# Functional Distribution, Land Ownership and Industrial Takeoff: The Role of Effective Demand 

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#### Abstract

This paper analyses how the distribution of land property rights affects industrial takeoff and aggregate income through the demand side. We study a stylized economy composed of two sectors, agriculture and manufacturing. The former produces a single subsistence good while the latter is constituted of a continuum of markets producing distinct commodities. Following Murphy et al. [20] we model industrialization as the introduction of an increasing returns technology in place of a constant returns one. However, we depart from their framework by assuming income to be distributed according to functional groups membership (landowners, capitalists, workers). We carry out an equilibrium analysis for different levels of land ownership concentration proving that, under the specified conditions, there is a non-monotonic relation between the distribution of land property rights and both industrialization and income. We clarify that non-monotonicity arises because of the way land ownership concentration affects the level and the distribution of profits among capitalists which, in turn, shape their demand. Our results suggest that i) both a too concentrated and a too diffused distribution of land property rights can be detrimental to industrialization, ii) land ownership affects the economic performance of an industrializing country by determining the demand of manufactures of both landowners and capitalists, iii) in terms of optimal land distribution there may be a tradeoff between income and industrialization.


## 1 Introduction

This paper analyses how the distribution of land property rights affects industrial takeoff and aggregate income by shaping the composition of effective demand. Our contribution is a variant of Murphy et al. [20] who proposed a model of early industrialization where the takeoff is sustained by domestic demand and the extent of industrialization is determined by the distribution of income. ${ }^{1}$ They studied the relationship between income distribution and the size of domestic demand assuming that individuals have hierarchical preferences and that a fraction of the labour force receives, besides wages, a share of profits and rents. In particular, the distribution of shares affects the composition of demand which, in turn, affects the profitability of mass production. Their analysis focuses on proving the necessity of a "middle class as the source of the buying power for domestic manufactures". ${ }^{2}$

Murphy et al. [20] assume that shares of profits and rents are distributed in such a way that individuals have the same quota of both. Instead, we assume a sharp functional division of property rights among social classes - land is owned by landowners, each firm is owned by a capitalist - which results in a functional division of income - workers earn only wages, rents go to landowners, profits go to capitalists. We do so for two main reasons. First, such an assumption is more reasonable: a widespread ownership of firms shares is typical of economies in advanced stages of industrialization while it is exceptional for countries which are about their takeoff. Secondly, our framework allows us to investigate how land distribution affects profits distribution which turns out to have an impact on industrialization and aggregate income. More precisely, we confirm the fundamental results provided by Murphy et al. [20] and in addition we show that not only the total amount of profits but also their distribution among capitalists matters.

### 1.1 Related literature

The importance of land distribution in economic development has been recently remarked by Deininger and Squire [8] using a new data set. They found that initial inequality of land distribution has a significant negative effect on subsequent growth rates. Only two of the 15 developing countries with land Gini coefficient above 70 could grow at more than $2.5 \%$ over the $1960-1992$ period. ${ }^{3}$ Deininger and Squire [8] provide two possible explanations of this relationship. The first recognizes that, whenever there are imperfections in asset markets, people without the necessary collateral may be prevented from undertaking the efficient level of investment. The second looks at the interaction between land ownership distribution and the political system, highlighting that individual ownership of assets affects people's preferences on political outcomes: under a democratic regime, inequality would be detrimental to growth because it induces preferences for higher taxation, therefore reducing incentives for investments. ${ }^{4}$ A later work provided by Galor et al. [12] analyses how the distribution of land property rights affects early growth via education. They argue that the more unequal is the distribution of land ownership the later

[^0]educational reforms are introduced, with a strong negative impact on the accumulation of human capital. ${ }^{5}$

Our contribution lies in this stream of literature, even though we analyse the link between inequality and growth on the demand side, investigating the direct effect of the distribution of land ownership on profits and, therefore, on income. Indeed, assuming hierarchical preferences, land ownership distribution matters for industrial takeoff because it determines the size of the market faced by each entrepreneur, directly shaping production incentives in the manufacturing sector. Murphy et al. [20], Baland and Ray [1], Eswaran and Kotwal [9] and Matsuyama [19] constitute few attempts to investigate the link between inequality and industrialization taking into account the composition of demand. ${ }^{6}$ The basic productive structure of those contributions is the dual economy studied by Rosenstein-Rodan [22], Lewis [15] [16] and Fleming [11] between 1940s and 1960s. Such literature was mainly focused on productivity improvements and their persistent effects. Here, instead, we leave aside the productivity issue in order to concentrate on the study of land ownership distribution as the lever of industrial takeoff.

In order to provide an intuition of the basic message of the paper let us consider the different economic performances of South Korea and Philippines. Lucas [17] showed that in the early 1960 's the two countries exhibited similar macroeconomic background under many respects showing about the same GDP per capita, schooling levels, population and urbanization. Nevertheless, during the following twenty-five years the former experienced sustained growth - about $6 \%$ - fully undertaking the industrialization process, while the latter grew at a speed of about one third - less than $2 \%$ - remaining mainly an agricultural economy. Lucas classified the case of Korea as a sort of productivity miracle. ${ }^{7}$

As Bénabou [3] pointed out, moving the attention to the distribution of income and land ownership, one finds no such similarities. Indeed, the two countries were sensibly different from a distributional point of view: South Korea had a much more equal distribution of both land property rights and income than Philippines. Remarkably, the ratio between income of the top $20 \%$ population and that of the bottom $20 \%$ - or even $40 \%$ was nearly twice bigger in Philippines. The Gini coefficient for land ownership was 38.7 in Korea in 1961 and 53.4 in Philippines in $1960 .^{8}$

These distributive differences contribute to explain the best economic performance of Korea over Philippines, particularly in the early years of industrialization. A more equal distribution of income and land ownership granted Korea a greater and more stable domestic demand of basic manufactures which made investments in mass production technologies more profitable. ${ }^{9}$

[^1]Clearly, there are other crucial factors which have been relevant as well. However, our point is rather simple and jointly applies with different explanations: if the industrial technology has increasing returns and domestic demand depends on the distribution of income, then the actual distribution of land property rights can affect both income growth and industrialization by determining the profitability of mass production.

### 1.2 An overview of the paper

The economy we describe is composed of two sectors: agriculture, which provides food, and manufacturing which is constituted by a continuum of markets each providing a different commodity. Consumption is assumed to be incremental in the sense that the higher is the income the greater the variety of goods consumed. This is consistent with the hypothesis of hierarchical preferences. Moreover, individuals have the same tastes which implies that they demand goods according to the same schedule of priorities.

Industrialization is conceived as the substitution of a traditional technology - showing constant returns to scale - with an industrial one - showing increasing returns to scale. In each manufacturing market, artisans using traditional technologies compete with each other driving profits to zero. A single artisan per market has access to the industrial technology. If she faces enough demand she can become an entrepreneur and monopolize the market making positive profits.

We first analyse economies where industrialization does not take place. Three different kinds can be distinguished: a) subsistence economies, where only food is produced and consumed and there are only landowners and land workers; b) small economies, where a manufacturing sector exists but the population is too small to make entrepreneurship and mass production profitable; c) traditional economies, where wages are at subsistence level but there is a manufacturing sector producing only for landowners. We take traditional economies as the standard case of non-industrialization, since it is a stylized picture of many non-industrial countries. Then, by comparing the effects of different distributions of land property rights we show that, ceteris paribus, the relationship between land concentration and income as well as that between land concentration and industrialization are non-monotonic. ${ }^{10}$

Different degrees of land ownership concentration produce quite different patterns of industrialization and income in equilibrium. We use two measures of the degree of industrialization: the number of markets which adopt the industrial technology, the extent of industrialization, and the number of workers hired by firms operating the industrial technology, the industrial employment. The maximum extent of industrialization is reached for an intermediate level of land concentration. In particular, this happens when no artisan adopts the traditional technology and demand in each market is just sufficient to cover start-up costs and grant profits equal to the opportunity cost of labour wage. Instead, the maximum income is generally obtained for a lower extent of industrialization - i.e. for a broader distribution of land property rights. The reason is that increasing returns are better exploited when demand is concentrated in less markets.

For what concerns maximum industrial employment, we find that it is obtained for a level of land ownership concentration which is in-between the level associated with the

[^2]maximum industrial extent and that associated with maximum income, possibly coinciding with either of them. Industrial employment increases with the intensity of exploitation of the increasing return technology - like income - but decreases with the number of industrialized markets - like industrial extent. However, a more intensive exploitation is often possible only at the cost of reducing the range of industrialized markets. So, more industrial employment does not necessarily coincide with more income or more industrialized markets. In Section IV and in the Appendix we illustrate a variety of cases.

Furthermore, we show that a too equal distribution of land property rights can be detrimental to both industrialization and income. Many "poor" landowners demand only few very basic manufactures, concentrating the benefits of industrialization into the hands of very few entrepreneurs. This induces a very unequal distribution between capitalists and everyone else with the consequence that the demand of manufactures produced with the industrial technology is rather low and mass production is not properly exploited.

The paper is organized as follows: section II presents the basic model; section III characterizes the equilibrium of the model; section IV compares equilibria with different distribution of land property rights; section V explores some extensions of the model; section VI contains concluding remarks.

## 2 The Model

### 2.1 Commodities and Consumption Patterns

There is a single homogeneous divisible agricultural good. For simplicity we label it food and use it as numeraire. Moreover, there is a continuum of manufactured goods represented by the open interval $[0, \infty) \in \Re$. Each good is denoted by its distance $q$ from the origin. The consumption pattern - or tastes, if one prefers - is assumed to be the same for each individual. There is a subsistence level of food consumption $\bar{\omega}$. After that, any unit of income is spent to buy the manufactured goods following their order in the interval.

Such assumption is meant to introduce in a simple way a common ranking of necessities: people first need to buy what is necessary to survive, then basic manufactures and durables which allow better life standards and, only after that, they buy luxuries. For simplicity, we assume that only one unit is bought of any manufactured good. In other terms, any individual with income $\omega \geq \bar{\omega}$ uses her first $\bar{\omega}$ of income to purchase food needed to survive and $(\omega-\bar{\omega})$ to purchase the manufactured goods. Any individual with $\omega<\bar{\omega}$ starves.

It is worth pointing out the intuitive consequences of our assumptions. First, individuals are almost identical for what concerns consumption decisions and they only differ in terms of income. Thus, a landowner and her servants would consume the same if given the same income. Second, any increase of income results in an increase of consumption variety. In particular, richer people buy the same bundle of poorer people plus some other commodities.

### 2.2 The Agricultural Sector

In order to produce food it is necessary to use land and labour. We abstract from land and assume it is always fully utilized in production. For the sake of simplicity, we also
assume all workers have the same skills - labour is homogenous - and perfect competition in the output side - no profits are earned.

Technology and Incomes. Given the amount of land utilized, labour has decreasing marginal productivity. Total production is determined by the function $F\left(L_{f}\right)$ where $L_{f}$ is the number of workers employed in agriculture. It is assumed $F^{\prime}>0, F^{\prime \prime}<0$. Agricultural wage $w_{f}$ is a function of agricultural employment with $w_{f}^{\prime}\left(L_{F}\right)<0$. This formalization is consistent with the case in which labour is paid its marginal product.

