Human Capital Dispersion and Incentives to Innovate

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Abstract

Do policies that alter the allocation of human capital across individuals affect the innovation capacity of an economy? To answer this question I extend Romer's growth model to allow for individual heterogeneity. I find that the value of an invention rises with equality. If skills and talents are evenly distributed, inventions are more widely adopted in production and users are willing to bid a higher price. Therefore more inequality is associated with a larger share of the population employed in the business of invention. But, somehow surprisingly, the analysis suggests that although an equal society values inventions more than an unequal one, it may produce fewer of them, or, equivalently, generates inventions of a lower quality. A calibration of the model suggests a weak, but positive, relationship between the rate of innovation and inequality.

Finally, in a two-country world, in which ideas, individuals, and capital circulate without restrictions, I find that the unequal economy tends to specialize into the business of innovation.

The main implication of the analysis is that an observed difference in the innovation rate between two countries with similar levels of education can hardly be attributed to variations in domestic human capital policies.

Keywords: human capital, inequality, innovation

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1   Introduction

Reforms of the education system are often dictated by the desire to foster cognitive abilities of students or by shifts in the notions of equality of opportunities and of social justice. For

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instance, some countries choose to track students into different school types, hierarchically structured by performance, as early as the age of ten (this is the case in Austria, Germany, Hungary, and the Slovak Republic), whereas others keep the entire lower secondary school system comprehensive or design some blend of the two systems. The choice of the structure of the education system affects the distribution of human capital. In an international comparative study, Hanushek and Wößmann (2006) found that grouping, for instance, increases the variance of pupils' attainments. The allocation of government funding among different levels of education is another, perhaps more direct, way in which educational policies affect the distribution of human capital. Castelló and Doménech (2002) documented an historical cross-country convergence of this type of inequality.

How do variations in the distribution of human capital affect a country's ability to innovate?

Policy makers seem to be quite attentive to news reporting increased gap in students achievement, although it is unlikely that their interest is driven by concern about the long-run performance of the economy. In 1982, an official inquiry by the Cockcroft Committee in England and Wales found that a seven year difference existed in pupils' mathematical attainments. This conclusion had fundamental consequences for the reorganization of the British school system. A National Curriculum established at the end of the 1980s set the target of containing 11-year-old pupils' attainment in the range of six-year span for 80 percent of the pupils[1]. The 2001 No Child Left Behind Act echoes similar concerns.

I will argue that any distributional change in human capital puts into motion forces affecting unavoidably both the demand and the supply of inventions[2]. The premise is that the more educated or talented individuals of a society supply inventions, which are used by producers to rend the production of a consumption good more efficient. The main insight of the analysis is that an increase in the dispersion of human capital causes a reduction in the value of innovations and in the number of inventors, but at the same time it also enhances inventors' productivity. In principle the innovation rate – which depends both on the number and quality of inventors – can be higher or lower in an economy with a more dispersed distribution, depending on the technical features of the sectors that produce and use inventions. Some calculations based on aggregate data suggest that institutional reforms – such as redistribution of resources from one educational level to another, or in the form of leveling pupils' achievements within a given level of education – are unlikely to alter significantly the innovation capacity of a closed economy, but do have major consequences

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[1] For a detailed discussion see Prais, 1993

[2] In the first part of the paper the terms *invention* and *innovation* will be used interchangeably, for whenever an idea is generated it always finds a use, and an improvement in output production is always based on some invention. The Schumpeterian distinction will become relevant only in sections (8) and (9) when a country can innovate by using foreign inventions.

on how much innovation is performed in an open economy.

To gain some intuition on the nature of the results, imagine that an individual, with human capital h, can produce a flow of output equal to $h^\alpha k^\beta$, where k denotes physical capital, and the parameters $\alpha + \beta < 1$ (diminishing returns). For a given price of capital, p, the (inverse) demand function of an individuals with human capital h is $p = \beta h^\alpha k^{\beta-1}$. In an economy with N individuals, each of whom is endowed with h_i for $i = 1, 2..N$, the total (direct) demand for capital will be $(p/\beta)^{1/(\beta-1)} \sum h_i^{\alpha/(1-\beta)}$.[3] In virtue of the diminishing returns, the total demand for capital is larger in the more equal economy. For instance consider two extreme cases one in which human capital is equally distributed and one in which it is concentrated in only one individual. In the former case the demand for capital is equal to $(p/\beta)^{1/(\beta-1)} N\bar{h}^{\alpha/(1-\beta)}$, where $\bar{h}$ is the average amount of human capital, whereas in the latter case it is $(p/\beta)^{1/(\beta-1)} (N\bar{h})^{\alpha/(1-\beta)}$. As long as there are diminishing returns this quantity is lower than the one computed for the equal economy. In the economy that I consider the price of capital is set by a monopolist who purchases a licence from an inventor granting him the exclusive right to produce machines. Therefore, in a more equal economy the capital producer expects to face a larger demand and earn bigger profits out of the use of the license. By the same token an inventor expects to receive more generous bids from capital producers in an equal economy. The descending curve of the first quadrant of Fig. (1) illustrates an hypothetical relationship between inequality and the market value of an invention. The fourth quadrant suggests that the higher the invention's value, the greater the number of people who want to be inventors (horizontal axis), for their rewards rise with it. But human capital inequality affects also inventors' productivity. The average human capital endowment of the top percentiles of the distribution is bigger in a society where human capital is spread out around the mean than one in which it is concentrated around (the same) mean. If inventors belong to the group of the more educated or talented individuals, their average productivity is then higher in a society where human capital is dispersed. The second quadrant of the figure illustrates such a relationship. In quadrant (III) two hypothetical 'innovation-isoquants' are plotted, A and B. Along an innovation-isoquant the rate of innovation is constant, whereas moving from one to another one below it (not shown) in the south-west area, would be equivalent to moving towards higher innovation rates – that is the rate of innovation increases both in the number of inventors and their quality. In the third quadrant the two dots x and y represent the combination of number of inventors and average quality of inventors associated with a high and low level of inequality, respectively. What is the shape of the isoquant? How can the two dots x and y be determined? The answer to these questions is one of the main concerns of the paper. If the correct isoquant

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[3]Let k_i bet the amount of capital demanded by individual i. The inverse demand function implies that $k_i = (\beta/p)^{1/(1-\beta)} h_i^{\alpha/(1-\beta)}$. Hence $\sum k_i = (\beta/p)^{1/(1-\beta)} \sum h_i^{\alpha/(1-\beta)}$.

Figure 1: Inequality and Innovation: A First Look
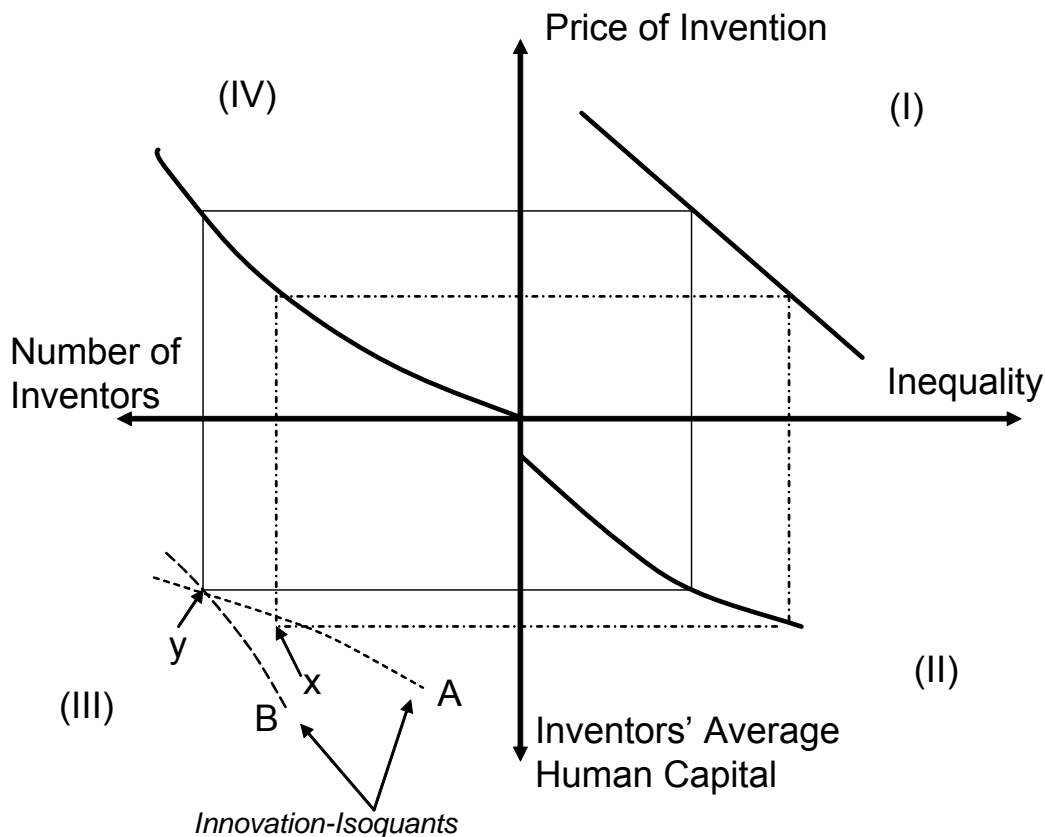
is A, then the point x lies on a lower (more towards south-west) isoquant, and therefore the unequal economy innovates at a faster pace. Conversely, if the correct isoquant is B the more equal economy is a better environment for spurring innovation. In sum, the more equal economy tends to have a higher invention's price and more inventors than the unequal one, but unfortunately the more pressing question – which one of the two economies innovates more – can be answered only after a more careful analysis of the forces behind the demand and supply of ideas.

Such analysis is the contribution of this paper. After proving the existence of a balanced growth path (BGP), I employ the calibration technique to study the long run performance of economy. One problem with calibrating the model is the absence of a consensus on what the distribution of human capital looks like. I circumvent the issue by comparing the simulated patterns of distributional changes in income, caused by shocks in the dispersion of human capital, with the actual US income distribution at different times.

The calibrated model suggests a modest but positive relationship between inequality and the innovation rate, in the order of 0.15% faster innovation as a response to a 0.1 increase in

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the income Gini coefficient (this ranges in the unit interval).[4] I then reconsider the effect of inequality on the innovation rate when ideas can freely move across countries, and these are identical but for the dispersion of human capital. The mechanisms described for the closed economy still hold, but give rise to a stronger positive relationship between inequality and innovation. More importantly, I find that the unequal economy tends to specialize in the innovation business, in the sense that the relative share of inventors and the relative supply of inventions are significantly larger than the ratio of the respective Gini coefficients.

The analysis is developed within the tradition of ideas-based models, as exemplified in Romer (1990), which I extend here to allow for individual heterogeneity. The growth literature has studied extensively the dynamic consequences of human capital. Lucas (1988) and Uzawa (1965) have hypothesized that the growth rate of the economy is driven by the accumulation of human capital, whereas Nelson and Phelps (1966) and Romer link such a rate to the stock of human capital, rather than its variation. However, this early literature has not addressed the question of how the distribution of human resources across different economic activities affects subsequent growth. Baumol (1990) marshalled a great variety of historical evidence showing that a society that allocates its best entrepreneurial talents into unproductive (and yet innovative) activities is likely to decline in the long run. His analysis suggests that linking the long run growth of the economy only to the overall supply of human capital, as it is done in the endogenous growth models, is unlikely to account for different historical experiences of countries that on average look similar.

A more detailed account of how this paper is linked to the literature is given in Section (2). Section (3) describes the extension of Romer's model, and proves the existence of an equilibrium for a generic human capital distribution. A first discussion of the links between human capital dispersion and innovation is provided in section (4) through three examples; in one Romer's key equilibrium condition is obtained as a special case; in the other two, such links are illustrated under alternative human capital distributions. Section (5) carries out the aggregation and characterizes the economy's balanced growth path. The calibration of the model is illustrated in section (7). The analysis is then extended in sections (8) and (9) to illustrate how the closed-economy results are modified in a world with free circulation of ideas. Some considerations on issues that remain open for further research are collected in section (10).

2   Review of the Literature

I draw from two different strands of literature. The motivation is built from studies that look at the trade-off between equality of opportunities and efficiency in promoting learning.

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[4]As it will be clarified in Section (2), this relationship is weaker than that found by Barro (2000).

But the paper's contribution is mostly to that long stream of studies that has explored the links between inequality, human capital, and economic growth.

There have been a variety of definitions of equality of opportunities. Atkinson (1980) concluded that there is a core agreement between philosophers and social scientists on its meaning: people's earnings should be strictly related to inborn abilities. His interpretation of this notion was not, however, that two individuals with the same innate abilities should earn the same, but rather that the 'ex-ante distribution of earnings is the same for all people with identical innate abilities' (p. 78), implying that the task of identifying discrimination, a situation in which earnings are systematically higher for people of one sex, race, or religious group, is quite daunting. I remain agnostic with respect to the issue of discrimination, for I am not concerned with the social circumstances that lead to variations in the distribution of human capital. It has been said that income segregation has been on the rise, and that this has led to a wider differences across schools' financial resources in places where education is locally financed. [5] Some argued that schools admission policies affect human capital distribution, as they have become better in tracking students by ability.[6] More recently the press has documented that an increasing fraction of both federal and state US financial aids to students is merit-based[7]. Clearly the school system can be a powerful instrument affecting equality of opportunities, for instance by providing better education to a selected group of students. As mentioned in the introduction some countries choose to track students according to performance, while others group them simply according to age. From a theoretical perspective it is not clear whether the peer effect that allow low ability students to learn from high ability students is strong enough to offset the potential loss of having homogenous classrooms, in which, arguably, a more focused curriculum and appropriately paced instruction are provided (Argys, Rees, and Brewer (1996); Dobbelsteen, Levin and Oosterbeek (2002)). But a large part of the empirical literature suggests that grouping increases the variance of pupils' attainments, although there is quite a lot of disagreement on whether it also enhances the average students' achievements. Recently Hanushek and

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[5]See Fernandez (2001) for a review of some of the most important contributions that study the links between sorting, education, and inequality.

