

## NCER Working Paper Series

# Forecasting Equicorrelation

Adam E Clements  
Christopher A Coleman-Fenn  
Daniel R Smith

Working Paper #72  
Revised August 2011

# FORECASTING EQUICORRELATIONS

ADAM E. CLEMENTS<sup>1</sup>, CHRISTOPHER A. COLEMAN-FENN<sup>1</sup>, and DANIEL R. SMITH<sup>1,2</sup>

<sup>1</sup>*School of Economics and Finance, Queensland University of Technology  
GPO Box 2434, Brisbane QLD 4001, Australia*

<sup>2</sup>*Beedie School of Business, Simon Fraser University  
8888 University Drive, Burnaby, BC, Canada, V5A 1S6*

August 25, 2011

## Abstract

We study the out-of-sample forecasting performance of several time-series models of equicorrelation, which is the average pairwise correlation between a number of assets. Building on the existing Dynamic Conditional Correlation and Linear Dynamic Equicorrelation models, we propose a model that uses proxies for equicorrelation based on high-frequency intraday data, and the level of equicorrelation implied by options prices. Using state-of-the-art statistical evaluation technology, we find that the use of both realized and implied equicorrelations outperform models that use daily data alone. However, the out-of-sample forecasting benefits of implied equicorrelation disappear when used in conjunction with the realized measures.

**Keywords:** Equicorrelation, Implied Correlation, Multivariate GARCH, DCC.

**JEL:** C32, C53, G17.

## Corresponding author

Adam Clements

School of Economics and Finance

Queensland University of Technology

GPO Box 2434, Brisbane, Qld, Australia 4001

email [a.clements@qut.edu.au](mailto:a.clements@qut.edu.au)

Ph +61 7 3138 2525, Fax +61 7 3138 1500.

# 1 Introduction

Recently the finance literature has seen renewed interest in the notion of equicorrelation, defined as the mean of the off-diagonal elements of a correlation matrix. Equicorrelation itself is not new, having been proposed by Elton and Gruber (1973) as a means to achieve superior portfolio allocation results due to reduced estimation error. More recently, Pollet and Wilson (2010) develop a theoretical argument, and provide supporting empirical evidence, that the average correlation of a stock market index is strongly related to future market returns, whereas stock market variance is not. Also, Driessen, Maenhout, and Vilkov (2009) show in an empirical exercise that the entire S&P 100 Index variance risk premium can be attributed to the correlation risk premium. Equicorrelation is also useful for portfolio managers interested in assessing the level of diversification among their assets. The equicorrelation of a portfolio is the only scalar measure we are aware of that summarises the degree of interdependence within a portfolio and hence its diversification benefits. Forecasts of equicorrelation may then provide portfolio managers with a simple guide to the interrelationships between their portfolio constituents which is more readily interpretable than forecasting each of the potentially numerous individual pairwise correlations.

The equicorrelation implied by option prices is also important. The return on a strategy known as dispersion trading, in which one goes long an option on a basket of assets and short options on each of the constituents, depends only on correlations after each of the individual options are delta hedged. It is common to assume that all of these correlations are equal, resulting in the value of the position depending upon the evolution of the implied equicorrelation (Engle and Kelly, 2008). Partially motivated by its use in dispersion trading, since July 2009, the Chicago Board of Exchange (CBOE) has published the Implied Correlation Index, the mean correlation of the S&P 500 Index for the preceding 22-trading-days. Therefore, in addition to being used in forming expectations of market returns, equicorrelation is also of use in popular derivatives trading strategies.

From an econometric perspective, the assumption of equicorrelation imposes structure on problems that are otherwise intractable. Many multivariate volatility models require the length of the time-series available to be significantly larger than the number of assets in the portfolio for the statistical results to be reliable, which is problematic for very large portfolios such as the S&P

500 Index. In the vast majority of models, the time-span available for estimation is limited to the shortest lived stock within the portfolio, which is conceivably quite short; for example, even the very large firm Kraft Inc. has only been a publicly traded firm since mid-2007.

Although we focus on equicorrelation forecasting, there has been significant prior interest in correlation forecasting more generally and a large range of competing candidate models exist that may also be applied to equicorrelation forecasting<sup>1</sup>. While there is a large number of Multivariate Generalised Autoregressive Conditional Heteroscedasticity (MGARCH) models, Silvennoinen and Teräsvirta (2009) note that an “ideal” time-series model of conditional covariance or correlation matrices faces competing requirements; while the specification must be flexible enough to model the dynamic structure of variances and covariances, it is also desirable to remain parsimonious for the purposes of estimation.

The Dynamic Conditional Correlation (DCC) model of Engle (2002), adapted for *consistent* estimation by Aielli (2009) in his cDCC model, allows for the forecasting of conditional correlations with the optimization of just two parameters while still retaining a reasonable degree of flexibility; it is on this model and variations thereof that we focus for generating our equicorrelation forecasts. In addition to meeting the criteria of flexibility and parsimony, the cDCC model has become the benchmark in the correlation forecasting literature and provides a natural starting point to begin discussing equicorrelation forecasting as the estimation framework for this model leads directly to that of two recently proposed equicorrelation models.

Motivated by the reasons outlined above, and to circumvent estimation issues that we will discuss shortly, Engle and Kelly (2008) propose two models of equicorrelation. One of these is based on the cDCC specification, the Dynamic Equicorrelation model; and the other is the Linear Dynamic Equicorrelation (LDECO) model. Both of these models are similar in functional form, but differ in the approach they take to modeling equicorrelation. This results in the LDECO model possessing additional flexibility as it allows the constituents of the portfolio to change over time, and even allows the number of portfolio constituents to change. The functional form of these models also plays an important role in being able to investigate potential avenues for improving

---

<sup>1</sup>It is beyond the scope of this paper to provide a thorough review of all of such models, see Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009) for a wide ranging overview and Laurent, Rombouts, and Violante (2010) for an extensive empirical comparison of out-of-sample forecast performance.

equicorrelation forecasts.

In the univariate volatility forecasting literature, it is now well-known that Realized Volatility (RV hereafter), which is defined as a sum of squared high-frequency intraday returns, provides a superior proxy for the latent volatility of an asset relative to the square of daily closing price returns (see, for example, Andersen, Bollerslev, Christoffersen, and Diebold, 2006). Further, when added as an exogenous regressor to GARCH models, Blair, Poon, and Taylor (2001) find that the co-efficient acting on RV is statistically significant, which implies RV has incremental explanatory power over the more noisy squared daily returns. The use of realized measures of latent variables has been extended into the multivariate setting by, for example, Barndorff-Nielsen, Hansen, Lunde, and Shephard (2010) and Corsi and Audrino (2007). These papers show that using high-frequency intraday data provides superior estimates of the level of latent covariance between assets, although one must be careful about market microstructure effects. A time-series model for correlation based on intraday data has been put forth by Corsi and Audrino (2007), who extend the univariate Heterogeneous Autoregressive (HAR) model for RV to its multivariate analogue, and demonstrate promising results in the bi-variate setting.

The existing equicorrelation proxy used in the LDECO model of Engle and Kelly (2008) is based upon the daily closing price returns of the portfolio constituents. The success of RV in the univariate framework, and the promising results for multivariate realized volatility just described motivate an investigation of whether a realized equicorrelation may be utilized to improve the forecast performance of the LDECO model. We propose three alternative proxies for equicorrelation that are based upon intraday data and may be substituted into the LDECO model in place of the daily returns based measure. This allows us to test whether high-frequency based proxies offer similar improvement in the equicorrelation setting to their benefit in the univariate volatility context. Two of these are based on the realized (co)variance technologies while the third is a non-parametric estimate of equicorrelation, the mean level of Spearman rank correlation.

In the univariate volatility forecasting literature it has also been shown that forecasts generated from the options market, implied volatility (IV), may contain information incremental to those based on physical market returns. The standard argument for the including IV is because options are priced with reference to a future-dated payoff, an efficient options market should in-

corporate both historical information as well as a forecast of information relevant to the pricing of the options. Poon and Granger (2003) report that IV based forecasts outperform time-series based forecasts in the majority of research that they reviewed. While this result is not directly related to equicorrelation, it does highlight a potential link between options markets and future levels of correlation.

These findings motivate our study of whether similar advantages may be found in the multivariate setting of conditional equicorrelation forecasting. By making the assumption of equicorrelation in the options market, it is possible to calculate the level of implied equicorrelation which may be used as a competitor to measures of equicorrelation based on historical data alone. Similar to including IV in univariate volatility models, the implied equicorrelation may be added to the LDECO specification to test for the marginal benefit in forecasting equicorrelation.

To summarize, motivated by prior results in the univariate volatility forecasting literature, we examine two potential improvements to the LDECO model; the use of realized equicorrelations in place of a daily returns based estimate, in a similar vein to RV in the univariate volatility literature; and a forecast of equicorrelation from the options market as an exogenous regressor, in a similar vein to IV in the univariate volatility literature. The two proposed sets of equicorrelation proxies may also be combined to analyse whether any improved forecasting ability over LDECO gained from incorporating IC disappears when one includes the realized equicorrelation.