Since profits are nil, income generated in agriculture is exhausted by land workers' wages and landowners' rents. Denoting with $R$ the total amount of rents earned, we have the account equation

$$
\begin{equation*}
R=F\left(L_{f}\right)-w_{f} L_{f} \tag{1}
\end{equation*}
$$

Land Ownership. Differently from Murphy et al. [20], we assume property rights of the land stock to be equally distributed among $M$ landowners. We also assume that the income of each landowner is equal to $R / M$ and, hence, is negatively related to their number. ${ }^{11}$ The idea is that, on average, the greater is the number of landowners the smaller is the area of land they posses and, therefore, the smaller the rent they earn. Although a non-uniform distribution of land property rights is the norm, our simplification works well as long as the average concentration is the relevant feature. In this sense, $M$ should be interpreted as a rough index of land property concentration. Finally, we abstract from the issue of productivity change due to variations in the extent of land ownership, such as that described in Banerjee et al. [2]. ${ }^{12}$

### 2.3 The Manufacturing Sector

We consider a continuum of markets where each of them is small with respect to the entire economy. The number of workers employed in the manufacturing sector as a whole is denoted by $L_{m}$ while the ruling wage is $w_{m}$.

Technology and Markets. Each commodity $q$ is produced with the same cost structure. Two technologies are available. The first, labelled traditional technology or TT, requires $\alpha$ units of labour in order to produce a unit of output. This represents the case in which commodities are produced by artisans who, at the same time, organize production and work as other wage-paid labourers. For this reason, the number of workers in TT markets includes also artisans. The second, labelled industrial technology or IT, requires $k$ units of labour to start up plus $\beta$ units of labour per unit of output produced, with $0<\beta<\alpha$. This represents the case where a former artisan becomes an entrepreneur exploiting the benefits of mass production.

[^3]Furthermore, we assume $(k+1)>(\alpha-\beta)$ which means that the amount $(\alpha-\beta)$ of labour saved producing one unit of output using IT is less than the fixed amount $k$ needed to introduce the IT plus the unit of labour provided by the artisan. Clearly, this is the only interesting case because if $(k+1) \leq(\alpha-\beta)$ IT never requires more units of labour with respect to TT and, hence, it is always preferred by artisans. Lastly, we denote by $E$ the number of entrepreneurs.

Notice that TT shows constant returns to scale while IT shows increasing returns. The difference between these two technologies represents the economic advantage of industrialization.

Competition and Income. A group of competing artisans is assumed to operate in each market $q$ of the economy. Given a wage $w_{m}$, any amount of commodities can be produced and sold at the unit price $\alpha w_{m}$. No profits are earned by artisans. Besides, in each market $q$ there exists one and only one artisan who knows IT. If she decides to be an entrepreneur she can become a monopolist by slightly undercutting the price $\alpha w_{m}$. In this case nobody buys the good produced with TT and profits of market $q$ are

$$
\begin{equation*}
\pi(q)=\left(p_{q}-\beta w_{m}\right) D_{q}-k w_{m} \tag{2}
\end{equation*}
$$

where $p_{q}$ is the price and $D_{q}$ is the demand.

### 2.4 Population and Labour Market .

Agricultural employment determines the ruling wage $w_{f}$. We assume perfect mobility of labour among sectors and markets so that $w_{f}=w_{m}=w$.

The active population is denoted by $L$ and each worker either supplies inelastically one unit of labour or becomes an entrepreneur. The total supply of labour is hence equal to $L-E$. Finally, the population is assumed to be fixed and equal to $N=L+M$ where $L=L_{f}+L_{m}+E$.

## 3 Equilibrium

In this section we characterize the equilibrium as a function of the number of landowners $M$, the wage level $w$, the available labour force $L$ and technology, denoted by $F$ for agriculture and by the vector $\tau \equiv(\alpha, \beta, k)$ for manufactures.

Since we want the economy to actually produce commodities, we assume that the ruling wage $w$ is not less than the subsistence level $\bar{\omega} .{ }^{13}$ For the sake of realism, the rent of a single landowner $R / M$ is not lower than $w$. The same holds for profits as artisans knowing the IT decide to become entrepreneurs if and only if $\pi_{q} \geq w$. The demand of food is given by

$$
\begin{equation*}
D_{f}=\left(L_{f}+L_{m}+E+M\right) \bar{\omega}=\bar{\omega} N \tag{3}
\end{equation*}
$$

while the supply of food is

$$
\begin{equation*}
S_{f}=F\left(L_{f}\right) \tag{4}
\end{equation*}
$$

[^4]As regards the manufacturing sector, we have to take into account how prices influence both aggregate demand and supply. The price of commodities produced with TT is, as mentioned above, $\alpha w$ as a consequence of competition among artisans. The price of commodities produced with IT is set by entrepreneurs in order to maximize profits. Since consumers buy manufactured goods following a well specified order and at most one of each kind, in any market the elasticity of demand with respect to the price is $0 .{ }^{14}$ Hence, entrepreneurs find convenient to rise prices as much as possible. However, the level $\alpha w$ constitutes an upper boundary because, for any price greater than that, nobody would buy commodities from them. Therefore, the price of each manufacturing commodity is $\alpha w$ independently of how many markets industrialize and which is the technology applied.

Besides, since poorer people simply consume a bundle of commodities which is a subset of richer ones, it cannot happen that for two markets $q^{\prime}$ and $q^{\prime \prime}$, such that $q^{\prime}<q^{\prime \prime}$, we have $D_{q^{\prime}}<D_{q^{\prime \prime}}$. Therefore, the demand faced by each manufacturing market is non-increasing in $q$. Moreover, entrepreneurs face the same cost structure, so in each sector they find convenient to start their business for the same level of $D_{q}$. The last two observations imply that there is a separating market $Q^{*}$ such that IT is introduced in any $0 \leq q \leq Q^{*}$ while in the remaining markets production is carried out by means of TT. So, we have that the aggregate demand of the manufacturing sector as a whole is

$$
\begin{equation*}
D_{m}=\frac{1}{\alpha w}\left[(R-\bar{\omega} M)+\left(L_{f}+L_{m}\right)(w-\bar{\omega})+\int_{0}^{Q^{*}}(\pi(q, \tau, w)-\bar{\omega}) \mathrm{d} q\right] \tag{5}
\end{equation*}
$$

and aggregate supply is

$$
\begin{equation*}
S_{m}=\int_{0}^{\bar{Q}} S_{q} \mathrm{~d} q \tag{6}
\end{equation*}
$$

where $\bar{Q}$ denotes the extent of the manufacturing sector and $S_{q}$ the supply of the market $q$. Finally, the demand of labour is

$$
\begin{equation*}
D_{l}=L_{f}+L_{m} \tag{7}
\end{equation*}
$$

and, as anticipated, the supply is

$$
\begin{equation*}
S_{l}=L-Q^{*} \tag{8}
\end{equation*}
$$

since the number of entrepreneurs is $E=Q^{*}$.
In equilibrium it must simultaneously hold that $D_{f}=S_{f}, D_{m}=S_{m}$ and $D_{l}=S_{l}$. We assume that the economy can sustain the whole population $N=(L+M)$, that is $F(L) \geq \bar{\omega} N$. Hence, from $D_{f}=S_{f}$, we get the equilibrium value of employment in agriculture

$$
\begin{equation*}
L_{f}^{*}=F^{-1}(\bar{\omega} N) \tag{9}
\end{equation*}
$$

which is fully determined as $F\left(L_{f}\right)$ is invertible with respect to $L_{f}$ and the parameters $N$ and $\bar{\omega}$ are given. In particular the equilibrium levels of wage, employment and output in the agricultural sector are independent of the equilibrium of the manufacturing sector

[^5]since the aggregate demand of food is $\bar{\omega} N$ in any case. From $D_{l}=S_{l}$ and $L_{f}^{*}$ we get the equilibrium value $\left(L-L_{f}^{*}\right)$ of people with a job in the manufacturing sector (workers, artisans or entrepreneurs). From $L_{f}^{*}$ we obtain $w\left(L_{f}^{*}\right)$; then, $M, F$ and $\tau$ determine the extent of the manufacturing sector $\bar{Q}$. We are left with only two unknowns, namely $Q^{*}$ and $L_{m}^{*}$. Exploiting equilibrium conditions, equation (5) can be written as
\[

$$
\begin{align*}
D_{m} & =\frac{1}{\alpha w}\left[R^{*}+\left(L_{f}^{*}+L_{m}\right) w+\int_{0}^{Q^{*}} \pi(q, \tau, w) \mathrm{d} q-\left(L_{f}^{*}+L_{m}+Q^{*}+M\right) \bar{\omega}\right]= \\
& =\frac{1}{\alpha w}\left[F\left(T, L_{f}^{*}\right)+L_{m} w+\int_{0}^{Q^{*}} \pi(q, \tau, w) \mathrm{d} q-(L+M) \bar{\omega}\right]= \\
& =\frac{1}{\alpha w}\left[L_{m} w+\int_{0}^{Q^{*}} \pi(q, \tau, w) \mathrm{d} q\right] \tag{10}
\end{align*}
$$
\]

where $R^{*}$ is the equilibrium level of aggregate rents. Now, exploiting $D_{m}=S_{m}$, equation (2) and $D_{q}=S_{q}$ for each $q \in(0, \bar{Q})$, we can equate the righthand sides of equations (6) and (10) to obtain

$$
\begin{align*}
L_{m}^{*} & =\alpha \int_{0}^{\bar{Q}} S_{q} \mathrm{~d} q-\int_{0}^{Q^{*}}\left((\alpha-\beta) D_{q}-k\right) \mathrm{d} q \\
& =\alpha \int_{Q^{*}}^{\bar{Q}} D_{q} \mathrm{~d} q+\beta \int_{0}^{Q^{*}} D_{q} \mathrm{~d} q+k Q^{*} \tag{11}
\end{align*}
$$

The first term of equation (11) represents the labour employed in markets using the TT while the sum of the second and the third terms represents the labour employed in the industrialized markets.

Since any entrepreneur in $q$ starts her business depending on the value of $D_{q}$, the extent of industrialization $Q^{*}$ is univocally determined by the continuum of demands in $(0, \bar{Q})$. Although we have not yet provided an expression for any of those demand functions, in the Appendix we illustrate the mechanism of profits formation and industrialization, showing that for each $q$ the demand $D_{q}$ is univocally determined by population, land ownership distribution and technology. So, the only real unknown variable is $L_{m}^{*}$, and equation (11) identifies the equilibrium.

Land Ownership, Profits and Industrialization. In order to give the intuition of the relation between income distribution and industrialization we focus on equilibrium outcomes for different land ownership concentrations.

Consider the economy we have described so far and assume that the agricultural sector is already in equilibrium. Denote with $\Omega_{m}$ the total expenditure in manufactures and with $\omega_{i}$ the income of individual $i$. Since every consumer who has already bought $\bar{\omega}$ units of food spends her remaining income to get a unit of each manufacture in the specified order, the demand $D_{q}$ faced by a generic market $q$ is determined by the number of individuals who earn enough income to buy at least commodity $q$ - namely $\left(\omega_{i}-\bar{\omega}\right) / \alpha w>q$.

Assume that workers are poor and consume only food, namely $w=\bar{\omega}$. Then, the distribution of land property rights shapes the demand for manufactures by determining
the income and the number of individuals who buy manufactures. For instance, if there are only few rich landowners the extent of the manufacturing sector is quite large, although the demand faced by each market is relatively small. On the contrary, if there is a large number of low income landowners, the extent of the manufacturing sector is smaller but the demand faced by each operating market is greater. The distribution of land property rights also affects the absolute level of $\Omega_{m}$. In particular, the higher is land ownership concentration the higher is $\Omega_{m}$ because less income is spent in food and therefore the fraction of rents spent in manufactures is higher.