For an explanation of why a decline in group inequality has not led to an equivalent reduction in the levels of segregation in cities with large minority population, see Sethi and Somanathan (2004)

[6]Herrnstein and Murray (1996, ch. 4) argue that sorting at the highest level of education became a widespread phenomenon sometimes in the late 1950s, when the finest college and university from being school for the local socioeconomic elite, became populated with some of the brightest minds attracted from all over the country.

[7]See June Kronholz. Wall Street Journal (Eastern edition). New York, N.Y.: Sep 23, 2002. p. B.1). The beginning of this new trend is usually associated with the legislation passed by the state of Georgia in 1993 when it launched the first state merit program, known as the HOPE scholarship, an attempt to reduce the flow of bright students to out-of-state colleges, and increase college attendance.

Wößmann (2006) found these results in an international comparison[8].

This paper is also related to the theoretical and empirical literature that has explored the links between inequality and growth. Three excellent papers surveying these links are Benabou (1996b), Aghion, Caroli, and Garcia-Peñalosa (1999), and Benabou (2004). The surprising evidence found by Perotti (1996), Alesina and Rodrik (1994), and Persson and Tabellini (1994), that equality and growth may go hand in hand spurred numerous works searching for explanations. Benabou (1996a), Durlauf (1996), Aghion and Bolton (1997), Piketty (1997), Benabou (2002), Galor and Moav (2004), building or extending previous works by Loury (1981), Evans and Jovanovic (1989), Banerjee and Newman (1993), and Galor and Zeira (1993), entertain the hypothesis that credit constraints limit the ability to invest in physical or human capital for people with little or no endowments. In presence of diminishing returns, redistribution (from rich to poor) causes an increase in the productivity of aggregate capital, and, as a result, the economy expands at a faster pace.[9] Similarly, I find that the value of an idea, which can be thought as the result of human capital investment, declines with inequality, and such decline is driven by the assumption of diminishing returns on capital (on a broad sense, physical and human capital). However, my conclusion is different from that of the credit-market imperfection literature, and it rather agrees with papers that predict a positive relationship between inequality and growth. Galor and Tsiddon (1997b) analyze specifically the link between human capital distribution and growth using a quite different argument than the one illustrated in this paper. Their main point is that at an early stage of development some level of educational inequality is not only beneficial but necessary for the economy to take-off. The knowledge acquired by a selected group of people generates a global externality, as it favors human capital accumulation in the rest of the population. In a related paper Galor and Tsiddon (1997a) conjectures that in a period of rapid technological changes, the human capital of parents becomes less important, social mobility increases and a there is more concentration of high-ability workers in technologically advanced sectors. This leads to a temporary increase in inequality during the dissemination phase of the major technological innovation, although, once the knowledge of the new technologies is spread, the initial conditions become important again, and inequality decreases, although it becomes more persistent. Their analysis focuses on the role of intergenerational

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[8]Other studies have followed different approaches. A selection of empirical papers focused on the UK school system can be found in Heath (1984). For the US see Slavin (1990). Meghir and Palme (2005) discuss reforms in Sweden and other European countries. It should be noticed that Figlio and Page (2002) are in disagreement with most of the literature that grouping raises the variance of pupils' achievements.

[9]This reasoning turns on its head Keynes's investment indivisibilities argument contained in the *Economic Consequences of the Peace* (1920). He held that the immense accumulation of capital at the turn of the century could have not be made with an equal distribution of wealth, for large investment projects could be carried out only because vast amount of resources were controlled by a small group of people.

mobility as the main driving force of inequality and growth acceleration, while my model emphasizes the allocation of human resources between production and invention activities. Also Saint-Paul and Verdier (1993) reach a similar conclusion but using a different type of argument. He hypothesizes that in unequal societies, the median voter, whose income is below the average, tends to elect representatives that favor high rates of taxation to finance spending in public education. This will raise human capital and hence growth. More recently, Aghion, Meghir, and Vandenbussche (2005) addressed the question of how the human capital distribution influences economic growth. They argue that the effect of education on growth is best understood by considering at the same time the features of the educational system – the relative weights of various levels of education – and the distance of the country from the technological frontier. They suggest that in countries close to the frontier, growth is mostly driven by tertiary education, whereas in countries far away from the frontier, primary and secondary education should play a central role in the expansion of the economy, as this is what is needed to adopt foreign technologies. Although they do not discuss the matter of income inequality, it is likely that such an extension would predict an increase in inequality as the country moves closer and closer to the technological frontier, because this movement is associated with a rising fraction of people gaining better education, and hence higher wages.

The panel data estimations performed by Kristin Forbes (2000) and by Robert Barro (2000) bring support to the theoretical results of a positive association between inequality and growth. Forbes, who extended Perotti (1996) estimation by including regional and time fixed-effects, estimates a surprisingly high value of 1.3 for the coefficient associated with the Gini value – that is, she predicts that an increase of the Gini coefficient of 0.1 raises the average annual growth rate in the subsequent 5-year period of 1.3 percent. Barro included in the panel regressions variables omitted in Perotti's and Forbes's regressions, such as terms of trade, the investment ratio, as well as rule-of-law and democracy indexes. He found a more modest positive correlation of 0.5 between growth and inequality (still measured by the Gini coefficient) for countries with a per capita GDP above $2070 (in 1985 US dollars) and a negative one of about the same magnitude for the remaining countries. Banerjee and Duflo (2003) attribute the conflicting conclusions reached by the empirical literature on the sign of the relationship between inequality and growth to the fact that this literature has imposed a linear structure. If the actual relationship is not linear – and they found that it is not – different variants of a linear specification, they argue, are likely to deliver a different sign for the estimated coefficient. Motivated by this evidence Bandyopadhyay and Basu (2005) provide a model and calibrate it using cross-country inequality data that reproduces the non-linear relationship between growth and inequality.

3 The Basic Model

The model that I propose is an extension of the well known Romer's R&D growth model, in which innovation activities are carried out by profit maximizing individuals and come in the form of an expansion in the variety of capital goods employed for the production of a final output. First I briefly summarize the main features of the theoretical framework and then illustrate the departures from it.

The economy consists of three sectors. One produces a good that can be either consumed or used in the manufacture of durable goods. Such a good is produced with the help of labor, human capital, and a number of durable goods that expands over time as more designs are created. Only labor and human capital are used in the creative process. In addition it is assumed that the creative process is facilitated by the positive spillovers of accumulated knowledge, and this is measured by the number of existing designs. Users bid up the renting or purchasing price of the design up to the point in which it is equal to the overall monopoly rent that the use of the design in production is expected to generate in the future. Spillovers are allowed in the phase in which blueprints are generated, but not when they are used in production.

In Romer (1990) individuals are endowed with the same amount of human capital. The objective of this section is to study the consequences of relaxing this assumption.

The economy is populated by infinitely-lived individuals of measure 1. Each individual is endowed with one unit of time, used either to produce a final good or to generate inventions. An individual employed in the final good sector operates a firm that produces a flow of output with the help of N durable goods according to the technology

$$y(h) = zh^{\alpha} \int_0^N q(i)^{\beta} di, \tag{1}$$

where $q(i)$ denotes the quantity of a durable good $i \in [0, N]$, h indicates the level of human capital of the individual who runs the firm, and α, β, and z are positive parameters.

The chief advantage of specifying the output function as in Eq. (1) is that the marginal productivity of durable good i does not depend on that of durable good j (for $i \neq j$), a feature that greatly simplifies the derivation of the demand function for durable goods. This production function is similar to the one proposed by Ethier (1982), Spence (1976), Dixit and Stiglitz (1977), and Romer (1990), except that here each individual employed in production runs his own firm, a departure dictated by the desire of relating the analytical results of this paper to the literature that studies the links between income growth and inequality under the assumption that agents have a limited access to the credit market.

Profit maximization leads to the optimal condition

$$z(h)^{\alpha} \beta q(i)^{\beta-1} = p(i), \tag{2}$$

where $p(i)$ is the price of one unit of intermediate input i. The previous expression can also be written as a (direct) demand function:

$$q(i) = (\frac{z\beta}{p(i)})^{1/(1-\beta)}(h)^{\alpha/(1-\beta)}. \tag{3}$$

3.1 The monopoly price

The market demand function for durable good i is derived by summing up all individual demand curves, which yields the revenue of the intermediate good producer i. Integrating equation (3) with respect to h, we get

$$X(i) = p(i)(\frac{z\beta}{p(i)})^{1/(1-\beta)}\int_0^{+\infty}(h)^{\alpha/(1-\beta)}d\tilde{F}(h), \tag{4}$$

where $\tilde{F}(h)$, yet to be determined, denotes the number of final good producers with a level of human capital equal or less than h. One unit of durable good i is obtained from η units of forgone consumption that can be rented at rate r. Then the flow of marginal cost of producing one unit of durable good i is the interest payments $r\eta$. It is easy to verify that the non-discriminatory monopoly rental price is the same for all durable goods, that is $p(i) = \bar{p} = r\eta/\beta$. A design can extract in every period the following rent:

$$\pi_t = (1-\beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r_t\eta})^{\frac{\beta}{(1-\beta)}}\int_0^{\infty}h^{\alpha/(1-\beta)}d\tilde{F}(h). \tag{5}$$

Therefore the value of a new design created at time t is the discounted stream of profits that the intermediate good producer can expect to gain from renting the durable good to a final good producer, that is

$$P_t = \int_t^{t+T}\pi_s e^{-R(t,s)}ds,$$

where π_s is defined in equation (5), T is the length of time during which the inventor extracts a rent from the blueprint, $R(t,s) = \int_t^s r_v dv$, and $r(v)$ is the instantaneous discount rate at time v.

If the interest rate is constant (a circumstance that will be verified in the equilibrium described below), the value of a new blue print is

$$P = \frac{1}{r}(1-e^{-rT})(1-\beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r\eta})^{\frac{\beta}{(1-\beta)}}\int_0^{\infty}h^{\alpha/(1-\beta)}d\tilde{F}(h). \tag{6}$$

In order to determine the allocation of individuals between output production and the the innovation sector, a reward function for the inventors must be specified.

Someone with human capital h can produce a flow of ideas equal to $\delta(h)^{\phi}N$, where δ is a productivity parameter and N is the number of existing designs. The underlying assumption

is that existing designs provide useful knowledge in elaborating a new design, and that such knowledge is accessible at not cost. The parameter $\phi > 0$ is the elasticity of the flow of ideas with respect to human capital. Thus the flow of income for an inventor with human capital h is

$$w_I(h) = \delta(h)^\phi NP. \tag{7}$$

3.2 Labor market equilibrium

A final good producer will choose an amount of durable goods according to equation (3) with $p(i)$ replaced by $r\eta/\beta$, if the design still commands a monopoly rent, and by the marginal cost $r\eta$ otherwise. Let $\bar{q}^m(h)$ and $\bar{q}^c(h)$ be the resulting demand function under the two circumstances, respectively. Let M measure the number of old vintages rented at competitive price. The flow of output produced by an individual with skills h employed in the final good sector then is

$$y(h) = zh^\alpha[M(\bar{q}^c(h))^\beta + (N - M)(\bar{q}^m(h))^\beta],$$

which can be rearranged as

$$y(h) = z^{\frac{1}{1-\beta}} h^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r\eta}\right)^{\beta/(1-\beta)} N[m + (1-m)(\beta)^{\beta/(1-\beta)}], \tag{8}$$

where $m = M/N$. One can show that the income of an individual h employed in the final good sector, denoted with $w_y(h)$, is simply $(1-\beta)y(h)$. Using Eqs (8) and (3) this expression becomes

$$w_y(h) = \chi N h^{\frac{\alpha}{1-\beta}}, \tag{9}$$

where $\chi = (1 - \beta)[m + (\beta)^{\beta/(1-\beta)}(1 - m)]z^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}$ – with m a variable to be determined.

At this point the all the elements needed to state a condition that indicates how people sort themselves between the occupation of inventors and final-good producers have been discussed. But the condition will crucially depend on the relationship between ϕ and $\frac{\alpha}{1-\beta}$. Hence, the following assumption is made.

(A1) The elasticity of ideas production with respect to human capital, ϕ, is larger than the ratio $\frac{\alpha}{1-\beta}$.

