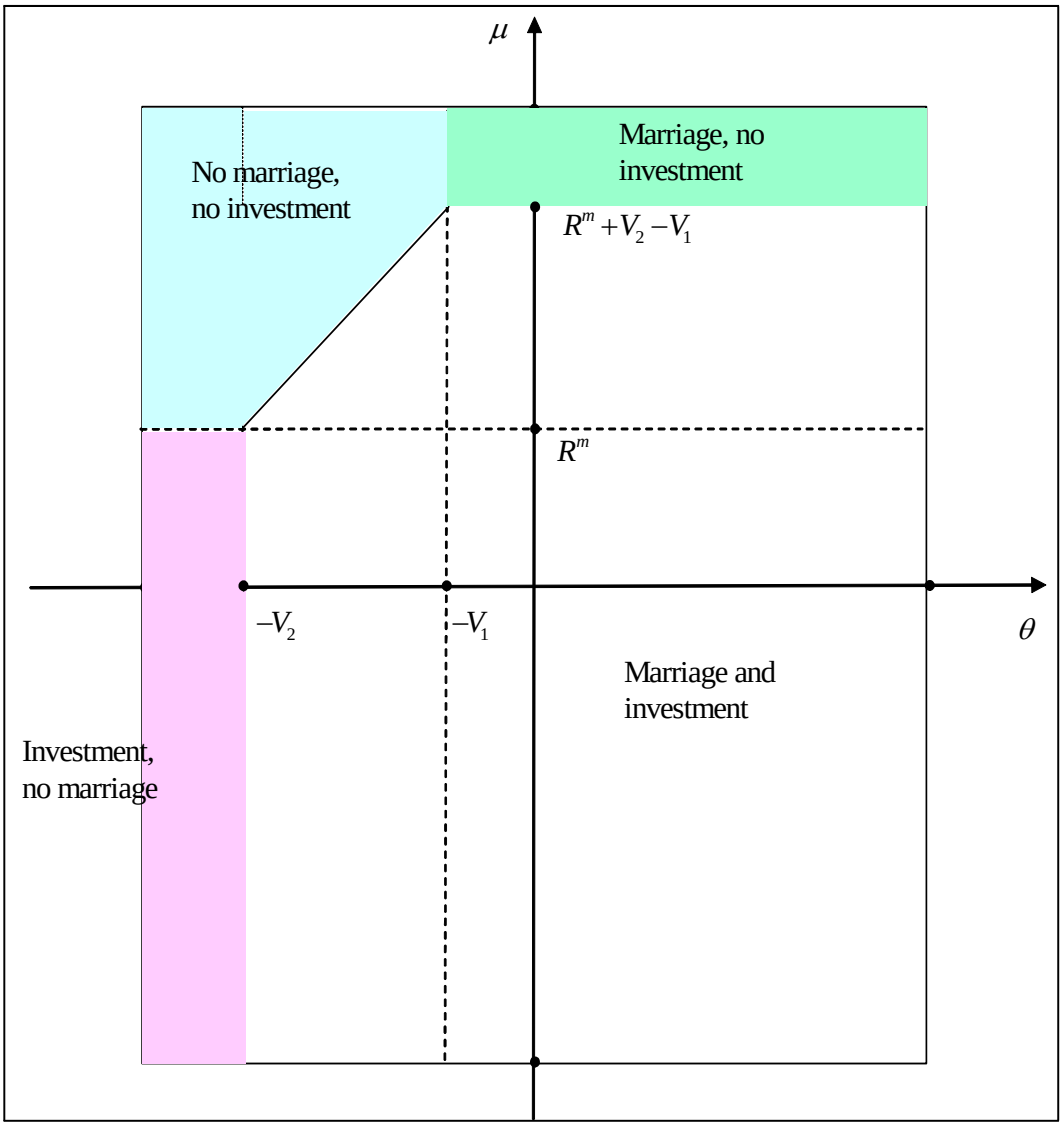

Figure 6: Equilibrium with Strictly Positive Assortative Matching