Since IT is introduced only if demand goes over a certain profitability threshold, a too concentrated ownership of land prevents the takeoff even if $\Omega_{m}$ is great. On the contrary, if land ownership is distributed so that the threshold is outdone, some markets industrialize and entrepreneurs make positive profits. This start a multiplicative process of demand sustained by entrepreneurs' earnings. Extra demand can well offset the negative effect of a lower aggregate demand by landowners with the result that a broader distribution of land is even more income enhancing. The multiplicative effect arises because the extra demand increases aggregate profits possibly inducing a further increase in $\Omega_{m} .{ }^{15}$ Such a process can go on for several steps - profits, new demand, new profits - but in each step the amount of new profits decreases because the new demand partially goes to cover production costs which are constituted by wages spent in food. In particular, the process ends when new generated profits fail to industrialize new markets or to generate extra demand for markets already industrialized.

Summing up, different distributions of land property rights determine, in equilibrium, different scenarios of industrialization and income: this supports the idea that land ownership distribution may be relevant to industrialization and income growth.

## 4 Analysis

### 4.1 Non-industrial Economies

We start illustrating those conditions which may prevent a country from industrializing. In order for a market $q$ to operate the IT profits $\pi_{q}$ must be not less than the ruling wage $w$. Since, as mentioned above, $\tau$ is the same in each market and $q^{\prime}<q^{\prime \prime}$ implies $D_{q^{\prime}} \geq D_{q^{\prime \prime}}$, a necessary and sufficient condition for no industrialization is $\pi_{0}<w$. From equation (2) we get

$$
\begin{equation*}
\pi_{0}<w \quad \Longleftrightarrow \quad D_{0}<\rho \tag{12}
\end{equation*}
$$

where $\rho \equiv(k+1) /(\alpha-\beta)$. Equation (12) states that in equilibrium no market industrializes if and only if the demand faced by the first market is less than the value which allows to cover start-up costs plus the opportunity cost of quitting the previous job. Neglecting profits - which however are nil in a no industrialization equilibrium - demand $D_{0}$ is given by

$$
D_{0}= \begin{cases}0 & \text { if } w=\bar{\omega}, R / M=\bar{\omega}  \tag{13}\\ M & \text { if } w=\bar{\omega}, R / M>\bar{\omega} \\ L+M & \text { if } w>\bar{\omega}, R / M>\bar{\omega}\end{cases}
$$

[^6]Any consumer who earns more than $\bar{\omega}$ demands at least the 0 -commodity which implies that for $R / M>\bar{\omega}$ demand $D_{0}$ is at least $M$ and for $R / M \geq w>\bar{\omega}$ it is $M+L$. So, the no industrialization condition of equation (12) can be satisfied by a variety of triples $(L, M, \tau)$. In the next paragraphs we group them in three classes of interest.

Subsistence economy. In a subsistence economy only food is produced and consumed and there is no manufacturing sector. In this case the ruling wage is $w=\bar{\omega}$ and $L, F$ and $M$ are such that $M+L=F(L) / \bar{\omega}$. Since $M+L=N$ from equation (9) we obtain $L_{f}^{*}=L$ meaning that all the labour force of the economy is employed in agriculture. From equations (7) and (8) we get $L_{m}^{*}=0, E=Q^{*}=0$.

Given the level of agricultural productivity, the number of landowners with respect to population is too high to allow for industrialization. The excessive dispersion of land property rights makes landowners' individual rents $R / M$ as low as $\bar{\omega}$, fully offsetting the benefits accruing from a wage equal to the subsistence level. As a consequence, no one demands manufactured goods and there is no manufacturing sector.

Traditional economy. In a traditional economy workers earn just what is needed to survive while few landowners are rich enough to demand manufactures. There exists a manufacturing sector but mass production is still not profitable and commodities are all produced with the traditional technology. In this case $w=\bar{\omega}$ and $L, F\left(L_{f}\right)$ and $M$ are such that $M+L<F(L) / \bar{\omega}$ and $M<\rho$. From equation (9) we get $L_{f}^{*}<L$ and $R / M>\bar{\omega} \cdot{ }^{16}$ Landowners spend $(R / M-\bar{\omega})$ in manufactures consuming commodities in $\left[0, Q_{R}\right]$ where $Q_{R}=(R-\bar{\omega} M) / \alpha \bar{\omega} M$. Since each operating market faces a demand $D_{q}=M<\rho$, no market industrializes. Hence, the extent of the manufacturing sector coincides with the extent of landowners demand, $\bar{Q}=Q_{R}$, as shown in Figure 1. From equations (7) and (8) we also get $L_{m}^{*}=L-L_{f}^{*}$ implying $E=Q^{*}=0$.

In this economy land concentration prevents industrialization because, although landowners are rich enough to demand manufactures, their number is not sufficient to make the introduction of IT profitable for entrepreneurs.

Small economy. In a small economy both workers and landowners are rich enough to demand manufactures but population is so small that the industrial technology is still not profitable. In this case $R / M \geq w>\bar{\omega}$ and $M+L<\rho$. As before, from equation (9) we get $L_{f}^{*}<L$. Notice that there is an upper bound for $w$ constituted by the level of wages which reduces the rent of each landowner to $\bar{\omega}$, namely $L \bar{\omega} / L_{f}^{*} .{ }^{17}$ So, for $w<L \bar{\omega} / L_{f}^{*}$ both workers and landowners demand manufactures. Let $Q_{L} \equiv(w-\bar{\omega}) / \alpha w$ be the extent of workers' demand and $Q_{R}$ be, as before, the extent of landowners demand. In a small economy markets in $\left[0, Q_{L}\right]$ face a demand equal to $D_{q}=(L+M)<\rho$ while

[^7]

Figure 1: Traditional economy
markets in $\left(Q_{L}, Q_{R}\right]$ get a demand equal to $M<\rho$, as shown in Figure 2. Hence, no market industrializes and the extent of the manufacturing sector is $\bar{Q}=Q_{R}$. Exploiting equations (7) and (8) we obtain $L_{m}^{*}=L-L_{f}^{*}$ and $E=Q^{*}=0 .{ }^{18}$

In this economy, industrialization is prevented by the small population size and not by distribution. Indeed, even if agricultural productivity is high enough to grant both workers and landowners a very high income, their small number makes mass production unprofitable. In this case the manufacturing sector may still flourish producing manufactures of great quality and value but no artisan tries to become an entrepreneur.


Figure 2: Small economy $\left(Q_{R}>Q_{L}\right)$

[^8]
### 4.2 Industrialization Driven by Landowners Demand

Workers of countries in an early stage of industrialization frequently experience subsistence wages. In order to focus on such a case we assume that $w=\bar{\omega}$. Besides, this simplification allows us to isolate the effects of land ownership distribution and, hence, to study it in greater detail. For the sake of completeness, in next section we sketch what happens for higher wages, although we leave its full analysis to further research.

We proceed comparing the equilibrium values of aggregate income, industrial extent and industrial employment which are obtained for different degrees of land concentration. Table 1 summarizes the equilibrium values for all the possible scenarios. Since calculations are not particularly enlightening and rather long, we collect them in the Appendix providing here only results and their interpretation.

As we have pointed out for traditional economies, if $M<\rho$ no artisan introduces the IT. Hence, both industrial extent and employment are nil. In this case, the income of the economy is equal to

$$
Y^{*}=R^{*}+\bar{\omega}\left(L_{f}^{*}+L_{m}^{*}\right)=R^{*}+\bar{\omega} N-\bar{\omega} M
$$

Since $N$ and $R^{*}$ are constant, $Y^{*}$ decreases in $M$ meaning that a more equal distribution of land property rights reduces aggregate income. The reason is that there are more landowners and hence less people work. On the demand side, aggregate demand of manufactures decreases in $M$ because the quota of rents spent on food increases.

For $M=\rho$ we have a sharp change. Assuming for simplicity that IT is introduced whenever it is not disadvantageous to do so, we have that industrial extent jumps to $Q^{*}=Q_{R}=\bar{Q}$ and no commodity is produced with the TT. Artisans operating in markets $\left[0, Q_{R}\right]$ who know the IT become entrepreneurs, although they still earn as much as a worker. Similarly, industrial employment jumps to

$$
L_{I T}^{*}=\beta M Q_{R}+k Q_{R}
$$

where the first term accounts for workers in direct production and the second one for those involved in start-up tasks. On the contrary, aggregate income which is equal to

$$
Y^{*}=R^{*}+\Pi^{*}+\bar{\omega} N-\bar{\omega}\left(M+Q_{R}\right)=R^{*}+\bar{\omega}(N-M)
$$

does not increase with respect to any equilibrium with no industrialization. This happens because increasing returns are exploited just enough to grant entrepreneurs an income of $\bar{\omega}$ which implies $\Pi^{*}=\bar{\omega} Q_{R}$.

For $M>\rho$, we have positive industrialization. Moreover, some entrepreneurs are rich enough to spend part of their profits in manufactures. This generates new demand starting the multiplicative mechanism described in the previous section. Furthermore, Since each entrepreneur makes profits depending on the units of output sold, different distributive scenarios are possible. In particular, there exists a level of $M$, which we denoted by $\mu$, such that: i) for $M=\mu$, all entrepreneurs and landowners are equally rich, ii) for $M<\mu$ entrepreneurs make heterogenous profits but all are poorer than landowners and iii) for $M>\mu$ entrepreneurs again make heterogenous profits but those operating in $\left[0, Q_{R}\right]$ are richer than landowners while the remaining are poorer. ${ }^{19}$

[^9]For $M \leq \mu$ the extent of industrialization $Q^{*}$ decreases in the number of landowners M. This happens because landowners are the richest consumers and, hence, the extent of industrialization coincides with the extent of landowners demand $Q_{R}$ which decreases in $M$. Since no one demands commodities beyond $Q_{R}$, no good is produced with the TT and $Q^{*}=Q_{R}=\bar{Q}$. For what concerns industrial employment we have

$$
\begin{equation*}
L_{I T}^{*}=\frac{R^{*}}{\bar{\omega}}-\left(M+Q_{R}\right) \tag{14}
\end{equation*}
$$

Notice that $L_{I T}^{*}$ is equal to the number of people who are not employed as agricultural workers, $R^{*} / \bar{\omega}=\left(N-L_{f}^{*}\right)$, minus the sum of landowners and entrepreneurs, $\left(M+Q_{R}\right)$. Since $Q_{R}$ diminishes in $M$ at a decreasing rate we have, from equation (14), that $L_{I T}^{*}$ can increase, decrease, or first increase and then decrease. Similarly, aggregate income is

$$
\begin{equation*}
Y^{*}=R^{*}+\bar{\omega} N+\Pi^{*}-\bar{\omega}\left(M+Q_{R}\right) \tag{15}
\end{equation*}
$$

and is not, in general, monotonic in $M .{ }^{20}$ In the range under consideration it can either increase or first increase and then decrease. The term $\Pi^{*}-\bar{\omega}\left(M+Q_{R}\right)$ determines its actual behaviour depending on two opposite effects induced by a greater $M$. On the one hand, the concentration of landowners demand in fewer markets allows a better exploitation of increasing returns while, on the other, a higher quota of rents spent in subsistence reduces aggregate landowners demand of manufactures. Whether the first or the second effect prevails - and, hence, if $Y^{*}$ increases or decreases - depends on aggregate profits. Indeed, although the number of entrepreneurs declines in $M$ we have that, on average, they become richer so that the total effect on $Y^{*}$ is ambiguous.

