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Attention Competition

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Abstract

I present a game-theoretic model where economic competition and attention competition are interdependent. On the one hand the effort to attract consumer attention depends on the value of attention to the firm which depends on the grade of price competition among all perceived firms. On the other hand attracting attention involves costs which must be covered by the earnings from competition. It is the task of this paper to clarify the consequences of such an interdependence between attention competition and economic competition for prices, attention effort and market structure as determined by the strategic equilibrium. Under limited attention the market as perceived by consumers and not the effective market is relevant to the firms which implies that prices also reflect the scarcity of attention. Less attentive consumers lead to higher prices but at the same time getting attention is more valuable which intensifies the competition for attention and leads to higher attention costs. I show that if attention competition is relatively inelastic or the commodities are strong substitutes then the gains from consumer inattention outweigh the costs of attracting attention which leads to higher profits and larger effective markets.

Keywords: Limited Attention, Competition, Pricing, Strategic equilibrium

JEL Classification: D43, D83, L13, C72

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1 Introduction and related literature

The main task of this paper is to provide a static game-theoretic framework that can account for the well-documented fact that decision-makers (consumers) acquire their resources (commodities) under limited attention.

Limited attention means the existence of an upper bound on how many different items are perceived by an individual. If an individual is confronted with more alternatives, an attention allocation problem exists: which alternatives are perceived (i.e. considered in the economic decision of the individual) and which not?

The model is solved in the special but empirically important case where the information sender (the firm) can directly affect the attention allocation of an attention-constrained consumer by increasing its effort to attract attention relative to the other senders. The possibility to increase their chance of perception leads the firms to engage in a competition for the scarce consumer attention - besides competing for the consumer budget in a conventional economic sense. Although the marketing science reports clear evidence for the existence of both limited attention and competition for attention (see section 1.2), questions regarding the implications of the interdependence between attention competition and conventional economic competition for market prices and market structure have not been posed previously. In this paper I provide first answers to such questions in a rather general setting. The framework may be adapted to more specific models (such as the Salop model of circular product differentiation) which then even allow a normative investigation of attention competition.

The paper is organised as follows. In the rest of section 1 I revise the relevant literature in psychology and the marketing science on attention which motivate the main two psychological building blocks of my model, namely the existence of limited attention and stimulus-driven attention allocation. Further, I discuss the related literature in economics and finance that has previously dealt with limited attention. In section 2 I give an informal and intuitive description of the attention problem and the main model of this paper (introduced in section 3) basically formalizes the ideas presented there. In section 4 I show that in case of symmtric firms a symmetric price-attention equilibrium exists both for a given number of firms or when the number of active firms is determined by free-entry. The (comparative-static) implications of attention competition are discussed in detail in section 5. In the appendix to this paper I take the formal analysis one step further by addressing questions concerning uniqueness and stability of the (symmetric) equilibrium.

1.1 Psychological literature on attention

In psychological work on attention there is a clear distinction between "whether attention is goal-driven, controlled in a top-down fashion, or stimulus driven, controlled in a bottom-up fashion" (e.g. Yantis (1998), p. 223).

The modern literature on experimental psychology provides evidence for both the active and the passive role of attention in the context of visual stimulus processing[1]: "goal-driven" and "stimulus driven" attention are important in explaining how subjects perform during visual search experiments. Nevertheless "stimulus-driven attentional control is both faster and more potent than goal-driven attentional control" (Yantis (1998), p. 251-252) - especially in experiments where attention cannot be focused on a certain region in advance. This literature argues that attention operates as a gating mechanism if multiple stimuli use the same neural pathway by "restricting the amount of information that is processed at once" (Mozer and Sitton (1998), p. 342) because parallel processing of information is not possible due to mental capacity limitations. Then the relative strength of a stimulus that enters the neural network of a person determines whether or not it is further transmitted into the neural network and eventually reaches the recognition network (Mozer and Sitton (1998), p. 343-356). That capacity limitations impose a "bottleneck" to visual stimulus processing is convincingly illustrated in experiments on visual search: response time of subjects to a stimulus is flat if few objects are displayed but increases exponentially with the number of displayed objects (Mozer and Sitton (1998), p. 378-340). Hence perception gets distracted under many signals which ultimately slows down response time. In explaining search time in visual search the "relative salience of a target" is much more important than "the occurrence of specific features" (Nothdurft (2000), p. 1184). Hence in order to generate a "pop-up" or "salience effect" the motion, color or luminance of an object matters only relatively to the local and global surrounding of the object. Experiments conducted to assess the spatial distribution of attention on the visual field suggest that attention works as a "spotlight": once a location is selected, all features at that location are processed and moved on to the recognition network (e.g. Kahneman and Henik (1981)) or at least "receive enhanced

[1] In such experiments subjects are usually required to identify the orientation of some geometric object that e.g. briefly flashes up where there might also be distractive stimuli present and response time of subjects is measured. See Yantis (Yantis (1998)) for an overview of such experiments.

processing” (Maunsell and Treue (2006)).

We may summarise the key messages of the psychological research in case of visual attention as follows:

P1) In the case of multiple stimuli relative signal strength matters whether a signal is processed or not.

P2) For a given spatial array of objects, if a region successfully attracts attention then all features of this region are considered.

Note that P1) also means that in case of multiple overlapping stimuli certain signals are inhibited[2] and not processed to the recognition network.

1.2 Limited attention in the marketing science

In the marketing literature it is a well-documented fact that the placement of a link on the screen of some online search site is a crucial determinant of the number of clicks that a site receives. For example using a dataset of a price listing service Baye et al. find that ”a firm receives about 17% fewer clicks for every competitor listed above it on the screen” (Baye et al. (2009), p. 938) but that the relationship between clicks and screen location is far from being linear. That ranking matters for clicks on search pages is also confirmed by Smith and Brynjolfsson (Smith and Brynjolfsson (2004)) in the context of online book markets and by Ghose and Yang in the context of sponsored search advertising (Ghose and Yang (2009)). Having sales data in case of memory modules Ellison and Ellison (Ellison and Ellison (2009), p. 442) report that on a price search engine ”moving from first to seventh on the list reduces a websites sales [...] by 83%”. Dreze and Zufryden, using data on the web traffic of Amazon and Ebay, point out that a site can increase its visibility to potential consumers most by altering its linkage in the web and thereby improving its indexing by search engines (Dreze and Zufryden (2004), p. 35). Most noteworthy is the finding that the first three positions get about 75% out of all clicks of the first ten positions on the screen (Baye et al. (2009), p. 945) and positions with rank larger than three do not differ substantially from each other. This finding is also confirmed by Ghose and Yang who additionally document a non-monotonic relationship between rank and profitability (Ghose

[2]See Milliken and Tipper (1998), p. 204-206 for experiments on the importance of inhibition in selection experiments.

and Yang (2009), p. 1614). This is explained by the fact that both costs and clicks decrease with a higher rank (a lower on-screen position) but that clicks decrease slower than costs. Hence it may be of less importance to be the first on the list but what matters is to be among the first few entries. A related finding of Smith and Brynjolfsson is that in case of the market for books being on the first page of a search page is of central relevance for receiving a high number of clicks and might even be more important than being the overall first in the list (Smith and Brynjolfsson (2004), p. 548-549). First-page dominance is also found by Jansen et al. (Jansen et al. (2000), p. 215). Finally, Pan et al. show in an eyeball-tracking experiment that the first entries on a Google search page get by far most viewing-time but there is no statistical difference between the first and the second item on the list (Pan et al. (2007), p. 814). In their online experiment Pan et al. further show that people frequently select the upper items of a list in a quality based search experiment even if these upper items systematically are of inferior quality. Hence the Pan finding suggests that people consider the first few alternatives in making their decisions *independent* of the overall quality of these alternatives.

The theoretical literature in marketing has recognised the empirical fact that consumers make their decision conditional on a subset of all possible alternatives. In such models consumers are believed to phase their decision in two parts using simple heuristics to screen all possible alternatives and then make a more thorough analysis of the reduced consideration set (Manrai and Andrews (1998); see Payne et al. (1993), Chapter 2, for an overview of different decision strategies). Hauser and Wernerfeld (Hauser and Wernerfelt (1990)) provide an overview of marketing studies that find empirical evidence for the existence of such consideration sets and emphasise that these sets usually are very small (a size of $3-6$ alternatives) compared to the complete set of possible alternatives[3]. Moreover, the consideration sets of supermarket shoppers seem to depend heavily on what is promoted strongly (Fader and McAlister (1990), p. 330-331). This is also supported by Mitra who reports that advertising does not alter the average size of the consideration set but it does influence which alternatives are in the consideration set in an experimental setting (Mitra (1995), p. 91). Mehta et al. use a structural model of quality-adjusted price search to estimate in-shop brand selection in case of liquid detergents and find that consumers frequently purchase low quality products "simply because they fail to notice the prices of the other brands" (Mehta et al. (2003), p. 76) and, at the same time, it

[3]In case of shampoos the mean consideration set contained only four shampoos out of more than 30 (Hauser and Wernerfelt (1990)).

is "the most feature-advertised and displayed brand". Similarly, Alba and Chattopadhyay find experimental evidence that higher salience of a brand inhibits the recall of other brands (Alba and Chattopadhyay (1986), p. 365).

The key messages of marketing and internet research are:

M1) The on-screen ranking of links determines the clicking distribution and is sale relevant.

M2) There appears to be a discontinuity and non-linearity between the clicks of the first few entries and the later entries of search pages and there is first-page relevance.

M3) Consideration sets of consumers exist and are small. The size of the set is not affected e.g. by advertising. Firms can influence their chance of consideration e.g. by means of promotion.

Note that P1) and P2) can explain M1) in the sense that people cannot process all information on a search screen in parallel and thus perceive the links from top to bottom. That flashy links by means of position or different color attract attention and hence clicks (the finding of Ghose and Yang (2009)) coincides with Nothdurft's (2000) observation that the relative salience of objects determines the speed of their detection and also is reflected by the fact that sponsored search advertising (purchasing advertising space and a different link color in the Google top- and sideframe) has become "the largest source of revenues for search engines" (Ghose and Yang (2009), p. 1605). Further P2) and M2) seem related as the empirical evidence suggests the clicking behavior to cluster around the first search page and the first entries of a page. Finally, M2) and M3) are related as the discontinuity could be explained by the fact that people consider only a small subset of all information and their selection rule follows a simple top-down logic. The facts documented in the last two sections suggest that:

1. Decision-makers have limited attention: They focus only on a subset of all available information.

2. Relative salience of the information (e.g. its on-screen ranking) crucially determines which information receives attention.

1.3 Limited attention in economics and finance

In a seminal paper Herbert Simon revises the notion of the “rational economic man” by introducing ”internal constraints” which account for the fact that besides the conventional budget constraint that represents economic scarcity an agent also faces physiological and psychological limitations to which he must obey (Simon (1955)). Consecutively, much theoretical and empirical effort was undertaken to comprehend what such bounds might imply for economic models. In many cases bounds on the information processing capabilities of agents have been found of central importance. The claim of these models usually is that acquiring and processing information imposes some cost on the agent. The existence of such deliberation costs then leads to a trade-off in decision-making (see Payne et al. (1993)): better decisions are available only at the expense of purchasing more information.

However, the information problem in a modern economy might be less one of getting information but one of receiving too much information. Maybe the central phenomen of the digital age is that ”Information has gone from scarce to superabundant[4]”.

Together with such limitations the ”wealth of information” then implies a ”scarcity of attention” (Simon (1971)). Managing scarce attention is seen to be the central task of modern business: ”If you want to be successful in the current economy, you’ve got to be good at getting attention” (Davenport and Beck (2001), p. 8).

This distinction of attention as an active or passive part of perception as was introduced in section 1.1 is reflected in the modern economic literature on attention. Taken to economic theory the dichotomy translates into the question whether the individual itself allocates its attention, i.e. the allocation problem is a part of the individuals optimization problem, or the attention allocation depends on the (relative) strength of different stimuli the individual is exposed to. Other than the strong evidence for the stimulus-driven attention allocation suggests the vast part of the economic literature on attention is concerned with goal-driven allocation meachanisms.

[4]Cited in: The Economist, ”Data, data everywhere. A special report on managing information” Feb 27th 2010, p.3.

1.3.1 Goal-driven attention allocation

In models of rational inattention (Reis (2006a) and Reis (2006b), also Gifford (2005)) information acquisition is costly and a part of the agent's maximization problem. In the dynamic context of these models the consumers only sporadically update their information which is interpreted as an agent being inattentive for some time concerning further information. Sims investigates the implications of rational inattention on macroeconomic topics such as the permanent income hypothesis using Shannon's information theory (Sims (2003)). Gabaix and Laibson develop a cost-benefit model that endogenously explains how subjects allocate their limited mental resources to different elements of a decision problem and provide experimental evidence in support of their directed cognition effect (Gabaix et al. (2003) and Gabaix et al. (2006)). In finance limited attention of market actors has been taken into account by different researchers. For example, Mondria et al. show that the magnitude of home bias in investments can be explained substantially by the allocation of investor's limited attention (Mondria et al. (2010)). Gifford presents a dynamic principal-agent model of venture capital where the venture capitalist (the agent) must decide in each period how much attention (or time) to allocate towards managing a current venture or towards funding new ventures (Gifford (1997)). Using an NYSE dataset from 2002 Corwin and Coughenour show that the ability of specialist traders to provide liquidity to a stock is negatively correlated with the attention required by the other stocks in the portfolio (Corwin and Coughenour (2008)). Peng shows that investors allocate their limited attention across sources of uncertainty to minimize total portfolio uncertainty (Peng (2005)), and Huberman (Huberman (2001)) as well as Barber and Odean (Barber and Odean (2008)) provide evidence that investors tend to focus on familiar stocks and that information may not be incorporated into prices until it attracts sufficient investor attention. These are all examples of economic models that deal with a goal-driven type of attention allocation.

1.3.2 Stimulus-driven attention allocation

The smaller part of the economic literature on attention focuses on stimulus-driven attention allocation. If attention is stimulus-driven this naturally moves the behaviour of the information sender into the centre of interest. For example, Falkinger (Falkinger (2007), Falkinger (2008)) works with a macroeconomic model where consumer attention is not a decision variable and attention is allocated only by the strength of the signals the consumers receive. Such a passive allocation process exists if and only if there is an abundance of information compared to a certain information overflow threshold. Such an economy is called information-rich as opposed to an information-poor economy where the receivers of information are capable of perceiving all information. Falkinger shows that whether an economy is information-rich or information poor is determined by fundamentals such as preferences, budget, information technology and production technology.

Hirshleifer and Teoh illustrate the implications of limited investor attention on stock prices if the manager of a firm can influence the perception of a financial report by disclosing additional material (Hirshleifer and Teoh (2003)). They show that managers have an incentive to include additional pro forma (non-GAAP) measures on earnings to improve the perception of the firm as inattentive investors treat pro forma earnings as if they were adjusted by the manager to be maximally informative. This failure of the inattentive investors to account for the strategic incentive of the manager implies that i) pro forma earnings are upward biased predictions of terminal cash flows and ii) stock prices are too high compared to a situation were reporting of pro forma earnings were forbidden.

Although the model of this paper is principally general enough to accomodate both types of attention allocation mechanisms I will, inspired by the strong empirical evidence, concentrate solely on the stimulus-driven mechanism throughout the paper.

2 An informal description of the attention problem

In this section I illustrate the attention problem in a schematic and intuitive way. The main part of this paper formalises these schematics and derives a game-theoretic model where firms (information senders) compete for consumer attention and consumer budget.

2.1 A simple consumer choice problem

Suppose for the sake of illustration that there are $n = 4$ firms. Each firm produces one commodity and sets a price y_j. Consumer i decides how much to purchase from each commodity. φ_i is the choice function of consumer i. How much of which commodity the consumer purchases (x_{ij}) depends on his preferences and budget v and on the prices of the commodities. This standard decision problem is illustrated in figure 1.

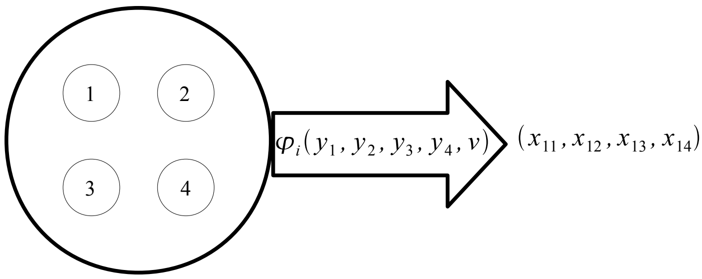

Figure 1: A classical consumer decision problem

2.1.1 Informative advertising and limited information

In models of informative advertising consumers are ex ante uninformed about existing commodities (e.g. Grossman and Shapiro (1984), Goeree (2008)). They learn about the commodities only by the information (ads) they receive from the firms and choose only among the commodities they are aware of. Such a situation is depicted in figure 2. In the figure we see that only firms

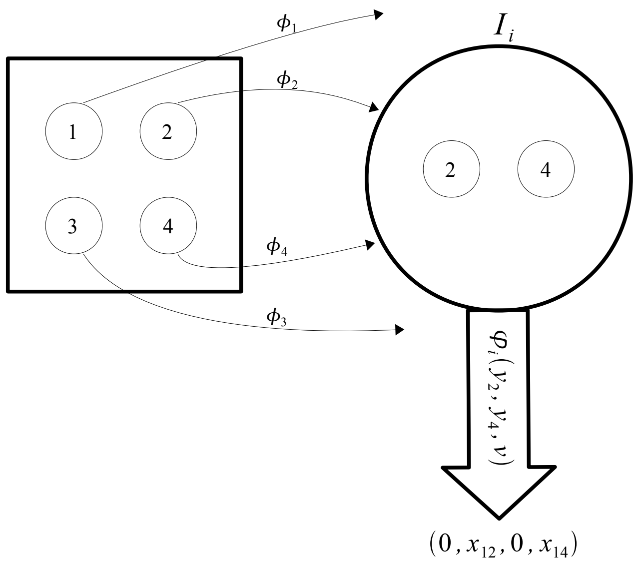

Figure 2: Limited information

two and four managed to inform consumer i. That is, the consumer received ads only from these firms but not of firm one and three and he decides only among those firms he is informed of. If the firms can choose how many consumers to inform (for example if they can decide in how many different newspaper to put an ad) then the information set I_i is endogenously determined and depends on the advertising efforts of the firms. In the figure this is suggested by the firm-specific variable ϕ_j. The literature on informative advertising says that a consumer who considers a set of alternatives that is smaller than the set of all existing alternatives has *limited information* (see e.g. Goeree (2008)).

