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MONEY AND THE DISPERSION OF RELATIVE PRICES

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ABSTRACT

A price dispersion equation is tested with data from the German hyperinflation. The equation is derived from a version of Lucas' (1973) and Barro's (1976) partial information-localized market models. In this extension, different excess demand elasticities across commodities imply a testable dispersion equation, in which the explanatory variable is the magnitude of the unperceived money growth. The testing of this hypothesis requires two preliminary steps. First, a price dispersion series is computed using an interesting set of data. It consists of monthly average wholesale prices of 68 commodities ranging from foods to metals, for the period of January, 1921 to July, 1923. The next step is the delicate one of measuring unperceived money growth. This estimation implies the postulation of an available information set and also a function relating the variables in this set to money creation. The function used was based on considerations related to government demand for revenue. The model receives support from the empirical analysis although it is evident that unincluded variables have important effects on price dispersion.

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The existence of a positive correlation between absolute price level variability and the dispersion of relative prices was observed by, for example, Mills (1927), Graham (1930), and recently by Vining and Elwertowski (1976). In his study of U.S. price behavior during the period 1920-26, Mills says:

> We have not however exhausted the possibility of discovering a relationship between price level and dispersion. It may be that dispersion depends upon the violence of the price change, regardless of direction. (1927, p. 284).

Graham finds in the post-World War I German hyperinflation an additional dynamic element of price behavior

> It is clear that with the initiation of an upward movement in general prices a series of lags in individual prices developed, that these lags tended quickly to disappear when stability of general prices was reached on a new level, or when general prices fell, but that they were nevertheless progressively eliminated even though the general price level continued to rise. (1930, p. 175).

This observation suggests that unexpected events may have an important role in the determination of price dispersion. Individual prices disperse at the beginning of an upward swing in the price level when the acceleration is presumably unexpected. As inflation continues the element of surprise wanes and prices tend to converge.

The studies cited above failed to offer an economic rationale for the observed statistical correlation. Recently, a theoretical explanation of the relationship between price level variance and relative price dispersion was offered by Barro (1976). Using a localized markets framework of the type described by Phelps (1970) and employed by Lucas (1973), Barro links the dispersion of relative prices to the variance of the money supply. The key elements of this model are, on the one hand, individuals possessing incomplete current information, and on the other, demand and supply in each market reacting to relative prices as they are locally perceived. Thus, agents are confronted with the problem of determining whether locally observed price movements are caused by general inflation or by shifts in relative excess demand. The larger the variance of the money supply, the more likely are agents to attribute local price movements to general inflation rather than relative shifts. Accordingly, as the money variance rises local price changes induce smaller supply and demand responses--that is, excess demand becomes less elastic. Consequently, stochastic shifts to local excess demand produce larger changes in individual prices, so that the dispersion of prices across markets tends to increase with the variance of money. In this specification of the model, in which all markets have the same structure, dispersion is unrelated to the magnitude of realized money shocks.

This paper modifies Barro's framework by interpreting each location to be the market of a specific commodity, characterized by a particular excess demand elasticity. Because elasticities vary across markets, aggregate shocks affect each commodity price differently. Therefore, in this modified setup price dispersion is positively related to the magnitude of these shocks.

The model also predicts that systematic or perceived money growth is neutral with respect to price relationships. Accordingly, a money shock in this model is defined to be the component of money growth that is currently unobservable and cannot be inferred from currently available information. Whereas the quotation from Graham suggested that sudden--presumably unexpected--shifts in money growth cause dispersion, in this model, unexpected monetary expansion disperses prices only if it is, at least partially, currently unperceived.

The main task of this study is to evaluate this hypothesis with data from the German hyperinflation, a period of predominantly monetary disturbances. The period considered runs from January 1921 to July 1923. The vertiginous monetary expansion initiated in August 1923 differentiates the last phase of the hyperinflation and thus it is not included in the sample.

The theoretical framework, presented in section I, neglects some important facets of the hyperinflation. Specifically, it ignores the foreign exchange market and the sustained divergence between the internal and external values of the mark,[1] which obviously are related to relative prices. Also ignored in the main text are changes in the velocity of monetary circulation. However, in a brief discussion, some general conditions are given under which price dispersion is neutral with respect to velocity changes. Finally, the only aggregate exogenous disturbances assumed to be affecting the economy are periodic infusions of new money made by the government. Real aggregate shocks are ignored. Empirically, they are probably of relatively minor importance, and can be considered to be part of the error term in the estimated equations.

The testing of the dispersion equation requires two important preliminary steps. First, a price dispersion series is computed in Section II using an interesting set of data. It consists of monthly averages of 68 commodity prices ranging from foods to metals, for the period of January 1921 through July 1923 (31 months). Data were unavailable for the months prior to January 1921. Next, a money growth equation is set up and estimated in section III. To do so, both an information set available to agents economy-wide and a functional form relating this set to money creation - are postulated and discussed. The explained part of money growth in the estimated equation is taken as a measure of the perceived rate of monetary expansion. Correspondingly, the unexplained part is interpreted as the money growth rate that could not be perceived from the assumed information set. These figures, in conjunction with the price dispersion series, are used in section IV to test the price dispersion equation.

Parks (1977) has also tested a model of price dispersion using pre- and post-World War II U.S. data. In Parks's model dispersion is explained by changes in real income and the unexpected part of inflation, as measured by the innovation in the inflation rate. In this specification expected inflation and changes in real income are treated as exogenous variables. He finds a strong positive correlation between unexpected inflation and price dispersion. His tests also suggest a separate but smaller effect of the actual inflation rate.

The present paper estimates an equation that relates price change dispersion to the exogenous shocks affecting the economy, in this case unperceived monetary injections. An additional monetary variable that is theoretically relevant for price dispersion is the variance of money shocks. An estimate for this variance is obtained from the money growth analysis and is included in the estimation. The model receives significant support from the empirical analysis. In particular, the variable measuring unperceived money growth has substantial explanatory power for price dispersion. The results also make clear that unincluded variables have important effects on price dispersion. Some of these are briefly considered in Section V.

I. The Model

The economy consists of an arbitrarily large number of physically separated markets indexed by z. In each location a specific commodity is produced and traded. At each date t the agents, assumed to be risk neutral, exchange money only for the commodity being traded in the market in which they are currently located. At date t+1, agents change location at random and the process is repeated. Consider now the information set available to the agents. It contains not only lagged values of all relevant variables, but also current information which is limited to the local market price $P_t(z)$, and some economy-wide shared knowledge about current variables related to money creation. Actual money growth, however, includes a random term which is assumed unknown.

The supply and demand for commodity z assume the log-linear forms:

$$y_t^s(z) = \alpha^s(z)[P_t(z)-EP_t] + \varepsilon_t^s(z) \tag{1}$$

$$y_t^d(z) = -\alpha^d[P_t(z)-EP_t] + [M_t-EP_t] + \varepsilon_t^d(z) \qquad \alpha^s(z) > 0,\ \alpha^d > 0 \tag{2}$$

The operator E is the mathematical expectation taken conditional on the information available in market z at time t. For each commodity z, $P_t(z) - EP_t$ is the locally perceived relative price. $\varepsilon_t^s(z)$ and $\varepsilon_t^d(z)$ represent relative shifts to supply and demand respectively. The excess demand shift, $\varepsilon_t(z) \equiv \varepsilon_t^d(z) - \varepsilon_t^s(z)$ is assumed serially uncorrelated, normally distributed with zero mean and variance σ_ε^2. This variance is assumed to to be equal in all markets. For each z, $\alpha^s(z)$ is the short run relative price elasticity of supply. Disparity in the supply elasticities of different goods follows from heterogeneous production functions. However, in the long run relative prices are assumed fixed because of perfect substitutability on the supply side. The long run is measured here by one period, after which all suppliers can shift to other markets.

Looking one period ahead, all the markets offer the same mean price, but as shown below the corresponding variances differ according to the excess demand elasticities. Because agents are risk neutral, they are indifferent between the markets and thus they choose a market for the next period randomly. There is an additional point related to the ex-ante variability of the individual prices. Intuitively, one would expect a market with more price variance to be less desirable because local information would yield price level estimates of lower precision. However, as shown below, this turns out not to be the case.

