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EXCHANGE RATE AND CURRENT ACCOUNT DYNAMICS UNDER RATIONAL EXPECTATIONS: AN ECONOMETRIC ANALYSIS

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ABSTRACT

An econometric portfolio balance model of an open economy, incorporating exchange rate, price, and current account dynamics, is derived and estimated. The usual stability conditions do not guarantee a unique rational expectations solution, and several proposals for resolving this situation are considered. Using constrained maximum likelihood methods, the model is estimated for Japan. The estimation results indicate that the model is quite successful in explaining the patterns found in the data. The model is estimated using several methods of resolving the question of non-uniqueness, and the results are compared.

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1. Introduction

Continued experience with flexible exchange rates for industrialized countries has stimulated a considerable amount of research on the relationship between the exchange rate, price level, and current account. Dornbusch (1976) and Mussa (1982) highlight the role of rational expectations and the relationship between the money supply, slow price adjustment, and exchange rate determination. Kouri (1976), Calvo and Rodriguez (1977), Dornbusch and Fischer (1980), Rodriguez (1980), and Mussa (1980) extend the rational expectations approach into a more general portfolio balance framewok to emphasize the effects of current account imbalances on the exchange rate. Recent work by Branson and Buiter (1983) considers both types of models while Eaton and Turnovsky (1983) and Buiter and Miller (1983) combine slow price and slow asset adjustment in a single model.

While this research has been motivated by empirical phenomena, there has been very little explicit econometric work on these models suitable for testing alternative hypotheses, estimating parameters, or conducting quantitative policy experiments. In this paper, an econometric portfolio balance model with rational expectations is constructed in the spirit of the theoretical research mentioned above. The model extends this earlier work by incorporating a stochastic structure within a framework that includes both portfolio balance and slow price adjustment, tracing out how the structure influences the dynamics of exchange rate expectations, and deriving the constraints between the portfolio balance, current account, and price level equations. By incorporating Mussa's (1982) price adjustment formulation into the portfolio balance framework, a model is derived which, while encompassing exchange rate, price, and current account dynamics, can be solved analytically. The model is capable of accounting for various patterns of correlation between the exchange rate and the current account.

A major focus of the paper is on the uniqueness of the rational expectations solution. Even after the imposition of the usual stability condition, non-uniqueness emerges in this model, not as an aberration, but as a quite plausible outcome. The methods for achieving a unique solution proposed by Taylor (1977) and McCallum (1983) are considered. It is shown that they coincide in this model.

The model is then estimated for Japan, using quarterly data since the advent of generalized floating in 1973. The estimation results accord well with the theory, with most of the structural and policy coefficients of the expected sign and significant. The estimates, which incorporate the constraints proposed by Taylor and McCallum to achieve a unique solution, are contrasted with a less restrictive set of estimates of a type proposed by Chow (1983). These do not constrain the model to any particular solution, but instead allow the solution to be determined by the data. We find that the two sets of estimates provide very similar results. The effects of various disturbances on the dynamics of the exchange rate, current account, and domestic price level are illustrated by the moving average representation. The model is estimated by constrained maximum likelihood methods recently used by Sargent (1978) and Taylor (1980) for closed economy rational expectations models.

The paper is organized as follows: Section 2 describes the model. Section 3 derives the rational expectations solution and considers various methods to achieve uniqueness. In Section 4, an estimatable form of the model is derived, the cross equation restrictions are imposed, and the model is estimated. The results from implementing McCallum and Taylor's and Chow's techniques are compared, and the dynamics of the model are illustrated by the moving average representation. Conclusions are presented in Section 5.

2. The Model

In this section, we construct a portfolio balance model of an open economy which incorporates a number of simplifying assumptions that enable us to focus on exchange rate, price, and current account dynamics. The country is small in world markets and produces a single (traded) good. Purchasing power parity is assumed to hold in the long run but, since demand for the good is not perfectly elastic, not instantaneously. Domestic residents hold two assets, domestic and foreign currencies, while foreigners hold only the foreign asset.[1] Assets do not pay interest.[2]

Equilibrium in the asset market is expressed by equating the supply of money with its demand, which depends on domestic income, domestic wealth, and the expected rate of depreciation,

$$(1) \qquad m_t - q_t = a_1(p_t + y_t - q_t) + a_2 w_t - a_3(\hat{e}_{t+1} - e_t) + \eta_{1t} ,$$

where m_t = the stock of domestic assets (money supply), q_t = the domestic consumer price index, p_t = the price of the domestic good, y_t = domestic real output, w_t = domestic real wealth, e_t = the exchange rate (domestic currency price of foreign exchange), and $\hat{e}_{t+1}$ = the expected exchange rate at time t + 1 conditional on information available at time t. All variables are expressed as logarithmic deviations from their steady state values. Real balances $(m_t - q_t)$ and real income $(p_t + y_t - q_t)$ are deflated by the consumer price index. The expected rate of depreciation is $\hat{e}_{t+1} - e_t$. The disturbance term (as well as the η's below) is a random variable, which may be serially correlated.[3]

