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TRADE AND UNEVEN GROWTH

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ABSTRACT

We consider trade between two countries of unequal size, where the creation of new intermediate inputs occurs in both. We assume that the knowledge gained from R&D in one country *does not* spillover to the other. Under autarky, the larger country would have a higher rate of product creation. When trade occurs in the final goods, we find that the smaller country has its rate of product creation slowed, even in the long run. In contrast, the larger country enjoys a temporary increase in its rate of R&D. We also examine the welfare consequences of trade in the final goods, which depend on whether the intermediate inputs are traded or not.

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1. Introduction

There has recently been a resurgence of interest in models of economic growth, prompted by the development of models in where the growth rate depends endogenously on the accumulation of human capital (Lucas, 1988) or product creation by firms (Romer, 1990). These models can be used to address a number of issues long discussed in the trade and development literature, including: conditions under which industrialization will occur in a country (Murphy, Shleifer and Vishny, 1989); formal analysis of the "product cycle" in trade (Grossman and Helpman, 1989c; Segerstrom, Anant and Dinopoulos, 1989); the effects of tariffs and quotas on growth (Dinopoulos, Oehmke and Segerstrom, 1989; Grossman and Helpman, 1989d); and other issues. In this paper we shall be concerned with the effect of international trade on the *rate of product development* in a country, using a model which is closely related to that of Grossman and Helpman (1989b) and Rivera-Batiz and Romer (1989).[1]

The basic outline of our model is as follows. Each country produces a final good which is traded internationally. The final goods are assembled from a range of intermediate inputs, whose number will grow endogenously over time. As in Ethier (1982), an increase in the range of intermediate inputs allows for more efficient production of the final goods. In the initial version of our model we shall suppose that the intermediate goods are not traded between countries, but as we later show, this assumption is easily relaxed.

The intermediate inputs themselves are produced using labor and the stock of knowledge within each country. Like Grossman and Helpman (1989b) and Rivera-Batiz and Romer (1989), we shall assume that this knowledge increases with the range of intermediate inputs developed: as the number of products

[1] At times we shall loosely refer to the rate of product development as the "rate of growth", and it is related to the growth in GNP. See footnote 6.

grows the fixed costs of creating new ones falls, which will allow continuous growth to occur. However, unlike these authors, we shall assume that the knowledge does not cross borders, but is only available to the firms within each country. This assumption will be the driving force behind our results.

Our assumption that knowledge of production techniques does not cross borders can be justified on several grounds. First, the same assumption is used in the Ricardian model of international trade, where production functions differ internationally. This assumption was dropped in the Heckscher-Ohlin model, where we instead assume identical technologies in all countries. But since Minhas (1962), the empirical evidence has often rejected this assumption. The most recent, comprehensive test of the Heckscher-Ohlin model is by Bowen, Leamer and Sveikauskis (1987), who find that the predictions of this model fail sadly, with evidence that technological differences across countries account for part of the failure.[2]

Second, our model generates the realistic result that countries will grow at different rates. This observation is actually one of the stylized facts put forth by Kaldor (1961), and is supported by more recent evidence.[3] Even on purely methodological grounds, we would argue that the analysis of a model with uneven growth rates across countries in of interest. Much of the existing literature on endogenous growth has focused on the case where a steady state or "balanced growth" solution exists, with both countries growing at the same

[2] See also the supporting evidence of Dollar, Wolff and Baumol (1988). It is still possible that technical knowledge does cross borders, but it used too slowly to give rise to identical production functions. We discuss the slow transmission of knowledge in section 6.

[3] Baumol (1986) has suggested that there is a convergence of growth rates among industrial countries, but this evidence is questioned on sample selection grounds by Romer (1989), from whom the reference to Kaldor (1961) is drawn.

exponential rate. This case naturally simplifies the analysis, but is not necessarily the most realistic. In our model, no such balanced growth solution will exist (unless the countries are identical).