We generate 22-trading-day ahead forecasts of equicorrelation using ten models that include existing time-series specifications and the extensions that we propose, which will all be specified in detail below. To evaluate the forecast performance of these models, we employ the Model Confidence Set (MCS) methodology of Hansen, Lunde, and Nason (2003, 2010). The MCS has been used previously in the univariate volatility context by, among others, Becker and Clements (2008) and in the multivariate setting by Laurent, Rombouts, and Violante (2010). An interesting result of the latter paper for the current context is that in turbulent times the DECO model, which is closely linked to the LDECO model employed here, dominates among DCC models, including those that relax the equicorrelation assumption and even include asymmetry terms.

We find that the proposed models of equicorrelation that include realized and implied corre-

lation separately both lead to superior in-sample fit and out-of-sample forecast performance over daily returns based information. While the specification that includes both of the discussed extensions perhaps unsurprisingly provides the best in-sample fit, for the purposes of out-of-sample forecasting, the use of realized equicorrelation alone is optimal for the data considered here.

The paper proceeds as follows. Section 2 provides an overview of the nesting framework and the models considered in this paper. Section 3 describes how forecasts of equicorrelation will be generated along with how the performance of the competing forecasts will be evaluated. Section 4 details the data, and the various proxies for equicorrelation employed. Section 5 presents and analyses the results and Section 6 concludes.

## 2 General Framework and Models Considered

Bollerslev (1990) and Engle and Kelly (2008) show that when modeling multivariate conditional covariances, it is useful to express the multivariate Gaussian density as

$$\mathbf{r}_{t|t-1} \sim N(0, \mathbf{H}_t), \quad \mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \quad (1)$$

where  $\mathbf{D}_t$  is the diagonal matrix of conditional standard deviations and  $\mathbf{R}_t$  is a conditional correlation matrix. The multivariate Gaussian log-likelihood function is then given by

$$\begin{aligned} L &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + \log|\mathbf{H}_t| + \mathbf{r}_t' \mathbf{H}_t^{-1} \mathbf{r}_t), \\ &= -\frac{1}{2} \sum_{t=1}^T (n \log(2\pi) + 2 \log|\mathbf{D}_t| + \mathbf{r}_t' \mathbf{D}_t^{-2} \mathbf{r}_t - \tilde{\mathbf{r}}_t' \tilde{\mathbf{r}}_t) \\ &\quad - \frac{1}{2} \sum_{t=1}^T (\log|\mathbf{R}_t| + \tilde{\mathbf{r}}_t' \mathbf{R}_t^{-1} \tilde{\mathbf{r}}_t), \\ &= L_{Vol}(\theta) + L_{Corr}(\theta, \Phi), \end{aligned} \quad (2)$$

where  $\tilde{\mathbf{r}}_t$  are volatility-standardized returns given by the  $n \times 1$  vector  $\tilde{\mathbf{r}}_t = \mathbf{D}_t^{-1} \mathbf{r}_t$ , and  $n$  is the number of assets under consideration.

Estimates of the correlation model parameters,  $\Phi$ , are obtained by maximizing the correlation component of the likelihood function,  $L_{Corr}(\theta, \Phi)$ . Traditional practice is to estimate volatility specific parameters,  $\theta$  in a first stage, followed by  $\Phi$ , which depend on the volatility specific parameters through the volatility standardized returns. We focus on forecasting equicorrelation and not on forecasting the univariate volatility of each asset. Hence, while numerous choices exist for maximizing the  $L_{Vol}$  component of the above log-likelihood function, it is of secondary importance here. We follow Engle and Kelly (2009) and model conditional variances using the GARCH(1,1) model. This allows us to focus directly on the maximizing the  $L_{Corr}$  component of the log-likelihood function; which is essentially a question of the most appropriate model choice for the evolution of the conditional correlation matrix,  $\mathbf{R}_t$ . Further, the in-sample log-likelihood results that are presented in Section 5.1 consist of comparisons of the  $L_{Corr}(\theta, \Phi)$  component of Equation (2) alone.

The first model for  $\mathbf{R}_t$  we discuss is the *consistent* Dynamic Conditional Correlation (DCC) model of Aielli (2009). This approach is flexible enough to model the dynamic structure over time, yet is parsimonious in that  $\Phi$  contains only two parameters to fully describe the evolution of pairwise correlations. It does require the estimation of  $n \times 3$  parameters for each of the univariate GARCH(1,1) models. Under the cDCC model, the conditional correlation matrix is given by

$$\mathbf{R}_t^{cDCC} = \tilde{\mathbf{Q}}_t^{-\frac{1}{2}} \mathbf{Q}_t \tilde{\mathbf{Q}}_t^{-\frac{1}{2}}, \quad (3)$$

where  $\mathbf{Q}_t$  has the following dynamics

$$\mathbf{Q}_t = \bar{\mathbf{Q}}(1 - \alpha - \beta) + \alpha \tilde{\mathbf{Q}}_{t-1}^{\frac{1}{2}} \tilde{\mathbf{r}}_{t-1} \tilde{\mathbf{r}}'_{t-1} \tilde{\mathbf{Q}}_{t-1}^{\frac{1}{2}} + \beta \mathbf{Q}_{t-1}, \quad (4)$$

where  $\bar{\mathbf{Q}}$  is the unconditional correlation matrix,  $\tilde{\mathbf{Q}}_t$  replaces the off-diagonal elements of  $\mathbf{Q}_t$  with zeros but maintains its principal diagonal<sup>2</sup>, and the following conditions must hold to ensure stationarity,  $\alpha > 0$ ,  $\beta > 0$ ,  $\alpha + \beta < 1$ .

Similar in structure to the univariate GARCH model, the cDCC model allows for an unconditional correlation matrix, or correlation targeting, as well as an innovation term on the lagged volatility-standardized residuals, and a persistence term for lagged values of  $\mathbf{Q}_t$ . The cDCC model

---

<sup>2</sup> $\tilde{\mathbf{Q}}_t = \mathbf{Q}_t \odot \mathbf{I}$ , where  $\mathbf{I}$  is the  $n \times n$  identity matrix, and  $\odot$  denotes the Hadamard product.



is attractive given its analytical tractability, flexibility, and low number of parameters; however, for the practical applications for which portfolio managers require solutions, the cDCC model falters as the optimization process requires finding the inverse and determinant of potentially very large matrices<sup>3</sup>. The calculation of these functions must be repeated at each time step for each iteration of the optimization algorithm, the estimation procedure can then quickly become very computationally burdensome.

## 2.1 The Dynamic Equicorrelation Model

To simplify computing the inverse and determinant in the log-likelihood function, Engle and Kelly (2008) make the simplifying assumption of equicorrelation in proposing an alternative means of modeling the conditional correlation matrix. At each point in time, it assumed that all off-diagonal elements of the conditional correlation matrix to the common scalar equicorrelation coefficient  $\rho_t$ . It is the dynamics of this equicorrelation that is the object of interest. Engle and Kelly (2008) suggest modeling  $\rho_t$  by using the cDCC specification to generate the conditional correlation matrix  $\mathbf{Q}_t$  and then taking the mean of its off-diagonal elements. This approach is termed the Dynamic Equicorrelation (DCC-DECO) model, and the scalar equicorrelation is formally defined by

$$\rho_t^{DECO} = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}}, \quad (5)$$

where  $q_{i,j,t}$  is the  $i, j$ th element of the matrix  $\mathbf{Q}_t$  from the cDCC model. This scalar equicorrelation is then used to create the conditional correlation matrix

$$\mathbf{R}_t = (1 - \rho_t)\mathbf{I}_n + \rho_t\mathbf{J}_n, \quad (6)$$

where  $\mathbf{J}_n$  is the  $n \times n$  matrix of ones and  $\mathbf{I}_n$  is the  $n$ -dimensional identity matrix.

The assumption of equicorrelation employed by the DECO model significantly decreases estimation time by allowing for analytical solutions to both the inverse and determinant of the conditional correlation matrix,  $\mathbf{R}_t$ , to be substituted into the log-likelihood function given in Equation

---

<sup>3</sup>While numerical techniques may exist for taking the inverse of a matrix such as Gauss-Jordan elimination, we are not aware of any such alternatives for computing the determinant of a matrix.

(2); these are given respectively by Equations (7) and (8) below

$$\mathbf{R}_t^{-1} = \frac{1}{1 - \rho_t} \left( \mathbf{I}_n - \frac{\rho_t}{(1 + [n - 1]\rho_t)} \mathbf{J}_n \right), \quad (7)$$

and

$$|\mathbf{R}_t| = (1 - \rho_t)^{n-1} (1 + [n - 1]\rho_t), \quad (8)$$

where  $\rho_t$  is the equicorrelation from Equation (5); the inverse  $\mathbf{R}_t^{-1}$  exists if and only if  $\rho_t \neq 1$  and  $\rho_t \neq \frac{-1}{n-1}$ , and  $\mathbf{R}_t$  is positive definite if and only if  $\rho_t \in \left( \frac{-1}{n-1}, 1 \right)$ .