On the demand side, the relative price elasticities are assumed constant across markets. The demand function also includes the term $M_t - EP_t$ which accounts for a real balance effect.

At the beginning of each period the stock of money in the economy is increased by transfers from the government to the public. This new money is assumed to be distributed equally across the markets. Within each market, however, the transfers are allocated randomly among a large number of agents. The rate of growth of the money stock, $m_t = M_t - M_{t-1}$, obeys

$$m_t = \sum_i \beta_i X_{it} + \tilde{m}_t \equiv g_t + \tilde{m}_t$$

where the X_{it}'s are variables (past or current) that can be observed in all locations and the β's are known coefficients. $\tilde{m}_t$ is a random variable with zero mean and variance σ_m^2. g_t is thus the expectation about money growth formed from all the economy-wide shared information. It can be considered the prior expectation. The posterior is formed using the additional information conveyed by the local price. Thus, while g_t is the same everywhere, the posterior expectation Em_t is conditional on location-specific information as well, and therefore varies across markets.

From equations (1) and (2) market clearing implies that

(3) $$P_t(z) = [1 - 1/(\alpha^s(z) + \alpha^d)]EP_t + [1/(\alpha^s(z) + \alpha^d)][M_t + \varepsilon_t(z)]$$

For each z, the sum $\alpha^s(z) + \alpha^d$ is the relative price elasticity of excess demand. Let $\lambda(z) \equiv 1/(\alpha^s(z) + \alpha^d)$. Each market has a constant $\lambda(z)$, but across markets, $\lambda(z)$ is distributed according to a given density function with average value λ and "variance" σ_λ^2. Consistent with the assumption that agents possess accurate knowledge about the structure of the economy, this distribution is assumed to be known.

Following Lucas (1973) and Barro (1976), the solution for prices in terms of exogenous variables is obtained using the method of undetermined coefficients. Given the log-linearity of the model, the solution for the aggregate price level has the form

(4) $$P_t = \Pi_1 M_{t-1} + \Pi_2 g_t + \Pi_3 \tilde{m}_t$$

Namely, the aggregate price level will be related to the current money stock, which is divided into its different components. Lagged values, if added to (4) yield zero coefficients. Since M_{t-1} and g_t are fully perceived at time t, taking the expectation of both sides yields

(5) $$EP_t = \Pi_1 M_{t-1} + \Pi_2 g_t + \Pi_3 E\tilde{m}_t$$

The conditional expectation of m_t is now computed. Rewrite (3) as:

$$\delta_t(z) = g_t + \tilde{m}_t + \varepsilon_t(z)$$

where

$$\delta_t(z) = [1/\lambda(z)]P_t(z) - [1/\lambda(z) - 1]EP_t - M_{t-1}$$

$\delta(z)$, the total disturbance affecting market z, is partly nominal and partly real. Agents perceive $\delta(z)$ and form their expectations about its components. Given the stochastic specification of m_t and $\varepsilon_t(z)$, the mean of the distribution of m_t conditional on $\delta(z)$ is

$$Em_t = g_t + \frac{\sigma_m^2}{\sigma_m^2 + \sigma_\varepsilon^2}[\delta_t(z) - g_t], \text{ or}$$

(6) $$Em_t = g_t + \frac{\sigma_m^2}{\sigma_m^2 + \sigma_\varepsilon^2}[\tilde{m}_t + \varepsilon_t(z)]$$

Observe that $\lambda(z)$ does not appear in (6). Since agents located in z know this elasticity, they are able to isolate the composite disturbance independently of $\lambda(z)$. Thus, while the <u>ex-ante</u> variance of prices depends on the particular elasticity (this follows from equation (26) below), the precision obtainable from the local information is independent of $\lambda(z)$. $P_t(z)$ would indeed convey less valuable information in higher price variance markets if the differential variability was due to a disparity in σ_ε^2. In this model, however, σ_ε^2 is the same across markets.

Substitute (6) into (5) and the resulting expression for EP_t into (3) to obtain

(7) $$EP_t = \Pi_1 M_{t-1} + \Pi_2 g_t + \Pi_3 \frac{\sigma_m^2}{\sigma_m^2 + \sigma_\varepsilon^2}[\tilde{m}_t + \varepsilon_t(z)]$$

$$(8)\qquad P_t(z) = [1-\lambda(z)]\{\Pi_1 M_{t-1} + \Pi_2 g_t + \Pi_3 \frac{\sigma_m^2}{\sigma_m^2+\sigma_\varepsilon^2}[\tilde{m}_t + \varepsilon_t(z)]\} +$$

$$\lambda(z)[M_{t-1} + g_t + \tilde{m}_t + \varepsilon_t(z)]$$

A new expression for the general price level can be computed from (8) by averaging with respect to the densities of $\lambda(z)$ and $\varepsilon_t(z)$

$$(9)\qquad P_t = (1-\lambda)[\Pi_1 M_{t-1} + \Pi_2 g_t + \Pi_3 \frac{\sigma_m^2}{\sigma_m^2+\sigma_\varepsilon^2}\tilde{m}_t] + \lambda[M_{t-1} + g_t + \tilde{m}_t]$$

Since equation (9) is identical to (4), the solution for Π_1, Π_2, and Π_3 is obtained by equating the corresponding coefficients in the two equations:

$$(10)\qquad \Pi_1 = 1$$

$$\Pi_2 = 1$$

$$\Pi_3 = \frac{\sigma_m^2 + \sigma_\varepsilon^2}{\sigma_m^2 + (1/\lambda)\sigma_\varepsilon^2}$$

Substituting (10) into (8), (9) and rearranging terms yields the solution for the individual commodity price and the average price level

$$(11)\qquad P_t(z) = M_{t-1} + g_t + \frac{\sigma_m^2 + \lambda(z)(1/\lambda)\sigma_\varepsilon^2}{\sigma_m^2 + (1/\lambda)\sigma_\varepsilon^2}[\tilde{m}_t + \varepsilon_t(z)]$$

$$(12) \qquad P_t = M_{t-1} + g_t + \frac{\sigma_m^2 + \lambda(z)(1/\lambda)\sigma_\varepsilon^2}{\sigma_m^2 + (1/\lambda)\sigma_\varepsilon^2} \tilde{m}_t$$

The resulting actual relative price is:

$$(13) \qquad P_t(z) - P_t = (1-\theta)\tilde{\lambda}(z)\tilde{m}_t \quad + [\theta + \lambda(z)(1 - \theta)]\varepsilon_t(z)$$

where $\tilde{\lambda}(z) \equiv \lambda(z) - \lambda$ and $\theta \equiv \frac{\sigma_m^2}{\sigma_m^2 + (1/\lambda)\sigma_\varepsilon^2}$.

The hypothesis expressed by equation (13) is that only the unperceived part of money growth can affect price relationships. Note that the realized values of the unperceived money growth appear in the relative price expression. This follows from the confusion between $\tilde{m}_t$ and $\varepsilon_t(z)$. Since in general $E\tilde{m}_t \neq \tilde{m}_t$, part of the money shocks in mistakenly perceived to be a shift in relative excess demand. The ensuing short run supply reactions differ across markets according to $\alpha^s(z)$, thus causing dispersion among actual prices. On the other hand, g_t is correctly identified as an aggregate disturbance and therefore cannot be confused with a relative shift of excess demand. The neutrality of perceived money follows from $y_t^s(z)$ and $y_t^d(z)$ being functions of the relative price, and from the one-to-one relationship between g_t and the expected price level (equations (7) and (10)). Given some value for g_t, the quantities along the supply and demand schedules are the same as before, for local nominal prices higher by an amount equal to the adjustment en EP_t--which equals g_t. Therefore, the market clears at a $P_t(z)$ which is higher by the same degree in all markets.[2]

The variance of relative prices at time t, defined as $\tau_t^2 \equiv \frac{1}{N}\sum_{z=1}^{N} [P_t(z)-P_t]^2$ where N is the 'very large' total number of markets in the economy, can be computed now from equation (13) [3]

$$\tau_t^2 = \{(1-\theta)^2\sigma_\lambda^2 + [\theta+\lambda(1-\theta)]^2\}\sigma_\varepsilon^2 + (1-\theta)^2\sigma_\lambda^2\tilde{m}_t^2 \quad (14)$$

An empirical test of this equation requires a measure of dispersion among prices or price indexes of different commodities. Mills (1927, Ch. III) discusses problems that the interpretation of this dispersion measure presents. For example, long run differential technological changes will cause prices to disperse over time. One would like to filter out such effects, because the focus here is on short run distortions caused by incomplete current information. The problem is alleviated by using rates of price change rather than price levels. Different trends do not affect the variation of price change dispersion over time--although alterations in these trends will. Thus, some of the long run relative price movements effect can be filtered from the dispersion measure. What remains can be considered to be captured by the random term in the dispersion equation.