For $M>\mu$ entrepreneurs in $\left[0, Q_{R}\right]$ demand commodities beyond $Q_{R}$. If their number is high enough - namely $Q_{R} \geq \rho$ - also the markets facing only their demand industrialize. Moreover, since $Q_{R}>\rho$, also entrepreneurs of these markets demand manufactures which, in turn, increases the earnings of some entrepreneurs in $\left[0, Q_{R}\right]$. Notice that these entrepreneurs are the richest among capitalists since their products are demanded by all who buy manufactures. In particular, the additional profits they earn transforms in demand of new kinds of commodities. So, if their number is greater than $\rho$, also the artisans producing these new commodities adopt IT and becomes entrepreneurs.

$$
\mu=\frac{\alpha(k+1)-\beta+\sqrt{(\alpha(k+1)-\beta)^{2}+4 \alpha \beta(\alpha-\beta) \frac{R}{\bar{w}}}}{2(\alpha-\beta) \alpha}
$$

[^10]Table 1 - Taxonomy of Equilibria.

| Number of <br> Landowners | $Q^{*}$ | $\Pi^{*}$ | $Y^{*}$ | $L_{I T}^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
| $M<\rho$ | 0 | 0 | $R *+\bar{\omega} N-\bar{\omega} M$ | 0 |
| $M=\rho$ | $Q_{R}$ | 0 | $R *+\bar{\omega} N-\bar{\omega} M$ | $\beta M Q_{R}+K Q_{R}$ |
| $\mu \geq M>\rho$ | $Q_{R}$ | $\left[(\alpha-\beta) \frac{\alpha}{\beta}(M-\rho)+1\right] \bar{\omega} Q_{R}$ | $R^{*}+\bar{\omega} N+\Pi^{*}-\bar{\omega}\left(M+Q_{R}\right)$ | $\frac{R^{*}}{\bar{\omega}}-\left(M+Q_{R}\right)$ |
| $M>\mu$ | $Q_{i^{*}}^{*} \S$ | $(\alpha-\beta) D_{i^{*}}^{\pi}(M>\mu) \bar{\omega}+\bar{\omega} Q_{i^{*}}^{*}$ | $R^{*}+\bar{\omega} N+\Pi^{*}-\bar{\omega}\left(M+Q^{*}\right)$ | $\frac{R^{*}}{\bar{\omega}}-\left(M+Q^{*}\right)-L_{T T}^{*}$ |

${ }^{\S}$ Where $Q_{i^{*}}^{*} \equiv(M-\rho) \sum_{j=1}^{i^{*}} x^{j}+Q_{R} x^{i^{*}}$.
${ }^{\dagger}$ Where $D_{i^{*}}^{\pi}$ is equation (39) derived in the appendix.

Such a process may go on for several rounds, each time industrializing an additional interval of markets. For simplicity, we refer to the number of such intervals as the number of steps and denote it by $i^{*}$. Clearly, $i^{*}$ is a non-increasing step function of $M$ but a greater $i^{*}$ does not imply a greater $Q^{*}$. In particular, $Q^{*}$ is determined by the income of the richest group of entrepreneurs among those of dimension not less than $\rho$ and can be written as

$$
\begin{equation*}
Q^{*}=(M-\rho) \sum_{j=1}^{i^{*}}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i^{*}} \tag{16}
\end{equation*}
$$

The first term accounts for the positive effect produced by the concentration of landowners demand in basic manufactures which allows the richest entrepreneurs to make more profits and, hence, to extend their demand. The magnitude of this effect increases, ceteris paribus, in the number of steps because more steps imply more groups of entrepreneurs demanding to the richest group. The second term accounts for the negative effect produced by the reduction of the extent of landowners demand which, other things being equal, reduces the number of industrialized markets and, hence, the demand faced by the richest group of entrepreneurs. The magnitude of this negative effect decreases in the number of steps because it is partially compensated by the industrialization of more markets which does not face landowners demand. ${ }^{21}$ As a result, the extent of industrialization can both increase and/or decrease in $M$, possibly showing discontinuous variations when $M$ reaches values which imply a decrease in $i^{*}$. Moreover, apart from such points of discontinuity, we have that $\bar{Q}>Q^{*}$. Indeed, the few richest entrepreneurs demand commodities produced with the TT and a traditional sector survives. ${ }^{22}$

Furthermore, industrial employment is equal to

$$
\begin{equation*}
L_{I T}^{*}=\frac{R^{*}}{\bar{\omega}}-\left(M+Q^{*}\right)-L_{T T}^{*} \tag{17}
\end{equation*}
$$

where $L_{T T}^{*}$ is the number of workers producing with TT. For any feasible value of $i^{*}$ and the associated range of $M, L_{T T}^{*}$ can both increase or decrease in $M$. So, taking into account the behaviour of $Q^{*}$, we have that also $L_{I T}^{*}$ can increase and/or decrease and possibly show discontinuities in coincidence with the reduction of $i^{*}$.

Finally, aggregate income is given by ${ }^{23}$

$$
\begin{equation*}
Y^{*}=R^{*}+\bar{\omega} N+\Pi^{*}-\bar{\omega}\left(M+Q^{*}\right) \tag{18}
\end{equation*}
$$

As for $M<\mu, Y^{*}$ can either decrease or first increase and then decrease. The intuition is fundamentally the same given for that case, although here income is more likely to decrease in $M$. For $\rho \leq M<\mu$ landowners are the richest group in society and no one demands commodities produced with TT. As a consequence, all profits are transformed in extra demand for industrial goods except what is spent in subsistence. On the contrary, for $M>\mu$ some entrepreneurs are the richest group in society and demand commodities

[^11]produced with TT so that the fraction of profits which generate additional income is lower. Therefore, in such a case it is more likely that $Y^{*}$ decreases in $M$, even when a greater $M$ implies greater profits for the richest entrepreneurs.

### 4.3 Maxima: Aggregate Income, Industrial Extent and Employment

So far, we have shown that aggregate income, industrial extent and industrial employment have a non-monotonic relationship with land concentration. The next step is to identify the values of $M$ which gives the maximal levels of these variables. Quite interestingly, it turns out that maxima are not achieved for the same distribution of land ownership. This suggests that there is a trade off between income and industrialization during the early stages of industrialization.

The maximum $Q^{*}$, which we denote by $\widehat{Q^{*}}$, is obtained for $M=\rho$. The reason is the following. For $M<\rho$ no market industrializes so $M$ must not be less than $\rho$. Notice that, for $M=\rho$, all workers of the manufacturing sector produce with the IT and industrial employment is the minimum which allows to industrialize until $\widehat{Q}^{*}$. Moreover, the maximum number of people employable as industrial workers is $\left(N-L_{f}^{*}-M-Q^{*}\right)$. Therefore, for $M>\rho$ it is impossible to have $Q^{*} \geq \widehat{Q^{*}}$ because there are not enough workers. So, the maximum extent of industrialization is obtained for the distribution of land which produces a demand of manufactures just sufficient to industrialize markets in $\left[0, Q_{R}\right]$, making entrepreneurs earn as much as workers and landowners the richest in society. Furthermore, comparative statics gives the expected results. A higher $k$ requires landowners to be in greater number in order to make IT profitable while, for the same reason, a higher $(\alpha-\beta)$ has an opposite effect.

On the contrary, the $M$ which gives the maximum $Y^{*}$, denoted by $\widehat{Y}^{*}$, depends on both $\tau$ and $F$. It may happen, for instance, that $Y^{*}$ is maximal when land is concentrated and landowners are the richest in society as well as when land is more equally distributed and the richest group is constituted by some entrepreneurs. It may also happen when landowners and entrepreneurs earn exactly the same. In any case, whatever is the technology, maximum income is achieved for a level of $M$ which is greater than that associated with $\widehat{Q^{*}}$. Indeed, for $M=\rho$ increasing returns are not exploited at all and income is even lower than in the traditional economy case. Hence, a more equal distribution of land increases $Y^{*}$ because, by inducing a greater concentration of demand in basic manufactures, allows a better exploitation of increasing returns. On the other hand, a too wide diffusion of land property rights may be detrimental. The concentration of landowners demand in few basic manufactures has the effect of concentrating most of the profits into the hands of few entrepreneurs. Since these are very rich with respect to the size of the industrial sector, they spend a substantial part of their earnings in manufactures produced with TT.

In general, the optimal land concentration can be greater, equal or lower than $\mu$. A higher $k$ increases the optimal $M$ because increases the profitability threshold of IT and, hence, requires a greater demand concentration for optimality. A higher $R^{*}$ has the same effect because increases the relative advantage of concentrating demand in few manufactures. On the contrary, a higher $\alpha$ reduces the optimal $M$ because it increases the relative price of manufactures, having the same effect on landowners demand as a reduction of rents. A higher $\beta$ may or may not have an effect but certainly increases $\mu$ because it reduces the profits earned for each unit sold and, hence, it increases the range of $M$ for which landowners are the richest. The reason of the ambiguous effect of $\beta$ on


Figure 3: An example with $i^{*}=2$.
the optimal $M$ is the following. As long as $M<\mu, \beta$ only affects the way profits accruing from landowners demand are increased by the multiplicative process. So, $\beta$ can affect the absolute value of $\widehat{Y}^{*}$ but not the optimal $M$. Instead, in the case that the optimal $M$ is grater than $\mu$, a higher $\beta$ reduces the former since it contributes to determine the demand received by markets beyond $Q_{R} .{ }^{24}$

Finally, the maximum $L_{I T}^{*}$, denoted by $\widehat{L}_{I T}^{*}$, is obtained for $M$ in $[\rho, \mu]$. The exact value depends, again, on $\tau$ and $F$. To see why $\widehat{L}_{I T}^{*}$ cannot happen for $M>\mu$ recall that $L_{I T}^{*}$ always decreases in $\left(M+Q^{*}\right)$ and that for $M=\mu$ we have $L_{T T}^{*}=0$. Hence, a necessary condition to have $\widehat{L}_{I T}^{*}$ with $M>\mu$ is that $M+Q^{*}(M) \leq \mu+Q^{*}(\mu)$ which implies $Q^{*}(M)<Q^{*}(\mu)$. So, the richest entrepreneurs must earn less than what they earn when $M=\mu$. This implies that the total demand faced by the industrial sector cannot be greater than in $M=\mu$ while employment in start-up tasks is certainly lower. Therefore, the $M$ which maximizes industrial employment cannot be greater than $\mu$.

More precisely, $\widehat{L}_{I T}^{*}$ is obtained for $M$ comprised between that associate with $\widehat{Q^{*}}$ and that associated with $\widehat{Y^{*}}$, possibly coinciding with either of them. The actual behaviour of $\left(M+Q_{R}\right)$ in the interval $[\rho, \mu]$ determines its exact position. For instance, when a greater number of landowners induces a shrinking of the interval of industrialized markets which never frees enough labour force to compensate the increased $M$, then we have no tradeoff

[^12]

Figure 4: Two more intervals of markets industrialize.
between industrial extent and employment and $\widehat{L}_{I T}^{*}$ is obtained for $M=\rho$. If, on the contrary, the reduction of $Q_{R}$ always compensates for a greater $M$, then $\widehat{L}_{I T}^{*}$ is obtained for $M=\mu$. In all other cases, $\left(M+Q_{R}\right)$ has its minimum in the interior of $[\rho, \mu]$ and the distribution of land ownership which maximizes industrial employment is strictly comprised between that maximizing industrial extent and the one maximizing income.