While this has the advantage of simplifying estimation, it still possesses some drawbacks that prevent it from being the model of choice here. Again consider the practical perspective of a portfolio manager, an important limitation of the cDCC, DECO, and MGARCH models in general is that they are unable to handle changes in portfolio composition or the number of assets in the portfolio. Consider, for example, the S&P 500 Index, where the portfolio constituents do change quite frequently.<sup>4</sup> Hence, Engle and Kelly (2008) propose a variation of the DECO model that accommodates changes in portfolio composition: the Linear DECO (LDECO) model, which may be written generally as

$$\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1}, \quad (9)$$

where  $X_t$  is a proxy for, or estimate of equicorrelation on day  $t$ .  $X_t$  serves as the surprise news about the level of equicorrelation allowing for time-variation, essentially fulfilling the same roll as do lagged squared returns in the basic GARCH model. In fact the functional form of the LDECO model is the same as that of the GARCH(1,1) model of univariate volatility; the co-efficient  $\alpha$  is the weight placed on the innovation term  $X_{t-1}$ , and  $\beta$  is the weight to be placed on the persistence term,  $\rho_{t-1}$ . This model is quite distinct from the cDCC model. While the DECO model uses individual elements derived directly from the full correlation matrices  $Q_t$  generated by the cDCC model, the LDECO model is an autoregressive form estimated on historical proxies of equicorrelation alone, no output from the cDCC or any other MGARCH model is required. The specification given in Equation (9) is quite general since it defines  $X_t$  only as a measure of equicorrelation, encompassing many alternative specifications of past equicorrelation. Hence a natural question is, what is the

---

<sup>4</sup>Between June 1st 2010 to June 1st 2011, 11 firms were removed from the Index.

best choice of the measure  $X_t$ ?

When initially proposing the LDECO model, Engle and Kelly (2008) note that “key in this approach is extracting a measurement of the equicorrelation in each time period using a statistic that is insensitive to the indexing of assets in the return vector” (pp. 13, Engle and Kelly, 2008). Engle and Kelly (2008) propose a statistic that they argue fulfills this criterion, which may be substituted into Equation (9) for the generic variable  $X_t$ , and is based on the volatility-standardized daily closing price returns of each of the portfolio constituents,

$$u_t = \frac{[(\sum_i \tilde{r}_{i,t})^2 - \sum_i (\tilde{r}_{i,t}^2)]/n(n-1)}{\sum_i (\tilde{r}_{i,t}^2)/n}. \quad (10)$$

The equicorrelation proxy,  $u_t$ , can be decomposed into an estimate of the covariance of returns, the numerator, and an estimate of the variance for all assets, the denominator. As  $\tilde{r}_{i,t}$  are volatility-standardized returns they should have unit variance and, therefore, the numerator should be a correlation estimate. However, the numerator is not technically restricted to lie in the range that ensures  $\mathbf{R}_t$  is positive definite, and it lacks robustness to deviations from unity for the conditional variance estimate (Engle and Kelly, 2008). However, Engle and Kelly (2008) demonstrate that the denominator of  $u_t$  standardizes this covariance estimate by an estimate of the common variance; this ensures that  $u_t$  lies within the range necessary for positive definiteness of the correlation matrix.

It is here that the first extension to the LDECO model is proposed. It has been demonstrated in prior research (see, for example, ABCD 2006 and references therein) that realized volatility-based approaches generally provide superior estimates of latent univariate volatility. Furthermore, recent work by, among others, Barndorff-Nielsen, Hansen, Lunde, and Shephard (2010), and Corsi and Audrino (2007) show that in multivariate contexts, using high-frequency intraday data provide superior estimates of the interrelationships between assets relative to alternatives such as closing price returns. These findings motivate us to examine whether high-frequency based proxies for equicorrelation generate superior forecasts relative to the daily returns estimate of Engle and Kelly (2008).

The argument for using realized correlation in a time-series model is not new. For example, Corsi and Audrino (2007) use realized correlations in their multivariate Heterogeneous Autore-

gressive model for conditional correlation matrices. However, to the best of our knowledge this is the first time realized correlations have been used when modeling equicorrelation. To incorporate realized equicorrelation in our models we first adapt the existing realized (co)variance technologies to generate an estimate of the realized equicorrelation. We use three alternative methods, which are each discussed below.

The first realized estimate of equicorrelation we propose uses the entire covariance matrix. While there are several ways to estimate realized covariance, we adopt the same approach as used in Laurent, Rombouts, and Violante (2010) in their empirical study<sup>5</sup>, one that is based on results from Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielson and Shephard (2004). The realized covariance (RCOV) on a given day is calculated as follows. We partition each trading day,  $t$ , into  $L_t$  distinct, non-overlapping trading intervals, denoted by  $l_t = 1, \dots, L_t$ , and define the  $n$ -vector of asset returns for the interval  $l_t$  by  $\mathbf{r}_{l_t,t}$ . The length of each of these  $l_t$  periods is allowed to vary such that all of the asset returns for a given period are non-zero, with the restriction that the minimum window length is 15-minutes<sup>6</sup> to minimize the Epps (1979) effect. The varying length of intervals is reflected in the time subscript notation of  $L_t$ , as each day may have a different number of total trading intervals<sup>7</sup>. The realized covariance matrix is then defined for a given day as the

---

<sup>5</sup>We note for completeness that Laurent, Rombouts, and Violante (2010) compare their results from the above approach (which is slightly different as they used fixed window lengths of 5 minutes in calculating RCOV rather than the adaptive window length employed here) with a realized kernel estimator and find qualitatively similar results. Hence, we don't resort to kernel-based estimators such that proposed by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2010). Further, the minimum window length in calculating RCOV was also set at 1-, 5-, and 30-minute horizons with no qualitative impact on the results.

<sup>6</sup>The choice of 15 minutes is partially motivated by the results of Sheppard (2006) who finds that a minimum length of 10 minutes is sufficient to get unbiased estimates of the correlation between constituents of the DJIA. The choice of 15 minutes rather than 10 is based on it resulting in a whole number of periods within the day.

<sup>7</sup>Although we allow for varying trading interval length, for the overwhelming majority of cases the trading interval is indeed 15-minutes. The average length of time for all assets to have non-zero returns is 2.57 minutes, with the maximum length of time being 108 minutes. This results in the calculation of the RCOV for the most part being closely aligned with the 15-minute fixed window used in the calculation of univariate realized volatility and Spearman rank correlations, which are discussed shortly.

sum of the outer products of the  $\mathbf{r}_{l_t,t}$  vectors,

$$\begin{aligned}
RCOV_t &= \sum_{l_t=1}^{L_t} \mathbf{r}_{l_t,t} \mathbf{r}'_{l_t,t}, \\
\tau_{l_t,t} &= \min \tau, \text{ for } l_t = 1, \dots, L_t, \\
\text{s.t.} \quad &r_{i,l_t,t} \neq 0, \forall i, \tau_{l_t,t} - \tau_{l_t-1,t} \geq 15,
\end{aligned} \tag{11}$$

where  $\tau_{l_t,t}$  is the time of the end of the  $l_t$ -th period of day  $t$ .

Following equation (5), the realized equicorrelation (REC) may be found by taking the mean of the off-diagonal elements of RCOV

$$REC_t = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{RCOV_{i,j,t}}{\sqrt{RCOV_{i,i,t}RCOV_{j,j,t}}}. \tag{12}$$

As Equation (12) produces a scalar level of equicorrelation, it may substituted in as the variable  $X_t$  in place of the  $u_t$  in Equation (9).

An alternative approach to computing realized equicorrelation is to rely solely on the realized volatilities of each of the assets within an index (portfolio) and the index itself using the portfolio variance identity. The RV of asset  $i$  on day  $t$  is given by

$$RV_{i,t}^{(m)} \equiv \sum_{k=1}^m r_{k,t}^2, \quad k = 1, \dots, m, \tag{13}$$

where  $r_{i,k,t}^2$  is the squared intraday log-return on asset  $i$  from period  $k-1$  to  $k$  for each of the  $m$  fixed-length<sup>8</sup> periods within day  $t$ . After calculating the RV of an index as well as the RVs for each of the index's constituents, a realized measure of equicorrelation may be constructed by using the portfolio variance identity

$$RV_{p,t} = \sum_{i=1}^n w_i^2 RV_{i,t}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n w_i w_j RV_{i,t} RV_{j,t} \rho_{i,j}, \tag{14}$$

---

<sup>8</sup>Based on the research of Hansen and Lunde (2006) and related articles, the RV is calculated based on 15-minute intervals; 1-, 5-, and 30-minute intervals are also used for robustness with no qualitative effect on the results.

and making the assumption of equicorrelation; re-arranging yields

$$DREC_t = \frac{RV_{p,t}^2 - \sum_{i=1}^n w_i^2 RV_{i,t}^2}{2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n w_i w_j RV_{i,t} RV_{j,t}}, \quad (15)$$

where  $RV_{p,t}$  is the RV of the index and  $w_i$  the weight in that index placed on asset  $i$ . This realized equicorrelation measure is defined as DREC, where the D denotes that it only uses the diagonal elements of a covariance matrix, i.e., the individual RVs. By using only individual RVs to estimate equicorrelation, a potential benefit of the DREC measure is that it avoids the Epps effect (Epps, 1979) altogether, in which estimates of covariance may be biased downwards due to asynchronous trading. This approach gives a scalar equicorrelation that may also be substituted in for the variable  $X_t$  discussed previously.