The variance of the rates of change in individual prices is calculated using equation (13) and the equivalent for t-1. This variance, defined as

$$\gamma_t^2 \equiv \frac{1}{N}\sum_{z=1}^{N}\{[P_t(z)-P_{t-1}(z)] - (P_t-P_{t-1})\}^2,$$ follows as

$$\gamma_t^2 = (1-\theta)^2\sigma_\lambda^2\sigma_\varepsilon^2 + 2[\theta+\lambda(1-\theta)^2]\sigma_\varepsilon^2 + (1-\theta)^2\sigma_\lambda^2(\tilde{m}_t-\tilde{m}_{t-1})^2 \quad (15)$$

Equation (15) is the final price dispersion equation that is generated by the model. Because it deals with the dispersion of price changes, the appropriate monetary shocks variable is the magnitude of changes in $\tilde{m}_t$.

Consider next the implied relationship between the variance of money shocks and the dispersion of relative prices. Barro's theoretical result was that σ_m^2 is positively correlated with relative price variability. However, the effect of σ_m^2 is ambiguous in this extent version of the model, since it has different and opposite effects on the three terms in the σ_t^2 expression. The second term on the right hand side of (15) is the remainder of the expression when all markets are alike; that is when, as in Barro's case, all have the same excess demand elasticity ($\sigma_\lambda^2 = 0$). This term corresponds to his relative price variance, which depends positively on σ_m^2 when $0 < \lambda < 1$. This condition is the counterpart to Barro's assumption that substitution effects dominate wealth effects.

The first term accounts for the positive interaction between the diversity in elasticities and the strength of the relative shifts. A term of this sort would be included also in the dispersion expression under full current information. In the present case of partial information, the fraction $(1-\theta)$ appears here because agents typically underestimate the magnitude of the relative shifts, thus diminishing their effect on price dispersion. Because this underestimation increases with σ_m^2 the first term is negatively related to the money variance. The other, more interesting negative effect of σ_m^2 appears in the third term, namely in the coefficient of $(\tilde{m}_t - \tilde{m}_{t-1})^2$. If σ_m^2 increases--or more precisely when the public perceives it doing so--money disturbances are less confused with real shifts, implying that a given shock induces smaller dispersion. This effect is a relative price equivalent of Lucas's hypothesis about the link between the variance of the nominal disturbances and the slope of the Phillips curve.

In the testing of equation (15), reported in section IV, an attempt is made to capture the different effects of σ_m^2 and its net influence on price dispersion. However, the procedure adopted does not indicate that shifts in σ_m^2 have an important effect.

II. Construction of the Price Dispersion Series

This section reports the computation of a measure of price dispersion for the hyperinflation in Germany during the period January 1921-July 1923. The data set, consisting of 68 series of monthly averages of wholesale commodity prices, is obtained from the German statistical yearbook, issues of 1921/22 and 1923 (see reference under Statistisches Reichsamt). Other series from this source, some reported only until December 1921 (7 commodities) and others beginning only in January 1922 (21 commodities) were deleted in order not to introduce a bias due to changes in the sample size and composition.

Prices are quoted from commodity exchanges of several German cities.[4] Each series, however, originates in a single location. The 68 commodities include 27 food stuffs, 19 textiles and leathers, and 22 metals, oils and coals. They are not finished goods but materials in a rather raw state. Because weights for the different commodities are unfortunately not available, unweighted rates of price change are used. Hopefully, the wide range of commodities in the sample approximates the general relative price instability during that period.

The individual price rates of change are computed as the first difference of the logarithms of the prices. Average values and variances are then calculated using

Table I

Mean and Variance of Wholesale Rates of Price Changes.
Germany, February 1921-July 1923.

Month	ΔP_t	γ_t^2	Number of Commodities
1921			
February	-.09	.015	63
March	-.05	.011	66
April	-.02	.007	66
May	-.01	.026	66
June	.05	.032	67
July	.04	.034	66
August	.19	.081	66
September	.19	.033	68
October	.24	.034	68
November	.38	.043	68
December	-.03	.062	66
1922			
January	.05	.018	66
February	.12	.013	68
March	.25	.011	65
April	.12	.018	65
May	.04	.013	66
June	.09	.007	66
July	.36	.017	67
August	.70	.097	68
September	.40	.071	66
October	.68	.064	66
November	.78	.038	66
December	.22	.052	66
1923			
January	.70	.064	66
February	.69	.099	66
March	-.17	.041	65
April	.11	.018	65
May	.50	.039	66
June	.82	.048	62
July	1.25	.151	63

Source: Based on monthly average price data from Statistisches Jahrbuch fur das Deutche Reich 1921/22, p. 282-83 and 1923, p. 286-89.

$$\Delta P_t = \frac{1}{N}\sum_i \Delta P_{it}$$

$$\gamma_t^2 = \frac{1}{N}\sum_i (\Delta P_{it})^2 - (\Delta P_t)^2$$

where P_{it} is the price of commodity i and $\Delta P_{it} = \log P_{it} - \log P_{it-1}$. Table I contains the computed values of ΔP_t and γ_t^2. Due to missing observations, the actual number of commodities included in the calculations varies slightly from month to month. The third column in table I indicates the number of commodities for which both P_{it} and P_{it-1} are available.

III. Estimation of the Unperceived Part of Money Growth

Determination of the unperceived component of money growth during the hyperinflation requires a specification of the information set assumed to have been available to the public and the functional form for calculating the conditional expectation of money growth. Consider the expectation conditioned on economy-wide or "global" information g_t. This term was defined in section I to be the prior expectation, and is distinguished from the posterior expectation because it does not incorporate the additional information derived from local price observations.

This global information is assumed to consist of the current government spending in foreign exchange units, S_t, the current exchange rate, e_t, and one month lagged data on the money stock, price level and all other macroeconomic variables. Not included is government revenue from taxation and other sources, because this variable depends on the current level of economic activity and is unlikely to be preannounced and to be widely known contemporaneously. It is natural to assume that the part of government expenditure consisting of the reparations to the Allied Powers was known in foreign exchange terms. With respect to the other expenditure, the implication is that nominal spending was observable and could be readily converted given the exchange rate.

The prior expectation of money growth is derived from the government monthly budget constraint, namely

$$(16) \quad M_t^o - M_{t-1}^o = S_t^o e_t^o - \text{(other forms of nominal government revenue)}$$

The superscript o indicates that the variables are not in logs but in their original form. Equation (16) indicates that creation of high-powered money equals the part of nominal expenditure that is not financed in some other way. M_t^o would correspond here to the end of month money stock. The other forms of government finance are taxes, net sale of bills, gold sold to the public, etc.

If this other revenue comprised an approximately fixed proportion of total expenditure over time, the budget equation above could be expressed as

$$(17) \quad M_t^o - M_{t-1}^o = kS_t^o e_t^o + \text{random term}$$

where k $(0<k<1)$ is the average fraction of the expenditure financed by money issue.

The first attempt to generate a perceived money growth series was made using an equation of this type. Dividing (17) through by M_{t-1}^o, money growth appears linearly related to $S_t^o e_t^o / M_{t-1}^o$. The three variables in this ratio are assumed currently known, and therefore this specification is consistent with the notion that the conditional expectation can be formed using only currently observable variables.