Indeed, industrial employment partly behaves like industrial extent because increases in the number of people working in start-up tasks and partly behaves like aggregate income because grows in the level of aggregate profits.

In order to give the reader the flavour of these findings we depict an example in Figure 4.3. It exemplifies the non-monotonic relation between land distribution and industrialization/income as well as the fact that maximal income, industrial extent and employment are not obtained for the same distribution of land property rights. Moreover, it shows that in the range $[\rho, \mu]$ there is a tradeoff between industrialization and income. More precisely, until $\lambda$ the tradeoff is between industrial extent on one side and industrial employment and income on the other. ${ }^{25}$ In $[\rho, \lambda]$, a more equal distribution of land concentrates landowners demand in such a way that the new workers needed for direct production are more than those who were previously needed for the start-up tasks of markets that no longer industrialize. So, income and industrial employment go the same way. On the contrary, in $[\lambda, \mu]$ a tradeoff exists between income and both industrial extent and employment. In this range, income increases despite the decrease in industrial employment because the total number of workers employed in direct production is still rising and industrial surplus grows. Industrial employment decreases because the reduction

[^13]of workers hired for start-up tasks is greater than new hirings for direct production, so that a better exploitation of increasing returns does not coincide anymore with a greater number of industrial workers.

For $M$ in $[\mu, \eta]$, land is distributed more widely and some entrepreneurs get richer than landowners. ${ }^{26}$ Apart from discontinuity points, there is again a tradeoff between industrial extent on one side and industrial employment and income on the other but with the opposite sign. Besides, the multiplicative mechanism induces the industrialization of new intervals of markets creating up to five different earning groups of entrepreneurs (as depicted in Figure 4). There are two intervals until the first discontinuity and one afterwards.

Finally, for $M$ greater than $\eta$ land ownership is so dispersed - and consequently landowners demand so concentrated - that there are only few and very rich entrepreneurs. Their number is so small that they demand many different manufactures but no single market receives enough demand to industrialize. In this range there is no longer tradeoff among income, industrial employment and industrial extent as they all decrease in $M$.

## 5 Extensions

### 5.1 Agricultural Productivity and Wages

Our analysis confirms the common wisdom that technical improvements in the agricultural sector can lead to industrialization. A slightly modified version of the model shows how a greater agricultural productivity can push the economy to a new equilibrium with more extensive industrialization and higher income level. Suppose $F=F\left(L_{f}, \gamma\right)$ where $\gamma$ is a productivity parameter. Assume $\mathrm{d} L_{f}^{*}(\gamma) / \mathrm{d} \gamma<0$ meaning that the equilibrium employment in agriculture decreases as a consequence of higher agricultural productivity. This implies that wages increase with productivity, that is $\mathrm{d} w(\gamma) / \mathrm{d} \gamma>0$. Any increase in agricultural productivity has two effects. The first is a reduction of $L_{f}^{*}$ which allows both a greater number of entrepreneurs $Q^{*}$ and a greater industrial employment $L_{I T}^{*}$. The second effect, on the contrary, is less recognized and is, to some extent, surprising. A higher agricultural productivity traduces in higher workers wages implying higher production costs and commodities prices. So, one expects that a higher $\gamma$ damages both traditional and industrial markets because, although it may increase profits of some entrepreneurs who benefit from higher prices, it reduces the extent of consumers demand. In general, this is not true. A higher $w$ can well benefit the manufacturing sector. Firstly, the impact of a higher $\gamma$ on landowners demand is ambiguous as $R$ may be affected in either ways. Secondly, richer workers demand more manufactures as $(w-\bar{\omega}) / \alpha w$ increases in $w$. Therefore, assuming workers outnumber landowners, it can easily happen that a higher $\gamma$ implies a higher demand for many markets with the result that both industrial extent and employment are greater.

This brief digression highlights the relevance of how the benefits accruing from a high agricultural productivity are shared between workers and landowners. Suppose agricultural productivity is very high and only few workers are employed in the agricultural sector. In this case there would be potential for a quite large industrial sector. However, if land is concentrated in the hands of few landowners and $w$ is close to the subsistence

[^14]level then in equilibrium no substantial industrialization takes place. The reason is that since few landowners are taking for themselves most of the benefits of the increased agricultural productivity, demand is highly dispersed and the manufacturing sector is large but only very few markets, if any, industrialize.

### 5.2 Exports

The model we have described so far provides insights also about the impact of export booms on industrialization. Naturally, there are many cases where domestic demand is not the unique source of potential purchases, even in traditional economies. Positive shocks on either international price or demand of tradables may induce export booms at the country level and help the industrialization process. Such positive effects, however, are not guaranteed. The case of Colombia reported by Harbison [13] is illuminating. During the years between 1850 and 1870 Colombia experienced a strong increase in revenues accruing from tobacco exports. Unfortunately, this did not result in any significant increase of domestic demand of manufactures and the small industrial sector of the country did not benefit from it. A second export boom took place in Colombia between 1880 and 1915 but this time it was coffee-driven. Interestingly, it was beneficial not only to the coffeerelated businesses but to the colombian industry as a whole. Harbison explanation of the different impact of the two booms points to the fact that tobacco was produced in huge land estates while coffee was mainly cultivated in small or medium fields. Since the first boom increased the income of few landlords, it had no substantial effect on the domestic demand of basic manufactures, mostly resulting in demand for luxuries and imports. This did not happen in the second boom because it rewarded a larger and poorer fraction of the population, increasing the aggregate demand of basic manufactures.

We shall distinguish between two types of export booms. The first one affects the manufacturing sector directly and takes place when there is an increase of international demand of manufactures produced at a country level with TT. In this case, the distribution of land property rights affects the equilibrium outcome by determining the extra demand needed to make mass production profitable. Consider a traditional economy and assume that technology $\tau$ is competitive in the sense that it allows producers to export with standard profits. ${ }^{27}$ The volume of exports needed to industrialize any market of this country is $(\rho-M)$. So, a greater $M$ makes smaller shocks capable of inducing industrialization in those markets which already produce with the TT. However, the extent of the manufacturing sector often decreases in $M$, reducing the number of markets which can benefit from the export boom. Which effect is more important may not be simple to establish analytically as it requires some kind of measure of expected industrialization benefits. ${ }^{28}$

The second type of export boom affects manufacturing markets indirectly and takes places when there is an increase of the revenues accruing from agricultural products sold abroad. Consider an economy exporting a certain amount of food denoted by $I$. Assume food is the only tradable good of the economy and local prices are unaffected by

[^15]international prices. The demand of food is then $(I+\bar{\omega} N)$ and the equilibrium aggregate rents are $R^{*}=I p_{f}+\bar{\omega} N-L_{f}^{*} \bar{\omega}$, where $p_{f}$ represents the international price of food. For the sake of exposition, we shall focus on two extreme cases: the boom driven only by an increase in prices and the one driven only by an increase of quantity demanded.

If $p_{f}$ increases with no substantial change in the volume of production then $R^{*}$ increases leaving wage and agricultural employment unchanged. Such an increase expands the extent of landowners demand accordingly. If land is not too concentrated - i.e. $M \geq \rho$ - then the increase in revenues from exports induces a sensible growth of the industrial sector. On the contrary, if property is quite concentrated - i.e. $M<\rho$ - nothing happens but an expansion of traditional production. These two stylized cases well represent Harbisons basic argument for the two opposite outcomes of the colombian export booms.

If the amount of food exported $I$ increases with no substantial effects on $p_{f}$, then several things can happen. If wages are both at the subsistence level and equal to marginal productivity, production cannot increase and the boom fails to take place. This is due to the fact that to produce more food more workers are needed but a greater $L_{f}^{*}$ would imply $w<\bar{\omega}$. If agricultural employment can increase to some extent without lowering wages, say leaving them constant, then $R$ increases as for the case of an increase in $p_{f}$. Finally, if $w>\bar{\omega}$ we have that the export boom increases both $R^{*}$ and $L_{f}^{*}$. Again, if $w$ is unaffected only $R$ increases. On the contrary, if $w$ decreases it may well happen that the export boom is detrimental to industrialization. The intuition is the following. Since $w>\bar{\omega}$, it must be that workers are demanding manufactures. With their wages reduced the extent of their demand shrinks accordingly even if manufactures prices decrease. If workers demand of manufactures is driven to zero and landowners does not compensate it - i.e. $(M<\rho)$ - the industrial sector disappears while the manufacturing sector as a whole expands dramatically. If $w$ is not reduced to the subsistence level but landowners demand alone is not sufficient to sustain mass production, then the industrial sector shrinks to the markets facing workers demand while the manufacturing sector as a whole expands. Finally, if landowners demand alone can sustain industrialization then the industrial sector may or may not expand depending on technology and on the actual distribution of profits. The same is true for the manufacturing sector as a whole.

## 6 Concluding Remarks

In this paper we have analysed how the distribution of land ownership affects income and industrialization through the demand side. In order to do this we have developed a modified version of the model of Murphy et al. [20]. The main novelty of our model is that we assume a functional distribution of income. The motivation for this choice is two-fold: on one side, we find that it is a better representation of an early industrializing country and, on the other, it allows us to investigate in greater detail the impact of land property rights distribution on income and industrialization.

Consistently with the general results about income distribution found in Murphy et al. [20], we have shown that the degree of land ownership concentration is in a nonmonotonic relation with both income and industrialization. We have also proved that, under quite general conditions, there may be a tradeoff between aggregate income and early industrialization. Indeed, their maximal values occur for different levels of land concentration and, in particular, we found that maximal income is associated with a more diffuse distribution of land ownership than maximal industrialization.

More important, we have shown that a high concentration of land ownership - a typical situation in many countries on the door of industrialization - can prevent the industrial takeoff. The reason is that it induces a very unequal distribution of income with the result that only few commodities of each kind are demanded and mass production is unprofitable. We have also shown that a very diffuse distribution of land may be detrimental to income and industrialization. Indeed, a widespread ownership of land allows the exploitation of scale economies in some markets of basic manufactures but their number is likely to be quite small due to the low income of landowners. As a consequence, since profits are not shared but each entrepreneur takes all earnings of the firm she manages, a very diffuse distribution of land concentrates profits into the hands of very few capitalists. Therefore, a very equal distribution of land property rights induces a very unequal distribution of income between capitalist and everyone else with the result that the demand of manufactures produced with the industrial technology is low and mass production is not exploited properly. In particular, our analysis shows that the degree of land concentration determines the distribution of profits because it determines the earnings of each entrepreneur in the first place, that is before the multiplicative process takes place. In our opinion these findings underline the relevance of how surplus is shared among the different social groups.

Few remarks about the nature of these results are worth doing. In our analysis there is no dynamics and all results come from a comparative statics exercise. Therefore, this study does not offer any reliable prediction about the impact of changes in land ownership distribution. So, although we recognize that land redistribution is a major source of reductions in land concentration, it has not been an explicit issue here. Indeed, a policy of land redistribution triggers a number of economic and social mechanisms which are not captured by our model and clearly require a dynamical analysis. In this sense, the present study can provide only weak policy suggestions. Nevertheless, the comparative statics we have carried out tells us something important. If a country is about the industrial takeoff we expect that, ceteris paribus, countries with a very concentrated land ownership perform worse than countries with a mild concentration. Going back to the example about South Korea and Philippines mentioned in the introduction, we understand that the more equal distribution of land in South Korea has helped its industrial takeoff by providing a domestic demand of basic manufactures since the very beginning of its development. In other words, we expect those countries that have successfully carried out a land reform to be in a better position for the industrial takeoff. Finally, since the analysis abstracts from the effects of industrialization in the long run, our findings must be intended as restricted only to countries in their early phase of industrialization.