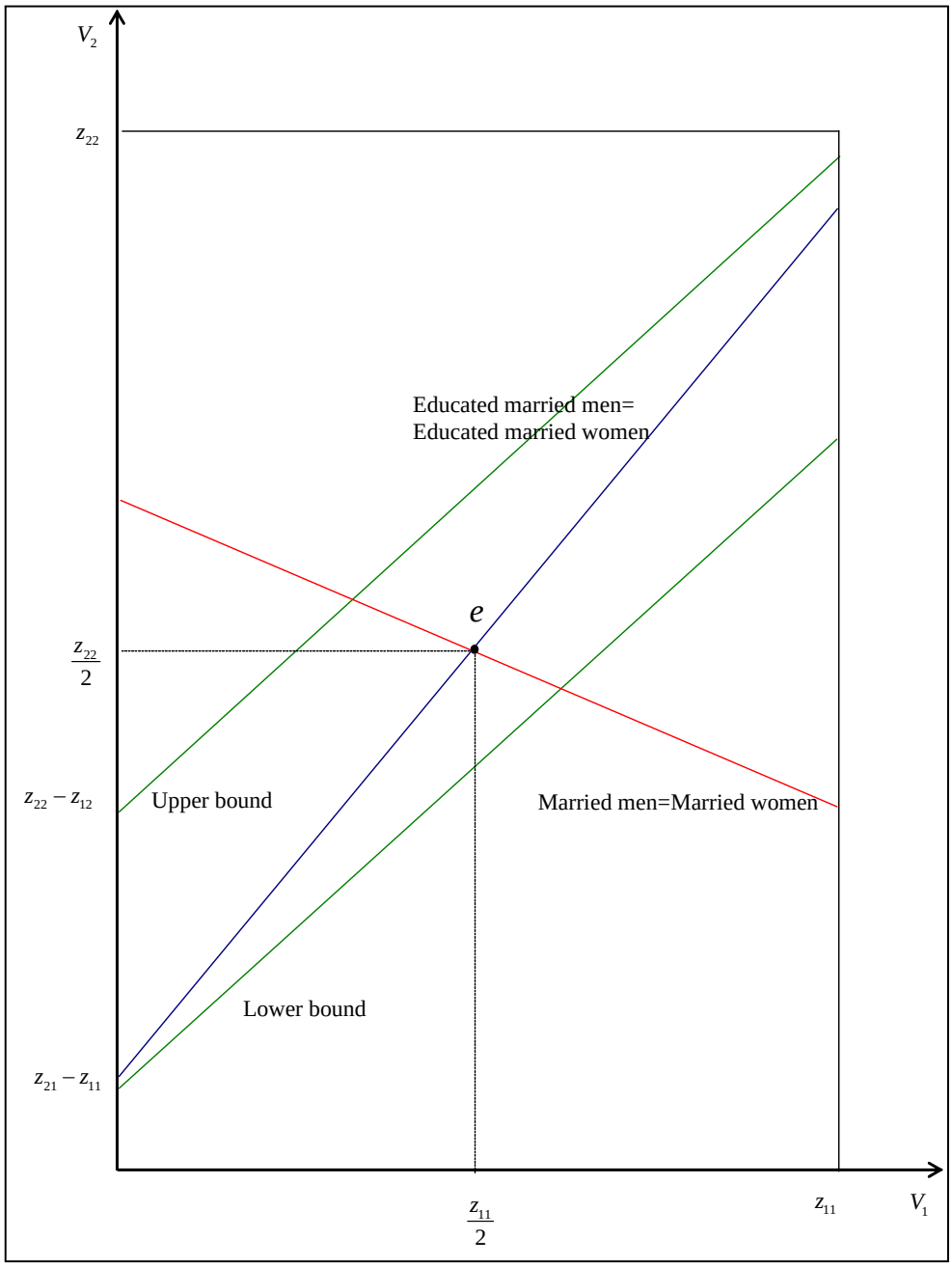

Figure 7: Mixed Equilibrium with More Educated Men than Women

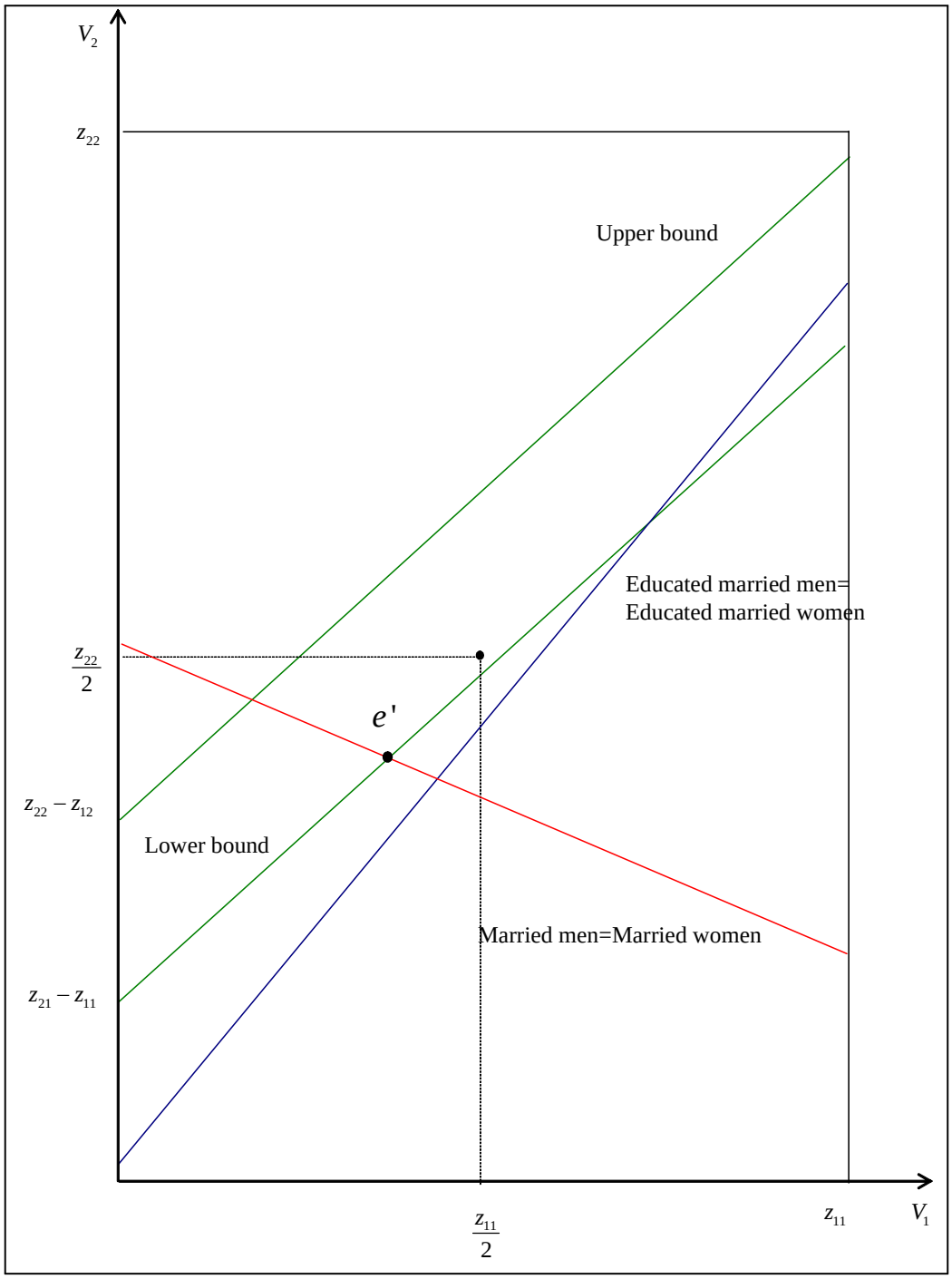

Figure 8: The Impact of an Increase in the Wage of Educated Men Combined with a Reduction in the Wage of Educated Women