However, a regression of this form,[5] including a constant, shows that k was probably not constant over the period. Specifically, the existence and pattern of residual serial correlation,[6] plus some additional considerations discussed below, suggest a nonlinear relationship between money issue and spending during that period.

Assuming then that the fraction k is not constant over time, the question is whether something can be said about its determinants. In order to suggest an answer to this question, rewrite equation (17) as

$$(18) \qquad \frac{M_t^o - M_{t-1}^o}{M_t^o} = k_t \frac{S_t^o}{M_t^o/e_t^o} + \text{random term}$$

Equation (18) preserves the positive correlation between money growth and the ratio of real expenditure to real cash balances, but unlike (17) the fraction k is now allowed to vary over time. It is now argued that k_t is itself correlated with M_t^o/e_t^o and S_t^o.

To examine this correlation, assume first that S_t^o is fixed at some value S^o. This level of real spending can be financed by different mixes of inflationary finance on the one hand, and taxation, debt issue, etc. on the other; where the amount to be collected by money issue is expressed as $k_t S^o$. In the usual diagram plotting the demand for real balances as a function of the inflation rate, $k_t S^o$ is measured in steady states by the area of the rectangle defined by μ--the inflation rate--and M_t^o/e_t^o.

Consider now an increment in μ. Real balances decline according to the money demand function, and the revenue from inflation, $k_t S^o$, increases as long as μ is below the rate that corresponds to a unitary demand elasticity for real balances. Because real spending is constant, k_t increases and hence the fraction of S^o financed by other means declines.

This shift from taxation to money issue can be viewed as the policy variable that brings about higher inflation rates. Classic works on the German hyperinflation, like those by Graham (1930) and Bresciani-Turroni (1937), describe an opposite direction of effect. Namely, the rate of depreciation of the currency had a negative effect on the real yield from taxation due to the interval of time existing between the occurrence of taxable transactions and the actual payment of the taxes.[7] The present discussion relies on the correlation between the fraction of expenditure financed by money issue and the inflation rate, rather than on a specific mechanism relating these two variables. This positive correlation implies that k_t and $1/(M_t^o/e_t^o)$ move in the same direction. However, this coincidental movement does not hold for all μ. When μ reaches the rate that maximizes the revenue from inflation k_t also reaches its highest level, and when it rises above that rate, k_t must decline. In other words, the correlation between k_t and $1/(M_t^o/e_t^o)$ turns negative in that range.

This decline in k_t implies that the revenue from other sources must go up. If tax collection and debt issue cannot be increased, (for example due to the negative effect of inflation mentioned above) spending must be partially financed by extraordinary means, such as sales of gold from the Central Bank's stock. In fact, the balance sheet of the German Central Bank shows that the stock of gold begins to decline significantly in April 1923, after being fairly stable since 1920.[8]

Given this behavior of k_t when S_t^o is constant, equation (18) can be approximated by the semilogarithmic form

$$\frac{M_t^o - M_{t-1}^o}{M_t^o} = \text{constant} + b' \log \left(\frac{1}{M_t^o/e_t^o}\right) + u' \qquad (19)$$

where b' is a positive coefficient, u' is a random term of zero mean, and the constant term is affected by the level of S^o. In this specification, the implicit fraction k_t increases along with $1/(M_t^o/e_t^o)$ at lower and middle ranges of this variable, but eventually declines when real balances fall below a certain value.[9]

Equation (19) acquires more empirical content if real spending is not fixed but rather fluctuates about a constant level. In fact, real government spending exhibits this pattern during the period under study. From equation (18), money growth varies positively with S_t^o. These fluctuations, which can be interpreted as temporary deviations from a "normal level", are assumed also to be correlated with the fraction k_t--while holding constant M_t^o/e_t^o, which captures the longer run trend in the finance mix. The assumption here is that given relatively high costs associated with temporary shifts in tax collection and debt issue, transitory movements in spending would be financed primarily by adjustments in money issue. A positive correlation between S_t^o and k_t would then result. However, a sufficiently high value of S_t^o could be presumed to require extraordinary finance of the sort previously mentioned, so that k_t might eventually decline.

Incorporating an approximation of this effect into equation (19) results in the following generalized expression,

$$\frac{M_t^o - M_{t-1}^o}{M_t^o} = a' + b' \log \left(\frac{1}{M_t^o/e_t^o}\right) + c' \log S_t^o + u'$$

which can be rewritten as

$$(20)\quad \frac{M_t^o - M_{t-1}^o}{M_t^o} = a' - b'[M_{t-1} + (M_t - M_{t-1}) - e_t] + c'S_t + u'$$

where variables without a superscript are again in logarithmic terms.

In order to proceed with the formulation of the prior expectation, it is convenient to replace the logarithmic growth rate $(M_t - M_{t-1})$ on the right hand side by the growth rate measured by $(M_t^o - M_{t-1}^o)/M_t^o$. While this rate is always lower than $M_t - M_{t-1}$, the gap widening the higher the growth rates, this effect can hopefully be captured approximately by the coefficients in the estimated equation. Then, equation (20) can be solved for $(M_t^o - M_{t-1}^o)/M_t^o$ to yield

$$(21)\quad \frac{M_t^o - M_{t-1}^o}{M_t^o} = \frac{a}{1+b} - \frac{b}{1+b}(M_{t-1} - e_t) + \frac{c}{1+b}S_t + \frac{1}{1+b}u_t$$

The prior conditional expectation is defined accordingly as

$$(22)\quad g_t \equiv (\frac{\hat{a}}{1+\hat{b}}) - (\frac{\hat{b}}{1+\hat{b}})(M_{t-1} - e_t) + (\frac{\hat{c}}{1+\hat{b}})S_t$$

The unperceived part of money growth $\tilde{m}_t$[10] is then computed by the difference between actual growth and g_t, namely

$$\tilde{m}_t = (M_t^o - M_{t-1}^o)/M_t^o - g_t$$

The coefficients in equation (22) are those which result from regressing $(M_t^o - M_{t-1}^o)/M_t^o$ on S_t and $(M_{t-1} - e_t)$. However, the exchange rate is not in general an exogenous variable in a money growth equation. A correlation between e_t and the error term u_t will exist via some unspecified condition for equilibrium in the foreign exchange market. Therefore, the cofficients in (22) do not correspond exactly to those in equation (21). This property is not a drawback. On the contrary, the bias in the estimated coefficients (relative to those in (21)) reflects the part of u_t that can be estimated from e_t. It therefore should be taken into account in calculating g_t.[11]

Turn now to the estimation of equation (21). There is a problem in matching the available data on money with those on prices for the German hyperinflation. Unlike the price series, which consist of monthly averages, the available data on the money stock until January 1923 are end of month figures.[12] From January 1923 onwards, four quotations per month are available. Thus, a proxy is constructed for the monthly average money stock. For the period January/July 1923 it contains averages of the beginning of month, end of month, and the three intermediate quotations available. Until December 1922, the monthly averages are approximated by linear interpolation of the end of month figures.

A further consideration arises. The estimation of equation (21) from monthly average data on the money stock, rather than end of month figures, means that both the current month's spending and that of the previous month should be considered. Spending financed by money issue during the previous month increases one-to-one the current monthly average stock but has, in general, a weaker effect on the prior month's average stock.[13] To account for this effect, lagged spending is incorporated into the framework of the semilogarithmic function in equation (32). First, define the variables S_t' and e_t' by $S_t' \equiv \log[\xi S_t^o + (1-\xi)\, S_{t-1}^o]$ and $e_t' \equiv \log\, [\xi S_t^o + (1-\xi)\, S_{t-1}^o]$. After substituting S_t' and e_t' for S_t and e_t in (32), ξ is estimated, simutaneously with the other coefficients in the equation, using a nonlinear maximum likelihood procedure under normally-distributed errors.[14]

The estimated nonlinear equation is

$$(23) \qquad \frac{M_t^o - M_{t-1}^o}{M_t^o} = \underset{(.071)}{.660} - \underset{(.007)}{.166}\,(M_{t-1} - e_t') + \underset{(.014)}{.115}\, S_t'$$

$$\hat{\xi} = \underset{(.11)}{.75}$$

$R^2 = .98$ D.W = 1.63 $\sigma = .026$ 33 observations

where the numbers in parenthesis are the standard errors of the coefficients. With respect to the number of observations, the starting month was taken as November 1920 in order to test lagged effects of monetary shocks on price dispersions. According to the argument of footnote 13, the value .75 for ξ suggests a pattern of spending that is biased towards the beginning of the month. In order to proceed with the analysis on the more familiar ground of linear equations estimation, ξ is assumed henceforth to equal .75. Given this value, the standard errors of the other coefficients, linearly estimated, are not materially different from those obtained above. In order of their appearence in (23) they are .068, .005, and .014. The standard error of the regression is now .0256. The other regression statistics and the coefficients remain obviously the same.

