

An Introduction to the New Zealand Treasury Model

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Abstract

The Treasury is the New Zealand government's lead advisor on economic and financial issues. Part of this advice consists of providing the government with forecasts of economic and fiscal variables. Economic forecasts are important, not only as a basis for forecasts of tax revenue, but also in informing the government of the macroeconomic environment in which proposed fiscal policy settings will operate. The New Zealand Treasury Model (NZTM) is an important part of the economic forecasting process at the Treasury. This paper has three purposes. The first is to give readers an idea of the key features of NZTM. The second is to detail major changes to the model since the last published documentation of the model (Szeto, 2002). These model developments have enhanced NZTM to provide more detailed forecasts. Key changes include the disaggregation of deflators into the various expenditure GDP components, the introduction of consumption and capital goods imports into the model (rather than just treating them as intermediate imports) and the disaggregation of the inflation equation into tradable and non-tradable components. The final purpose of this paper is to outline briefly NZTM's role in the Treasury's forecasting process.

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C68 – Computable general equilibrium model
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K E Y W O R D S

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An Introduction to the New Zealand Treasury Model

1 Introduction

Roberta Piermartini and Robert Teh, two economists at the WTO, urge modellers to “demystify” their creations, making it clear to their audience what makes their models tick. A failure to do this, they argue, “risks bringing a useful analytical tool into disrepute and may even induce unwarranted cynicism...” (The Economist, 13 July 2006)

The New Zealand Treasury is mandated by section 26O of the Public Finance Act (1989) to prepare economic forecasts to be presented by the Minister of Finance each financial year. These economic forecasts are important, not only as a basis for forecasting tax revenue, but also in informing the government of the macroeconomic conditions in which proposed fiscal policy settings will operate. The New Zealand Treasury Model (NZTM) is an important part of the economic forecasting process at the Treasury. The purpose of this paper is three-fold. One is to provide an accessible introduction to NZTM without too much technical detail of the structure of a large computable general equilibrium (CGE) model. In describing the model, we have attempted to be as non-technical as possible, with a more technical description available in Szeto (2002). The second purpose of the paper is to provide an update of changes to the model since the last documentation (Szeto, 2002). The final purpose of this paper is to describe briefly the role NZTM has in the Treasury’s economic forecasting process.

NZTM seeks to describe the behaviour of, and interactions between, four sectors of the economy:

- *Households* who consume goods and services from New Zealand and overseas firms and supply labour
- *Firms* who maximise profits subject to available technology by employing labour, importing intermediate goods, investing in capital and producing goods and services for the domestic market and export
- *Trade and financial linkages with the rest of the world:* the rest of the world buys New Zealand’s exports, sells us our imports and lends to and borrows from New Zealand. Consistent with our small open economy status, New Zealand is a price-taker with respect to the rest of the world, and
- *Government* who consumes, invests, employs, taxes and transfers.

1.1 The evolution of NZTM

After a lengthy period of development, NZTM has become an integral part of the Treasury forecasting process. Szeto (2002) outlined the early history of NZTM which began as the New Zealand Model (NZM). NZM was based on the Murphy Model of Australia (Powell and Murphy, 1997), but adjusted to allow for differences in data and institutional structures.

The model was then renamed NZTM following a major redevelopment of the model. Key changes, according to Szeto (2002), were the introduction of:

- the relative price structure
- the inflation-targeting framework for monetary policy
- the analytical framework of real equilibrium exchange rate determination, and
- the demand-pull framework of inflation determination.

Following these developments, NZTM began to be used to produce forecasts. Initially, these forecasts were used for developing scenarios around the main forecast track (developed using other methods) and more recently to produce the main forecast track itself. The use of NZTM as a forecasting tool has spurred new developments which we will outline in this paper, notably the disaggregation of deflators into the various expenditure GDP components, the introduction of consumption and capital goods imports into the model (rather than just treating them as intermediate imports) and the disaggregation of the inflation equation into tradable and non-tradable components.

1.2 The remainder of the paper

The remainder of this paper is structured as follows. First, we give a high level description of the two-tiered structure of the model: the steady-state model, the dynamic model and how these two models interact. The structure of the steady-state and dynamic models, and their interaction, are crucial in understanding the properties of the model and therefore how the growth paths of variables evolve in the model. Once we have described the structure of NZTM at a general level, we look at the equations that describe the behaviour of the major sectors of the economy. Section 3 describes firms' production decisions. Section 4 describes how firms interact with the rest of the economy and how their production decisions feed through into firms' investment, exports, imports and the labour market. We then look at how the model determines monetary conditions and price deflators in Sections 5 and 6 respectively. Section 7 describes NZTM's role in the forecast process. Section 8 concludes.

2 The general structure of NZTM

NZTM consists of two parts: the steady-state model and the dynamic model. The growth path for the economy is determined by the interaction of the model's steady-state and dynamic equations. The steady state is a long-run state to which the key variables converge, while the dynamic equations describe how the economy moves to steady state. We outline these two components below.

2.1 The steady state¹

2.1.1 General features

The steady-state model of NZTM provides an explicit estimate of the future long-run value that each variable normally converges to. The explicit statement of the steady-state level is a key point of difference from some other models where the level is not explicitly defined but rather variables are expressed as deviation from trend.

The steady state of the model can be broadly characterised as consisting of two parts, the production block (ie, the supply side) and the demand side. Later in the paper we will outline in more detail the steady-state structure of the production block and demand-side. However, for our current purposes, it is sufficient to think of the steady state as when the economy is on a balanced growth path in the Solow growth-model² sense, with output growth dependent only on productivity and population growth. The balanced growth path has the implication that the capital-to-output ratio is constant.

A second feature of the steady state in NZTM is that the economy is in internal and external balance. Internal balance is a condition that the values of all variables are such that the unemployment rate is equal to the non-accelerating inflation rate of unemployment (NAIRU) and the domestic goods market is in equilibrium (supply equals demand). External balance is the requirement that New Zealand's net external indebtedness with the rest of the world is at a certain level as a share of GDP.³ These two balance requirements, sometimes called collectively macroeconomic balance,³ play a key role in linking the supply and demand sides of the model.

¹ Steady state in NZTM is a solution to a system of equations, such that all the equations are simultaneously satisfied. We impose only the values of a few variables in the steady state; these are listed in Appendix 3. Section 7, which talks about NZTM in the forecasting environment, will explain how these exogenous assumptions are arrived at. Key individual equations in the steady state will be discussed in sections 3 to 4, and all steady-state equations are listed in Appendix 1.

² See Solow (1956).

³ The macroeconomic-balance approach, which is based on the simultaneous achievement of internal and external balance, goes back to Meade (1951).

2.1.2 The role of the real exchange rate in achieving simultaneous external and internal balance

In steady state, the real exchange rate has an important role in achieving internal and external balance simultaneously and thus achieving steady state. Simultaneous achievement of internal and external balance can be thought of as occurring in three steps:⁴

1. In steady state, both the real current account deficit (*cad*) and the real net foreign asset position (*nfa*) return to their steady-state GDP ratio. The steady-state growth rate of real GDP is the sum of the growth rate of working-age population and labour-augmented productivity. Therefore, the level of real net foreign assets will need to grow at the steady-state GDP growth rate to ensure its steady-state GDP ratio remains constant and at target. By definition, a decrease in net foreign assets implies net financial inflows (*nff*), which equals the current account deficit. Given the change in real net assets is known, we get the steady-state current account deficit (see equation 2.1.1).

$$-I^* \Delta nfa = nff = cad \quad (2.1.1)$$

2. If we decompose the current account deficit into the trade balance (*nx*), net income balance (*nib*) and net transfers (*ntr*) we get the following identity:

$$cad = -I^* (nx + nib + ntr) \quad (2.1.2)$$

In steady state, the net income balance (*nib*) can be thought of as being “given” for this purpose, in that it is the target level of net foreign assets multiplied by the world interest rate plus a risk premium. We can also assume net transfers are given. Therefore, rearranging (2.1.2) and taking *nib* and *ntr* as given, we get the steady-state trade balance (*nx*):

$$nx^* = -I^* (cad + nib + ntr) \quad (2.1.3)$$

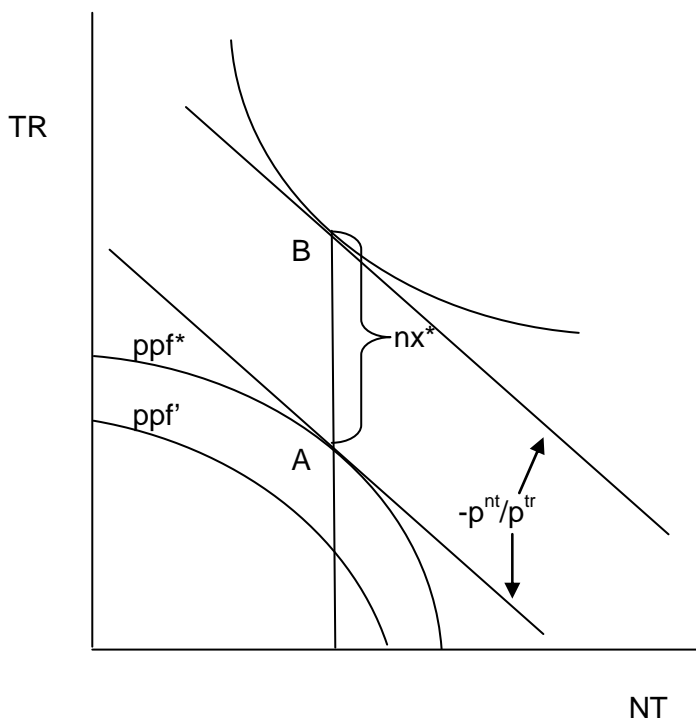
3. To illustrate this, we divide the productive economy into two sectors: tradables (*TR*) and non-tradables (*NT*).⁵ For a given level of resource use, the production of tradables and non-tradables can be shown by the production possibility frontier. In Figure 1, *ppf** represents the production possibility frontier in a fully employed economy (such that unemployment is at its NAIU level, one requirement of internal balance). This is opposed to *ppf'* with the unemployment rate higher than the NAIU. Thus, to be consistent with internal balance, the economy will need to produce somewhere on *ppf**. Where the economy produces on *ppf** depends on the real exchange rate (the relative price of non-tradables to tradables, p^{nt}/p^t). There is also the nation’s mix of aggregate expenditure on tradables and non-tradables, which is represented by the national indifference curve in Figure 1. The amounts of

⁴ Section 5.3.1 outlines the equations that determine the real exchange rate.

⁵ The Salter and Swan model (see Swan, 1955 and Salter, 1959) provides a framework to determine the equilibrium value of real exchange rates when the home country produces two types of goods, namely traded goods and non-traded goods. In the Salter and Swan model, exportables and importables are treated jointly as a single class of goods (traded goods) on the grounds that a small country cannot affect its terms of trade and the terms of trade remained unchanged. As described in Section 4, the tradable sector in NZTM is further divided into exports and imports, reflecting the importance of the terms of trade in the New Zealand economy.

tradables and non-tradables consumed also depend on the real exchange rate. By definition, a country's production and demand of non-tradables must be the same, while the difference between tradables production and consumption equals the trade balance. For external balance to hold, the real exchange rate must be equal to the negative of the slope of the two parallel lines evaluated at points A and B so that the trade balance is equal to the steady-state trade balance derived above, whilst ensuring that the economy is producing on the production possibility frontier where the unemployment rate is equal to the NAIRU.

Figure 1: Modified Salter-Swan diagram showing macroeconomic balance



2.2 Dynamic path

If prices and wages were fully flexible and all factors of production, such as capital and labour, could move instantaneously, the economy would always operate at steady state (ie, the values of all variables would always be at their steady-state value).

However, the economy is one where frictions exist (for example, adjustment costs and imperfect information). As a result, prices are sticky (ie, do not adjust instantaneously) and resources are not fully mobile and therefore it takes time (in the absence of further shocks) for the economy to get to steady state. These frictions are modelled in NZTM by variables seeking to adjust partially (as opposed to full adjustment in a frictionless world) towards steady-state values (x^{ss}). NZTM also introduces frictions through partial adjustment towards medium-run values (x^m), representing profit or utility maximising values in the medium run, as well as the use of lags of dependent variables. The medium-run profit maximising values are determined by the first order conditions for profit maximisation of the dynamic production block of the business sector, which will be described further in Section 3.

In general, the dynamic equations in NZTM have the following structure:

$$x_t = \alpha x_{t-1} - \lambda f(x_{t-1} - x^{ss}) + \omega f(x_{t-1} - x^{mr}) + \gamma \mathbf{z} \quad (2.3.1)$$

That is, the dynamic value of a generic variable x_t is a function of its lag(s), its steady-state value, its medium-run value and a vector of relative prices (\mathbf{z} ; for example, interest and exchange rates). Note that not all equations contain all the elements stated above. We discuss the components of equation (2.3.1) in turn below:

(i) *Feedback from the steady state*

The steady state of the model gives long-run values of the following variables which are used in the dynamic part of the model:

- Private sector potential output and, given government spending is exogenous, economy-wide potential output. These variables are private sector and economy-wide output in the steady state
- Real hourly wage
- Real exchange rate
- Inventory levels
- The relative price of residential investment
- Equilibrium housing stock
- Equilibrium volumes of non-commodity export goods and export services, and
- The relative price of both non-commodity export goods and export services.

When the economy is not in equilibrium, the current levels of some (if not all) variables are different from their steady-state levels. The deviation of the above variables from their steady-state level is a key driver in restoring the economy to long-run equilibrium and is therefore also a key driver of dynamics in some equations. The parameter λ measures the speed of adjustment towards achieving that steady state value.

For some equations, the dynamic value of x_t is a function of its steady-state stock variable rather than its steady-state flow variable. If the *actual* value of the relevant steady-state stock variable (X_t) falls short of, or exceeds, its *required* steady-state level (X^{ss}), flows must adjust to ensure that in the long run the gap is closed. For example, if the housing stock is currently below its steady-state value, all else equal, growth in residential investment will need to be faster (compared to steady-state growth). Over time, this increased investment will see the difference between the housing stock and the steady-state housing stock fall, until (all else equal) the housing stock is at its steady-state level and therefore the current rate and the steady-state rate of residential investment are equal. Table 1 presents some of the key stocks in the model and the flows which adjust to achieve the steady-state values.

Table 1: How flows in economy adjust to achieve steady-state stocks

	Steady-state variable	Steady-state value is achieved through adjusting...
Households	Desired level of the housing stock	Residential investment
Government	Optimal level of gross debt consistent with the government's fiscal strategy	The personal tax rate
External balance	Households have a desired level of net financial wealth which in turn determines the desired level of net foreign asset position.	The real exchange rate (see section 2.1.1)
Internal balance	Target level of unemployment (the non-accelerating inflation rate of unemployment, NAIRU ⁶)	Wages, inflation and monetary policy

(ii) Feedback from medium-run values

In the dynamic model, as in the steady state, there is a production block (discussed in more detail in Section 3) that determines the firms' profit-maximising input and output mixes. However, the presence of adjustment costs means adjustment towards the input and output mixes that maximise profit in the dynamic version of the production block is not instantaneous. Examples of adjustment costs include the costs of hiring and firing labour and changing the production mix. Therefore, equations that are dependent on production-block decisions (for example, labour market decisions and exporting and importing decisions) feature a partial adjustment towards these dynamic production block values (which we call medium-run values, x^{MR}). Private consumption also adjusts towards a medium-run value, which can be (informally)⁷ thought of as the utility-maximising level of consumption in the Permanent Income Hypothesis sense. We will discuss this in more detail in Section 4.6.

(iii) Persistence

Lags serve two functions in the model. The first is to introduce momentum. A positive coefficient on a lag term ensures that strong growth in the previous quarter flows through to the next quarter. Lags also provide another way of introducing frictions and lags also mean the adjustment to shocks in the economy takes time to flow through.

(iv) Relative prices

Relative prices are a key driver of decision making in NZTM. The relative price of inputs determines the amounts of different inputs used in production, and the relative price of outputs determines the output mix. In demand equations, an increase in the real exchange rate (the relative price of New Zealand goods and services to foreign goods and services) will encourage more importing, while an increase in interest rates (the relative price of current spending as opposed to saving) will discourage expenditure. Note, with the obvious exception of interest rates, relative prices are typically expressed relative to the price of "other goods" (*pydo*) which includes all non-commodity goods and services produced by the private sector.

⁶ Strictly speaking, NAIRU is not a stock variable.

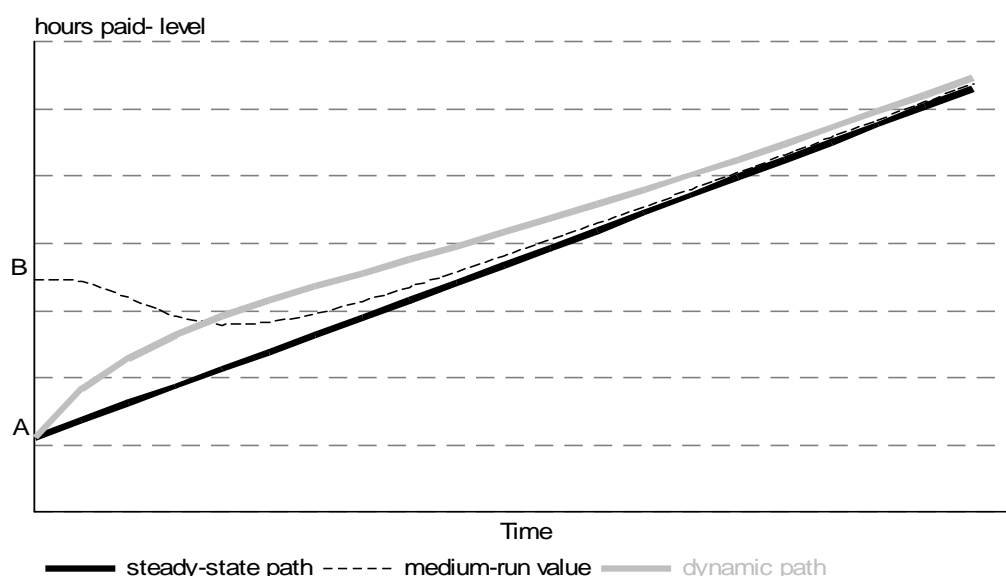
⁷ We use the term "informally" as the medium-run level of consumption is not explicitly derived from the utility-maximising problem.

2.3 The interaction between the dynamic path and the steady state

To illustrate the interaction between the dynamic path, the medium run and the steady state, Figure 2 shows the response of employment to a temporary increase in consumption. For the purposes of exposition, this can be thought of as occurring in three parts, outlined below:

- With higher demand, the profit-maximising level of labour demand from the dynamic production block (the medium-run variable) rises immediately from A to B. Although the firms would like to increase employment immediately to point B, the actual employment (the dynamic path) remains at point A at the time of shock owing to the unanticipated nature of the shock. Furthermore, the temporary nature of the shock means that the shock has no impact on the steady state in this simulation.
- In the medium run, increased consumption leads to excess demand, adding inflationary pressures. As a result, the monetary authority tightens monetary conditions, which eventually weaken demand. As demand weakens, the profit maximising level of employment begins to fall. Although the medium-run value begins to decrease, actual employment continues to increase and converges on its medium-run value because its current value is below the profit-maximising level.
- In the long run, the dynamic variable and the medium-run variable converge to the steady state. When the economy reaches the steady-state level, the dynamic variable will just grow at the steady-state rate (ie, productivity and population growth).

Figure 2: Stylised diagram of the impact on employment of a consumption shock



2.4 Assigning coefficient values in NZTM

Before we describe individual equations, we make one more general remark about how model coefficients are set. Given the difficulties involved in estimating empirically a model of this size (particularly in the presence of possible structural breaks), the majority of the dynamic equations and steady-state equations in NZTM are calibrated. The major exception is the production block, which is statistically estimated. While calibration means that parameter values may seem *ad-hoc*, it appears necessary to produce sensible dynamic properties. Calibration in NZTM is either based on theory (for example, the inflation expectations process, see later), empirical studies (for example, the consumption function is based on Goh and Downing, 2002) or something that gives sensible dynamic paths in response to shocks. The convention we have adopted through this paper is to denote coefficients as $pa_{_}$.

Having described the general structure of the steady-state and dynamic equations, we now start to describe various parts of the model in more detail. The first part is the equations that relate to the production block of the business sector.

3 The production block

3.1 The production block

In this section we examine the production block for the private business sector. The production block is the name given to the component of the model that determines the combination of the representative firm's inputs and outputs that will maximise profits within the private business sector of the economy. The advantage of dividing output into private sector output and public sector output is to give a more accurate picture of how the government sector affects the economy – that is, the government sector not only uses private sector goods and services but also absorbs resources, especially labour, directly. Coefficients in the production block are estimated. Szeto (2001) provides further details on the production block and how it is estimated.

The production block combines three inputs (capital k , labour n , and imported intermediates imo) to produce goods and services (gross output, t). All the goods and services produced by the private business sector are classified into two distinct groups: commodity export production ($cexps$) and non-commodity goods and services (“other goods”, ydo).

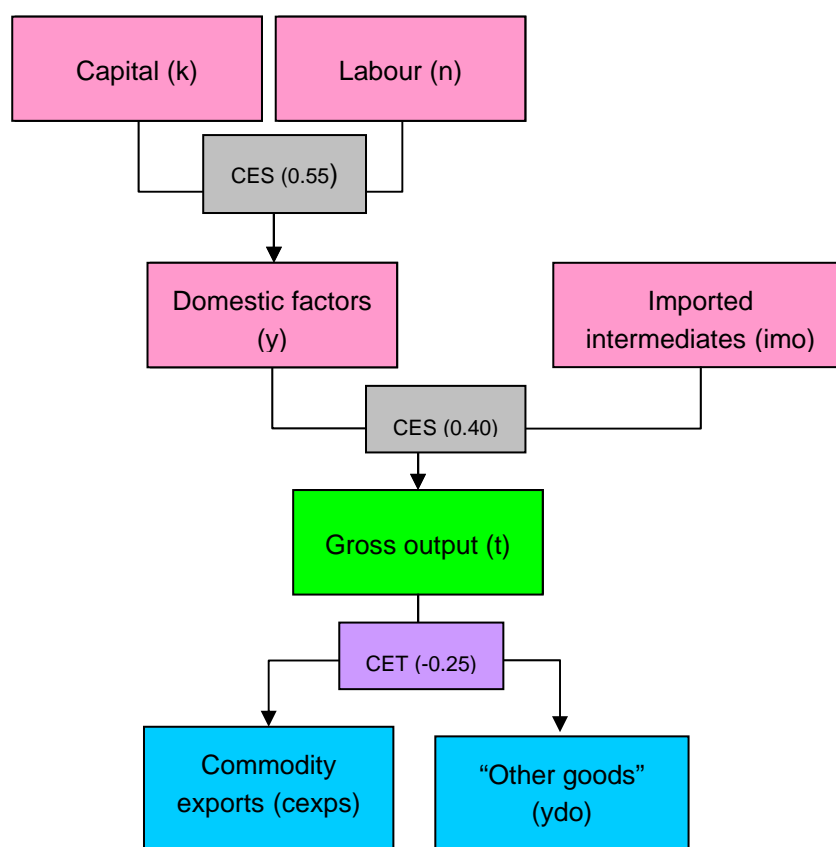
In the past, NZTM made the assumption that all imports were intermediate inputs in the production process. One of the key changes since Szeto (2002) is the disaggregation of imports into four different components: consumption goods and services, capital goods, household overseas spending and intermediates. Disaggregation of imports helps the Treasury to tell a more detailed forecast story, as well as being useful for the purposes of modelling inflation, which we will discuss more in Section 5.1.