As well as enabling the use of realized (co)variance technologies, the availability of high-frequency intraday data allows for a third approach to measuring equicorrelation. As both the REC and DREC measures are calculated from raw intraday returns, they may be excessively influenced by large shocks in returns or excess volatility in a small number of the constituent stocks. A non-parametric approach insensitive to the magnitude of the largest and smallest returns is the Spearman rank correlation, which examines how the rankings of returns are related throughout the course of the trading day<sup>9</sup>. The Spearman rank correlation between two assets  $i$  and  $j$ ,  $SR_{i,j}$ , is calculated from 15-minute log-returns

$$SR_{i,j,t} = 1 - \frac{6 \sum_{k=1}^m d_k^2}{m(m-1)}, \quad (16)$$

where  $d_k$  is the difference in rankings of returns for period  $k$  for each of the  $m$  15-minute periods of day  $t$ , the time indexing employed for the Spearman rank calculation is identical to the notation for the calculation of the realized variance. We then construct the Spearman rank equicorrelation (SREC) as the average of the off-diagonal elements of this matrix of Spearman rank correlations:

$$SREC_t = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^n SR_{i,j,t}, \quad (17)$$

---

<sup>9</sup>We thank Andrew Harvey for suggesting this approach.

In total, we consider three alternative proxies for equicorrelation in addition to the original measure proposed by Engle and Kelly (2008). Rather than simply using returns based on daily closing prices we consider three measures that use high-frequency intraday data; REC uses a realized covariance approach, DREC uses the index and individual asset realized volatilities, and SREC uses a non-parametric ranking of returns. Recall the functional form we use to model equicorrelation:  $\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1}$  (Equation (9)), which nests all these models by using each of these four alternative measures of the equicorrelation  $X_{t-1}$ :

$$\begin{aligned}
\text{LDECO: } \rho_t &= \omega + \alpha u_{t-1} + \beta \rho_{t-1}, \\
\text{REC: } \rho_t &= \omega + \alpha \text{REC}_{t-1} + \beta \rho_{t-1}, \\
\text{DREC: } \rho_t &= \omega + \alpha \text{DREC}_{t-1} + \beta \rho_{t-1}, \\
\text{SREC: } \rho_t &= \omega + \alpha \text{SREC}_{t-1} + \beta \rho_{t-1}.
\end{aligned} \tag{18}$$

## 2.2 Incorporating Implied Equicorrelation

In addition to using historical return-based proxies for equicorrelation, we investigate whether the information from options markets offers forecasting power over and above that contained in historical returns alone. Our motivation for this extension to LDECO again comes from an appeal to the univariate volatility literature, in which numerous papers demonstrate the advantages of implied volatility in generating forecasts; see Poon and Granger (2003), for example, for a review. Similar to the use of the *VIX* in the univariate context, it is possible to construct a model-free measure of implied equicorrelation (IC) from options data; which may be incorporated into the LDECO model.

It is well known that a model-free estimate of the implied volatility of a stock index that has options traded on it, e.g., the DJIA, can be constructed, i.e. the *VXD*<sup>10</sup>. For each of the constituent stocks in an index on which options trade, a similar model-free estimate of its IV may be found, denoted as  $IV_{i,t}$  for the implied volatility of asset  $i$ . Recalling the portfolio variance identity used in calculating the DREC earlier, and invoking the assumption of equicorrelation, model-free estimates

---

<sup>10</sup>The *VXD* is the DJIA equivalent of the potentially more well known *VIX* for the S&P 500 Index; a model-free, risk-neutral, option implied forecast of the mean annualised volatility of the index over a fixed 22 trading day horizon.

of index and individual asset IVs may be made used the to form the IC,

$$IC_t = \frac{IV_{p,t}^2 - \sum_{j=1}^n w_j^2 IV_{j,t}^2}{2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n w_i w_j IV_{i,t} IV_{j,t}}, \quad (19)$$

where  $IV_{p,t}$  is the annualised *implied* 22-day-ahead standard deviation of the index,  $w_i$  is the portfolio weight given to asset  $i$ , and  $s_i$  is the annualised *implied* 22-day-ahead standard deviation of asset  $i$ .

Engle and Kelly (2008) calculate the IC for the S&P500 index and show that it closely matches the fitted equicorrelation from both the cDCC-DECO and LDECO models, although they do not use the IC directly in model estimation or forecasting. Further, the IC has been used previously by Castren and Mazzotta (2005) in a bivariate setting of exchange rates and they find that a combination forecast of IC and an MGARCH model is preferred, these conclusions are based on the in-sample adjusted  $R^2$  only as they do not conduct a forecasting exercise. These results, and the publishing of IC for the S&P 500 Index by the CBOE<sup>11</sup>, motivate an investigation of the incremental information content of the IC relative to the various proposed measures of the equicorrelation,  $X_t$ .

Following Blair, Poon and Taylor (2001) who consider the role of the  $VIX$  in a univariate GARCH model of volatility, it is proposed that the LDECO specification be extended to include IC as an additional variable. This combines the information contained within the historical returns series of the portfolio constituents with the information implied by the options market,

$$\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1} + \gamma IC_{t-1}, \quad (20)$$

where  $X_t$  may be any of the previously proposed measures of equicorrelation, which leads to

$$\begin{aligned} \text{LDECO-IC: } \rho_t &= \omega + \alpha u_{t-1} + \beta \rho_{t-1} + \gamma IC_{t-1}, \\ \text{REC-IC: } \rho_t &= \omega + \alpha REC_{t-1} + \beta \rho_{t-1} + \gamma IC_{t-1}, \\ \text{DREC-IC: } \rho_t &= \omega + \alpha DREC_{t-1} + \beta \rho_{t-1} + \gamma IC_{t-1}, \\ \text{SREC-IC: } \rho_t &= \omega + \alpha SREC_{t-1} + \beta \rho_{t-1} + \gamma IC_{t-1}, \\ \text{IC: } \rho_t &= \omega + \gamma IC_{t-1}. \end{aligned} \quad (21)$$

---

<sup>11</sup>Details on the implied equicorrelation published by the Chicago Board of Exchange is available online at the CBOE S&P 500 Implied Correlation Index micro site (2009).



These models retain the useful property of having analytical solutions for the inverse and determinant of  $\mathbf{R}_t$  as given by Equations (7) and (8) respectively as equicorrelation is assumed in calculating IC and Equation (20) is a linear combination of two equicorrelation measures. Hence, the proposed model is easily incorporated into the previously defined general framework in Section 2 and may be estimated by quasi-maximum likelihood methods by optimising Equation (2). In a fully efficient options market, the co-efficient on  $X_t$  is expected to be statistically indistinguishable from zero as the historical information should be incorporated by options market participants in generating their IC forecast.

For any given definition of  $X_t$ , the model descriptions in Equation (21) above clearly nest those in Equation (18); for example, the REC-IC model nests the REC model. A standard likelihood ratio test of these two models allows for an examination of the improvement in model fit yielded by the inclusion of the IC, but only for those pairs of models using the same definition of  $X_t$ . Any improvements between non-nested model such as the REC-IC and DREC models may not be attributable to the IC term and hence standard likelihood ratio test are not applicable, and an alternative method of comparing these models is required. To compare the in-sample fit of non-nested models we use the non-nested likelihood ratio test of Vuong (1989). Where two non-nested models are competing to explain the same variable,  $\rho_t$  in our case, Vuong (1989) demonstrates that under certain regularity conditions the variable

$$T^{-1/2}LR_T/\hat{\xi}_T \xrightarrow{D} N(0, 1), \quad (22)$$

where  $LR_T = L_T^i - L_T^j$  is the difference in log-likelihood between models  $i$  and  $j$ , and  $\hat{\xi}_T$  is the variance of the likelihood ratio statistic:

$$\hat{\xi}_T = \frac{1}{T} \sum_{t=1}^T \left[ \log \frac{f_i(\rho_t)}{f_j(\rho_t)} \right]^2 - \left[ \frac{1}{T} \sum_{t=1}^T \log \frac{f_i(\rho_t)}{f_j(\rho_t)} \right]^2, \quad (23)$$

and  $f_i(\rho_t)$  here is the calculated  $L_{Corr}$  component of Equation (2) for model  $i$  for each of its fitted values of  $\rho_t$ .

### 3 Forecast Evaluation

We are interested in evaluating the forecasting performance of these various models of equicorrelation. We do this across a range of forecast horizons and use the Model Confidence Set methodology to compare the statistical performance of the respective forecasts.

#### 3.1 Generating Forecasts

In addition to an in-sample comparison of model performance, we generate multi-step-ahead point forecasts of equicorrelation up to a 22-day forecast horizon, the horizon over which the IC is defined. Unlike variance and covariance, however, one cannot aggregate correlation through time and each point forecast must be evaluated individually, rather than the total 22-day correlation. So we evaluate the forecasting performance of each of the models for each  $k$ -day ahead forecast,  $\forall k = 1, \dots, 22$  days.