Lemma 1 *Under assumption (A1), only people with the highest level of human capital are engaged in invention activities.*

Proof. Let an individual with human capital $\bar{h}$ be indifferent between being employed as an inventor or as a final good producer. Then $\bar{h}$ must be that value so that $w_I(\bar{h}) = w_y(\bar{h})$, or equivalently

$$\delta\bar{h}^\phi\bar{P} = \bar{\chi}\bar{h}^{\frac{\alpha}{1-\beta}}, \tag{10}$$

where a bar on P and χ indicates the value of these variables when the indifferent individual is $h = \bar{h}$. The above equality implies that $\bar{P} = \frac{1}{\delta}\bar{\chi}\bar{h}^{\frac{\alpha}{1-\beta}-\phi}$. Let individual L be endowed with a level of human capital $h_L < \bar{h}$. Eqs. (7) and (10) imply that $L's$ wage is $w_I(h_L) = \delta(h_L)^\phi N\frac{1}{\delta}\bar{\chi}\bar{h}^{\frac{\alpha}{1-\beta}-\phi}$ as an inventor, whereas, according to Eq. (9) L would earn $w_y(h_L) = \bar{\chi}N(h_L)^{\frac{\alpha}{1-\beta}}$ as a final good producer. The ratio of the two types of income is $(\bar{h}/h_L)^{\frac{\alpha}{1-\beta}-\phi}$. Since $\frac{\alpha}{1-\beta} < \phi$ under (A1), $w_I(\bar{h}_L) < w_y(\bar{h}_L)$. Likewise, it can be proved that $w_I(h_H) > w_y(h_H)$, where $h_H > \bar{h}$. Hence the claim of the lemma. ∎

Next I show how to obtain $\bar{h}$, for a given interest rate and under the condition (A1)[10]. Let $F(h)$ be the cumulative distribution function over h, and let (S1) be the following system of equations:

$$\bar{P} = \frac{1}{\delta}\bar{\chi}\bar{h}^{\frac{\alpha}{1-\beta}-\phi}, \tag{11}$$

$$\bar{P} = \frac{1}{r}(1 - e^{-rT})(1-\beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r\eta})^{\frac{\beta}{(1-\beta)}}\int_0^{\bar{h}} h^{\alpha/(1-\beta)}dF(h) \tag{12}$$

$$\bar{g} = \delta\int_{\bar{h}}^\infty h^\phi dF(h) \tag{13}$$

$$\bar{\chi} = [\exp(-\bar{g}T) + (\beta)^{\beta/(1-\beta)}(1 - \exp(-\bar{g}T))](1-\beta)z^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}. \tag{14}$$

Proposition 1 *Under the assumption (A1) there exists an $\bar{h}$ which solves the system (S1).*

Proof. Let the functions $\Pi(\hat{h}), P(\hat{h}), g(\hat{h})$ be defined by the right-hand side of equations (11), (12) and (13), respectively, with the variable $\bar{h}$ being replaced by $\hat{h}$, and let the function $\chi(g(\hat{h}))$, be defined by the right side of equation (14), with $\bar{g}$ being replaced by $g(\hat{h})$. In order to find an equilibrium I study, in order, the behavior of $P(\hat{h})$ and that of the product of $\hat{h}^{\frac{\alpha}{1-\beta}-\phi}$ with $\frac{1}{\delta}\chi(g(\hat{h}))$. By inspecting the right side of equation (12) one realizes that: i)$P'(\hat{h}) > 0$, ii) $\lim_{\hat{h}\to 0} P(\hat{h}) = 0$, and the $\lim_{\hat{h}\to+\infty} P(\hat{h}) = C$, where $C = \frac{1}{r}(1 - e^{-rT})(1-\beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r\eta})^{\frac{\beta}{(1-\beta)}}\int_0^{+\infty} h^{\alpha/(1-\beta)}dF(h)$. It remains to be verified that the function $\Pi(\hat{h}) = \frac{1}{\delta}\chi(g(\hat{h}))\hat{h}^{\frac{\alpha}{1-\beta}-\phi}$ crosses $P(\hat{h})$ for some $\hat{h}$. The value of the expression $\Pi(\hat{h})$ at the extremes of the support of the cdf $F(h)$ is best understood by verifying that of $\chi(g(\hat{h}))$ and $\hat{h}^{\frac{\alpha}{1-\beta}-\phi}$, since it is the product of these two expressions divided by δ.

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[10]In principle one can determine $\bar{h}$ under the more general condition that $\frac{\alpha}{1-\beta} \neq \phi$. however I focus the attention on the case in which (A1) applies, as such a restriction will be imposed on the equilibria to be discussed in the coming sections.

Notice that: i) $\lim_{\hat{h}\to 0} \hat{h}^{\frac{\alpha}{1-\beta}-\phi} = +\infty$; ii) the $\lim_{\hat{h}\to\infty} \hat{h}^{\frac{\alpha}{1-\beta}-\phi} = 0$; iii) $\lim_{\hat{h}\to 0} \chi(g(\hat{h})) = (\beta)^{\beta/(1-\beta)}(1-\beta)z^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}$; iv) $\lim_{\hat{h}\to\infty} \chi(g(\hat{h})) = (1-\beta)z^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}$. Therefore, $\lim_{\hat{h}\to+\infty} \Pi(\hat{h}) = 0$ and $\lim_{\hat{h}\to 0} \Pi(\hat{h}) = +\infty$. Because both $P(\hat{h})$ and $\Pi(\hat{h})$ are continuous there must be a value $\hat{h} = \bar{h}$ where $P(\bar{h})$ and $\Pi(\bar{h})$. Hence the claim. ∎

By combining Eqs. (11), (12), and (14) the labor market equilibrium condition can be restated as

$$\underbrace{\bar{h}^\phi \delta \frac{1}{r}(1-e^{-rT})(\beta)^{1/(1-\beta)} \int_0^{\bar{h}} h^{\alpha/(1-\beta)} dF(h)}_{\approx P(\bar{h}) \text{ [i.e. the value of a design]}} = [\exp(-\bar{g}T) + (\beta)^{\beta/(1-\beta)}(1-\exp(-\bar{g}T))]\bar{h}^{\frac{\alpha}{1-\beta}},$$

(15)

where the left-hand side is $w_I(\bar{h})/N$ and the right-hand side $w_y(\bar{h})/N$. This is a key equation of the model. Notice that it does not depend on the parameters η or on z, (for the same general equilibrium considerations that in Romer's model the corresponding equilibrium equation is not affected by the technological parameters –see Romer (1990), S93).

4   Discussion: Human Capital Inequality and the Rate of Innovation

The link between inequality and innovation will be illustrated by starting from the extreme case in which all individuals are identical, as in Romer (1990). Then the scenario is slightly modified to allow for only two types of people, one of whom has a higher level of human capital than the other. In a third scenario the distribution is uniform in given interval. Interestingly, most of the insights emerge already when moving from homogenous individuals to two-types of individuals. Before proceeding I must note that although in all the three scenarios the interest rate is exogenously given[11], none of the qualitative results that will emerge are affected by this assumption.

4.0.1   Example 1: Romer as a special case

Consider a degenerate frequency distribution $f(h)$ with a mass of probability one at $h = H$, and let $T \to +\infty$. The equilibrium equation in Eq. (15) implies that

$$\frac{1}{r}H^{\frac{\alpha}{1-\beta}}(1-l_F) = \frac{1}{\delta\beta}(H)^{\frac{\alpha}{1-\beta}-\phi},$$

(16)

where l is the number of inventors. The previous equation leads to

$$l = 1 - \frac{1}{\tau\delta\beta H^\phi}.$$

(17)

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[11] The determination of the interest rate requires the definition and characterization of a balance growth path, which will be done in section (5).

Following Eq. (13) the rate of innovation is

$$g = \delta H^{\phi} - \frac{r}{\delta \beta}. \tag{18}$$

which is the same as the key equation in Romer (1990) – that is equation (11') at pag. S92– provided that $\phi = 1$.

4.0.2 Example 2: Two Types of Individuals of Equal Size

The economy is populated by two types of individuals. Let H_1 and H_2 denote the per-capital human capital of individual of type-1 and type-2, where $H_1 = H(1 - \varepsilon)$, $H_2 = H(1 + \varepsilon)$.and $\epsilon > 0$.The population is equally split between the two types of individuals. From Lemma 1 we know that the most skilled individuals are inventors. Let l_2 be the fraction of type-2 individual who are inventors. Then the labor market equilibrium condition is either

$$\tau[\frac{1}{2}H_1^{\frac{\alpha}{1-\beta}} + \frac{1}{2}H_2^{\frac{\alpha}{1-\beta}}(1 - l_2)] = \frac{1}{\delta \beta}(H_2)^{\frac{\alpha}{1-\beta} - \phi}, \tag{19}$$

if the parameters of the model imply $l_2 < 1/2$, or

$$\tau \frac{1}{2}H_1^{\frac{\alpha}{1-\beta}}(1 - l_{1,G}) = \frac{1}{\delta \beta}(H_1)^{\frac{\alpha}{1-\beta} - \phi}, \tag{20}$$

otherwise. Since the number of inventors is likely to be less than half of the population I continue the exposition based on Eq. (19). This implies that

$$\frac{1}{2}l_2 = \frac{1}{2} - [\frac{1}{\tau \delta \beta H_2^{\phi}} - \frac{1}{2}(\frac{1}{\gamma})^{\frac{\alpha}{1-\beta}}], \tag{21}$$

where $\gamma = \frac{1+\epsilon}{1-\epsilon}$, is a measure of human capital inequality. This equation suggests that higher inequality produces two opposite effects on the share of individuals engaged in innovative activity. On the one side type-2 individuals are more skilled and therefore find it more attractive to be inventors – first term inside the square brackets. On the other side higher inequality reduces the demand for intermediate products, and consequently both the value of an innovation and the inventor's reward decline. In Fig. (2.A), where $\frac{1}{2}l_2$ is plotted against ϵ, the former effect dominates the latter one for small departures from an equal situation (small ϵ). But as the economy becomes more unequal a smaller share of population choose to be employed in innovative activities.[12] Therefore an inverted-U shape curve may emerge. The intuition is that for low level of inequality more dispersion induces some type-2 individuals

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[12]One can easly study how variations in the parameters influence the behavior of the plot. From Eq. (21) we get

$$\frac{1}{2}l_{2,G} = \frac{1}{2} + \frac{1}{2}\gamma^{\frac{\alpha}{1-\beta}} - \frac{1}{\delta \beta \tau H_2^{\phi}},$$

to move into the invention business, even if the demand for innovation shrinks, because the productivity differential effect $(h^{\phi-\frac{\alpha}{1-\beta}})$ prevails.

The blueprint's value. Eq. (6) can be rearranged as:

$$P = \tau(1-\beta)\beta z^{\frac{1}{(1-\beta)}}\left(\frac{1}{r\eta}\right)^{\frac{\beta}{(1-\beta)}} H^{\frac{\alpha}{1-\beta}}\left[\underbrace{\frac{1}{2}(1-\epsilon)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}}}_{\text{inequality}} - l_2\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}}\right], \qquad (22)$$

which indicates that a wider dispersion of human capital affects the value of innovation in two ways:

a) The first two terms inside the square brackets capture the negative effect of inequality – total output declines with inequality, and therefore the rent of the inventor goes down as well. The descending curve in Fig. (2.B) illustrates this aspect.

b) The term $\frac{1}{2}l_2$, enters with a negative sign to emphasize that the inventors are a group of people that in principle could be employed into the final output sector. Therefore the larger the level of human capital of type-2 individuals, the larger the potential market loss for new designs, the lower the price of a design. The inverted U-shaped curve in Fig. (2.B) plots the last term of the square brackets $(l_2\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}})$. It closely traces the behavior of the plot in panel A. The overall effect of ϵ on P is highlighted by the plot in panel D. For the set of parameters in use the 'inequality' effect dominates, so that a larger ϵ is associated with a lower blueprint's value.

Finally, the growth rate of innovation is given by

$$g = \delta H_2^{\phi}\frac{1}{2}l_2.$$

Since $\frac{1}{2}l_2$ declines in ϵ, at least when this is large enough – see plot A–, and since H_2 is always increasing ϵ, the rate of innovation may go either way when inequality rises: The quality of inventors (H_2) is higher, but inequality reduces the design's value and some inventors abandon the innovation business. To understand how these two competing forces operate, it is useful to rearrange the above expression as

$$g = \delta H^{\phi}(1+\epsilon)^{\phi-\frac{\alpha}{1-\beta}}\left[\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1-\epsilon)^{\frac{\alpha}{1-\beta}}\right] - \frac{1}{\beta\tau} \qquad (23)$$

where $\gamma = H_1/H_2$. The partial derivative of the right-hand side this equation with respect to ϵ is

$$\frac{\alpha}{1-\beta}\frac{1}{2}\gamma^{\frac{\alpha}{1-\beta}-1}\gamma' + \phi\frac{1}{\delta\beta\tau H^{\phi}(1+\epsilon)^{\phi+1}},$$

where γ' indicates the partial of γ with respect to ϵ; the partial is approximately equal to -2. Therefore, $\frac{1}{2}l_{2,G}(\epsilon)$ is increasing in ϵ when this assumes relatively low values, that is in the region that approximately satisfies the condition

$$\frac{\delta\beta\tau H^{\phi}(1+\epsilon)^{\phi+1}}{[(1-\epsilon)/(1+\epsilon)]^{1-\frac{\alpha}{1-\beta}}} < \frac{\phi}{\frac{\alpha}{1-\beta}}$$

and is decreasing otherwise.

The negative effect of inequality is accounted by the expression in the square bracket, whereas the positive effect is captured by the term in front of the square bracket. Clearly if there are constant return to scale in the final good sector ($\frac{\alpha}{1-\beta} = 1$), a wider dispersion in human capital has always a positive effect on the rate of innovation under the assumption that $\phi > \frac{\alpha}{1-\beta}$. But if $\frac{\alpha}{1-\beta} < 1$, the innovation rate can go up or down, depending on the parameter values. (One can verify that the partial derivative of the right-hand side of Eq. (23) with respect to ϵ is positive for $\frac{2(1+\epsilon)}{[(1-2\epsilon)^{1-\alpha}+(1-2\epsilon)]} < \frac{\phi}{\frac{\alpha}{1-\beta}}$, approximately, and negative otherwise). Intuitively, as we move away from a situation of complete equality, the reduction in the design's value caused by the rising inequality is small and only few innovators, if any, switch activity to become final good producers. The better quality of inventors is the prevailing force and therefore the result is a higher innovation rate. Conversely, as inequality becomes more pronounced, the reduction in the number of inventors is so substantial that it cannot be compensated by the higher productivity of the residual inventors. Fig(2.C), which plots g_G against ε, shows the inverted-U relationship between the innovation rate and inequality just described.