Another change to the production block is that we do not aggregate commodity exports and non-commodity exports into total export goods. Instead, we treat commodity export goods as a separate output and aggregate the rest of the output as “other goods”, reflecting that it is more suitable to group non-commodity exports with other domestically produced goods because of the nature of the production process.

The production block is composed of two nested constant elasticity of substitution (CES) functions and a constant elasticity of transformation (CET) function. In order to maximise profits, the representative firm can be thought of as making three decisions (summarised in Figure 3):

- i the mix of capital and labour (known collectively as domestic factors) they use
- ii the mix of domestic factor input (y) and imported intermediates they use
- iii the mix of commodity exports and “other goods” they produce.

Figure 3: The production block of the business sector



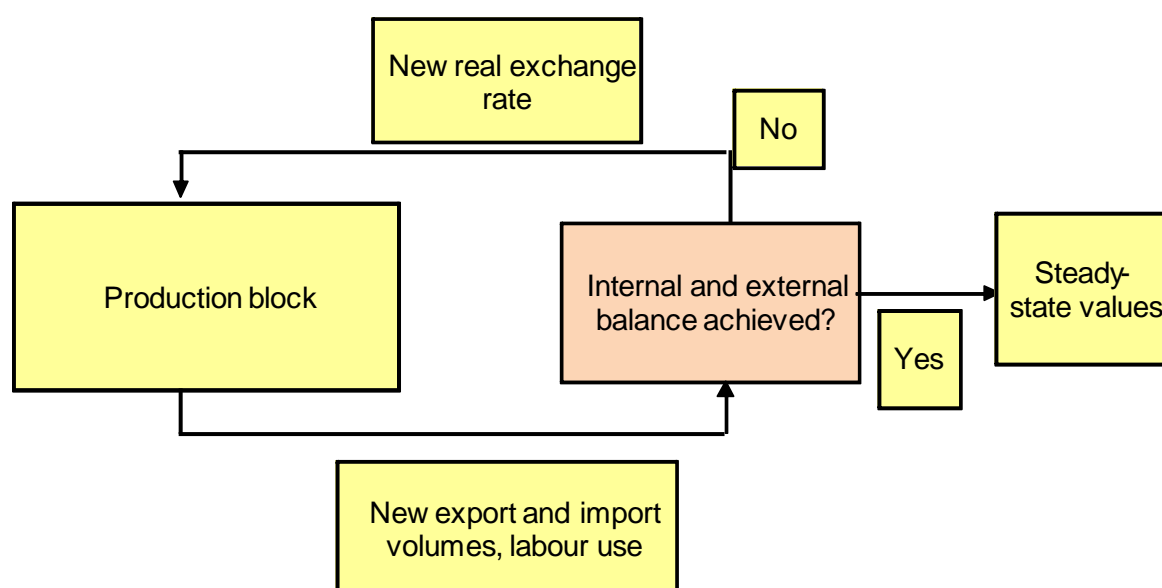
3.1.1 The production block and the rest of the economy

For ease of exposition, we treat some variables as “exogenous” to the production block in this chapter. For reasons we will discuss below, these variables are not strictly exogenous as their values are dependent on what happens in the production block. The relative prices (real exchange rate) of imported intermediates and commodity exports are examples of such variables.

To explain this point, consider solving the steady state of the model as an iterative process (see Figure 4).⁸ For a given real exchange rate, the production block will give us commodity export and imported intermediate volumes as well as the firms’ demand for labour. These values may or may not be consistent with external balance and internal balance. If external and internal balance is not achieved with this real exchange rate, the real exchange rate will need to adjust until these critical balances are met (see Section 2.2 for explanation of the role of the real exchange rate in simultaneously achieving internal and external balance). Given the foreign price of the tradable goods is exogenous, it will need to be the nominal exchange rate or the domestic price level that adjusts. For example, if with the current real exchange rate, external debt is greater than target, the relative price of tradable goods to domestic goods will need to increase to encourage the production of exports and slow the use of imported intermediates. Therefore, the relative prices of imported intermediates and commodity exports (real exchange rates) are simultaneously determined within the whole model.

⁸ This is an abstraction as steady-state values are simultaneously determined.

Figure 4: The production block and internal and external balance



3.1.2 The input decisions

1. The mix of capital and labour

The profit-maximising firm in NZTM seeks to choose the amount of capital (k) and labour (hours paid, n) that minimises the cost of production for a given use of domestic factors.⁹ The functional form of the production function is a CES (constant elasticity of substitution) equation. This means the rate of response to the relative price changes is constant, regardless of the level of the capital/labour ratio. The current estimation of the production block means a 1 percent increase (decrease) in the price of labour (w) relative to the price of capital leads to 0.55 percent decrease (increase) in labour/capital ratio.

Figure 5 shows how the input decisions are made, and therefore how medium-run variables are determined in the model. In Figure 5, $c_0(y_0)$ represents the isocost curve. In the medium run, the amount of capital is “exogenous” to the block, as capital accumulation takes time, and the wage and the demand for “other goods” are also determined outside (or exogenous to) the production block. Given the level of capital is fixed, the medium-run value of labour input (n_0^{mr}) is determined by the intersection of a vertical line drawn at the given level of capital (k_0). The rental/wage ratio (ar^0) is also determined by the slope of the isocost curve where the firm uses k_0 units of capital. Given the wage rate, the medium-run value of the rental price of capital can be found by the factor-price ratio.

Figure 6 shows the impact that an increase in labour productivity has on the input decisions in the model in both the medium run and steady state respectively. In the medium run, an increase in labour productivity will lead to the inward movement of the isocost function from $c_0(y_0)$ to $c_1(y_0)$, which implies less labour needed to produce the same amount of output for a given level of capital. Given the level of capital is fixed in the medium run at k_0 , higher labour productivity sees the medium-run value of labour input fall from n_0^{mr} to n_1^{mr} .

⁹ A standard result from microeconomic theory is that the bundle of inputs that minimises the cost of producing a given level of output, is also the profit-maximising combination of inputs.

However, in the steady state, both wages and capital are no longer exogenous to the production block. On the other hand, hours paid become exogenous (based on working-age population) as does the rate of return on capital. The rental price of capital is determined mainly by the neutral long-term interest rate and the risk premium for investment. In the steady state, firms will employ more capital ($k_1 - k_0$) to increase their output from y_0 to y_1 because of higher labour productivity. The new steady-state value of capital is equal to k_1 . Given the factor-price ratio (ar_1), hours paid, and the rental price of capital, we can work out the new steady-state wage.

The production function in NZTM incorporates a labour-augmenting technological process. Labour-augmenting technological progress is technological innovation that makes labour more productive. Solow (1956) showed that a production function with labour-augmenting technological process implies the growth rate of the capital to labour ratio equals the rate of technological progress. In other words, if the technology is assumed to improve over time, it implies the capital-to-labour ratio is also increasing over time. Figure 7 shows that this is consistent with the New Zealand data and therefore the labour-augmented production function is a reasonable assumption in the New Zealand context. Furthermore, Barro and Sala-i-Martin (1995) show that labour-augmenting technological change is consistent with the existence of a steady state in the neoclassical growth model.

Figure 5: Determination of employment and wages

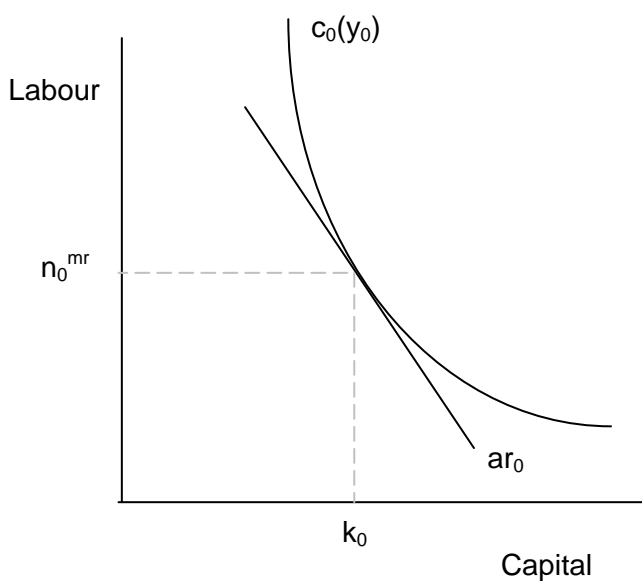


Figure 6: Impact of higher labour productivity

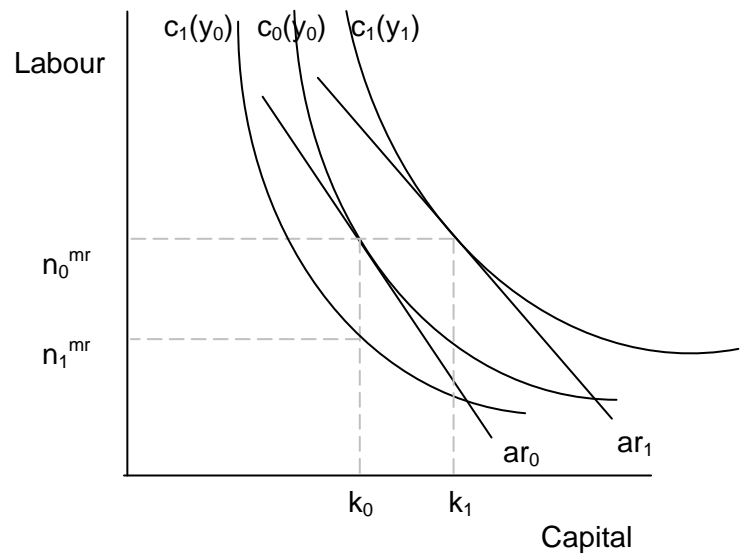
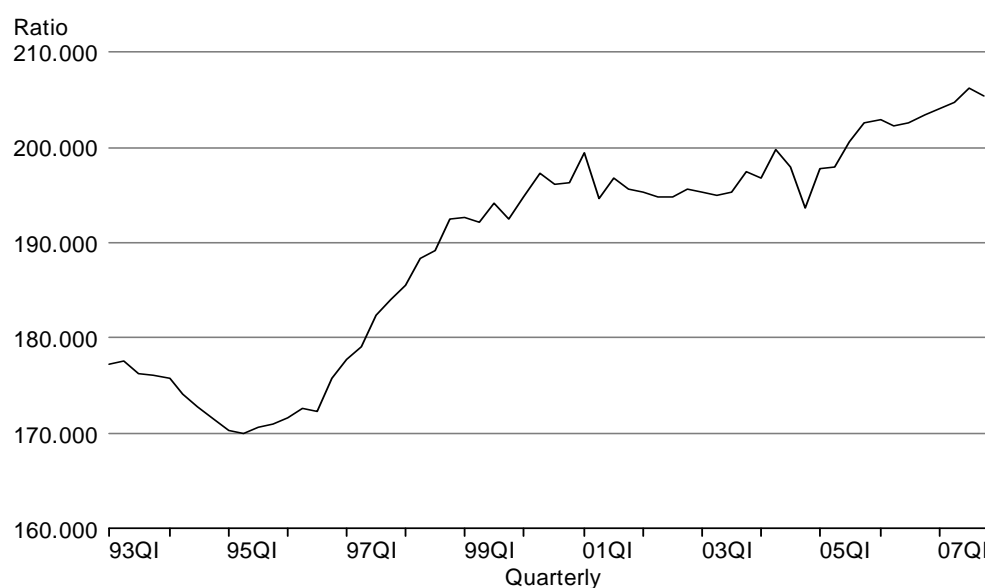


Figure 7: Ratio of business capital stock to private sector hours paid



Source: Statistics New Zealand, Treasury

2. The mix of domestic factors and imported intermediates they use

Similar to the first decision, the firm determines its split between domestic factors and imported intermediate goods that minimises the cost of producing a given level of gross output (t) based on the relative price of domestic factors to imported intermediates. In the current estimation of the production block, a 1 percent increase (decrease) in the price of domestic factor inputs (py) relative to imported intermediates (p_{mo}) leads to a 0.40 percent decrease (increase) in the ratio of domestic factors to imported intermediates. Again, the functional form implies a constant elasticity of substitution, so again the ratio of domestic factors to imported intermediates does not matter.

In the past, the model allowed for the increasing real share of imported intermediates in the production process. However, the results of the current estimation suggest that the increasing trend is very small, reflecting that imported intermediates no longer include all imports. There is an equivalent term in the exporting equation and the new estimate is very close to zero.

3.1.3 The output decisions

Firms in NZTM choose to produce either commodity exports or “other goods”. The “other goods” production is then split into non-commodity exports and domestically consumed goods and services (this occurs outside the production block and is discussed more fully in Section 4.4). The rationale for having two distinct outputs in the model is that commodity exports and “other goods” are not easily transformable in production because a significant proportion of New Zealand’s commodity goods are primary-based.

An increase in the relative price of commodity exports to “other goods” will result in an increase in commodity export production relative to “other goods” production. The current estimation of the production block means a 1 percent increase (decrease) in the relative price of commodity exports to “other goods” leads to a 0.29 percent increase (decrease) in the supply of commodity exports relative to “other goods”. Similar to the input equations, the functional form of this equation is such that it implies a constant elasticity of transformation – this means that regardless of the ratio of commodity export production to “other goods” production, the proportionate rate of change of the output ratio to the proportionate rate of change of the relative price changes is constant. For a more detailed description of the production block, refer to Powell and Murphy (1997).

4 The firm and the rest of the model

To summarise the interaction the production block has with the demand side of the model and the labour market, consider Tables 2 and 3 and Figures 8 and 9 which outline the variables that are inputs into the production block from the labour market and the demand side of the model and those which are outputs from the production block that feed into other parts of the model.

Table 2: Inputs to and outputs of the production block – dynamic (short-run) equations

Endogenous to the production block which feed into other parts of the model	Exogenous to the production block (come from other parts of the model or are imposed)
Medium-run private sector hours paid	Demand for “other goods”
Medium-run relative price of “other goods”	Private sector business capital stock
Medium-run commodity export production	Wages relative to the price of “other goods”
Medium-run demand for imported intermediates	The relative price of commodity exports and of imported intermediates
The relative rental price of capital	

Table 3: Inputs to and outputs of the production block – steady (long-run) state

Endogenous to the production block which feed into other parts of the model	Exogenous to the production block (come from other parts of the model or are imposed)
Demand for “other goods”	Required rate of return on capital relative to the price of “other goods”
Private sector business capital stock	Private sector hours paid
Wages relative to the price of “other goods”	The relative price of commodity exports and of imported intermediates
Commodity export production	The relative price of “other goods”
Imported intermediates	

Figure 8: The production block and the rest of the model (dynamic model)

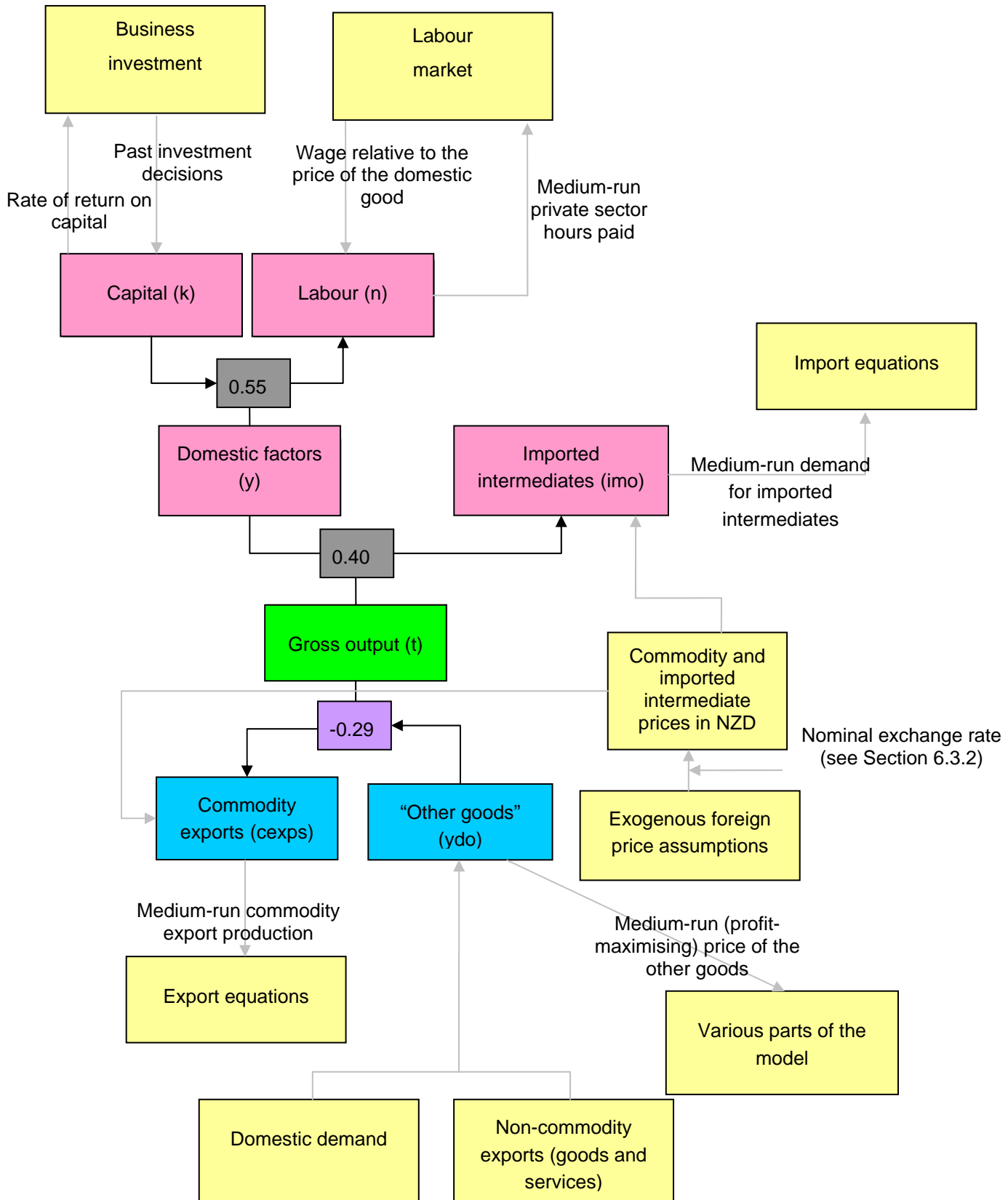
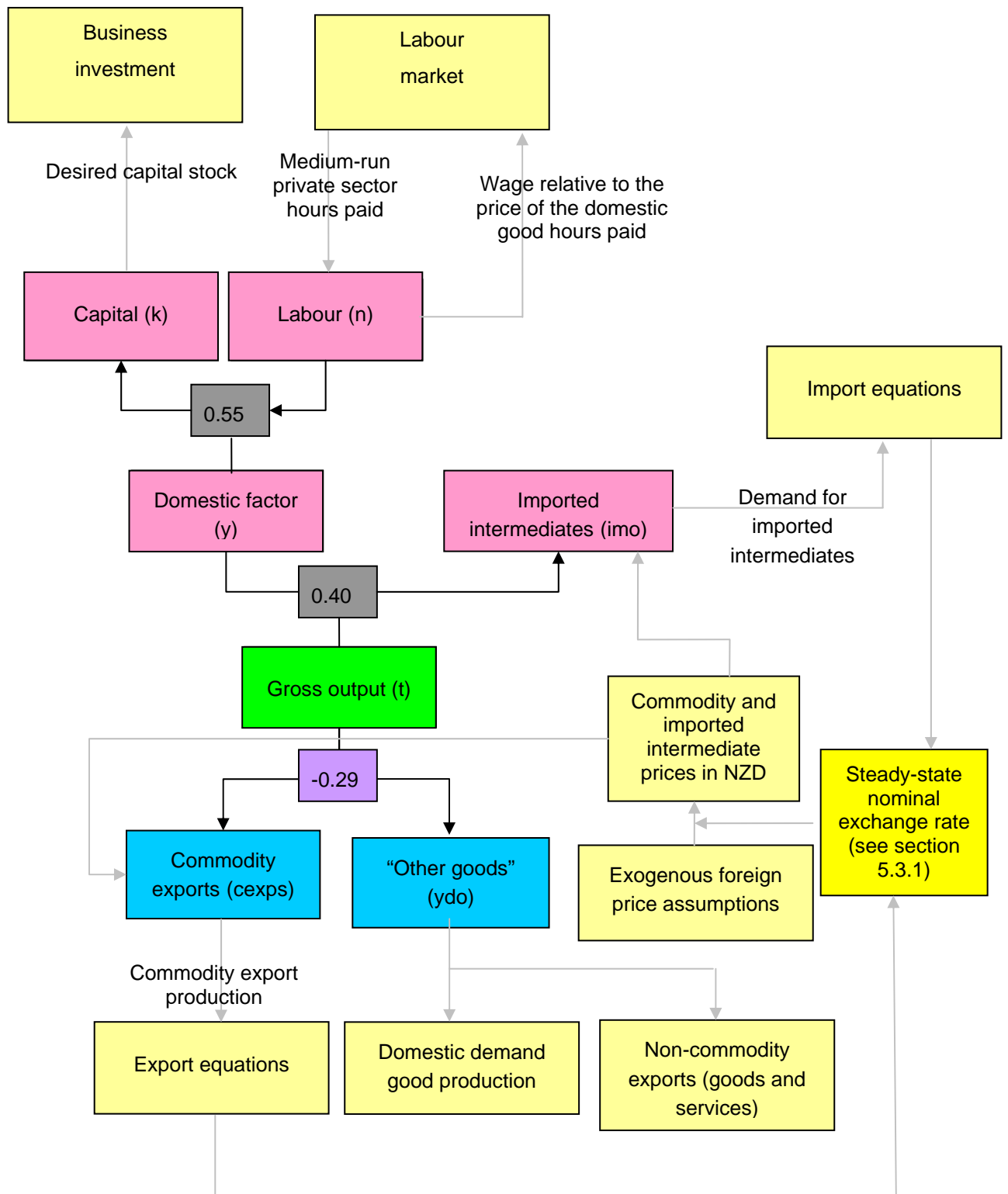


Figure 9: Production block and the rest of the model (steady state)



In the remainder of Section 4 we outline some of the equations (business investment, exports and labour market) that depend on the production block values.