The consumer price index is a weighted average of the price of the domestic good and the price of the foreign good expressed in terms of domestic currency $(e_t + p_t^*)$. The weight (b_1) represents the share of the domestic good in domestic consumption,

$$q_t = b_1 p_t + (1 - b_1)(e_t + p_t^*) \ . \tag{2}$$

Domestic nominal wealth is a weighted average of the stock of domestic assets and domestic holdings of the stock of foreign assets (f_t), expressed in terms of domestic currency $(e_t + f_t)$. The weight (b_2) represents the share of the domestic asset in domestic wealth. Real wealth is nominal wealth deflated by the consumer price index,[4]

$$w_t = b_2 m_t + (1 - b_2)(e_t + f_t) - q_t \ . \tag{3}$$

The demand for domestic assets is assumed to increase with income and wealth and to decrease with the expected rate of depreciation. The asset market equilibrium equation incorporates current realizations of variables which, as will be shown below, include current shocks. This is a discrete time approximation of continuous portfolio balance with variables that change over the period.

The rate of increase of domestic holdings of foreign assets ($f_t - f_{t-1}$) is equal to the current account surplus because foreigners do not hold domestic currency. The current account surplus depends on domestic income, foreign income (y^*), and domestic wealth,[5]

$$f_t - f_{t-1} = a_4(p_{t-1} + y_{t-1} - q_{t-1}) + a_5 y^*_{t-1} - a_6 w_{t-1} + \varepsilon_{2t} \ . \qquad (4)$$

The effect of an increase in domestic income on the current account is ambiguous. The two traditional perspectives are that higher income, by increasing imports, causes a deficit, and that higher income, by raising absorption less than proportionately, causes a surplus. Recent work on the current account in a utility maximizing framework, such as Sachs (1981) and Obstfeld (1983), does not give us a determinate answer because, as will be seen below, output will depend on relative prices. Analogously, the effect of foreign income on the current account is also indeterminate. An increase in domestic wealth is assumed to decrease savings and to cause a deficit.

The specification of the current account equation assumes that export and import decisions are made at the beginning of the period, based on the values of domestic income, foreign income, and wealth at the end of the previous period. Since trade does not take place instantaneously, the current account evolves during the period. Thus the stock of foreign assets is pre-determined in the sense that it is not affected by the current realizations of other variables. It is affected, however, by the current realization of the disturbance term ε_{2t}.

Following Mussa (1982), the domestic rate of inflation is assumed to equal the expected rate of change of the equilibrium price level plus some proportion (< 1) of the difference between equilibrium and actual prices,[6]

$$(5) \qquad p_t - p_{t-1} = (\hat{e}_t + \hat{p}^*_t) - (e_{t-1} + p^*_{t-1}) + a_7(e_{t-1} + p^*_{t-1} - p_{t-1}) + \varepsilon_{3t} \; ,$$

where $(\hat{e}_t, \hat{p}^*_t)$ is the expectation of the (exchange rate, foreign price level) for period t, conditional on information available at the end of period $t - 1$. The equilibrium price level is postulated to be the price level at which purchasing power parity is satisfied, and thus equals $e_t + p^*_t$. Domestic prices, like the stock of foreign assets, are predetermined in the sense that they are unaffected by current realizations of other variables, although they are affected by the realization of ε_{3t}.

Domestic output is assumed to depend on the relative price of foreign goods $(e_t + p^*_t - p_t)$ and the level of foreign output,

$$(6) \qquad y_t = a_8 e_t + a_9 p^*_t - a_{10} p_t + a_{11} y^*_t + \eta_{t5} \; .$$

While an increase in the relative price of foreign goods would normally be expected to increase output, this effect may depend on whether relative price changes reflect movements in price levels or in the exchange rate. Since the exchange rate is determined in auction markets, changes in relative prices caused by exchange rate movements may not be a good signal of future relative prices, and thus output may be unresponsive to such changes. Changes in relative prices caused by price level movements should, since prices are determined in contract markets, be a better signal of future relative prices, and thus output should be responsive to these changes. For this reason, we place separate coefficients on the three terms comprising relative prices. The sign of a_8 is ambiguous: a_9 and a_{10} are assumed to be positive. As with the current account, the effect of an increase in foreign income on domestic output is ambiguous. η_{4t} is a random disturbance.

The money supply is modeled as a reaction function which responds to the exchange rate, domestic price level, foreign price level, and stock of foreign assets,

$$(7) \qquad m_t = a_{12}e_t + a_{13}p_t + a_{14}p_t^* + a_{15}f_t + \eta_{5t} .$$

This formulation encompasses a number of possibilities. The money supply could respond to movements in the real exchange rate (relative prices of foreign goods). This would constrain $a_{12} = a_{14} = -a_{13}$. In Taylor (1984) the money supply responds to the consumer price index; this produces a very different set of constraints. We allow the money supply to respond to the stock of foreign assets to incorporate the possibility that the monetary authorities will intervene in the foreign exchange markets in response to the current account balance. It is assumed that $a_{15} \geq 0$.