To generate a multi-period forecast, we assume that  $\mathbb{E}_t[X_{t+k}] \approx \mathbb{E}_t[\rho_{t+k}]$ , which can then be used to generate recursive forecasts,

$$\mathbb{E}_t[\rho_{t+k}] = \omega + (\alpha + \beta)\mathbb{E}_t[\rho_{t+k-1}] + \gamma\mathbb{E}_t[IC_{t+k-1}]. \quad (24)$$

Unfortunately we have no *a priori* guidance as to the dynamics of IC and hence no way to forecast it, so we assume a simple AR(1) process. Under such dynamics, the multi-period forecast of IC is given by

$$\mathbb{E}_t[IC_{t+K}] = \theta_1^K IC_t + \mu(1 - \theta_1^K), \quad (25)$$

where  $\mu$  is the drift term in the AR(1) process and  $\theta_1$  is the co-efficient acting on the lagged value of  $IC_t$ . Recursively substituting Equation (24) into Equation (25) leads to the following expression

for multi-step-ahead forecasts,

$$\begin{aligned} \rho_{t+K} &= \omega \left[ \frac{1 - (\alpha + \beta)^{(K-1)}}{1 - \alpha - \beta} \right] + (\alpha + \beta)^{K-1} \rho_{t+1} \\ &+ \gamma \sum_{k=0}^{K-2} (\alpha + \beta)^{K-2-k} \left[ \mu \left[ \frac{1 - \theta^{(k+1)}}{1 - \theta} \right] + \theta^{(k+1)} IC_t \right]. \end{aligned}$$

In discussing potential avenues for forecasting equicorrelation, our focus has been on using alternative historical measures of equicorrelation directly in the estimation procedure; however, there does exist a natural alternative to this approach. Rather than estimating prior levels of equicorrelation and forecasting using Equation (9), it is possible to generate forecasts of a more general covariance matrix without the equicorrelation restriction imposed. One may take the equicorrelation forecast as the mean of the off-diagonal elements of this less restricted matrix; that is, the equicorrelation restriction may be imposed *post hoc* to the estimation procedure. Even though the forecast object will still be the level of equicorrelation, we argue this is a more flexible approach in generating the forecast; each of the correlation pairs is allowed to evolve in a less restricted framework. The chosen model for this alternative approach is the cDCC model of Aielli (2009) given its benchmark status in the literature; forecasts generated in this fashion will be denoted  $\overline{cDCC}$ .

Finally, we note that each of the models are estimated over a rolling fixed estimation length<sup>12</sup> of 1000-trading-days. After allowing for a 1000-trading-day initial estimation window, 941 out-of-sample forecasts are generated for the 22-day-ahead horizon; while more forecasts could have been generated for the shorter forecast horizons, we decided to keep the sample size the same across all statistical analyses.

### 3.2 Statistical Evaluation of Forecasts

In order to statistically evaluate the relative forecast performance of the models considered, we require an estimate of the “true” equicorrelation on each of the days for which point forecasts are generated in order to gauge their accuracy. We have already argued in favor of realized equicorre-

---

<sup>12</sup>Expanding window estimation was also carried out with no qualitative difference in results.

lation as a superior measure of the daily relationship between assets of interest when constructing the REC measure for use within the LDECO model and the majority of our results will be discussed with this measure in mind. However, as a robustness check, we also use the Engle and Kelly (2008) measure of equicorrelation defined in Equation (10),  $u_t$ , the diagonal realized equicorrelation (DREC) defined in Equation (15), and the Spearman rank equicorrelation defined in (16),  $SREC$ , as the “true” equicorrelation values; the results are qualitatively similar across all measures.

We employ the Model Confidence Set (MCS) approach (Hansen, Lunde and Nason; 2003, 2010) to examine the forecast performance of each of the models considered. A forecast loss measure is central to the MCS methodology. Although there are many options available, the loss functions we use are mean-square-error (MSE) and QLIKE,

$$MSE_k^i = (\rho_{t+k} - f_{t,k}^i)^2, \quad (26)$$

$$QLIKE_k^i = \log(f_{t,k}^i) + \frac{\rho_{t+k}}{f_{t,k}^i}, \quad (27)$$

where  $f_{t,k}^i$ ,  $i = 1, \dots, \mathcal{M}$  are individual forecasts (formed at time  $t$  for  $k$ -days ahead) obtained from an initial set of  $\mathcal{M}$  individual models, and  $\rho_{t+k}$  is the measure of true equicorrelation.

While these loss functions allow forecasts to be ranked, they give no indication of whether the top performing model is statistically superior to any of the lower-ranked models. The MCS approach allows for such conclusions to be drawn. The construction of a MCS is an iterative procedure that requires sequential testing of equal predictive accuracy (EPA) between competing forecasts. The procedure begins with a set of  $\mathcal{M}$  individual forecasts to which tests of EPA are applied. Any forecast found to be statistically inferior is eliminated leaving  $\mathcal{M}^*$  forecasts such that  $\mathcal{M}^* \subset \mathcal{M}$ . This iterative procedure is repeated until EPA cannot be rejected and hence the remaining  $\mathcal{M}^*$  forecasts are of EPA at a given level of confidence. MCS results are presented in the form of *p-values* for an individual forecast being a member of the final MCS,  $\mathcal{M}^*$ . The *p-values* relate to the rejection of the null hypothesis that a forecast is a member of  $\mathcal{M}^*$ , hence the smaller a *p-value* the less likely a forecast is a member of the MCS. We refer the reader to Hansen, Lunde and Nason (2003, 2010) for technical details regarding the implementation of the MCS methodology.

## 4 Data

Our results are based on the DJIA over the period starting on the 1st of November 2001 through to the 30th of October 2009, giving 1964 observations<sup>13</sup>. Our data comes from three sources: the OptionsMetrics IvyDB US database for calculating model-free implied volatilities for individual stocks, the CBOE for the daily closing values of the  $VXD$  index, and ThomsonReuters Tick History for minute-by-minute intraday prices used in calculating the realized equicorrelation measures.

Similar to the more commonly known  $VIX$  for the S&P 500 Index, the  $VXD$  is a model-free 22-day-ahead at-the-money implied volatility forecast for the DJIA. To fix ideas, the day  $t$  implied equicorrelation is given by

$$IC_t = \frac{VXD_t^2 - \sum_{j=1}^n w_{j,t}^2 RV_{j,t}^2}{2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n w_{i,t} w_{j,t} RV_{i,t} RV_{j,t}}, \quad (28)$$

where the weights now have a  $t$  subscript to denote that the constituents of the index vary through time<sup>14</sup>.

For comparative purposes, the entire sample of the five equicorrelation proxies used in this paper are plotted in Figure 1; the  $u_t$  measure proposed by Engle and Kelly (2008), the implied equicorrelation, the realized diagonal equicorrelation, the realized equicorrelation and the Spearman rank equicorrelation.

Figure 1 about here.

From the plot of  $u_t$  in Panel A of Figure 1, we observe that the measure proposed by Engle and Kelly (2008) is quite noisy, perhaps more noisy than one would expect of the mean correlation of thirty of the largest US firms. However, to demonstrate that this measure is still quite persistent, the centered 44-day-moving-average, the mean equicorrelation of data one month either side of a given day, is plotted in white.

---

<sup>13</sup>We choose the DJIA as we are able to obtain the implied volatilities of each of its constituent stocks for each day of the sample and are therefore able to calculate the implied equicorrelation with certainty. This is not true of the S&P 500 Index, for which the CBOE publishes its IC based on an approximation from the largest 50 stocks within the index, as not all of its constituent stocks have listed options traded.

<sup>14</sup>Although the DJIA is relatively more stable than, say, the S&P 100, only 17 of the original 30 constituents remain in the index consistently for our sample period.

As can be seen in Panel B of Figure 1, IC derived from the options market appears to be significantly less noisy than any of the alternatives. It also appears to track the realized measures quite closely, which augurs well for the out-of-sample forecasting exercise given these measures are used as alternative “true” equicorrelation proxies. Further anecdotal support for the use of IC in equicorrelation forecasting comes from the fact that the  $IC_t$  tends to peak in times of market turmoil; when large indices fall, the majority of assets suffer losses and this is reflected in a high level of correlation across assets.

Panels C, D and E of Figure 1 plot the realized equicorrelation measures and the Spearman rank equicorrelation over the sample period. We observe that the measures follow similar dynamics, as realized diagonal equicorrelation is the least noisy of the three it may possess more power in distinguishing between the out-of-sample forecast performance of the competing models.

To reinforce the point regarding the relative noisiness of  $u_t$ , descriptive statistics are provided for each of the series in Table 1. It can be seen that the  $u_t$  is extremely noisy, with its standard deviation of 0.2681 larger than its mean of 0.2631;  $u_t$  is also weakly correlated with all of the alternative estimates. The standard deviation of the other four measures are significantly smaller, all falling between 0.105 and 0.135, and they are more highly correlated with each other. The two realized equicorrelations DREC and REC, are somewhat different in their means at 0.3556 and 0.2782 respectively, with the standard deviation of the DREC measure slightly smaller, they are unsurprisingly highly correlated with each other at 0.7237. The mean of IC, at 0.4218, is higher than all of the other measures and probably reflects a correlation risk premium being priced in the derivatives market. Finally, we note that estimates of equicorrelation from the physical market are not highly correlated with the IC from the options market, so there should not be any adverse effects from multicollinearity by including multiple estimates of equicorrelation.

Table 1 about here.

## 5 Results

We now study the performance of these proxies for equicorrelation to fit in-sample the DJIA, and forecasts the average correlation out-of-sample.

### 5.1 In-Sample Estimation Results

There are two broad questions we address in this Section. Firstly, of the proposed alternatives, what is the optimal choice for the equicorrelation measure  $X_t$ ? Secondly, does the information contained within IC lead to superior model fit above those models based on historical returns alone? Addressing these questions will shed light on whether the incremental value from the addition of IC term is robust to the choice of  $X_t$ . This analysis is based on the entire sample available.