Further research should provide a detailed analysis of the role of agricultural productivity and non-subsistence wages, taking into account the issue of unemployment. Moreover, this framework may well fit the analysis of the impact of distribution of property rights over natural resources, both exhaustible and renewable.

## A Appendix

## A. 1 Derivation of $\mu$

In order to obtain the expression for $\mu$ provided in footnote 19 we must impose the condition that entrepreneurs and landowners in equilibrium have the same income. For any $M>\rho$, the $Q_{R}$ entrepreneurs who receive the demand of landowners will demand manufactures in the interval $\left[0, Q_{1}\right]$ where
$Q_{1}=(M-\rho)(\alpha-\beta) / \alpha$. Assuming that $Q_{1}<Q_{R}$, entrepreneurs in $\left[0, Q_{1}\right]$ face additional demand and their profits will be equal to $\left[\left(M+Q_{R}\right)(\alpha-\beta)-k\right] \bar{\omega}$. Equalizing the latter expression to the income of each landowner $R / M$ we obtain

$$
\begin{align*}
& {\left[\left(M+Q_{R}\right)(\alpha-\beta)-k\right] \bar{\omega}=\frac{R}{M}} \\
& \alpha(\alpha-\beta) M^{2}-[\alpha(k+1)-\beta] M-\beta \frac{R}{\bar{\omega}}=0 \tag{19}
\end{align*}
$$

where the solutions to equation (19) are

$$
\begin{equation*}
\frac{\alpha(k+1)-\beta \pm \sqrt{(\alpha(k+1)-\beta)^{2}+4 \alpha \beta(\alpha-\beta) \frac{R}{\bar{\omega}}}}{2(\alpha-\beta) \alpha} \tag{20}
\end{equation*}
$$

Notice that $(\alpha(k+1)-\beta)$ is positive and greater than one by assumptions about technology, so we have one strictly positive and one strictly negative solution. We name $\mu$ the positive one.

## A. 2 Equilibria: $M \leq \mu$

Define $Q_{2}$ the extent of demand of the group of entrepreneurs in $\left[0, Q_{1}\right]$. Then, $M \leq \mu$ implies $Q_{2} \leq Q_{R}$. Since entrepreneurs in $\left[0, Q_{1}\right]$ face the highest demand, and hence earn the highest profits, no entrepreneur demands manufactures beyond $Q_{R}$. Moreover, in equilibrium entrepreneurs in $\left[0, Q_{1}\right]$ receive the demand of every entrepreneur, including themselves, so their total demand is $\left(Q_{R}+M\right)$. Hence, entrepreneurs in $\left(Q_{1}, Q_{2}\right]$ earn additional profits and demand manufactures until $Q_{3}=\left(M+Q_{2}\right)(\alpha-\beta) / \alpha$. By iterating this mechanism, we can calculate the equilibrium demand and profits of each market.

Points $Q_{1}, Q_{3}, Q_{5}, \ldots$ and $Q_{2}, Q_{4}, Q_{6}, \ldots$ can be written as follows

$$
\begin{aligned}
Q_{1} & =(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right) \\
Q_{2} & =(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right) \\
Q_{3} & =(M-\rho)\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}\right] \\
Q_{4} & =(M-\rho)\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}\right]+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{2} \\
Q_{5} & =(M-\rho)\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}+\left(\frac{\alpha-\beta}{\alpha}\right)^{3}\right] \\
Q_{6} & =(M-\rho)\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}+\left(\frac{\alpha-\beta}{\alpha}\right)^{3}\right]+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{3} \\
\ldots & =\ldots
\end{aligned}
$$

Hence, the expressions for a generic $Q_{2 i}$ (even index) and $Q_{2 i+1}$ (odd index) are

$$
\begin{align*}
Q_{2 i} & =(M-\rho) \sum_{j=1}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i} \\
& =(M-\rho) \sum_{j=0}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i}-M^{k}  \tag{21}\\
Q_{2 i+1} & =(M-\rho) \sum_{j=1}^{i+1}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}=(M-\rho) \sum_{j=0}^{i+1}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}-(M-\rho) \tag{22}
\end{align*}
$$

where $Q_{0} \equiv Q_{R}$. As $i$ goes to $\infty$ expressions (21) and (22) converge to the same value from above and below, respectively

$$
\begin{aligned}
\lim _{i \rightarrow \infty} Q_{2 i} & =\lim _{i \rightarrow \infty}(M-\rho) \sum_{j=0}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i}-(M-\rho)=(M-\rho)\left(\frac{\alpha-\beta}{\beta}\right) \\
\lim _{i \rightarrow \infty} Q_{2 i+1} & =\lim _{i \rightarrow \infty}(M-\rho) \sum_{j=0}^{i+1}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}-(M-\rho)=(M-\rho)\left(\frac{\alpha-\beta}{\beta}\right)
\end{aligned}
$$

Therefore, for any value of $M \leq \mu$ the equilibrium demand of each market is uniquely determined and the calculation of the equilibrium profits of each entrepreneur is straightforward. In the case $M=\mu$ we get $Q_{R}=Q_{2} i$ for every $i \geq 1$.

Figure 5 and Figure 6 give a graphical representation of industrialization for $M<\mu$ and for $M=\mu$ respectively.


Figure 5: $M<\mu$

## A. $3 \quad M \leq \mu$ : Industrial Extent, Industrial Employment and Aggregate Income

Since no entrepreneur demands beyond market $Q_{R}$ the extent of industrialization is $Q^{*}=Q_{R}$. Industrial employment is given by

$$
L_{I T}=N-L_{f}-L_{T T}-E-M
$$

In an equilibrium with positive industrialization, since i) $L_{T T}^{*}=0$ and ii) $E=Q^{*}=Q_{R}$, we have

$$
\begin{equation*}
L_{I T}^{*}=N-L_{f}^{*}-Q_{R}-M \tag{23}
\end{equation*}
$$

Taking into account equation (1) and the fact that $F\left(L_{f}^{*}\right)=\bar{\omega} N$ we obtain

$$
\begin{equation*}
N-L_{f}^{*}=\frac{R}{\bar{\omega}} \tag{24}
\end{equation*}
$$

From equations (23) and (24) we get equation (14).
Aggregate income is given by the sum of of rents, profits and wages

$$
Y=R+\Pi+w\left(L_{f}+L_{m}\right)
$$



Figure 6: $M=\mu$

Since $\left(L_{f}+L_{m}\right)=N-E-M$, in equilibrium we obtain equation (15). The expression for $\Pi^{*}$ is derived by adding the profits of each entrepreneur, calculated on the basis of the demand she faces in equilibrium. In order to calculate aggregate demand, we first derive the length of the generic intervals

$$
\begin{align*}
Q_{2 i}-Q_{2 i+2} & =(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)^{i}+Q_{R} \frac{\beta}{\alpha}\left(\frac{\alpha-\beta}{\alpha}\right)^{i}  \tag{25}\\
Q_{2 i+1}-Q_{2 i-1} & =(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)^{i+1} \tag{26}
\end{align*}
$$

where $Q_{-1} \equiv 0$. Multiplying the length of each interval of markets by the demand exceeding $\rho$ that they face, we get the aggregate demand which generates profits exceeding subsistence. The demand faced by each interval of markets is

$$
\begin{aligned}
D_{q \in\left(Q_{2 i-1}, Q_{2 i+1}\right]} & =M+Q_{2 i} \\
D_{q \in\left(Q_{2 i+2}, Q_{2 i}\right]} & =M+Q_{2 i-1}
\end{aligned}
$$

So aggregate demand is

$$
\begin{equation*}
D^{\pi}(M \leq \mu)=\sum_{i=0}^{\infty}\left[\left(Q_{2 i+1}-Q_{2 i-1}\right)\left((M-\rho)+Q_{2 i}\right)+\left(Q_{2 i}-Q_{2 i+2}\right)\left((M-\rho)+Q_{2 i-1}\right)\right] \tag{27}
\end{equation*}
$$

Defining $x \equiv(\alpha-\beta) /(\alpha)$ and plugging (21), (22), (25) and (26) into expression (27), we can rewrite the latter as

$$
\begin{align*}
& \sum_{i=0}^{\infty}\left[(M-\rho) x^{i+1}\left((M-\rho) \sum_{j=0}^{i} x^{j}+Q_{R} x^{i}\right)+(M-\rho) \sum_{j=0}^{i} x^{j}\left(Q_{R} \frac{\beta}{\alpha} x^{i}-(M-\rho) x^{i+1}\right)\right]= \\
= & \left(M^{k}\right)^{2} \sum_{i=0}^{\infty}\left(x^{i+1} \sum_{j=0}^{i} x^{j}-x^{i+1} \sum_{j=0}^{i} x^{j}\right)+(M-\rho) Q_{R} \sum_{i=0}^{\infty}\left[x^{2 i+1}+\frac{\beta}{\alpha} x^{i} \sum_{j=0}^{i} x^{j}\right]= \\
= & (M-\rho) Q_{R} x\left[\frac{1}{1-x^{2}}+\frac{\beta}{\alpha} \sum_{i=0}^{\infty}\left(x^{i} \sum_{j=0}^{i} x^{j}\right)\right] \tag{28}
\end{align*}
$$

In order to solve the remaining series in (28) notice that

$$
\begin{aligned}
\sum_{i=0}^{\infty}\left(x^{i} \sum_{j=0}^{i} x^{j}\right) & =1+\left(x+x^{2}\right)+\left(x^{2}+x^{3}+x^{4}\right)+\ldots= \\
& =1+x+2 x^{2}+2 x^{3}+3 x^{4}+3 x^{5}+4 x^{6}+4 x^{7}+\ldots= \\
& =(1+x)\left(1+2 x^{2}+3 x^{4}+4 x^{6}+5 x^{8}+\ldots\right)= \\
& =(1+x) \sum_{i=0}^{\infty}(i+1) x^{2 i}=(1+x)\left(\frac{1}{1-x^{2}}+\sum_{i=0}^{\infty} i x^{2 i}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\sum_{i=0}^{\infty} i x^{2 i} & =0+x^{2}+2 x^{4}+3 x^{6}+4 x^{8}+\ldots= \\
& =\left(0+x^{4}+2 x^{6}+3 x^{8}+\ldots\right)+\left(x^{2}+x^{4}+x^{6}+x^{8}+\ldots\right)= \\
& =x^{2} \sum_{i=0}^{\infty} i x^{2 i}+\left(\sum_{i=0}^{\infty} x^{2 i}-1\right)
\end{aligned}
$$

Hence, we have

$$
\sum_{i=0}^{\infty} i x^{2 i}=\frac{\sum_{i=0}^{\infty} x^{2 i}-1}{1-x^{2}}=\left(\frac{x}{1-x^{2}}\right)^{2}
$$

that in turn gives

$$
\begin{equation*}
\sum_{i=0}^{\infty}\left(x^{i} \sum_{j=0}^{i} x^{j}\right)=(1+x)\left[\frac{1}{1-x^{2}}+\left(\frac{x}{1-x^{2}}\right)^{2}\right]=\frac{1+x}{\left(1-x^{2}\right)^{2}} \tag{29}
\end{equation*}
$$

From (29) and (28), making some rearrangements, we get

$$
D^{\pi}(M \leq \mu)=(M-\rho) Q_{R}\left(\frac{\alpha(\alpha-\beta)}{\beta(2 \alpha-\beta)}+\frac{\alpha^{2}}{\beta(2 \alpha-\beta)}\right)=(M-\rho) Q_{R} \frac{\alpha}{\beta}
$$

Finally, taking into account entrepreneurs expenditure in food, we obtain the aggregate profits $\Pi^{*}$ of equation (15), that is

$$
\Pi^{*}=(\alpha-\beta) \frac{\alpha}{\beta}(M-\rho) \bar{w} Q_{R}+\bar{w} Q_{R}
$$

Notice that profits are equal to the units of manufactures demanded beyond those needed to introduce IT, i.e. $(M-\rho) Q_{R}$, times the profit earned for each unit sold, i.e. $(\alpha-\beta) \bar{\omega}$, times $\alpha / \beta$ which accounts for the multiplicative process we have described plus the subsistence of the entrepreneurs $\bar{\omega} Q_{R}$.