The model is completed by, based on the small country assumption, assuming that foreign prices and output are determined exogenously,

$$(8) \qquad p_t^* = a_{16}p_{t-1}^* + \eta_{6t} ,$$

$$y_t^* = a_{17}y_{t-1}^* + \eta_{7t} .$$

The first order autoregressive process is specified for simplicity. The disturbance terms, as above, can be serially correlated. It is assumed that a_{16} and $a_{17} < 1$.

3. The Rational Expectations Solution

We now proceed to solve the model and consider the implications of the rational expectations assumption. Most research on exchange rate dynamics, cited above, incorporates either slow price or slow asset adjustment, but not both. This produces models with two first order differential (or difference) equations, involving one predetermined and one non-predetermined variable, which can always be solved analytically. Incorporating slow price and slow asset adjustment produces a third order system, which in general cannot be solved analytically. Buiter and Miller (1983) use numerical simulation while Eaton and Turnovsky (1983) are able to determine the signs of the characteristic roots, although not their values. The essential difference between Eaton and Turnovsky's paper and ours is that, following Dornbusch (1976), they assume that prices adjust in response to the relative price of foreign and domestic goods.[7] Although Mussa's rule, which we adopt, is seemingly more complicated, it allows us to determine the characteristic roots exactly. While this is necessary for estimation, the theoretical implications of the two models are very similar.

The first step towards a solution is to write the structural equations in the form of five first order stochastic difference equations. Using (2), (3), (6), and (7), the portfolio balance equation (1) can be written,

$$\hat{e}_{t+1} = \delta_1 e_t + \delta_2 f_t + \delta_3 p_t + \delta_4 p^*_t + \delta_5 y^*_t + u_{1t} , \tag{9}$$

where $\delta_1 = [(1 - a_1)(1 - b_1) + a_1 a_8 + a_2(b_1 + b_2(a_{12} - 1)) - a_{12} + a_3]/a_3$,

$\delta_2 = [a_2(1 + b_2(a_{15} - 1)) - a_{15}]/a_3$,

$\delta_3 = [b_1 + a_1(1 - b_1) - a_1 a_{10} + a_2(b_2 a_{13} - b_1) - a_{13}]/a_3$,

$\delta_4 = [(1 - a_1)(1 - b_1) + a_1 a_9 + a_2(b_2 a_{14} + b_1 - 1) - a_{14}]/a_3$, and

$\delta_5 = [a_1 a_{11}]/a_3$.

The disturbance term u_{1t}, as well as the other u's below, is a linear combination of the η's.

The current account equation (2) becomes,

$$f_t = \gamma_1 e_{t-1} + \gamma_2 f_{t-1} + \gamma_3 p_{t-1} + \gamma_4 p^*_{t-1} + \gamma_5 y^*_{t-1} + u_{2t} , \tag{10}$$

where $\gamma_1 = a_4(a_8 - 1 + b_1) - a_6(b_1 + b_2(a_{12} - 1))$,

$\gamma_2 = 1 - a_6(1 + b_2(a_{15} - 1))$,

$\gamma_3 = -(a_4(a_{10} - 1 + b_1) + a_6(b_2 a_{13} - b_1))$,

$\gamma_4 = a_4(a_9 - 1 + b_1) - a_6(b_2 a_{14} + b_1 - 1)$, and

$\gamma_5 = a_4 a_{11} + a_5$.

The price adjustment equation (5) becomes,

$$p_t = \tau_1 e_{t-1} + \tau_2 f_{t-1} + \tau_3 p_{t-1} + \tau_4 p^*_{t-1} + \tau_5 y^*_{t-1} + u_{3t} , \tag{11}$$

where $\tau_1 = \delta_1 + a_7 - 1$,

$$\tau_2 = \delta_2 \ ,$$

$$\tau_3 = \delta_3 - a_7 + 1 \ ,$$

$$\tau_4 = \delta_4 + a_7 + a_{16} - 1 \ , \text{ and}$$

$$\tau_5 = \delta_5 \ .$$

The equations for the exogenous variables (8) are unchanged.

A rational expectations solution for the model involves finding distribution functions for e_t, f_t, p_t, p_t^*, and y_t^* that satisfy (8)-(11). The solution technique used is a multivariate version of the method of undetermined coefficients used by Muth (1961) and described in detail by Taylor (1985). It involves representing the variables in general infinite moving average form, substituting these general forms into the structural equations, and then solving the resultant identities for the coefficients.

An infinite order moving average representation can be written,

$$(12) \qquad z_{kt} = \sum_{j=1}^{5} \sum_{i=0}^{\infty} \Pi_{kji} \, u_{jt-i} \ ,$$

where $z_{kt} = (z_{1t}, z_{2t}, z_{3t}, z_{4t}, z_{5t})' = (e_t, f_t, p_t, p_t^*, y_t^*)'$,

$u_{jt} = (u_{1t}, u_{2t}, u_{3t}, \eta_{6t}, \eta_{7t})'$, and

Π_{kj} is a 5×5 matrix of coefficients .

The representations of all of the variables incorporate current disturbances. The exchange rate is determined in asset markets and is assumed to reflect all available information including, under the assumption of perfect current information, all contemporaneous shocks. The other variables are represented by end of period values, and thus incorporate current disturbances.