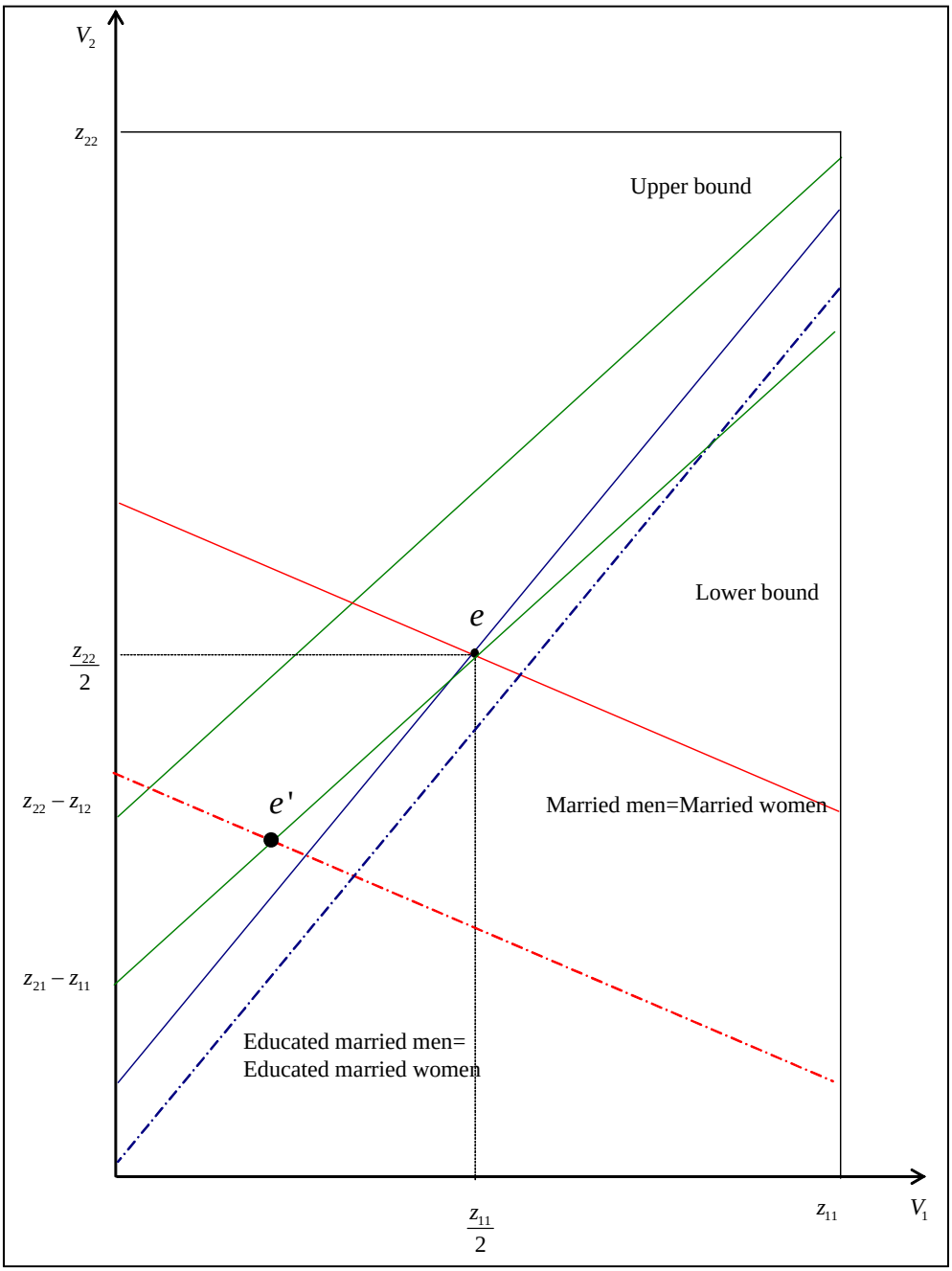

Figure 9: The Impact of an Increase in the Wife's Work at Home

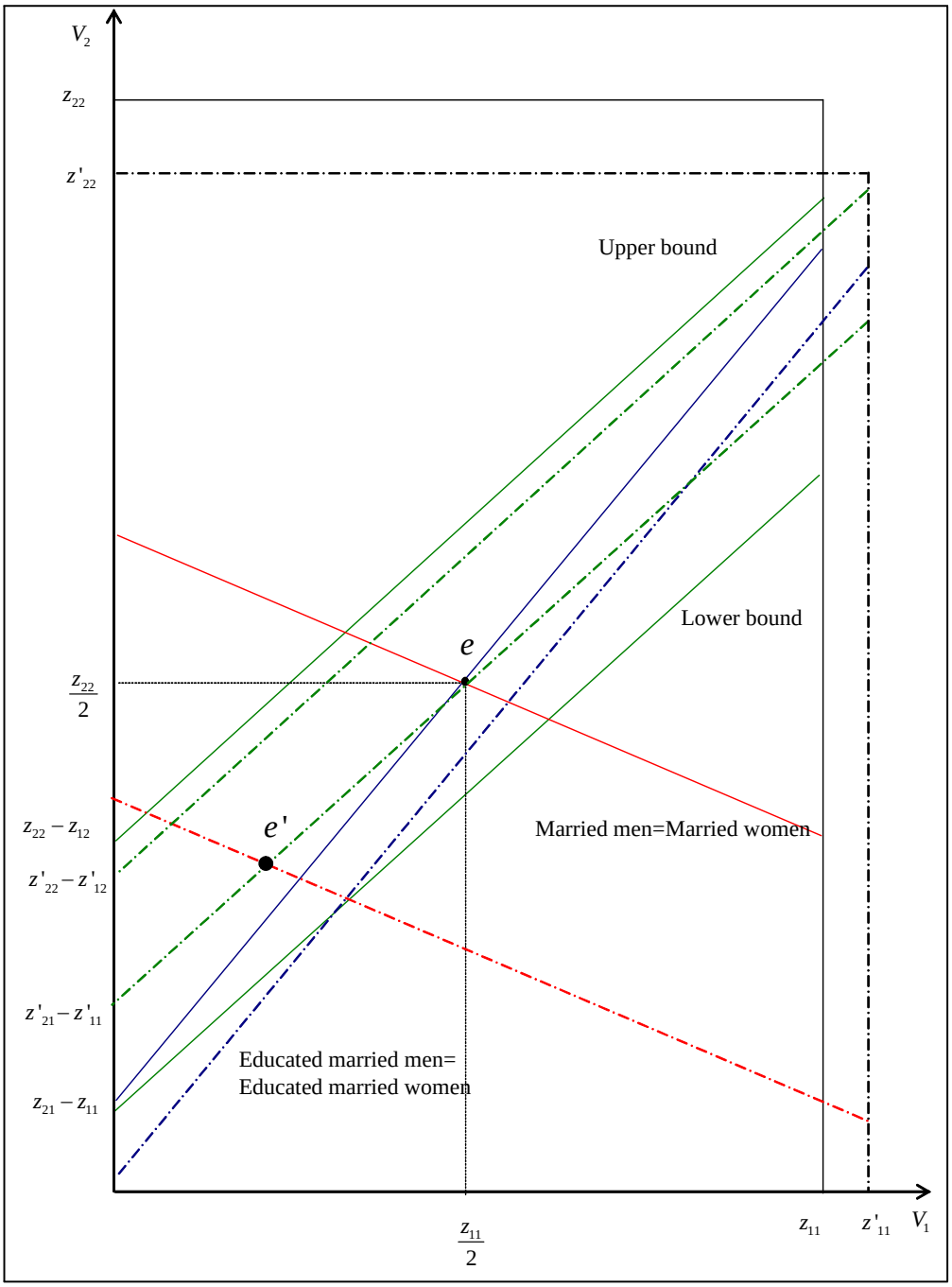

Figure 10: The Impact of an Increase in the Wife'sWork at Home Combined with a Reduction in the Wage of Uneducated Women