2.1.2 Scarce attention

In this paper limited attention means that an upper bound, $R < \infty$, on how much information can be perceived by a consumer exists. Suppose for the sake of illustration that $R = 2$. If a consumer receives information of not more than two firms then he considers all alternatives he is informed about in making his decision. Such a situation is captured by figure 2. Now suppose consumer i received information from firms one, two and four but not from firm three, as depicted in figure 3. Limited attention means the the consumer bases his decision on a subset of I_i that contains $R = 2$ different alternatives. As figure 3 suggests there are three possibilities to pick two alternatives out of the set of three alternatives and $P_{i\alpha}$ for $\alpha = 1, 2, 3$ with $P_{i1} + P_{i2} + P_{i3} = 1$ denotes the probability with which consumer i observes a certain attention set $\tilde{A}_{i\alpha}$. In this paper I will argue that the firms which are a part of I_i have means to influence the probability distribution (P_{i1}, P_{i2}, P_{i3}). Formally, this means that the probability vector (P_{i1}, P_{i2}, P_{i3}) will depend on the (relative) choice of attention effort of the firms. That is, firms can influence their *chance of perception*.

Note however that other mechanisms that determine the vector can be imagined. In the case of recommender systems the probability to observe a certain subset would depend on how the alternatives in I_i are rated by the community. Or we could think of a filter mechanism that excludes or includes certain alternatives with a certain probability. For example, in figure 3 a 1-pass filter would pass only subsets containing alternative 1, i.e. $P_{i3} = 0$. In the case of goal driven attention allocation we can think of the vector as determined by the individual itself. This makes sense e.g. if the alternatives in I_i are different assets (rather than commodities) and the individual can only manage (e.g. compare) these assets pairwise. Then the probability vector could be interpreted e.g. as the fraction of time an individual decides to spend by comparing the different pairs of assets in his portfolio. However, in this paper I stick to the case where the attention probabilities represent the effort of the attention-seeking firms. I think that this properly captures how people who use the internet, especially search engines, make their decision: they are presented with a large list of alternatives and consider only a few alternatives

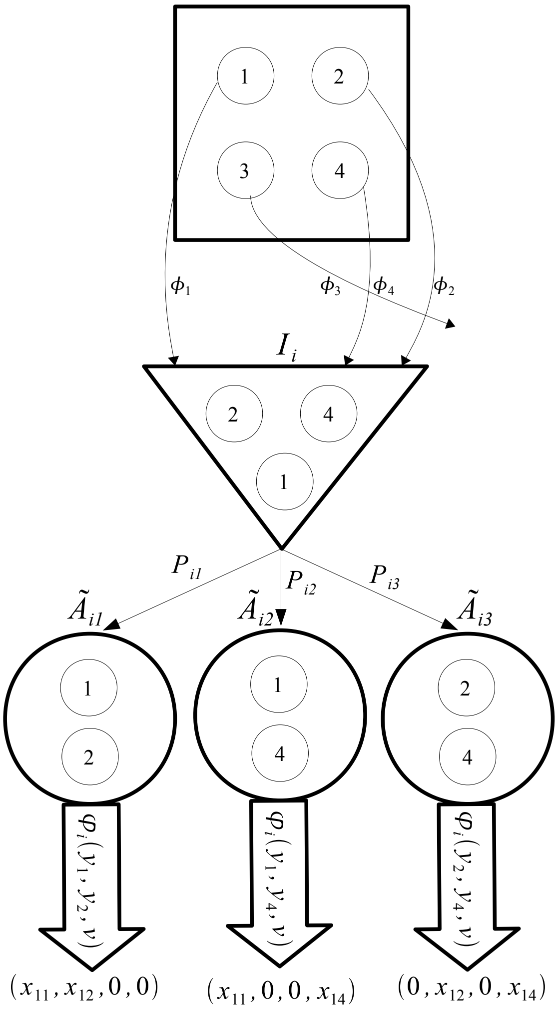

Figure 3: Limited attention

- especially those alternatives that have top positions or are highlighted by some other means (e.g. in the Google sideframe).

Note that by the notion of the literature on informative advertising the consumer in figure 3 also has limited information - but for a very different reason: the cause is not that he received only little information but rather that he received more information than he considers. My contribution essentially shows that if limited information is caused by limited attention rather than scarce information this generates a very different strategic environment and implies very different results compared to standard models of informative advertising.

As figure 3 suggests in order to earn money from a consumer a firm must achieve three goals: i) the firm must inform the consumer (e.g. receive an index by a Google search bot), ii) it must be perceived by a consumer (e.g. because it has a high index and a top on-screen placement) and iii) it must provide a satisfying offer in a conventional economic sense compared to all other firms that the consumer perceives[5]. One contribution of this paper is to provide a specification of how the firms in I_i can influence their chance of perception. This means that the probability distribution (P_{i1}, P_{i2}, P_{i3}) in figure 3 will be endogenously determined and depends on the strategic attention efforts of the firms in I_i. What I want the model to capture is that if in figure 3 firm one's messages are *relatively conspicuous* compared to the messages of firms two and four then we should have $P_{i1} + P_{i2} \approx 1$.

To make the difference between figure 2 and figure 3 as clear as possible suppose firms can advertise only by posting ads in different newspapers. Consumers do not read all newspapers. If a firm places its ads in more newspapers this increases the number of people that could see the ad. Put differently, this increases the number of information sets that contain the firm. Suppose now a consumer reads a newspaper that contains many ads of different firms. Then it is reasonable to assume that a large or colorful ad or an ad placed on the front page has a larger chance of getting the readers attention than a small ad somewhere on the last pages. That is, contrary to the theory of informative advertising, a firm that advertises in many newspapers but has very non-salient ads may go unnoticed by most consumers. In essence, it is this struggle for attention which is the main concern of this paper.

3 A formal model of attention and price competition

3.1 The Internet economy

For many people the internet nowadays is the main source of information and, as e.g. the stories of Google's or facebook's success document, the internet also offers great business opportunities. But the internet, especially a search page such as Google, also provides the most convincing example of the prevalence of limited attention. A typical Google search query usually results in an enormous number of hits[6]. As suggested by the observations documented in the introduction, the typical user of a search engine focuses on the first few entries (or on the first few pages) instead of threading through the entire list. Moreover, this information is accessible for any

[5] Suppose that consumer i perceives the set $\tilde{A}_{i1}$ in figure 3. Then firm one competes with firm two for the consumer budget v.

[6] For example, the keyword "Hawaii" gives approximately 123'000'000 hits and the query "Hawaii vacations" gives approximately 8'160'000 hits (January 09, 2011).

consumer in the world provided he has access to the internet. This clearly illustrates that the contemporaneous (and future) information problem of an advertising firm is less one of reaching more consumers (more information sets) but one of making its messages salient. In the Google example this e.g. means to be listed on the first page of a search query.[7] In this paper I extrapolate this observation by assuming that every active firm that pays a fixed cost $F > 0$ reaches the entire population Δ. Hence all consumers have the same information set: $I_i = I$. In the Google example this means that if a firm designs a web page and puts it online (achieving this requires some fixed amount of money) it is found and indexed by a Google bot. However, whether the firm has a high or a low index (a top rank position or not) depends on further investments[8] of the firm. Setting $I_i = I$ appears also to be adequate for mature product classes. For such products, e.g. Coke and Pepsi, the role of advertising is not information provision about the features but "keeping a product top-of-mind" (Iyer et al. (2005), p. 464) for which "these companies spend a significant amount of their budget on reminder-advertising". I simply call an economy where a firm is either in all or in no information set an "internet economy". If a firm pays the setup cost F then I call the firm active. Suppose there are n active firms indexed by $1, ..., n$ in the internet economy. Then $n = |I|$.

3.2 Attention and price competition in the internet economy

In this section I formalise the schematics of figure 3. First, I develop a model that determines how the attention efforts of the active firms influence their probability of being perceived by a consumer. Then I combine this model with a model of imperfect price competition. I will provide a simple example that accompanies and illustrates the abstract theory developed in this section. In the end of section 3 we will have a completely specified model of n active firms that simultaneously and non-cooperatively choose their attention effort and their price. I will restrict myself to the case of symmetric firms. Thus formally we will have to deal with a static two-dimensional symmetric n-player game.

[7] As a Web commentator puts it: "Simply, you want to be found at the common meeting point...page 1 of a Google search" (see www.optimum7.com).

[8] For example, the firm can try to get more cross-links to its webpage or can bid for a placement in the Google sideframe. In both cases there is some kind of variable cost involved.

3.2.1 Attention probabilities

Suppose there are n active firms in the internet economy and hence $n = |I|$. The power set of I is denoted by $\mathbb{P} \equiv \mathbb{P}(I)$ and I call an element $A \in \mathbb{P}$ an attention set. Limited attention means that only attention sets not larger than some $1 < R < \infty$ are considered by a consumer. I denote the attention set of consumer i (the set of alternatives that the consumer effectively perceives) by $\tilde{A}_i \subset I$. Limited attention means that $\left|\tilde{A}_i\right| \leq R$. The attention constraints of the consumers are strictly binding if and only if $n > R$. If $n \leq R$ then we have $\tilde{A}_i = I$ for all consumers in the internet economy. If however $n > R$ then we have $\tilde{A}_i \subsetneq I$ and $\tilde{A}_i \neq \tilde{A}_h$ is possible for any two consumers. In case of a binding attention constraint literally spoken only some commodities can make it into the attention set $\tilde{A}_i$ of a consumer and the next question is which alternatives succeed in doing so. To answer this question we need to specify a rule that for a given I tells whether $j \in I$ implies $j \in \tilde{A}_i$ or not. The general way to map a larger set into a smaller one is to specify a probability distribution on the set of all possible attention sets. Define the indicator variable z by

$$z = \min\{R, n\} \tag{1}$$

Let $\mathcal{A}(I, z) \subset \mathbb{P}$ be the set of all subsets of I with size z.[9] For every consumer i I now define a function P_i on $\mathcal{A}$ that assigns to every possible attention set A of $\mathcal{A}$ with size z a probability[10] of realising this particular set:

$$P_i : \mathcal{A} \to s^{|\mathcal{A}|} \quad , A \mapsto P_{iA} \tag{2}$$

where $s^{|\mathcal{A}|}$ is the $(|\mathcal{A}| - 1)$-dimensional simplex[11] and $A \in \mathcal{A}$ is an attention set of size z. Thus P_{iA} is the probability that $\tilde{A}_i = A$, i.e. P_{iA} is the probability that consumer i perceives the attention set A. Note that we have $z = n$ if $R \geq n$. But then we must have $\mathcal{A} = I$ and $|\mathcal{A}| = 1$. The only possible function satisfying (2) in this case is the unit function which gives $\tilde{A}_i = I$ for all consumers. Hence the entire apparatus I am about to construct only comes into play when we have $n > R$, i.e. when the attention constraints of the consumers bind.

Before continuing it should be noted that by definition of (2) I only allow the attention sets to be proper sets (instead of multisets). This can be justified by the assumption that attention-

[9] Because by definition of the internet economy we have $I_i = I$ for all consumers the set $\mathcal{A}$ of possible attention sets is the same for all consumers. This would not be the case if we allowed for $I_i \neq I_h$.

[10] Of course, the probability of a certain set could also be one ore zero.

[11] $s^{|\mathcal{A}|} \equiv \left\{ P \in \mathbb{R}_+^{|\mathcal{A}|} : \sum\limits_{A \in \mathcal{A}} P_{iA} = 1 \right\}$.

constrained decison-makers cannot be fooled in the sense that they always recognize and consider R truly distinct alternatives.[12]

The probability p_{ij} that firm j is perceived by consumer i given that $j \in I$ is calculated by

$$p_{ij} = \sum_{A \in \mathcal{A}} P_{iA} \, 1 \left[j \in A \right] \tag{3}$$

where $1 \left[j \in A \right]$ is a variable indicating whether alternative j is in attention set A or not. The following proposition establishes an aggregate relationship on P_i and p_{ij}.

Proposition 1 *If P_i is a probability function as defined by* (2) *then* $\sum_{j \in I} p_{ij} = z$.

<u>Proof</u>: Appendix B (8.2)

Conversely, it generally only is possible to construct P_i from the set of p_{ij} where $j \in I$ in certain trivial cases.[13] The vector $p_i^A \equiv (p_{ij})_{j=1,...,n}$ for $j = 1, ..., n$ can be interpreted as the distribution of attention of consumer i. Likewise, if there are Δ consumers then the n-vector

$$p^A \equiv \frac{\sum_{i=1}^{\Delta} p_i^A}{\Delta}$$

is the average distribution of attention in the market. Then the j-th entry of p^A corresponds to the average fraction of consumers in the economy that consider an attention set containing j.

3.2.2 Limited attention and economic competition: the problem of the firm

From an economic perspective the function P_i becomes interesting if actions chosen by the market agents such as advertising efforts by firms (or search efforts by consumers) affect the probability distribution. Throughout this paper I will maintain the assumption that P_i is determined only by the attention efforts of the firms in I. Suppose every active firm j can choose its attention effort $f_j \geq 0$. I denote by f_{-j} the vector of attention efforts chosen by all active firms other than j. Let $\mathcal{F} \equiv (f_1, ..., f_n) \in \mathbb{R}_+^n$ denote the vector of attention efforts. I assume that the attention probabilities P_A depend on $\mathcal{F}$. Further, for any active firm j define the set $B_j \equiv \{A \in \mathcal{A} : j \in A\}$.

[12] This means that even if a firm j by some reason has a high chance of perception, i.e. $\sum_{A \in \mathcal{A}: j \in A} P_{iA} \approx 1$, then for $R > 1$ this does not mean that it is the only firm that is perceived.

[13] A nice property of the symmetric game is that P_i can be deduced from p_{ij}.

Assumption 1 (Relative salience) *For all consumers $P_i = P$ where the function P is defined by* (2). *If* $\sum_{A \in B_j} P_A < 1$ *then* $P_A(\mathcal{F})$ *is an increasing function of* f_j *for all* $A \in B_j$.

The first part of assumption 1 means that all consumers have the same function P which determines the probability that a particular attention set $A \in \mathcal{A}$ is perceived. This assumption is mainly for simplicity. The second part of assumption 1 means that under limited attention $(n > R)$ a firm can positively influence the chance that a consumers perceives an attention set A with $j \in A$.

Lemma 1 *Under assumption 1 we have for any active firm j and any consumer i that $p_{ij} = p_j$. If $R \geq n$ then $p_j = 1$ for any active firm j. If $n > R$ then $p_j = p_j(f_j, f_{-j})$ where $p_j(f_j, f_{-j})$ is an increasing function of f_j if $p_j < 1$.*

Proof: Appendix B (8.3)

To formalise the interdependence of economic competition and attention competition I assume that every firm non-cooperatively chooses one variable y_j, henceforth interpreted as its price. Let $Y_A \equiv (y_{u1}, ..., y_{uz}) \in \mathbb{R}^z$ with $uk \in A$ denote the vector of prices of those firms that belong to A. For every $A \in \mathcal{A}$ define a function $V^j(Y_A) : \mathbb{R}^z \to \mathbb{R}^1$ with the property that $V^j(Y_A) = 0$ if $j \notin A$. Then $V^j(Y_A)$ is the value of attention set A to firm j and summarises the value earned by firm j from the economic competition between the firms in attention set A. As an example think of price competition with substitute products and linear production costs. Then

$$V^j(Y_A) = (y_j - c)x_j(Y_A) \tag{4}$$

where $x_j(Y_A)$ is the demand function of a consumer facing attention set A with price vector Y_A. The second assumption I impose with the goal of eventually writing down the payoff function of firm j is the following:

Assumption 2 (Separability) *For any active firm j and any $A \in B_j$ the function $V^j(Y_A)$ is independent of $(f_1, ..., f_n)$.*

This is a very important assumption. It means that conditional on an attention set consumers make decisions *independent* of all attentional activities.[14] Taken to the Google example the as-

[14]This is a crucial difference between attention competition as understood by my contribution and persuasive advertising: persuasive advertising would mean that the advertising effort of a firm could influence the function

sumption means that the rank of an alternative only influences its chance of perception but does not convey more economically relevant information to the consumer. Formally, this assumption enables me to separate the competition for an attention set from the economic competition within an attention set and leads to an analytically highly tractable structure of such a problem. Under assumptions 1 and 2 the expected profit function of firm j is

$$\Pi^j\left((y_1, f_1), ..., (y_j, f_j), ..., (y_n, f_n)\right) = \sum_{A \in B_j} P_A(\mathcal{F}) V^j(Y_A) \Delta - F - C(f_j) \tag{5}$$

where $C(f_j)$ denotes the cost firm j must incur under effort level f_j. The properties of C will be discussed below. The fixed setup cost $F > 0$ can be thought of as summarizing infrastructure costs for production and IT.

In case of (4) the profit function (5) becomes

$$\Pi^j = (y_j - c) \sum_{A \in B_j} P_A(\mathcal{F}) x_j(A) \Delta - F - C(f_j) \tag{6}$$

Concerning the costs of attention I assume the following properties to hold for $f \in [0, \infty)$

$$C(0) = 0, \;\; C(f > 0) \in (0, \infty), \;\; C'(0) \in [0, \infty), \;\; C'(f > 0) \in (0, \infty), \;\; C''(f) \in [0, \infty) \tag{7}$$

An example which provides useful later is given by

$$C(f) = \theta f^\eta \qquad \theta > 0 \quad \eta \geq 1 \tag{8}$$

As the elasticity of the cost function will play a major role for the comparative statics of the model it is appropriate to discuss the cost function in greater detail which is accomplished in the remainder of this section. In case of sponsored search advertising firms can purchase certain keywords (called adwords) by offering a cost-per-click (CPC) rate in an ongoing online auction. For a purchased keyword the rank of the link within the frame for sponsored ads depends on the relative bid and on the PageRank measure of the page[15]. Thus for a given set of keywords and identifying attention probabilities with the on-screen ranking we might expect constant unit

V^j. Attention competition means that the chance of perception depends on firm actions but has no further influence on the choice behavior of consumers.

[15] In case of Google the CPC corresponds to the next highest bid plus one cent.

costs to be reasonable. Imagine a situation where all initial bids are the same for a fixed set of keywords. Assuming that only relative bids determine the ranking the on-screen position then is random and the probability to be among the first R ranks is R/n for every competitor. Then if all competitors double their bids my firm can get the same chance as before (R/n) only if I also double my bid which simply implies doubling advertising expenditure. However, as Williamson and Rusmevichientong point out, sponsored search advertising is highly complex as it is a multidimensional issue if the set of keywords is not fixed and novel keywords are associated with an unknown number of click-through rates (Williamson and Rusmevichientong (2006), p. 260). Nonlinear costs of attention may arise e.g. if retrieving new click-generating keywords gets more difficult the more keywords are already employed. Nonlinearity is more generally also supported by the finding of Nothdurft that it gets increasingly difficult to generate a higher salience (pop-up effect) of some visual stimulus at higher levels of salience (Nothdurft (2000), p. 1195): combining the salience of two features (e.g. color and movement of a visual object) does not lead to the sum of the saliences of the two features for the same background. Finally, in case of adword competition how narrow the set of possible keywords is might be a market-specific feature which then implies different elasticities of the cost functions for different markets. The market for a specialised set of screwdrivers might encompass less critical keywords than the market of holiday destinations.