Solution of the model involves substituting equation (12), as well as similar expressions for $\hat{e}_{t+1}$ and z_{t-1}, into (8)-(11) and solving the resultant set of identities for the Π's. For the purpose of exposition, we present the solution for the case where the disturbances are serially uncorrelated.[8] Performing the substitutions, we obtain,

$$\text{(13)} \qquad \begin{aligned} \Pi_{111} &= \delta_1 \Pi_{110} + 1 & \qquad \Pi_{1j1} &= \delta_1 \Pi_{1j0} + \delta_j \\ \Pi_{211} &= \gamma_1 \Pi_{110} & \qquad \Pi_{2j1} &= \gamma_1 \Pi_{1j0} + \gamma_j \qquad (j = 2, \ldots, 5) \\ \Pi_{311} &= \tau_1 \Pi_{110} & \qquad \Pi_{3j1} &= \tau_1 \Pi_{1j0} + \tau_j \end{aligned}$$

$$\text{(14)} \qquad \Pi_{kji+1} = Q\Pi_{kji} \qquad \begin{aligned} &(j = 1, \ldots, 5) \\ &(i = 1, \ldots, \infty) \end{aligned}$$

where $\Pi_k = (\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5)'$, and

$$Q = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 \\ \tau_1 & \tau_2 & \tau_3 & \tau_4 & \tau_5 \\ 0 & 0 & 0 & a_{16} & 0 \\ 0 & 0 & 0 & 0 & a_{17} \end{bmatrix}$$

Each block of equations in (13) consists of three equations and four unknowns, and thus does not have a unique solution without further specification. This indeterminancy occurs in most rational expectations models that include expectations of the future values of the variables, and we proceed in the usual manner. The general solution to (14) is,

$$\Pi_{kji} = \sum_{h=1}^{5} C_h D_{hk} \lambda_h^i \, , \qquad \text{(for } j = 1, \ldots, 5) \, , \tag{15}$$

where the λ's are the characteristic roots,

the D's are the characteristic vectors (with $D_{h1} = 1$ by construction), and

the C's are arbitrary constants.

The characteristic roots of Q are the roots of the 3×3 matrix in the upper left corner, a_{16}, and a_{17}. The relation between the δ's and the τ's, based on Mussa's price adjustment formulation, allow us to solve for the roots. They are, $1 - a_7$ (which is equal to $\tau_3 - \delta_3$), and the roots of the matrix,

$$Q_1 = \begin{bmatrix} \delta_1 + \delta_3 & \delta_2 \\ \gamma_1 + \gamma_3 & \gamma_2 \end{bmatrix} ,$$

which are,

$$\lambda_1, \lambda_2 = \frac{(\delta_1 + \delta_3 + \gamma_2) \pm [(\delta_1 + \delta_3 + \gamma_2)^2 - 4((\delta_1 + \delta_3)\gamma_2 - \delta_2(\gamma_1 + \gamma_3))]^{1/2}}{2} .$$

In order to illustrate the questions involving uniqueness, it is useful to characterize the roots as $\lambda_1 = \delta_1 + \delta_2 + x$ and $\lambda_2 = \gamma_2 - x$, where $x \gtreqless 0$ as $\delta_2(\gamma_1 + \gamma_3) \gtreqless 0$. The other roots, $\lambda_3 = 1 - a_7$, $\lambda_4 = a_{16}$, and $\lambda_5 = a_{17}$, are assumed to be stable (< 1). Unless there is a destabilizing parameter value for

a_{15} or a large, negative value for $\delta_2(\gamma_1 + \gamma_3)$, λ_2 will also be stable. If $\lambda_1 > 1$, the usual assumption, that the conditionally expected time paths of the variables be stable, is imposed by setting C_1, the coefficient of the unstable root in (15), equal to zero. This allows all of the Π's to be uniquely determined.[9]

There is no reason, however, to presume that $\lambda_1 > 1$ in this model. If $\lambda_1 < 1$, the requirement that the conditionally expected time paths of the variables be stable does not imply any additional conditions, and thus does not provide a unique solution. Procedures for achieving a unique solution have been suggested by Taylor (1977) and McCallum (1983). The first part of McCallum's procedure is to require that a minimal set of state variables be employed in agents forecasting rules. This can be implemented by setting either C_1 or C_2 equal to zero, and thus does not yield a unique solution. The second part of his procedure is to require that the solution be valid for all admissable parameter values. Consider the case where the determinant of Q_1 is equal to zero. Then $x = \gamma_2$ so that $\lambda_1 = \delta_1 + \delta_3 + \gamma_2$ while $\lambda_2 = 0$. The minimal set of state variables criterion implies that zero is the appropriate root in this case, thus it is necessary to set $C_1 = 0$, and a unique solution can be derived. Taylor's procedure provides the same answer. If a solution to the model is defined as setting either C_1 or C_2 equal to zero, then Taylor's condition, that the asymptotic variance of the variables be minimized, is implemented by setting C_1, the coefficient on the largest of the two roots, equal to zero. It should be noted that these two procedures do not always coincide. In particular, both McCallum (1983) and Taylor (1985) discuss how they differ when applied to Taylor's (1977) model.