To begin addressing the question of the optimal choice for the equicorrelation measure  $X_t$ , the results for the restricted models are presented first. These models exclude the information content of the IC by enforcing the restriction of  $\gamma = 0$  in Equation (20); their parameter estimates and respective  $L_{Corr}$  terms from Equation (2) are presented in Panel A of Table 2. We observe that the best in-sample fit of the restricted models is given by the choice of REC as the equicorrelation proxy. It generates the highest log-likelihood function value and the associated co-efficient is also statistically significant at more than three robust standard errors from zero. The worst in-sample fit is given by the  $u_t$  proposed by Engle and Kelly (2008), while the relevant co-efficient is statistically insignificant at approximately 1.5 robust standard errors from zero.

Table 2 about here.

The relative log-likelihood values of the restricted models may be assessed through the Vuong likelihood ratio test results in Panel A of Table 3. At traditional levels of significance only one claim may be made: that using the REC measure offers statistically significant improvement over the  $u_t$  and DREC definitions of  $X_t$ , it cannot be statistically separated from the Spearman rank equicorrelation measure. No other proposed measure of equicorrelation offer a significant difference in the  $L_{Corr}$  term relative to its competitors.

Table 3 about here.

By incorporating information contained within IC in relaxing the restriction that  $\gamma = 0$  in Equation (20), an interesting pattern emerges from the results presented in Panel B of Table 2. We observe that none of the estimated coefficients of the proposed proxies for equicorrelation,  $X_t$ , are statistically significant; in each case the standard error is of larger magnitude than the parameter estimate. This would suggest that the choice of  $X_t$  is irrelevant as they all lack significant explanatory power in this setting. This result is confirmed by the Vuong likelihood ratio test results presented in Panel B of Table 3. After including IC, none of the values for  $L_{Corr}$  are statistically different from each other. The only specification that is consistently dominated is the specification using information from the options market alone,  $\rho_t = \omega + \gamma IC_t$ .

Overall, the above discussion demonstrates that the choice for the  $X_t$  is important only if IC is excluded. If IC is included, then the choice for  $X_t$  is irrelevant as all of these models will possess statistically indistinguishable in-sample fits. If the model is based on historical information alone, then the choice of REC will dominate the  $u_t$  and DREC proxies, but not SREC. As the relevance of the choice of  $X_t$  is dependent on the inclusion of the IC term, whether IC itself warrants inclusion is now addressed. In Panel B of Table 2  $p$ -values of standard likelihood ratio tests are reported for the restriction that  $\gamma = 0$ , in all cases the relevant  $p$ -values are smaller than 0.10. This result combined with insignificant estimates of  $\alpha$ , leads us to conclude that the IC subsumes the information content of all of the alternative proxies of equicorrelation that are based on historical data alone.

In addition, the Vuong likelihood ratio test results in Table 4 reinforce the fact that the inclusion of the IC term improves model fit. The prior Vuong statistics separately compared the relative performance of the restricted models given in Equation (18) with the results given in Panel A of Table 3, while the results presented in Panel B of Table 3 were for unrestricted models given in Equation(21); both of these sets of results focus solely on the choice of  $X_t$  measure. Comparing the restricted against the unrestricted models via the Vuong likelihood ratio test allows for an examination of the statistical improvement in model fit offered by the inclusion of the IC. Table 4 compares the log-likelihood values of those models that include the IC term with those that do not, we find that all of the models which include the IC term dominate those that do not. Even the  $\rho_t = \omega + \gamma IC_t$  specification outperforms all of the restricted models; each of the calculated



$p$ -values is less than 0.01, suggesting clear rejection of the null hypothesis that the models have equal log-likelihood.

Table 4 about here.

These results demonstrate that including IC improves model fit. The unrestricted models all have insignificant co-efficient estimates for the historical measures of equicorrelation and significant parameter estimates for the IC term. Further, all of the Vuong likelihood ratio test results suggest that the including IC leads to significantly improved model fit over models that do not include information from the options market.

Plots of the fitted equicorrelations,  $\rho_t$  given a range of the candidate models<sup>15</sup> are shown in Figure 2. The plots reinforce the prior results in that the choice of proxy for  $X_t$  is irrelevant if the IC term is included, as the estimates of  $\rho_t$  in Panels B, D, and F are remarkably similar. However, there are a clear differences in  $\rho_t$  when IC is excluded as seen in Panles A, C and E. The fitted  $\rho_t$  when  $u_t$  is used for  $X_t$  appear to miss variations in the underlying object of interest, which would explain its poor performance revealed by earlier results in Table 2.

Figure 2 about here.

To summarise the in-sample results, all of the proposed extensions to the original LDECO model of Engle and Kelly (2008) yield a higher log-likelihood. However, only the REC proxy offers statistically significant improvement over the original measure among the restricted models that do not include information from the options market. If IC is included, then this also results in a statistically significant improvement over the original Engle and Kelly (2008) specification. However, it also means the choice of equicorrelation proxy is rendered redundant as none of the unrestricted models can be separated by the Vuong likelihood ratio test. Overall, these results are consistent with the univariate volatility forecasting literature. Realized proxies for volatility offer improvements over estimates of volatility based on closing price returns, and option based information is beneficial.

---

<sup>15</sup>The univariate model of IC is excluded as it is obvious from the results in Table 2 that a persistence term is highly significant. Further, models incorporating the SREC measure are also excluded as they are qualitatively similar to the REC models.

## 5.2 Out-of-sample Forecast Results

The results from the MCS procedure are presented in Table 5 with the forecast performance evaluated using the Mean Square Error loss function under the range statistic using the REC as the measure of “true” equicorrelation<sup>16</sup>. This table presents a summary of the  $p$ -values of rejecting the null hypothesis that the relevant model is not a member of the MCS; the higher the  $p$ -value, the more likely that the relevant specification belongs in the set of statistically superior models. The statistics are summarized by a spectrum of “ticks” reflecting the probability of a model being included in the MCS; a blank entry indicates a  $p$ -value between 0.00 and 0.05,  $\checkmark$  indicates a  $p$ -value between 0.05 and 0.10,  $\checkmark\checkmark$  between 0.10 and 0.20, and  $\checkmark\checkmark\checkmark$  greater than 0.20. From the summarized results in Table 5, two main observations may be made.

Overall, the REC proxy for equicorrelation generates the best out-of-sample forecasts (under both loss functions, test statistics and all measures of “true” equicorrelation; however) at short forecast horizons. As the forecast horizon increases, it becomes increasingly difficult to statistically distinguish between the competing models. Beyond the 5 day forecast horizon, the only models to be excluded under any loss function, test statistic or equicorrelation proxy are the  $\overline{cDCC}$  model, and those models that use either the  $u_t$  or DREC; that is, those models that use either daily data or the diagonal realized equicorrelation<sup>17</sup>.

Note that in the robustness check of using four measures of “true” equicorrelation, there is one exception to the REC specification providing the best forecast. In results not reported here, the DREC specification provides the best forecast under the DREC target for equicorrelation, at the one-day horizon under both loss functions and test statistics. For all other loss functions, tests statistics, time horizons, and measures of equicorrelation<sup>18</sup> the REC measure provides the superior out-of-sample forecast, and we therefore believe that our finding that REC is the superior measure is a robust result.

---

<sup>16</sup>Results are qualitatively similar for both the QLIKE and MSE loss functions under both the range and semi-quadratic test statistics. Further, the use of the alternative measures of “true” equicorrelation, the DREC, SREC and  $u_t$  measures, do not qualitatively alter the results. For brevity’s sake, only one set of representative results are presented here, with the remainder available from the authors upon request.

<sup>17</sup>Recall that the cDCC model uses daily closing price volatility-standardized returns in its estimation.

<sup>18</sup>These results are not presented to conserve space, but are available from the authors upon request.

It is important to note from Table 5 that those models that rely on daily closing price returns are typically the worst performing. Both the original LDECO model using  $u_t$ , and  $\overline{cDCC}$ , generally yield inferior forecasts to those specifications that use realized and implied equicorrelation. In fact, a simple linear regression on IC typically yields superior forecasts to those models solely using daily returns. These out-of-sample forecasting results confirm the in-sample estimation results, as well as corroborating a larger amount of the univariate literature that shows that realized estimates of latent volatility are superior to those based on daily closing price returns.

However, when examining those models that incorporate realized measures of equicorrelation (DREC, REC or SREC), forecast performance generally deteriorates over the longer term upon the inclusion of the IC measure. This may reflect the fact that the chosen AR(1) specification for IC does not adequately match its true dynamics. However, as REC dominates at all time horizons, even an improved forecasting method for IC would not reverse the rankings of the models. Perhaps if the REC-IC model dominated for shorter horizons before the REC specification became the superior forecast, a more thorough investigation of the dynamics of IC would be warranted, but this is not believed to be the case here. In either scenario, the superior in-sample fit by including IC is not replicated in the out-of-sample forecasting exercise, where information from the physical market alone generates the best forecasts.

## 6 Conclusions

We have analyzed the in-sample fit and out-of-sample forecasting performance of ten candidate models of equicorrelation after adapting the LDECO model to utilize realized and implied proxies for equicorrelation. We find that the proxy for equicorrelation based on realized covariance technology provided superior in-sample fit to all of the alternative historical estimates considered. This difference is statistically significant in the restricted models where no option implied information was included, but the inclusion of implied equicorrelation rendered the choice of realized proxy irrelevant. In fact, all of the historical based estimates of equicorrelation were statistically insignificant when implied equicorrelation was added as an exogenous regressor to the LDECO specification. This result may be used to argue for the informational efficiency of the options market. Further,

this finding is similar to the majority of research in the univariate volatility forecasting literature where option implied measures have information incremental to historical measures of volatility, particularly squared daily returns.