3.3 The symmetric price-attention game

Let $S_y \equiv [c, y^{max}]$, $S_f \equiv [0, \infty)$ and $S \equiv [c, y^{max}] \times [0, \infty)$. Further, I assume (5) to be symmetric, i.e.

$$\Pi^j \left((y_1, f_1), ..., (y_j, f_j), ..., (y_n, f_n)\right) = \Pi^{\sigma(j)} \left((y_{\sigma(1)}, f_{\sigma(1)}), ..., (y_{\sigma(j)}, f_{\sigma(j)}), ..., (y_{\sigma(n)}, f_{\sigma(n)})\right)$$

where σ is a permutation of $\{1, ..., n\}$ All active firms simultaneously and non-cooperatively choose their strategy, the pair $(y_j, f_j) \in S$ in order to maximize (5) and take (y_{-j}, f_{-j}) as given. Hence (n, S^n, Π) is a static symmetric game. In the main part of this paper I restrict myself to the case of symmetric equilibria.[16] To establish a symmetric equilibrium I apply the symmetric opponents form approach. In the next section I derive the symmetric opponent form of (5). It is convenient to impose all further assumptions relevant for the analysis of the symmetric

[16] Appendix A explores the possibility of asymmetric equilibria in the symmetric game.

equilibrium directly on the symmetric opponent form.

3.3.1 The symmetric opponent form

To derive the symmetric opponent form of the profit function of (5) I take firm j as the representative firm and set $y_j = y$ and $y_g = \bar{y}$ as well as $f_j = f$ and $f_g = \bar{f}$ for any $g \neq j$. Then $V^j(A) = V^j(A')$ if $A, A' \in B_j$ and I define $V(y, \bar{y}, z) \equiv V^j(Y_A : A \in B_j)$ (remember that the number of arguments in V^j is z) and[17]

$$p\left(f, \bar{f}, n, R\right) \equiv \begin{cases} 1 & n \leq R \\ p_j(f, f_{-j})|_{f_g = \bar{f}\ \forall g \neq j} & n > R \end{cases}$$

Lemma 2 *If $n > R$ the symmetric opponent form of* (5) *is*

$$\Pi(y, f) = p(f, \bar{f}, n, R) V(y, \bar{y}, R)\, \Delta - F - C(f) \tag{9}$$

If $R \geq n$ the symmetric opponent form of (5) *is*

$$\Pi(y, f) = V(y, \bar{y}, n)\, \Delta - F - C(f) \tag{10}$$

Proof: Appendix B (8.4)

Note that $p(f, \bar{f}, n, R)\Delta$ is the fraction of consumers which perceive firm j. That is, $p(f, \bar{f}, n, R)\Delta$ is the number of realised attention sets $\tilde{A}$ that contain j.

Because $C(f)$ is an injective function of f it is possible to rewrite (9) directly in terms of attention costs rather than in terms of attention effort f. For $f = \rho(\omega)$ with $C(\rho(\omega)) = \omega$ and $\bar{f} = \rho(\bar{\omega})$ with $C(\rho(\bar{\omega})) = \bar{\omega}$ we get

$$\Pi(y, \omega) = p\left(\rho(\omega), \rho(\bar{\omega}), n, R\right) V(y, y, R)\Delta - F - \omega$$

This reformulation makes sense as firms rather observe attention costs than attention effort of their opponents. However, it is convenient to solve the model in terms of effort f rather than in terms of attention cost and I continue to use this specification.

[17] For $R < n$ the function $p(f, \bar{f}, n, R)$ generally depends on R because there are $|\mathcal{A}| = \binom{n}{R}$ possible attention sets and the probability assignment as defined by (2) depends on how many attention sets there are.

It is possible to establish a simple relationship between the probability p that firm j is perceived by a consumer and the probability, $\bar{p}$, that one of its opponents is perceived. Let p_g be the probability that $g \neq j$ is perceived by a consumer. Then $f_g = \bar{f}$ for all $g \neq j$ implies that $p_g = \bar{p}$ for all $g \neq j$.[18] Proposition 1 shows that

$$\bar{p} = \frac{R-p}{n-1} \qquad R < n \tag{3'}$$

3.3.2 Assumptions on the symmetric opponent form

In this section I impose and discuss the main assumption on the value function $V(y, \bar{y}, z)$ and the probability function $p(f, \bar{f}, n, R)$. I take these assumptions to be satisfied in the main part of this paper.

Assumption 3 *The function $V(y, \bar{y}, z)$ is twice continuously differentiable in y, $\bar{y}$ for $y, \bar{y} \in S_y$ and decreasing in z. $V_2(y, \bar{y}, z) > 0$.*

The assumption $V_2(y, \bar{y}, z) > 0$ means that higher prices of the opponents ceteris paribus increase the revenue that the firm can extract from its attention sets. In case of (4) this means that higher prices of the opponents ceteris paribus increases demand (as the opponents loose some consumers to the firm) and hence also revenue for the firm. The assumption that $V(y, \bar{y}, z)$ decreases in z means that if consumers perceive more alternatives then ceteris paribus the firm earns less value from the consumer. In case of (4) a standard intuition for this is that consumers allocate their budget over the set of all firms they perceive. If they perceive more firms then, for a fixed budget v, they divide their budget over more firms which leaves less budget for a single firm.

As an example for a function that satisfies assumption 3 suppose consumers have CES-preferences over all z commodities they perceive and $c > 0$. Then it is straightforward to show that

$$V(y, \bar{y}, z) = (y - c) \underbrace{\frac{vy^{-\sigma}}{y^{1-\sigma} + (z-1)\bar{y}^{1-\sigma}}}_{x(y,\bar{y},z)} \tag{11}$$

[18] Moreover, if we know $p(f, \bar{f}, n)$ then we can determine $P_A(\mathcal{F})$ for all $A \in B_j$ as well as $P_{A'}(\mathcal{F})$ where $A' \notin B_j$ (see the remark in section 8.4 of the appendix). This means that, if we restrict ourselves to the symmetric opponent form, whenever we determine $p(f, \bar{f}, n)$ then the probability set function P is also determined which means that every possible attention set $A \in \mathcal{A}$ has a well defined probability of being realised.

where $x(y, \bar{y}, z)$ denotes demand of a consumer for commodity j who perceives j and also $z-1$ other firms. The parameter $\sigma > 1$ denotes the elasticity of substitution[19] and $v > 0$ is a consumer's budget.

Assumption 4 *Let $p \in [0,1]$ denote the probability of firm j of being in the attention set of a consumer.*

a)

$$p = p\left(f, \bar{f}, n, R\right) \tag{12}$$

If $n > R$, $f, \bar{f} > 0$ and $p < 1$ then the following properties of the function $p\left(f, \bar{f}, n, R\right)$ are postulated:

b) $p\left(\lambda f, \lambda \bar{f}, n, R\right) = p\left(f, \bar{f}, n, R\right)$ for $\lambda > 0$.

c) $p\left(f, \bar{f}, n, R\right)$ is twice continuously differentiable in f, $\bar{f}$ and

$$p_1(f, \bar{f}, n, R) > 0 \quad p_{11}(f, \bar{f}, n, R) < 0 \quad p_2(f, \bar{f}, n, R) < 0$$

Moreover, $p(f, \bar{f}, n, R)$ is strictly increasing in R and strictly decreasing in n.

Because of assumption c) $p(f, \bar{f}, n, R)$ is an increasing strongly concave function of f. Note that $p_1 > 0$ directly follows from lemma 1 if we assume the function $p(f, \bar{f}, n, R)$ to be differentiable in f. The assumption that $p_2 < 0$ means that attention efforts impose a negative externality of senders on each other.[20] The assumption that $p(f, \bar{f}, n, R)$ is strictly increasing in R means that the probability to be perceived ceteris paribus *increases* with the size of the attention set which is intuitively clear: if people perceive more (less) alternatives then, ceteris paribus, the probability of a single firm to be perceived should increase (decrease). Similarly, the assumption that $p(f, \bar{f}, n, R)$ decreases in n means that if there are more (less) active firms then, ceteris paribus, the chance of a single firm to be perceived should decrease (increase).

[19] $V_2(y, \bar{y}, z) > 0$ follows because $\sigma > 1$.

[20] Generally, a positive effect of aggregate attention effort, $\Sigma = \sum_{j=1}^{n} f_j$ on a market could be imagined if Δ were not exogenously fixed but depends on Σ. Think of a market where by some reason fierce attention competition between the firms for those consumers that are already aware of the market leads to a high Σ. But this may imply that information about the market leads to a strong general coverage in the media which may attract new consumers to that market *without* the firms explicitly wooing them. If there is an aggregate effect of attention competition in the sense that louder markets attract more consumers (have a higher Δ) then attention driven spillover effects between markets may exist. This is the subject of my mimeo "Attention markets".

If $n > R$ and all active firms choose the same effort level f then the probability of being perceived by a consumer is the same for all firms:

Lemma 3 *If $n > R$ and $f = \bar{f}$ then $p = \bar{p} = R/n$.*

Proof: $f = \bar{f}$ implies $p = \bar{p}$. The result then follows immediately from (3').

■

3.3.3 An example: The Attention Contest Function

In this section I present an important example of a function $p(f, \bar{f}, n, R)$ that satisfies assumption 4. Besides having a nice intuition this function also has some analytically convenient properties. Let

$$p(f, \bar{f}, n, R) = 1 - \prod_{i=1}^{R} \left(1 - \frac{f}{f + (n-i)\bar{f}}\right) \tag{13}$$

Throughout this paper I refer to (13) as the (symmetric) Attention Contest Function (ACF).

Proposition 2 *The symmetric ACF satisfies all properties from assumption 4.*

Proof: Appendix B (8.5)

Note that we cannot partially differentiate the ACF as stated in (13) with respect to R because the function is not continuous in R.[21]

The remainder of this section provides an intuitive derivation of the ACF. The following derivation is from the perspective of firm j. Let

$$F_n = \frac{f_j}{\sum\limits_{k=1}^{n} f_k} \tag{14}$$

measure the relative effort of firm j. Now assume that getting a consumer's attention can be described by a stochastic process similar to making random draws without repetition in a simple urn model. Hence the consumer will draw R times from this urn which corresponds to selecting R different alternatives. Firm j is in the attention set of a consumer if it gets either the first, the second ... or the R-th draw. F_n corresponds to the probability of getting the first of the R draws.

[21] It is formally possible to extend (13) to a continuous function in R by using a Gamma function expansion. See the appendix for details.

This probability depends (positively) on the mass of firm j's ball (f_j) versus the aggregate mass of all balls in the urn ($\sum_{k=1}^{n} f_k$). If $f_k = \bar{f}$ and $f_j = f$ then (14) becomes[22]

$$F_n = \frac{f}{f + (n-1)\bar{f}}$$

and $1 - F_n$ corresponds to the probability of not getting the first draw. Similarly,

$$(1 - F_n)(1 - F_{n-1}) = \left(1 - \frac{f}{f + (n-1)\bar{f}}\right)\left(1 - \frac{f}{f + (n-2)\bar{f}}\right)$$

is the probability of not getting the first and second draw (which obviously never is larger than $1 - F_n$) and generally $\prod_{i=1}^{R} (1 - F_{1+n-i})$ is the probability of not getting any of the R draws. Consequently, the probability of being in one of the R draws is given by (13).

3.3.4 Best response of the representative firm

In this section I discuss and illustrate the best-response function of the representative firm. Suppose that $n > R$. Assuming an interior solution the two first-order conditions of (9) are

$$\begin{aligned} &V_1(y, \bar{y}, R) = 0 \\ &p_1\left(f, \bar{f}, n, R\right) \Delta V(y, \bar{y}, R) = C'(f) \end{aligned} \tag{15}$$

From (15) we see that $y = y(\bar{y}, R)$ and $f = f(\bar{f}, \bar{y}, n, R, \Delta)$. This means that the strategic choice of price of our firm is independent of attention efforts but it depends on the perceived market size (R) and not on the total amount of information that a consumer receives (which is $|I| = n$). Also note that f depends on n because the marginal probability of perception depends on n and f depends on R because *both* the marginal probability and the value function V depend on R. This will have an important consequence for the comparative statics of the symmetric equilibrium later. Because the marginal probability of perception decreases in f (assumption 4) and $V_2(y, \bar{y}, R) > 0$ (assumption 3) we immediately get $f'(\Delta) > 0$ and also $f'(\bar{y}) > 0$.[23] More consumers (a larger audience) means more potential budgets which increases the marginal value of attention and leads to a higher effort level. Similarly, if $\bar{y}$ increases this means that our firm

[22] In Appendix A (7.1) I discuss the ACF without imposing $f_k = \bar{f}$. This is technically necessary when dealing with the possibility of asymmetric equilibria.

[23] $f'(\bar{y}) = \frac{p_1 V_2 \Delta}{C'' - p_{11} V \Delta} > 0$

can earn more value from every attention set that it is contained in which increases the marginal value of attention and hence also effort level f. In the CES-example (11) we get $y'(\bar{y}) > 0$ and also $y'(R) < 0$ (see section 8.6, Part I, for the calculations). The last result means that, because consumers are less attentive, our firm sets a higher price and originates from the fact that if a consumer has a fixed budget but perceives less commodities then he spends his funds over fewer firms. This means that, ceteris paribus, the demand for the commodity of our firm increases and the optimal action of the firm then is to exploit this by setting a higher price.[24] How does f depend on $\bar{f}$? Formally, the answer to this question depends on how $\bar{f}$ affects the marginal chance of perception $p_1(f, \bar{f}, n, R)$:

$$f'(\bar{f}) = \frac{p_{12}(f, \bar{f}, n, R)\overbrace{V(y, \bar{y}, R)\Delta}^{>0}}{\underbrace{C''(f) - p_{11}(f, \bar{f}, n, R)V(y, \bar{y}, R)\Delta}_{>0}}$$

Intuitively, we could imagine $\bar{f}$ to have a complementary effect on f by a defensive-type of argument: if all opponents increase their effort level then the representative firm must also increase its effort level in order to remain salient. But we could also expect that more "noise" means that it gets more difficult for our firm to influence its chance of perception. In such a case we would expect that $p_1(f, \bar{f}, n, R)$ decreases in $\bar{f}$. It turns out that the ACF incorporates both features which implies a non-monotonic relationship between $\bar{f}$ and f. Figure 4 depicts f as a function of $\bar{f}$ and n in case of the ACF with $R = 2$ and cost function[25] (8). What these

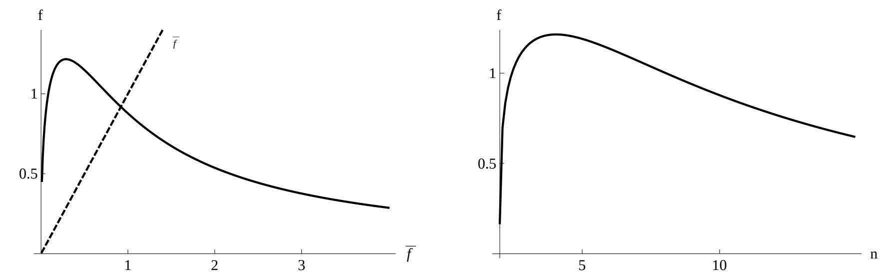

Figure 4: ACF ($n = 10$ left, $\bar{f} = 1$ right)

[24] Note that such an effect cannot exist in Falkinger's model of limited attention (which also uses the CES demand function) as firms have no mass and the price does not depend on the measure of the perceived market size (Falkinger (2008)).

[25] I impose $V(y, \bar{y}, 2)\Delta = 10$, $\theta = 1$ and $\eta = 2$. The pictures do not change qualitatively if other numerical values are used (or if $R > 2$).

pictures generally tell us is that with the ACF if there is little aggregate attentional activity of the opponents ("low noise"), because $\bar{f}$ is small or n is small (close to R), then f increases in the noise whereas if the level of noise is already high then the opposite occurs.

4 Existence of a symmetric equilibrium

In this section I derive the conditions which assert the existence of exactly one symmetric equilibrium in the price-attention game as described in section 3.3. I start with the case where n is given exogenously and then proceed to the case where n is endogenously determined by the free-entry condition of the two-stage game. I show that the CES-ACF example has exactly one symmetric equilibrium.

4.1 Symmetric equilibrium for exogenous n

The fundamentals of the symmetric game are the parameter set $\{n, R, \Delta\}$, the cost function $C(f)$, the value function $V(y, \bar{y}, z)$ with $z = min\{R, n\}$ as defined by assumption 3 and the probability function $p(f, \bar{f}, n, R)$ as defined by assumption 4. In a symmetric equilibrium all n firms choose the same strategy (y, f). To find a symmetric equilibrium we can rely on the SOFA.

Suppose that $n > R$. Assuming an interior solution, the first-order conditions of the representative firm's optimisation problem are given by (15). Then, a solution (y, f) to

$$\begin{aligned} V_1(y, y, R) &= 0 \\ p_1(f, f, n, R)\,\Delta V(y, y, R) &= C'(f) \end{aligned} \tag{16}$$

corresponds to an interior symmetric equilibrium of the game.

If $R \geq n$ then it is easy to see from (10) and (7) that a symmetric equilibrium (y, f) of the game with $y \in (c, y^{max})$ must have $f = 0$ and satisfies $V_1(y, y, n) = 0$.

To summarise: for given $R, n > 1$ and $\Delta > 0$ a symmetric equilibrium (y, f) of the price-attention game satisfies

$$V_1(y, y, z) = 0$$

$$f : \begin{cases} p_1(f, f, n, R)\Delta V(y, y, R) - C'(f) = 0 & R < n \\ f = 0 & R \geq n \end{cases} \tag{17}$$

$$z = \min\{R, n\}$$

Assumption 5 *a) The following boundary conditions are satisfied:*

i) For any $z > 1$ we have $V_1(c, c, z) > 0$ and $V_1(y^{max}, y^{max}, z) < 0$.

ii) If $n > R$ we have $\lim_{f \to 0} p_1(f, f, n, R) = \infty$ and $\lim_{f \to \infty} p_1(f, f, n, R) = 0$

b) For any $z > 1$ and $y \in (c, y^{max})$ we have $V(y, y, z) \in (0, \infty)$, $V_{11}(y, y, z) < 0$ and[26]

$$V_1(y, y, z) = 0 \quad \Rightarrow \quad -\frac{\partial}{\partial y}(V_1(y, y, z)) > 0 \tag{18}$$

c) For any $R > 1$ and any n with $n > R$ and $f \in (0, \infty)$

$$-\frac{\partial}{\partial f}(p_1(f, f, n, R)) > 0 \tag{19}$$

The boundary conditions exclude the possibility of symmetric boundary equilibria which simplifies the comparative-static analysis later. The assumptions of monotony (18) and (19) exclude the possibility of multiple symmetric equilibria.