An alternative method for estimating models of this type has been proposed by Chow (1983). He suggests that, instead of constraining the model to achieve a unique solution, the parameter C_1 be estimated along with the other parameters of the model. Thus the "solution" of the model will be based on whatever value of C_1 maximizes the likelihood function. Evans and Honkapohja (1984) also discuss estimators of this type.

We conclude this section by considering the causes of non-uniqueness in this model. One possible factor is if a depreciation of the exchange rate reduces output ($a_8 < 0$). Calvo (1983) considers the implications of this for non-uniqueness. A second factor that operates through its effect on domestic output is if an increase in the domestic price level reduces domestic output ($a_{10} > 0$), which is the expected case. Monetary policy which accommodates either exchange rate and/or domestic price movements (a_{12} and/or $a_{13} > 0$) also contributes towards non-uniqueness. This is also plausible. It can be produced by, for example, monetary policy which accommodates the consumer price index. Thus non-uniqueness of the rational expectations solution can be the result of reasonable parameter values.

4. Estimation of the Model

We now examine the extent to which the model developed above can explain phenomena in the current flexible exchange rate period. The cross equation restrictions imposed by the theory constrain the exchange rate, stock of foreign assets, and price level to follow a unique path after a shock. Estimation of the model subject to these constraints provides a joint test of the portfolio balance approach with slow price adjustment and the rational expectations assumption.

The model is estimated for Japan, using quarterly data from 1973 (II) to 1983 (IV). The various configurations of the exchange rate and the current account experienced by Japan in the 1970's and the 1980's seem to be amenable to explanation by the portfolio balance approach. Japan also appears to fit the model's assumptions (that foreign residents do not hold domestic assets and that assets are imperfect substitutes) better than most other industrialized countries with flexible exchange rates.[10] In addition, the assumption that assets do not pay interest makes estimation of the model inappropriate for countries where high capital mobility makes interest rate differentials an important determinant of exchange rates.

The exchange rates used are weighted average exchange rates, with the weights derived from the International Monetary Fund's Multilateral Exchange Rate Model (MERM). The weights represent the effect on the country's trade balance (calculated from the MERM) of a 1 percent change in the value of each foreign currency in terms of the home currency. The data for stocks of foreign assets are computed by taking a base figure for the middle of the period and then adding quarterly current account balances forwards and backwards. To the extent that current account balances cannot be exactly identified with changes in domestic holdings of foreign assets, the model will account for the former at the expense of the latter. The price level used is the GNP deflator, real income is real GNP, and the money supply is M1. Foreign variables are constructed by taking weighted averages, with the weights corresponding to the

MERM weights.[11] The share of domestic goods in domestic consumption for Japan (b_1) is .86, while the share of domestic assets in domestic wealth is .76.[12] To remove nonstationarity in the variables, all data (after taking logarithms) are first-differenced.[13]

Estimation of the model requires deriving the reduced form, which in turn necessitates making some assumptions about the structure of the error terms. We assume that they are generated by first order autoregressive processes, $u_{jt} = \alpha_j u_{jt-1} + \psi_{jt}$, $j = 1, \ldots, 5$, where the ψ's are serially uncorrelated. We then take the infinite moving average representation implicit in the above autoregressive process and truncate it at third order for u_1 and fourth order for the others.[14] This produces a first order autoregressive fourth order moving average model. Assuming, as above, that expectations are determined rationally and solving by the method of undetermined coefficients, the reduced form of (8)-(11) is derived,

$$(16) \qquad z_t = Az_{t-1} + B(L)\nu_t \ ,$$

where $z_t = (e_t, f_t, p_t, p_t^*, y_t^*)'$,

$\nu_t = (\nu_{1t}, \nu_{2t}, \nu_{3t}, \nu_{4t}, \nu_{5t})'$, and

A and B are 5×5 matrices. The elements of A and B are non-linear combinations of the δ's, γ's, and τ's (which in turn are combinations of the a's), and the α's. The ν's are combinations of the ψ's, written so as to make the zero lag coefficient matrix the identity matrix. The constraints on the parameters are generated by the form of the structural equations, the assumption of rational expectations, and the imposition of the uniqueness conditions.

The model to be estimated consists of equations (6), (7), and (16). These are estimated jointly in accordance with the Lucas critique. We make the same assumptions with regard to the structure of the disturbance terms for output (η_{4t}) and the money supply (η_{5t}) as we do for the others. (α_6 is the serial correlation term for output, α_7 for the money supply.) There are 98 constrained parameters in the complete model, 14 autoregressive, 76 moving average, and 8 simultaneous, which are combinations of 24 fundamental parameters, 17 structural (a's) and 7 serial correlation (α's).