4.1 Business investment

The steady-state level of capital (kbf) comes from the production block. As implied by the balanced growth path, the growth rate of the capital stock is equal to population plus productivity growth (gr_1). Business investment ($ibfr$) is such that capital grows at this rate and any depreciated capital is replaced. dr_eq is the steady-state depreciation rate.

$$kbf(+1) = kbf * exp(gr_1) \quad (4.1.1)$$

$$ibfr = kbf(+1) - kbf * exp(-1 * dr_eq) \quad (4.1.2)$$

where exp is the exponential operator.

In the dynamic part of the model, business investment will adjust to close the gap between the current capital stock and the steady-state capital stock. In the model, this adjustment occurs using the gap between the actual rate of return and the required rate of return. If the current level of capital is below the steady-state stock of capital, then the actual rate of return on capital (ar , determined in the production block) will exceed the required rate of return, meaning firms will invest more (relative to steady-state investment). Eventually, the actual rate of return converges on the required rate of return as the higher level of business investment means the dynamic value of the capital stock converges to the steady-state capital stock. The required rate of return is the rate that covers depreciation (dr_eq) plus the opportunity cost of capital (the steady-state real interest rate, ri_eq , plus a risk premium, $rp1$).

In the dynamic model, business investment also depends on the cost of borrowing to fund investment (as captured by the yield curve, $ycurve$), the relative price of capital imports ($rpmca$) and its lag ($ibfr(-1)$), which introduces frictions into the adjustment process.

$$\begin{aligned} \log(ibfr) = & \log((pa15_1 * (ibfr(-1) * exp(gr(-1))) \\ & + (1 * pa15_1) * ((exp(beta * 0.25 * popgr_eq) - exp(-1 * dr_eq)) * KBF)) \\ & + pa15_2 * tobinq(-2) + pa15_3 * ycurve(-2) \quad (4.1.3) \\ & + pa15_4 * \log\left(\frac{rpmca(-1)}{rpmca(-2)}\right) + pa15_5 * \log\left(\frac{rpmca(-2)}{rpmca(-3)}\right) \\ & + pa15_6 * \log\left(\frac{rpmca(-3)}{rpmca(-4)}\right) \end{aligned}$$

$$tobinq = (ar - (dr_eq + ri_eq + rp1)) \quad (4.1.4)$$

4.2 Capital and intermediate imports

The capital goods import equations determine the share of capital imports ($imca$) relative to the sum of business investment and government investment ($ggifr_eq$). In the steady state (4.2.1) and dynamic model (4.2.2), the share of capital imports depends on the relative price of capital ($rpmca$). The relative price of capital is the exogenous foreign

price of capital goods converted into New Zealand dollar terms by the nominal exchange rate (see Section 5) deflated by “other goods” prices (Section 6).

Steady-state equations

$$\log\left(\frac{imca}{ibfr + ggifr_eq}\right) = pa_401 + pa_402 * \log(rpmca) \quad (4.2.1)$$

Dynamic equations

$$\begin{aligned} \log\left(\frac{imca}{ibfr + ggifr_eq}\right) - \log\left(\frac{imca(-1)}{ibfr(-1) + ggifr_eq(-1)}\right) &= pa9_1 * (\log(imca(-1)/ibfr(-1) \\ &+ ggifr(-1))) - (pa401 + pa402 * \log(rpmca(-1))) \end{aligned} \quad (4.2.2)$$

Another flow we want to model is the amount of intermediate goods imported (*imo*). This is mainly dependent on the firms’ demand for intermediate imports from the production block (*imsr*). In steady state, firms’ demand for imported intermediates is identical to firms’ demand from the steady-state production block.

$$imo = imsr \quad (4.2.3)$$

In the dynamic part of the model, imports of intermediates partially adjust towards firms’ demand for intermediate imports from the dynamic production block (*imsr*), that is the profit-maximising medium-run value. The partial adjustment (rather than a full adjustment) towards the medium-run value of intermediate imports reflects that firms cannot achieve their profit-maximising level of intermediate imports straight away as it takes time to change the amount of goods imported (for example, due to contractual arrangements and transport lags). The lag, *imo(-1)*, also induces frictions in this adjustment. The parameter *beta2* is introduced to allow for the real share of imports to increase over time as the economy becomes more open.¹⁰

$$\begin{aligned} \log(imo) &= pa8_1 * \log(\exp((gr - 1) + (-1 * beta2 * 0.25)) * imsr(-1)) + \\ &(1 - pa8_1) * \log(\exp((gr - 1) + (-1 * beta2 * 0.25)) * imo(-1)) \end{aligned} \quad (4.2.4)$$

4.3 Commodity exports

Analogous to the imported intermediate equation, in steady state commodity exports (*cexps*) equals its profit-maximising value from the steady-state production block (*exrsr*).

$$cexps = exrsr \quad (4.3.1)$$

In the dynamic model, analogous to the intermediate imports equation, commodity exports (*cexps*) adjust towards the medium-run profit-maximising level of exports from the dynamic production block (*exrsr*). The extent to which commodity production can be varied is restricted by biological limits,¹¹ and therefore the adjustment parameter towards

¹⁰ The parameter *beta2* is estimated to be smaller than those estimated earlier (in absolute value) because imported intermediates no longer include capital and consumption goods. Both *beta2* and *beta1* are set to zero in the forecasting period.

¹¹ For example, in the very short-run a cow can produce only so much extra milk. Forestry, with its long growing times, is an extreme example of just how long these adjustments can be.

the profit-maximising level of commodity exports ($pa5_1$) has quite a low value (currently 0.1). In the absence of further shocks and any other adjustments, it takes about 7 quarters to make half of the adjustment. A lagged term is also used to introduce slow adjustment ($cexps(-1)$). $Beta1$ allows for the real share of commodity exports to change but the new estimate of this parameter is close to zero suggesting that the real share of commodity exports as a percentage of total private sector output is rather constant.

$$\begin{aligned} \log(cexps) = & pa5_1 * \log((\exp((gr - 1) + (-1 * beta1 * 0.25)))^3 * exrsr(-3)) + \\ & (1 - pa5_1) * \log(\exp((gr - 1) + (-1 * beta1 * 0.25)) * cexps(-1)) \end{aligned} \quad (4.3.2)$$

4.4 Non-commodity exports

The following equations determine how the composition of the “other good” (from the production block) is split between goods and services supplied to the domestic economy and exports of non-commodity goods and services.

4.4.1 Non-commodity goods exports

In steady state, the representative firm decides the ratio of non-commodity goods exports ($ncexpg$) to the production of “other goods” (ydo) based on the price received for exports of non-commodity goods relative to the price of ydo . The relative price of $ncexpg$ ($rpexncg$) is an exogenous foreign price ($pexncgf$) multiplied by the nominal exchange rate (e) and deflated by $pydo$.

$$\log(ncexpg / ydo) = pa603 + pa604 * \log(rpexncg) \quad (4.4.1)$$

In the dynamic model, the supply of non-commodity goods exports ($ncexpg$) adjusts towards its steady-state value ($encexpg$). A relative price term, the deviation of the relative price ($rpexncg$) from its steady-state relative price ($erpenxcg$), also drives dynamics, reflecting the fact that when the New Zealand dollar price of exports is higher than the price received domestically, firms will prefer to supply the export market (note that the relative prices enter with a lag reflecting that the supply response to higher prices takes time). The closing of the gap between the current relative price and the steady-state value is a key mechanism to get the dynamic values of these variables to converge to their steady-state values over time. If the relative price is currently higher than its steady-state value, this will increase the supply of the non-commodity goods exports (relative to its steady-state value), decreasing our external indebtedness and increasing the associated real exchange rate. A higher real exchange rate will decrease the relative price of exports in New Zealand dollar terms, meaning the relative price moves towards the steady-state relative price and thus helps ensure non-commodity goods exports converge to their steady-state value.

$$\begin{aligned} \log(ncexpg) = & pa7_1 * \log((\exp(gr - 1))^2 * encexpg(-2)) + \\ & (1 - pa7_1) * \log(\exp(gr - 1) * ncexpg(-1)) + pa7_2 * \log\left(\frac{rpexncg(-3)}{erpenxcg(-3)}\right) \end{aligned} \quad (4.4.2)$$

4.4.2 Services exports

In steady state, the ratio of services exports to production for domestic demand is dependent on the relative price of export services to $pydo$ ($rpexncs$) and a trend factor (TF). The trend factor reflects that travel services, which make up around half of services exports, have generally been increasing over time owing to rising global incomes promoting more spending on luxury goods such as travel. In the forecasting environment, the trend factor continues to operate for the first five years of the forecasting period. As with the exports of non-commodity goods equation, the relative price of export services is an exogenous foreign price ($pexncsf$) multiplied by the nominal exchange rate (e) and deflated by $pydo$.

$$\log(ncexps / ydo) = pa601 + pa602 * \log(rpexncs) + pa605 * TF \quad (4.4.3)$$

The relative price term in equations (4.4.3) plays a similar role as in the other export equations.

In the dynamic equation, as with the exports of non-commodity goods equation, the closing of the gap between current relative prices ($rpexncs$) and their steady-state ($erpexncs$) values is a key mechanism to get the dynamic values of these variables to converge to their steady-state values over time. Also like the exports of non-commodity goods equation, the services exports equation also features an adjustment towards its steady-state value ($encexps$).

$$\begin{aligned} \log(ncexps) = & pa6_1 * \log((exp(gr - 1))^2 * encexps(-2)) + \\ & (1 - pa6_1) * \log(exp(gr - 1) * ncexps(-1)) + pa6_2 * \log\left(\frac{rpexncs(-2)}{erpexncs(-2)}\right) \\ & + pa6_3 * \log\left(\frac{rpexncs(-3)}{erpexncs(-3)}\right) + pa6_4 * \log\left(\frac{rpexncs(-4)}{erpexncs(-4)}\right) \\ & + pa6_5 * \log\left(\frac{rpexncs(-5)}{erpexncs(-5)}\right) + pa6_6 * \log\left(\frac{rpexncs(-6)}{erpexncs(-6)}\right) \end{aligned} \quad (4.4.4)$$

4.5 Labour market

4.5.1 Employment and participation

(i) Steady-state equations

The labour force (nts) is equal to the product of the participation rate and the working-age population. In steady state, the participation rate ($partt$, based on filtering historical data, see Section 7.3) is an exogenous assumption, as is the working-age population.

$$nts = (partt / 100) * rpop3_eq * (pop3_eq + pop4_eq) \quad (4.5.1)$$

where $rpop3_eq$ is the ratio of Household Labour Force Survey (HLFS) working-age population to Statistics New Zealand estimated resident working age population.

In steady state, unemployment (urt) is set equal to the NAIRU which is exogenous. Guy and Szeto (2004) provided evidence that the NAIRU may change over time depending on the structure of the economy and government policy. This and an exogenous assumption with respect to government employment (ngg_eq) form an identity with private sector employment (nsr):

$$urt = nairu \quad (4.5.2)$$

$$nsr + ngg_eq = \left(1 - \frac{urt}{100}\right) * nts \quad (4.5.3)$$

If we multiply private sector employment by an exogenous hours paid per week assumption ($prhr_eq$) (multiplied by 13 to get hours paid per quarter) we get aggregate hours paid in the private sector ($nthpr1$), which is used as an input into the production block.

$$nthpr1 = nsr * prhr_eq * 13 / 1000 \quad (4.5.4)$$

(ii) Dynamic equations

The main driver of hours paid ($nthpr1$) in the economy is the firm's demand for hours from the dynamic production block (ie, the medium-run profit-maximising value, $nthsr$). This adjustment term towards the production block value and a lag ($nthpr1(-1)$) are used to introduce a sluggish response towards the firms' profit maximising number of hours paid. The reason hours paid do not immediately match the firm's demand in the model can be justified by the presence of search costs (time and effort it takes to find new workers) introducing frictions into the labour market. The profit margin, as proxied by the profit-maximising price of the domestic good relative to the current price of the domestic good ($rpydmr$), is included to capture the idea that when profits are high, firms will demand more labour.

$$\begin{aligned} \log(nthpr1) = & pal_1 * \log(nthsr(-1) * \exp(popgr_eq)) \\ & + (1 - pal_1) * \log(nthpr1(-1) * \exp(popgr_eq)) + pal_2 * \log(rpydmr(-2)) \end{aligned} \quad (4.5.5)$$

Knowing aggregate hours paid, and using an exogenous assumption of hours paid per person per week ($prhr_a$; multiplied by 13 to get hours paid per person per quarter), we can calculate the number of people employed in the private sector (nsr) using the following identity:

$$nsr = \left(\frac{nthpr1}{prhr_eq * 13 / 1000} \right) \quad (4.5.6)$$

Adding government employment to this (an exogenous assumption) we get the number of people employed in the whole economy.

$$nt = nsr + ngg_eq \quad (4.5.7)$$

To derive the labour force, we need to know the participation rate. In the dynamic model, the participation rate depends on its lag, adjustment toward its steady-state value as well as the extent to which unemployment is above or below the NAIRU – capturing the idea that a labour market with relatively more unemployment will mean fewer people participating due to a discouraged worker effect.¹²

$$\begin{aligned} partt = & pa2_1 * partt(-1) + pa2_2 * partt_eq + pa2_3 * (nairu - urt) \\ & + pa2_4 * (nairu(-1) - urt(-1)) \end{aligned} \quad (4.5.8)$$

We can calculate the labour force using equation (4.5.1). Unemployment can be calculated using the identity that unemployment is equal to the labour force minus those employed.

$$urt = (100 * (1 - \frac{nt}{nts})) \quad (4.5.9)$$

4.5.2 Wages

The steady-state value of real average earnings (*rwa*) is based on the hourly wage (*rhw*) from the production block (see Section 3.1) and an exogenous hours paid per week (*prhr_eq*; multiplied by 13 to get hours paid per quarter).

$$rwa = rhw * \frac{prhr_eq * 13}{1000} \quad (4.5.10)$$

In the dynamic model, wage growth (*inf_hw*) depends on:

- an exogenous labour productivity assumption (*a1*, see section 8.3 for how this is determined) – if the marginal product of labour increases, labour should be paid more
- the ratio of the profit-maximising price level of *ydo* to its actual level (*rpydmr*)¹³ – this variable is used as a proxy for the profitability of firms (ie how close the current price of the domestic good is to its profit-maximising price). Relatively low values of *rpydmr* mean firms can afford to pay workers more
- previous inflation (*inf*, second lag) and inflation expectations (*infe*, first lag) – to reflect the role inflation plays in wage setting, note both these variables are lagged to reflect the fact that wage bargaining tends to be backward looking
- tightness of the labour market (proxied by the derivation of unemployment from the NAIRU) – this is a proxy for the bargaining power of workers, and
- an equilibrium adjustment term (*rhw/erhw*).

¹² The discouraged worker effect hypothesises that the experience of unsuccessful job search (more likely in a labour market with high unemployment) increases the propensity to withdraw from the labour force.

¹³ The profit-maximising price of the domestic good comes from the production block.

$$\begin{aligned}
inf_hw = & pa18_1 * inf(-2) + (1 - pa18_1) * infe(-1) + pa18_5 * \log(a1/a1(-1)) \\
& + pa18_5 * \log(a1(-1)/a1(-2)) + pa18_5 * \log(a1(-2)/a1(-3)) \\
& + pa18_5 * \log(a1(-3)/a1(-4)) + pa18_2 * \log(rpydmr(-1)) \\
& + pa18_3 * \log(rhw(-2)/erhw(-2)) + pa18_4 * (urt(-2) - nairu)
\end{aligned}
\tag{4.5.11}$$

4.6 Private consumption

In steady state, private non-housing consumption (*conor*) is related to real after-tax labour income (*rincome*), real wealth (*rwealth*) and the relative price of non-housing consumption goods and services (*rpyd_c*). The relative price term means that the higher the relative price of consumption, the lower is real consumption.

$$\begin{aligned}
\log(conor) = & pa12_0 + pa12_1 * \log(rincome) \\
& + (1 - pa12_1) * \log(rwealth) - \log(rpyd_c)
\end{aligned}
\tag{4.6.1}$$

The elasticities on real income and wealth are currently set at 0.8 and 0.2 respectively.

The dynamic specification of the private consumption equations is based on the work of Downing and Goh (2002). Private consumption (*conor*) is assumed to adjust towards a level of medium-run consumption (*conord*; not to be confused with the steady-state value of consumption). The partial adjustment mechanism towards medium-run consumption is consistent with the Permanent Income/ Life-cycle Hypothesis (see Friedman, 1957; Modigliani and Blumberg, 1954) where consumers spread existing resource to achieve a smooth consumption profile over their life. Medium-run consumption depends on current net wealth, current after-tax labour income and households' desired debt level (*pagdpr_eq*, see section 4.6.1).

$$\begin{aligned}
\log(conord) = & pa12_0 + pa12_1 * \log(rincome) + (1 - pa12_1) * \log(rwealth) \\
& - \log(rpyd_c) + pa12_2 * \sum_{i=0}^{i=3} (pagdpr_eq(-i) - pagdpr_eq(-i-1))
\end{aligned}
\tag{4.6.2}$$

In addition to the partial adjustment towards medium-run consumption, other variables which affect the dynamics of consumption in the short-run are interest rates and the relative price of imports. Interest rates (specifically the yield curve, *ycurve*)¹⁴ negatively impact consumption with a lag of two quarters. The change in the price of imports relative to the price of domestic goods (*rpm*) also enters with lags of two and three quarters reflecting that cheaper imports will promote consumption. The yield curve and the relative price of consumption imports are lagged to reflect the fact that monetary policy and import price pass-through take time. There is also a lagged term (*conor(-1)*) to introduce habit persistence in consumption contributing to real rigidities in economic adjustment.

¹⁴ The yield curve is the 90 day interest rate less the 10 year interest rate.

$$\begin{aligned}
\log(\text{conor}) &= \text{pa13_1} * \log(\text{conord}) + (1 - \text{pa13_1}) * \log(\text{conor}(-1)) \\
&+ \text{pa13_2} * (\text{ycurve}(-2)) + \text{pa13_3} * (\text{ycurve}(-3)) + \text{pa13_4} * \log\left(\frac{\text{rpm}(-1)}{\text{rpm}(-2)}\right) \\
&+ \text{pa13_5} * \log\left(\frac{\text{rpm}(-2)}{\text{rpm}(-3)}\right)
\end{aligned}
\tag{4.6.3}$$

4.6.1 Desired debt

One of the major changes in NZTM since Szeto (2002) is it incorporates changes in the desired debt level in the determination of medium-run consumption. The motivation behind the desired debt level term is the observation that, in recent years, consumption has grown faster than labour income and wealth growth, implying that households are willing to borrow more.¹⁵ Changes in the desired debt level can be used to induce a temporary change in consumption in the short and medium term. However, there is no free lunch as any increase in consumption due to a change in the desired debt level will lead to less net wealth, reducing consumption in the long run.

4.6.2 The link between dynamic consumption and steady-state

There are two key mechanisms in getting dynamic consumption to equal steady-state consumption in the long run. The first is that consumption in the dynamic model depends on net wealth. In the dynamic equations, if the level of consumption is currently above its steady-state level and all other variables such as income and net wealth are at their equilibrium, households' net financial asset position must be in decline (ie, they are borrowing to fund their consumption). Declining net financial assets mean net wealth must decrease (net wealth = net financial assets + net non-financial assets) and therefore through equations (4.6.2) and (4.6.3) so must consumption growth in the dynamic model.

The real exchange rate also plays a role in the adjustment. Periods of consumption growth above steady-state growth will be funded by increased borrowing overseas. In order to keep external balance, the real exchange rate will need to depreciate, discouraging consumption as imports of consumption goods and services are now relatively more expensive.

Eventually consumption will have slowed through both the lower real exchange rate and lower net wealth. As consumption slows, the net financial asset position should improve, allowing consumption to grow at its steady-state rate.

4.6.3 Consumption of housing services

In both steady-state and the dynamic equations, consumption of housing services (*conh*) grows in proportion to the rate of growth of the housing stock (*kh*). However, as Figure 10 shows, the proportion of housing services to the housing stock over history has been falling, thus equations (4.6.4) and (4.6.5) are supplemented with a trend term (*TF*).

¹⁵ Alternatively it can also be used to induce precautionary saving in uncertain times. The financial crisis of 2008/09 is one such period.