Proposition 3 *If assumption 5 is satisfied then for any $R, n > 1$ there exists a unique vector (y, f) that solves* (17)*. Hence there exists only one symmetric equilibrium. In the equilibrium we have $y \in (c, y^{max})$. If $R \geq n$ we have $f = 0$ and $y = y(n)$. If $n > R$ then $y = y(R)$ and $f = f(n, R, \Delta) \in (0, \infty)$.*

Proof: Appendix B (8.7)

Hence if $R \geq n$ then all active firms choose the minimal effort level $f = 0$. Because in this case the consumer perceives all alternatives he is informed of there is no benefit from choosing a higher level of f.

[26]Note that $\frac{\partial}{\partial y}(V_1(y, y, z)) = V_{11}(y, y, z) + V_{12}(y, y, z)$, i.e. $\frac{\partial}{\partial y}(V_1(y, y, z))$ is the *total derivative* of $V_1(y, y, z)$ with respect to y. Similarly, $\frac{\partial}{\partial f}(p_1(f, f, n, R))$ is that total derivative of $p_1(f, f, n, R)$ with respect to f.

A nice property of the ACF is that this functional form for $p(f, \bar{f}, n, R)$ satisfies the boundary conditions in assumption 5 as well as condition (19).

Lemma 4 *Suppose $n > R$ and the function $p(f, \bar{f}, n, R)$ is given by the ACF* (13). *Then for $f > 0$*

$$p_1(f, f, n, R) = \frac{n-R}{nf} \sum_{i=1}^{R} \frac{1}{1+n-i} \tag{20}$$

The ACF satisfies assumption 5. A good approximation to (20) *if n is large relative to R is given by*

$$p_1(f, f, n, R) \cong \frac{n-R}{n^2 f} R \tag{21}$$

Proof: Appendix B (8.8)

Almost all results in this paper are proved by using (20) and not its approximation. In any case the results hold also if (21) were used instead and the proofs usually are less technical. In applications we can use (21) rather than (20) because (21) is differentiable also in R and generally simpler to work with. Because the ACF satisfies assumption 5 then whenever we combine the ACF with a function $V(y, y, R)$ that satisfies assumption 5 we always end up with a single symmetric equilibrium (for $n > R$).

Corollary 1 *If for $n > R$ the function $p(f, \bar{f}, n, R)$ is given by the ACF and $V(y, y, R)$ satisfies assumption 5 then there exists exactly one symmetric equilibrium $(y, f) \in Int(S)$. f is approximately determined by*

$$\frac{n-R}{n^2 f} R \Delta V(y, y, R) = C'(f) \tag{22}$$

Proof: Follows from proposition 3, lemma 4 and (21).

■

4.2 Free-entry equilibrium: existence and uniqueness

In the symmetric equilibrium the level of profits is determined by

$$\Pi(y, f, n) = \begin{cases} \Delta V(y, y, n) - F - C(f) & n \leq R \\ \frac{R}{n} \Delta V(y, y, R) - F - C(f) & n > R \end{cases} \tag{23}$$

for any given $R > 1$. The entry game then is a two-stage game where firms first decide whether or not to enter. A firm that chooses to enter pays the fixed costs $F > 0$. All firms that chose to enter then play the symmetric price-attention game from the last section. In a pure subgame perfect equilibrium (SPE) given any decision in the first stage a Nash equilibrium follows in the second stage. In a free-entry equilibrium each active firm makes non-negative profits ($\Pi(n) \geq 0$) and further entry would result in negative profits ($\Pi(n+1) < 0$). Concerning the Nash-equilibrium of the second stage we already know that for any given $n, R > 1$ there exists a single symmetric equilibrium. Ignoring the integer-value problem of n the free-entry equilibrium (y, f, n) then is described by

$$V_1\left(y, y, min\left\{R, n\right\}\right) = 0 \tag{24}$$

$$f : \begin{cases} p_1(f, f, n, R)\Delta V(y, y, R) - C'(f) = 0 & R < n \\ f = 0 & R \geq n \end{cases} \tag{25}$$

$$\Pi(f, y, n) = 0 \tag{26}$$

With symmetry we can only hope to determine n, the number of active firms in a SPE and not which firm enters and which firm stays out. I call a SPE unique if there only is one vector (y, f, n) that solves (24) - (26). The fundamentals of the symmetric game with free entry are the parameter set $\{R, \Delta\}$, the cost function $C(f)$, the value function $V(y, \bar{y}, z)$ with $z = min\left\{R, n\right\}$ as defined by assumption 3 and the probability function $p(f, \bar{f}, n, R)$ as defined by 4.

Assumption 6 *The following properties are imposed:*

a) The function $V(y, y, z)$ is twice continuously differentiable in z and $V_3 < 0$ as well as $V_{13} < 0$.

b) The function $p_1(f, f, n, R)$ is continuously differentiable in n

c) For $f > 0$ we have $\lim\limits_{n \to R^+} p_1(f, f, n, R) = 0$

The assumption $V_3(y, y, z) < 0$ is just the differentiable version of assumption 3 and the last assumption means that the marginal value of an attention set decreases if more alternatives are being considered. Let $\hat{y}$ be implicitly defined by $V_1(\hat{y}, \hat{y}, 2) = 0$.

Proposition 4 *Let $2 \leq R < \infty$. Suppose that assumptions 5 and 6 are satisfied and*

$$\Delta V(\hat{y}, \hat{y}, 2)/F \geq 1 \tag{27}$$

If the condition

$$\frac{\frac{\partial}{\partial f}\left(p_1(f, f, n, R)\right)}{p_1(f, f, n, R)} < p_{13}(f, f, n, R)\frac{n^2}{R} \tag{28}$$

holds then the free-entry game has a single symmetric SPE (y, f, n) with $2 \leq n < \infty$

Proof: Appendix B (8.9)

To understand intuitively why condition (28) excludes the possibility of multiple symmetric equilibria note that (28) is satisfied if $p_{13} > 0$. Suppose that $p_{13} > 0$ holds for any $n > n'$ and assume that (y', f', n') is an equilibrium of the free-entry game with $n' > R$. From (16) it is not difficult to see that $p_{13} > 0$ implies that $f'(n) > 0$ for $n \geq n'$. That is, equilibrium attention effort increases if we *exogenously* increase n. From (23) we see that an exogenous increase of n has two effects on the value of $\Pi(y(n), f(n), n)$: equilibrium profits decrease in n because R/n decreases in n. This is the direct effect of n on the profit level and originates from the fact that more senders means that *equilibrium* chances of being perceived decrease for a single firm which means less (expected) revenue. But there also is an indirect effect on the level of profits because $C(f)$ depends on n in the equilibrium. $f'(n) > 0$ implies that attention costs increase in n which means that profits decrease in n also over this channel. Thus we see that $p_{13} > 0$ unambiguously implies profits to decrease in n for $n > n'$. But as we have n' in the free-entry equilibrium this means that profits must be negative for any $n > n'$. Consequently, there cannot exist a free-entry equilibrium with $n > n'$. This is illustrated in the left picture of figure 5. If

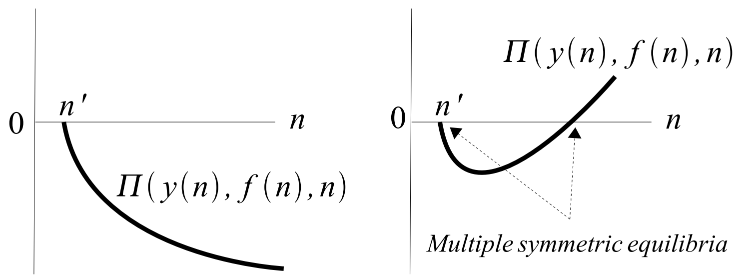

Figure 5: The possibility of multiple symmetric equilibria under limited attention

however we have $p_{13} < 0$ then the indirect effect of a change of n on profits is positive as a higher n means lower f and thus lower costs $C(f)$. If this indirect effect is very strong then profits might also increase in n. In this case we might have multiple symmetric equilibria as is illustrated in the right picture of figure 5.

A very nice property of the ACF is that condition (28) holds.

Corollary 2 *Let $2 \leq R < \infty$. The ACF (see (20)) satisfies assumption 6. If $V(y, y, z)$ satisfies assumptions 5 and 6, the function $p(f, f, n, R)$ is given by the ACF and condition (27) holds then the free-entry game has a single symmetric SPE (y, f, n) with $2 \leq n < \infty$*

Proof: The ACF satisfies assumption 5 because of lemma 4. It is not hard to see that the ACF also satisfies b) and c) of assumption 6. Finally, the ACF also satisfies condition (28) (see lemma 7 in the appendix (8.1)). The claim then follows from proposition 4.

■

4.3 The ACF-CES example

Suppose that $n > R$ is given exogenously. If $p(f, \bar{f}, n, R)$ is given by the ACF and $V(y, \bar{y}, z)$ by the CES example (11) then exactly one symmetric equilibrium (y, f) exists and is approximately[27] determined by (see 8.6, Part II, for the calculations)

$$y = c + \frac{cR}{(R-1)(\sigma-1)} \tag{29}$$

$$\frac{(n-R)Rv\Delta}{fn^2(1+(R-1)\sigma)} = C'(f) \tag{30}$$

Moreover, we have

$$V(y, y, z) = \frac{y-c}{y}\frac{v}{z} \tag{31}$$

Hence $V_3 < 0$ and $V_{13} < 0$ hold. Suppose that $V(\hat{y}, \hat{y}, 2) = \frac{v\Delta}{1+\sigma} \geq F$. Then by corollary 2 we may conclude that in the ACF-CES example there exists exactly one symmetric SPE with $n \geq 2$.

[27]Note that (29) is exact and (30) follows from using (21).

5 Comparative statics

In this section I discuss the comparative-static results of the symmetric price-attention equilibrium. I begin with an exogenous number of firms and then proceed to the case where n is determined endogenously in a free-entry equilibrium. I always take assumptions 5 and 6 to be satisfied. I extend the cost function from (7) by a parameter α, i.e. $C = C(f, \alpha)$ with the property that $C_2(f, \alpha) > 0$ and $C_{12}(f, \alpha) > 0$. For notational simplicity I set $C' \equiv C_1(f, \alpha)$ and $C'' \equiv C_{11}(f, \alpha)$.

5.1 Comparative statics for an exogenous number of firms

In this section I derive the comparative statics of the symmetric price-attention game where n is given exogenously. I only discuss the case where $n > R$ is given exogenously. The equilibrium conditions are given by (16). The game has the following set of exogenous parameters: $\{R, n, \Delta, F, \alpha\}$.

Proposition 5 *The comparative statics of* (16) *are given by* $y = y(\underset{-}{R})$ *and* $f = f(\underset{?}{R}, \underset{?}{n}, \underset{+}{\Delta}, \underset{-}{\alpha})$.

Proof: Appendix B (8.10)

The only parameter that matters for equilibrium prices is the size of the attention set. $y'(R) < 0$ results because $V_{13}(y, y, R) < 0$. This assumption means that the equilibrium marginal value that can be earned from an attention set decreases if people compare more alternatives. This is a very intuitive assumption and many examples (such as the CES example) satisfy this property. In the CES case this follows because consumers spend their fixed budget v over more alternatives if they perceive a larger set which means that less budget is available for the single firm. To offset this loss of demand the best-response is to decrease the price which explains why equilibrium prices are lower if the perceived market is larger.

However, the fact that y is determined only by the perceived market size and does not depend on other parameters implies that the aggregate level of attention competition nf (e.g. advertising intensity) observed may be a bad predictor of market power. Suppose we have two almost identical economies $\mathcal{E}_1 = \{R, n, \Delta, F, \alpha_1\}$ and $\mathcal{E}_2 = \{R, n, \Delta, F, \alpha_2\}$ with $\alpha_1 > \alpha_2$. Then $nf_1 < nf_2$. The classical theory of informative advertising holds that higher aggregate advertising levels should deteriorate market power of the firms which in equilibrium manifests itself in lower prices. The intuition behind this theory is that *limited information* of consumers because of scarce information leaves the firm with market power (see Goeree (2008)). In such a case means to increase advertising intensity would be beneficial for consumers as competition is reinforced and prices are decreased. We see that under limited attention this conjecture does not hold as consumers have limited information because of their limited attention capacities and *not* because of scarce information.

Search models usually predict lower equilibrium prices if search costs are reduced and electronic marketplaces or the internet with its powerful search engines are generally thought of dramatically lowering the search costs (e.g. Bakos (1997)). In such models this usually means that people become aware of more (or better) alternatives. However, in many cases the anticipated price-reduction was a lot smaller as suggested by these models (Ellison and Ellison (2005), p.149) or on the contrary online prices even were found to be higher than offline prices (e.g. Lee (1998)). The model of this paper provides an explanation why such gaps between theory and empirics exist. Search models do not distinguish the information set from the attention set. If we agree to $V_{13} < 0$ then larger attention sets will always imply lower prices in such models. If however the attention set is a subset of the information set with fixed size then larger information sets have no (or at least less) impact on equilibrium prices. The massive evidence on top-rank clicking-behaviour and also on the fact that people often do not use more than one search query (e.g. Jansen et al. (2000), p. 212) means that people concentrate their decision on just a few alternatives which suggests that not search cost theories but an attention theory adequately models the contemporaneous and future information problems.

As is shown in the appendix (see 8.10) we have $sign(f'(n)) = sign\,(p_{13})$ and $sign\,(f'(R)) = sign\left(p_1 V_2 V_{13} - \frac{\partial}{\partial y}\,(V(y,y,R))\,(p_{14}V + p_1 V_3)\right)$. These expressions cannot be signed without further assumptions. In case of a change of R there are two effects at work[28]. First, a decrease of R increases the equilibrium value of an attention set because i) an attention set must be shared with fewer competitors and ii) equilibrium prices are higher[29]. This set value effect increases the marginal revenue of attention and increases attention levels f. Second, the marginal probability of attention also depends on R. Depending on the sign of p_{14} the effect of R on p_1 may either reinforce or offset the set value effect. A sufficient condition for $f'(R) < 0$ is that $\frac{p_{14}R}{p_1} < \frac{-V_3 R}{V}$ holds in equilibrium. If $V(y,y,R) = \tilde{V}(y)/R$ with $\tilde{V}(y)' > 0$ then[30] the RHS of this inequality

[28]Recall the discussion of strategic behavior in section 3.3.4

[29]Formally: $\frac{dV}{dy} = V_3 + V_2 y'(R) < 0$.

[30]This is a property that many demand functions share, e.g. demand derived from the circular ideal variety model. Also the CES example has this property: $V(y,y,R) = \frac{v(y-c)}{Ry} = \frac{(y-c)}{y}\frac{v}{R}$.

is one. As for the ACF $p_1(f, f, n, R)$ is a strongly concave function of R which goes through the origin (see (21) in case of the approximation and lemma 6 in the appendix for the general case) we may conclude that $f'(R) < 0$ in such a case because $\frac{p_{14}R}{p_1} < 1$. Hence less attention implies higher prices and higher effort levels (stronger attention competition) in this example.
Suppose now that n increases exogenously. We see that the sign of $f'(n)$ depends entirely on how the marginal probability p_1 depends on n. This is different from the case of an exogenous change of R as a change of n has no set value effect. In case of the ACF it can be shown (see 7 in the appendix (8.1)) that $p_1(f, f, n, R)$ is a hump-shaped function of n. Consequently, $f(n)$ takes on a hump-shaped form in this example.
Summarising, we see that limited attention implies equilibrium prices to be independent of the size of the information set (n) which is different from models of informative advertising. In equilibrium, prices reflect the scarcity of attention in the sense that smaller attention sets mean higher prices. On the other side limited attention implies that a competition for attention emerges ($f > 0$ if $n > R$). If consumers are less attentive this means that the firms can earn higher values. But this makes attention more valuable for the firms which tends to increase attention levels - and attention costs. In case of the ACF this is the dominant effect which means that less attention always leads to higher attention costs. In this case we have two effects on profits if consumers are more inattentive: i) the set value effect which states that more inattention (lower R) increases profits and ii) the fact that, because getting attention is more valuable, the competition for attention is reinforced and leads to higher attention costs.
In the next section I investigate the implications of these two conflictive effects for the equilibrium number of firms that survive the price-attention competition.