To sum up, a relatively small departure from a situation of equality does induce more innovation whereas large deviations from it generate the opposite result. The intuition is quite straightforward: at first the 'quality' effect – researchers are endowed with higher human capital – more than compensate the 'quantity effect' – a reduction in the size of the R&D sector– if any. But as ϵ increases the 'quantity' effects kicks in and it more than offsets the higher per-innovator yielding, bringing down the innovation rate. Finally, notice that the level of inequality that maximizes the rate of innovation does not depend on the average level of human capital. Indeed, finding the optimal value of ϵ in Eq. (23) is equivalent at finding the optimal value of the expression $(1+\epsilon)^{\phi - \frac{\alpha}{1-\beta}}[\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1-\epsilon)^{\frac{\alpha}{1-\beta}}]$ which depends only on the parameters of the elasticities ϕ, α, and β.

### 4.0.3 Example 3: Uniform Distribution

The *pdf* of human capital is uniform in an interval of length d with mean equal to μ. The extremes of the support of the distribution then are $\mu - d/2$ and $\mu + d/2$, where d conveniently measures how dispersed the distribution is around the mean. Under these circumstances, the equilibrium equation (15), for $T \rightarrow +\infty$, becomes

$$\frac{\beta}{r}\frac{1}{d}\int_{\mu-d/2}^{\bar{h}} h^{\alpha/(1-\beta)}dh = \frac{1}{\delta}\bar{h}^{\frac{\alpha}{1-\beta}-\phi}, \tag{24}$$

the population share of inventors is $1/2 + \frac{1}{d}(\mu - \bar{h})$, whereas the rate of innovation is equal to $\frac{\delta}{d}\int_{\bar{h}}^{\mu+d/2} h^\phi dh$, or

$$\bar{g} = \frac{\delta}{d(1+\phi)}[(\mu + d/2)^{1+\phi} - \bar{h}^{1+\phi}].$$

16

Figure 2: High vs. Low Skills
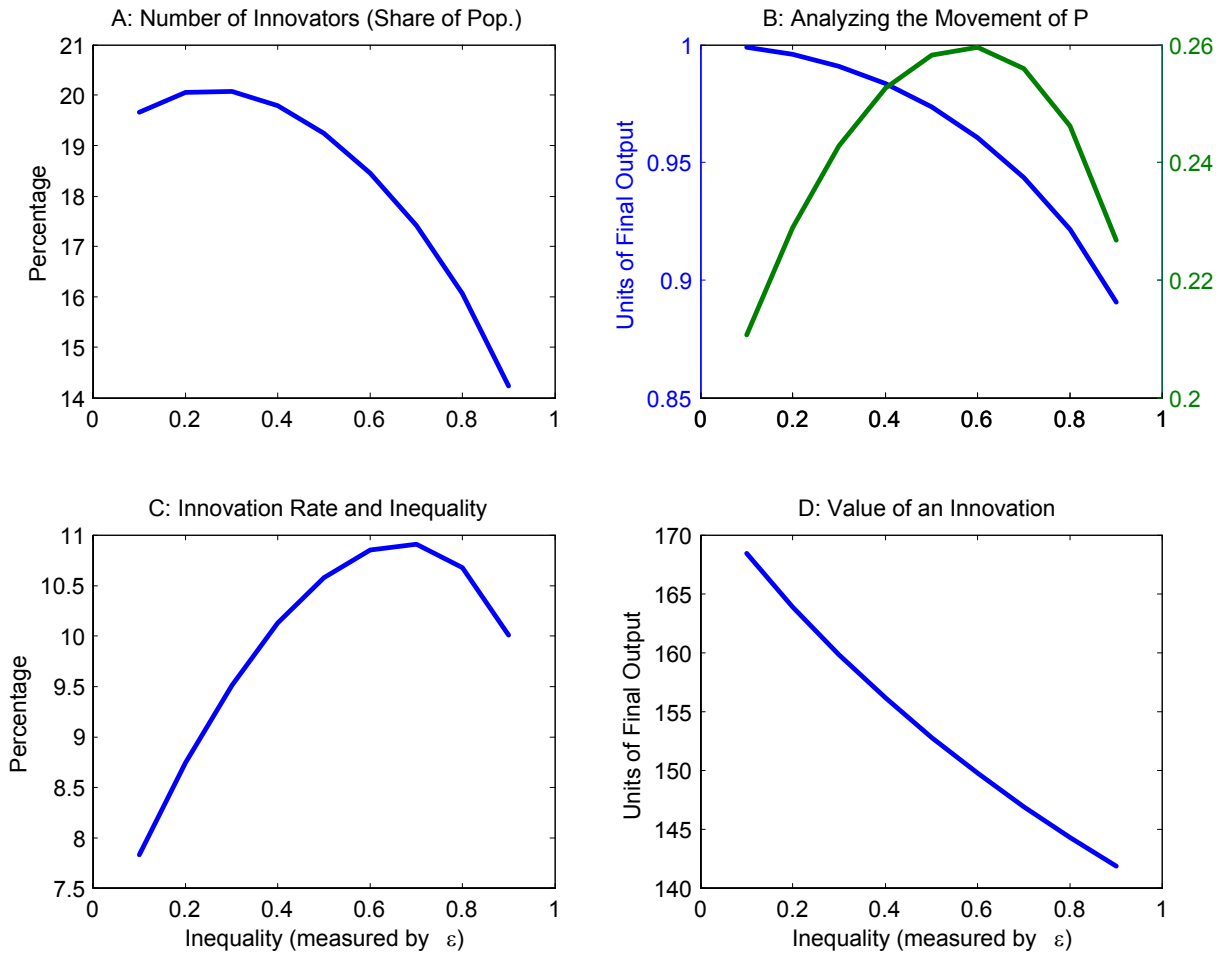
Parameters: $H = 10; \beta = 0.27; \alpha = 0.53; \delta = 0.003; \phi = 1.04; \eta = z = 1.$

Finally, the value of an innovation, following Eq. (6), is

$$P = \frac{1}{r}(1 - e^{-rT})(1 - \beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r\eta})^{\frac{\beta}{(1-\beta)}}\frac{1}{d}\int_{\mu-d/2}^{\bar{h}} h^{\alpha/(1-\beta)}dh.$$

In all above expressions $\bar{h}$ satisfies Eq. (24). Fig (3) proposes a similar set of plots as Fig. (2) except that this time the variable capturing inequality (running on the horizontal axis) is the parameter d. Plot (A) confirms that after a certain threshold, increasing levels of inequality cause a reduction in the share of population employed as inventors. The same line of reasoning developed in the previous example can be applied now to explain the inverted-U shape of the curve in plot A. Inequality has an adverse effect on the design's price, inducing people to leave the business of innovation, but it also makes more marked the productivity advantage of an inventor *vis-à-vis* an output producer's. Indeed if the difference between ϕ and $\alpha/(1 - \beta)$, increases, the maximizer of the plot in Panel (A) moves to the right. Conversely as $\phi - \alpha/(1 - \beta)$ gets smaller a larger section of the plot becomes descending, because the design's price effect becomes the dominant force in the individuals' occupational choice. However, for the given set of parameters, the decline in the number of inventors is not big enough to compensate the innovators' increased productivity, which eventually is the prevailing force in determining the innovation rate.

Conclusion 1 *Variations in human capital inequality influence individuals' occupational choices between innovative and non-innovative activities and the innovation rate of the economy. However the intervening mechanisms that link changes in the dispersion of human capital and innovation activities operate in an ambiguous way. More human capital inequality per se reduces the value of an invention, for the demand of the intermediate products embodying a given invention declines. Therefore the would-be inventors are more likely to turn their attention to output production if the expected rent that they can extract from the invention is small. But on the other side inventors' productivity is higher in an unequal economy as they receive a finer education.*
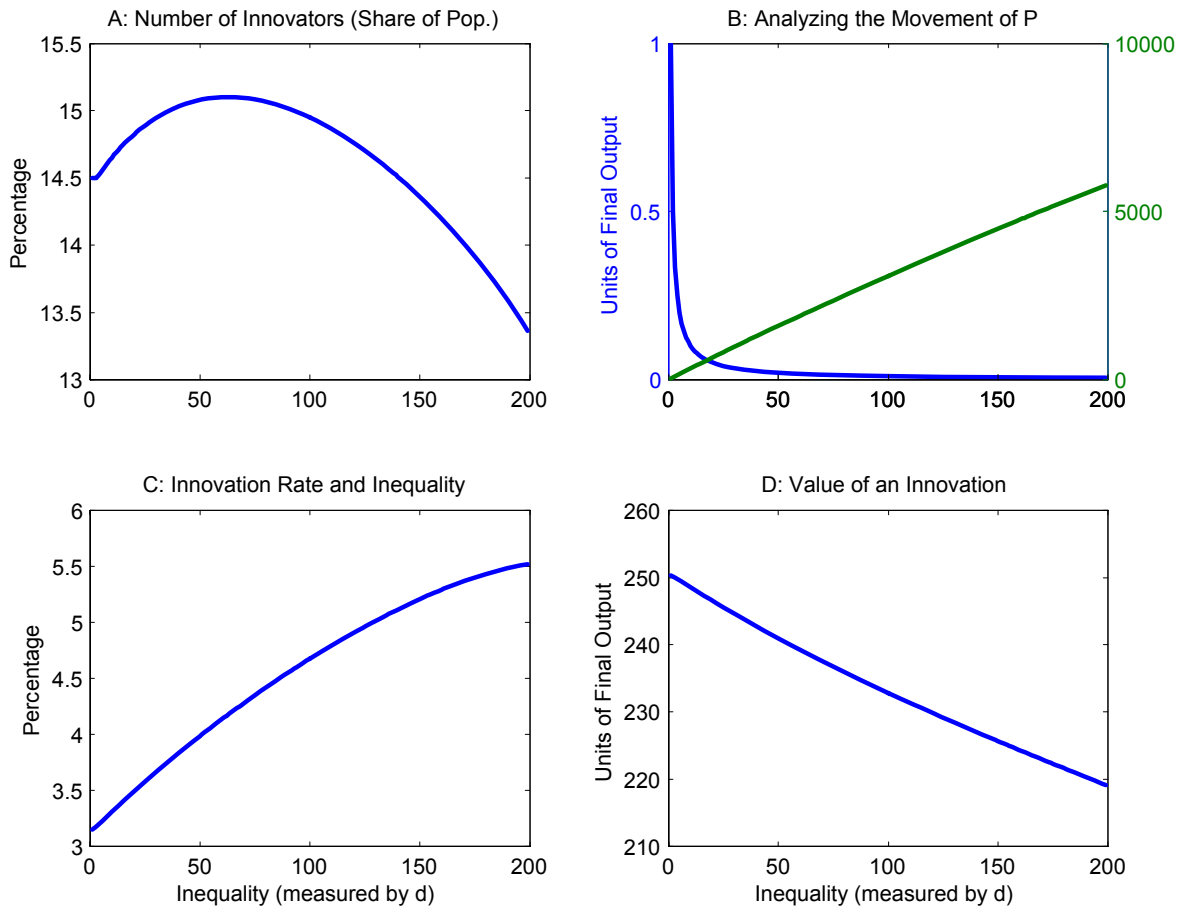
Section (7) attempts to solve the ambiguities that emerged in the previous two examples by calibrating the model on the balanced growth path.

5 Balanced Growth Path

5.1 Production Side

The aggregate stock of capital can be computed by summing up all the intermediate goods in use and multiplying the resulting quantity by η so that it is expressed in terms of units of consumption goods (recall that η is the quantity of consumption goods required to build on unit

Figure 3: Uniform Distribution

A: Number of Innovators (Share of Pop.)

B: Analyzing the Movement of P

C: Innovation Rate and Inequality

D: Value of an Innovation

Parameters: $H = 100; \beta = 0.27; \alpha = 0.58; \delta = 0.0018; \phi = 1.04; \eta = z = 1.$

of capital). Integrating Eq. (3) with respect to h in the interval $[0, \bar{h}]$ one computes the overall demand for one type of intermediate goods, which is equal to $(\frac{z\beta}{p(i)})^{1/(1-\beta)} \int_0^{\bar{h}} (h)^{\alpha/(1-\beta)} dF(h)$. There are N intermediate goods, of which M are priced at the marginal cost $(p(i) = r\eta)$ and the remaining $N - M$ at the monopoly price $r\eta/r$. Hence

$$K = \eta(\frac{z\beta}{r\eta})^{1/(1-\beta)} N[m + (1-m)(\beta)^{1/(1-\beta)}] \int_0^{\bar{h}} h^{\alpha/(1-\beta)} dF(h). \tag{25}$$

where $m = M/N$. Similarly, final output is calculated by using equation Eq. (8):

$$Y = z^{\frac{1}{1-\beta}}(\frac{\beta}{r\eta})^{\beta/(1-\beta)} N[m + (1-m)(\beta)^{\beta/(1-\beta)}] \int_0^{\bar{h}} h^{\frac{\alpha}{1-\beta}} dF(h), \tag{26}$$

Notice that if $\bar{h}, r$, and m are constant, a situation which holds on the balanced growth path, then

$$Y = AK, \tag{27}$$

where $A = \frac{r}{\beta} \frac{m+(1-m)(\beta)^{\beta/(1-\beta)}}{m+(1-m)(\beta)^{1/(1-\beta)}}$.