Steady-state equations

$$conh = ksratio_eq * kh \quad (4.6.4)$$

$$\log(ksratio_eq) = pa501 + pa502 * TF \quad (4.6.5)$$

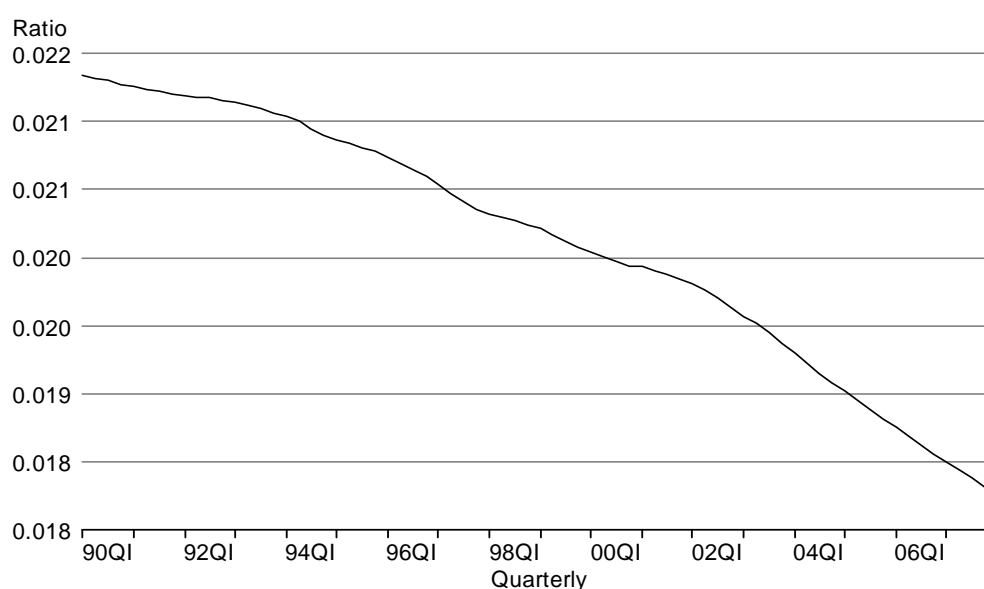
Dynamic equations

$$conh = ksratio * kh \quad (4.6.6)$$

$$ksratio = ksratio_eq \quad (4.6.7)$$

where *ksratio* and *ksratio_eq* are the ratios of consumption of housing services to the housing stock.

Figure 10: Ratio of consumption of housing services to housing stock



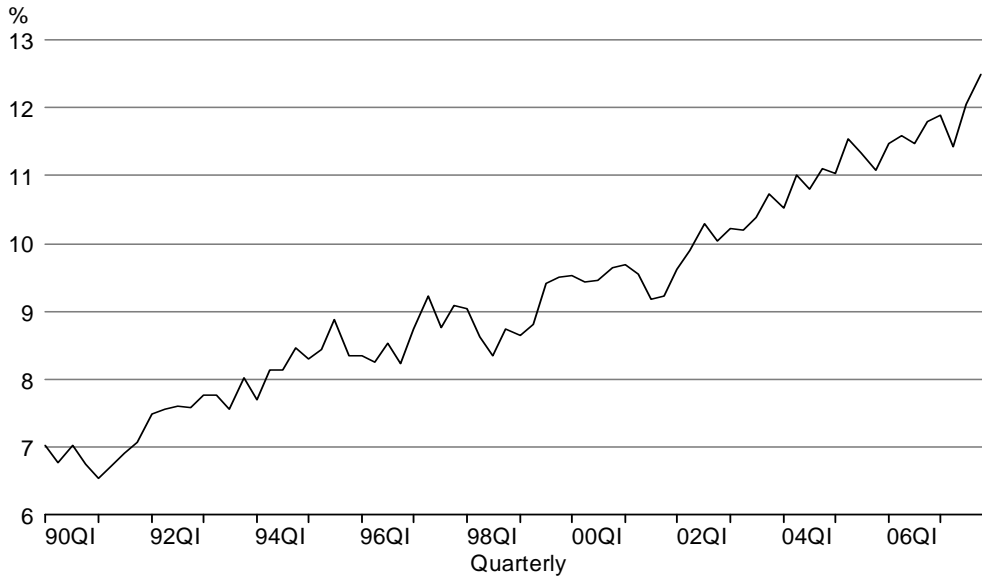
Sources: Statistics New Zealand, Treasury

4.7 Imports of private consumption goods and services

Further to the addition of the desired debt term, another important change since Szeto (2002) is the disaggregation of imports into different components (consumption, capital goods etc). In both the steady-state and dynamic model, imports of both consumption goods (*imc*) and services (*imcs*; household overseas holiday expenditure) are expressed as a share of total consumption. It is therefore useful to think about the consumption import equations as determining the share of imported consumption goods and services of total consumption (ie, the penetration ratio) rather than the level of consumption imports (which will also vary with the overall level of consumption). In both the dynamic and the steady-state model, a decrease in the price (relative to the domestic price) of imported consumption goods (*rpmc*) and services (*rpmcs*) will increase this ratio as it is now relatively cheaper to source goods and services offshore than domestically. The relative price of each of imported consumption goods and services is calculated by taking the respective foreign prices (an exogenous input) and multiplying it by the nominal exchange

rate and the inverse of the domestic price level. The trend term in both the dynamic and steady-state imported consumption goods equation (TF) reflects New Zealand's increased preference for imported goods since the removal of import controls, as well as their lower relative price because of higher productivity in the tradables sector (see Figure 11).¹⁶

Figure 11: Real consumption goods import share of total private consumption



Source: Statistics New Zealand

In the dynamic model, both imported consumption goods and services are also sensitive to changes in the nominal exchange rate ($etwit$). The sensitivity of imported consumption services partly reflects the sensitivity of New Zealand outbound travel (a large component of imported consumption services) to exchange rate movements (see Stephenson *et al.*, 2007).

Steady-state equations

$$\log(imc / conor) = pa404 + pa405 * \log(rpmc) + pa406 * TF \tag{4.7.1}$$

$$\log(imcs / conor) = pa407 + pa408 * \log(rpmcs) \tag{4.7.2}$$

¹⁶ This is the open extension of Baumol (1967).

Dynamic equations

$$\log\left(\frac{imc}{conor}\right) - \log\left(\frac{imc(-1)}{conor(-1)}\right) = pa10_1 * \left(\log\left(\frac{imc(-1)}{conor(-1)}\right) - (pa404 + pa405 * \log(rpms(-2)) + pa406 * TF(-2))\right) + pa10_2 * \log\left(\frac{etwit(-2)}{etwit(-3)}\right) \quad (4.7.3)$$

$$\log\left(\frac{imcs}{conor}\right) - \log\left(\frac{imcs(-1)}{conor(-1)}\right) = pa11_1 * \left(\log\left(\frac{imcs(-1)}{conor(-1)}\right) - (pa407 + pa408 * \log(rpms(-3)))\right) + pa11_2 * \log\left(\frac{etwit(-2)}{etwit(-3)}\right) \quad (4.7.4)$$

4.8 Residential investment

In steady state, residential investment (*ihr*) depends on growth in population and productivity (*gr_1*), as well as investment required to replace depreciated stock (*drrb_eq*). Population growth (*popgr_eq*) reflects both natural increases and migration (both exogenous assumptions) with the larger the population the more housing required. The productivity term (*cbetas*) is used as a proxy for real incomes; rising real incomes are likely to be associated with increased demand for housing, with more people likely to seek holiday homes, better quality homes or other second homes as they become wealthier.

$$ihr = kh * (exp(gr_1) - exp(-1 * drrb_eq)) \quad (4.8.1)$$

$$gr_1 = \log\left(\frac{cbetas(-1)}{cbetas(-2)}\right) + popgr_eq \quad (4.8.2)$$

In the dynamic part of the model, residential investment seeks to partially close the gap between the actual housing stock (*kh*) and the steady-state housing stock (*ekh*). The use of such a relationship to drive dynamics is consistent with work done on the housing market by the Department of Prime Minister and Cabinet (DPMC) who concluded: “Supply responses in the housing market tend to be slow as it takes time to turn undeveloped land into new houses or to subdivide land. While the response was slow, the construction industry has responded to population growth” (DPMC, 2008, p.1). In the dynamic equations, monetary policy is also assumed to impact on residential investment in the form of a yield curve (*ycurve*), with looser monetary policy leading to more residential investment relative to periods when monetary policy is tight.

$$\log(ihr) = \log((pa14_1 * (ihr(-1) * kh / kh(-1))) + (1 - pa14_1) * ((exp(beta * 0.25 + popgr_eq) - exp(-1 * drrb_eq)) * kh)) + pa14_2 * \log(kh(-1) / ekh(-1)) + pa14_3 * ycurve(-2) + pa14_4 * ycurve(-3) \quad (4.8.3)$$

5 Monetary conditions

5.1 Inflation

A change in the model since documentation in Szeto (2002) is inflation (*inf*) is now forecast as separate tradable (*inftr*) and non-tradable components (*infnt*). These components are then weighted (based on the respective weightings in the CPI) to give an overall inflation figure. The inflation equations in NZTM are similar to the Reserve Bank's *Forecasting and Policy System* (see Hargreaves *et al.*, 2006) with the key exception that expectations are assumed to be formed differently in the two models (see section 5.2).

$$inf = pa20_2*(infnt + infnt_c) + (1 - pa20_2)*(inftr + inftr_c) \quad (5.1.1)$$

where *infnt_c* and *inftr_c* are the average non-tradable inflation rate and the average tradable inflation rate respectively.

Non-tradable inflation depends on inflation expectations (*infe*) and the first and second lags of the output gap (*lgap*),¹⁷ implying firms set their domestic prices on what they expect inflation to be plus any adjustment firms choose to make based on demanded resource pressures. The delayed response of inflation to resource pressures (ie, output gap) reflects sticky prices. Sticky prices in response to resource pressures arise because the costs of changing prices mean that firms are reluctant to change prices too often. The term *algap* introduces asymmetry into the Phillips curve relationship (the relationship between the output gap and inflation) meaning an increase in a negative output gap will have less impact on inflation in absolute terms than the equivalent increase in a positive output gap (see Razzak, 1997, for evidence of such a relationship in the New Zealand context).

$$\begin{aligned} infnt = & (infe - inf_tar) + pa20_3 * lgap(-1) + pa20_4 * lgap(-2) \\ & + pa20_3 * algap(-1) \end{aligned} \quad (5.1.2)$$

where *inf_tar* is the central bank's inflation target and if *lgap*>0 then *algap* = *lgap* otherwise *algap* = 0.

Tradable inflation depends on changes in the exogenous world price of intermediate imports (*pimof*) and consumption imports (*pimcf*), the trade-weighted exchange rate (*etwit*), inflation expectations and the output gap. Inflation expectations are included to reflect that importers do have some price-setting ability (or more correctly, margin-setting) based on what they expect price changes to be. The output gap is assumed to influence tradable inflation given that a lot of non-tradables resources are involved in distributing and retailing tradable goods within New Zealand – therefore pressure on New Zealand resources can influence the domestic price of imported goods sold here. The coefficient (*pa20_5*) on inflation expectations is set at 0.5 in NZTM, compared to 1.0 in the Reserve Bank of New Zealand's *Forecasting and Policy System* (Hargreaves *et al.*, 2006) reflecting a different judgement on the relative impact of domestic versus international influences on tradables prices.

¹⁷ The output gap is positive when growth in actual output exceeds potential output (equivalent to steady-state output growth).

$$\begin{aligned}
infr = & pa20_5 * (infe - inf_tar) + \beta \alpha \sum_{i=0}^{i=3} (\log(pimcf(-i) / pimcf(-i-1)) - inf_tar) \\
& + \beta (1 - \alpha) \sum_{i=0}^{i=3} (\log(pimof(-i) / pimof(-i-1)) - inf_tar) \\
& + \beta \sum_{i=0}^{i=3} \log(etwit(-i) / etwit(-i-1)) + pa20_6 * lgap(-3)
\end{aligned}
\tag{5.1.3}$$

where $\alpha=0.15$ and $\beta=(0.18/4)$

Note that inflation sourced from imports is disaggregated into inflation that comes from consumption good imports and inflation from intermediate imports. As discussed, previously NZTM assumed that all imports were intermediate materials in the production process, meaning it did not explicitly model imported consumption goods and services. One clear advantage of the inclusion of consumption imports is that it allows a way that fluctuations in imported consumption goods prices can influence inflation. The importance of the impact of consumption import prices on inflation cannot be understated. The early part of this decade was characterised by the emergence of China and other economies (for example, India) as exporters of low-priced goods and thereby lowering inflationary pressures in countries that import from them, such as New Zealand.

The tradables inflation equation imposes a restriction that a 1% increase in the nominal exchange rate has the same long-run effect on CPI as a 1% decrease in world import prices. Combining all the coefficients suggests that a 10% increase in the nominal exchange rate would lead to about a 0.2% decrease in CPI within the first quarter and a 0.8% decrease in CPI in the long run.

5.2 Inflation expectations

Inflation expectations (*infe*) are formed by a weighted average of current inflation and inflation expectations one period ahead (weighted 0.05 and 0.95 respectively).

$$I + infe = pa20_1 * (I + inf) + (1 - pa20_1) * (I + infe(+I))
\tag{5.2.1}$$

The inflation expectation process in NZTM is based on the expectations theory of the term structure of interest rates. In particular, the expectations theory of the term structure of interest rates states that the yields on financial assets of different maturities are related primarily by market expectations of future yields, otherwise arbitrage is possible. For example, if the risk premium is zero (or constant) then the expected 40-quarter (ie, 10 year) nominal interest rate is equal to the average over the next 40 quarters of the one-quarter interest rates expected to prevail. If this did not hold (for example the long-term 10-year rate was lower) one could simply borrow at the 10-year rate and invest continually in short term rates and make a riskless profit. NZTM slightly modifies this theory to simplify computation, as well as removing the artificial cut off dates by giving expectations further into the future less weight (rather than equal as implied by the theory). Powell and Murphy (1997) show that under the assumptions outlined above:

$$rl = b * E(rl(+I)) + (1 - b) * rcs
\tag{5.2.2}$$

where b is the distributed *lead* (as opposed to *lag*), rl is the 10-year interest rate, r_{cs} is the 90-day rate and E is the expectations operator. Powell and Murphy (1997) show that under certain assumptions (namely the mean lead is 20 quarters) b is equal to 0.95.

Assuming a constant real interest rate equation and remembering the nominal interest rate is equal to the real interest rate plus inflation expectations, it can be shown that:

$$infe = 0.95 * infe(+1) + 0.05 * inf \quad (5.2.3)$$

This is the NZTM inflation expectation process.

5.3 Exchange rate

5.3.1 Steady-state exchange rate

In steady state, imports and exports are at a level so that the economy is in external balance. There is a level of the real exchange rate that will achieve this (see Section 2.1). In practical terms, the real exchange rate is expressed as the relative price of all tradable goods such as the relative price of imported intermediates ($rpmo$) and the relative price of commodity export goods ($rpexc$). This framework is based on Dornbush (1974), reflecting the importance of changes in the terms of trade faced by the New Zealand economy.

The identity in Section 2.1.2 states that net capital inflows should equal the current account deficit, is the key in determining the real exchange rate. Equation (5.3.1) is based on this identity. In steady state, net debt with the rest of the world (as a percentage of GDP, $rfdebt$) must be at its external balance level. The aggregate net debt with the rest of the world must therefore grow at steady-state GDP (ie, population and productivity growth, gr_1) and therefore so must the current account deficit (the expression of the right hand side of 5.3.1; see Table 4 for variable names). Thus the value of the real exchange rate must be such that the relative prices and quantities of exports and imports (which as we saw in sections 4.2 to 4.4 depend on relative prices) ensure the identity in equation 5.3.1 holds.

$$\begin{aligned} rfdebt * (exp(gr_1) - 1) = r_{dos} \equiv & -1 * (rpexc * cexp + rpexcnc * ncexp \\ & - (\frac{rpmo}{1 + pol5_eq}) * imsr + (\frac{rpmc}{1 + pol5_eq}) * imc + (\frac{rpmcs}{1 + pol5_eq}) * imcs \quad (5.3.1) \\ & + (\frac{rpmca}{1 + pol5_eq}) * imca + rtrospr + rtrpuos + ryospu + rmtransfer_eq) \end{aligned}$$

Table 4: Notation for current account deficit identity

Variable	Label	Variable	Label
<i>rfdebt</i>	Real foreign net debt	<i>rpmc</i>	Relative price of imported consumption goods
<i>exp</i>	Exponential operator	<i>imc</i>	Imported consumption services
<i>gr_1</i>	Lag of the growth rate in productivity and population growth	<i>rpmcs</i>	Relative price of imported consumption services
<i>rdos</i>	Real net capital inflow	<i>imcs</i>	Imported consumption goods
<i>rpexc</i>	Relative price of commodity export	<i>rpmca</i>	Relative price of imported capital goods
<i>cexp</i>	Commodity exports	<i>imca</i>	Imported capital goods
<i>rpexcnc</i>	Relative price of non-commodity exports	<i>rtrspr</i>	Real net transfers from the foreign sector to the private sector
<i>ncexp</i>	Non commodity exports	<i>rtrpuos</i>	Real net transfers from the private sector to the foreign sector
<i>rpmo</i>	Relative price of intermediate exports	<i>ryospu</i>	Real net transfers from the foreign sector to the public sector
<i>pol5_eq</i>	Customs tax rate (ie, tax on imports)	<i>rmtransfer</i>	Real migrants net transfers
<i>imsr</i>	Profit-maximising level of intermediate imports from the production block		

In general, the relative price of each traded good/service is expressed as:

$$\text{The relative price} = \frac{\text{foreign price}}{e * pydo}.$$

For example, the relative price of commodity export goods is given by:

$$rpexc = \frac{pexcf}{e * pydo} \tag{5.3.2}$$

where *pexcf* = foreign price of commodity exports, *e* = the nominal exchange rate and *pydo* is the price deflator of the other goods.

Given the foreign price is exogenous, the relative price is determined jointly by the nominal exchange rate and the domestic price level. Thus, equation (5.3.2) implies that the solution to equation (5.3.1) can be determined by the real exchange rate index (*rer*) where $rer = e * pydo$.

5.3.2 Exchange rates in the dynamic model

Equation (5.3.3) shows the medium-run value of the real exchange rate index (re) will adjust towards the steady-state value of the real exchange rate index (ere). The equation also contains an adjustment in the actual level of net foreign debt as a percent of real GDP ($dgdpr$) with the level of net foreign debt as a percent of real GDP that is consistent with external balance ($fdgdpr$). If actual debt (as a percentage of GDP) is above target, the real exchange rate will depreciate. $eregr$ is a growth term that ensures convergence in the model.

$$\begin{aligned}
 re = & 0.25 * ere + 0.25 * ere(-1) * \exp(eregr) + 0.25 * ere(-2) * \exp\left(\sum_0^1 eregr(-i)\right) \\
 & + 0.25 * ere(-3) * \exp\left(\sum_0^2 eregr(-i)\right) + pa3_1 * \frac{dgdpr(+7)}{fdgdpr(+7)} + pa3_2 * \frac{dgdpr(+8)}{fdgdpr(+8)}
 \end{aligned}
 \tag{5.3.3}$$

where \exp is the exponential operator.

Equation (5.3.4) states that the nominal exchange rate will adjust to close the gap between the lagged value of the real exchange rate index (rer) and the medium-run value of the real exchange rate index (re), with exchange rate dynamics also being influenced by uncovered interest parity (UIP). The UIP condition means if the New Zealand short-term interest rate (r_{cs}) is higher than the sum of the world short-term interest rate (r_{csf}) and the sovereign risk premium (srp) for New Zealand 90-day bank bills, the magnitude of the expected depreciation in the nominal exchange rate (and therefore loss of profit on the investment in New Zealand dollar terms) is large enough to offset the increased return on New Zealand investment.

Exchange rate dynamics are not fully governed by UIP as theory would suggest given its mixed empirical success (see King, 1998 and Stephens, 2004). Stephens (2004) also found that interest rates may not be driving the exchange rate cycle in the early 2000s with dynamics being governed more by other factors. The assumption of UIP will hold in equation (5.3.4) if both $pa4_1$ and $pa4_2$ in the exchange rate equation are set to zero.

$$\begin{aligned}
 \log(e) = & pa4_1 * \log(e(-1)) + (1 - pa4_1) * \log(e(+1)) + \log\left(\left(1 + \frac{r_{cs}}{400}\right) / \left(1 + \frac{r_{csf} + srp}{400}\right)\right) \\
 & + pa4_2 * \left(\frac{rer(-1)}{re(-1)}\right)
 \end{aligned}
 \tag{5.3.4}$$

Equation (5.3.5) reconciles the model nominal exchange rate (e) with the nominal exchange rate on a trade weighted Index ($etwti$).

$$etwti = e * \exp(c4800 + c4801 * Trend)
 \tag{5.3.5}$$

5.4 Interest rates

The current framework in New Zealand means that the central bank sets monetary policy using the official cash rate as the policy instrument to adjust the short-term nominal interest rate (r_{cs}). This adjustment in turn affects the slope of the yield curve ($r_{cs}-r_l$).¹⁸ In setting monetary policy, the central bank looks forward at an equally weighted average of the fifth through seventh leads of the deviation of annual inflation from the central bank's target (cpi_tar , currently 2%). The focus on the fifth through seventh leads of annual inflation reflects the medium-run focus of inflation targeting in New Zealand. Equation (5.4.1) also contains a term representing the difference between the 90 day bill rate and its lag to reflect how the central banks set monetary policy under uncertainty. In the presence of full information about the future, the central bank would set the policy rate at a rate that would achieve its inflation target. However, in the absence of full information the central bank is more risk averse and avoids extreme changes in interest rates - the lag term in (5.4.1) helping to achieve this dynamic.

$$r_{cs} - r_l = pa19_1 * \sum_{i=5}^{i=7} (inf_cpix(+i) - cpi_tar) + pa19_4 * (r_{cs} - r_{cs}(-1)) \quad (5.4.1)$$

where

$$inf_cpix = exp\left(\sum_{i=0}^{i=3} inf(-i)\right) - 1$$

and

10-year bond rates (r_l) are determined using the term structure of interest rates (see the discussion in section 5.2 on inflation expectations).

$$r_l = 0.05 * r_{cs} + 0.95 * r_l(+1) \quad (5.4.2)$$

¹⁸ See Black et al. (1997) for an explanation of the advantages of using the yield curve specification.

6 Deflators

This section describes how various GDP deflators are computed in NZTM. The disaggregation of the GDP deflator into more detailed components is a major change since Szeto (2002). Behaviour in NZTM is determined by relative prices. For example, in the production block, when the price of commodity exports is higher relative to the price of “other goods” (*ydo*), the firm chooses to supply more commodity exports (see Section 4). In NZTM, we choose “other goods” price (*pydo*) to be the numeraire. As a result, the relative price of *pydo* is equal to one. Once *pydo* is determined, any individual price deflator can be backed out from its corresponding relative price.

6.1 Relative prices

“Other goods” (*ydo*) is a model variable that basically describes the amount of domestic production in the economy (with the exception of commodities). *pydo* is the deflator for the *ydo* variable.

$$ydo = (conor / (1 + pol4ww) - imc - imcs) + (ibfr + ggifr - imca) + ihr + ggcor + ncexp \quad (6.1.1)$$

where the first term and second term represent domestically produced non-housing consumption and non-residential investment respectively and notation for equation (6.1.1) is given in the following table.