Using maximum likelihood techniques, we can estimate the structural coefficients, the serial correlation coefficients, and the covariance matrix of the shocks. The reduced form (16) is a vector ARMA (1,4) model with nonlinear constraints on the parameters. Under the assumption that ν_t is a multivariate normal vector with $E\nu_t\nu_t' = \Omega$, maximum likelihood estimates (conditional on the initial disturbances being set to zero) under the restrictions imposed by the model are obtained by maximizing,

$$L(\beta) = (2\Pi)^{-T}\ \Omega^{-T/2}\ \exp(-1/2 \sum_{i=1}^{T} \hat{\nu}_t'\ \Omega^{-1}\ \hat{\nu}_t) ,$$

where T is the number of observations.

Maximum likelihood estimates of β with Ω unknown can be obtained by minimizing the determinant of Ω with respect to the a's and the α's, subject to the constraints, where $\hat{\Omega} = \frac{1}{T}\sum_{i=1}^{T} \hat{\nu}_t \hat{\nu}_t'$ is a sample covariance matrix of ν_t. Nonlinear minimization routines can be used to find the minimum even though the determinant is a highly nonlinear function of the parameters. The technique used here is the Davidon-Fletcher-Powell algorithm.

The maximum likelihood estimates of the parameters are given in Table 1 along with their asymptotic "t-ratios", the ratio of these coefficients to their standard errors computed from the inverse of the second derivative matrix of the likelihood function. The results of the estimation are very successful. Most of the structural coefficients have the expected sign, are of reasonable magnitude, and are significant. In the portfolio balance equation, the coefficients on income (a_1), wealth (a_2), and the expected rate of depreciation (a_3) are all positive, plausible, and significant. For the current account equation, the coefficient on domestic income (a_4) is positive and insignificant, while for foreign income (a_5) it is negative and insignificant. Both of these parameters, however, are theoretically ambiguous. The coefficient on wealth (a_6) is, as expected, positive and significant. Mussa's price adjustment equation appears to fit well, with the coefficient on the difference between equilibrium and actual prices (a_7) positive and significant.

Foreign (a_9) and domestic (a_{10}) prices have the expected affects on domestic output, with both coefficients positive and a_{10} significant. The coefficient on the exchange rate (a_8) is negative and significant, while foreign output (a_{11}) has no affect on domestic output at all. The money supply is accommodative with respect to the exchange rate (a_{12}), the domestic price level (a_{13}), the foreign price level (a_{14}), and the stock of foreign assets (a_{15}), with all four coefficients being positive and significant. The signs of these coefficients are not consistent with the hypothesis that the money supply responds to the real exchange rate. While the signs are consistent with the money supply accommodating the consumer price index, the magnitudes are not.

The response of the money supply to the exchange rate and the stock of foreign assets indicates that intervention is important.

The autoregressive parameter on foreign prices (a_{16}) is positive and significant while the coefficient for foreign output (a_{17}) is positive and insignificant. Only the serial correlation coefficient for the exchange rate (α_1) is positive while only the coefficients for domestic (α_3) and foreign (α_4) prices are significant. In summary, the estimated values of all of the coefficients for which the theory predicts a determinate sign are of that sign, are significant, and are plausible.[15]

We now examine the parameter values implied by the estimates, which are also given in Table 1. Both λ_1 and λ_2 are stable (< 1), and C_1, the coefficient of λ_1, is set to zero by the arguments of McCallum and Taylor described above to provide a unique solution. All four possible factors contribute to the non-uniqueness. A depreciation of the exchange rate and an increase in the domestic price level both reduce domestic output, while monetary policy is accommodative with respect to both the exchange rate and the domestic price level.

Estimates of the model using Chow's technique are presented in Table 2. The value of C_1 is -.19 (-1.54) which, while not significant, has a large enough value and a small enough standard error so that the possibility that C_1 does not equal zero should be taken seriously. The structural parameters, however, do not change very much and the parameter values implied by the estimates (the δ's, γ's, τ's, and λ's) are very similar. In summary, the less restrictive estimates using Chow's technique indicate that the constraints imposed above to achieve a unique solution are not seriously at odds with the data.

One illustrative method of examining the results of the estimation is to consider the moving average representations for the constrained model. These give the response over time of the exchange rate, stock of foreign assets, and domestic price level to disturbances of the exchange rate, stock of foreign assets, domestic price level, foreign price level, and foreign output, and are presented in Table 3. The representations for the less constrained model (Chow's method) are very similar and are not presented. These representations are conditional on the assumption that the correlations between the disturbances are zero, so that an innovation in one variable can be considered independently of movements in the others. While some of the correlations are low, some are high enough to suggest that these representations be interpreted with caution.[16] Sims (1979), Sargent (1978) and Taylor (1980) have used the moving average representation in other contexts.

A disturbance that depreciates the exchange rate causes a deficit. Over time, the exchange rate appreciates and the current account is in surplus back to the steady state. The maximum depreciation and deficits are attained quickly, and the adjustment is fairly smooth. The domestic price level increases slightly. A surplus shock to the current account produces appreciation of the exchange rate and a small decrease in the domestic price level. The responses to current account and to exchange rate shocks are of comparable magnitude. An increase in the domestic price level depreciates the exchange rate and causes cycles of current account surplus and deficits. An increase in the foreign price level first causes depreciation and deficits, but the deficits continue after the depreciation is reversed. Shocks to foreign output do not have strong affects. The correlation between the exchange rate and the current account is positive (deficits associated with depreciation and vice versa) in response to shocks of the exchange rate and stock of foreign assets, and mixed in response to shocks of the domestic and foreign price levels.