To describe our most important results, let us measure labor force of each country in terms of efficiency units in the R&D activity, i.e. the number of new intermediate inputs which could be developed by the population in a given year. Then when trade is opened between countries of different size, we shall find that the smaller country has its growth rate of new products permanently slowed: even in the long run, this growth rate does not approach its autarky value. In contrast, the larger country enjoys a higher rate of product creation, while approaching its autarky rate in the long run. From a welfare perspective, the effect of these results depends on whether the intermediate inputs are traded or not.

Several other papers are related to the issues addressed here. Boldrin and Scheinkman (1988), Krugman (1988), Lucas (1988) and Young (1989) all analyse models where trade may be detrimental to growth in a country. These papers rely on some form of learning by doing, where the technology in each industry is affected by past production in that, and possibly other, industries. While our model of endogenous product development is quite different in detail, the results are remarkably similar to those of Young (1989). A numerical analysis of trade and product development is provided by Markusen (1989), and the analysis of this paper was in fact prompted by the desire to obtain analytical solutions to the questions he posed.[4]

In the next section we describe our model and determine the equilibrium conditions. In section 3 we show how these conditions can be reduced to

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[4] Markusen was primarily concerned with determining whether trade would eliminate R&D in one country, as we discuss briefly in section 4.

certain second-order differential equations. We argue that there is a stable solution, which describes the equilibrium of the economies. In section 4, the rates of product development are characterized, with special attention to their limiting values. In section 5 we provide the welfare analysis. Section 6 discusses generalizations of the model and gives conclusions.

2. The Model

The model we shall use is a simplified version of Grossman and Helpman (1989b), so our presentation will be brief. There are two countries, labelled by $i=1,2$, with labor as the only resource. Let n_i denote the number (measure) of intermediate inputs available in country i at time t, where we suppress t as an explicit argument. We shall initially suppose that the intermediate inputs are *not* traded, so each country uses only its own varieties; this assumption will be relaxed in section 6. Denote the quantity of each intermediate input by $x_i(\omega)$, where ω is an index of varieties. By symmetry of our model, $x_i(\omega)$ will be constant across all varieties in each country, so that $x_i(\omega) \equiv x_i$. Letting y_i denote the output of the final good in each country, the production function for the final goods are given by:

$$y_i = \left(\int_0^{n_i} x_i^{\alpha} \, d\omega \right)^{1/\alpha} = n_i^{1/\alpha} x_i , \qquad 0 < \alpha < 1, \tag{1}$$

where the elasticity of substitution among the intermediate inputs is $1/(1-\alpha)$.

Let p_{yi} denote the price of the final good produced in country i, and p_{xi} denote the price of each intermediate input available there. An important variable in our analysis will be the share of world expenditure spent on the products of country i. We will let s_i denote the share of world expenditure on *final* products from country i, $s_i \equiv p_{yi}y_i/(p_{y1}y_1 + p_{y2}y_2)$. Because each final product is produced with only the intermediate inputs of that country, the value of output $p_{yi}y_i$ equals the cost of inputs $p_{xi}X_i$, where $X_i \equiv n_ix_i$ denotes the aggregate quantity of intermediate inputs in country i. It follows that the share of world expenditure on final products is *equal* to the share on intermediate inputs, $s_i = p_{xi}X_i/(p_{x1}X_1 + p_{x2}X_2)$. We will use this result frequently.

The final goods y_i enter the utility functions of consumers, while the number of intermediate inputs created each period is determined by the R&D activities of firms. We specify next the problems solved by consumers (section 2.1) and firms (section 2.2), from which the equilibrium conditions for the economies can be determined.

2.1. Consumers

Consumers in both countries have the identical utility function,

$$U_t = \int_t^{\infty} e^{-\rho(\tau-t)} \log[u(y_1(\tau),y_2(\tau))]\, d\tau \quad , \tag{2}$$

where $y_i(\tau)$ is the chosen consumption of the final good from country i at time τ, i=1,2. We shall suppose that the instantaneous utility function $u(y_1,y_2)$ takes on a CES form,

$$u(y_1,y_2) = [y_1^{\beta} + y_2^{\beta}]^{1/\beta} , \qquad 0 < \beta < 1, \tag{3}$$

where the elasticity of substitution is $1/(1-\beta)$. Results for $\beta = 1$ can be obtained as a limiting case of our analysis. We ignore $\beta \leq 0$, however, since in that case zero consumption of either final good would give utility of $-\infty$ in (2), so that neither country could survive in autarky.