In the out-of-sample forecasting results, realized equicorrelation provided superior performance. It was the best performing model at all horizons and under both the QLIKE and MSE loss functions; with only one exception to its dominance. Further, it typically generated more accurate forecasts than models including implied equicorrelation. That is, the in-sample benefits of including implied equicorrelation did not translate to the out-of-sample analysis. However, the implied equicorrelation based models did typically outperform those models based on daily returns based estimates of equicorrelation. Again, these results resemble those in the univariate volatility literature where implied volatility typically outperform squared daily returns, but do not dominate realized volatility.

## References

- Aielli, G.P., Dynamic Conditional Correlations: On Properties and Estimation, 2009. SSRN Working Paper, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1507743](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1507743)
- Andersen, T. G., Bollerslev, T., Christoffersen, P. F., and Diebold, F. X., Volatility and Correlation Forecasting, 2006. In G. Elliot, C. Granger, and A. Timmermann, *Handbook of Economic Forecasting*, North Holland Press.
- Andersen, T., Bollerslev, T., Diebold, F. X., and Labys, P., (2003): "Modeling and Forecasting Realized Volatility", *Econometrica*, **71**, 579625.
- Barndorff-Nielson, O. E., Hansen, P. R., Lunde, A., Shephard, N., Designing Realized Kernels to Measure the ex post Variation of Equity Prices in the Presence of Noise, 2008. *Econometrica* 76-(6), 1481-1536.
- Barndorff-Nielson, O. E., Hansen, P. R., Lunde, A., Shephard, N., Multivariate Realised Kernels: Consistent Positive Semi-Definite Estimators of the Covariation of Equity Prices with Noise and Non-Synchronous Trading, Working Paper, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1154144](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1154144)
- Barndorff-Nielson, O. E., Shephard, N., Econometric analysis of realized volatility and its use in estimating stochastic volatility models, 2002. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 64-(2), 253-280.
- Barndorff-Nielson, O. E., Shephard, N., Estimating quadratic variation using realized variance, 2002. *Journal of Applied Econometrics* 17-(5), 457-477.
- Barndorff-Nielson, O. E., Shephard, N., Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics, 2004. *Econometrica* 72-(3), 885-925.
- Bauwens, L., Laurent, S., Rombouts, J. V. K., Multivariate GARCH Models: A Survey, 2006. *Journal of Applied Econometrics* 21, 79-109.
- Becker, R., Clements, A., Are combination forecasts of S&P 500 volatility statistically superior?, 2008. *International Journal of Forecasting* 24, 122-133.

- Becker, R., Clements A. E., Coleman-Fenn, C. A., Forecast performance of implied volatility and the impact of the volatility risk premium, 2009. NCER Working Paper Series, 45. <http://www.ncer.edu.au/papers/documents/WPNo45.pdf>
- Blair, B. J., Poon, S-H., Taylor, S. J., Forecasting S&P 100 volatility: the incremental information content of implied volatilities and high-frequency index returns, 2001. *Journal of Econometrics* 105, 5-26.
- Bollerslev, T., Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH model, 1990. *Review of Economics and Statistics* 72, 498-505.
- Bollerslev, T., Engle, R. F, Wooldridge, J. M., A Capital Asset Pricing Model with Time-Varying Covariances, 1988. *Journal of Political Economy* 96-(1), 116-131.
- Castren, O., Mazzotta, S., Foreign Exchange Option and Returns Based Correlation Forecasts: Evaluation and Two Applications, 2005. European Central Bank Working Paper Series, 447. <http://www.ecb.int/pub/pdf/scpwps/ecbwp447.pdf>
- Chicago Board of Exchange, CBOE S&P 500 Implied Correlation Index, 2009. <http://www.cboe.com/micro/impliedcorrelation/ImpliedCorrelationIndicator.pdf>
- Corsi, F., A Simple Approximate Long-Memory Model of Realized Volatility, 2009. *Journal of Financial Econometrics* 7-(2), 174-196.
- Corsi, F., Audrino, F., Realized Correlation Tick-by-Tick, 2007. Universität St. Gallen Discussion Paper 2007-02, <http://www.vwa.unisg.ch/RePEc/usg/dp2007/DP02-Au.pdf>
- Driessen, J., Maenhout, P., and Vilkov, G., Option Implied Correlations and Correlation Risk, 2009. *Journal of Finance* 64-(3), 1061-1519.
- Elton, E. J., Gruber, M. J., Estimating the Dependence Structure of Share Prices-Implications for Portfolio Selection, 1973. *Journal of Finance* 28, 1203-1232.
- Engle, R. F., Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, 2002. *Journal of Business and Economic Statistics* 20-(3), 339-350

- Engle, R. F., Kelly, B. T., Dynamic Equicorrelation, 2008. NYU Working Paper, <http://pages.stern.nyu.edu/~rengle/Dynamic%20Equicorrelation.pdf>
- Engle, R. F., Kelly, B. T., Dynamic Equicorrelation, 2009. SSRN Working Paper, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1354525](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1354525)
- Epps, T. W., (1979) "Comovements in Stock Prices in the Very Short Run", *Journal of the American Statistical Association*, **74**, 291-298.
- Hansen, P. R., Lunde, A., Realized Variance and Market Microstructure Noise, 2006. *Journal of Business and Economic Statistics* 24, 127-218
- Hansen, P. R., Lunde, A., Nason, J. M., (2003). "Choosing the best volatility models: The model confidence set approach", *Oxford Bulletin of Economics and Statistics* 65, 839-861.
- Hansen, P. R., Lunde, A., Nason, J. M., The Model Confidence Set, 2010. SSRN Working Paper, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=522382](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=522382)
- Jorion, P., Predicting Volatility in the Foreign Exchange Market, 1995. *Journal of Finance* 50, 507-528.
- Laurent, S., Rombouts, J. V. K., and Violante, F., On the Forecasting Accuracy of Multivariate GARCH Models, 2010. CIRPEE Working paper, [http://www.cirpee.org/fileadmin/documents/Cahiers\\_2010/CIRPEE10-21.pdf](http://www.cirpee.org/fileadmin/documents/Cahiers_2010/CIRPEE10-21.pdf)
- Mincer, J., Zarnowitz, V., The evaluation of economic forecasts, 1969. In *Economic Forecasts and Expectations*, Columbia University Press.
- Pollet, J. M., and Wilson, M., Average correlation and stock market returns, 2010. *Journal of Financial Economics* 96, 364-380.
- Poon S-H., Granger C. W. J., Forecasting volatility in financial markets: a review, 2003. *Journal of Economic Literature* 41, 478-539.
- Sheppard, K., (2006), "Realized Covariance and Scrambling", Working Paper, <http://www.kevinsheppard.com/images/7/7e/Scrambling.pdf>

Silvennoinen, A., Teräsvirta, T., Multivariate GARCH models, 2009. In *Handbook of Financial Time Series*, Springer.

Vuong, Q. H., Likelihood ratio tests for model selection and non-nested hypotheses, 1989. *Econometrica* 57, 307-333.



Table 1: Descriptive Statistics for Alternative Equicorrelation Measures  $X_t$

We present the mean, standard deviation, and correlation matrix between four equicorrelation innovation variables: the Linear DECO variable  $u_t$  (Equation (10)), the average pairwise Realized Equicorrelation  $REC_t$  (Equation (12)), a portfolio-based Realized Equicorrelation  $DREC_t$  (Equation (15)), and the Spearman Realized Equicorrelation  $SREC_t$  (Equation (17)); and the implied Equicorrelation from options prices  $IC_t$  (Equation (19)).

$X_t$	Mean	Std	Correlation Matrix				
			$u_t$	$IC_t$	$REC_t$	$DREC_t$	$SREC_t$
$u_t$	0.2631	0.2681	1	0.2075	0.2112	0.1624	0.1680
$IC_t$	0.4218	0.1152	0.2075	1	0.4649	0.5047	0.4429
$REC_t$	0.3556	0.1347	0.2112	0.4649	1	0.7237	0.6932
$DREC_t$	0.2782	0.1054	0.1624	0.5047	0.7237	1	0.5523
$SREC_t$	0.3363	0.1221	0.1680	0.4429	0.6932	0.5523	1

Table 2: In-sample Equicorrelation Model Parameter Estimates

We report parameter estimates for the full in-sample window (robust standard errors are given in parentheses) and values of the log-likelihood ( $L_{Corr}$ , which is given in equation (2)) for nine candidate models. The models can be categorized into two sub-sets of the general specification:  $\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1} + \gamma IC_t$ , where  $X_{t-1}$  is one of four innovations in equicorrelation (the Linear DECO variable  $u_t$  (Equation (10)), the average pairwise Realized Equicorrelation  $REC_t$  (Equation (12)), a portfolio-based Realized Equicorrelation  $DREC_t$  (Equation (15))), and the Spearman Realized Equicorrelation  $SREC_t$  (Equation (17)) and the implied equicorrelation  $IC_t$  (Equation (19)). Panel A presents the restricted models where  $\gamma = 0$ , while Panel B presents the unrestricted model; one exception exists in which we use implied correlation alone:  $\rho_t = \omega + \gamma IC_t$ . In Panel B, the  $p$ -values of a likelihood ratio test of  $\gamma = 0$  are reported in parentheses under the relevant log-likelihood  $L_{Corr}$  value.