Table 5: Notation for “other goods” identity

Variable	Label	Variable	Label
<i>conor</i>	Real private consumption (non-housing)	<i>ggifr</i>	Real government investment
<i>pol4ww</i>	The average rate of tax on consumption of non-housing	<i>imca</i>	Real imports of capital goods
<i>imc</i>	Real imports of consumption goods	<i>ihr</i>	Real housing investment
<i>imcs</i>	Real imports of consumption services	<i>ggcor</i>	Real total government consumption (non-wage)
<i>ibfr</i>	Real private investment (non-housing)	<i>ncexp</i>	Non commodity exports

Therefore, the relative price of *pydo* is equal to the weighted average of the relative prices of domestically produced non-housing consumption (*rpyd_ca*), domestically produced non-resident investment (*rpyd_ia*), residential investment (*rpyd_h*), government consumption (*rpyd_gc*) and non-commodity export goods and services (*rpexnc*).

$$rpydo = dw_1 * rpyd_ca + dw_2 * rpyd_ia + dw_3 * rpyd_h + dw_4 * rpyd_gc + dw_5 * rpexnc \quad (6.1.2)$$

where dw_i are the weights.

The relative prices of non-housing consumption and non-residential investment are given by equation (6.1.3) and (6.1.4) respectively.

$$rpyd_c = dw_6 * rpyd_ca + dw_7 * rpmc + dw_8 * rpmcs \quad (6.1.3)$$

$$rpyd_i = dw_9 * rpyd_ia + dw_{10} * rpmca \quad (6.1.4)$$

where $rpmc$, $rpmcs$ and $rpmca$ are the relative prices of consumption goods imports, household overseas expenditure and capital goods imports respectively.

With the assumption of $rpyd_ca = rpyd_ia$, substituting (6.1.3) into equation (6.1.2) yields the following:

$$rpydo = (dw_1 + dw_2) * (rpyd_c - dw_7 * rpmc - dw_8 * rpmcs) / dw_6 + dw_3 * rpyd_h + dw_4 * rpyd_gc + dw_5 * rpexnc \quad (6.1.5)$$

Rearranging (6.1.5), the relative price of non-housing consumption can be written as:

$$rpyd_c = (1 - dw_3 * rpyd_h - dw_4 * rpyd_gc - dw_5 * rpexnc) * \frac{dw_6}{dw_1 + dw_2} + dw_7 * rpmc + dw_8 * rpmcs \quad (6.1.6)$$

The relative price of residential investment depends on its own lag and its equilibrium price ($erpyd_h$):

$$\ln(rpyd_h) = dw_{11} * \ln(rpyd_h(-1)) + (1 - dw_{11}) * \ln(erpyd_h) \quad (6.1.7)$$

With the exception of the relative price of export services, the relative prices of all export and import components are given by the exogenous values of each output, measured at world prices, divided by the nominal exchange rate index (e) and $pydo$. For example, the relative price of consumption goods imports is:

$$rpmc = \frac{pimcf}{e * pydo} \quad (6.1.8)$$

where $pimcf$ is the world price of consumption goods imports.

Reflecting that New Zealand service firms that export (predominantly tourism firms) have the power to set the price in the short run,¹⁹ the relative price of export services can deviate from its equilibrium price ($erpexncs$):

$$\ln(rpexncs) = dw_{12} * \ln(rpexncs(-1)) + (1 - dw_{12}) * (\ln(erpexncs) - dw_{13} * (\ln(erpexncs(-1)) - \ln(rpexncs(-1)))) \quad (6.1.9)$$

Equation (6.1.9) can be rewritten as:

$$\ln(rpexncs) = dw_{12} * \ln(rpexncs(-1)) + (1 - dw_{12}) * (\ln(erpexncs) - dw_{13} * (\ln(e(-1) * pydo(-1)) - \ln(ere(-1)))) + z_rpexncs \quad (6.1.10)$$

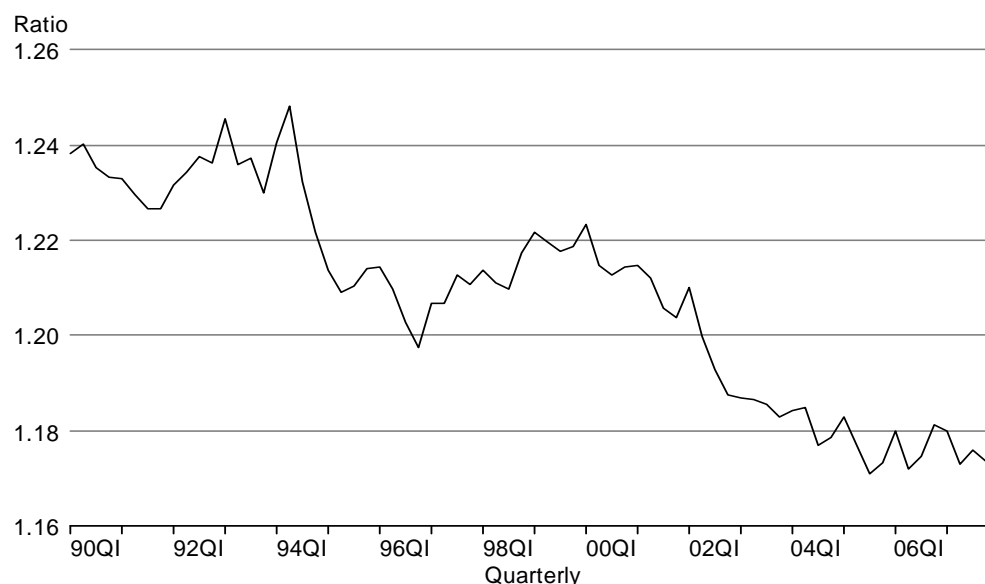
where ere is the equilibrium steady-state composite value of e and $pydo$.

¹⁹ This assumption can be justified by the fact that service firms tend to sell differentiated products and therefore are closer to monopolistic competitive, than perfectly competitive (if they sold the same product).

6.2 Prices

We now have various relative prices for each expenditure component (consumption, investment etc), but what we are interested in is absolute prices. Thus we would like a measure of the price of “other goods” (*pydo*) to multiply the individual relative prices to get the absolute price level. To do this we first model the non-housing consumption deflator (*pyd_c*) using the CPI excluding the housing component (*cpixh*). In theory, both price indices are designed to measure the same component of the consumption sector. Figure 12 shows the ratio of *pyd_c* to *cpixh*, which suggests a linear time trend over the sample period. The linear time trend could be due to different methods in constructing the indices. The CPI is based on a fixed-weight formula and the consumption deflator is based on a chain-linking formula. Currently, the CPI now has an expression base of June 2006 quarter = 1000. Chain linking means constructing price measures by cumulating movements in short-term indices with different base periods.

Figure 12: The ratio of the consumption deflator to the CPI ex housing



Source: Statistics New Zealand, New Zealand Treasury

Thus we express the non-housing consumption deflator relative to the CPI (excluding housing) with a constant and trend term to capture the differences:

$$\ln\left[\frac{pyd_c}{cpixh}\right] = pa701 + pa702 * TF + z_pydc \quad (7.11)$$

The CPI (*cpix*) is determined elsewhere in the model (see section 5). Given the CPI we can use the following relationship to estimate *cpixh*.

$$cpix = dw_{14} * cpixh + (1 - dw_{14}) * cpihouse \quad (7.12)$$

where *cpihouse* is a price index of the CPI housing component.

Once *pyd_c* is estimated using equation (7.11), we can then use the non-housing consumption deflator to calculate the “other goods” price level (using $rpyd_c = pyd_c / pydo$). Finally we can use *pydo* to back out the rest of the individual deflators (business investment etc) from the relative prices for each component.

7 NZTM in the forecasting process

7.1 The role of NZTM in the forecasting process

In a typical year the Treasury produces two sets of forecasts, the *Budget Economic and Fiscal Update* and the *Half Year Economic and Fiscal Update*. As discussed in the introduction, NZTM has played an increasing role in the formation of these forecasts. Mawson (2005) notes the advantage of using a model to forecast is:

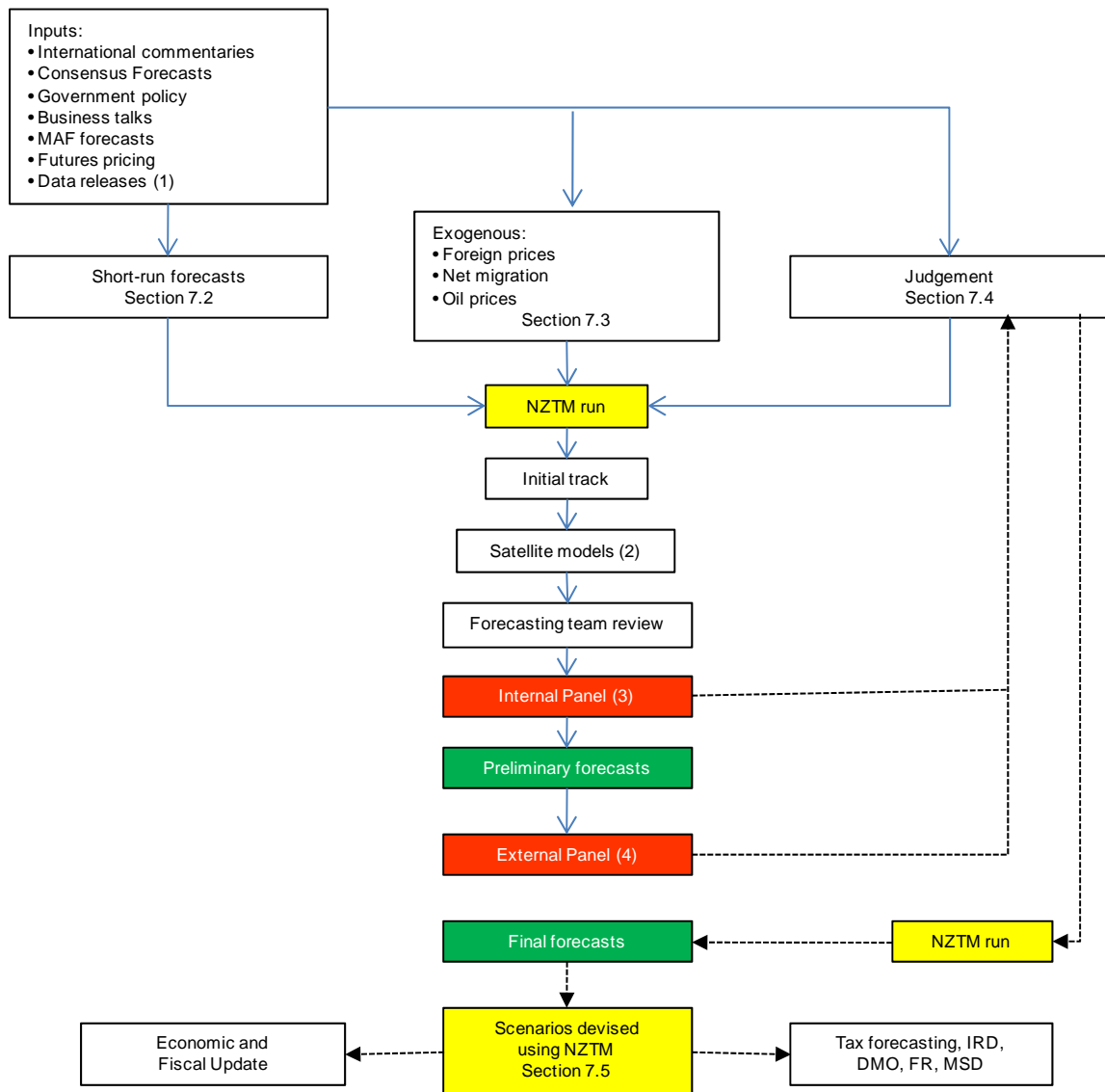
- To require forecasters to be explicit about how different variables relate to each other
- To ensure internal consistency, meaning changes to the forecast path are flowed through consistently, and
- To force constraints and long-run anchors to be adhered to.

The interaction of the model with the rest of forecasting process can be stylised using the following diagram (see Figure 13). Sections 7.2-7.5 discuss each of the different interactions between the model and the forecast process.

7.2 Short-run forecasts

The Treasury uses a set of indicator models to forecast the short-term (typically the first, second and sometimes third quarters after the latest release of GDP). The indicator models are designed to use available macroeconomic data (typically from Statistics New Zealand but also from consumer and business confidence surveys and other sources) as well as the sector analysts' judgement to formulate short-term forecasts. At the Treasury, these indicator models are typically single equation models (for example, using building consents and house sales to predict residential investment). Recently the Treasury has also employed factor models in the style of Matheson (2006) to forecast GDP one to four quarters ahead. In addition, the judgement component is informed by information obtained by visiting businesses or other information available. These forecasts are then put into NZTM as if they are historical data

Figure 13: NZTM and the forecast process



- (1) For example from Statistics New Zealand, the Reserve Bank, Datastream and consumer and business confidence surveys.
- (2) These take aggregate model output and disaggregate it as a consistency check on model output. For example, estimating a household saving rate by constructing a Household Income Outlay Account.
- (3) This is a panel of senior Treasury staff that review the forecasts.
- (4) This is a panel of external experts which include academic scholars and ex-public and private sector forecasters.

7.3 Exogenous assumptions

Before either the dynamic model or the steady-state model can be calculated, a number of the values of certain exogenous variables must be specified. Key exogenous variables are:²⁰

- Export and import price tracks in foreign price terms
- Government consumption, government investment and government employment
- The NAIRU, average hours paid per week and labour force participation rate
- Productivity variables

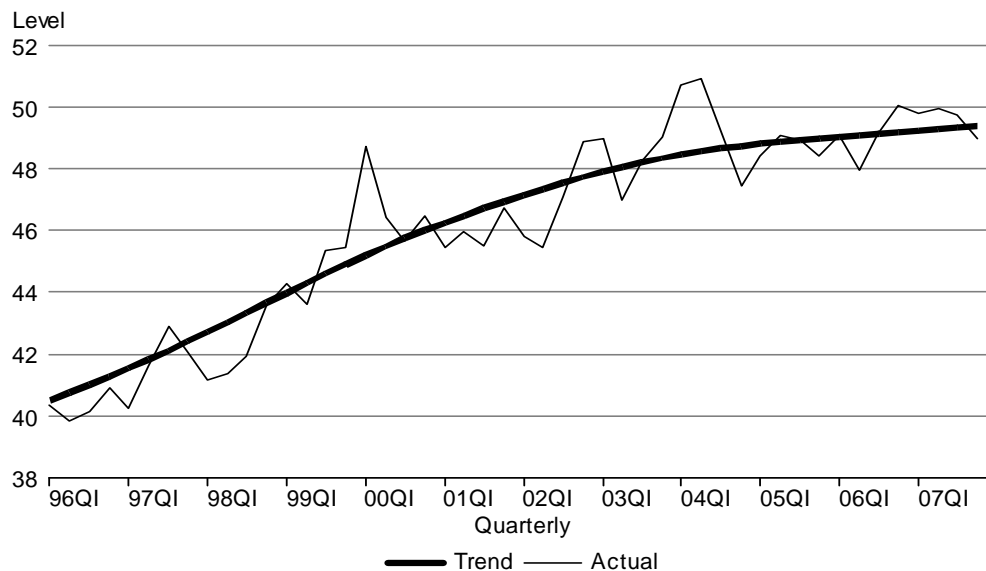
²⁰ A full list of exogenous variables is available in the Appendix.

- Desired debt level of households
- Population growth, and
- Net foreign debt target.

Export and import price tracks in foreign price terms, as well as the government expenditure track, are supplied by members of the Treasury forecasting team responsible for the external and government sectors respectively. The exogenous nature of foreign prices is consistent with New Zealand’s small open economy.

The time path for the NAIRU, the participation rate, productivity variables and the desired debt level of households are determined by filtering the data. Filtering is a process of using statistical methods either to decompose a historical time series into its trend and cyclical components, for a series where the variable of interest is observable; or, in the case where the variable is not observable (ie, not measured; for example, the NAIRU),²¹ use other observable variables to estimate their value. For observable variables we can then use the recent trend of the variable of interest as the basis for inputting into the model as the future track of that variable in the forecast period.²² Figure 14 shows the filtered trend over history for the labour productivity series.

Figure 14: Estimation of the historical trend of the labour productivity series



²¹ The process for determining the NAIRU is documented in Guy and Szeto (2004).

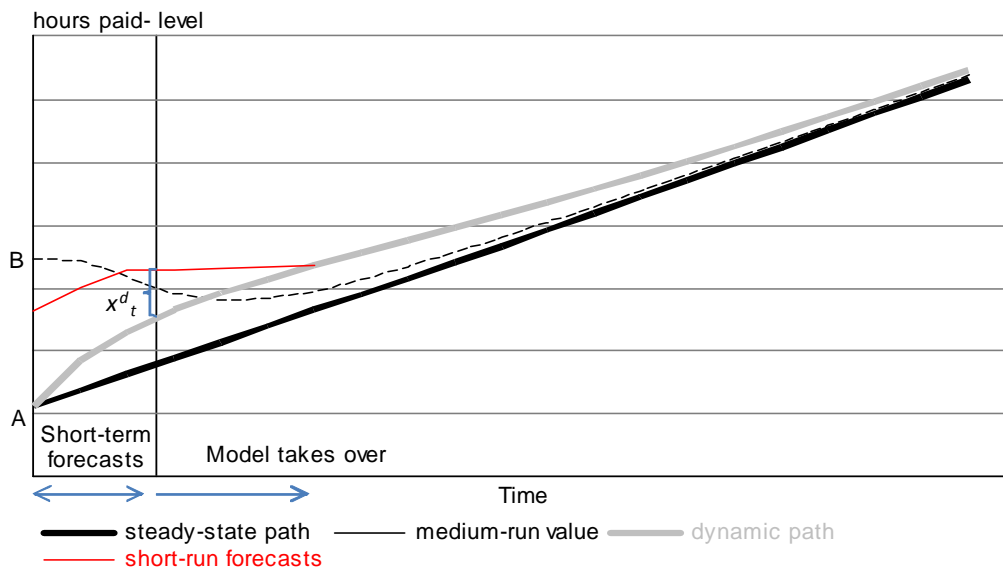
²² We described the recent trend as being the basis for the forecast. We used the term “basis” as sometimes the trend is adjusted in the forecast period to ensure that forecast trends and end points (ie, where the track ends up at the end of the forecast period) are not too dissimilar to previous forecasts. The rationale for this is we do not want large changes in the forecasts because of radically different estimation of trends; especially given the well-known sensitivity of filtering techniques to the end point of the sample they are estimated over.

7.4 Judgement

A final factor that can affect the dynamic path is judgement. When used as part of Treasury’s forecasting process, judgement has generally been applied when there are situations that the model is not particularly well equipped to handle occur. Recent examples when we have imposed judgement include the impact of the Emissions Trading Scheme on inflation and the impacts on the international investment position from higher borrowing costs due to the financial market turmoil that arose from the US sub-prime mortgage crisis. Judgement may also be required to deal with data anomalies.

Another situation where judgement is used is to smooth the adjustment between short-run forecasts (see Section 7.2) and the model’s forecast of the dynamic path. Referring to the stylised diagram of the model below (Figure 15), it is most likely that the short-run forecast value provided by the Treasury forecasting team for these quarters will differ from the model’s dynamic path value; we denote this difference as “residual” (x_t^d) in Figure 15). We need to make some assessment of the adjustment back to the model’s dynamic path, x_t^d . If we allow adjustment straight back to the model’s dynamic value, then we are implicitly assuming that previous deviation was due to factors specific to the previous quarter. If, on the other hand, the sector analyst’s forecast has some information value (ie, reflects something the model is not formulated to capture) and thus needs to be taken into account in the model’s forecast period, we will give the “residual” some time to unwind (as in Figure 15).

Figure 15: Stylised diagram showing the connection between the sector forecast, the dynamic path and the steady-state path

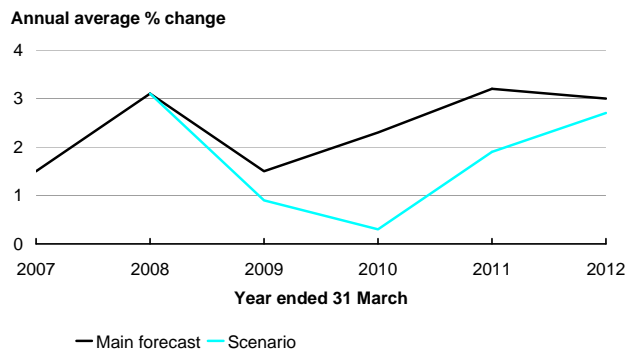


7.5 Scenarios

As we have discussed, the central forecast presented incorporates a number of key exogenous assumptions and judgements about how various forces affecting the economy will evolve. These judgements reflect the balancing of both positive and negative risks facing the economy to arrive at our best assessment of how it is likely to develop. Given the implications for tax revenue, it is important to give ministers an idea of the impact on the economy if key assumptions were different. The model is useful for this type of analysis as it allows Treasury to vary one or more key parameters/assumptions, holding the rest constant and to flow the changes through consistently. A recent example where using the model to run an alternative scenario was to look at the impact on the New Zealand economy if the financial market crisis originating from the 2007 sub-prime crisis was more severe and protracted than we had assumed in our main forecast (see New Zealand Government, 2008 for more information). To run such a scenario in the model involved (relative to the central forecast):

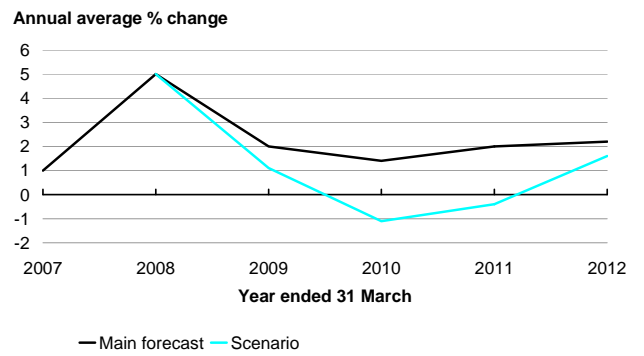
- Imposing a higher interest rate track as such a situation would result in an increased risk premium
- Imposing weaker demand for exports, particularly commodities and tourist services as global growth and therefore incomes will be lower
- Imposing weaker private consumption and business investment as New Zealand households become more cautious in their spending behaviour (ie, engage in precautionary saving) and firms become reluctant to invest in a more uncertain environment.