Since $u(y_1,y_2)$ is homogeneous of degree one, the corresponding expenditure function can be written as $E = \pi(p_{y1},p_{y2})u$, where π can be thought of as a cost of living index (it is formally the unit-cost function for (3)). It follows that utility can be expressed as $u = E/\pi(p_{y1},p_{y2})$, or expenditure deflated by the cost of living index. Substituting this into (2), we can write utility as,

$$U_t = \int_t^{\infty} e^{-\rho(\tau-t)} [\log E(\tau) - \log\pi(p_{y1},p_{y2})] \, d\tau \; . \tag{4}$$

Consumers maximize (4) subject to a budget constraint stating that the present discounted value of expenditure cannot exceed the present discounted value of labor income plus initial assets. We shall suppose that consumers face an *integrated* world capital market, so there is a single, endogenous interest rate at which citizens of either country can borrow or lend. We will let $R(t)$ denote the *cumulative* interest factor from time 0 to t ($R(0) = 1$), so that $\dot{R}(t)$ is the instantaneous interest rate at time t. Then the first order condition for maximizing (4) is,

$$\dot{E}/E = \dot{R} - \rho \; . \tag{5}$$

Thus, the path of expenditure will be rising (falling) as the interest rate is greater (less) than the consumers' discount rate. Savings takes the form of riskless equities issued by firms to finance their R&D activities, as we describe next.

2.2. Firms

Grossman and Helpman (1989b) and Rivera-Batiz and Romer (1989) assume that the technical knowledge available in the world is directly related to the product development which has occurred in *both* countries. In contrast, we shall assume that this knowledge does not cross borders. Letting technical knowledge be denoted by K_i, i=1,2, we shall assume that this knowledge increases in proportion to the number of products already developed in each country, so that $K_i \equiv n_i$. This formulation presumes that there are no diminishing returns in the accumulation of knowledge, and will allow continuous growth to occur.

The labor cost of developing a new product in each country is given by $a_{ni}/K_i \equiv a_{ni}/n_i$. It will be convenient to measure labor in each country in terms of efficiency units in R&D, so that $a_{ni} \equiv 1$, i=1,2. The cost of creating a new intermediate input is then simply $1/n_i$. Firms finance this R&D expenditure by issuing equities, which provide a share of the future stream of profits as a return.

Let the cost of producing each unit of the intermediate input be $w_i a_{xi}$, where w_i denotes the wage in country i=1,2. The demand for intermediate inputs is derived from the CES production function in (1). The producers of the intermediates in each country engage in monopolistic competition, so the prices p_{xi} will be a constant markup over marginal costs:

$$p_{xi} = w_i a_{xi}/\alpha , \qquad i=1,2. \tag{6}$$

It follows that the instantaneous profits from producing the intermediate input are $(p_{xi}-w_i a_{xi})x_i = [(1-\alpha)/\alpha]w_i a_{xi} X_i/n_i$, where we earlier let $X_i \equiv n_i x_i$ denote the aggregate supply of intermediates in each country. At each instant of time, the development of new intermediate inputs will occur until the present discounted value of profits is zero, that is:

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} \left(\frac{1-\alpha}{\alpha}\right) \frac{w_i a_{xi} X_i}{n_i} \, d\tau = w_i/n_i \,. \tag{7}$$

where the right side of (7) is the cost of developing a new input.