5.2 Comparative statics in the free-entry game

In this section I discuss the consequences of limited attention for the equilibrium (y, f, n) of the free-entry game. The equilibrium conditions are given by (23) - (26). The game has the following set of exogenous parameters: $\{R, \Delta, F, \alpha\}$. In the entire section I assume that (27) and (28) are satisfied and $R \geq 2$. To understand intuitively how n depends on the these parameters it is sufficient to understand how the profit function depends on the parameters because $sign\left(\Pi'(\chi)\right) = sign\left(n'(\chi)\right)$ for $\chi \in \{R, \Delta, F, \alpha\}$

5.2.1 Comparative statics and conditions for endogenous limited attention

An attention equilibrium (an equilibrium with $n > R$) occurs endogenously in the free-entry game if the system

$$\begin{aligned} &V_1(\tilde{y}, \tilde{y}, \tilde{n}) = 0 \\ &\Delta V(\tilde{y}, \tilde{y}, \tilde{n}) - F = 0 \end{aligned} \tag{32}$$

has a solution $(\tilde{y}, \tilde{n})$ with $\tilde{n} > R$. If $\tilde{n} \leq R$ then $(\tilde{y}, 0, \tilde{n})$ corresponds to the equilibrium of the game; a conventional equilibrium occurs. If $\tilde{n} > R$ then there cannot be a conventional symmetric equilibrium. But as exactly one symmetric equilibrium exists this implies that an attention equilibrium must occur. To illustrate the situation graphically note that $\tilde{n} = \tilde{n}(\Delta, F) = R$ for fixed $R \geq 2$ implies that $\frac{d\Delta}{dF} = -\frac{\tilde{n}_2(\Delta,F)}{\tilde{n}_1(\Delta,F)}$. If the Implicit Function Theorem is applied to (32) it is straightforward to show that $\frac{d\Delta}{dF} = \frac{1}{V(\tilde{y},\tilde{y},\tilde{n})} > 0$. Figure 6 illustrates the situation. In the figure we see that lower infrastructure costs F or a higher market potential Δ rather

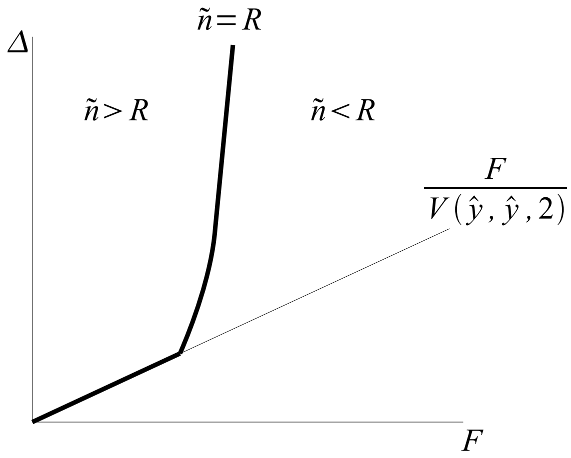

Figure 6: Attention equilibrium or conventional equilibrium?

impose that an attention equilibrium occurs ($\tilde{n} > R$). Generally, the conditions that imply an attention equilibrium are very similar to the conditions that imply an information-rich economy in Falkinger's model (Falkinger (2008), p. 1604-1605). If we use the CES-example we gain two further parameters: consumer budget $v > 0$ and the elasticity of substitution $\sigma > 1$. Suppose that $V(\hat{y}, \hat{y}, 2)\Delta = \frac{v\Delta}{1+\sigma} \geq F$. Then $v\Delta > F$. It is an easy exercise to solve (32) in the CES case for $\tilde{n}$:

$$\tilde{n} = \frac{v\Delta}{F\sigma} + \frac{\sigma - 1}{\sigma}$$

Hence we have $\tilde{n}'(v) > 0$ and $\tilde{n}'(\sigma) = \frac{F - v\Delta}{F\sigma^2} < 0$. This means that if the consumers have a higher budget or the commodities are less substitutable then an attention equilibrium is more likely to occur.

5.2.2 Comparative statics for limited attention equilibria

In this section I discuss the comparative statics of the symmetric free entry equilibrium if an attention equilibrium occurs endogenously. As in the previous section the comparative statics can be signed unambiguously in many cases if the function $p(f, \bar{f}, n, R)$ is given by the ACF[31]. I show that both f and n increase in the market potential Δ. This means that aggregate attention costs $nC(f)$ also increase in market potential which is consistent with the US-data on the number of consumers and advertising expenditure. Moreover, I show that an increase of the cost parameter α has ambiguous effects on the equilibrium number of firms. A positive relationship between equilibrium profits and the cost parameter α is possible which means that higher costs of attention could lead to a *higher* number of active firms which is an interesting result. Finally, I show that under the ACF the effects of the attention parameter R on f are negative also under free entry, but the effects on n are ambiguous and depend crucially on how much more value can be earned from less attentive consumers compared to how much attention costs increase because of the higher efforts to attract attention.
To simplify notation I set $V'' \equiv \frac{\partial}{\partial y}(V_1(y, y, R))$ and $p'' \equiv \frac{\partial}{\partial f}(p_1(f, f, n, R))$. For the general calculations see the appendix (8.11).

Effects of limited attention on prices

Consider two economies $\mathcal{E}_1 = \{R, \Delta_1, F_1, \alpha_1\}$ and $\mathcal{E}_2 = \{R, \Delta_2, F_2, \alpha_2\}$. Assume parameter values such that in the free entry equilibria of these economies we have $n_1 > R > n_2$. Hence in economy one an attention equilibrium occurs whereas in economy two a conventional equilibrium occurs. Then it is straightforward to verify that $y_1 = y_1(R)$ but $y_2 = y_2(\underset{+}{F_2}, \underset{-}{\Delta_2})$. This difference occurs because in an attention equilibrium the competitive entry effect on prices is absent. As in the last section we have $y_1'(R) < 0$.

The sign of $f'(\Delta)$ and $n'(\Delta)$

We know from the last section that the $f(n)$ equilibrium locus can be non-monotonic because of

[31]See lemma 7 in the appendix (8.1) for the properties of the ACF.

$p_{13}(f, f, n, R) \lesseqgtr 0$ and hence the comparative statics of f in the case of free-entry is not ex ante clear. It can be shown that $sign\,(f'(\Delta)) = sign\left(\frac{p_1(f,f,n,R)}{n} + p_{13}(f, f, n, R)\right)$. That is the sign of $f'(n)$ depends (positively) on the direct effect of Δ on the marginal value of perception but also on how the marginal probability of perception depends on n. In case of the ACF it can be shown that $f'(\Delta) > 0$ (see 7). Thus if the market potential increases so does the competition for attention. Further the ACF implies $n'(\Delta) > 0$ because $sign\,(n'(\Delta)) = sign\left(-\frac{R}{n}p'' - p_1^2 + \frac{\omega}{n}\right)$ which is unambiguously positive (see 7). Hence with the ACF the direct effect on equilibrium profits of a higher market potential always dominates the increase of the costs of attention competition. We know from the data that the number of US-consumers doubled from 1950 - 2005. At the same time advertising expenditure more than doubled. This is in line with the comparative static predictions of my model as both f and n increase with Δ under the ACF. Hence aggregate advertising expenditure $nC(f)$ (or $nF + nC(f)$) unambiguously increases if Δ increases.[32]

The sign of $f'(F)$ and $n'(F)$

We always have $n'(F) < 0$ unambiguously. In case of the ACF we have that f depends in a hump-shaped way on F because $sign\,(f'(F)) = sign\,(-p_{13})$. The different ways in which Δ and F affect f under free-entry originate in the fact that Δ also positively affects the marginal revenue of attention which increases f. In the case of a change of F attention efforts are only affected by the change of n according to the entry or exit decisions and we know from the last section that $f(n)$ is hump-shaped with the ACF.

The sign of $f'(\alpha)$ and $n'(\alpha)$

Let $\varepsilon \equiv \frac{p''f}{p_1} < 0$, $\mu \equiv \frac{C''f}{C'} \geq 0$ and $\beta \equiv \frac{C_{12}f}{C_2} > 0$. Suppose that $p(f, \bar{f}, n, R)$ is given by the ACF. Then it is straightforward to show (use lemma 7 b)) that a sufficient condition for $f'(\alpha) < 0$ is given by $\beta + \varepsilon \geq 0$. In case of the ACF we have $\varepsilon = -1$ (see the proof of 7 b)). Hence if $\beta \geq 1$ then $f'(\alpha) < 0$. If the cost function is of the type $C(f, \alpha) = \alpha c(f)$ then the sufficient condition for $f'(\alpha) < 0$ reduces to $\frac{c'(f)f}{c(f)} \geq 1$. But $c(0) = 0$, $c'(f) > 0$ and $c''(f) \geq 0$ imply $\frac{c'(f)f}{c(f)} \geq 1$. Hence the ACF together with $C(f, \alpha) = \alpha c(f)$ unambiguously imply that $f'(\alpha) < 0$. Further, it is possible to show that $sign(n'(\alpha)) = sign(\mu - \varepsilon - \beta)$ which is ambiguous. Hence it is possible to have $n'(\alpha) > 0$, which means that increasing costs of attention might imply a *higher* number

[32] The same holds in the CES example if consumer budget v increases. See section 5.2.3.

of active firms and is an interesting result. This occurs if the reduction of f, due to the higher marginal costs of attention, lead to lower equilibrium attention costs at the firm level. With the ACF and $C(f,\alpha) = \alpha c(f)$ we get $sign(n'(\alpha)) = sign(\mu + 1 - \beta)$. If $c(f) = f^\eta$ and $\eta \geq 1$ then we have $\mu = \eta - 1$ and $\beta = \eta$. Thus in this case $n'(\alpha) = 0$. Hence in this example α affects only the level of f but has no effect on the equilibrium number of active firms.

The sign of $f'(R)$ and $n'(R)$

A decrease in R (consumers are more inattentive) leads to higher value of the attention sets and thus also to a higher marginal return on attention which increases f. But the marginal probability to make it into the attention set also depends on R. It is possible to determine the sign of $f'(R)$ unambiguously if we use the ACF and assume that $V(y, y, R) = \tilde{V}(y)/R$. Then we get

$$sign\left(f'(R)\right) = sign\left(V_{13}RV_2\left(p_{13} + \frac{p_1}{n}\right) - \frac{V''\tilde{V}(y)}{n}\left(p_{14} - \frac{p_1}{R}\right)\right)$$

and hence $f'(R) < 0$ follows unambiguously also with endogenous limited attention (because $p_{14} - \frac{p_1}{R} < 0$ and $p_{13} + \frac{p_1}{n} > 0$, see lemmata 6 and 7 in the appendix).

To discuss the effect of limited attention on the equilibrium number of firms I set $V(y, y, R) = \tilde{V}(y)/R$ and $C(f) \equiv \theta f^\eta$ with $\theta > 0$ and $\eta \geq 1$. Then with (21) the two equilibrium equations (25) and (26) become

$$\begin{array}{l} \frac{n-R}{n^2\eta}\Delta\tilde{V}(y) = C(f) \\ \frac{\Delta\tilde{V}(y)}{n} = F + C(f) \end{array} \tag{33}$$

Regarding the $n(R)$-locus two conflictive effects can be eyeballed from the second equation of (33) which originate from the interaction between attention competition and economic competition. First, as less attentive consumers compare fewer alternatives this loosens price competition which leads to higher equilibrium prices and higher revenues (see lemma 5). Hence by the revenue effect we would expect profits and thus n to increase under limited attention. Second, because getting attention is of more value to firms, the competition for attention is intensified ($f'(R) < 0$) and attention costs increase. Thus the cost effect suggests n to decrease under limited attention. Manipulation of (33) shows that a single equation determines $n(R)$:

$$\frac{\Delta\tilde{V}(y(R))}{n^2\eta}\left(n(\eta - 1) + R\right) = F \tag{34}$$

Using the implicit function theorem on (34) we can deduce that

$$sign(n'(R)) = sign\left(R + \left(R + n\left(\eta - 1\right)\right)\varepsilon_v\right) \tag{35}$$

with $\tilde{v}(R) \equiv \tilde{V}(y(R))$ and $\varepsilon_v \equiv \frac{\tilde{v}'(R)R}{\tilde{v}(R)} < 0$.[33] Expression (35) nicely confronts the two conflictive effects with each other. We see that the elasticity η and ε_v are important determinants of the curve $n(R)$. I first discuss the impact of ε_v. If $\varepsilon_v \approx 0$ e.g. because price competition is such that equilibrium prices respond only weakly to a change of R then $n'(R) > 0$. In this case less attention increases the revenue from the attention sets only weakly. At the same time competition for the sets is increased which implies higher costs of attention. As the additional costs outweigh the additional revenue gains profits decrease which leads to a smaller market. In the absence of price competition ($V(y, y, R) = v/R$) we unambiguously get $n'(R) > 0$ ($\varepsilon_v = 0$ because in this case $y'(R) = 0$). In order for the revenue effect to dominate the cost effect if consumer attention declines, the value of the attention set must respond sufficiently elastic to R. If this is the case (e.g. because commodities are strong substitutes and hence economic competition is intense, see section 5.2.3 below) then limited attention implies larger markets. To explain the impact of η on $n(R)$ note that a high value of η means that a change of f implies a strong change of marginal costs which reduces the reagibility of f. In this sense η controls the elasticity of attention competition: a high value of η means that f reacts less responsive to any exogenous change. To see this use (33) to find

$$f = \left(\frac{n-R}{n^2\theta\eta}\tilde{v}(R)\right)^{\frac{1}{\eta}} = \left(\frac{\tau}{\eta}\right)^{\frac{1}{\eta}} \qquad \tau \equiv \frac{n-R}{n^2\theta}\tilde{v}(R) \tag{36}$$

Hence $f'(\tau)\tau/f(\tau) = 1/\eta$. A higher value of η means that the equilibrium f does not react strongly to a change of R which also means that attention costs do not change much. Thus the revenue effect is more likely to dominate the cost effect if η is large. Moreover, we can deduce from (34) that $n'(\eta) > 0$ which by (33) means that equilibrium costs $C(f)$ must decrease in η. From (34) we see that

$$\lim_{\eta\to\infty} n(\eta) = \frac{\Delta\tilde{V}(y)}{F}$$

Similarly, if $\theta = 0$ such that attention competition is free we get $f^* \to \infty$ and $n = \Delta\tilde{V}(y)/F$.

[33] The expression is negative because $\tilde{v}'(R) = \tilde{V}'\left(y(R)\right)y'(R)$ and $y'(R) < 0$ but $\tilde{V}'(y(R)) > 0$.

In both cases equilibrium profits are only determined by the value of the attention set. As in both cases $C(f) = 0$ this can be thought of an upper bound on the market size (for given R). To summarise we note that if the economic competition is weak ($\varepsilon_v \approx 0$), e.g. because of weak substitutes, then firms cannot extract much additional revenue from a decline of attention and the cost effect tends to dominate which implies smaller markets if consumers are less attentive. If however the economic competition is strong or attention competition is inelastic (high η) then less attentive consumers might imply higher profits and larger markets.

5.2.3 The comparative statics of the ACF-CES example

We can use the general results of section 5.2.2 to discuss the comparrative statics of the ACF-CES example considered earlier. Suppose the parameter values are such that a free-entry equilibrium with $n > R$ occurs endogenously. From section 4.3 we know that a single symmetric SPE exists in this example. As we use the ACF we immediately know from the previous section that $f'(\Delta) > 0$, $f'(R) < 0$ and $f(F)$ is non-monotonic (hump-shaped). Also $n'(\Delta) > 0$ and $n'(F) < 0$ follow directly from the last section. Because in the CES-example we have $V(y, y, R) = \frac{y-c}{y}\frac{v}{R}$ (see (31)) we can use (35) to examine $n'(R)$ in the ACF-CES case. It is straightforward to show that $\tilde{v}(R) = \frac{vR}{1+\sigma(R-1)}$ and $\varepsilon_v = \frac{(1-\sigma)}{1+(R-1)\sigma} \in (-1, 0)$. Hence for $\eta = 1$ we get $sign(n'(R)) = sign\,(R(1+\varepsilon_v)$ and thus $n'(R) > 0$. In this case less attentive consumers imply a very fierce attention competition that involves high attention costs and leads to exit. However, if η is large we could also get $n'(R) < 0$, i.e. the gains from consumer inattention more than cover the increased attention costs. This leads to firm entry. Numerical evaluations of the ACF-CES example confirm that, depending on the parameter constellation[34], both cases may occur as figure 7 suggests. Finally, it is clear that in the ACF-CES example consumer budget has the same comparative statics as Δ, i.e. $y'(v) = 0$, $f'(v) > 0$ and $n'(v) > 0$. A larger value of σ means that commodities are stronger substitutes and hence the economic competition is more intense. It can be shown that $y'(\sigma) < 0$, $f'(\sigma) < 0$ and $n'(\sigma) < 0$ (see 8.6, part III).

[34] It can be shown that in the CES case we have $\varepsilon_v'(\sigma) < 0$ which means that if the varieties are stronger substitutes (larger σ) then, as suggested in the main text, $n'(R) < 0$ is more likely to occur.

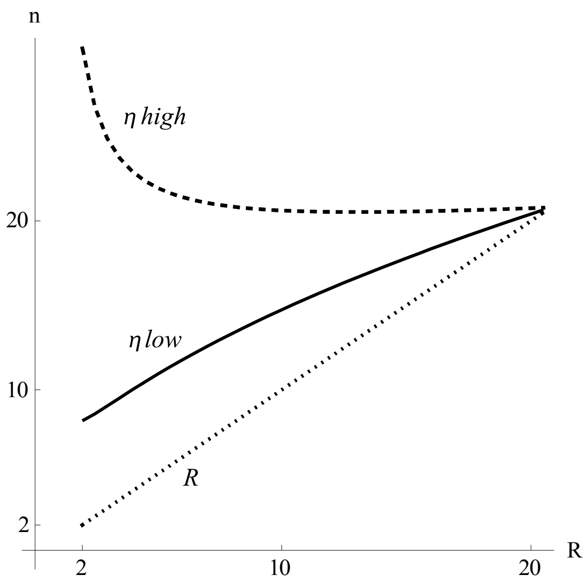

Figure 7: The ACF-CES example

6 Conclusion

To my knowledge, this is the first contribution that develops a rigorous game-theoretic setting which allows to combine attention competition with economic competition.

In the model firms can influence their *chance* of perception which depends on the own attention effort level but also on the effort level of all competitors and is endogenously determined.

In the free-entry equilibrium of the symmetric price-attention game prices, attention effort levels and the number of active firms are endogenously determined. I derive general conditions which assert that only one symmetric equilibrium exists and show that my main example of the attention contest, the ACF, satisfies these properties.

Competition for attention emerges if and only if the information is superabundant relative to perception capabilities, i.e. if the equilibrium number of information senders is higher than the exogenous attention threshold. To the information senders the market as perceived by consumers and not the effective market matters which is reflected in equilibrium prices: besides reflecting e.g. traditional economic market power prices also reflect the scarcity of attention. This important finding can explain e.g. why online prices have not dropped as much as expected after the introduction of the Internet - a fact that search models and models of informative advertising have significant difficulties to explain.

As firms can gain from inattentive consumers but at the same time need to cover the costs of getting attention there exist two conflictive effects that determine the market structure. If either attention competition is inelastic or the economic competition is strong (commodities are strong substitutes) then the positive revenue effect of limited attention dominates the costs of getting attention which leads to higher profits and larger markets.

Specific models of price competition give more insight on the consequences of price-attention competition. For example, it is possible to apply my setting to Salop's model of circular product differentiation - which also allows to examine the normative implications of price-attention competition (see Hefti (2011a), chapter 4).

The model can be extended in several ways. One interesting case extends the basic model by letting firms also choose the reach of their information campaign (as was sketched in section 2, see Hefti (2011a), chapters 2 and 4). In the case of the Internet economy as considered in this paper the reach was exogenously given to all active firms. In the more general case firms can choose how many consumers (how many attention sets) to reach (e.g. in how many newspapers to advertise) versus their probability to get a consumers' attention (e.g. where to place an ad in a given newspaper). All main results of this paper also hold in this extended setting. Moreover, the pro-competitive effect of informative advertising vanishes under limited attention and policy implications taken from models of informative advertising that ignore limited attention might have adverse effects if limited attention matters.