5. Conclusions

This paper has shown that, by incorporating rational expectations explicitly into the portfolio balance framework, a model of an open economy can be constructed that is capable of accounting for the dynamics of the exchange rate and the current account. By deriving the cross equation constraints implied by the rational expectations solution and then estimating the model subject to these constraints, the paper provides a rigorous joint test of the portfolio balance approach and the rational expectations assumption.

The estimation results for Japan are quite successful. They accord well with the theory and are capable of explaining various patterns of correlation between the exchange rate and the current account. One noteworthy result is that the usual stability condition does not provide a unique solution to the model. Several methods for resolving this problem are implemented and compared. The results provide strong econometric evidence of the importance of simultaneous determination of the exchange rate, the price level, the current account, and expectations in open economy macroeconomic models.

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Footnotes

1. The assumption that foreigners hold only the foreign asset is common in portfolio balance models. If relaxed, a two-country model would be needed. If, in that context, the assumption were made that each country's residents preferred their own asset, the results would be similar to the present model.
2. Allowing foreign assets to pay an exogenously determined rate of interest, as in Dornbusch and Fischer (1980), does not affect the theoretical results. Since the model contains no interest bearing assets, it can be considered a currency substitution model in the sense used by Calvo and Rodriguez (1977). We call it a portfolio balance model becuase the term "currency substitution" has come to imply trade in currencies in addition to other assets.
3. It is assumed that the η's are not contemporaneously correlated.
4. This log-linear approximation for wealth has been used by Driskill (1980) and by Eaton and Turnovsky (1983).
5. An alternative specification would be to have domestic income and wealth denominated in terms of foreign currency. The model with this equation cannot be distinguished, either theoretically or empirically, from the specification in the text. It is not possible to have both the current account and domestic output (specified below) depend on relative prices and still econometrically identify the parameters. We do not differentiate between foreign output and income.

6. Mussa argues that his formulation has a better microeconomic rationale and more sensible steady state properties than Dornbusch's.

7. Dornbusch assumes that prices adjust in response to excess demand in the goods market. In his model, although not in Eaton and Turnovsky's nor in ours, that is equivalent to having prices adjust in response to the relative price of foreign and domestic goods.

8. If the disturbances were modeled as moving average processes, equation (13) would contain terms up to the order of the moving average. Equation (14) would begin with the next term and the remainder of the solution, including the characteristic roots and the uniqueness criteria, would be unchanged.

9. This criterion is also known as the "no speculative bubbles" or "finite variance" condition.

10. While it is certainly not true that no Japanese currency is held by foreigners, the yen is not an international currency in the sense that the dollar or sterling is. Relaxing this assumption is discussed in footnote 1.

11. The data is taken from International Financial Statistics. All data is seasonally adjusted except for the exchange rate (which does not exhibit a strong seasonal pattern). The countries used to construct the foreign variables (weights in parentheses) are: Canada (.04), France (.10), Germany (.16), Italy (.05), United Kingdom (.05), and United States (.60).

12. These weights are averages over the sample period. Theoretically, b_1 should represent the share of importable goods in domestic consumption, rather than imported goods, and should therefore be estimated. It is

impossible, however, to econometrically identify the parameters in equation (1) if b_1 is also estimated.

13. In addition, the constants were removed from the first-differenced data so that the variables would have zero means. We also estimated the model with the data detrended by regression on a constant, linear time trend, and trend squared. The results of this estimation indicated that the detrending was not sufficient to remove nonstationarity for the exchange rate.

14. The disturbances need to be written as finite order moving averages for the model to be estimated. We generate these from first order autoregressive processes in order to limit the number of parameters to be estimated. The moving average representation for u_1 is truncated one order lower than the others so as not to impose arbitrary zero restrictions on u_{24}, ..., u_{54}. The ARMA (1,4) model was chosen because it eliminated most of the serial correlation among the residuals.

15. While it would be desirable to test the cross equation restrictions by using a likelihood ratio test, the lack of degrees of freedom in the unrestricted system makes this impossible.

16. Let σ_{ij} be the sample correlation coefficient between variables i and j. Then $\sigma_{ef} = .44$, $\sigma_{ep} = -.16$, $\sigma_{ep*} = -.31$, $\sigma_{ey*} = .53$, $\sigma_{fp} = -.03$, $\sigma_{fp*} = -.37$, $\sigma_{fy*} = .80$, $\sigma_{pp*} = -.01$, and $\sigma_{py*} = .08$.