Differentiating (7) with respect to t, and dividing by w_i/n_i, we obtain one equilibrium condition for the economies:

$$\left(\frac{1-\alpha}{\alpha}\right) a_{xi} X_i + \frac{\dot{w}_i}{w_i} - \frac{\dot{n}_i}{n_i} = \dot{R} \,. \tag{8}$$

As discussed by Grossman and Helpman (1989b), (8) can be interpreted as a no-arbitrage condition, which equates the rate of interest with the return on assets of an input producing firm. The first term on the left is the instantaneous profits relative to the costs of R&D (dividends); while the remaining terms on the left are the rate of change in present discounted profits in (7) (capital gains).

The second equilibrium condition for the economies equates total demand with the supply of labor. Let the endowment of labor in each economy be denoted by L_i, i=1,2. Since the labor required to produce one new intermediate input is $1/n_i$, the total labor devoted to R&D is $\dot{n}_i/n_i$. The labor devoted to the actual production of existing intermediate inputs is $a_{xi}X_i$, so equilibrium in the labor market requires that,

$$\dot{n}_i/n_i + a_{xi} X_i = L_i \,. \tag{9}$$

In the next section we show how (8) and (9) can be simplified to obtain differential equations governing the rate of product development in each country.

3. Equations of Motion

To simplify the equilibrium conditions, let $\mu_i \equiv \dot{n}_i/n_i$ denote the rate of product development in each country. We can solve for $X_i = (L_i - \mu_i)/a_{xi}$ from (9), and substitute into (8) obtaining a single equation for each country:

$$\mu_i = (1-\alpha)L_i + \alpha(\dot{w}_i/w_i) - \alpha\dot{R} . \tag{10}$$

Equation (10) is particularly useful in understanding the factors governing long-run growth. Suppose that each country is in *autarky*, and set $w_i \equiv 1$ by choice of numeraire so that $\dot{w}_i = 0$. Consider a steady-state growth path where expenditure is constant, so from (5), the instantaneous interest rate $\dot{R}$ equals the discount rate ρ. Then it follows from (10) that the long-run growth rate in each country is:

$$\mu_i = g_i \equiv (1-\alpha)L_i - \alpha\rho > 0, \tag{11}$$

where we assume that this expression is positive. We shall refer to (11) as the autarky growth rates of the countries, denoted by g_i.[5] The *larger* country, as measured by the effective labor force L_i, then grows faster in autarky.[6] This reflects the fact the the larger country will have a greater variety of intermediate inputs, more knowledge, and thus lower costs of R&D.

We shall assume that the two countries are of different size, and let country 1 be *larger* in terms of the effective labor force. In autarky, then,

[5] In fact, it can be argued that the autarky economy must jump immediately to the growth rate g_i. For a single country, the term $\dot{s}_i/s_i$ is zero in (14), and by inspection the only stable solution is $\mu_i = g_i$ for all t. As discussed below (14), the unstable solutions with $\mu_i \to L_i$ or $\mu_i \to 0$ are not equilibria.

[6] Using (1) and (9), final output y_i can be written as $n_i^{(1-\alpha)/\alpha}(L_i-\mu_i)/a_{xi}$. Then in autarky, final output grows at the rate $(1-\alpha)g_i/\alpha$.

country 1 will be growing faster than the country 2. In order to solve for the dynamics after opening trade, it will not be appropriate to focus on a steady-state solution where $\dot{R} = \rho$, since this solution will not exist. Instead, we need to express the instantaneous interest factor $\dot{R}$ which appears in (8) in terms of underlying variables in the system.

Recall from our discussion in section 2 that s_i denotes the share of world expenditure on *either* final or intermediate goods. Choosing the latter measure $s_i = p_{xi}X_i/E$, and so expenditure can be written as $E = p_{x1}X_1/s_1 = p_{x2}X_2/s_2$. Since $(\dot{p}_{xi}/p_{xi}) = (\dot{w}_i/w_i)$ from (6), the interest rate is,

$$\dot{R} = \dot{E}/E + \rho = \left(\frac{\dot{w}_i}{w_i} + \frac{\dot{X}_i}{X_i}\right) - \frac{\dot{s}_i}{s_i} + \rho, \quad \text{for } i = 1,2. \tag{12}$$

where we have used (5). It is convenient to eliminate $\dot{X}_i$ from (12). To do so, differentiate (9) to obtain:

$$\dot{X}_i/X_i = -\dot{\mu}_i/(L_i - \mu_i) . \tag{13}$$

Substituting (13) into (12), we can express the interest rate in terms of underlying variables.