Panel A: Comparison of Models with Restriction $\gamma = 0$					
$X_t$ measure	$\omega$	$\alpha$	$\beta$	$\gamma$	$L_{Corr}$
$u_t$	0.0778 (0.0592)	0.0798 (0.0535)	0.7523 (0.178)	- -	-8756.1362
REC	0.0422 (0.0182)	0.1240 (0.0399)	0.7769 (0.0636)	- -	-8722.9397
DREC	0.1058 (0.0616)	0.2181 (0.1311)	0.5832 (0.2372)	- -	-8730.4046
SREC	0.0365 (0.0120)	0.0686 (0.0213)	0.8497 (0.0392)	- -	-8747.8650
Panel B: Comparison of Models without Restriction $\gamma = 0$					
$u_t$ -IC	0.0210 (0.0150)	0.0121 (0.0260)	0.7462 (0.0907)	0.1920 (0.0626)	-8613.6772 (0.0375)
REC-IC	0.0164 (0.0140)	0.0199 (0.0288)	0.7613 (0.0671)	0.1779 (0.0517)	-8613.1755 (0.0989)
DREC-IC	0.0202 (0.0340)	0.0258 (0.0345)	0.7615 (0.2551)	0.1872 (0.0625)	-8614.3331 (0.0870)
SREC-IC	0.0161 (0.0147)	0.0143 (0.0255)	0.7728 (0.0681)	0.1737 (0.0587)	-8613.6491 (0.0493)
IC	0.1362 (0.0407)	- -	- -	0.6563 (0.0875)	-8638.6055

Table 3: Non-nested Likelihood Ratio Test Comparisons Between  $X_t$  Variables.

We report the results of non-nested Vuong likelihood ratio tests to compare the in-sample performance of the different equicorrelation models,  $\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1} + \gamma IC_t$ , where  $X_{t-1}$  are the four alternative equicorrelation innovation terms (the Linear DECO variable  $u_t$  (Equation (10)), the average pairwise Realized Equicorrelation  $REC_t$  (Equation (12)), a portfolio-based Realized Equicorrelation  $DREC_t$  (Equation (15)), and the Spearman Realized Equicorrelation  $SREC_t$  (Equation (17)) and  $IC_t$  is the implied equicorrelation (Equation (19))). The focus is on the relative performance of the various  $X_t$  rather than comparing restricted models ( $\gamma = 0$ ) with unrestricted models. As the models are non-nested, we use Vuong likelihood ratio statistic outlined in Equation (22),  $p$ -values are reported in parentheses. The Vuong statistic of row  $i$  and column  $j$  is positive if model  $i$  has a superior in-sample fit to model  $j$ . In each case,  $H_0 : L_{Corr}^i = L_{Corr}^j$  or that the in-sample fit of each model is equal;  $H_1 : L_{Corr}^i > L_{Corr}^j$  or that model  $i$  offers superior in-sample fit.

Panel A: Comparison of Models That Impose the Restriction $\gamma = 0$					
$X_t$ measure	$u_t$	REC	DREC	SREC	
$u_t$	-	-1.4368 (0.9246)	-1.1868 (0.8823)	-0.4648 (0.6789)	
REC	1.4368 (0.0753)	-	1.6566 (0.0488)	0.3182 (0.3751)	
DREC	0.4648 (0.3210)	-1.6566 (0.9511)	-	-0.7740 (0.7805)	
SREC	1.1868 (0.1176)	-0.3182 (0.6248)	0.7740 (0.2194)	-	
Panel B: Comparison of Models Without the Restriction $\gamma = 0$					
	$u_t$ -IC	REC-IC	DREC-IC	SREC-IC	IC
$u_t$ -IC	-	-0.1592 (0.5632)	0.2472 (0.4023)	-0.0103 (.5041)	1.4104 (0.0792)
REC-IC	0.1592 (0.4367)	-	0.3504 (0.3630)	0.2270 (0.4102)	1.4523 (0.0732)
DREC-IC	-0.2472 (0.5976)	-0.3504 (0.6369)	-	-0.2539 (0.6002)	1.4230 (0.0773)
SREC-IC	0.0103 (0.4958)	-0.2270 (0.5897)	0.2539 (0.3997)	-	1.3788 (0.0839)
IC	-1.4104 (0.9207)	-1.4523 (0.9267)	-1.4230 (0.9226)	-1.3788 (0.9160)	-

Table 4: Non-nested Likelihood Ratio Test Comparisons Between Restricted ( $\gamma = 0$ ) and Unrestricted models With Different  $X_t$  Variables.

We report the results of non-nested Vuong likelihood ratio tests to compare the in-sample performance of the unrestricted equicorrelation models,  $\rho_t = \omega + \alpha X_{t-1} + \beta \rho_{t-1} + \gamma IC_t$  against restricted models,  $\gamma = 0$  where  $X_{t-1}$  are the four alternative equicorrelation innovation terms (the Linear DECO variable  $u_t$  (Equation (10)), the average pairwise Realized Equicorrelation  $REC_t$  (Equation (12)), a portfolio-based Realized Equicorrelation  $DREC_t$  (Equation (15)), and the Spearman Realized Equicorrelation  $SREC_t$  (Equation (17)) and  $IC_t$  is the implied equicorrelation (Equation (19))). The focus is on the relative performance of the restricted models ( $\gamma = 0$ ) with unrestricted models where the definition of  $X_t$  used differs between the pairs of models compared. As the models are non-nested, we use Vuong likelihood ratio statistic outlined in Equation (22),  $p$ -values are reported in parentheses. The Vuong statistic of row  $i$  and column  $j$  is positive if model  $i$  has a superior in-sample fit to model  $j$ . In each case,  $H_0 : L_{Corr}^i = L_{Corr}^j$  or that the in-sample fit of each model is equal;  $H_1 : L_{Corr}^i > L_{Corr}^j$  or that model  $i$  offers superior in-sample fit.

Comparison of Restricted and Unrestricted Models				
	$u_t$	REC	DREC	SREC
$u_t$ -IC	-	3.1116 (0.0009)	3.1911 (0.0007)	3.2282 (.0006)
REC-IC	3.3672 (0.0003)	-	3.1847 (0.0007)	3.2563 (0.0005)
DREC-IC	3.3182 (0.004)	3.0345 (0.0012)	-	3.1733 (0.0007)
SREC-IC	3.3860 (0.0003)	3.1643 (0.0007)	3.1831 (0.0007)	-
IC	2.7071 (0.0033)	2.3342 (0.0097)	2.6358 (0.0041)	2.5764 (0.0049)



Figure 1: Equicorrelation proxies

Plots of the five equicorrelation proxies across the entire sample period of 1st November 2001 through to 30th of October 2009. In Panel A, a centered moving average is also shown in addition to the daily equicorrelation estimate,  $u_t$

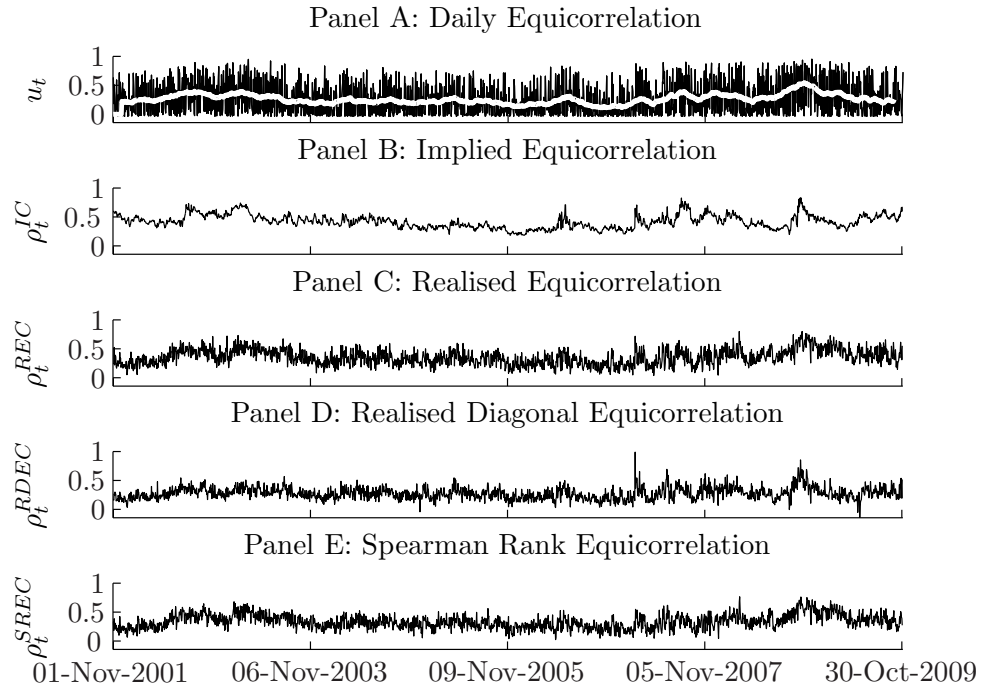


Figure 2: In-sample fitted equicorrelations

Plots of the fitted equicorrelations,  $\rho_t$ , for the entire full sample period given each of the six competing models.