I

6 Savings

To close the model the consumer preferences need to be specified. I assume a utility function with constant elasticity of intertemporal substitution

$$\int_0^\infty \frac{c_t^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} dt,$$

which implies that the intertemporal optimization condition for a consumer faced with the an interest rate r_t is

$$g_c(t) = \frac{1}{\sigma}(r_t - \rho), \tag{28}$$

where $g_c(t)$ is the annual growth rate of per capita consumption and $\frac{1}{\sigma}$ is the elasticity of substitution.

6.1 Balanced Growth Path

I want to characterize an equilibrium in which the variables K, Y, and N, grow at constant exponential rates, and both $\bar{h}$ and the interest rate are constant. By inspecting Eq. (28) one realizes that if the interest rate is constant, $g_c(t)$ is constant too. Furthermore A also remains constant – $m(\bar{h})$ depends on $\bar{h}$ and on r both of which are required to stay constant. From equation (27) we learn that K and Y must grow at the same rate. The clearing condition

$Y = C + \dot{K}$ implies that also aggregate consumption grows at the same rate of K and Y. Population is constant by assumption. Therefore aggregate and per capita consumption grow at the same rate. Hence the growth rate of K, Y and c are equal to each other

$$g_K = g_Y = g_c.$$

Finally equation (25) implies that the ratio K/N remains constant along the balanced growth path. We can thus conclude that on the balanced growth path

$$g_K = g_Y = g_c = \bar{g}, \tag{29}$$

where $\bar{g}$ is determined by the system of equations (11)–(14) along with equation (28) under the assumption that $r(t) = r$.

7 Calibration

The objective of the experiment is to compare economies that are on their balanced growth path and differ only in the dispersion of human capital with the purpose of answering the following three questions. Does a more equal economy: a) have a high share of the population engaged in the business of invention? b) spun innovation at a faster pace? c) place a higher value on a new idea?

7.1 Measuring Human Capital Dispersion

Perhaps the most challenging aspect of the experiment is the choice of an appropriate human capital distribution. Psychologists have for long time have been busy in determining the IQs distribution for different segment of the society. For instance it has been proposed a normal distribution with mean of 100 and standard deviation of 15 to represent the IQ distribution of England pupils at the age of 15th, and to standardize the IQs distribution of other countries to the English one[13]. But there is hardly any consensus on the significancy of IQ tests in measuring human cognitive abilities. Even if they were accurate, it is not clear that cognitive ability is the most appropriate way of measuring the kind of human capital that serves to improve production efficiency or to facilitate technology adoption. Cognitive abilities may only be one input into the process of accumulating human capital, and again there is quite a lot of disagreement on how the 'production function' of human capital looks like. Carneiro and Heckman (2004) question the notion that human capital is to be strictly linked to cognitive abilities. They survey studies suggesting that other attributes, such as

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[13]Lynn and Vanhanen (2002) created an IQ data set for 81 countries on the basis of an extensive survey of psychological studies.

perseverance, dependability, optimisms are important predictors of grades in school and that these traits are also among the most valued ones by employers. Nevertheless the Mincerian and Beckerian literature uses extensively educational data as a measure of human capital. Several types of education-based data are available to measure distributional changes. Table (1) contains information on pupils' achievements in mathematics at the age of 15, recorded in a recent survey conducted by the Programme for International Student Assessment (PISA). In several instances countries with similar mean scores have shown substantial differences in variance. For instance the mean score of Japanese and Canadians pupils is about the same but the estimated variance of Japanese pupils' score is 30 percent larger than the Canadian pupils'. A similar observation can be made for Germany and Ireland. The existence of such dispersion within the same educational level might partly account for Juhn, Murphy, and Piere (1993) observation that the majority of the increases in U. S. wage inequality in the 1980s was due to unobserved attributes of workers belonging to the same educational or demographic group, provided that knowledge acquired in school played a larger role in production. As mentioned in section (2), the dispersion in pupils' educational achievements maybe magnified if the educational system is highly meritocratic in the sense that it functions as effective screening process whereby pupils in the right tail of the distribution end up into higher levels of education, and this is of a high-quality variety. Table (2), which records annual expenditures in tertiary education relative to non-tertiary education for most OECD countries, informs us that the commitment in tertiary education is much larger in the US both in absolute and in relative terms than any of the other country included in the survey.

For the sake of the experiment instead of taking one specific view on the process of human capital formation I propose a Gamma distribution that, appropriately parametrized, yields an income distribution close to the one observed in the US in recent years.

7.2   Parameters

The choice of the baseline values for the vector of parameters $(z, \eta, \rho, \sigma, \beta, \alpha, T, \phi, \delta)$ is more straightforward. The productivity parameters associated with the production of final output and capital, z and η, respectively, do not play any role in the equilibrium condition (15); therefore I set both of them equal 1. The preferences parameters $\rho = 0.02$ and $\sigma = 2$ in line with many other studies. The output elasticity to the capital goods it is rarely considered below 0.25 and is often set at around 0.3. I start with a baseline $\beta = 0.27$. The parameter α sets the returns on human capital. I will assume mild diminishing returns to scale in the final sector, so that inequality can generate effects similar to the one illustrated in examples 1 and 2, section (4). Hence $\alpha = 1 - \beta - 0.05$. The length of monopoly pricing is $T = 20$, in line with the patents' legislation in many countries. There are no ready estimates for ϕ and δ. Nevertheless there is a constraint given by (A1): $\phi > \alpha/(1-\beta)$. I will restrict my attention

Rank	Country	Mean score	Variance as a percentage of the OECD average
1	Finland	544.29	81.16
2	Korea	542.23	99.28
3	Netherlands	537.82	91.90
4	Japan	534.14	116.31
5	Canada	532.49	88.75
6	Belgium	529.29	121.76
7	Switzerland	526.55	111.04
8	Australia	524.27	105.15
9	New Zealand	523.49	110.06
10	Czech Republic	516.46	99.87
11	Iceland	515.11	94.52
12	Denmark	514.29	96.47
13	France	510.80	NA
14	Sweden	509.05	103.34
15	Austria	505.61	98.39
16	Germany	502.99	108.29
17	Ireland	502.84	83.94
18	Slovak Republic	498.18	98.66
19	Norway	495.19	98.13
20	Luxembourg	493.21	98.13
21	Poland	490.24	94.70
22	Hungary	490.01	101.55
23	Spain	485.11	90.80
24	United States	482.89	104.93
25	Portugal	466.02	88.99
26	Italy	465.66	106.52
27	Greece	444.91	101.84
28	Turkey	423.42	127.45
29	Mexico	385.22	84.89
	OECD average	500.00	100.00

Source: OECD (2005) Table A6.1; OECD PISA (2003), Table 4.1a.

Table 1: Dispersion in Educational Achievement

Rank (last Col.)	Country	Non-tertiary education	All tertiary education	Tertiary/Non-Tertiary
		(Percentage of GDP)		(Ratio)
1	United States	4.1	2.6	0.64
2	Korea	4.1	2.2	0.54
3	Greece	2.7	1.2	0.46
4	Finland	3.9	1.8	0.45
5	Denmark	4.2	1.9	0.45
6	Turkey	2.6	1.2	0.45
7	Ireland	3.1	1.3	0.41
8	Spain	3.2	1.2	0.39
9	Sweden	4.6	1.8	0.38
10	Australia	4.2	1.6	0.38
11	Hungary	3.3	1.2	0.37
12	Netherlands	3.4	1.3	0.37
13	Poland	4.1	1.5	0.37
14	Japan	3.0	1.1	0.37
15	Norway	4.3	1.5	0.34
16	Mexico	4.1	1.4	0.34
17	Czech Republic	2.9	0.9	0.32
18	Belgium	4.3	1.4	0.32
19	Slovak Republic	2.8	0.9	0.31
20	New Zealand	4.9	1.5	0.31
21	Germany	3.6	1.1	0.30
22	Switzerland	4.6	1.4	0.30
23	Austria	3.8	1.1	0.29
24	Italy	3.5	0.9	0.27
25	United Kingdom	4.3	1.1	0.27
26	France	4.2	1.1	0.26
27	Portugal	4.2	1.0	0.24
28	Iceland	5.7	1.1	0.19
	Canada	NA	NA	NA
	Luxembourg	3.9	NA	NA

Source: Author's elab. based on OECD (2005), Table B2.1c

Table 2: Expenditure in Education in OECD countries, by level of education (2002)

H	β	α	δ	ϕ	η	z	T	σ	ρ
100	0.27	$1 - \beta - 0.05$	0.00105	1.05	1	1	20	2	0.02

Table 3: Baseline Parameters

to cases where there are constant or mild increasing returns on human capital for inventors, considering ϕ ranging between 1 and 1.05. Once this parameter is set, and the average of human capital is normalized to 100, the parameter δ becomes the tuning instrument to set the innovation rate of the economy. This parameter is set to 0.00105, as in combination with the above specified parameters yields a rate of innovation of about 2%, very close to the average growth rate of US per capita income in the last 15 years.

7.3   Results

Plot (4) is a graphical illustration of Lemma 1: All individuals above a threshold level of human capital are inventors, and the rest final good producers. A point in the descending line in Fig. (5.A) represents the number of inventors, obtained as integral of the distribution for values above $\bar{h}$, for a given variance of the distribution. As the variance increases the right tail of the distribution becomes heavier, but at the same time $\bar{h}$ may move to the right because innovations are worth less. Indeed in the experiment $\bar{h}$ increases so much that the number of innovators declines notewithstanding the heavier tail. Not surprisingly, then, the average quality of the inventors relative to that of producers rises with inequality, both because there are more of outstanding individuals, and because $\bar{h}$ moves to the right.

The implied values of the real interest rate for each level of inequality is shown in plot C. This rate does not exceed the Mehra and Prescott (1985)'s estimate of the average stock returns in the post-war period (seven percent) which presumably includes some compensation for risk taking. Plot D confirms the intuition that inventors move to the final-goods sector when inequality increases, because the value of a design falls.

The most important plot is that contained in panel B, which shows that movement of the rate of innovation against the Gini coefficient. The slope of the schedule is about 0.0145, meaning that an increase in the Gini value by 0.1 is associated with a faster rate of innovation of some 0.145 percentage points. To appreciate the importance of a 0.1 increase in the Gini value, perhaps it is useful to say that change that occurred in US in the late 1970s and 1980s, which alarmed many social scientists, is measured between a variation of 0.05 and 0.1 –see Table (4)– of the Gini scale. It also correspond to one standard deviation of the Gini coefficients computed for a large number of countries by Barro (2000) and Forbes (2000). The slope of the rate of innovation-inequality schedule is smaller than Barro's estimated coefficient relating the growth rate of per capita income to inequality, which he found to be

Figure 4: Gamma Distribution
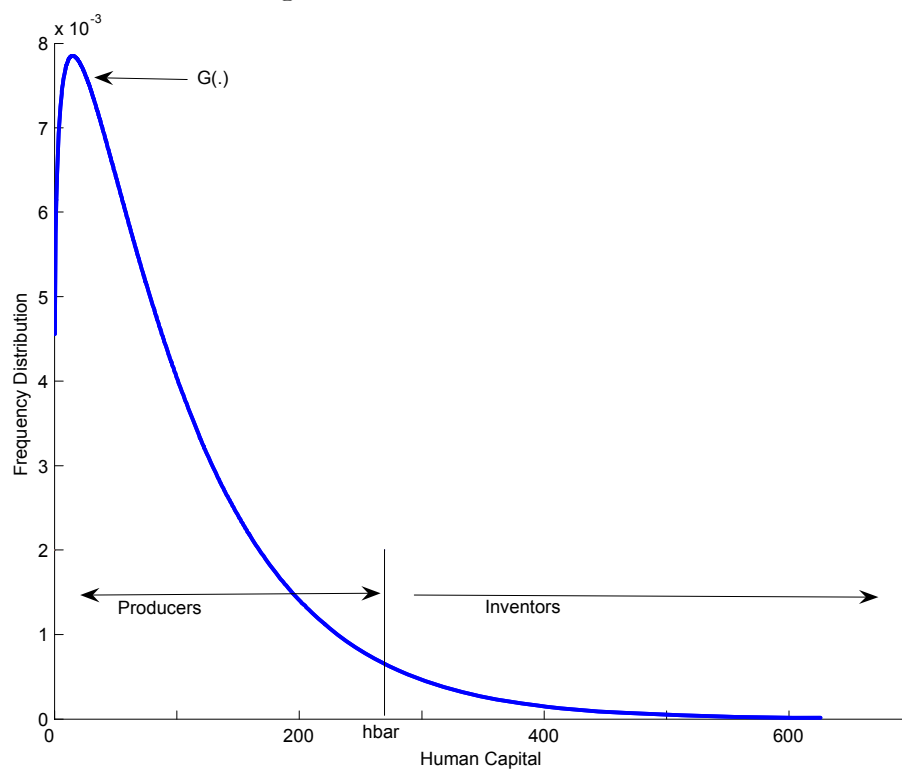
Parameters: a=1.1765 and b=85