Figure 16: Real GDP



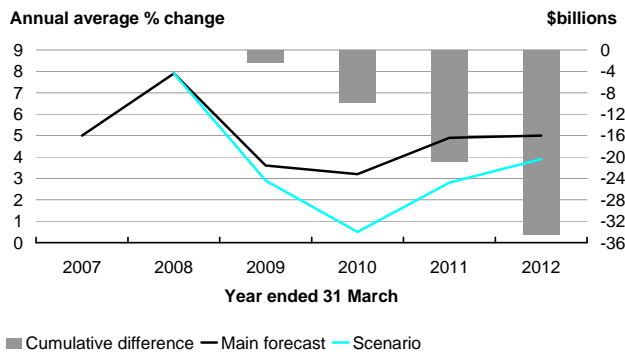
Sources: Statistics New Zealand, The Treasury

Figure 17: Real GNE



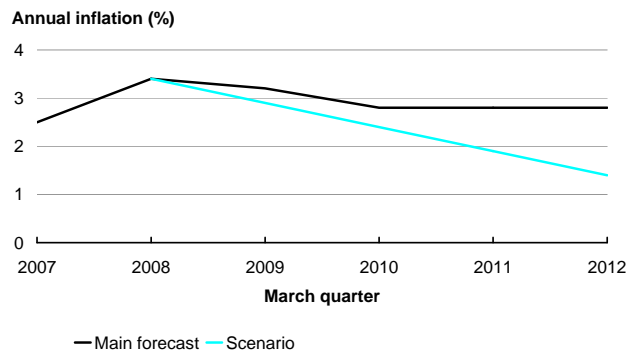
Sources: Statistics New Zealand, The Treasury

Figure 18: Nominal GDP



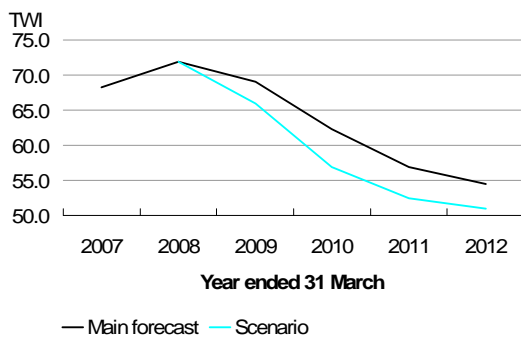
Sources: Statistics New Zealand, The Treasury

Figure 19: Inflation



Sources: Statistics New Zealand, The Treasury

Figure 20: Trade weighted index



Sources: Reserve Bank, The Treasury

Figures 16 to 18 show that such a situation would lead to lower real GDP and GNE, with this slower domestic activity reducing inflation pressures (offsetting the inflationary pressures of the falling exchange rate; see Figures 19 and 20). Lower inflation and real activity would lead to lower nominal GDP and therefore tax revenue.

8 Conclusion

8.1 Conclusion

In this paper we gave a brief overview of the general structure of NZTM as well as some of its equations. Additionally, we outlined additional changes to the model since the previous documentation of the model (Szeto, 2002). The majority of these changes reflect developments to make the model more amenable to forecasting, reflecting NZTM's increasing role as part of the New Zealand Treasury's forecasting process. Salient changes include splitting the inflation equations into tradables/non-tradables components, the use of a desired household debt term to allow households to temporarily consume faster than their income and wealth would imply (reflecting the recent experience in New Zealand) and the introduction of more disaggregation, particularly of import components and deflators.

Any model should evolve and we expect that this paper will need to be updated as the equations in NZTM are continually modified to reflect changes both in the sciences of economics and economic modelling and the structure of the New Zealand economy.

References

- Barro, R. and Sala-i-Martin X. (1995). *Economic Growth*. McGraw Hill.
- Baumol, W. (1967). 'Macroeconomics of unbalanced growth: The anatomy of urban crisis', *The American Economic Review* vol. 57(3), pp. 415-426.
- Black, R., Cassino, V., Drew, A., Hansen E., Hunt B., Rose D. and A. Scott (1997). 'The Forecasting and Policy System: the core model', Reserve Bank of New Zealand, Research Paper no. 43, August 1997.
- Dornbusch, R. (1974). 'Tariffs and nontraded goods', *Journal of International Economics* vol. 4(2), pp. 177-185.
- DPMC (2008). 'Final Report of the House Prices Unit: House Price Increases and Housing in New Zealand', Report prepared by the House Prices Unit, Department of the Prime Minister and Cabinet.
- Friedman, M. (1957). 'A Theory of the consumption function', National Bureau of Economic Research, Princeton.
- Goh, K. and Downing R. (2002). 'Modelling New Zealand consumption expenditure over the 1990s', New Zealand Treasury, working paper no. 02/19.
- Guy, M. and Szeto K. (2004). 'Estimating a New Zealand NAIRU', New Zealand Treasury, working paper no.04/10.
- Hargreaves, D., Hodgetts, B. and Kite H. (2006). 'Modelling New Zealand inflation in a Phillips curve', *Reserve Bank of New Zealand Bulletin* vol. 69(3), pp 23-37.
- King, A. (1998). 'Uncovered interest parity: New Zealand's post-deregulation experience', *Applied Financial Economics* vol. 8(5), pp. 495-503.
- Matheson, T. (2006). 'Factor model forecasts for New Zealand', *International Journal of Central Banking* vol. 2(2), pp. 169-238.
- Mawson, P. (2005). *NZTM: Overview presentation*. Presentation to the New Zealand Treasury's Forecasting and Monitoring team.
- Meade, J. (1951). *The Balance of Payments*. Oxford University Press.
- Modigliani, F. and Blumberg R. (1954). 'Utility Analysis and the Consumption Function: An interpretation of cross-section data', in *Post Keynesian Economics* K K Kurihara (eds.), London: George Allen and Unwin.
- New Zealand Government (2008). *Budget 2008: Economic and Fiscal Update*. New Zealand Government, Wellington, New Zealand.
- Powell, A. and Murphy C. (1997). *Inside a modern macroeconometric model: A Guide to the Murphy model*. Lecture notes in economics and mathematical systems, Springer, Australia.

- Razzak, W. (1997). 'The inflation-output trade-off: Is the Phillips Curve symmetric? A Policy Lesson from New Zealand', Reserve Bank of New Zealand Discussion Paper No G97/2.
- Salter, W. (1959). 'Internal and external balance: the role of expenditure effects', *The Economic Record* vol. 35(71), pp.226–238.
- Solow, R. M., (1956). 'A contribution to the theory of economic growth', *Quarterly Journal of Economics* vol.70(1), pp. 65-94.
- Stephens, D. (2004). 'The equilibrium exchange rate according to PPP and UIP', Reserve Bank of New Zealand, Discussion Paper no. 2004/03.
- Stephenson, J., Viat, S., Walton, M., Wang, A. (2007), 'Exchange rate and tourism relationships in New Zealand', *Report to the Ministry of Tourism*, New Zealand Institute of Economic Research.
- Swan, T. (1955). 'Longer run problems of the balance of payments', in *The Australian Economy: A Volume of Readings* H.Arndt and M. Corden (eds.), Cheshire Press, Melbourne.
- Szeto, K. (2001). 'An econometric analysis of a production function for New Zealand', New Zealand Treasury, working paper no. 01/31.
- Szeto, K. (2002). 'A dynamic CGE model of the New Zealand economy'. New Zealand Treasury, working paper no. 02/07.

Appendix 1: Steady-state equations

Please note that the definitions of variable names are available in Appendix 3.

Notation:

Endogenous variables are in bold.

Trend variables are denoted by `_eq` at the end.

Policy variables are denoted by POL at the beginning.

Parameters are denoted by pa at the beginning.

Lag variables are denoted by (-k) after the variable name; lags of one quarter are denoted by (-1).

Lead variables are denoted by (+k) after the variable name; leads of one quarter are denoted by (+1).

The production block

Profit-maximising conditions

$$\mathbf{rptlr} = ((\mathbf{rpexc}/a5)**(\theta/(\theta-1)) + (\mathbf{rpydo}/a6a)**(\theta/(\theta-1)))**((\theta-1)/\theta)$$

$$\mathbf{rpylr} = a4a * (\mathbf{rptlr}**(\delta/(\delta-1)) - (\mathbf{rpmo}/a3)**(\delta/(\delta-1)))**((\delta-1)/\delta)$$

$$\mathbf{rhw} = (a1) * (\mathbf{rpylr}**(\rho/(\rho-1)) - (\mathbf{rpklr}/a2a)**(\rho/(\rho-1)))**((\rho-1)/\rho)$$

$$\mathbf{ysr} = 1000000 * (((a1) * \mathbf{nthsr}/1000000)**\rho + (a2a * \mathbf{kbf}/1000000)**\rho)**(1/\rho)$$

$$\mathbf{kbf} = 1/a2a * (a1) * \mathbf{nthsr} * ((\mathbf{rpylr}/(\mathbf{rhw}/(a1)))**(\rho/(\rho-1)) - 1)**(1/\rho)$$

$$\mathbf{ydo} = \mathbf{tsr}/a6a * (\mathbf{rpydo}/(a6a * \mathbf{rptlr}))**(-1/(1-\theta))$$

$$\mathbf{tsr} = ((a4a * \mathbf{ysr})**\delta + ((a3) * \mathbf{imsr})**\delta)**(1/\delta)$$

$$\mathbf{exrsr} = \mathbf{tsr}/(a5) * (\mathbf{rpexc}/((a5) * \mathbf{rptlr}))**(1/(\theta-1))$$

$$\mathbf{rpmo} = (a3) * \mathbf{rptlr} * ((\mathbf{imsr} * (a3)/(\mathbf{ysr} * a4a))**(-\delta) + 1)**((1-\delta)/\delta)$$

$$\mathbf{rpklr} = \mathbf{rpydo} * \mathbf{ar}$$

$$\mathbf{a1} = a1(-1) * \exp(\beta * 0.25)$$

$$\mathbf{a3} = a3(-1) * \exp(\beta * 0.25)$$

$$\mathbf{a5} = a5(-1) * \exp(\beta * 0.25)$$

$$\mathbf{pna10} = \mathbf{ydo} + \mathbf{exrsr} - \mathbf{imsr}$$

$$\mathbf{gna10} = \mathbf{rhwpu_eq} * \mathbf{puhr_eq} * 13/1000 * \mathbf{ngg_eq}$$

$$\mathbf{gr} = (\beta * 0.25 + \mathbf{popgr_eq} + 1)$$

$$\mathbf{gr_1} = (\mathbf{gr} - 1)$$

Labour market

$$ngggr = popgr_eq$$

$$partt = 100 * nts / (rpop3_eq * (pop3_eq + pop4_eq))$$

$$partt = partt_eq$$

$$urt = nairu$$

$$urt = 100 * (1 - (nsr + ngg_eq) / nts)$$

$$nt = nsr + ngg_eq$$

$$ngg_eq = ngg_eq(-1) * \exp(ngggr)$$

$$nthpr1 = nthsr$$

$$nthsr = nsr * prhr_eq * 13 / 1000$$

$$rwa = rhw * prhr_eq * 13 / 1000$$

Real exchange rate

$$rdos = -1 * (rpexc * cexp + rpexnc * ncexp - (rpmo / (1 + pol5_eq)) * imsr - (rpmc / (1 + pol5_eq)) * imc - (rpmcs / (1 + pol5_eq)) * imcs - (rpmca / (1 + pol5_eq)) * imca + rtrospr - rtrpuos + ryospr + ryospu + rmtransfer_eq)$$

$$re = (1 + pol5_eq) * (pimof) / rpmo$$

$$re = (1 + pol5_eq) * (pimcf) / rpmc$$

$$re = (1 + pol5_eq) * (pimcsf) / rpmcs$$

$$re = (1 + pol5_eq) * (pimcaf) / rpmca$$

$$rpm = (rpmo * imo + rpmc * imc + rpmcs * imcs + rpmca * imca) / im$$

$$rer = re$$

$$rer = e * pydo$$

$$rpexc = (pexcf) / re$$

$$rpexncg = (pexncgf) / re$$

$$rpexncs = (pexncsf) / re$$

$$rpexnc = (ncexps * rpexncs + ncexp * rpexncg) / ncexp$$

$$etwit = e * \exp(c4800)$$

Deflators

$$I = (dw_1 + dw_2) * (rpyd_c - dw_7 * rpmc - dw_8 * rpmcs) / dw_6 + dw_3 * rpyd_h + dw_4 * rpexnc + dw_5 * rpyd_gc$$

$$rpyd_i = (dw_9 * ((rpyd_c - dw_7 * rpmc - dw_8 * rpmcs) / dw_6) + dw_{10} * rpmca)$$

$$\log(rpyd_h) = (pa801 + pa803 * TF)$$

$$pydo = pydo(-1) * \exp(\inf_pydo)$$

International capital flows

$$rdos = rfdebt * \exp(gr_1) - rfdebt$$

$$rfdebt1 = rfdebt + rdos$$

$$(rb - rzpa - rzp + rapa + rap - 0.3 * rnzsals) = pagdpr_eq * rgdp$$

$$rpfaa = (rb - rzpa - rzp + rapa + rap - 0.3 * rnzsals)$$

$$dgdpr = rfdebt / rgdp$$

$$fdgdpr = dgdpr$$

$$rtnziaa = tnziar_eq * rgdp$$

$$rfdebt = rtfianza - rtnziaa$$

$$rtfianza = rzpa + rzp + rzga + rzg$$

$$rzp = fpdratio_eq * (rzp + rzpa)$$

$$rapa = rtnziaa * 0.2$$

$$rap = 0.8 * rtnziaa - 0.7 * rnzsals$$

$$ryospr = -((rif_eq + ereturn) * (rzp) + (ri_eq + ereturn) * rzpa) \\ + (rif_eq + ereturn) * (rap) + (ri_eq + ereturn) * rapa$$

$$ryospu = -((rif_eq) * (rzg) + (ri_eq) * rzga) + (rif_eq) * (0.7 * (rnzsals))$$

Government block

$$rypupr = (ri_eq + ereturn) * rb + ratb_eq * rgdp - (ri_eq + ereturn) * 0.3 * rnzsals$$

$$rb = pubder - (rzga + rzg)$$

$$pubder = (rpubde_eq) * rgdp$$

$$rcogz = (rpyd_gc * ggcor_eq + rhwpu_eq * puhr_eq * 13/1000 * ngg_eq) * cratio_eq \\ + rpyd_gi * ggifr_eq * iratio_eq$$

$$rtax = (poll) * (rwa * (nsr) + rhwpu_eq * puhr_eq * 13/1000 * ngg_eq)$$

$$+ pol4_eq * rpyd_c / (1 + pol4_eq) * conor$$

$$+ pol5_eq * (rpmo * imsr + rpmc * imc + rpmcs * imcs + rpmca * imca) + pol3_eq * rgdp$$

$$rpubdot = (rcogz + rtrpupr) - rtax + pol12_eq * rgdp - ryospu$$

$$rpubs = iratio_eq * ggifr_eq * rpyd_gi - (rpubdot + rypupr)$$

$$pubder1 - pubder = rnm - (rpubs + (-1 * (rpyd_gi * ggifr_eq * iratio_eq) - rnnppi)) - ricc$$

$$pubder1 = pubder * \exp(gr_1)$$

$$rtrospr = pol14_eq * rgdp$$

$$rtrpuos = pol12_eq * rgdp$$

$$rtrpupr = (1 - poll) * rwa * trbase$$

$$trbase = pol8_eq * pop4_eq + pol9_eq * unb + pol11_eq * (pop1_eq + pop2_eq + pop3_eq + pop4_eq)$$

$$unb = (1 + difd_eq) * nts - (nsr + ngg_eq)$$

$$rzga = dgdratio_eq * pubder$$

$$rzg = fgdratio_eq * pubder$$

$$rpudos1 = (rzga - rzga(-1)) + (rzg - rzg(-1)) - 0.7 * (rnzsal - rnzsal(-1))$$

$$ggcor_eq = ggcor_eq(-1) * \exp(\log(a1/a1(-1))) + popgr_eq$$

$$ggifr_eq = ggifr_eq(-1) * \exp(\log(a1/a1(-1))) + popgr_eq$$

Domestic demand

$$ydo = conor / (1 + pol4ww) + (ihr + ibfr + ggcor_eq + ggifr_eq) + iinr + ncexp - imc - imcs - imca$$

$$yd = ydo + imc + imcs + imca$$

$$cond = conor + conh$$

$$con = cond$$

$$rincome = (1 - pol1) * (rwa * (nsr) + rhwpu_eq * puhr_eq * 13/1000 * ngg_eq) + rtrpupr + rtrospr + rmtransfer_eq$$

$$rwealth = (rb - rzpa - rzp + rapa + rap - 0.3 * rnzsal) + kh * rpyd_h$$

$$rlpcon = \log((conor * rpyd_c + conh * rpconh) / cond)$$

$$\log(conor) = coef1 + (pa12_1) * \log(rincome) + (1 - pa12_1) * \log(rwealth) - \log(rpyd_c)$$

$$\log(rpconh / rpyd_c) = pa201 + pa202 * \log(conh / conor) + pa204 * TF$$

$$kh = cyratio_eq * (pop3_eq + pop4_eq)$$

$$kh = conh / ksratio_eq$$

$$\log(ksratio_eq) = c501 + c502 * TF$$

$$ihr = kh * (\exp(gr_1) - \exp(-1 * drrb_eq))$$

$$rp = ksratio_eq * (rpconh / rpyd_h - pol7_eq) - (drrb_eq + ri_eq)$$

$$rp1 = ar - (dr_eq + ri_eq)$$

$$kh1 = ihr + kh * \exp(-1 * drrb_eq)$$

$$kbf1 = kbf * \exp(gr_1)$$

$$ibfr = kbf1 - kbf * \exp(-1 * dr_eq)$$

$$kinr1 = kinr + iinr$$

$$iinr = ssratio_eq * yd$$

$$kinr = (\text{if } kinr_eq > 0 \text{ then } kinr_eq \text{ else } kinr1(-1))$$

Exports

$$ncexp = ncexpg + ncexps$$

$$\log(ncexps/ydo) = pa601 + pa602 * \log(rpexncs) + c605 * TF$$

$$\log(ncexpg/ydo) = pa603 + pa604 * \log(rpexncg)$$

$$cexps = exrsr$$

$$cexp = cexps - iie$$

$$iie = cesratio_eq * cexps$$

$$texp = cexp + ncexp$$

$$kie = (\text{if } kie_eq > 0 \text{ then } kie_eq \text{ else } kie1(-1))$$

$$kie1 = kie + iie$$

Imports

$$imo = imsr$$

$$\log(imca/(ibfr + ggifr_eq)) = pa401 + pa402 * \log(rpmca)$$

$$\log(imc/conor) = pa404 + pa405 * \log(rpmc) + pa406 * TF$$

$$\log(imcs/conor) = pa407 + pa408 * \log(rpmcs)$$

$$im = imo + imc + imcs + imca$$

Add ups

$$rgdp = rpexc * cexps + rpexnc * ncexp + iinr * rpydo + conor * rpyd_c + ihr * rpyd_h + ibfr * rpyd_i + ggifr_eq * rpyd_gi + ggcors_eq * rpyd_gc + rhwpu_eq * puhr_eq * 13/1000 * ngg_eq + rponch * conh - (rpmo * imo + rpmc * imc + rpmcs * imcs + rpmca * imca) / (1 + pol5_eq)$$

$$na14 = cexps + conor + (ihr + ibfr + ggcors_eq + ggifr_eq) + iinr + ncexp - im + conh + rhwpu_eq * puhr_eq * 13/1000 * ngg_eq / rppsw$$

$$tbal = rpexc * cexp + rpexnc * ncexp - (rpmo * imo + rpmc * imc + rpmcs * imcs + rpmca * imca) / (1 + pol5_eq)$$

$$ibal = ryospr + ryospu$$

$$tfbal = rtrospr - rtrpuos$$

$$ri_eq = (\text{if } hist > 0 \text{ then } ri_e \text{ else } (1 + rl/400) / \exp(inf) - 1)$$

$$rif_eq = (\text{if } hist > 0 \text{ then } rif_e \text{ else } (1 + rlfb/400) / \exp(inf) - 1)$$

Appendix 2: Dynamic equations

Please note that the definitions of variable names are available in Appendix 4 and all the endogenous variables are in bold.