The pattern of the residuals from equation (23), which are reported in the appendix, table III, suggests that their variance increased during the sample period as inflation progressed. A method similar to that proposed by Glejser (1969), is adopted to correct for the apparent heteroscedasticity by assuming a specific model for the variance of the error term. In this procedure the variance is postulated to be determined by a set of variables $\{z_i\}$ in the linear form:

$$\sigma^2_{mt} = \sum_i \omega_i z_{it} \tag{24}$$

If the values of the true money shocks $\tilde{m}^*_t$ were available, one could use them as follows. Since the expectation of $\tilde{m}^{*2}_t$ is σ^2_{mt}, it follows that

$$\tilde{m}^{*2}_t = \sigma^2_{mt} + v_t$$

where v_t is of zero mean. Combining the last two equations yields

$$\tilde{m}^{*2}_t = \sum_i \omega_i z_{it} + v_t \tag{25}$$

Estimates of the coefficients in equation (24) could be obtained by regressing $\tilde{m}^{*2}_t$ on the z_i variables. The heteroscedasticity problem is also present here but it will be ignored in what follows.

The values of $\tilde{m}^*_t$ are however unknown; only the estimated residuals $\tilde{m}_t$ are available from the O.L.S. money growth equation. Since $\tilde{m}^2_t$ converges to $\tilde{m}^{*2}_t$ asymptotically, the variance estimated using $\tilde{m}^2_t$ values obtained from small samples will be biased. This bias is neglected hoping that it is of relatively small importance.

In order to proceed with the implementation of this procedure the set of the z_i variables must be specified. The presumption is that the same variables used to explain the growth rates are also correlated with the variances. Thus S_t', e_t' and M_{t-1} are candidates. The lagged squared residual $\tilde{m}^2_{t-1}$ is also included as an explanatory variable. It is presumably capturing the effect of serially-correlated omitted variables, and perhaps a direct correlation between σ^2_{mt-1} and the current variance. The estimated variance equation is

$$(26) \qquad \tilde{m}_t^2 = \underset{(.003)}{-.007} - \underset{(.00047)}{.00038}\, S_t' - \underset{(.00031)}{.00026}\, e_t' + \underset{(.00047)}{.00090}\, M_{t-1} - \underset{(.130)}{.281}\, \tilde{m}^2_{t-1}$$

$R^2 = .57$ D.W. = 2.5 $\sigma = .001$ 33 observations[15]

Nothing in this procedure for estimating the series of money variances guarantees that all the fitted values from equation (26) would be positive. Indeed, two of the fitted values have a negative sign. In order to use the estimated series as a measure of variances, these two negative values are replaced with the smallest positive value in the series. The series of the square roots of these estimates are reported in column (4) of table III.

Using this series as weights for the corresponding observations, equation (23) is reestimated with the following results:

$$(27) \qquad (M^o_t - M^o_{t-1})/M^o_t = \underset{(.047)}{.700} - \underset{(.006)}{.154}\, (M_{t-1} - e_t') + \underset{(.012)}{.094}\, S_t'$$

$R^2 = .95$ D.W. = 1.8 $\sigma = 1.01$ 33 observations

Observe that the coefficient of S_t' here is somewhat lower than in the O.L.S. equation and that the coefficient of $(M_{t-1} - e_t')$ is somewhat higher.

The general form of the equation is, however, robust to this transformation of the data.

The next step is to test the stability of the coefficients in equation (27) across two subperiods. Stability of the coefficients has particular relevance here. If the equation is approximately stable, it seems easier to assume that it was known from the beginning of the period and that perceptions about money growth were formed using the same equation during the entire sample.[16] The period is divided into an approximately non-accelerating money supply period until May 1922 and an accelerating phase beginning in June 1922. An F-test applied to these two sub-periods yields the statistic $F^3_{27} = 1.7$, with a corresponding 5 percent critical value of 3.0. Therefore, the hypothesis of stable coefficients across these two sub-periods cannot be rejected at the 5 percent significance level.

A regression in which the coefficients of M_{t-1} and e'_t are unconstrained produce coefficients of similar magnitude for the two variables. The F-test for the linear constraint of equal coefficients yielded the statistic $F^1_{29} = 2.3$, where the 5 percent critical value is 4.2.

V. Empirical Test of the Dispersion Equation

The tests of the price dispersion model in equation (15) are performed using the dispersion series computed in section II and the unperceived monetary shocks as measured by the residuals in equation (27).

For convenience equation (15) is rewritten here

$$(15) \qquad \gamma_t^2 = \{(1-\theta)^2\sigma_\lambda^2 + 2[\theta + \lambda(1-\theta)]^2\}\, \sigma_\varepsilon^2 + (1-\theta)^2\sigma_\lambda^2(\tilde{m}_t - \tilde{m}_{t-1})^2$$

Importantly, this equation has a simple linear form--and can therefore be tested by an ordinary least squares procedure--only under constant money and relative shocks variances. However, the analysis of money growth in the previous section suggested that σ_m^2 increased during the hyperinflation. If this is indeed the case, it would not be appropriate to test the model with a specification that relates price dispersion to money shocks with a constant coefficient.

Two different procedures are adopted here to deal with the possibility of a changing money variance. The first uses a linear approximation in which the variance of money--as measured by the $\hat{\sigma}_m^2$ series from section III--is kept constant by including it additively in the equation. As discussed in section I, σ_m^2 has different and opposite effects on γ_t^2, and therefore on a a-priori basis the coefficient of $\hat{\sigma}_m^2$ could take either sign. The other attempt is to estimate (15) as a nonlinear equation. The results of this procedure, reported later in the section, are quite poor.

The estimated equation in which $\hat{\sigma}_m^2$ is added linearly is:

$$\text{(28)} \qquad \gamma_t^2 = \underset{(.010)}{.033} + \underset{(3.2)}{17.4}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(8.6)}{15.8}\,\hat{\sigma}_{mt}^2$$

$R^2 = .59$ D.W. = 1.2 $\sigma = .022$ 30 observations

The monetary shocks appear to have considerable explanatory power for price dispersion. The coefficient of $\hat{\sigma}_{mt}^2$ is negative, and therefore suggests a dominant Lucas-type effect of the money variance on price dispersion. That is, the degree of dispersion associated with given shocks diminishes the higher their variance. The explanatory power of $\hat{\sigma}_{mt}^2$ however, is fairly low; its coefficient is significantly different from zero at the 5% level but fails to be so at the 2.5% level.

Theoretically, the one month lagged money variance $\hat{\sigma}^2_{mt-1}$ belongs also in the equation. However, when included, its coefficient is insignificant with a t-statistic of .8.

The Durbin- Watson statistic indicates autocorrelated residuals, which may be caused by omitted real variables (like changes in the pattern of government spending,[17] in income distribution, etc.) that are serially correlated, or by the fact that the $\tilde{m}_t$ variable includes an estimation error. In order to check whether the degree of significance of the estimated coefficients in (28) is affected by this autocorrelation, the equation is reestimated using the Cochrane-Orcutt technique. The results are quite similar to those obtained before,

$$(29) \qquad \Upsilon_t^2 = \underset{(.008)}{.038} + \underset{(2.9)}{16.8}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(9.8)}{18.5}\,\hat{\sigma}^2_{mt}$$

$R^2 = .67$ D.W. = 2.2 $\sigma = .020$ 29 observations.