We are now in a position to summarize the dynamics of the system. Substituting (12) and (13) into (10) it turns out that the wage terms vanish, and we obtain two differential equations governing the rate of product development in each country:

$$\mu_i - \alpha\dot{\mu}_i/(L_i - \mu_i) = g_i + \alpha(\dot{s}_i/s_i) . \tag{14}$$

The change in market shares which appears on the right of (14) is endogenous, depending on the prices and variety of goods available from each country. To think about the nature of solutions to (14), however, it is convenient to think of the market shares as exogenous functions of time (taking on their solution values, for example). Then (14) is a system of nonautonomous *first-order* differential equations in $\mu_i = \dot{n}_i/n_i$, or *second-order* differential equations in n_i. This means that we are free to specify the initial number of products $n_i(0)$ in each country, but *also* have the initial rates of product development $\mu_i(0)$ as free parameters.

Why does this system not determine the initial rates of product development? It turns out that for many initial values $\mu_i(0)$ the solutions to (14) are unstable, implying that either $\mu_i \to L_i$ as $t \to \infty$, or that $\mu_i \to 0$ in finite time. The former solution means that nearly all the resources in country i are absorbed in R&D, with expenditure on final goods approaching zero. Residents of that country would be accumulating increasing amounts of assets from firms, which would violate their transversality condition. We therefore rule out solutions in which $\mu_i \to L_i$ as equilibria.

The other solution we shall rule out is where $\mu_i \to 0$ in both countries in finite time. Adapting an argument from Grossman and Helpman (1989a), we can argue that this path would violate the optimality conditions of firms. Note that if R&D ceased in both countries, then expenditure would be constant (setting either wage as numeraire), so $\dot{R}=\rho$ from (5). Instantaneous profits for input producing firms are $[(1-\alpha)/\alpha]a_{xi}X_i/n_i = [(1-\alpha)/\alpha]L_i/n_i$, since $\dot{n}_i=0$ in (9). Since we have assumed that $g_i=[(1-\alpha)L_i-\alpha\rho]>0$, it follows that profits exceed ρ/n_i. Then the present discounted value of profits (using $\dot{R}=\rho$) exceeds $1/n_i$, which is the fixed cost of creating a new input, so this is not an equilibrium. In the next section we solve the differential equations (14), while ignoring solutions where either $\mu_i \to L_i$ as $t \to \infty$, or $\mu_i \to 0$ in both countries.

4. Growth Paths

To analyse the effects of trade on product development, we proceed in several steps. First, we shall solve the differential equations (14), treating the market shares as exogenous functions of time. Second, we shall determine the equilibrium path of the market shares. Third, we shall then characterize the rates of product development in each country.

Recalling that $\mu_i = \dot{n}_i/n_i$, we can multiply (14) by $1/\alpha$ and directly integrate to obtain,

$$n_i^{1/\alpha}(L_i - \mu_i) = \alpha k_i s_i e^{(g_i/\alpha)t} , \qquad (15)$$

where the $k_i > 0$ are arbitrary constants of integration, i=1,2. Multiply both sides of (15) by the factor $-(1/\alpha)e^{-L_i t/\alpha}$. This allows us to again integrate both sides from 0 to t, obtaining,

$$n_i^{1/\alpha} e^{-L_i t/\alpha} = n_i(0)^{1/\alpha} - k_i \int_0^t s_i e^{-(L_i+\rho)\tau} d\tau . \qquad (16)$$