Production block

Profit maximising conditions

$$\mathbf{nthsr} = (1000000/a1) * ((\mathbf{ysr}/1000000)**\rho - (a2a*kb/1000000)**\rho)**(1/\rho)$$

$$\mathbf{rpysr} = rhw/a1 * (((a1)*\mathbf{nthsr}/(a2a*kb)) **(-\rho)+1)**((\rho-1)/\rho)$$

$$\mathbf{rptsr} = ((\mathbf{rpysr}/a4a)**(\delta/(\delta-1)) + (\mathbf{rpmo}/(a3+a3shock))**(\delta/(\delta-1)))**((\delta-1)/\delta)$$

$$\mathbf{rpydmr} = a6a * (\mathbf{rptsr}**(\theta/(\theta-1)) - (\mathbf{rpexc}/(a5+a5shock))**(\theta/(\theta-1)))**((\theta-1)/\theta)$$

$$\mathbf{tsr} = a6a * \mathbf{ydo} * (\mathbf{rpydmr}/(a6a*\mathbf{rptsr}))**1/(1-\theta)$$

$$\mathbf{ysr} = 1/a4a * (\mathbf{tsr}**\delta - ((a3+a3shock)*\mathbf{imsr})**\delta)**1/\delta$$

$$\mathbf{exrsr} = \mathbf{tsr}/(a5+a5shock) * (\mathbf{rpexc}/((a5+a5shock)*\mathbf{rptsr}))**1/(\theta-1)$$

$$\mathbf{imsr} = a4a * \mathbf{ysr}/(a3+a3shock) * ((\mathbf{rpmo}/((a3+a3shock)*\mathbf{rptsr}))**(\delta/(1-\delta)) - 1) **(-1/\delta)$$

$$\mathbf{a1} = a1(-1) * \exp(\beta * 0.25)$$

$$\mathbf{a3} = a3(-1) * \exp(\beta * 0.25)$$

$$\mathbf{a5} = a5(-1) * \exp(\beta * 0.25)$$

Labour market

$$\log(\mathbf{nthpr1}) = (pa1_1 * \log((\mathbf{nthsr}(-1)) * \exp(\mathbf{popgr_eq})) + (1-pa1_1) * \log((\mathbf{nthpr1}(-1)) * \exp(\mathbf{popgr_eq}))) + pa1_2 * \log(\mathbf{rpydmr}(-2))$$

$$\mathbf{nthpr1} = \mathbf{nsr} * \mathbf{prhr_eq} * 13/1000$$

$$\mathbf{nt} = \mathbf{nsr} + \mathbf{ngg_eq}$$

$$\mathbf{partt} = pa2_1 * \mathbf{partt}(-1) + pa2_2 * (\mathbf{partt_eq}) + pa2_3 * (\mathbf{nairu_urt}) + pa2_4 * (\mathbf{nairu}(-1) - \mathbf{urt}(-1))$$

$$\log(\mathbf{nts}) = \log(\mathbf{partt}/100 * (\mathbf{rpop3_eq} * (\mathbf{pop3_eq} + \mathbf{pop4_eq})))$$

$$\mathbf{urt} = 100 * (1 - \mathbf{nt}/\mathbf{nts})$$

Real exchange rates

$$\mathbf{re} = 0.25 * \mathbf{ere} + 0.25 * \mathbf{ere}(-1) * \exp(\mathbf{eregr}) + 0.25 * \mathbf{ere}(-2) * \exp(\mathbf{eregr} + \mathbf{eregr}(-1))$$

$$+ 0.25 * \mathbf{ere}(-3) * \exp(\mathbf{eregr} + \mathbf{eregr}(-1) + \mathbf{eregr}(-2)) + pa3_1 * \mathbf{ldgdpr}(+7) + pa3_2 * \mathbf{ldgdpr}(+8)$$

$$\log(\mathbf{e}) = pa4_1 * \log(\mathbf{e}(-1)) + (1-pa4_1) * \log(\mathbf{e}(+1)) + \log((1+\mathbf{rcs}/400)/(1+(\mathbf{rcsf}+\mathbf{srp})/400))$$

$$+ pa4_2 * \log(\mathbf{rer}(-1)/\mathbf{re}(-1))$$

$$\mathbf{rer} = \mathbf{e} * \mathbf{pydo}$$

Export sector

$$\log(\mathbf{cexps}) = pa5_1 * \log(\exp((gr-1) + (-1 * beta1 * 0.25)))^{**3} * exrsr(-3)$$

$$+ (1 - pa5_1) * \log(\exp((gr-1) + (-1 * beta1 * 0.25))) * cexps(-1)$$

$$\log(\mathbf{ncexps}) = pa6_1 * \log(\exp((gr-1)))^{**2} * encexps(-2) + (1 - pa6_1) * \log(\exp((gr-1))) * ncexps(-1) + pa6_2 * \log(rpexncs(-2)/erpexncs(-2)) + pa6_3 * \log(rpexncs(-3)/erpexncs(-3)) + pa6_4 * \log(rpexncs(-4)/erpexncs(-4)) + pa6_5 * \log(rpexncs(-5)/erpexncs(-5)) + pa6_6 * \log(rpexncs(-6)/erpexncs(-6))$$

$$\log(\mathbf{ncexpg}) = pa7_1 * \log(\exp((gr-1)))^{**2} * encexpg(-2) + (1 - pa7_1) * \log(\exp((gr-1))) * ncexpg(-1) + pa7_2 * \log(rpexncg(-3)/erpexncg(-3))$$

$$\mathbf{ncexp} = \mathbf{ncexps} + \mathbf{ncexpg}$$

$$\log(\mathbf{cexp}) = \log((1 - cesratio_eq) * \mathbf{cexps}) + 0.3 * \log(kie(-1)/ekie(-1))$$

$$\mathbf{ie} = \mathbf{cexps} - \mathbf{cexp}$$

$$\mathbf{texp} = \mathbf{ncexp} + \mathbf{cexp}$$

Import sector

$$\log(\mathbf{imo}) = pa8_1 * \log(\exp((gr-1) + (-1 * beta2 * 0.25))) * imsr(-1) + (1 - pa8_1) * \log(\exp((gr-1) + (-1 * beta2 * 0.25))) * imo(-1)$$

$$\log(\mathbf{imca}/(\mathbf{ibfr} + \mathbf{ggifr_eq})) - \log(\mathbf{imca}(-1)/(\mathbf{ibfr}(-1) + \mathbf{ggifr_eq}(-1))) = pa9_1 * (\log(\mathbf{imca}(-1)/(\mathbf{ibfr}(-1) + \mathbf{ggifr_eq}(-1)))) - (pa401 + pa402 * \log(\mathbf{rpmca}(-1)))$$

$$\log(\mathbf{imc}/(\mathbf{conor})) - \log(\mathbf{imc}(-1)/(\mathbf{conor}(-1))) = pa10_1 * (\log(\mathbf{imc}(-1)/(\mathbf{conor}(-1)))) - (pa404 + pa405 * \log(\mathbf{rpmc}(-2)) + pa406 * \mathbf{TF}(-2)) + pa10_2 * \log(\mathbf{etwit}(-2)/\mathbf{etwit}(-3))$$

$$\log(\mathbf{imcs}/(\mathbf{conor})) - \log(\mathbf{imcs}(-1)/(\mathbf{conor}(-1))) = pa11_1 * (\log(\mathbf{imcs}(-1)/(\mathbf{conor}(-1)))) - (pa407 + pa408 * \log(\mathbf{rpmcs}(-3))) + pa11_2 * \log(\mathbf{etwit}(-2)/\mathbf{etwit}(-3))$$

$$\mathbf{im} = \mathbf{imo} + \mathbf{imc} + \mathbf{imcs} + \mathbf{imca}$$

External sector

$$\mathbf{rdos} = -1 * (\mathbf{tbal} + \mathbf{ibal} + \mathbf{tfbal} + \mathbf{rmtransfer_eq})$$

$$\mathbf{tbal} = \mathbf{rpexc} * \mathbf{cexp} + \mathbf{rpexc} * \mathbf{ncexp} - \mathbf{im} * \mathbf{rpm} / (1 + \mathbf{pol5_eq})$$

$$\mathbf{ibal} = \mathbf{ryospr} + \mathbf{ryospu}$$

$$\mathbf{tfbal} = \mathbf{rtrospr} - \mathbf{rtrpuos}$$

$$\mathbf{rfdebt1} = \mathbf{rfdebt} + \mathbf{rdos}$$

$$\mathbf{rfdebt} = \mathbf{rfdebt1}(-1)$$

$$\mathbf{dgdpr} = \mathbf{rfdebt} / \mathbf{rgdp}$$

$$\mathbf{rtnziaa} = \mathbf{tnziar_eq} * \mathbf{rgdp}$$

$$\mathbf{rapa} = \mathbf{rtnziaa} * 0.2$$

$$\mathbf{rap} = 0.8 * \mathbf{rtnziaa} - 0.7 * \mathbf{rnzsal}$$

$$\mathbf{rfdebt} = \mathbf{rtfinza} - \mathbf{rtnziaa}$$

$$\mathbf{rtfinza} = \mathbf{rzpa} + \mathbf{rzp} + \mathbf{rzga} + \mathbf{rzg}$$

$$rzp = fpdratio_eq * (rzp + rzpa)$$

$$rtrospr = poll4_eq * rgdp$$

$$ryospr = -((rif_eq + ereturn) * (rzp) + (ri_eq + ereturn) * rzpa) + (rif_eq + ereturn) * (rap) + (ri_eq + ereturn) * rapa$$

$$ryospu = -((rif_eq) * (rzg) + (ri_eq) * rzga) + (rif_eq) * (0.7 * (rnzsal))$$

Domestic demand

$$\log(\mathit{conord}) = coef1 + (pa12_1) * \log(\mathit{rincome}) + (1 - pa12_1) * \log(\mathit{rwealth}) - \log(\mathit{rpyd_c}) + pa12_2 * (\mathit{pagdpr_eq} - \mathit{pagdpr_eq}(-1)) +$$

$$pa12_2 * (\mathit{pagdpr_eq}(-1) - \mathit{pagdpr_eq}(-2)) + pa12_2 * (\mathit{pagdpr_eq}(-2) - \mathit{pagdpr_eq}(-3)) + pa12_2 * (\mathit{pagdpr_eq}(-3) - \mathit{pagdpr_eq}(-4))$$

$$\mathit{rincome} = (1 - pol1) * (\mathit{rwa} * (\mathit{nsr}) + \mathit{rhwpu_eq} * \mathit{puhr_eq} * 13 / 1000 * \mathit{ngg_eq}) + \mathit{rtrpupr} + \mathit{rtrospr} + \mathit{rmtransfer_eq}$$

$$\mathit{rwealth} = (\mathit{rb} - \mathit{rzpa} - \mathit{rzp} + \mathit{rapa} + \mathit{rap} - 0.3 * \mathit{rnzsal}) + \mathit{kh} * (\mathit{rpyd_h})$$

$$\mathit{rpfaa} = (\mathit{rb} - \mathit{rzpa} - \mathit{rzp} + \mathit{rapa} + \mathit{rap} - 0.3 * \mathit{rnzsal})$$

$$\mathit{rlpcon} = \log((\mathit{conor} * \mathit{rpyd_c} + \mathit{conh} * \mathit{rpconh}) / \mathit{con})$$

$$\log(\mathit{conor}) = pa13_1 * \log(\mathit{conord}) + (1 - pa13_1) * \log(\mathit{conor}(-1) * \exp(\mathit{gr} - 1)) + pa13_2 * (\mathit{ycurve}(-2)) + pa13_3 * (\mathit{ycurve}(-3)) + pa13_4 * \log(\mathit{rpm}(-1) / \mathit{rpm}(-2)) + pa13_5 * \log(\mathit{rpm}(-2) / \mathit{rpm}(-3))$$

$$\mathit{con} = \mathit{conor} + \mathit{conh}$$

$$\mathit{conh} = \mathit{ksratio} * \mathit{kh}$$

$$\mathit{ksratio} = \mathit{ksratio_eq}$$

$$\log(\mathit{ihr}) = \log((pa14_1 * (\mathit{ihr}(-1) * \mathit{kh} / \mathit{kh}(-1))) + (1 - pa14_1) * ((\exp(\mathit{beta} * 0.25 + \mathit{popgr_eq}) - \exp(-1 * \mathit{drrb_eq})) * \mathit{kh})) + pa14_3 * \mathit{ycurve}(-2) + pa14_4 * \mathit{ycurve}(-3) + pa14_2 * \log(\mathit{kh}(-1) / \mathit{ekh}(-1)))$$

$$\mathit{kh} = \mathit{kh1}(-1)$$

$$\mathit{kh1} = \mathit{ihr} + \mathit{kh} * \exp(-1 * \mathit{drrb_eq})$$

$$\log(\mathit{rpconhd}) = c201 + c202 * \log(\mathit{conh} / \mathit{conor}) + \log(\mathit{rpyd_c}) + c204 * \mathit{TF}$$

$$\mathit{rpconh} = 0.7 * \mathit{rpconh}(-1) + 0.3 * \mathit{rpconhd}$$

$$\mathit{gr} = (0.25 * \log(\mathit{a1} / \mathit{a1}(-1)) + 0.25 * \log(\mathit{a1}(-1) / \mathit{a1}(-2)) + 0.25 * \log(\mathit{a1}(-2) / \mathit{a1}(-3)) + 0.25 * \log(\mathit{a1}(-3) / \mathit{a1}(-4))) + \mathit{popgr_eq} + 1$$

$$\log(\mathit{ibfr}) = \log((pa15_1 * (\mathit{ibfr}(-1) * \exp(\mathit{gr} - 1))) + (1 - pa15_1) * ((\exp(\mathit{beta} * 0.25 + \mathit{popgr_eq}) - \exp(-1 * \mathit{dr_eq})) * \mathit{kbfr})) + pa15_2 * \mathit{tobinq}(-2) + pa15_3 * \mathit{ycurve}(-2) + pa15_4 * \log(\mathit{rpmca}(-1) / \mathit{rpmca}(-2)) + pa15_5 * \log(\mathit{rpmca}(-2) / \mathit{rpmca}(-3)) + pa15_6 * \log(\mathit{rpmca}(-3) / \mathit{rpmca}(-4)))$$

$$\mathit{tobinq} = ((\mathit{ar} - (\mathit{dr_eq} + \mathit{ri_eq} + \mathit{rp1})))$$

$$\mathit{ar} = 0.2 * (\mathit{rpklr}) + 0.2 * \mathit{ar}(-1) + 0.2 * \mathit{ar}(-2) + 0.2 * \mathit{ar}(-3) + 0.2 * \mathit{ar}(-4)$$

$$\mathit{kbf1} = \mathit{kbfr} * \exp(-1 * \mathit{dr_eq}) + \mathit{ibfr}$$

$$\mathit{kbfr} = \mathit{kbf1}(-1)$$

$$rptlr = ((rpexc/(a5+a5shock))^{**}(theta/(theta-1)) + (rpydmr/a6a)^{**}(theta/(theta-1)))^{**}((theta-1)/theta)$$

$$rpylr = a4a*(rptlr^{**}(delta/(delta-1)) - (rpmo/(a3+a3shock))^{**}(delta/(delta-1)))^{**}((delta-1)/delta)$$

$$rhw = (a1)*(rpylr^{**}(rho/(rho-1)) - (rpklr/a2a)^{**}(rho/(rho-1)))^{**}((rho-1)/rho)$$

$$iinr = yd*ssratio_eq - 0.2*(kinr(-1) - ekinr(-1))$$

$$kinr1 = iinr + kinr$$

$$kinr = kinr1(-1)$$

$$kie = kie1(-1)$$

$$kie1 = kie + iie$$

$$\log(yd) = \log(conor/(1+pol4ww)) + (ihr + ibfr + ggcor_eq + ggifr_eq) + iinr + ncexp$$

$$ydo = yd - imc - imcs - imca$$

Government sector

$$rypupr = (ri_eq + ereturn)*rb + ratb_eq*rgdp - (ri_eq + ereturn)*0.3*rnzsal$$

$$rtrpuos = pol12_eq*ergdp$$

$$rtrpupr = (1 - pol1)*rwa*trbase$$

$$trbase = pol8_eq*pop4_eq + pol9_eq*unb + pol11_eq*(pop1_eq + pop2_eq + pop3_eq + pop4_eq)$$

$$unb = (1 + difd_eq)*nts - (nt)$$

$$rcogz = (rpyd_gc*ggcor_eq + rhwpu_eq*puhr_eq*13/1000*ngg_eq)*cratio_eq + rpyd_gi*ggifr_eq*iratio_eq$$

$$rtax = (pol1)*(rwa*(nsr) + rhwpu_eq*puhr_eq*13/1000*ngg_eq) + pol4_eq*rpyd_c/(1 + pol4_eq)*conor + pol5_eq*rpm*im + pol3_eq*rgdp$$

$$rpubdot = (rcogz + rtrpupr) - rtax + pol12_eq*rgdp - ryospu$$

$$rpubs = iratio_eq*ggifr_eq*rpyd_gi - (rpubdot + rypupr)$$

$$rzga = dgdratio_eq*pubder$$

$$rzg = fgdratio_eq*pubder$$

$$rb = pubder - (rzga + rzg)$$

$$pol1 = (1 - twt)*(pol1(-1) + pa16_1*(rpubde_eq - rpubde_eq(-1))/1000 + pa16_2*(pubder(8)/ergdp(8) - rpubde_eq(8)) + pa16_3*((pubder(9)/ergdp(9) - rpubde_eq(9)) - (pubder(8)/ergdp(8) - rpubde_eq(8))) + twt*(pol1(-1)))$$

$$pubder1 - pubder = rnmc - (rpubs + (-1)*(rpyd_gi*ggifr_eq*iratio_eq) - rnnppi) - ricc$$

$$pubder = pubder1(-1)$$

Inflation, prices and interest rates

$$pna10 = cexp + iie + yd - im$$

$$pna10_eq = epna10$$

$$gna10 = rhwpu_eq * puhr_eq * 13/1000 * ngg_eq // + conh * rpconh$$

$$lgap = \log((pna10 + gna10) / (pna10_eq + egna10))$$

$$inf_tar = 0.8 * inf_tar(-1) + 0.2 * inf_tar_eq$$

$$(1 + infe) = (pa20_1 * (1 + inf) + (1 - pa20_1) * (1 + infe(1)))$$

$$inf = (pa20_2) * (infnt + infnt_c) + (1 - pa20_2) * (inftr + inftr_c)$$

$$infnt = (infe - inf_tar) + pa20_3 * lgap(-1) + pa20_4 * lgap(-2) + pa20_3 * algap(-1)$$

$$algap = \text{if } lgap > 0 \text{ then } lgap \text{ else } 0$$

$$inftr = pa20_5 * (infe - inf_tar) + \beta \alpha * ((\log(pimcf/pimcf(-1)) - inf_tar) + (\log(pimcf(-1)/pimcf(-2)) - inf_tar) + (\log(pimcf(-2)/pimcf(-3)) - inf_tar) + (\log(pimcf(-3)/pimcf(-4)) - inf_tar))$$

$$+ \beta(1 - \alpha) * ((\log(pimof/pimof(-1)) - inf_tar) + (\log(pimof(-1)/pimof(-2)) - inf_tar)$$

$$+ (\log(pimof(-2)/pimof(-3)) - inf_tar) + (\log(pimof(-3)/pimof(-4)) - inf_tar))$$

$$+ \beta * (\log(etwit/etwit(-1)) + \log(etwit(-1)/etwit(-2)) + \log(etwit(-2)/etwit(-3)) + \log(etwit(-3)/etwit(-4))) + pa20_6 * lgap(-1)$$

$$inf_hw = pa18_1 * inf(-2) + (1 - pa18_1) * infe(-1) + 0.25 * \log(a1/a1(-1)) + 0.25 * \log(a1(-1)/a1(-2)) + 0.25 * \log(a1(-2)/a1(-3)) + 0.25 * \log(a1(-3)/a1(-4)) + pa18_2 * \log(rpydmr(-1))$$

$$+ pa18_3 * \log(rhw(-2)/erhw(-2)) + pa18_4 * (urt(-2) - nairu)$$

$$hw = hw(-1) * \exp(inf_hw)$$

$$rhw = hw/pydo$$

$$rwa = rhw * prhr_eq * 13/1000$$

$$pydo = pydo(-1) * \exp(inf_pydo)$$

$$rpm = rpm(-1) * \exp(inf_pm - inf_pydo)$$

$$inf_pmc = \log(pimcf/pimcf(-1)) - \log(e/e(-1))$$

$$inf_pmcs = \log(pimcsf/pimcsf(-1)) - \log(e/e(-1))$$

$$inf_pmo = \log(pimof/pimof(-1)) - \log(e/e(-1))$$

$$inf_pmca = \log(pimcaf/pimcaf(-1)) - \log(e/e(-1))$$

$$inf_pexncg = \log(pexncgf/pexncgf(-1)) - \log(e/e(-1))$$

$$rpexncs = rpexncs(-1) * \exp(inf_pexncs - inf_pydo)$$

$$inf_pexnc = dw_{15} * inf_pexncs + (1 - dw_{15}) * inf_pexncg$$

$$inf_pydh = \log(pyd_h/pyd_h(-1))$$

$$inf_pconh = \log(pconh/pconh(-1))$$

$$1 = (dw_1 + dw_2) * (rpyd_c - dw_7 * rpmc - dw_8 * rpmcs) / dw_6 + dw_3 * rpyd_h + dw_4 * rpexnc + dw_5 * rpyd_gc$$

$$rpyd_i=(dw_9*((rpyd_c-dw_7*rpmc-dw_8*rpmcs)/dw_6)+dw_{10}*rpmca)$$

$$\log(rpyd_h)=dw_{11}*\log(rpyd_h(-1))+(1-dw_{11})*\log(erpyd_h)$$

$$\log(pyd_c/cpixh)=pa701+pa702*TF$$

$$inf_cpihouse=0.3*inf_pydh+0.7*inf_pconh$$

$$cpihouse=cpihouse(-1)*\exp(inf_cpihouse)$$

$$cpix=dw_{14}*cpixh+(1-dw_{14})*cpihouse$$

$$pydo=pyd_c/rpyd_c$$

$$rpmo=((1+pol5_eq)*(pimof)/(rer))$$

$$rpmc=((1+pol5_eq)*pimcf/(rer))$$

$$rpmcs=((1+pol5_eq)*pimcsf/(rer))$$

$$rpmca=((1+pol5_eq)*pimcaf/(rer))$$

$$\log(rpexncs)=(1-dw_{12})*(\log(erpexncs)-dw_{13}*\log(rer(-1)/ere(-1)))+dw_{12}*\log(rpexncs(-1))$$

$$rpexncg=((pexncgf)/rer)$$

$$rpexc=((pexcf)/rer)$$

$$rpm=(rpmo*imo+rpmc*imc+rpmcs*imcs+rpmca*imca)/im$$

$$rpexnc=(ncexps*rpexncs+ncexpg*rpexncg)/ncexp$$

$$rptex=(rpexc*pexc+rpexnc*pexnc)/(pexc+pexnc)$$

$$rcs=(1-rcswt)*(pa19_1*(inf_cpix(5)-cpi_tar)+pa19_2*(inf_cpix(6)-cpi_tar)+pa19_3*(inf_cpix(7)-cpi_tar)+pa19_4*(rcs-rcs(-1))+rl)$$

$$cpix=cpix(-1)*\exp(inf)$$

$$inf_cpix=\exp(inf+inf(-1)+inf(-2)+inf(-3))-1$$

$$ri=(1+rl/400)/\exp(infe)-1$$

$$rl=(1-rlwt)*((1-0.95)*rcs+0.95*rl(1))+rlwt*rl_ad$$

$$ycurve=rcs-rl$$

$$\log(e)=pa4_1*\log(e(-1))+(1-pa4_1)*(\log(e(+1))+\log((1+rcs/400)/(1+(rcsf+srp)/400)))+pa4_2*\log(rer(-1)/re(-1))$$

$$etwit=(rer/pydo)*\exp(c4800)$$

$$rs=(1+rcs/400)/\exp(infe)-1$$

$$rsf=(1+rcsf/400)/\exp(inf_tar)-1$$

Add ups

$$\mathbf{rgdp} = \mathbf{rpexc} * \mathbf{cexps} + \mathbf{rpexc} * \mathbf{ncexp} + \mathbf{iinr} * \mathbf{rpydo} + \mathbf{conor} * \mathbf{rpyd_c} + \mathbf{ihr} * \mathbf{rpyd_h} + \mathbf{ibfr} * \mathbf{rpyd_i} + \mathbf{ggifr_eq} * \mathbf{rpyd_gi} + \mathbf{ggcor_eq} * \mathbf{rpyd_gc} + \mathbf{rhwpu_eq} * \mathbf{puhr_eq} * 13/1000 * \mathbf{ngg_eq} + \mathbf{rpconh} * \mathbf{conh} - \mathbf{rpm} * \mathbf{im} / (1 + \mathbf{pol5_eq})$$

$$\mathbf{gdpz} = \mathbf{rgdp} * \mathbf{pydo}$$

$$\mathbf{ldgdpr} = \log(\mathbf{dgdpr} / \mathbf{fdgdpr})$$

$$\mathbf{na14} = \mathbf{cexps} + \mathbf{conor} + (\mathbf{ihr} + \mathbf{ibfr} + \mathbf{ggcor_eq} + \mathbf{ggifr_eq}) + \mathbf{iinr} + \mathbf{ncexp} - \mathbf{im} + \mathbf{conh} + \mathbf{rhwpu_eq} * \mathbf{puhr_eq} * 13/1000 * \mathbf{ngg_eq} / \mathbf{rppsw}$$

$$\mathbf{na15} = \mathbf{conh}$$

$$\mathbf{na1} = \mathbf{conh} + \mathbf{conor}$$

$$\mathbf{na2} = (\mathbf{ggcor_eq} + \mathbf{rhwpu_eq} * \mathbf{puhr_eq} * 13/1000 * \mathbf{ngg_eq} / \mathbf{rppsw})$$

$$\mathbf{na3} = \mathbf{ihr}$$

$$\mathbf{na4} = \mathbf{ibfr}$$

$$\mathbf{na7} = \mathbf{ggifr_eq}$$

$$\mathbf{na10} = (\mathbf{na14} + \mathbf{balitem})$$

$$\mathbf{na9} = \mathbf{iie}$$

$$\mathbf{na8} = \mathbf{iinr}$$

$$\mathbf{na11} = \mathbf{na1} + \mathbf{na3} + \mathbf{na4} + \mathbf{na2} + \mathbf{na7} + \mathbf{na8} + \mathbf{na9}$$