## A. 4 Equilibria: $M>\mu$

For $M>\mu$ we have $Q_{2}>Q_{R}$. As before, markets in $\left[0, Q_{R}\right]$ industrialize. In order to determine whether or not other markets introduce the IT, we must compare the demand they face with the threshold value $\rho$.

The sequence of $Q_{2 i+1}$ is formally as in 22 but stops beyond $Q_{R}$. The sequence of $Q_{2 i}$ is constant and equal to

$$
Q_{2 i}=Q_{2}=(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)
$$

and it stops as soon as $Q_{2 i+1}$ stops. So far, each market in $\left[0, Q_{R}\right]$ faces the same demand $\left(Q_{R}+M\right)$ and each entrepreneur in $\left[0, Q_{R}\right]$ earns the same profits. In order to take into account the multiplicative process triggered by industrialization beyond $Q_{R}$, let us simplify notation and preserve the intuition about the sequence of $Q s$. Both $Q_{1}$ and $Q_{0}$ are set equal to $Q_{R}$, and $Q_{2}$ denotes the extent of demand of the richest entrepreneurs, no matter where the " $Q_{1}$ " defined for the previous case falls. So, $\left(Q_{R}, Q_{2}\right.$ ] is the first interval to receive only entrepreneurs demand, which industrializes if and only if $Q_{R}>\rho$. If this happens, we call $Q_{3}$ the extent of demand of entrepreneurs in $\left(Q_{R}, Q_{2}\right]$. Similarly, $Q_{4}$ indicate the new extent of demand of entrepreneurs in $\left[0, Q_{3}\right]$, and so on. Therefore, points $Q_{1}, Q_{3}, Q_{5}, \ldots$ and $Q_{2}, Q_{4}, Q_{6}, \ldots$ are given by

$$
\begin{aligned}
Q_{1} & =Q_{R} \\
Q_{2} & =(M-\rho)\left(\frac{\alpha-\beta}{\alpha}\right)+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right) \\
Q_{3} & =Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)-\rho\left(\frac{\alpha-\beta}{\alpha}\right) \\
Q_{4} & =(M-\rho)\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}\right]+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{2} \\
Q_{5} & =Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{2}-\rho\left[\frac{\alpha-\beta}{\alpha}+\left(\frac{\alpha-\beta}{\alpha}\right)^{2}\right] \\
\ldots & =\ldots
\end{aligned}
$$

from which we get the expressions for the generic $Q_{2 i}$ and $Q_{2 i+1}$

$$
\begin{align*}
Q_{2 i} & =(M-\rho) \sum_{j=1}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i}  \tag{30}\\
Q_{2 i+1} & =Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i}-\rho \sum_{j=1}^{i}\left(\frac{\alpha-\beta}{\alpha}\right)^{j} \tag{31}
\end{align*}
$$

As previously mentioned, markets in $\left(Q_{R}, Q_{2}\right]$ industrialize if and only if

$$
\begin{equation*}
Q_{R} \geq \rho \tag{32}
\end{equation*}
$$

If (32) holds with strict inequality, new demand is generated and entrepreneurs in $\left[0, Q_{3}\right]$ earn new profits and extend their demand of manufactures beyond $Q_{2}$ until $Q_{4}$. Then, markets in $\left(Q_{2}, Q_{4}\right]$ industrialize if and only if

$$
\begin{aligned}
Q_{3} & \geq \rho \Leftrightarrow \\
\Leftrightarrow Q_{R} & \geq \rho\left(1+\frac{\alpha}{\alpha-\beta}\right)
\end{aligned}
$$

By iteration, we get that the number of steps - that is the number of new industrialized intervals of markets - is given by the minimum value of $i$, denoted by $i^{*}$, such that


Figure 7: Scenario 2, Sub-case II: $Q^{*}=Q_{2}, \bar{Q}=Q_{4}, i^{*}=1$

$$
\begin{equation*}
Q_{R}<\rho \sum_{j=0}^{i}\left(\frac{\alpha}{\alpha-\beta}\right)^{j} \tag{33}
\end{equation*}
$$

Since $\alpha /(\alpha-\beta)$ is greater than 1 there always exists a finite value of $i$ such that inequality (33) is satisfied. Figure 7 shows the case of $i^{*}=1$ while Figure 4 in the main body refers to the case of two steps. Moreover, since $Q_{R}$ is a decreasing function of $M, i^{*}$ takes its maximum value for $M$ infinitesimally greater than $\mu$ and is non-increasing step function afterwards.

## A. $5 \quad M>\mu$ : Industrial Extent, Industrial Employment and Aggregate Income

Given $i^{*}$ the extent of industrialization is

$$
\begin{equation*}
Q^{*}=Q_{2 i^{*}}=(M-\rho) \sum_{j=1}^{i^{*}}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i^{*}} \tag{34}
\end{equation*}
$$

Similarly the extent of the manufacturing sector is

$$
\begin{equation*}
\bar{Q}=Q_{2\left(i^{*}+1\right)}=(M-\rho) \sum_{j=1}^{i^{*}+1}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}+Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i^{*}+1} \tag{35}
\end{equation*}
$$

and the last group of markets which receive only the demand of the richest entrepreneurs employs the TT. However, if the last group of entrepreneurs who industrialize faces a demand equal to $\rho$, then the extent of the manufacturing sector coincides with the extent of industrialization and traditional production disappears.

The equilibrium level of $L_{T T}^{*}$ is equal to the number of units produced with TT multiplied by the labour coefficient $\alpha$, that is

$$
\begin{align*}
L_{T T}^{*} & =\alpha\left(\bar{Q}-Q^{*}\right) Q_{2 i^{*}+1} \\
& =\left(\frac{\alpha-\beta}{\alpha}\right)^{i^{*}}\left[(M-\rho)(\alpha-\beta)-\beta Q_{R}\right]\left[Q_{R}\left(\frac{\alpha-\beta}{\alpha}\right)^{i^{*}}-\rho \sum_{j=1}^{i^{*}}\left(\frac{\alpha-\beta}{\alpha}\right)^{j}\right] \tag{36}
\end{align*}
$$

where $\left(\bar{Q}-Q^{*}\right) Q_{2 i^{*}+1}$ is the number of traditional manufactures demanded. By plugging equation (36) into (17) we get $L_{I T}^{*}$.

For any $i^{*}$, aggregate profits are obtained by summing the profits earned by each entrepreneur. If there is no step the net total demand is

$$
\begin{equation*}
D_{0}^{\pi}=Q_{R}\left(M+Q_{R}-\rho\right) \tag{37}
\end{equation*}
$$

where $\pi$ is used to denote the fact that we are referring only to demand generating new profits - labelled net demand, hereafter. When there is one step net demand is

$$
\begin{align*}
D_{1}^{\pi} & =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left(Q_{R}-\rho\right)+\left(Q_{R}-\rho\right)\left(Q_{2}-Q_{R}\right) x \\
& =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left[Q_{R}(1+x)-\rho(1+x)\right] \tag{38}
\end{align*}
$$

where, as before $x=(\alpha-\beta) / \alpha$. Looking at the first line, the second term accounts for net demand of new entrepreneurs beyond $Q_{R}:\left(Q_{2}-Q_{R}\right)$ is the number of markets while $\left(Q_{R}-\rho\right)$ is the net demand for each of them. The third term accounts for the additional demand received by the first group of entrepreneurs: $\left(Q_{R}-\rho\right) x=Q_{3}$ is the number of markets and $\left(Q_{2}-Q_{R}\right)$ is their additional demand. With two steps, the new markets receiving the demand of the richest entrepreneurs introduce the IT. Their net demand grants entrepreneurs in $\left[0, Q_{5}\right]$ additional profits, where $Q_{5}<Q_{3}$. The aggregate net demand generated with two steps is

$$
\begin{aligned}
D_{2}^{\pi} & =D_{1}^{\pi}+\left(Q_{2}-Q_{R}\right) x\left[\left(Q_{R}-\rho\right) x-\rho\right]+\left(Q_{2}-Q_{R}\right) x^{2}\left[\left(Q_{R}-\rho\right) x-\rho\right] \\
& =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left[Q_{R}\left(1+x+x^{2}+x^{3}\right)-\rho\left(1+2 x+2 x^{2}+x^{3}\right)\right] \\
& =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left[Q_{R}\left(1+x+x^{2}+x^{3}\right)-\rho(1+x)\left(1+x+x^{2}\right)\right]
\end{aligned}
$$

In the case of three steps we have

$$
\begin{aligned}
D_{3}^{\pi} & =D_{2}^{\pi}+\left(Q_{2}-Q_{R}\right) x^{2}\left\{\left[\left(Q_{R}-\rho\right) x-\rho\right] x-\rho\right\}+\left(Q_{2}-Q_{R}\right) x^{3}\left\{\left[\left(Q_{R}-\rho\right) x-\rho\right] x-\rho\right\} \\
& =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left[Q_{R}\left(1+x+x^{2}+x^{3}+x^{4}+x^{5}\right)-\rho\left(1+2 x+3 x^{2}+3 x^{3}+2 x^{4}+x^{5}\right)\right] \\
& =D_{0}^{\pi}+\left(Q_{2}-Q_{R}\right)\left[Q_{R}\left(\sum_{j=0}^{5} x^{j}\right)-\rho\left(1+x+x^{2}\right)\left(1+x+x^{2}+x^{3}\right)\right]
\end{aligned}
$$

Hence, for a generic $i^{*}$ we get the following expression

$$
\begin{equation*}
D_{i^{*}}^{\pi}=Q_{R}\left(M+Q_{R}-\rho\right)+\left(Q_{2}-Q_{R}\right)\left(Q_{R} \sum_{j=0}^{2 i^{*}-1} x^{j}-\rho \sum_{j=0}^{i^{*}-1} x^{j} \sum_{j=0}^{i^{*}} x^{j}\right) \tag{39}
\end{equation*}
$$

Therefore, $\Pi^{*}$ of equation (18) is

$$
\begin{equation*}
\Pi^{*}=(\alpha-\beta)\left[Q_{R}\left(M+Q_{R}-\rho\right)+\left(Q_{2}-Q_{R}\right)\left(Q_{R} \sum_{j=0}^{2 i^{*}-1} x^{j}-\rho \sum_{j=0}^{i^{*}-1} x^{j} \sum_{j=0}^{i^{*}} x^{j}\right)\right] \bar{\omega}+\bar{\omega} Q^{*} \tag{40}
\end{equation*}
$$

## A. 6 Maxima

As we proved in the main body of the article, the maximum extent of industrialization obtained for $M=\rho$ is

$$
\begin{equation*}
\widehat{Q}^{*}=\frac{R-\bar{\omega} \rho}{\alpha \bar{\omega} \rho}=\frac{1}{\alpha}\left(\frac{R}{\bar{\omega}} \frac{\alpha-\beta}{k+1}-1\right) \tag{41}
\end{equation*}
$$