Figure 5: Case with mild Decreasing Returns to Scale
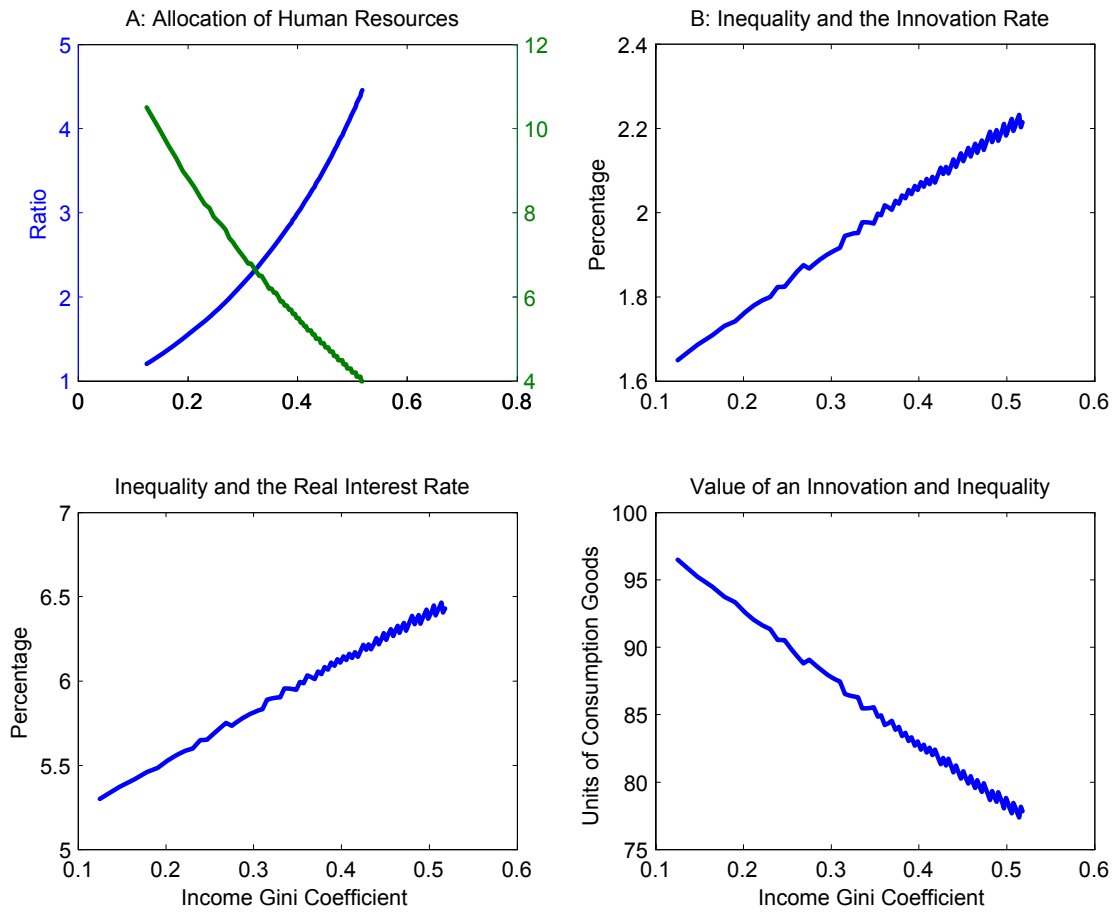
Note: For parameters see Table (3).

0.05, for high- and medium-income countries.

Table (4) reports summary statistics that describe the evolution of household inequality in the US since 1967. From the table emerges that the rise of the Gini value is due to the dramatic increase in the percentage of income accruing to the top 20% and top 5% of the population and to the lower appropriations accounted by the bottom two fifths of the population. The appropriateness of the Gamma distribution will be judged on its ability to replicate the type of data collected in the table, which add to the number of constraints to be followed in the calibration exercise. I will proceed in two steps. First the parameters a and b of the Gamma distribution are varied to the point in which the Gini coefficient is close to the one estimated by DeNavas-Watt et al. (2003) for the year of interest. Then the fraction of income accounted by each fifth of the population, ordered according to the level of income, is compared with that reported in Table (4).[14]

The income Gini value will depend on the choice of the parameters of the Gamma distribution (a, b), which however are bound by the constraint $a = 100/b$ (because $H = 100$). Therefore I search for the value of b that delivers the Gini coefficient recorded in 1967 and in 2001. Table (5) compares the calibrated distribution of income with the actual one and also reports the calibrated innovation rate and real interest rate (columns g and r) which are equal to some 2% and 6%, respectively. Between 1991 and 2004 the average annual growth rate of per capita gross domestic product has been 1.96%,the average interest rate on the 10-year US Treasury security was 5.86%, and the average annual inflation rate, computed with the CPI-U index, was about 2.6%. However the returns on funds invested in stocks are likely to be higher than the government bonds' returns of 3.3% ($\approx 5.86\% - 2.6\%$). Prescott and Mehra (1985) estimated an average stocks return of 7% for the postwar period. The interest rate implied by the model is in between these two figures.

From the table emerges that in both years the models overestimate by about 4% the income of the top 5%. It also overestimates, though by a lower magnitude, the income accruing to the bottom two fifths of households. In other words the model predicts too much equality in the bottom part of the distribution and too much inequality in the top part. If we keep the type of distribution given, then only a fine-tuning of the parameters α and ϕ, can reduce the differences between the data and the calibration results. The bottom part of the distribution is employed in output production. An increase in α magnifies income inequality for a given distribution of human capital. Conversely, less inequality among the

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[14]The model implies a tight relationship between the distribution of human capital and that of income. Of course in reality the link between the two distributions needs not to be working as described in the model, as this does not consider explicitly the government's tax and spending policy. Nevertheless, government educational spending and other redistributive policies are usually highly correlated. If human capital disparity is inversely related to the amount of government funding of public education, then calibrating a model without considering the role of fiscal policies probably would not affect the results dramatically.

Year	Lowest	Second	Third	Fourth	Highest	top 5%	Gini
2002	3.5	8.8	14.8	23.3	49.7	21.7	0.462
1990	3.9	9.6	15.9	24	46.6	18.6	0.428
1980	4.3	10.3	16.9	24.9	43.7	15.8	0.403
1970	4.1	10.8	17.4	24.5	43.3	16.6	0.396
1967	4	10.8	17.3	24.2	43.8	17.5	0.399

Source: Table A-3, Carmen DeNavas-Walt et al. (2003)

Table 4: Share of Aggregate Income Received by Each Fifth and Top 5 percent; Gini Coefficient

	Lowest	Second	Third	Fourth	Highest	top 5%	Gini	g	r
US in 2002	3.5	8.8	14.8	23.3	49.7	21.7	0.462		
Calibration	3.95	9.16	14.57	22.81	50.5	25.52	0.463	2.16	6.33
Difference	-0.45	-0.36	0.23	0.49	-0.8	-3.82			
US in 1967	4	10.8	17.3	24.2	43.8	17.5	0.399		
Calibration	5.77	10.97	15.72	21.68	45.86	21.99	0.399	2.05	6.11
Difference	-1.77	-0.17	1.58	2.52	-2.06	-4.49			

Source: Table (4) and Author's Calculation. For Parameters see Table (3)

Note: Columns g and r report the calibrated innovation and real interest rates, resp.

Table 5: Matching the Income Distribution

richest can only be obtained with a decline in ϕ.

The calibration exercise presented so far assumed that an innovation is protected for 20 years, in line with current patent legislation. A simulation with a longer T would have a small effect on the slope of the schedule. For instance if T goes from 20 to 40 years, the slope becomes about half of the original one. Of course a larger T shifts upward the growth schedule, as more individuals flock into the invention business lured by the longer rent periods. Perhaps surprisingly, variations in T do not affect significantly the price of the innovation. Although a higher T makes an invention worth more because inventors extract monopoly rents for a longer period of time the fact that more people migrates into this sector from the final-goods sector causes the demand of intermediate inputs to shrink, and inventions' users are less willing to bid for a high price.

7.4 Sensitivity Analysis and the Dual Role of Human Capital

How sensitive are the values of the Gini coefficient and of the rate of innovation to variations in α, β and ϕ?

Figure 6: The Dual Role of Human Capital
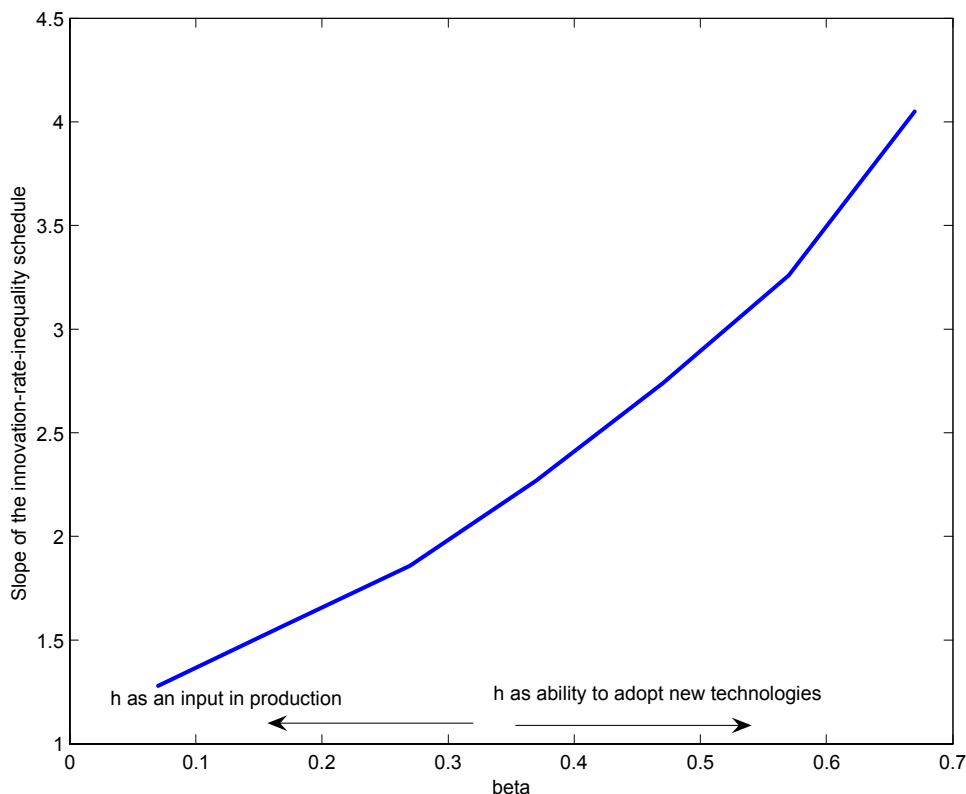
In Romer's model human capital enters the production function as an input. Keeping constant the quantity of capital, an individual with higher knowledge is more efficient in production. Nelson and Phelps (1966) however contended that the most important role of education and knowledge acquisition is not so much in improving the workers' efficiency given the quality and quantity of capital, but rather facilitating the adoption of new technologies. Benhabib and Spiegel (1994) report evidence in favor of this view. In a cross-country regressions they found that growth of GDP per capita from 1965 to 1985 was not significantly affected by the rise of average educational attainments but it was positively associated with the level of education in 1965. Fortunately the model is compatible with this view of human capital as well. One can embrace the idea that knowledge is essential in technology adoption but plays a minor role in improving production efficiency by attributing a small value to α, the output elasticity to human capital, and a large one to the parameter β. Is the weak relationship between inequality and innovation any different when β is large relative to α? It is.

Fig. (6) plots the 'slope' of the innovation-rate-inequality schedule against β. It shows that as β increases (and correspondingly α declines) the positive relationship between inequality and innovation becomes stronger and stronger. The intuition is quite straightfor-

	Lowest	Second	Third	Fourth	Highest	top 5%	Gini	g	r
US in 2002	3.5	8.8	14.8	23.3	49.7	21.7	0.462		
Calibration with β=0.27	3.95	9.16	14.57	22.81	50.5	25.52	0.463	2.16	6.33
Difference	-0.45	-0.36	0.23	0.49	-0.8	-3.82			
Calibration with β=0.67	4.53	9.82	14.98	21.64	49.03	25.16	0.463	1.13	4.26
Difference	-1.03	-1.02	-0.18	1.66	0.67	-3.46			

Source: Author's Elaboration and Table (4)

Note: The parameters' values are the same as Table (5)

Table 6: Distribution of Income with High and Low beta

ward. The relationship between the dispersion of human capital and income depends crucially on the value of α. When this is small, a given dispersion in human capital translates into a smaller dispersion of income. Therefore to generate a given income Gini value a wider dispersion of human capital is required as α gets smaller. In other words the weaker the link between human capital and income inequality the steeper the rate-of-innovation-inequality schedule.

In Table (6) the calibrated distribution of income is reported for $\beta = 0.27$ (baseline case) and $\beta = 0.67$. In the latter case human capital is relatively more important in facilitating technological adoption than it is in the former case. Notice that the parameters of the underlying human capital distributions are adjusted so as to match the calibrated Gini value with the one estimated by the Census. The table shows that even if β varies considerably (0.4), it is still possible to replicate the observed distribution of income.

Another verification involves a reduction of α, keeping constant $\beta = 0.27$, so as to test the sensitivity of the innovation inequality relationship to variations in the degree of diminishing returns of the production function to both physical and human capital, measured by $\alpha + \beta$. It turns out that the slope is similar to Barro's estimated correlation of 0.5 when the sum of these coefficients is 0.75. Setting a and b so that the Gini value is close to that estimated for the year 2002 (0.46), the computed fractions of income accounted by the each quintile, reported in the second raw of Table (7) are significantly further away from the actual ones (first raw). The second part of the table proposes another such exercise trying to use ϕ to fine-tune the slope to Barro's estimate. Again, for a combination of a and b that delivers a Gini close to 0.46, the distribution seems to be quite far off the mark, except for the second lowest quintile and the top 5 percent.