Using equation (16) we can calculate $\mu_i = \dot{n}_i/n_i$. We want to rule out solutions in which $\mu_i \to L_i$ as $t \to \infty$, or where $\mu_i \to 0$ in finite time. This is done by choosing k_i as,[7]

$$k_i = n_i(0)^{1/\alpha} / \int_0^\infty s_i e^{-(L_i+\rho)\tau} d\tau . \qquad (17)$$

By differentiating (16), and substituting (17), the resulting rates of product development are:

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[7] Calculating μ_i from (16), it is not difficult to show other values of $k_i > 0$ imply that μ_i approaches L_i or zero.

$$\mu_i = L_i - \left(\frac{\alpha s_i e^{-(L_i+\rho)t}}{\int_t^\infty s_i e^{-(L_i+\rho)\tau} d\tau} \right). \tag{18}$$

Note that in this expression for the rates of product development, the market shares are endogenous. In order to determine the evolution of the market shares along the equilbrium path we need to consider the demand side of the model, which we do next.

The prices of the two final goods in each country will equal their marginal cost, given the production function in (1). All varieties of the intermediate inputs have the same price in each country, as shown in (6). Using the unit-costs corresponding to the CES production function, the price of each final good is then,

$$p_{yi} = (a_{xi} w_i / \alpha) n_i^{-(1-\alpha)/\alpha} . \tag{19}$$

Given that the utility function $u(y_1, y_2)$ is CES, the relative market shares for the two final goods are determined by $s_1/s_2 = (p_{y1}/p_{y2})^{-\beta/(1-\beta)}$. However, the expenditure on each final good equals the value of intermediate inputs used in its production, and so $s_1/s_2 = p_{x1}X_1/p_{x2}X_2 = a_{x1}w_1X_1/a_{x2}w_2X_2$. In this expression we can substitute $X_i = (L_i - \mu_i)/a_{xi}$ from (9). Combining these various results with (19), we can express the relative market shares as:

$$\frac{s_1}{s_2} = \left(\frac{w_1}{w_2}\right)\left(\frac{L_1-\mu_1}{L_2-\mu_2}\right) = \left(\frac{a_{x1}w_1}{a_{x2}w_2}\right)^{\frac{-\beta}{(1-\beta)}} \left(\frac{n_1}{n_2}\right)^{\gamma}, \qquad \gamma \equiv \frac{(1-\alpha)\beta}{(1-\beta)\alpha} . \tag{20}$$

To fully determine the path of markets shares, solve for the relative wage w_1/w_2 from the latter equality in (20), and substitute this expression back into (20). Making use of (15)-(18), we can obtain the following equation depending solely on the market shares:

$$\frac{s_1}{s_2} = \left(\frac{a_{x2}}{a_{x1}}\right)^{\beta/(1-\beta)} \left(\frac{n_1(0)}{n_2(0)}\right)^{\gamma} e^{\gamma(1-\alpha)(L_1-L_2)t} \cdot$$

$$\left(\frac{\int_0^\infty e^{-(L_2+\rho)\tau} s_2\, d\tau}{\int_0^\infty e^{-(L_1+\rho)\tau} s_1\, d\tau}\right)^{\frac{(1-\alpha)\beta}{(1-\beta)}} \left(\frac{\int_t^\infty e^{(L_2+\rho)(t-\tau)} s_2\, d\tau}{\int_t^\infty e^{(L_1+\rho)(t-\tau)} s_1\, d\tau}\right)^{\frac{\alpha\beta}{(1-\beta)}} . \qquad (21)$$

Noting that $s_2 = 1-s_1$, equation (21) depends only on the function $s_1(t)$ for $t \in [0,\infty)$. Two features of this equation should be noted. First, we shall be assuming that there exists a path of the market share $s_1(t)$ which satisfies (21). As an example, consider the special case $\alpha = \beta/(1+\beta)$. Then it can be verified that (21) is satisfied by:

$$s_i(t) = \left(\frac{s_i(0)e^{L_i t}}{s_1(0)e^{L_1 t} + s_2(0)e^{L_2 t}}\right), \qquad i=1,2, \qquad (22a)$$

with,

$$s_1(0)/s_2(0) = (a_{x2}/a_{x1})^{\beta}[n_1(0)/n_2(0)]. \qquad (22b)$$

We believe that solutions to (21) occur more generally, but do not prove this here.[8]