A different extension is to relax the assumption that the attention distribution is independent of the price distribution. This would certainly be the case with low-price filters. Nevertheless, my model still can be used to address such problems simply by letting the attention probability vector be determined by relative prices. The implications of such pricing filters, especially in the context of quality goods, is the subject of current research.

7 Appendix A: On uniqueness and stability

In section 4 I have presented the assumptions on the value function V and the probability function p which exclude the possibility of multiple symmetric equilibria in the symmetric price-attention game. I have shown that the ACF satisfies all assumptions on p.

In this section I ask the more ambitious question whether there can be asymmetric equilibria in the symmetric game. Intuitively, the interaction of economic competition with attention competition suggests reasons why we could expect asymmetric "specialisation" equilibria to exist. We can imagine some firms to specialise in the economic competition. These firms set a low price and do not advertise a lot (i.e. choose a low attention effort level). The other firms specialise in attention competition: they choose a high price and high attention levels. A firm in the first group knows that its chance of perception is smaller than the chance of an attention specialist but i) it needs less revenue to cover attention costs and ii) it may earn more revenue than an attention specialist from those attention sets both are in. The firms in the second group are in more attention sets which could cover their higher attention expenditure.

I now derive the conditions that exclude the possibility of asymmetric equilibria in the price-attention game. To achieve this goal we cannot work with the symmetric opponent form of the profit function. But because of the complex nature of the price-attention game (every profit function has his own "context" B_j, see (5)) it is clear that standard approaches to uniqueness such as the univalence approach or the index theorem are not feasible because we would have to evaluate at $(2N) \times (2N)$-matrix! Fortunately, there exists an approach to uniqueness that separates between the possibility of multiple symmetric equilibria and the possibility of asymmetric equilibria. This approach is developed in Hefti (2011b). The proof of the inexistence of asymmetric equilibria in the symmetric price-attention game in this section is an application of this methodology. Further, I briefly discuss stability of the symmetric equilibrium under iterative adjustments.

The proof is organized as follows. First, I consider a reduced version of the game where the attention distribution is given exogenously but prices are chosen simultaneously. I show that, under certain assumption, there only is one equilibrium in this pricing game, the symmetric equilibrium, where all firms choose the same price y independent of the attention distribution. Then it suffices to find conditions that assert that the pure attention game for given and identical prices y only has one equilibrium, the symmetric equilibrium.

7.1 Uniqueness of the symmetric equilibrium

First, I state the basic assumptions regarding price competition that are maintained throughout appendix A. Let $S_y \equiv [c, y^{max}]$ as before. Suppose that the attention distribution P as defined by (2) is exogenously fixed with the property that $P_A > 0$ for all $A \in \mathcal{A}$. A firm can only choose its price y_j. I consider the following profit function:

$$\Pi(y_j) = \sum_{A \in B_j} P_A V^j(A)\Delta \tag{37}$$

Suppose that all n firms simultaneously and non-cooperatively choose their price according to (37). Then $\left(n, S_y^n, \Pi\right)$ is an n-player pricing game. I impose the following assumptions on the value functions $V^j(Y_A)$ with $A \in B_j$:

Assumption 7 *Let $A \in B_j \subset \mathcal{A}$ and $|A| = z > 1$. Then*

a) For all $A \in B_j$ and $j = 1, ..., n$: $V^j(Y_A) = V^{\sigma(j)}(Y_{\sigma(A)})$, where σ is a permutation of A. $V^j(Y_A) > 0$ if $y_j = y_g = y \in Int(S_y)$ for all $g \in A$.

b) $V^j(Y_A) \in C^2\left(Int(S_y^z), \mathbb{R}\right)$, $V^j(Y_A)$ is strongly quasiconcave in y and $\frac{\partial V^j(Y_A)}{\partial y_g} > 0$ for $g \in A$.

c) $\left.\frac{\partial V^j(Y_A)}{\partial y}\right|_{y=c} > 0$ and $\left.\frac{\partial V^j(Y_A)}{\partial y}\right|_{y=y^{\max}} < 0$.

d) For $y_g \in (c, y^{\max})$ with $g \neq j$:

$$\sum_{A \in B_j} P_A \frac{\partial V^j(Y_A)}{\partial y_j} = 0 \quad \Rightarrow \quad \sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_1 y_g} > 0$$

The last assumption means that prices are strategic complements. In case of an equal distribution of attention, i.e. $P(A) = P(A') = \begin{pmatrix} n \\ R \end{pmatrix}^{-1}$ for all $A, A' \in \mathcal{A}$, the symmetric opponents form of (37) is

$$\Pi(y) = \begin{cases} \frac{R}{n} V(y, \bar{y}, R)\Delta & R < n \\ V(y, \bar{y}, n)\Delta & R \geq n \end{cases} \tag{38}$$

To see this in the case where $R < n$ note that $|B_j| = \begin{pmatrix} n-1 \\ R-1 \end{pmatrix}$. Then if $y_g = \bar{y}$ for any $g \neq j$ we have

$$\sum_{A \in B_j} P_A V^j(Y_A)\Delta = \frac{1}{\begin{pmatrix} n \\ R \end{pmatrix}} \begin{pmatrix} n-1 \\ R-1 \end{pmatrix} V(y, \bar{y}, R)\Delta = \frac{R}{n} V(y, \bar{y}, R)\Delta$$

Lemma 5 *Let $P(A) = P(A')$ for all $A, A' \in \mathcal{A}$ and take assumption 7 as well as condition* (18) *to be satisfied. Then the interior symmetric equilibrium $y^* \in (c, y^{max})$ is the unique equilibrium of the pricing game.*

Proof: Appendix B (8.12)

Note that because $\frac{\partial}{\partial y}\left(V_1(y, y, z)\right) = V_{11}(y, y, z) + V_{12}(y, y, z)$ the inequality in condition (18) can be rewritten as $-V_{11}(y, y, z) > V_{12}(y, y, z)$. If even

$$V_1(y^*, y^*, z) = 0 \;\Rightarrow\; -V_{11}(y^*, y^*, z) > |V_{12}(y^*, y^*, z)| \tag{39}$$

holds then the symmetric equilibrium y^* is locally contraction-stable (stable under iterative adjustments; see Hefti (2011b)).

Lemma 5 implies that if $R \geq n$ the general proof of uniqueness in the symmetric price-attention game is already finished because there cannot be an equilibrium other than (y^*, f) with $f = 0$ and y^* defined by $V_1(y^*, y^*, n) = 0$.

From now on I always set $n > R$. Suppose that the attention distribution function P is still exogenously given but $P_A > 0$ may be different for different $A \in \mathcal{A}$. An interior equilibrium $(y_1, ..., y_n)$ of the pricing game satisfies

$$\begin{pmatrix} \sum\limits_{A \in B_1} P_A \frac{\partial V^1(Y_A)}{\partial y_1} \\ \vdots \\ \sum\limits_{A \in B_n} P_A \frac{\partial V^n(Y_A)}{\partial y_n} \end{pmatrix} \equiv D_y\left(y_1, ..., y_n\right) = 0 \tag{40}$$

The following proposition shows that under a further assumption the only interior equilibrium price vector is symmetric and independent of the attention distribution.

Proposition 6 *Suppose that $n > R$ and $P_A > 0$ for all $A \in \mathcal{A}$. Then under assumption 7 and condition* (18) *there exists a single symmetric equilibrium y^* that is independent of the distribution of P_A. Further, if for any feasible y_g with $g \neq j$ and any $A' \in B_j$ with $j, g \in A'$*

$$\sum_{A \in B_j} P_A \frac{\partial V^j(Y_A)}{\partial y_j} = 0 \Rightarrow -\frac{\partial^2 V^j\left(Y_{A'}\right)}{\partial y_j^2} > (R-1)\left|\frac{\partial^2 V^j\left(Y_{A'}\right)}{\partial y_j \partial y_g}\right| \tag{41}$$

holds then y^* is the unique equilibrium of the pricing game.

Proof: Appendix B (8.13)

Note that if the inequality in condition (41) holds even without the requirement that $\sum\limits_{A \in B_j} P_A \frac{\partial V^j(Y_A)}{\partial y_j} = 0$ then uniqueness follows directly from the contraction mapping theorem (see Vives (1999), p. 47).[35]

Now I return to the symmetric price-attention game from the main text.

Proposition 7 (Sufficient conditions for uniqueness) *Suppose $n > R$ and assumptions 4 and 7 as well as (41) and (19) are satisfied. If for all $A \in B_j$, $f_1, ..., f_n \in (0, \infty)$ and any fixed player $g \neq j$ the condition*

$$-\frac{\partial^2 P_A(\mathcal{F})}{\partial f_j^2} + \frac{\partial^2 P_A(\mathcal{F})}{\partial f_j \partial f_g} > 0 \tag{42}$$

holds then the symmetric equilibrium (y, f) determined by (16) is the unique equilibrium of the symmetric price-attention game.

Proof: Appendix B (8.14)

Note that if all presuppositions of proposition 7 are satisfied for any $n \geq 2$ and (27) and (28) hold then there only is one equilibrium (y, f, n) in the free-entry game and the equilibrium is symmetric.

From the main text we already know that the ACF does not generate multiple symmetric equilibria. But does the ACF also satisfy (42)?

Consider (14) and suppose firm j did not get the first draw. Then one of firm j's competitors must have succeeded in doing so. The aggregate mass of the balls remaining in the urn has changed conditional on the ball drawn out. Suppose for example that the ball of firm $g \neq j$ has been drawn. Then conditional on this event firm j's chance of getting the next draw is $\frac{f_j}{\sum\limits_{k=1}^{n} f_k - f_g}$. We can use such a line of reasoning to calculate the probabilities P_A for all $A \in \mathcal{A}$.

To illustrate this, suppose $R = 2$ and $n = 3$. Then $\mathcal{A} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$. Let $A = \{1,2\}$. Then $P_A(\mathcal{F}) = P_A(f_1, f_2, f_3)$ and

$$P_A(f_1, f_2, f_3) = \frac{f_1 f_2}{f_1 + f_2 + f_3} \left(\frac{1}{f_2 + f_3} + \frac{1}{f_1 + f_3} \right)$$

[35]In the CES example it can be shown by applying proposition (2) for e.g. $\sigma = 2$, $R = 2$ and $n = 3$ that in this case the joint best-response function is not a global contraction but condition (41) holds resulting in a unique price-equilibrium.

Corollary 3 *Let $R = 2$. The ACF satisfies condition*(42)*.*

Proof: Appendix B (8.15)

Remark: The procedure of the proof of 3 can be applied to show that (42) is satisfied also if $R = 3$. I conjecture that (42) holds for any $R > 2$ under the ACF.

7.2 An example

I now provide an example of a function V^j that satisfies assumption 7 and condition (41). Let j denote the representative firm. Then for any $A \in B_j$ let

$$V^j(Y_A) = (y_j - c)x^j(Y_A)$$

with

$$x^j(Y_A) = \max\left\{0, \frac{1}{1+(z-1)\gamma}\left(1 - \frac{1+(z-2)\gamma}{1-\gamma}y_j + \frac{\gamma}{1-\gamma}\sum_{i\in A: i\neq j} y_i\right)\right\} \tag{43}$$

where $\gamma \in (0,1)$ controls the degree of substitutability[36] between perceived commodities and $z = min\{R, n\}$. This demand function is often used in oligopolistic theory and IO models and can be microfounded by assuming a quasilinear upper tier utility function and quadratic subutility (see e.g. Vives (1999) p. 145-146). For simplicity, I set $c = 0$ and $\Delta = 1$.

Theorem 1 *Suppose $S_y = [0,1]$. Then with* (43) *the symmetric equilibrium $y^* \in (0,1)$ of the pricing game is unique and independent of the attention distribution.*

Proof:

In step 1 I show that assumption 7 is satisfied. In step 2 I show that (41) is satisfied.

Step 1: Assume $j \in A \in B_j$.

a) Permutation-invariance is obvious. Let $y_j = y_g = y \in (0,1)$ for all $g \in A$. Then $V^j(Y_A) = \frac{(1-y)y}{1+(z-1)\gamma} > 0$.

b) We have $\frac{\partial^2 V^j(Y_A)}{\partial y_j^2} = -2\frac{1+(z-2)\gamma}{(1-\gamma)(1+(z-1)\gamma)} < 0$ because $z > 1$ and $\gamma < 1$. Hence $V^j(Y_A)$ is strongly concave and thus also strongly quasiconcave.

[36] $\gamma \to 1$ means that commodities are perfect substitutes whereas $\gamma \to 0$ means that commodities are independent.

c) We have

$$\left.\frac{\partial V^j(Y_A)}{\partial y_j}\right|_{y=0} = \frac{1+\frac{\gamma}{1-\gamma}\sum\limits_{i\in A:i\neq j} y_i}{1+(z-1)\gamma} > 0$$

and

$$\left.\frac{\partial V^j(Y_A)}{\partial y_j}\right|_{y=1} = -\frac{1+(2z-3)\gamma-\gamma\sum\limits_{i\in A:i\neq j} y_i}{(1-\gamma)\left(1+(z-1)\gamma\right)}$$

Thus $\left.\frac{\partial V^j(Y_A)}{\partial y_j}\right|_{y=1} < 0$ because $1+\gamma(z-2)>0$.

d) Because $\frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} = \frac{\gamma}{(1-\gamma)(1+(z-1)\gamma)} > 0$ for any $g \neq j$ with $g \in A$ the game is strictly supermodular in prices. Hence prices are strategic complements.

Step 2:

Assume $g \in A$. Then

$$\begin{aligned} -\frac{\partial^2 V^j(Y_A)}{\partial y_j^2} > (z-1)\left|\frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g}\right| \quad &\Leftrightarrow \quad 2\frac{1+(z-2)\gamma}{(1-\gamma)(1+(z-1)\gamma)} > \frac{\gamma(z-1)}{(1-\gamma)(1+(z-1)\gamma)} \\ &\Leftrightarrow \quad 2+\gamma\left(z-3\right) > 0 \end{aligned}$$

which holds as $z>1$ and $\gamma<1$. Hence condition (41) from proposition 6 holds and so does condition (18). This implies the existence of a unique interior equilibrium for any $n, R > 1$ that is independent of the attention distribution. Moreover, the equilibrium is symmetric.

■

If $R=2$ we know from corollary 4 that if we use the ACF to determine the attention probability distribution the price-attention game only has a unique symmetric equilibrium.

7.3 Stability

This section provides conditions which imply that the symmetric equilibrium is a local contraction.

Proposition 8 *The symmetric equilibrium (y, f) of the price-attention game is locally contraction-stable if $V(y, \bar{y}, R)$ is a local contraction at y (condition (39) holds) and for $n > R$*

$$\Delta \frac{p_1(f,f,n,R)V_2(y,y,R) + \left|p_{12}(f,f,n,R)\right| V(y,y,R)}{-\left(p_{11}(f,f,n,R)\Delta V(y,y,R) - C''(f)\right)} < 1 \tag{44}$$

Proof: See Hefti (2011b).

$\blacksquare$

Corollary 4 *Suppose $n > R$ and $V_2(y, y, R) = 0$ and let $C(f) = \theta f^\eta$ where $\eta \geq 2$ and $p(f, \bar{f}, n, R)$ is given by the ACF. Then the symmetric equilibrium is locally contraction-stable.*

Proof: Appendix B (8.16)

Note that e.g. with $\eta = 1$ the contraction condition (44) need not hold generally for the ACF. In any case we require $V_2(y, y, R)$ to be small. Intuitively, the contraction condition rather holds with higher η as then attention competition is less elastic (see (36)) and hence firms only respond weakly to small deviations around the symmetric equilibrium.

8 Appendix B

8.1 Properties of the ACF

Lemma 6 *There exists a continuously differentiable extension to (13) which is given by*

$$p^C(f, f, n, R) = 1 - \frac{\Gamma(n)}{\Gamma(n-R)} \frac{\Gamma\left(\frac{f}{\bar{f}} + n - R\right)}{\Gamma\left(\frac{f}{\bar{f}} + n\right)} \tag{45}$$

where $\Gamma(x)$ denotes the Gammafunction evaluated at x and R and n are real numbers with $R < n$.

$$p_1^C(f, f, n, R) = \frac{n-R}{nf} \left(\psi\left(1+n\right) - \psi\left(1+n-R\right)\right) \tag{46}$$

where $\psi(.)$ is the digamma function. If R_0 is an integer then

$$p_1^C(f, f, n, R_0) = \frac{n - R_0}{nf} \sum_{i=1}^{R_0} \frac{1}{1+n-i}$$

and $p_1^C(f, f, n, R_0)$ is strictly concave in R around R_0.

Proof:

First note that

$$\prod_{i=1}^{R} \left((n-i)\bar{f}\right) = \bar{f}^R \frac{(n-1)!}{(n-R)!}(n-R) \tag{47}$$

The Gamma-function $\Gamma(x)$, which is continuously differentiable on $x \in \mathbb{R}^+$, has the properties that $n! = \Gamma(n+1)$ if n is a positive integers and $\Gamma(x+1) = x\Gamma(x)$. Hence (47) becomes

$$\bar{f}^R \frac{\Gamma(n)}{\Gamma(n-R)}$$

Equivalently,

$$\begin{aligned} \prod_{i=1}^{R} \left(f + (n-i)\bar{f}\right) &= \bar{f}^R \prod_{i=1}^{R} \left(\tfrac{f}{\bar{f}} + n - i\right) \\ &= \bar{f}^R \frac{\Gamma\left(\frac{f}{\bar{f}}+n\right)}{\Gamma\left(\frac{f}{\bar{f}}+n-R\right)} \end{aligned} \tag{48}$$

Hence

$$p^C(f, f, n, R) = 1 - \frac{\Gamma(n)}{\Gamma(n-R)} \frac{\Gamma\left(\frac{f}{\bar{f}} + n - R\right)}{\Gamma\left(\frac{f}{\bar{f}} + n\right)}$$

which is continous for $R < n$ and $P(f) \overset{R \to n^-}{\longrightarrow} 1$ as $\Gamma(n-R) \overset{R \to n^-}{\longrightarrow} \infty$. As the derivation makes clear we have $p(f, \bar{f}, n, R) = p^C(f, \bar{f}, n, R)$ whenever R and n are integers. Differentiation of (45) at $f = \bar{f} > 0$ then yields (46). If R_0 is an integer we have

$$\psi\,(1+n) = \psi\,(1+n-R_0) + \sum_{i=1}^{R_0} \frac{1}{1+n-i}$$

as also the digamma function is recursive with $\psi(1+x) = \psi(x) + \frac{1}{x}$. Hence

$$p_1^C(f, f, n, R_0) = \frac{n-R_0}{nf} \sum_{i=1}^{R_0} \frac{1}{1+n-i}$$

which corresponds to (20). But know is is formally possible to take the derivative and

$$p_{14}^C(f, f, n, R_0) = \frac{1}{nf} \left((n-R_0) \frac{1}{(1+n-R_0)^2} - \sum \frac{1}{1+n-i} \right)$$

But

$$p_{14}^C(f, f, n, R_0) - \frac{p_1^C(f, f, n, R_0)}{R_0} = \frac{n-R_0}{(1+n-R_0)^2} - \frac{n}{R_0} \sum_{i=1}^{R_0} \frac{1}{1+n-i}$$

which is negative as $\frac{n-R_0}{(1+n-R_0)^2} < 1$ but $\sum_{i=1}^{R_0} \frac{1}{1+n-i} = \frac{1}{n} + \frac{1}{n-1} + ... + \frac{1}{1+n-R_0} > \frac{1}{n} + \frac{1}{n} + ... + \frac{1}{n} = \frac{R_0}{n}$. Hence $p_1^C(f, f, n, R_0)$ is strictly concave in R around R_0.[37]

$\blacksquare$

Lemma 7 *With the ACF based on* (20) *the following properties hold:*

a) $\frac{p_1(f,f,n,R)}{n} + p_{13}(f, f, n, R) > 0$

b) $\frac{p''(f,f,n,R)}{p_1(f,f,n,R)} < p_{13}(f, f, n, R)\frac{n^2}{R}$

c) $\frac{p''(f,f,n,R)}{p_1(f,f,n,R)} < -\frac{n}{R}p_1(f, f, n, R)$

d) $p_1(f, f, n, R)$ *for given* $f > 0$ *is a hump-shaped function of* n*.*

where $p''(f, f, n, R) \equiv \frac{\partial p_1(f,f,n,R)}{\partial f}$.