Conclusion 2 *Under parametrization of the production functions that imply only mild deviations from constant returns to scale ($\alpha + \beta > 0.95$ and $\phi < 1.05$, with $\phi > \alpha + \beta$), the*

31

slope of the rate-of-innovation-inequality schedule is about 0.15, assuming that human capital enters significantly (α high) as an input in production (that is a change in the Gini coefficient of 0.1 is associated with a 0.15% change of the innovation rate). When human capital is viewed as knowledge that facilitates the adoption of technologies, the slope more than doubles and becomes close to Barro's estimate, but the distribution of income gets further away from the actual one. Similarly, it is possible to increase the slope innovation-inequality to 0.5 by introducing severe diminishing returns on capital ($\alpha + \beta = 0.75$) or significant increasing returns in the R&D sector ($\phi = 1.2$). But in both scenarios, however, the resulting distribution of income is far away from the actual one.

Next I will argue that a more radical transformation of the framework – that is opening-up the economy to international flow of ideas – makes the innovation rate more sensitive to distributional changes.

8 International Flow of Innovations: Some Considerations

Up to now the analysis of the relationship between the distribution of human capital and the intensity of innovation activity was carried out under the premise that the economy was close to the rest of the world. The objective of this section is to relax the assumption to study situations in which ideas, workers, and output are free to move on the international market. I will limit my discussion to a special case in which the world is formed by two countries, identical except for inequality in human capital.[15] It will also be assumed that ideas generated in either country generates positive spillovers for the creation of more ideas in the future. Inventors are inspired either by observing the functioning of existing machines, or by studying the designs on the basis of which they are built, or by a combination of both. This notion of spillovers is different from the popular one which holds that trade is the main vehicle for knowledge transmission– see for instance in Grossman and Helpman, 1991, Ch.6, and Long and Wong (1997). I am abstracting from the effect of trade on innovation not because trade is irrelevant for the matter I am investigating – quite to the contrary it has been amply documented that trade is positively correlated with innovation, although the causal relationship is still debated – but to keep the analysis focused on the motive that inventions are the outcome of explicit investment decisions. Specifically I will assume:

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[15]However, I will abstract from the governments' strategic behavior in taking part into international agreements on the protection of intellectual property rights. This analysis has been recently carried out by Grossman and Lay (2004). See also Chin and Grossman (1990), and Deardorff (1992).

a) An innovation is equally protected in both countries, no matter the residency of the inventor;

b) The two economies have the same size and use the same technology, both in the manufacturing sector and in the R&D sector;

c) Preferences over the consumption good, the discount rate, and the intertemporal elasticity of substitution of the consumption good is the same for all individuals in either country.

d) Ideas are excludable in the manufacturing sector, but not in the R&D sector. Inventors of both countries have access to all previous blueprints, no matter where they have been developed.

Under these circumstances all aggregate variables are jointly determined in the two economies. The instantaneous diffusion of inventions makes the rate of innovation identical in the two countries – though, it will be clarified, these might differ substantially in their research effort. The two economies also share the same real interest and the same price of an invention. And the compensation of two individuals equally skilled is the same. If the two countries differ in the dispersion of human capital, however, differences in the distribution of income will persists. Indeed an interesting insight that will emerge is that an economy's capacity to innovate depends on its level of inequality *vis-à-vis* that prevailing in the other country.

In what follows I single out the main modifications of the closed-economy model that are needed to determine the innovation rate in the two-country world. I will refer to the two economies (or countries) as $F-$ and $G-$economy (or country).

First of all, the value of an innovation is increased by the expansion of the market. Equation (6) is replaced by

$$P = \frac{1}{r}(1 - e^{-rT})(r\eta)^{\frac{-\beta}{1-\beta}}\beta(z)^{1/(1-\beta)}[\int_0^\infty h^{\alpha/(1-\beta)}d\tilde{F}(h) + \int_0^\infty h^{\alpha/(1-\beta)}d\tilde{G}(h)]. \qquad (30)$$

where $\tilde{G}(h)$ is a measure of the final good producers in the $G-$economy with human capital equal or less than h.

Secondly, since the two countries share the same technology and inventions are equally protected in both countries, no labor migration occurs. The labor market equilibrium condition is still (15) except that the value of the design, P, is now computed using (30). Therefore, the equilibrium that pins down $\bar{h}$, for a given interest rate and for a certain expected growth rate of innovation $\bar{g}$, is

$$\frac{1}{r}(1-e^{-rT})(\beta)^{1/(1-\beta)}[\int_0^{\bar{h}} h^{\alpha/(1-\beta)}dF(h)+\int_0^{\bar{h}} h^{\alpha/(1-\beta)}dG(h)] = \frac{\bar{h}^{\frac{\alpha}{1-\beta}-\phi}}{\delta}[e^{-\bar{g}T}+(\beta)^{\beta/(1-\beta)}(1-e^{-\bar{g}T})].$$
$$(31)$$

Finally, the innovation rate is now determined by the joint-effort of innovators from both

countries. Equation (13) is then replaced by

$$\bar{g} = \delta(\lambda)[\int_{\bar{h}}^{\infty} h^{\phi} dF(h) + \int_{\bar{h}}^{\infty} h^{\phi} dG(h)]. \tag{32}$$

This specification of the creation process follows directly from the above point (e), which says that the amount of knowledge spills over among inventors regardless of their nationality. Notice that the level of human capital of the individual who indifferent between being an inventor or a final-good producer is the same in both countries ($h = \bar{h}$). An obvious consequence of this feature is that the country that has a human capital distribution with a relatively heavier right-tail will account for a larger proportion of inventions. Next section illustrates this point by proposing an extension of one example contained in section (4.0.3).

8.1 Randomization in a two-country model

An increase in the dispersion of human capital, in the second-order sense, is more likely to affect the innovation capability of an economy in an open or closed world? I will try to answer this question proposing a randomization exercise similar to the one illustrated in section (4.0.2): there is no inequality at all in country F, and some inequality in country G. The answer is quite different if the shock affects country F or country G.

8.1.1 The unequal country becomes more so

Lemma 1 established that people with the highest level of skills innovate. Therefore only residents of the G-economy with high level of skills will be employed in the innovation business. The labor market equilibrium condition that holds when people can choose the type of employment in either country is

$$H^{\frac{\alpha}{1-\beta}} + \frac{1}{2}H_1^{\frac{\alpha}{1-\beta}} + H_2^{\frac{\alpha}{1-\beta}}\frac{1}{2}(1 - l_{2,G}) = \frac{1}{\tau\delta\beta}H_2^{\frac{\alpha}{1-\beta}-\phi}, \tag{33}$$

where H_1 and H_2 are defined as in section (4.0.2) and the rate of innovation implied by Eq. (32) is

$$g = \delta(\frac{1}{2}l_{2,G})H(1 + \epsilon). \tag{34}$$

The first term on the left-hand side of Eq. (33) accounts for the fact that an inventor resident in the G-economy collects monopoly rents in the F-economy as well. Therefore the quantity $\frac{1}{2}l_{2,G}$ is larger than the one that we would observe in a closed $G-$economy. However, none of the F-economy's residents works as an inventor, and the world overall has fewer inventors. To see this recalls that the two labor market equilibrium equations under autarky are

$$(1 - l_F) = \frac{1}{\tau\delta\beta}H^{-\phi} \tag{35}$$

and

$$\frac{1}{2} + \frac{1}{2}(1 - l_{2,G}) = \frac{1}{\tau\delta\beta}[H(1+\epsilon)]^{-\phi} + \frac{1}{2}(1 - \gamma^{\frac{\alpha}{1-\beta}}), \tag{36}$$

where $\gamma = H_1/H_2$. The sum of the left sides of Eqs. (35) and (36) is the total number of non-inventors in the two economies. Eq. (33) can be rearranged as

$$1 + \frac{1}{2} + \frac{1}{2}(1 - l_{2,G}) = \frac{1}{\tau\delta\beta}[H(1+\epsilon)]^{-\phi} + 1 - \left(\frac{H}{H_2}\right)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1 - \gamma^{\frac{\alpha}{1-\beta}}) \tag{37}$$

It is easy to verify that the sum of the right-hand sides of Eqs (35) and (36) is smaller than the right-hand side of Eq. (37); therefore a larger fraction of population is engaged in non-invention activities in a world with open economies.

Finally by combining Eqs. (35) and (36) we get the rate of innovation in an integrated world

$$g_W = \delta H^{\phi}(1+\epsilon)^{\phi - \frac{\alpha}{1-\beta}}\left[\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1-\epsilon)^{\frac{\alpha}{1-\beta}} + 1\right] - \frac{1}{\beta\tau}, \tag{38}$$

which is the same as equation (23) except for the additional number one appearing inside the square brackets. Clearly the model has a 'scale effect', as designs serve a larger population when the economy is open. It is easy to verify that $g_W > g_G$ and that $g_W > g_F$.

How does a shock that causes a rises in human inequality in the G-economy – for instance some educational redistributive programs are eliminated – affect g_W? As in the closed economy there are two competing forces, one related to the decreasing returns in production, which lower the value of an innovation, and one with the higher productivity of more educated inventors that raises the innovation rate. This latter force is more powerful in an open economy, because of the scale effect.

Notice that in the range for which there is a positive relationship between g_W and ϵ the F-economy benefits from a larger inequality in the G-economy, at least in the long run, for it enjoys a larger growth rate and is still an economy with no inequality.

8.1.2 The equal country becomes more unequal

Let ϵ_F and ϵ_G be the percentage of the mean value of human capital in the F- and G-economy, respectively in the randomization example introduced in section (4.0.2). By lemma 1 if $\epsilon_F < \epsilon_G$, still innovation occurs exclusively in the G-economy, although the innovation rate g_w is lower – in Eq. (38) the number 1 is replaced by $\frac{1}{2}(1 + \epsilon_F)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1 - \epsilon_F)^{\frac{\alpha}{1-\beta}} < 1$ and ϵ_G substitutes ϵ. Conversely if $\epsilon_F > \epsilon_G$ the role of the G and F economy are reversed – all innovation activities are carried out in the F-economy. When $\epsilon_F = \epsilon_G$ a situation of indeterminacy arises.

Figure 7: Comparing Gamma Distributions
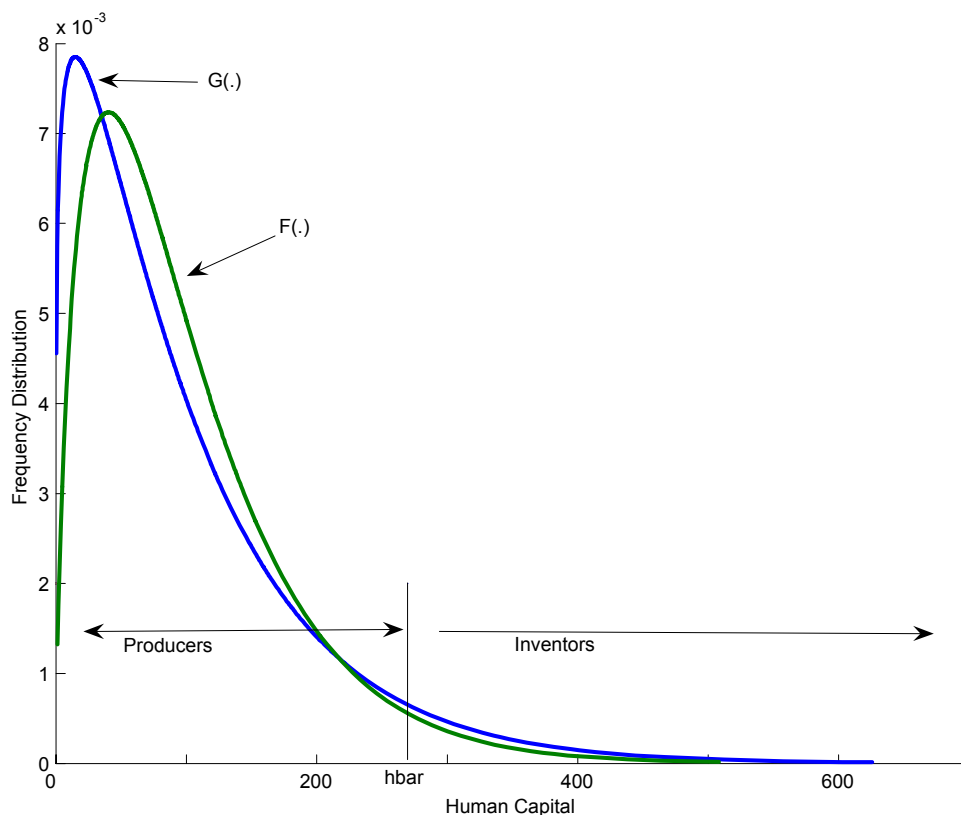
9 Calibration in a two-country model

The above example suggested that a country's innovation capacity depends on its level of inequality *relative* to the other country's. How much does it matter that countries differ in the dispersion of human capital? I propose to answer this question keeping section (7)'s assumption that human capital follows a Gamma distribution, now in both countries. Let a_i and b_i, for $i = \{F, G\}$ denote the parameters of the Gamma distribution. Since $H = 100$, then $a_F b_F = a_G b_G = 100$. Fig. (7) plots the distribution of the economies when and $b_F < b_G$ (lower dispersion in the F-economy). The four plots in Fig. (9) are generated with the same parameters underlying Fig. (5)'s plots, and assuming a very high correlations of inequality in the two countries ($b_F = 0.65b_G$). The Gini coefficient of income running on all four horizontal axes is that of the $G-$economy. Likewise the share and quality of inventors refer to the G-economy only, whereas the remaining variables (the innovation rate, the interest rate, and the price of an invention, whose behavior is depicted in Plots B, C, and D, respectively) are the same for both countries.