Table 1

Maximum Likelihood Estimates of the Structural Parameters

Parameter	Estimate	Asymptotic "t-ratio"	Parameter	Estimate	Asymptotic "t-ratio"
a_1	.63	18.71	a_{13}	.39	25.29
a_2	.71	127.98	a_{14}	.55	15.49
a_3	.27	12.28	a_{15}	.40	2.63
a_4	.17	.63	a_{16}	.84	163.60
a_5	-.08	-.60	a_{17}	.16	1.21
a_6	.77	15.71	α_1	.17	.90
a_7	.32	17.64	α_2	-.08	-.88
a_8	-.10	-11.89	α_3	-.39	-3.14
a_9	.05	.93	α_4	-.42	-3.45
a_{10}	.10	11.15	α_5	-.03	-.21
a_{11}	-.01	-.34	α_6	-.04	-.37
a_{12}	.29	15.25	α_7	-.16	-1.31

Parameter Values Implied by the Estimates

Parameter	Estimate	Parameter	Estimate
δ_1	.73	τ_1	.05
δ_2	-.05	τ_2	-.05
δ_3	.10	τ_3	.78
δ_4	-.74	τ_4	-.58
δ_5	-.01	τ_5	-.01
γ_1	-.29	λ_1	.82
γ_2	.58	λ_2	.60
γ_3	.35	λ_3	.68
γ_4	-.14	λ_4	.84
γ_5	-.08	λ_5	.16

Log Likelihood

838.474

Table 2

Maximum Likelihood Estimates of the Structural Parameters - Chow's Method

Parameter	Estimate	Asymptotic "t-ratio"	Parameter	Estimate	Asymptotic "t-ratio"
a_1	.72	36.73	a_{14}	.54	26.08
a_2	.71	99.46	a_{15}	.39	2.56
a_3	.28	18.15	a_{16}	.83	90.76
a_4	.12	.67	a_{17}	.18	1.29
a_5	-.06	-.51	α_1	-.01	-.11
a_6	.80	22.37	α_2	-.08	-.89
a_7	.35	20.12	α_3	-.38	-3.15
a_8	-.10	-15.19	α_4	-.42	-3.55
a_9	.03	.49	α_5	-.03	-.27
a_{10}	.10	12.26	α_6	-.15	-.33
a_{11}	-.01	-1.05	α_7	-.15	-1.21
a_{12}	.25	24.23	C_1	-.19	-1.54
a_{13}	.39	34.30			

Parameter Values Implied by the Estimates

Parameter	Estimate	Parameter	Estimate
δ_1	.70	τ_1	.05
δ_2	-.04	τ_2	-.04
δ_3	.12	τ_3	.77
δ_4	-.79	τ_4	-.60
δ_5	-.04	τ_5	-.04
γ_1	-.27	λ_1	.80
γ_2	.57	λ_2	.59
γ_3	.37	λ_3	.65
γ_4	-.14	λ_4	.84
γ_5	-.06	λ_5	.18

Log Likelihood

839.481

Table 3

Moving Average Representations Implied by the Constrained Model

Quarter	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
0	1.00	.00	.00	.00	.00	.00	1.00	.00	.00	.00	.00	.00	1.00	.00	.00
1	.54	-.17	.71	1.65	-.01	-.29	.51	.35	-.14	-.08	.05	-.05	.38	-.58	-.02
2	.38	-.18	.65	1.26	-.01	-.30	.33	.14	-.82	-.06	.08	-.07	.48	-.60	-.01
3	.25	-.18	.67	1.11	.00	-.24	.23	.01	-1.34	-.04	.10	-.08	.39	-.63	.00
4	.20	-.16	.55	.62	.00	-.18	.16	-.07	-1.40	-.02	.10	-.08	.33	-.57	.00
5	.08	-.19	.71	.99	.00	-.15	.22	.24	-1.96	-.01	-.04	-.06	.67	-.48	.01
6	.13	-.12	.37	.10	.00	-.13	.09	-.29	-1.25	-.01	.03	-.06	.10	-.70	.00
7	.13	-.11	.37	.10	.00	-.09	.07	-.05	-1.12	-.01	.06	-.06	.25	-.57	.00
8	.12	-.10	.31	.01	.00	-.06	.05	-.13	-.86	.00	.07	-.06	.17	-.46	.00
9	.11	-.09	.28	-.02	.00	-.04	.04	-.08	-.68	.00	.07	-.06	.18	-.37	.00
10	.04	-.11	.51	.37	.00	-.07	.08	.06	-.93	.00	-.02	-.05	.39	-.30	.00
11	.07	-.07	.17	-.13	.00	-.06	.03	-.16	-.56	.00	.02	-.04	.05	-.42	.00
12	.07	-.06	.22	-.10	.00	-.04	.03	-.05	-.48	.00	.04	-.04	.14	-.33	.00

Response of the exchange rate to a unit innovation in the:
(1,1): Exchange Rate
(1,2): Stock of Foreign Assets
(1,3): Domestic Price Level
(1,4): Foreign Price Level
(1,5): Foreign Real Output

Response of the stock of foreign assets to a unit innovation in the:
(2,1): Exchange Rate
(2,2): Stock of Foreign Assets
(2,3): Domestic Price Level
(2,4): Foreign Price Level
(2,5): Foreign Real Output

Response of the domestic price level to a unit innovation in the:
(3,1): Exchange Rate
(3,2): Stock of Foreign Assets
(3,3): Domestic Price Level
(3,4): Foreign Price Level
(3,5): Foreign Real Output

The infinite order moving average representations are truncated after 12 quarters.