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8 Note that from (20) we can solve for s_i as a function of n_i, μ_i, and various parameters. Substituting this into (18), we obtain two differential equations in n_i and $\dot{n}_i$, where $\dot{n}_i \geq 0$ should be imposed. For any values of k_i in a certain compact set these equations can be solved, and $\tilde{k}_i$ defined by (17) are in the same set. The equilibrium we are interested in corresponds to the fixed point

Second, in our derivation of (21) we have been assuming that $\mu_i > 0$ in both countries. This means that when the market shares s_i satisfying (21) are substituted into (18), we must have $\mu_i > 0$, $i=1,2$. If instead R&D stops in one country, then the equations determining the equilibrium are different than what we have presented. If $\mu_2 = 0$, for example, then this is substituted into (20), and only the formula for μ_1 is taken from (18). As a result, equation (21) takes on a somewhat different form. We will discuss the conditions under which R&D will stop in one country, after first solving the case where R&D continues in both.

The general properties of the path of s_1 can be deduced from the fact that $e^{\gamma(1-\alpha)(L_1-L_2)t}$ appears as "forcing term" in (21), tending to *increase* the market share of country 1. This rising market share allows us to determine a number of results about the long run rates of product development:

Proposition 1

Suppose that $\mu_i > 0$, $i=1,2$. Then there exists $T \geq 0$ (depending on the parameters of (21)) such that:

(a) $\dot{s}_1 > 0$ for $t \geq T$, and $\lim_{t\to\infty} s_1 = 1$;

(b) $\mu_1 > g_1$ for $t \geq T$, and $\lim_{t\to\infty} \mu_1 = g_1$;

(c) $g_2 > \mu_2 > g_2 - [(1-\alpha)^2/\alpha](L_1 - L_2)$ for $t \geq T$, and

$$\lim_{t\to\infty} \mu_2 = g_2 - \Delta(L_1-L_2), \quad \text{with } \Delta = \frac{(1-\alpha)^2}{\alpha}\left[\frac{(1-\beta)}{\alpha\beta} + 1\right]^{-1}.$$

$k_i = \bar{k}_i$. While it appears that such a fixed point will exist, it is difficult to determine whether $\dot{n}_i > 0$ or not. If so, then the corresponding path $s_1(t)$ satisfies (21) by construction.

Referring back to the differential equations (14), we see that the growth rate μ_i of an economy is positively related to the change in its market share $\dot{s}_i/s_i$. Thus, $\dot{s}_1 > 0$ will tend to quicken product development in country 1. Since $s_1 \rightarrow 1$ then $\dot{s}_1/s_1 \rightarrow 0$, so the positive impact of the rising market share on growth in country 1 must be transitory, as indicated in (b). Country 1 will approach its autarky rate of product development from above. This result is illustrated in Figure 1, along the path labelled $\mu_1(t)$.

In country 2, which is smaller in terms of the effective labor force, its falling market share s_2 will slow the rate of product development. Since $s_2 \rightarrow 0$ it turns out that $\dot{s}_2/s_2$ approaches a strictly negative value. Then the growth rate in country 2 is permanently less than in autarky, by an amount which depends on the difference in the labor force of the two countries, as indicated in (c). We are not able to determine in general whether μ_2 approaches its long run value from above or below. However, notice that when $\beta = 1$ then the lower bound in (c) equals the limiting value of μ_2. In this case μ_2 must approach its long run value from above, as illustrated along the path $\mu_2(t)$ in Figure 1.