The remaining parameters' values are the same as the ones used in the one-country calibration exercise.

	Lowest	Second	Third	Fourth	Highest	top 5%	Gini
US in 2002	3.5	8.8	14.8	23.3	49.7	21.7	0.462
α+β=0.75; φ=1.05	7.44	9.95	11.74	13.71	57.16	27.76	0.463
Difference	-3.94	-1.15	3.06	9.59	-7.46	-6.06	
α+β=0.95; φ=1.20	6.73	9.16	10.96	12.98	60.17	19.97	0.467
Difference	-3.23	-0.36	3.84	10.32	-10.47	1.73	

Source: Author's Elaboration and Table (4).

Note: The parameters' values not reported are in Table (5).

Table 7: Sensitivity Analysis: Comparing Calibrated Model with US Income Distribution

Figure 8: Inequality in Open and Closed Economies

Note: For parameters see Table 3.

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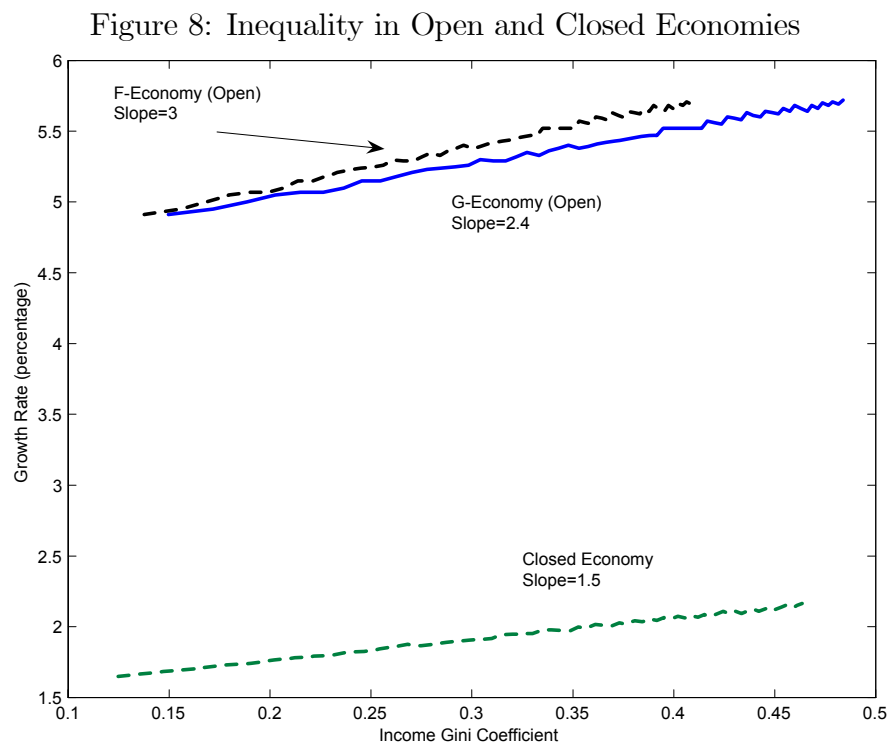
One feature of the Romer's model is the scale effect predicted as two economies become integrated. Fig. (8) gives an idea of the magnitude of the scale effect in the baseline case ($\beta = 0.27$). For a given level of inequality, the world growth rate is between two and three times larger than the one calculated for the closed economy, a quite unlikely outcome to be observed when two economies integrate. The magnitude of the scale effect partly is due to important simplifying assumptions: instantaneous diffusion of inventions; no 'stepping on the toe effect' – inventors in the two countries might be working on similar problems. One way to offset the sale effect is by reducing the length of protection of an invention. Although in principle there is no reason to think that governments pass legislation that weakens intellectual propriety rights as a consequence of opening-up their economies, it is likely that competition among inventors rends obsolete existing inventions at a faster rate. To obtain a rate of innovation for the consolidated economies similar to the one calculated for the closed economy, T must be slashed by a bit more than a half (from 20 to 9), when $\beta = 0.27$. The scale effect becomes less severe at some higher values of β, but remains still strong. For instance when $\beta = .5$ a reduction in T from 20 to about 12 is needed to generate an innovation-rate-inequality-schedule (not shown) for the open economy that tracks closely the one calculated for the closed economy (the dashed bottom curve in Fig. (8)).

To easy the comparison between open vs. closed economy Fig. (9) reproduces the plots in Fig. (5) for an open economy. A quick inspection of the plots reveals that ideas are worth more when the economy is open (plots (D)), that the innovation rate, and consequently the interest rate, [16] are larger in the open economy (plots (B) and (C)) and that a bigger fraction of the population are involved in inventions in the more unequal economy when this opens its borders (plot (A)). How are invention activities being reallocated between the two countries as these get more integrated? I will answer to this question under the assumption that integration does not yield scale effects (that is, running the two-country economy with a lower T). Fig. (10) shows the fraction of the country's population engaged in the business of inventions against inequality, when the economy is closed – top curve– and when it is open – bottom two curves. Notice that the graph is built in a way that for a given value of Gini coefficient the rate of innovation is roughly the same in the closed and open economy ($T = 20$ when the economy is closed, and $T = 9$ otherwise). Not surprisingly a smaller share of the population is needed to generate a given amount of innovation when ideas freely circulate. More important is the observation that the number of inventors is between two and three times larger in the G-economy than in the F-economy, although the Gini coefficients differ only by some 15%. As already noted, because there is free movement of workers the threshold in $\bar{h}$ will be shared by both countries. This feature combined with the assumption that the G-economy's human capital distribution has a heavier tail than the
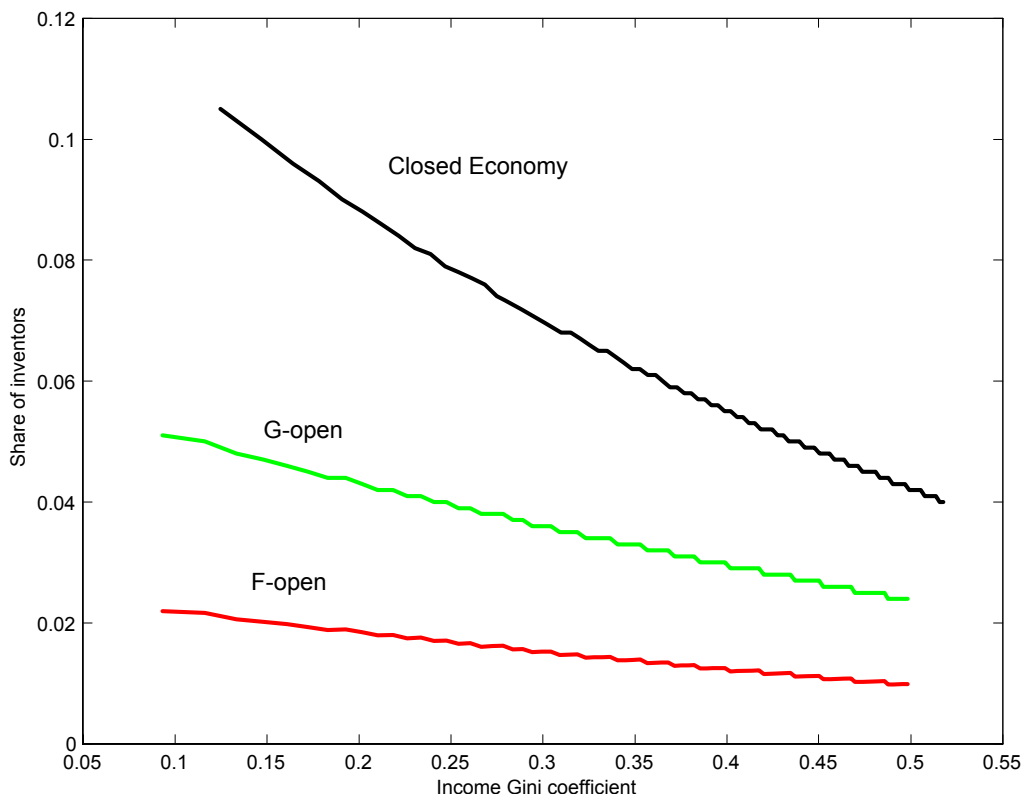
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[16] The link between the innovation rate and the interest rate is based on Eqs. (28) and (29).

Figure 9: Inequality and Innovation in the Open G-economy
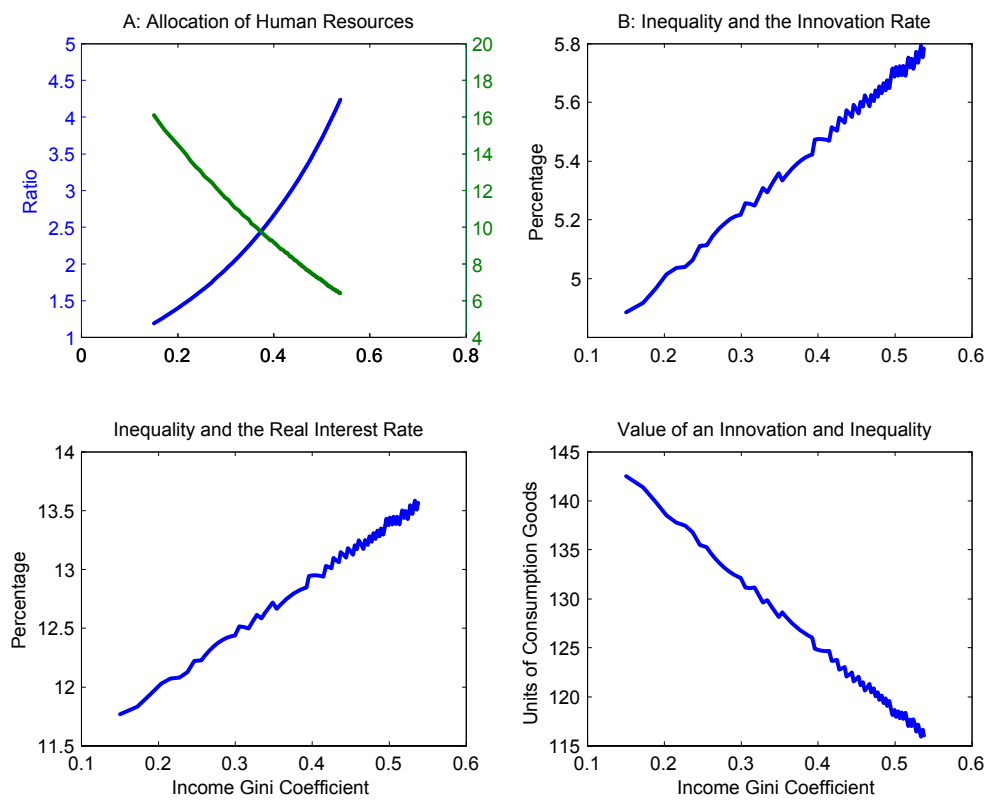
Note: For parameters's values see Table 3; $b_F = 0.65b_G$;'slope'=2.4)

Figure 10: Comparing Commitment in Innovation

Note: T equal 9 and 20 in the open and closed case, resp. (no scale effect)

F's, accounts for the G's stronger commitment in inventions, and explains the G's steeper slope of associated in the plot. I summarize the two main insights if this section as follows.

Conclusion 3 *1) The association between inequality and the innovation rate becomes stronger when closed economies open up their borders and allow free circulation of ideas. 2) The unequal economy tends to specialize in business of invention, whereas the equal economy is a net importer of blueprints.*

10 Conclusion

This paper has suggested that policies that affect the dispersion of human capital have consequences on the market of inventions. In presence of diminishing returns, more inequality reduces the demand for a new blueprint as well as its market value. But if inequality is caused by a wider disparity in knowledge acquisition, inventors are more productive in a more unequal economy, for they are better trained. A simple example in which people were classified into high- and low-skilled groups, suggested that for low and medium levels

of inequality, the relationship between inequality and the rate of innovation is positive, and negative otherwise. However, once the model was calibrated to the US economy the sign of the relationship was always positive, within some reasonable parameters' values. In particular for a set of preferred parameters the schedule of the innovation rate plotted against the Gini value computed on income was about 0.15, that is about one third of the partial correlation estimated by Barro (2000) between the average rate of per capita income and the Gini coefficient for a set of medium- and high-income countries, and about a tenth of a similar estimation performed by Forbes (2000). Perhaps the most important aspect of the calibration exercise is that in a sensitivity analysis the relationship would remain positive for a wide range of parameters.

Motivated by the desire to study the link between innovation and inequality the paper has contributed to the theory of endogenous growth by showing that the landmark Romer's paper on endogenous growth is a useful framework to analyze issues such as inequality, for which individuals' heterogeneity is needed.

Some indications of how inequality affects the market for ideas, and hence growth, in an open economy were given, but under very special circumstances of one world that consists of two countries differing only for the variance in human capital. Other aspects, such as size, initial conditions, terms of trade, and relative distance from the technological frontier, if included in the analysis, may alter the result about opening-up the economies so far obtained –the more unequal economy takes the leadership in the innovation market – and are left for future research.

The question of how much the calibration is sensitive to alternative specification of externalities also remains open. For instance, in the so-called second-generation of growth models[17], the production of new ideas does not depend linearly on the stock of existing knowledge, but is strictly concave, thus reducing the role played by knowledge spillovers.

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[17]See Jones (1999) for a review.

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