$\hat{\rho} = \underset{(.17)}{.36}$

Including $\hat{\sigma}^2_{mt-1}$ in this regression yielded at t-statistic of only 1.0 for its coefficient. The possibility of lagged effects of monetary shocks on price dispersion was explored by including the variable $(\tilde{m}_{t-1} - \tilde{m}_{t-2})^2$ in the equation, but the estimated coefficient was found statistically insignificant. The t-ratio was 0.3 in the O.L.S. regression and 1.4 using the Cochrane-Orcutt technique.

The next step is to see whether the dispersion equation is stable over the entire period. In fact, the inclusion of the $\hat{\sigma}^2_{mt}$ variable is an attempt to control one source of instability in the coefficients. In order to carry

Table II

Estimated Price Change Dispersion Equations

Feb. 1921 - May 1922

O.L.S.

$$\underset{(.012)}{.030} + \underset{(15.8)}{18.3}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(32.2)}{28.2}\,\hat{\sigma}^2_{mt}$$

$R^2 = .22$ D.W. = 1.2 $\sigma = .019$

Cochrane-Orcutt

$$\underset{(.012)}{.035} + \underset{(13.5)}{27.7}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(32.0)}{43.1}\,\hat{\sigma}^2_{mt}$$

$R^2 = .47$ D.W. = 2.5 $\sigma = .016$

Feb. 1921 - Dec. 1922

O.L.S.

$$\underset{(.009}{.024} + \underset{(12.1)}{34.4}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(19.0)}{7.3}\,\hat{\sigma}^2_{mt}$$

$R^2 = .29$ D.W. = 1.1 $\sigma = .022$

Cochrane-Orcutt

$$\underset{(.012)}{.041} + \underset{(9.0)}{30.7}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(21.2)}{35.1}\,\hat{\sigma}^2_{mt}$$

$R^2 = 53$ D.W. = 2.2 $\sigma = .018$

Feb. 1921 - July 1923

O.L.S.

$$\underset{(.010)}{.033} + \underset{(3.2)}{17.4}\,(m_t - m_{t-1})^2 - \underset{(8.6)}{15.8}\,\hat{\sigma}^2_{mt}$$

$R^2 = .59$ D.W. = 1.2 $\sigma = .022$

Cochrane-Orcutt

$$\underset{(.008)}{.038} + \underset{(2.9)}{16.8}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(9.8)}{18.5}\,\hat{\sigma}^2_{mt}$$

$R^2 = .67$ D.W. = 2.2 $\sigma = .020$

June 1922 - July 1923

O.L.S.

$$\underset{(.012)}{.054} + \underset{(3.6)}{17.5}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(11.6)}{28.1}\,\hat{\sigma}^2_{mt}$$

$R^2 = .70$ D.W. = 1.6 $\sigma = .022$

Jan. 1923 - July 1923

O.L.S.

$$\underset{(.026)}{.036} + \underset{(3.2)}{16.6}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(15.0)}{16.4}\,\hat{\sigma}^2_{mt}$$

$R^2 = .87$ D.W. = 3.1 $\sigma = .020$

out this test the sample is divided first after May 1922, estimating the equation separately for the two subperiods. This partition of the sample is the same one adopted previously to test the stability of the money growth equation. Then, the exercise is repeated dividing the sample at the end of 1922. The aim of this partition is to see how the model performs after removing from the sample the 7 months of 1923, which had a much more unstable monetary growth.

The results of these regressions, reported in table II, can be summarized as follows. When the sample is divided in May 1922 (column (1)), the results for the first subperiod are fairly weak. Both coefficients are insignificant in the O.L.S. equation. Using the Cochrane-Orcutt technique the coefficient of $(\tilde{m}_t - \tilde{m}_{t-1})^2$ turns out significant at the 5% level, although that corresponding to $\hat{\sigma}^2_{mt}$ is still insignificant. During the second subperiod--from June 1922 to July 1923--the statistical performance of the equation is much stronger.

Column (2) reports the equations estimated for the periods through and after December 1922. Observe that the removal of the 1923 portion of the sample worsens the performance of the model as judged by the size of the t-ratios. However, the coefficient of the monetary shocks still remain quite significant.

Formal F-tests fail to reject the hypothesis of stable coefficients across the mentioned subperiods. When the sample is divided in May 1922, the resulting statistic in $F^3_{24} = 1.8$, while the 5% critical value is 3.0. Partitioning the sample at the end of 1922 yields the statistic of only 0.8.

The other approach adopted to test the dispersion equation was to treat it as a nonlinear relationship. When σ^2_m is changing over time, equation

(15) generalizes to

$$(30)\qquad \gamma_t^2 = \sigma_\varepsilon^2 \sigma_\lambda^2 \left\{ \left[\frac{\sigma_\varepsilon^2}{\lambda\sigma_{mt}^2+\sigma_\varepsilon^2}\right]^2 + \left[\frac{\sigma_\varepsilon^2}{\lambda\sigma_{mt-1}^2+\sigma_\varepsilon^2}\right]^2 \right\} + \sigma_\varepsilon^2 \left\{ \left[\frac{\lambda\sigma_{mt}^2+\lambda\sigma_\varepsilon^2}{\lambda\sigma_{mt}^2+\sigma_\varepsilon^2}\right] \left[\frac{\lambda\sigma_{mt-1}^2+\lambda\sigma_\varepsilon^2}{\lambda\sigma_{mt-1}^2+\sigma_\varepsilon^2}\right] \right\} + \sigma_\lambda^2 \left[\frac{\sigma_\varepsilon^2}{\lambda\sigma_{mt}^2+\sigma_\varepsilon^2}\tilde{m}_t - \frac{\sigma_\varepsilon^2}{\lambda\sigma_{mt-1}^2+\sigma_\varepsilon^2}\tilde{m}_{t-1}\right]^2$$

As discussed in section I, the third term in this equation reflects the negative effect of σ_m^2 on the impact of monetary shocks. The second term corresponds to Barro's (1976) relative price variance expression that is affected positively by σ_m^2 when $0<\lambda<1$. Recall that λ is defined as the average of $1/[\alpha^s(z) + \alpha^d]$ across markets, and $\alpha^s(z)+\alpha^d$ is the excess demand elasticity of commodity z. Using again the $\hat{\sigma}_m^2$ series, the parameters of equation (30)-- λ, σ_ε^2 and σ_λ^2 --are estimated using a nonlinear least-squares procedure with fairly weak results. The additional structure given to the equation seems to be rejected by the data, as judged by a higher sum of squared errors than in the linear equation.

Observe that a value of zero for λ reduces the equation to the linear form of equation (15), in which the money variance is constant. The estimated value for this parameter was .056 with a standard error of .065.[18] The interpretation of this result is not that the average $1/[\alpha^s(z)+\alpha]$ is likely to be close to zero. Instead, it suggests that the detailed specification of equation (30) is too stringent. For example, if the variance of the relative excess demand shifts, σ_ε^2 , also changes over time, this is more of a problem in this approach, since σ_ε^2 itself is being estimated as a constant.

V. Actual money growth and inflation in the dispersion equation

In this section additional variables that were mentioned in the literature as being related to price dispersion are tried in the equation. There is no rigorous theoretical justification for their inclusion. Thus, only loose verbal explanations are given. Also, the variables related to inflation are clearly not exogenous and therefore any observed correlation cannot imply causality.

Actual money growth and price dispersion: A variable that can be considered exogenous and, if one assumption of section I is violated, in principle also relevant for price dispersion is the actual money growth. Changes in the money stock can affect the dispersion of prices (even when perceived) if the new money is spread unevenly across the economy, thereby affecting relative demand in different sectors. This type of effect was discussed by Cairnes (1873) with respect to gold discoveries. In the framework of the present model this type of effect could be represented by changes in the relative excess demand variance σ_ε^2 . Since here the focus is on price change dispersion, the corresponding variable in this context is the change in the growth rate, or the degree of acceleration/deceleration in the money stock.