Since we have two different functions for aggregate income depending on the relation between $M$ and $\mu$, we must calculate maximum income in both cases. Taking into account equations (15) and (30) for $M<\mu$ we have

$$
\begin{align*}
Y^{*} & =R+\bar{\omega} N+(\alpha-\beta)(M-\rho) \frac{R-\bar{\omega} M}{\beta M}-\bar{\omega} M \\
& =\frac{\alpha}{\beta} R+\bar{\omega} N-\frac{\alpha}{\beta} \bar{\omega} M-\frac{k+1}{\beta} \frac{R}{M}+\frac{k+1}{\beta} \bar{\omega} \tag{42}
\end{align*}
$$

In order to maximize (42) with respect to $M$ we calculate the first order condition.

$$
\begin{align*}
\frac{\partial Y^{*}}{\partial M} & =\frac{k+1}{\beta} \frac{R}{M^{2}}-\frac{\alpha}{\beta} \bar{\omega}=0 \\
& \Longleftrightarrow M^{2}=\frac{k+1}{\alpha} \frac{R}{\bar{\omega}} \tag{43}
\end{align*}
$$

The only positive solution is

$$
\begin{equation*}
M=\sqrt{\frac{k+1}{\alpha} \frac{R}{\bar{\omega}}} \tag{44}
\end{equation*}
$$

If $\mu>\sqrt{(k+1) R / \alpha \bar{\omega}}$ the maximum income is obtained for a concentration of land property rights such that landowners are richer than entrepreneurs. In this case from equations (42) and (44) we get the expression $\widehat{Y}^{*}$

$$
\begin{align*}
\widehat{Y}^{*} & =\frac{\alpha}{\beta} R+\bar{\omega} N+\frac{k+1}{\beta} \bar{\omega}-2 \frac{\sqrt{\alpha \bar{\omega}(k+1) R}}{\beta} \\
& =\bar{\omega} N+\left(\sqrt{\frac{\alpha}{\beta} R}-\sqrt{\frac{k+1}{\beta} \bar{\omega}}\right)^{2} \tag{45}
\end{align*}
$$

If instead $\mu \leq \sqrt{(k+1) R / \alpha \bar{\omega}}$ the maximum income is reached for a $M \geq \mu$. In this case, the value of $M$ which maximizes aggregate income must be calculated numerically since the function changes depending on the number of steps.

As we pointed out in the main body, maximum industrial employment $\widehat{L}_{I T}^{*}$ is obtained for $M$ in $[\rho, \mu]$. $A b$ absurdo consider that there is a distribution of land ownership such that $\widehat{L}_{I T}^{*}$ is obtained for $M>\mu$. A necessary condition for $M$ to maximize $L_{I T}^{*}$ when $M>\mu$ is that $\left(M+Q^{*}\right)$ does not increase in $M$, since i) $L_{I T}^{*}=N-L_{f}^{*}-L_{T T}^{*}-\left(M+Q^{*}\right)$, ii) $L_{T T}^{*}=0$ for $M \leq \mu$ and iii) $L_{T T}^{*} \geq 0$ for $M>\mu$. For $M=\mu$ industrial labour amounts to the units of labour necessary to produce the demanded units of industrial goods, hence

$$
\begin{equation*}
L_{I T}^{*}(\mu)=\beta Q_{\mu}^{*}\left(Q_{\mu}^{*}+\mu\right)+k Q_{\mu}^{*} \tag{46}
\end{equation*}
$$

where $Q_{\mu}^{*}\left(Q_{\mu}^{*}+\mu\right)$ is aggregate demand of manufactures. When $M>\mu$ the richest group of entrepreneurs receives demand by $\left(M+Q_{M}^{*}\right)$ individuals, but other industrialized markets will receive less. However we overestimate the aggregate industrial demand and, as before, we write industrial labour as

$$
\begin{equation*}
L_{I T}^{*}(M)=\beta Q_{M}^{*}\left(Q_{M}^{*}+M\right)+k Q_{M}^{*} \tag{47}
\end{equation*}
$$

We can say with certainty that (46) is greater then (47) since as $M$ increases $\left(M+Q^{*}\right)$ does not increase and $Q^{*}$ decreases. Therefore, $Q_{\mu}^{*}>Q_{M}^{*}$ for $M>\mu$ implying that it is impossible that $\widehat{L}_{I T}^{*}$ is obtained for $M>\mu$.

In the interval $[\rho, \mu]$ there is no market operating with the TT, therefore industrial employment is given by equation (14). In order to maximize (14) with respect to $M$ we derive the first order condition

$$
\frac{\partial L_{I T}^{*}}{\partial M}=0 \Longleftrightarrow M=\sqrt{\frac{R}{\alpha \bar{\omega}}}
$$

If $\sqrt{R /(\alpha \bar{\omega})} \leq \rho$, then maximal industrial employment is obtained for $M=\rho$ also maximizing the extent of industrialization. If $\sqrt{R /(\alpha \bar{\omega})} \geq \mu$, then $\widehat{L}_{I T}^{*}$ is obtained for $M=\mu$ which may also maximize aggregate
income. Finally if $\rho<\sqrt{R /(\alpha \bar{\omega})}<\mu$ the maximum industrial employment is obtained for a level of $M$ strictly lower than that which maximizes aggregate income, that is $\sqrt{(k+1) R / \alpha \bar{\omega}}$. In this last case

$$
\begin{equation*}
\widehat{L}_{I T}^{*}=\left(\sqrt{\frac{R}{\bar{\omega}}}-\sqrt{\frac{1}{\alpha}}\right)^{2} \tag{48}
\end{equation*}
$$

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[^0]:    ${ }^{1}$ This paper is also inspired by Fiaschi and Signorino [10] who analyse an extended version of Murphy et al. [20].
    ${ }^{2}$ see Murphy et al. [20] p. 538.
    ${ }^{3}$ See Deininger and Squire [8], p.260. Greater details about the data set can be found in Deininger and Squire [7].
    ${ }^{4}$ See Bénabou [3] and Bollettini and Ottaviano [4] for references to recent contributions on the general link between inequality and growth.

[^1]:    ${ }^{5}$ They provide empirical evidence from US in the period 1880-1920.
    ${ }^{6}$ More recently, Zweimüller [23] and Mani [18] try to consider explicitly the growth process by investigating how hierarchical demand influences technological progress.
    ${ }^{7}$ See Lucas [17].
    ${ }^{8}$ This latter difference is the effect of the land reform undertaken by the Government of South Korea in 1949 which took the name of Agricultural Land Reform Amendment Act (ALRAA). It consisted mostly of the redistribution of land previously owned by Japanese people. ALRAA reduced the number of tenants to nearly zero in a couple of years (see Jeon and Kim [14]).
    ${ }^{9}$ Chenery and Syrquin [6], Chenery et al. [5] provide further empirical evidence of the relevance of domestic demand for industrialization. Using a sample of rapidly growing economies they show that the expansion of domestic demand accounts for a large part of the increase of domestic income. For the biggest countries in their sample, domestic demand explains more than 70 percent of the increase of domestic income, while in small countries (under 20 million people) the percentage diminishes until a minimum of 50 percent. See also Murphy et al. [21] section II.

[^2]:    ${ }^{10}$ Differently from Rosenstein-Rodan [22], in this model there is no direct spillover accruing from industrialization. Indeed, only one equilibrium is determined for any given set of parameters and there is no coordination problem.

[^3]:    ${ }^{11}$ Murphy et al. [20] do not consider the existence of landowners as individuals: in their model, agricultural production - as the industrial one - is organized by firms which divide their profits among a certain number of shareholders.
    ${ }^{12}$ The qualitative results of our model can be obtained also by allowing for an increase in productivity due to the reduced dimension of land property. However, the analysis would become more complicated and would somehow obscure the mechanism we want to highlight.

[^4]:    ${ }^{13}$ If $w<\bar{\omega}$ no worker would supply labour as the wage she earns - unique source of income - would not be sufficient to survival; so the economy would not be viable.

[^5]:    ${ }^{14}$ Notice that being the manufacturing sector a continuum of markets, the consumers' income is always entirely spent.

[^6]:    ${ }^{15}$ The precise outcome depends on how profits are distributed among entrepreneurs. We investigate this issue in the following sections and in the Appendix.

[^7]:    ${ }^{16}$ Since $L_{f}^{*}<L$, in equilibrium we have

    $$
    M<L+M-L_{f}^{*} \Longleftrightarrow \bar{\omega}<\frac{(L+M) \bar{\omega}-L_{f}^{*} \bar{\omega}}{M} \Longleftrightarrow \bar{\omega}<\frac{F\left(L_{f}^{*}\right)-L_{f}^{*} \bar{\omega}}{M}=\frac{R}{M}
    $$

    ${ }^{17}$ From (1) and imposing $R / M>\bar{\omega}$ we obtain

    $$
    \frac{R}{M}=\frac{F\left(L_{f}^{*}\right)-w L_{f}^{*}}{M}=\frac{\bar{\omega}(M+L)-w L_{f}^{*}}{M}>\bar{\omega} \Longleftrightarrow \bar{\omega} L-w L_{f}^{*}>0
    $$

[^8]:    ${ }^{18}$ For completeness notice that by assuming $R / M \geq w$ we have ruled out the case of $\bar{\omega}<w=L \bar{\omega} / L_{f}^{*}$ where only workers demand manufactures. In such a case, $D_{q}=L$ for any $q$ in $\left[0, Q_{L}\right]$ and an extent of the manufacturing sector equal to $\bar{Q}=Q_{L}$.

[^9]:    ${ }^{19}$ In the Appendix we show that

[^10]:    ${ }^{20}$ The expression for $\Pi^{*}$ is quite complicated and adds very little by itself. It can be found in the Appendix.

[^11]:    ${ }^{21}$ Of course, there exists a level of $M$ such that $i^{*}=0$. In such a case no market beyond $Q_{R}$ industrializes and $Q^{*}=Q_{R}$ as for $M<\mu$; moreover, there are commodities produced with TT and the extent of the manufacturing sector $\bar{Q}$ is still equal to the extent of entrepreneurs demand (they earn the same profits).
    ${ }^{22}$ In the discontinuity points where a change in $I^{*}$ takes place we have that the richest group of entrepreneurs has not less than $\rho$ members so that $\bar{Q}=Q^{*}$ and production with TT disappears.
    ${ }^{23}$ Again, the expression for $\Pi^{*}$ is provided in the Appendix.

[^12]:    ${ }^{24}$ See the Appendix for more details.

[^13]:    ${ }^{25}$ When $M=\lambda$, the industrial employment $\hat{L}_{I T}^{*}$ is maximized and lies in between the maximum level of income and the maximum level of industrial extent.

[^14]:    ${ }^{26}$ When $M=\eta$, we have that $Q_{R}=\rho$. A higher $M$ would imply that the number of entrepreneurs which receives the demand of landowners would not be sufficient to industrialized markets beyond $Q_{R}$.

[^15]:    ${ }^{27}$ In this brief discussion it is assumed that all necessary conditions for producing tradables are met and that imports play no significant role. Moreover, we abstract from the extra labour force which may be needed to meet demand.
    ${ }^{28}$ For instance, measuring the area between the line identified by $\rho$ and the segment representing landowner demand (see Figure 1) does not work because it does not take into account the effects of new demand generated by the profits of the industrialized markets.