Proof:

a) As

$$p_{13}(f, f, n, R) = \frac{1}{nf}\left(\frac{R}{n}\sum_{i=1}^{R}\frac{1}{1+n-i} - (n-R)\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2\right) \tag{49}$$

we have

$$\frac{p_1}{n} + p_{13} > 0 \;\Leftrightarrow\; \sum_{i=1}^{R}\frac{1}{1+n-i} - (n-R)\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2 > 0 \tag{50}$$

which holds because

$$\sum_{i=1}^{R}\frac{1}{1+n-i} - (n-R)\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2 = \sum_{i=1}^{R}\left(\frac{1+R-i}{(1+n-i)^2}\right) > 0$$

b) From (20) we see that $\frac{p''}{p_1} = -\frac{1}{f}$. Then using (49) gives

$$\frac{p''}{p_1} < p_{13}\frac{n^2}{R} \;\Leftrightarrow\; -\frac{R}{n^2} < \frac{R}{n^2}\sum_{i=1}^{R}\frac{1}{1+n-i} - \frac{(n-R)}{n}\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2$$

Reformulation gives

$$\frac{R}{n}\left(\sum_{i=1}^{R}\frac{1}{1+n-i} + 1\right) - (n-R)\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2 > 0$$

[37] It is an easy numerical excersise to show that this result generalises for an arbitrary real-valued R_0.

or equivalently

$$\frac{R}{n}\left(\sum_{i=1}^{R}\frac{1}{1+n-i}+1\right)-(n-R)\sum_{i=1}^{R}\left(\frac{1}{1+n-i}\right)^2+\sum_{i=1}^{R}\frac{1}{1+n-i}-\sum_{i=1}^{R}\frac{1}{1+n-i}>0$$

But then because of (50) we only need to show

$$\frac{R}{n}\left(\sum_{i=1}^{R}\frac{1}{1+n-i}+1\right)-\sum_{i=1}^{R}\frac{1}{1+n-i}>0$$

or

$$R-\sum_{i=1}^{R}\frac{1}{1+n-i}(n-R)>0 \tag{51}$$

which is true because

$$R-\sum_{i=1}^{R}\frac{1}{1+n-i}(n-R)=\sum_{i=1}^{R}\left(1-\frac{n-R}{1+n-i}\right)=\sum_{i=1}^{R}\left(\frac{1+R-i}{1+n-i}\right)>0$$

c) Because $\frac{p''}{p_1}=-\frac{1}{f}$ we have

$$\frac{p''}{p_1}<-\frac{n}{R}p_1 \;\Leftrightarrow\; 1>\frac{n-R}{R}\sum_{i=1}^{R}\frac{1}{1+n-i}$$

or equivalently

$$R-(n-R)\sum_{i=1}^{R}\frac{1}{1+n-i}>0$$

which holds because of (51).

d) Let $n>R$ and use (49) to find

$$sign\left(p_{13}(f,f,n,R)\right)=sign\left(\sum_{i=1}^{R}\left(2R-n+\tfrac{R}{n}(1-i)\right)\right)$$
$$=sign\left(\underbrace{(2R-n)+\frac{R}{n}-\frac{R(R+1)}{2n}}_{=\psi(n)}\right)$$

But $n>R$ implies that $\psi'(n)=-1-\frac{R}{n^2}+\frac{R(R+1)}{2n^2}<0$. Moreover, we have $\psi(R)=\frac{R+1}{2}>0$ and $\psi(2R)=\frac{1}{2}-\frac{(R+1)}{4}<0$ because $R>1$. These findings imply that $p_1(f,f,n,R)$ must be hump-shaped in n.

$\blacksquare$

8.2 Proof of proposition 1

Use (3) to find

$$\sum_{j\in I} p_{ij} = \sum_{j\in I}\left(\sum_{A\in\mathcal{A}} P_{iA}\,1\,[j\in A]\right) = \sum_{A\in\mathcal{A}} P_{iA}\sum_{j\in I} 1\,[j\in A]$$
$$= \sum_{j\in I} 1\,[j\in A] = z$$

$\blacksquare$

8.3 Proof of lemma 1

The first part follows directly from (3). If $R \geq n$ then $p_j = 1$ for any active firm because there only is one attention set, namely I itself, which is perceived with probability one. Now suppose $n > R$. Then (3) implies $p_j = \sum_{A\in B_j} P_A(\mathcal{F})$ which gives $p_j = p_j(f_j, f_{-j})$. Now suppose that $p_j < 1$. Then $\sum_{A\in B_j} P_A(\mathcal{F}) < 1$ which by assumption 1 implies that $P_A(\mathcal{F})$ increases in f_j for all $A \in B_j$. But this implies that $p_j(f_j, f_{-j})$ increases in f_j.

$\blacksquare$

8.4 Proof of lemma 2

Suppose $n > R$. Because $f_g = \bar{f}$ for any $g \neq j$ we have $P_A(\mathcal{F}) = P_{A'}(\mathcal{F})$ for $A, A' \in B_j$. Let $s \equiv |B_j|$ and suppose $j \in A$. Then $\sum_{A\in B_j} P_A(\mathcal{F}) = sP_A(\mathcal{F})$ for $A \in B_j$. Lemma 1 and (3) imply that $sP_A(\mathcal{F}) = p(f, \bar{f}, n, R)$. Hence

$$\sum_{A\in B_j} P_A(\mathcal{F})V^j(Y_A)\Delta = V(y, \bar{y}, R)\Delta \sum_{A\in B_j} P_A(\mathcal{F})$$
$$= p(f, \bar{f}, n, R)V(y, \bar{y}, R)\Delta$$

If this is used in (5) we get (9). If $R \geq n$ then $p_j = 1$ and $V^j(Y_A) = V(y, \bar{y}, n)$ which then implies (10).

$\blacksquare$

Remark: Hence if we know $p(f, \bar{f}, n, R)$ then we also know $P_A(\mathcal{F})$ for any $A \in B_j$. But then we also can find $P_{A'}(\mathcal{F})$ for any $A' \notin B_j$. To see this let $B_{-j} \equiv \{A' \in \mathcal{A} : j \notin A'\}$. Then we have $P_{A'}(\mathcal{F}) = P_{A''}(\mathcal{F})$ for any $A', A'' \in B_{-j}$. Then

$$\begin{aligned}\sum_{A' \in B_{-j}} P_{A'}(\mathcal{F}) &= 1 - \sum_{A \in B_j} P_A(\mathcal{F}) \\ &= 1 - sP_A(\mathcal{F}) = 1 - p(f, \bar{f}, n, R)\end{aligned}$$

8.5 Proof of proposition 2

Let $n > R$. Define

$$G(f) \equiv \prod_{i=1}^{R} g(f, i) \tag{52}$$

with

$$g(f, i) \equiv \frac{(n-i)\bar{f}}{f + (n-i)\bar{f}}$$

a) Obviously $G(f) \in [0, 1]$ for $f, \bar{f} > 0$. Hence $p \in [0, 1]$. b) Relativity is obvious. c) With

$$-\frac{\partial G(f)}{\partial f} = -G(f) \sum_{i=1}^{R} \frac{\partial g(f, i)}{\partial f} \frac{1}{g(f, i)}$$

and

$$\begin{aligned}\frac{\partial g(f,i)}{\partial f} \frac{1}{g(f,i)} &= -\frac{(n-i)\bar{f}}{\left(f+(n-i)\bar{f}\right)^2} \frac{f+(n-i)\bar{f}}{(n-i)\bar{f}} \\ &= -\frac{1}{f+(n-i)\bar{f}}\end{aligned}$$

we get

$$p_1(f, \bar{f}, n, R) = -\frac{\partial G(f)}{\partial f} = G(f) \sum_{i=1}^{R} \frac{1}{f + (n-i)\bar{f}} > 0 \tag{53}$$

The same reasoning gives

$$p_2(f, \bar{f}, n, R) = -G(f) \sum_{i=1}^{R} \frac{\partial g(f, i)}{\partial \bar{f}} \frac{1}{g(f, i)}$$

But as

$$\frac{\partial g(f, i)}{\partial \bar{f}} = \frac{f(n-i)}{\left(f + \bar{f}(n-i)\right)^2} > 0$$

we have $p_2(f,\bar{f},n,R)<0$. Similarly, $p_3(f,\bar{f},n,R)<0$ as

$$\frac{\partial g(f,i)}{\partial n}=\frac{f\bar{f}}{\left(f+\bar{f}(n-i)\right)^2}>0$$

Suppose $n\geq R+1$. The (strong) monotonicity of p in R can be seen from

$$G(f,R+1)-G(f,R)=-G(f,R)\left(\frac{f}{f+\bar{f}\left(n-(R+1)\right)}\right)<0$$

Finally, I show that $p_{11}(f,\bar{f},n,R)<0$.

$$\begin{aligned}
p_{11}(f,\bar{f},n,R) &= -\tfrac{\partial^2 G(f)}{\partial f^2}\\
&= -\left(\tfrac{\partial G(f)}{\partial f}\sum_{i=1}^{R}\tfrac{\partial g(f,i)}{\partial f}\tfrac{1}{g(f,i)}+G(f)\sum_{i=1}^{R}\left(\tfrac{\partial^2 g(f,i)}{\partial f^2}g(f,i)-\left(\tfrac{\partial g(f,i)}{\partial f}\right)^2\right)\tfrac{1}{g(f,i)^2}\right)\\
&= -G(f)\left(\left[\left(\sum_{i=1}^{R}\tfrac{\partial g(f,i)}{\partial f}\tfrac{1}{g(f,i)}\right)^2-\sum_{i=1}^{R}\left(\tfrac{\partial g(f,i)}{\partial f}\tfrac{1}{g(f,i)}\right)^2\right]+\sum_{i=1}^{R}\left(\tfrac{\partial^2 g(f,i)}{\partial f^2}g(f,i)\right)\tfrac{1}{g(f,i)^2}\right)
\end{aligned}\tag{54}$$

But as $\left(\sum_{i=1}^{R}a_i\right)^2>\sum_{i=1}^{R}a_i^2$ for $a_i>0$ and $\frac{\partial^2 g(f,i)}{\partial f^2}>0$ the result follows immediately.

■

8.6 The ACF-CES example: calculations

Part I:

In case of (11) $V_1(y,\bar{y},R)=0$ implies

$$y^{\sigma}(R-1)(\sigma-1)=c\left(\bar{y}^{\sigma-1}+y^{\sigma-1}(R-1)\sigma\right)\tag{55}$$

which is equivalent to

$$y^{\sigma-1}(R-1)\left(y(\sigma-1)-c\sigma\right)=c\bar{y}^{\sigma-1}$$

which for $\bar{y}\geq c$ implies that $y>\frac{c\sigma}{\sigma-1}$ and hence also $y>c$. Applying the Implicit Function Theorem to (55) an rearranging gives

$$y'(\bar{y})=c\frac{(y\bar{y})^{2-\sigma}}{\sigma(R-1)(y-c)}>0$$

and

$$y'(R) = y\frac{y(1-\sigma)+c\sigma}{(R-1)(\sigma-1)\sigma(y-c)} < 0$$

Part II:

Suppose that $n > R$. Use $\bar{y} = y$ in (55). This gives (29). Moreover,

$$V(y, y, R) = (y-c)\frac{v}{yR} = \frac{v}{1+(R-1)\sigma} \tag{56}$$

Using this in (22) gives (30).

Part III:

$y'(R)$ follows directly from (29). From proposition 9 (see 8.11) we can deduce $sign(f'(\sigma)) = sign\left(V_{1\sigma}(p_{13} + \frac{p_1}{n}\right)$ and thus with the ACF $f'(\sigma) < 0$ follows because of $V_{1\sigma} = -\frac{(y-c)v(R-1)}{R^2y^2} < 0$. Similarly, we get $sign(n'(\sigma)) = sign\left(-V_{1\sigma}\left(np_1^2 + Rp'' - \omega\right)\right)$ and because of the ACF we have $n'(\sigma) < 0$.

8.7 Proof of proposition 3

Because the function $V(y, \bar{y}, z)$ is twice continuously differentiable in $y, \bar{y}$, the function $V_1(y, y, z)$ must be continuous in $y \in (c, y^{max})$ for any $z > 1$. Because of the boundary condition a) i) in assumption 5 the equation $V_1(y, y, z) = 0$ must then have a solution $y = y(z)$ for any $z > 1$ and $y \in (c, y^{max})$. Moreover, because of (18) the solution to $V_1(y, y, z) = 0$ must be unique for any given $z > 1$ (this is an index theorem result; see Hefti (2011b)). Hence if $R \geq n$ then there exists exactly one symmetric equilibrium (y, f) with $f = 0$ and $y = y(n) \in (c, y^{max})$.

If $n > R$ repeating the previous argument shows that there exists a unique solution $y = y(R) \in (c, y^{max})$ to $V_1(y, y, R) = 0$. Because $p_1(f, f, n, R)$ is continuous for $f \in (0, \infty)$ (a consequence of assumption 4) and because of the boundary conditions a) ii) in assumption 5, $V(y, y, z) \in (0, \infty)$ and (7) the equation

$$\psi(f) \equiv p_1(f, f, n, R)V(y, y, R)\Delta - C'(f) = 0$$

must have a solution $f = f(n, R, \Delta) \in (0, \infty)$. Condition (19) together with (7) implies that $\psi(f)$ is strictly decreasing in $f \in (0, \infty)$ which means that the solution must be unique. Consequently, there exists a unique vector (y, f) that solves (17).

(Second-order conditions) If $R \geq n$ then $V_{11}(y, y, n) < 0$ implies that second-order conditions of the representative firm's maximisation problem in (10) are satisfied at $y = y(n)$. Similarly, if $n > R$ then the second-order conditions of (9) evaluated at $y = y(R)$ and $f = f(n, R, \Delta)$ are satisfied because $\Pi_{11} = \frac{R}{n}V_{11}(y, y, R)\Delta < 0$, $\Pi_{22} = p_{11}(f, f, n, R)V(y, y, R)\Delta - C''(f) < 0$ and $\Pi_{12} = p_1(f, f, n, R)V_1(y, y, R)\Delta = 0$.

Thus the existence of exactly one symmetric equilibrium has been proved.

■

8.8 Proof of lemma 4

Let $n > R$. If we set $f = \bar{f} > 0$ then (52) gives $G(f) = \frac{n-R}{n}$. Using this in (53) gives (20). Moreover, (20) shows that the ACF satisfies assumption 5. Note that

$$\sum_{i=1}^{R} \frac{1}{1+n-i} = \sum_{j=0}^{R-1} \frac{1}{n-j} = H(n) - H(n-R) \tag{57}$$

where $H(x)$ is the x-th harmonic number. It is well known that the sequence $(H(x) - Ln(x))_{x \in \mathbb{N}_+}$ is monotonically decreasing and converges to the Euler-Mascheroni constant γ. Hence

$$H(n) - Ln(n) = \gamma + e(n)$$

where $e(n)$ is an error term with $\frac{de}{dn} < 0$. Thus the maximal error occurs at $n = 1$ and has $e(1) = 1 - \gamma < 1/2$. Because of the subtraction in (57) the constant γ cancels out. Further $e(n-R)$ is large if R is close to n. Thus the overall numerical error in setting $H(x) = Ln(x) + \gamma$ in (57), $e = e(n) - e(n-R)$ is bounded in absolute value by one and is small if n large and R small. Thus

$$\sum_{i=1}^{R} \frac{1}{1+n-i} \cong Ln(n) - Ln(n-R) = Ln\left(\frac{n}{n-R}\right)$$

By defining $x \equiv R/n$ we have

$$Ln\left(\frac{n}{n-R}\right) = Ln\left(\frac{1}{1-x}\right)$$

Applying a first-order Taylor expansion at $x_0 = 0$ then gives

$$Ln\left(\frac{1}{1-x}\right) = x + r_1(0, x)$$

with $|r_1(0,x)| \leq \frac{x^2}{1-x}$. Thus again if R is relatively small compared to n we can expect our approximation to perform numerically well. Thus the suggested approximation (21) is nummerically accurate whenever R/n is small.

■

8.9 Proof of proposition 4

Existence:

From (23) we see that profits are continuous in (y, f, n), because assumption 6 implies that $V(y, y, z)$ with $z = min\{R, n\}$ is continuous in n. Further we have $\Pi(\hat{y}, 0, 2) \geq 0$ by presupposition. $n \to \infty$ implies $n > R$. Hence $\lim_{n\to\infty} \Pi(y, f, n) = -F - C(f) < 0$. From proposition 3 we know that for any given $n \geq 2$ a solution (y, f) to (24) - (25) exists. Moreover, because of lemma 8 (see below) $(y, f) = (y(n), f(n))$ is a continuous function of n. Hence $\Pi(y(n), f(n), n)$ is a continuous function of n and consequently there must exist $n' \in [2, \infty)$ such that $\Pi(y(n'), f(n'), n') = 0$ which proves existence.