Appendix 3: Steady-state model variable list

Symbolname	Label
A1	labour efficiency scale parameter
A3	imports efficiency scale parameter
A5	exports inefficiency scale parameter
AR	actual rate of return on capital
CESRATIO_EQ	The proportion of commodity goods produced in time t , that are exported in time t ie, the ratio of CEXP to CEXPS
CEXP	commodity exports
CEXPS	commodity exports supply (i.e, CEXP + IIE)
COEF1	estimated constant term in the consumption equation
CON	real consumption (model-basis)
COND	equilibrium consumption
CONH	real consumption of housing
CONOR	real private consumption (non-housing)
CRATIO_EQ	ratio of Core Crown consumption (Crown Accounts) to total Government consumption (SNA)
CYRATIO_EQ	ratio of KH to (POP3+POP4)
DGDPR	real foreign debt to RGDP
DGDRATIO_EQ	ratio of RZA to PUBDER
DIFD_EQ	benefit-survey unemployment rates
DR_EQ	depreciation rate for business capital stock
DRRB_EQ	depreciation rate for housing stock
E	trade weighted exchange rate index used in model
ERETURN	premium on equity
ETWIT	trade weighted exchange rate
EXRSR	short-run equilibrium exports
FDGDPR	real foreign debt to real GDP ratio
FGDRATIO_EQ	ratio of ZG/RE to PUBDER
FPDRATIO_EQ	ratio of private foreign debt contracted in foreign currency to total private foreign debt
GGCOR_EQ	real total Govt consumption (non-wage) -trend
GGIFR_EQ	real Govt investment
GNA10	government sector output
GR	equilibrium real growth factor
GR_1	GR-1
HIST	Dummy variable for the observatoin period
IBAL	investment balances
IBFR	real private investment (non-housing)
IHR	real housing investment
IIE	investment in inventory (commodity good)
IINR	investment in inventory (non-commodity good)
IM	Imports
IMC	imports of consumption goods
IMCA	imports of capital goods
IMCS	household overseas spending

Symbolname	Label
IMO	imports of intermedate goods and others
IMSR	short-run equilibrium IM
INF	inflation as meased by change in CPI
INF_PYDO	inflation as meased by change in price of domestic good
IRATIO_EQ	ratio of Core Crown investment (cash flow statement) to total Government investment (SNA)
KBF	business capital stock (start of period)
KBF1	business capital stock (end of period)
KH	housing stock (start of period)
KH1	housing stock (end of period)
KIE	stock of commodity goods (start of period)
KIE_EQ	stock of commodity goods (start of period) -trend
KIE1	stock of commodity goods (end of period)
KINR	stock of non-commodity goods (start of period)
KINR_EQ	stock of non-commodity goods (start of period) - trend
KINR1	stock of non-commodity goods (end of period)
KSRATIO_EQ	ratio of CONH to KH
NA14	real expenditure based GDP
NA2	real total govt consumption
NAIRU	long-run unemployment rate
NCEXP	non commodity exports
NCEXPB	non commodity goods exports
NCEXPS	non commodity services exports
NGG	total Govt employment
NGG_EQ	steady-state employment - general government -trend
NGGGR	growth rate for NGG
NSR	private sector employment NT-NGG
NT	total employment
NTHPR1	quarterly hours paid for the private sector
NTHSR	quarterly hours paid for the private sector -medium run variable
NTS	labour force
PAGDPR_EQ	net real household financial asset to GDP ratio
PARTT	labour force participation rate
PARTT_EQ	steady-state labour force participation rate
PEXCF	commodity export prices (in foreign currency)
PEXNCF	non-commodity export prices (in foreign currency)
PEXNCGF	non-commodity goods export prices (in foreign currency)
PEXNCSF	non-commodity services export prices (in foreign currency)
PIMCAF	capital goods import prices (in foreign currency)
PIMCF	consumption goods import prices (in foreign currency)
PIMCSF	household overseas spending import prices (in foreign currency)
PIMOF	intermediate import prices (in foreign currency)
PNA10	private sector production
POL1	income tax rate
POL11_EQ	transfers rate - other

Symbolname	Label
POL12_EQ	transfers rate - public sector to foreign sector
POL14_EQ	transfers rate - foreign sector to private sector
POL3_EQ	rate of lump sum tax
POL4_EQ	rate of tax on consumption of non-housing
POL4WW	mean of POL4
POL5_EQ	rate of tax on imports
POL7_EQ	rate of tax on consumption of housing
POL8_EQ	transfers rate - superannuitants
POL9_EQ	transfers rate - unemployment
POP1_EQ	steady-state population - aged 0-4
POP2_EQ	steady-state population - aged 5-14
POP3_EQ	steady-state population - aged 15-64
POP4_EQ	steady-state population - aged 65+
POPGR_EQ	working-age population trend growth rate
PRHR_EQ	average weekly paid hrs for private sector employees
PUBDER	real public liabilities
PUBDER1	real public liabilities at the end of the quarter
PUHR_EQ	average weekly paid hrs for public sector employees
PYDO	The price of PYDO
RAP	NZ private international asset in foreign currency at the beginning of the quarter (expressed at \$nz)
RAPA	NZ private international asset in nz currency at the beginning of the quarter
RATB_EQ	non-interest income from public to private sector
RB	real domestic bonds + cumulated equity disinvestment
RCOGZ	Core Crown spending including investment
RCSF	foreign (weighted) 90 day interest rate
RDOS	real net capital inflow
RE	equilibrium real exchange rate index
RER	real exchange rate index
RFDEBT	total NZ net foreign real debt at the start of the quarter
RFDEBT1	total NZ net foreign real debt at the end of the quarter
RGDP	Model real GDP – deflated by pydo
RHW	private sector hourly rate
RHWPU_EQ	public sector hourly rate (trend)
RI_EQ	real interest rates (trend)
RICC	real issues of currency
RIF_E	real foreign interest rates
RIF_EQ	real foreign interest rates (trend)
RINCOME	real household income
RL	10-yr bond rate
RLFB	foreign 10-yr bond rate
RLPCON	price of consumption (model-basis)
RMTRANSFER_EQ	real migrant transfer
RNMC	net movement in cash (cash flow statement in fiscal data)
RNNPPI	net purchase of non physical investment

Symbolname	Label
RNNPPI_EQ	net purchase of non physical investment
RNZSAL	real assets of NZS fund at the beginning of the quarter
RP	risk premium for residential investment
RP1	risk premium for business investment
RPCONH	relative price of housing services
RPEXC	relative commodity export prices
RPEXNC	relative non-commodity export prices
RPEXNCG	relative non-commodity goods export prices
RPEXNCS	relative non-commodity services export prices
RPFAA	real financial assets owned by the private sector at the beginning of the quarter
RPKLR	real long-run equilibrium price of capital services
RPM	the relative price of imports
RPMC	the relative price of consumption goods imports
RPMCA	the relative price of capital goods imports
RPMCS	the relative price of household overseas spending
RPMO	the relative price of intermediate imports
RPOP3_EQ	$POP/(POP3+POP4)$
RPPSW	public sector wages deflator
RPTLR	real long-run equilibrium price of gross output
R PUBDE_EQ	ratio of public debt to GDP
R PUBDOT	net Govt deficit
RPUBS	real government saving (crown accounts)
R PUDOS1	real net public capital inflow (previous quarter)
R PYD_C	relative price of CONOR (non-housing consumption)
R PYD_GC	relative price of GGCOR
R PYD_GI	relative price of GGIFR
R PYD_H	relative price of IHR
R PYD_I	relative price of IBFR
R PYDO	relative price of PYDO = 1
R PYLR	long-run equilibrium price of primary factors
RTAX	total tax take (per quarter)
RTFINZA	foreign investment stock in nz at the beginning of the quarter
RTNZIAA	NZ investment aboard stock at the beginning of the quarter
RTROSPR	real net transfers from foreign sector to private sector
RTRPUOS	net transfers from public sector to foreign sector
RTRPUPR	net transfers from public sector to private sector
RWA	real average earnings (nat. ac. basis): incl. payroll tax
RWEALTH	real household wealth
RYOSPR	real net income from foreign sector to private sector
RYOSPU	real net income from foreign sector to public sector
RYPUPR	real net income from public to private sector
RZG	Govt foreign debt denominated in foreign currency (expressed in \$NZ) at the beginning of the quarter)
RZGA	real public foreign debt denominated in \$NZ

Symbolname	Label
RZP	private foreign debt denominated in foreign currency (expressed in \$NZ) at the beginning of the quarter)
RZPA	real private foreign debt denominated in \$NZ
SSRATIO_EQ	inventory investment to sale ratio for domestic good
TBAL	trade balances
TEXP	real total exports
TFBAL	transfer balances
TNZIAR_EQ	ratio of TNZIAA to GDPZ
TRBASE	transfers base (total number of average wages paid in benefits)
TF	time trend
TSR	quantity of gross output
UNB	sa no of people in unemployment benefit (start of quarter)
URT	unemployment rate
YD	YDO+IMC+IMCS+IMCA
YDO	gross output of the private sector excluding commodity exports
YSR	quantity of primary factors

Appendix 4: Dynamic model variable list

Symbolname	Label
A1	labour efficiency scale parameter
A3	imports efficiency scale parameter
A5	commodity exports inefficiency scale parameter
ALGAP	asymmetric output gap
AR	actual rate of return on capital
CESRATIO_EQ	ratio of CEXP to CEXPS
CEXP	commodity exports
CEXPS	commodity exports supply (i.e, CEXP + IIE)
CON	real consumption (model-basis)
CONH	real consumption of housing
CONOR	real private consumption (non-housing)
CONORD	real equilibrium consumption
CPI_TAR	annual CPI inflation target - 0.014884
CPIHOUSE	CPI for housing group
CPIX	consumer price index
CPIXH	CPI for non-housing group
CRATIO_EQ	ratio of Core Crown consumption to total Government consumption
DGDPR	real foreign debt to RGDP
DGDRATIO_EQ	ratio of RZA to PUBDER
DIFD_EQ	benefit-survey unemployment rates
DR_EQ	depreciation rate for business capital stock
DRRB_EQ	depreciation rate for housing stock
E	trade weighted exchange rate index used in model
EGNA10	equilibrium government sector output
EKH	equilibrium KH
EKIE	equilibrium KIE
EKINR	equilibrium KINR
ENCEXPG	equilibrium NCEXPG
ENCEXPS	equilibrium NCEXPS
EPNA10	equilibrium PNA10
ERE	equilibrium RE
EREGR	growth factor for RE (model variable)
ERETURN	premium on equity
ERGOP	equilibrium RGDP
ERHW	equilibrium RHW
ERPEXNCG	equilibrium RPEXNCG
ERPEXNCS	equilibrium RPEXNCS
ERPVD_H	equilibrium RPYD_H
ETWIT	trade weighted exchange rate
EXRSR	short-run equilibrium exports
FDGDP	real foreign debt to real GDP ratio
FGDRATIO_EQ	ratio of ZG/RE to PUBDER

Symbolname	Label
FPDRATIO_EQ	ratio of private foreign debt contracted in foreign currency to total private foreign debt
GDPZ	nominal expenditure gdp
GGCOR_EQ	real total Govt consumption (non-wage) -trend
GGIFR_EQ	real Govt investment
GNA10	government sector output
GR	equilibrium real growth factor
HW	private sector hourly rate
IBAL	investment balances
IBFR	real private investment (non-housing)
IHR	real housing investment
IIE	investment in inventory (commodity good)
IINR	investment in inventory (non-commodity good)
IM	imports
IMC	imports of consumption goods
IMCA	imports of capital goods
IMCS	household overseas spending
IMO	imports of intermedate goods and others
IMSR	short-run equilibrium IM
INF	quarterly inflation rate of CPI
INF_CPIHOUSE	quarterly inflation rate of CPIHOUSE
INF_CPIX	inflation rate of CPI
INF_HW	quarterly inflaton rate of HW
INF_PCONH	quarterly inflation rate of housing services (CONH) deflator
INF_PEXNC	quarterly inflaton rate of PEXNC
INF_PEXNCG	quarterly inflation rate of PEXNCG
INF_PEXNCS	quarterly inflation rate of PEXNCS
INF_PM	quarterly inflation rate of PM
INF_PMC	quarterly inflation rate of PMC
INF_PMCA	quarterly inflation rate of PMCA
INF_PMCS	quarterly inflation rate of PMO
INF_PMO	quarterly inflation rate of PYDH
INF_PYDH	quarterly inflation rate of PYDO
INF_PYDO	inflation as meased by change in price of domestic good
INF_TAR	inflation target
INF_TAR_EQ	steady-state inflation target
INFE	inflation expectation
INFNT	quarterly inflation rate of CPI non-tradables
INFNT_C	average non-tradable inflation rate
INFTR	quarterly inflation rate of CPI tradables
INFTR_C	average tradable inflation rate
IRATIO_EQ	ratio of Core Crown investment (cash flow statement) to total Government investment (SNA)
KBF	business capital stock (start of period)
KBF1	business capital stock (end of period)
KH	housing stock (start of period)

Symbolname	Label
KH1	housing stock (end of period)
KIE	stock of commodity goods (start of period)
KIE1	stock of commodity goods (end of period)
KINR	stock of non-commodity goods (start of period)
KINR1	stock of non-commodity goods (end of period)
KSRATIO	ratio of CONH to KH
KSRATIO_EQ	ratio of CONH to KH
LDGDPR	LOG(DGDPR/FDGDPR)
LGAP	private sector production output gap
NA1	private consumption
NA10	gdp (production-based estimate)
NA11	real imports (=IM)
NA14	real expenditure based GDP
NA15	private consumption - housing
NA2	real total govt consumption
NA3	private investment - residential buildings
NA4	private business gross fixed capital formation
NA7	public gross fixed capital formation
NA8	inventory investment - non-commodity
NA9	inventory investment - commodities
NAIRU	long-run unemployment rate
NCEXP	non commodity exports
NCEXPB	non commodity goods exports
NCEXPS	non commodity services exports
NGG_EQ	steady-state employment - general government -trend
NSR	private sector employment NT-NGG
NT	total employment
NTHPR1	quarterly hours paid for the private sector
NTHSR	quarterly hours paid for the private sector -medium run variable
NTS	labour force
PAGDPR_EQ	net real household financial asset to GDP ratio
PARTT	labour force participation rate
PARTT_EQ	steady-state labour force participation rate
PCONH	nominal price of housing services
PEXC	commodity export prices
PEXCF	commodity export prices (in foreign currency)
PEXNC	non-commodity export prices
PEXNCGF	non-commodity goods export prices (in foreign currency)
PIMCAF	capital goods import prices (in foreign currency)
PIMCF	consumption goods import prices (in foreign currency)
PIMCSF	household overseas spending import prices (in foreign currency)
PIMOF	intermediate import prices (in foreign currency)
PNA10	private sector production
PNA10_EQ	medium-run equilibrium private sector production

Symbolname	Label
POL1	income tax rate
POL11_EQ	transfers rate - other
POL12_EQ	transfers rate - public sector to foreign sector
POL14_EQ	transfers rate - foreign sector to private sector
POL3_EQ	rate of lump sum tax
POL4_EQ	rate of tax on consumption of non-housing
POL4WW	mean of POL4
POL5_EQ	rate of tax on imports
POL7_EQ	rate of tax on consumption of housing
POL8_EQ	transfers rate - superannuitants
POL9_EQ	transfers rate - unemployment
POP1_EQ	steady state population - aged 0-4
POP2_EQ	steady state population - aged 5-14
POP3_EQ	steady state population - aged 15-64
POP4_EQ	steady state population - aged 65+
POPGR_EQ	working-age population trend growth rate
PRHR_EQ	average weekly paid hrs for private sector employees
PUBDER	real public liabilities
PUBDER1	real public liabilities at the end of the quarter
PUHR_EQ	average weekly paid hrs for public sector employees
PYD_C	deflator for CONOR (non-housing consumption)
PYD_H	deffator for IHR
PYDO	The price of PYDO
RAP	NZ private international asset in foreign currency at the beginning of the quarter (expressed at \$nz)
RAPA	NZ private international asset in nz currency at the beginning of the quarter
RATB_EQ	non-interest income from public to private sector
RB	real domestic bonds + cumulated equity disinvestment
RCOGZ	Core Crown spending including investment
RCS	90-day bank bills
RCSF	foreign (weighted) 90 day interest rate
RDOS	real net capital inflow
RE	equilibrium real exchange rate index
RER	real exchange rate index
RFDEBT	NZ net foreign real debt at the start of the quarter
RFDEBT1	total NZ net foreign real debt at the end of the quarter
RGDP	Model real GDP – deflated by pydo
RHW	private sector hourly rate
RHWPU_EQ	public sector hourly rate (trend)
RI	real interest rates
RI_EQ	real interest rates (trend)
RICC	real issues of currency
RIF_EQ	real foreign interest rates (trend)
RINCOME	real household income
RL	10-yr bond rate

Symbolname	Label
RLPCON	price of consumption (model-basis)
RMTRANSFER_EQ	real migrant transfer
RNMC	net movement in cash (cash flow statement in fiscal data)
RNNPPI	net purchase of non physical investment
RNZSAL	real assets of NZS fund at the beginning of the quarter
RP	risk premium for residential investment
RP1	risk premium for business investment
RPCONH	relative price of housing services
RPCONHD	equilibrium real price of housing services
RPEXC	relative commodity export prices
RPEXNC	relative non-commodity export prices
RPEXNCG	relative non-commodity goods export prices
RPEXNCS	relative non-commodity services export prices
RPFAA	real financial assets owned by the private sector at the beginning of the quarter
RPKLR	real long-run equilibrium price of capital services
RPM	the relative price of imports
RPMC	the relative price of consumption goods imports
RPMCA	the relative price of capital goods imports
RPMCS	the relative price of household overseas spending
RPMO	the relative price of intermediate imports
RPOP3_EQ	$POP/(POP3+POP4)$
RPPSW	public sector wages deflator
RPTEX	the relative price of exports
RPTLR	real long-run equilibrium price of gross output
RPTSR	real short-run equilibrium price of gross output
RPUBDE_EQ	ratio of public debt to GDP
RPUBDOT	net Govt deficit
RPUBS	real government saving (crown accounts)
RPYD_C	deflator for CONOR (non-housing consumption)
RPYD_GC	deflator for GGCOR
RPYD_GI	deflator for GGIFR
RPYD_H	deffator for IHR
RPYD_I	deflator for IBFR
RPYDMR	real short-run equilibrium PYD
RPYDO	Relative price of PYDO = 1
RPYLR	long-run equilibrium price of primary factors
RPYSR	real short-run equilibrium price of primary factors
RS	real 90-day rate
RSF	real foreign 90-day rate
RTAX	total tax take (per quarter)
RTFINZA	foreign investment stock in nz at the beginning of the quarter
RTNZIAA	NZ investment aboard stock at the beginning of the quarter
RTOSPR	real net transfers from foreign sector to private sector
RTRPUOS	net transfers from public sector to foreign sector

Symbolname	Label
RTRPUPR	net transfers from public sector to private sector
RWA	real average earnings (nat. ac. basis): incl. payroll tax
RWEALTH	real household wealth
RYOSPR	real net income from foreign sector to private sector
RYOSPU	real net income from foreign sector to public sector
RYPUPR	real net income from public to private sector
RZG	Govt foreign debt denominated in foreign currency (expressed in \$NZ) at the beginning of the quarter)
RZGA	real public foreign debt denominated in \$NZ
RZP	private foreign debt denominated in foreign currency (expressed in \$NZ) at the beginning of the quarter)
RZPA	real private foreign debt denominated in \$NZ
SSRATIO_EQ	inventory investment to sale ratio for domestic good
TBAL	trade balances
TEXP	real total exports
TFBAL	transfer balances
TNZIAR_EQ	ratio of TNZIAA to GDPZ
TOBINQ	actual rate of returns on business investment - required rate of returns
TRBASE	transfers base (total number of average wages paid in benefits)
TF	time trend
TSR	quantity of gross output
UNB	sa no of people in unemployment benefit (start of quarter)
URT	unemployment rate
YCURVE	yield curve
YD	YDO+IMC+IMCS+IMCA
YDO	gross output of the private sector excluding commodity exports
YSR	quantity of primary factors