In order to test this sort of effect, and also to see whether $(\tilde{m}_t - \tilde{m}_{t-1})^2$ is only a proxy for changes in actual money growth, price dispersion was regressed on $[(M_t^o - M_{t-1}^o)/M_t - (M_{t-1}^o - M_{t-2}^o)/M_{t-1}]^2$ with the following results

$$\Upsilon_t^2 = \underset{(.006)}{.033} + \underset{(.6)}{2.1}\left(\frac{M_t^o - M_{t-1}^o}{M_t^o} - \frac{M_{t-1}^o - M_{t-2}^o}{M_{t-1}^o}\right)^2$$

$R^2 = .33$ D.W. = 1.5 $\sigma = .028$ 30 observations

The monetary acceleration /deceleration variable has a statistically significant correlation with price dispersion. Remarkably however, its explanatory power vanishes when $(\tilde{m}_t - \tilde{m}_{t-1})^2$ and $\hat{\sigma}^2_{mt}$ are also included in the regression. The equation including the three variables is

$$\Upsilon_t^2 = .034 + \underset{(4.0)}{16.6}(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(9.0)}{16.5}\hat{\sigma}^2_{mt} + \underset{(.77)}{.25}\left(\frac{M_t^o - M_{t-1}^o}{M_t^o} - \frac{M_{t-1}^o - M_{t-2}^o}{M_{t-1}^o}\right)$$

$R^2 = .59$ D.W. = 1.3 $\sigma = .022$ 30 observations

This result denies the existence of any effect of relative demands following the introduction of new money during this period. It supports the hypothesis that money affects relative prices only if it is currently unperceived.

Inflation and price dispersion: In his analysis of price behavior during the hyperinflation, Graham describes a positive correlation between the acceleration of price level and the dispersion of prices. Mills's findings in his study of U.S. prices suggest that dispersion is positively correlated with both acceleration or deceleration of the price level.

If the acceleration/deceleration in the price level is related to an unperceived monetary expansion or contraction, the theory tested here predicts that the correlation mentioned above should be captured by a variable measuring unperceived money growth. To test whether this is the case here, the variable $(\mu_t - \mu_{t-1})^2$ is also included in the equation--where μ_t is the inflation rate from month t-1 to month t computed from the wholesale price index (see table III). The results suggest that there is a separate correlation between $(\mu_t - \mu_{t-1})^2$ and price dispersion. The equation estimated by O.L.S. is

$$\underset{}{\gamma_t^2} = \underset{(.005)}{.032} + \underset{(3.0)}{18.6}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(8.7)}{24.2}\,\hat{\sigma}^2_{mt} + \underset{(.031)}{.074}\,(\mu_t - \mu_{t-1})^2$$

$R^2 = 67$ D.W. = .9 $\sigma = .020$ 30 observations

Given the low D.W.-statistic, the equation was reestimated by Cochrane-Orcutt,

$$\gamma_t^2 = \underset{(.008)}{.033} + \underset{(2.6)}{19.5}\,(\tilde{m}_t - \tilde{m}_{t-1})^2 - \underset{(9.0)}{25.3}\,\hat{\sigma}^2_{mt} + \underset{(.022)}{.077}\,(\mu_t - \mu_{t-1})^2$$

$R^2 = .77$ D.W. = 2.00 $\sigma = .017$ 29 observations

$\hat{\rho} = \underset{(.16)}{.49}$

Not only does $(\mu_t - \mu_{t-1})^2$ have a statistically significant correlation with price dispersion, but its inclusion in the equation also sharpens the performance of the original two variables. Possible explanations of this correlation could be related for example, to income redistribution following from unanticipated inflation, or substitution between money and certain commodities as stores of value when their relative costs changes.

Sheshinski and Weiss (1977) consider a model of a monopolistic firm in which costs involved in changing the price of the commodity produced generate discrete periodic price adjustments whose magnitude increases with the inflation rate. They suggest that if the timing of these adjustments is

Sheshinski, E. and Weiss, Y., "Inflation and Costs of Price Adjustment," *The Review of Economic Studies*, 137(June 1977): 287 - 303.

Sonderhefte zur Wirtschaft und Statistic: *Zahlen zur Geldenwertung in Dutchland 1914 bis 1923*, Berlin 1925.

Statistisches Reichsamt: *Statistisches Jahrbuch fur das Deutche Reich*, 1921/22 and 1923.

Stigler, G. J. and Kindahl, J.K., *The Behavior of Industrial Prices*, New York 1970.

Vining, D.R. and Elwertowski, T.C., "The Relationship between Relative Prices and the General Price Level," *American Economic Review*, 66(September 1976): 699 - 708.

Footnotes

[1]See for example Bresciani-Turroni.

[2]In order to consider the effects of changes in the velocity of money circulation on price dispersion, I worked out a similar simple model in which there is some current public information about future money growth. This information may be conveyed by political or military events that are believed to have implications for the future state of government finances. The prediction of future monetary expansion, which generates inflationary expectations, affects the velocity of circulation in the current period.

With respect to relative prices, if the knowledge about future money growth is shared economy-wide, they will be unaffected by the change in velocity. This neutral effect follows from the same mechanism determining the neutrality of perceived money. Since all agents share the same knowledge and are assumed to use it in the same model to predict its effects, they will equally adjust their EP_t according to the change in velocity taking place. A "one-time-jump" in all prices therefore occurs, without affecting their dispersion.

[3]For this computation, since $\varepsilon_t(z)$ and $[\varepsilon_t(z)]^2$ are independent of $\lambda(z)$, $[\lambda(z)]^2$ and $\tilde{\lambda}(z)$, the following equalities are used:

$$(1/N)\Sigma\varepsilon_t(z).\lambda(z) = 0.\lambda = 0, (1/N)\Sigma[\varepsilon_t(z)]^2[\tilde{\lambda}(z)]^2 = \sigma_\varepsilon^2\sigma_\lambda^2, \text{ and}$$

$$(1/N)\Sigma[\varepsilon_t(z)]^2[\lambda(z)]^2 = \sigma_\varepsilon^2(\sigma_\lambda^2 + \lambda^2).$$

[4]Given this source, these data do not present the problem of reported wholesale price data in the U.S., discussed by Stigler and Kindahl (1970), that they do not always reflect discounts from list prices.

[5]The estimated O.L.S. equation is the following (see below for the inclusion of the lagged spending variable).

$$\frac{M^o_t - M^o_{t-1}}{M^o_{t-1}} = \underset{(.019)}{-.049} + \underset{(.033)}{.317}\,\frac{S^o_t e^o_t}{M^o_{t-1}} + \underset{(.089)}{.381}\,\frac{S^o_{t-1} e^o_{t-1}}{M^o_{t-1}}$$

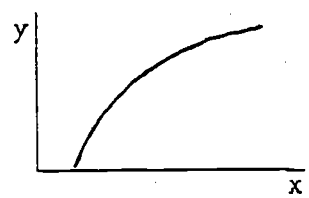
$R^2 = .95$ D.W. = 1.0 $\sigma = .067$

[6]The residuals are generally negative at the beginning and the end of the period, approximately the low and high values of money growth, and generally positive for the rest of the sample.

[7]See e.g., Bresciani-Turroni, p. 66 and Graham, p. 44.

[8]See Sonderhefte zur Wirtschaft und Statistic p.53.

[9]The graph of the semilogarithmic function $y = \alpha + \beta \log(x)$ is



[10]The unperceived growth $\tilde{m}_t$ does not correspond exactly to the error term u_t. On this point, see below.

[11]For example, in the general linear model $y = X\beta + u$, where the variables in X are correlated with u, the estimated vector of coefficients is

$$\hat{\beta} = (X'X)^{-1}X'y = (X'X)^{-1}X'(X\beta+u), \text{ or}$$

$$\hat{\beta} = \beta + (X'X)^{-1}X'u$$

where $(X'X)^{-1}X'u$ is the regression coefficient of u on X. The prediction of y_t given the values of the vector x_t is accordingly

$$y_t = x'_t[\beta + (X'X)^{-1}X'u]$$

i.e., it is composed of the systematic part $x'_t\beta$, plus the conditional expectation of u_t given x_t.