Uniqueness:

Suppose $n \geq> 2$ is given exogenously. Let $(y(n), f(n))$ denote the solution to (17). Because of proposition 3 and lemma 8 we now that $(y(n), f(n))$ is unique for any given n and that the vector $(y(n), f(n))$ is a continuously differentiable function of n if $n \neq R$. Define $\tilde{\Pi}(n) \equiv \Pi(y(n), f(n), n)$ where the function Π is given by (23). Then $\tilde{\Pi}(n)$ is a continuously differentiable function of n if $n \neq R$. But by the definition of $\tilde{\Pi}$ the vector $(y(n'), f(n'), n')$ is a symmetric equilibrium of the free entry game if and only if we have $\tilde{\Pi}(n') = 0$. If we have $\tilde{\Pi}'(n) < 0$ for all $n \neq R$ that satisfy $\tilde{\Pi}(n) = 0$ $(*)$ then there can be at most one $n \geq 2$ that solves $\tilde{\Pi}(n) = 0$. I now show that (28) is a sufficient condition for $(*)$.

Let $n < R$. Then

$$\tilde{\Pi}'(n) = \Delta \left(\underbrace{-\frac{V_2(y,y,n)V_{13}(y,y,n)}{\frac{\partial}{\partial y}\left(V_1(y,y,n)\right)}}_{<0} + \underbrace{V_3(y,y,n)}_{<0} \right) < 0$$

where $\tilde{\Pi}'(n)$ is evaluated at $\tilde{\Pi}(n) = 0$. Consequently, condition $(*)$ holds if $n < R$.

Now let $n > R$. Then

$$\tilde{\Pi}'(n) = -\frac{R}{n^2} - p_1(f, f, n, R)f'(n)$$

where $\tilde{\Pi}'(n)$ is evaluated at $\tilde{\Pi}(n) = 0$. But (see 8.10)

$$f'(n) = -\frac{p_{13}(f, f, n, R)\Delta V(y, y, R)}{\frac{\partial}{\partial f}\left(p_1(f, f, n, R)\right)\Delta V(y, y, R) - C''(f)}$$

This implies

$$\tilde{\Pi}'(n) = -\frac{R}{n^2} + p_1 \frac{p_{13}\Delta V}{\frac{\partial}{\partial f}\left(p_1\right)\Delta V - C''}$$

Then we have $\tilde{\Pi}'(n) < 0$ if

$$p_{13}\frac{n^2}{R} > \frac{\frac{\partial}{\partial f}\left(p_1\right)}{p_1} - \frac{C''}{\Delta V p_1} \tag{58}$$

Because $\frac{C''}{\Delta V p_1} \geq 0$ condition (28) is a sufficient condition for (58). Consequently, condition $(*)$ also holds if $n > R$.

Thus there only is one vector (y, f, n) that solves (24)-(26). Hence there only is one symmetric equilibrium.

■

Lemma 8 *Let* $R, n > 1$. *Suppose assumptions 5 and 6 are satisfied. Then the equilibrium vector* (y, f) *defined by* (17) *is a continuous function of* n *and continuously differentiable in* n *except at* $n = R$.

Proof: Because of proposition 3 $\exists!(y, f)$ that solves (17).

Case 1: $n < R$. Then $f = 0$ and hence $f'(n) = 0$. Because of (18) the IFT tells us that $y(n)$ is continuously differentiable in n. Consequently, (y, f) is continuously differentiable in n.

Case 2: $n > R$. Let $\tilde{J}$ denote the Jacobian of (16) with respect to (y, f):

$$\tilde{J} = \begin{pmatrix} \frac{\partial}{\partial y}\left(V_1(y, y, R)\right) & 0 \\ p_1(f, f, n, R)\Delta \frac{\partial}{\partial y}\left(V(y, y, R)\right) & \frac{\partial}{\partial f}\left(p_1(f, f, n, R)\right)\Delta V(y, y, R) - C''(f) \end{pmatrix}$$

But then

$$Det(\tilde{J}) = \frac{\partial}{\partial y}\left(V_1(y, y, R)\right)\left(\frac{\partial}{\partial f}\left(p_1(f, f, n, R)\right)\Delta V(y, y, R) - C''(f)\right) > 0$$

which by the IFT means that (y, f) is continuously differentiable in n.

Case 3: $n = R$. Because

$$y = \begin{cases} y(R) & R < n \\ y(n) & R \geq n \end{cases}$$

y is continuous in n but not differentiable at $n = R$. I now show that f is continuous in n also at $n = R$. We have

$$f = \begin{cases} f(n, R, \Delta) & R < n \\ 0 & R \geq n \end{cases}$$

Let

$$\psi(f) = p_1(f, f, n, R)\Delta V(y, y, R) - C'(f)$$

Then the Kuhn-Tucker necessary condition corresponding to the representative firm's opimization problem with respect to f is

$$\psi(f) + \lambda = 0 \qquad \lambda f = 0 \qquad f, \lambda \geq 0$$

Suppose $f > 0$. Then

$$\lim_{n \to R^+} \psi(f) = -C'(f) + \lambda$$

But then $f > 0$ implies $C'(f) > 0$ which implies $\lambda > 0$, a contradiction to optimality. Consequently, we must have $\lim\limits_{n \to R^+} f(n, R, \Delta) = 0$. Hence f is continuous at $n = R$.

■

8.10 Proof of proposition 5

The Jacobian J of (16) is given by

$$J = \begin{pmatrix} V'' & 0 \\ p_1 \Delta V_2 & p'' \Delta V - C'' \end{pmatrix}$$

where $V'' \equiv \frac{\partial}{\partial y}(V_1(y, y, R))$ and $p'' \equiv \frac{\partial}{\partial f}(p_1(f, f, n, R))$. Hence $Det(J) = V''(p''\Delta V - C'') > 0$. Let $\psi \equiv p_1(f, f, n, R)\Delta V(y, y, R) - C'(f, \alpha)$. Then by Cramer's rule we have

$$sign(y'(\chi)) = sign\left(\frac{-V_{1\chi}}{V''}\right) = sign\left(V_{1\chi}\right)$$

and

$$sign(f'(\chi)) = sign\left(\frac{p_1\Delta V_2 V_{1\chi} - \psi_\chi V''}{Det(J)}\right) = sign\left(p_1\Delta V_2 V_{1\chi} - \psi_\chi V''\right)$$

where $\chi \in \{R, n, \Delta, F, \alpha\}$. Then it is easy to see that $y'(R) < 0$, $f'(\Delta) > 0$ and $f'(\alpha) < 0$. If $\chi = R$ we get

$$sign(f'(R)) = sign(\underbrace{p_1 V_2 V_{13}}_{<0} - \underbrace{V''}_{<0}\underbrace{(p_{14}V + p_1 V_3)}_{?})$$

If $\chi = n$ we get $sign(f'(n)) = sign(p_{13})$. Hence the sign of $f'(R)$ and $f'(n)$ can only be determined under further assumptions.

■

8.11 Comparative statics under free entry: calculations

Assume parameter values such that an attention equilibrium ($n > R$) occurs endogenously. The Jacobian of (24) - (26) with respect to (y, f, n) is

$$J = \begin{pmatrix} V'' & 0 & 0 \\ p_1\Delta V_2 & p''\Delta V - C'' & p_{13}\Delta V \\ \frac{R}{n}\Delta V_2 & -p_1\Delta V & -\frac{R}{n^2}\Delta V \end{pmatrix} \tag{59}$$

Then after some manipulation

$$sign\left(Det(J)\right) = sign\left(\frac{p''}{p_1} - p_{13}\frac{n^2}{R}\right)$$

But then (28) implies that $Det(J) < 0$. Let $\chi \in \{R, \Delta, F, \alpha\}$.

Proposition 9 *Suppose an attention equilibrium ($n > R$) occurs endogenously. Then the comparative statics are given by*

$$y'(\chi) = \tfrac{V_{1\chi}\Delta^2 V^2}{Det(J)}\left(\tfrac{R}{n^2}p'' - p_1 p_{13} - \tfrac{\omega}{n^2}\right)$$

$$f'(\chi) = \tfrac{\Delta V}{Det(J)}\left(V''\left(\tfrac{R\Pi_{f\chi}}{n^2} + \Pi_\chi p_{13}\right) - V_{1\chi}\Delta\tfrac{RV_2}{n}\left(p_{13} + \tfrac{p_1}{n}\right)\right)$$

$$n'(\chi) = \tfrac{\Delta V}{Det(J)}\left(\tfrac{V_{1\chi}\Delta V_2}{n}\left(np_1^2 + Rp'' - \omega\right) - \tfrac{V''}{R}\left(Rp''\Pi_\chi + p_1 R\Pi_{f\chi} - \Pi_\chi\omega\right)\right)$$

where $\Pi_\chi \equiv \frac{\partial}{\partial\chi}\left(\frac{R}{n}V(y,y,R) - F - C(f,\alpha)\right)$, $\Pi_{f\chi} \equiv \frac{\partial}{\partial\chi}\left(p_1(f,f,n,R)\Delta V(y,y,R) - C'(f,\alpha)\right)$, $\omega \equiv \frac{C''(f,\alpha)R}{\Delta V} \geq 0$ *and* J *is the Jacobian* (59) *with* $Det(J) < 0$*.*

Proof:

Follows immediately by applying Cramer's rule to

$$J\begin{pmatrix} y'(\chi) \\ f'(\chi) \\ n'(\chi) \end{pmatrix} = -\begin{pmatrix} V_{1\chi} \\ \Pi_{f\chi} \\ \Pi_\chi \end{pmatrix}$$

■

8.12 Proof of lemma 5

First note that under assumption 7 and by the definition of $V(y,\bar{y},z)$ (see 3.3.1) we have a symmetric differentiable game that permits only interior equilibria and asserts the existence of a symmetric equilibrium. Any symmetric equilibrium must satisfy $V_1(y^*,y^*,z) = 0$ with $y \in Int(S_y)$. Strategic complementarity excludes asymmetric equilibria. Condition (18) corresponds to the condition that rules out multiple symmetric equilibria (see Hefti (2011b)).

■

8.13 Proof of proposition 6

Let $y_1 = ... = y_n = y$. Then by the symmetry of V^j we have

$$\frac{\partial V^j(Y_A)}{\partial y_j} = \frac{\partial V^j(Y_{A'})}{\partial y_j} = V_1(y,y,R)$$

for all $A, A' \in B_j$. Hence

$$\sum_{A\in B_j} P_A \frac{\partial V^j(Y_A)}{\partial y_j} = V_1(y,y,R)\sum_{A\in B_j} P_A$$

But lemma 5 implies the existence of a unique y^* with $V_1(y^*,y^*,R) = 0$. The inexistence of multiple symmetric equilibria for any non-degenerate distribution of P_A follows from (40). Assumption 7 implies that no boundary equilibria exist. To proof the claim of uniqueness I apply the index theorem (see Vives (1999), p. 48), which requires the Jacobian of $-D_y$ to have a positive determinant whenever $D_y = 0$ holds. The j-th row of this Jacobian is

$$\left(-\sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_1} \quad \cdots \quad -\sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j^2} \quad \cdots \quad -\sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_n} \right)$$

I now show that (41) implies

$$-\sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j^2} > \sum_{g \neq j} \left| \sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} \right| \tag{60}$$

The triangle inequality implies

$$\sum_{A \in B_j} P_A \sum_{g \neq j} \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} \right| = \sum_{g \neq j} \sum_{A \in B_j} P_A \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} \right| \geq \sum_{g \neq j} \left| \sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} \right|$$

Symmetry of $V^j(\cdot)$ implies for any $A \in B_j$ that $\exists g(A) \neq j \in A$ such that

$$\sum_{g \neq j} \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_g} \right| \leq (R-1) \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_{g(A)}} \right|$$

Hence a sufficient condition for (60) to hold is

$$-\sum_{A \in B_j} P_A \frac{\partial^2 V^j(Y_A)}{\partial y_j^2} > \sum_{A \in B_j} P_A (R-1) \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_{g(A)}} \right|$$

which is equivalent to

$$\sum_{A \in B_j} P_A \left(-\frac{\partial^2 V^j(Y_A)}{\partial y_j^2} - (R-1) \left| \frac{\partial^2 V^j(Y_A)}{\partial y_j \partial y_{g(A)}} \right| \right) > 0 \tag{61}$$

But (61) is obviously implied by condition (41). Symmetry further implies that whenever (60) holds then a similar statement holds for any other row of the $n \times n$ - Jacobian of $-D_y$. But then $-D_y$ must have a dominant diagonal whenever $D_y = 0$ and hence $-D_y$ must have a positive determinant[38] if $D_y = 0$. But then by the index theorem we conclude that the symmetric equilibrium must be unique.

[38]This follows because $Det(-D_y) = r_1 \cdot ... \cdot r_n$ and all eigenvalues r_k must have positive real parts.

$\blacksquare$

8.14 Proof of proposition 7

Because (41) is required to hold the attention distribution does not affect the equilibrium price by proposition 6. The equilibrium price is determined vy $V_1(y, y, R) = 0$. Thus we only need to show that for the equilibrium price y the attention competition does not generate any asymmetric equilibria. Because $\Pi_{f_j f_j} = \sum_{A \in B_j} \frac{\partial P_A(\mathcal{F})}{\partial f_j^2} V(Y_A) - C''(f_j)$ and $\Pi_{f_j f_g} = \sum_{A \in B_j} \frac{\partial P_A(\mathcal{F})}{\partial f_j f_g} V(Y_A)$ condition (42) implies that $-\Pi_{f_j f_j} + \Pi_{f_j f_g} > 0$. But this means that the slope of player j's best resonse function with respect to f_g must be larger than -1:

$$\frac{\partial f_j(f_1, ..., f_n)}{\partial f_g} = -\frac{\Pi_{f_j f_g}}{\Pi_{f_j f_j}} > -1$$

But then there cannot be any asymmetric attention equilibria (see Hefti (2011b)). Condition (19) rules out the possibility of multiple symmetric equilibria. Consequently, there can only be one equilibrium in the price-attention game, the symmetric equilibrium.

$\blacksquare$

8.15 Proof of corollary 3

For simplicity I set $j = 1$ and $g = 2$. The following argument obviously remains the same if any different players were used.

I use the following decomposition of B_1. Let $\hat{B}_2 \equiv \{A \in B_1 : 1 \in A \wedge 2 \in A\}$ and $\hat{B}_{-2} \equiv \{A \in B_1 : 1 \in A \wedge 2 \notin A\}$. Hence

$$\Pi^1 = \sum_{A \in \hat{B}_1} P_A V(Y_A) + \sum_{A \in \hat{B}_2} P_A V(Y_A) - C(f_1) - F$$

Thus according to proposition 7 we need to proof

$$-\frac{\partial^2 P_A}{\partial f_1^2} + \frac{\partial^2 P_A}{\partial f_1 \partial f_2} > 0 \tag{62}$$

for $f_1, ..., f_n \in (0, \infty)$ and any $A \in B_1$. Suppose $A \in \hat{B}_{-2}$. Then P_A takes on the form

$$P_A = \frac{f_1 f_h}{\sum f_j} \left(\frac{1}{\sum f_j - f_1} + \frac{1}{\sum f_j - f_h} \right)$$

where $3 \leq h \leq n$. Now suppose $A' \in \hat{B}_2$. Then

$$P_{A'} = \frac{f_1 f_2}{\sum f_j} \left(\frac{1}{\sum f_j - f_1} + \frac{1}{\sum f_j - f_2} \right)$$

In fact with $R = 2$ there can only be one such set: $A' = \{1, 2\}$. Differentiation yields:

$$-\frac{\partial^2}{\partial f_1^2} P_A + \frac{\partial^2}{\partial f_1 \partial f_2} P_A = \frac{1}{\left(\sum f_j - f_h\right)^2} - \frac{1}{\left(\sum f_j\right)^2} > 0$$

and

$$-\frac{\partial^2}{\partial f_1^2} P_{A'} + \frac{\partial^2}{\partial f_1 \partial f_2} P_{A'} = \frac{2\left(\sum f_j - f_1 - f_2\right)}{\left(\sum f_j - f_2\right)^3} > 0$$

Consequently, (62) is satisfied for any $A \in B_1$.

■

8.16 Proof of corollary 4

For $C(f) = \theta f^\eta$ we have

$$C''(f) = \frac{(\eta - 1)}{f} C'(f) \tag{63}$$

Using $V_2 = 0$, (63) and (16) in (44) gives

$$\frac{|p_{12}(f, f, R, n)|}{-\left(p_{11}(f, f, R, n) - \frac{(\eta-1)}{f} p_1(f, f, n, R)\right)} < 1 \tag{64}$$

Further calculate

$$p_{12}(f, f, R, n) = G(f) \left(\left(\sum_{i=1}^{R} \frac{1}{f(1+n-i)} \right)^2 - \sum_{i=1}^{R} \frac{n-i}{f^2 (1+n-i)^2} \right) \tag{65}$$

and from (54)

$$p_{11}(f,f,R,n) = -G(f)\left(\left(\sum_{i=1}^{R}\frac{1}{f(1+n-i)}\right)^2 + \sum_{i=1}^{R}\frac{1}{f^2(1+n-i)^2}\right) \tag{66}$$

With (20) and $V_2 = 0$ condition (44) is equivalent to

$$\begin{aligned}\left|\underbrace{\left(\left(\sum_{i=1}^{R}\frac{1}{1+n-i}\right)^2 - \sum_{i=1}^{R}\frac{n-i}{(1+n-i)^2}\right)}_{=K}\right| &< \left(\left(\sum_{i=1}^{R}\frac{1}{(1+n-i)}\right)^2 + \sum_{i=1}^{R}\frac{1}{(1+n-i)^2}\right)\\ &+(\eta-1)\sum_{i=1}^{R}\frac{1}{(1+n-i)}\end{aligned} \tag{67}$$

which obviously is rather satisfied for larger η. Thus I set $\eta = 2$. If $K \geq 0$ then (67) obviously holds. If $K < 0$ then (67) also holds because

$$\begin{aligned}0 &< 2\left(\sum_{i=1}^{R}\frac{1}{(1+n-i)}\right)^2 + \sum_{i=1}^{R}\frac{1-n+i}{(1+n-i)^2} + \sum_{i=1}^{R}\frac{1}{(1+n-i)}\\ &= 2\left(\sum_{i=1}^{R}\frac{1}{(1+n-i)}\right)^2 + \sum_{i=1}^{R}\frac{1-n+i}{(1+n-i)^2} + \sum_{i=1}^{R}\frac{1+n-i}{(1+n-i)^2}\\ &= 2\left(\sum_{i=1}^{R}\frac{1}{(1+n-i)}\right)^2 + \sum_{i=1}^{R}\frac{2}{(1+n-i)^2}\end{aligned}$$

■

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