[12]Another problem stems from the form of the prior expectation, which as it stands requires the use of end of month money stocks. See the discussion below.

[13]This effect can be seen by considering first a case where each month's spending is spread evenly over the month, and say, it is financed only by the issue of new money, In this case the money stock grows linearly at, in general, different rates within each monthly period. Then, if $M^o_{t-2,end}$ denotes the money stock at the end of month t-2, the montly average for t-1 equals $M_{t-2,end} + \frac{1}{2} S^o_{t-1}e^o_{t-1}$, and that corresponding to month t equals $M^o_{t-2,end} + S^o_{t-1},e^o_{t-1} + \frac{1}{2} S^o_t e^o_t$. Thus the increase in the monthly average from t-1 to t equals $\frac{1}{2} S^o_{t-1}e^o_t + \frac{1}{2} S^o_t e^o_t$. Namely, spending evenly spread over each month would imply equal weights for current and lagged spending in the money growth equation.

Alternatively if spending is concentrated at the beginning of the month, the relative weight of lagged expenditure would be lower. At the extreme, for example, if all spending is made only on the first day of each month both t-1 and t monthly averages increase equally by the amount of the t-1 expenditure. In this case lagged spending does not belong in the money growth equation.

[14]This procedure is from the TSP Regression Package.

[15]In order to estimate the 33 variances needed to reestimate equation (34), the $\breve{m}_t$ series were obtained from running the money growth equation after adding the additional observation of October 1920.

[16]Estimation of a money growth equation in a similar context, using the entire sample (for U.S., 1941 - 1973), was discussed and performed in Barro (1977).

[17]The magnitude of changes in the amount of real government spending, however, does not have any significant explanatory power.

[18]$\hat{\sigma}_\lambda$ was 17.7 with a standard error of 8.4 and $\hat{\sigma}^2_\varepsilon$ was .0008 with a standard error of .0004.

[19]However, the empirical implications of Sheshinski and Weiss's analysis for price dispersion do not seem clear to me. An ambiguity arises because of the probably positive effect of inflation on the frequency of price changes that they derived. If the length of the observation period is kept constant, a higher frequency of price change may diminish the measured dispersion of price changes. The possibility of a negative effect of inflation on price dispersion in this framework can be seen in the following example. Assume that the optimal frequency of price adjustments for all firms is two months, and that part of the firms adjust their prices during odd-numbered months and the rest during even-numbered months. Using monthly data, dispersion of price changes will depend on the magnitude of price changes corresponding to the group of firms currently adjusting prices. Now assume that inflation increases; as a consequence the magnitude of price adjustments goes up, and also the optimal frequency is increased--say, to one per month. Since now all the firms adjust prices during the same month the dispersion of price changes collapes to zero, in spite of the larger individual price changes.

Table III

Values of Money Growth and Inflation

	$(M_t^o - M_{t-1}^o)/M_t^o$	$\widehat{(M_t^o - M_{t-1}^o)/M_t^o}$	(1)-(2)	$\hat{\sigma}_m$	g_t	m_t	μ_t
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Nov. 1922	.010	.003	.008	.008(*)	.006	.004	.029
Dec.	.026	.025	.001	.011	.024	.002	-.047
Jan. 1921	.008	-.006	.014	.014	-.003	.011	.001
Feb.	-.011	.010	-.021	.010	.008	-.019	-.045
Mar.	.010	.009	.001	.008(*)	.008	.002	-.028
Apr.	.007	.005	.002	.012	.004	.003	-.009
May	.008	-.009	.017	.014	-.007	.015	-.014
June	.022	.019	.003	.009	.017	.005	.043
July	.029	.012	.017	.014	.012	.016	.044
Aug.	.022	.042	-.020	.009	.038	-.016	.294
Sept.	.045	.036	.009	.008	.036	.009	.075
Oct.	.057	.041	.016	.017	.046	.011	.174
Nov.	.069	.076	-.007	.018	.083	-.014	.328
Dec.	.100	.117	-.017	.013	.113	-.013	.021
Jan. 1922	.061	.065	-.003	.017	.067	-.006	.050
Feb.	.024	.037	-.013	.023	.044	-.021	.113
Mar.	.060	.074	-.014	.020	.079	-.019	.281
Apr.	.075	.072	.003	.021	.078	-.003	.158
May	.070	.071	-.001	.024	.076	-.006	.015
June	.088	.059	.028	.026	.066	.022	.085
July	.106	.066	.040	.025	.077	.029	.358
Aug.	.158	.161	-.002	.020	.166	-.008	.646
Sept.	.221	.260	-.038	.023	.251	-.029	.402
Oct.	.285	.310	-.025	.016	.300	-.015	.679
Nov.	.349	.347	.002	.027	.339	.010	.712
Dec.	.393	.372	.021	.032	.355	.038	.245
Jan. 1923	.339	.367	-.027	.036	.355	-.016	.636
Feb.	.431	.380	.050	.038	.367	.063	.696
Mar.	.385	.364	.021	.034	.342	.043	-.133
Apr.	.252	.275	-.024	.047	.261	-.009	.064
May	.205	.268	-.064	.050	.260	-.056	.450
June	.371	.399	-.028	.037	.378	-.007	.864
July	.572	.518	.054	.049	.488	.084	1.350

Notes to Table III:

$\widehat{(M_t^o - M_{t-1}^o)/M_t^o}$ is the estimated value of money growth from equation (34).

$\hat{\sigma}_m$: square root of the estimated value from equation (26). The entries with an asterisk are those with a negative fitted value that were replaced by the smallest positive value in the series.

g_t: estimated value from the weighted least-squares regression (equation 38), where the series $\hat{\sigma}_m$ are used as weights.

$\tilde{m}_t \equiv (M^o_t - M^o_{t-1})/M^o_t - g_t$.

μ_t: first difference of the logs of the Wholesale Price Index. Data obtained from Sonderhefte zur Wirtschaft und Statistic.

Table IV

Values (in logarithms) of Real Expenditure, Exchange Rate and Money Stock

	S_t	e_t	M_t
Oct. 1920	5.85	2.79	11.24
Nov.	6.44	2.91	11.25
Dec.	6.58	2.86	11.28
Jan. 1921	6.42	2.74	11.29
Feb.	6.80	2.68	11.28
Mar.	6.66	2.70	11.29
Apr.	6.65	2.71	11.30
May	6.53	2.69	11.30
June	6.76	2.80	11.33
July	6.43	2.91	11.36
Aug.	6.76	3.00	11.38
Sept.	6.23	3.22	11.42
Oct.	5.95	3.58	11.48
Nov.	5.54	4.14	11.55
Dec.	6.44	3.82	11.66
Jan. 1922	5.96	3.82	11.72
Feb.	5.84	3.90	11.75
Mar.	5.85	4.22	11.81
Apr.	5.77	4.24	11.88
May	5.93	4.24	11.96
June	5.75	4.33	12.05
July	5.34	4.77	12.16
Aug.	5.32	5.60	12.33
Sept.	6.08	5.86	12.58
Oct.	5.60	6.63	12.92
Nov.	5.30	7.44	13.35
Dec.	6.08	7.50	13.85
Jan. 1923	5.32	8.36	14.26
Feb.	5.63	8.80	14.82
Mar.	6.55	8.53	15.31
Apr.	6.15	8.67	15.60
May	5.65	9.34	15.83
June	6.21	10.17	16.29
July	6.16	11.34	17.14

Notes to table III:

S_t: log of the monthly government expenditure in millions of gold marks. Source: Bresciani-Turroni, p. 436-37.

e_t: log of the monthly average exchange rate of the gold marks in millions of paper marks. Source: Bresciani-Turroni: p. 441.

M_t: log of the monthly average money stock in millions of paper marks. Source: based on fixed days quotations from Sonderhefte zur Wirtschaft und Statistic, p. 45-47.