Note that in drawing Figure 1 we have assumed that the lower bound for μ_2, $g_2 - [(1-\alpha)^2/\alpha)](L_1 - L_2)$, is positive. This condition guarantees that growth will continue in both countries after the opening of trade. As either Δ grows or the two country become more different in size, then it becomes less likely that R&D will continue in country 2. Suppose, for example, that $g_2 - \Delta(L_1 - L_2) < 0$. If it also happened that $\mu_2(0) > 0$, then R&D would occur in country 2 for some finite period of time, and then cease. On the other hand, since we know from Proposition 1 that $\mu_2(0)$ falls below g_2, it is possible that R&D in country 2 would cease upon the opening of trade.

While the results of Proposition 1 characterize the path of market shares for large t, we are also interested in determining properties of s_i for all time.

It turns out that s_1 will be monotonically increasing if either the initial number of products in country 2 relative to country 1 is sufficiently small, if α lies in the range $\beta/(1+\beta) \leq \alpha \leq 0.5$. Then we have:

<u>Proposition 2</u>

Suppose that $\mu_i > 0$, $i = 1,2$, and either:

(i) $n_2(0)/n_1(0) \leq N$ (where $N > 0$ depends on the parameters of (21)); or,

(ii) $\beta/(1+\beta) \leq \alpha \leq 0.5$.

Then $\dot{s}_1 > 0$ for all $t \geq 0$, and the results of Proposition 1 hold for $T = 0$.

Thus, under either conditions (i) or (ii) we can simply replace T by zer in Figure 1, and obtain the paths of product development for all $t \geq 0$. The pr of (i) relies an interesting feature of our model. Suppose that instead of opening trade at time 0, we wait until time T; in the interval [0,T] we let t countries grow at their autarky rates. Then it turns out that the values of s and μ_i for $t > T$ are identical to what they would have been if the countries h traded during [0,T]. That is, delaying the opening of trade has not effect at on the rates of R&D or market shares after trade is opened. This means that for any initial values $n_i(0)$, $i=1,2$, N can be computed as the ratio $n_2(T)/n_1(T)$ the countries grew in autarky over [0,T].

Condition (ii) was illustrated by (22), which is the equilibrium market shares when $\alpha = \beta/(1+\beta)$. By inspection, s_1 is increasing for all t in (22), an the other results in Proposition 1 then apply for $T = 0$. The line $\alpha = \beta/(1+\beta)$ i shown in Figure 2, and the region $\beta/(1+\beta) \leq \alpha \leq 0.5$ is labelled as A. When deriving the welfare implications in the next section, we will be supposing that either (α, β) falls in A, or $n_2(0)/n_1(0) \leq N$, so that Proposition 2 applies

Welfare

Could the slower rate of product development in the small country, as npared to autarky, ever lead to welfare losses due to trade? To address this ꞊stion, observe that for the utility function in (4) there are two sources of ꞊ns (or loss) from trade: intertemporal gains, by choosing a path of expen- ꞊ure E_i which differs from autarky due to the global capital market; and ꞊ratemporal or "static" gains, by having prices for final goods which differ ꞊m autarky. In this paper we will not attempt to quantify the magnitude of tertemporal gains.[9] Instead, we will focus on the intratemporal gains or sses, by simply comparing the prices faced by each country under free trade ꞊th their values if the economies had continued in autarky. We shall assume at the results of Proposition 1 apply with $T=0$.

Prices for final goods affect utility through the price index $\pi(p_{y1}, p_{y2})$. reduction in the price of each final good indicates a fall in the price index π, ꞊d a rise in instantaneous utility. We need to determine whether the opening trade leads to such a fall in final goods prices for each country, or not.

First consider country 1. Choosing $w_1 \equiv 1$ as the numeraire, the prices of nal goods are given by (19). From Proposition 1 we know that this country ꞊periences a higher rate of product development with trade than in autarky. It ꞊llows immediately that p_{y1} with trade is less than in autarky, and so the ꞊untry experiences gains even in the consumption of its own final good. In ꞊dition, country 1 has available the final good from country 2, and so we ꞊nclude that the price index $\pi(p_{y1}, p_{y2})$ must be lower under free trade than in

While we normally think of countries as gaining from intertemporal trade, e do not assert that this result holds here. The difficulty is that the ꞊terest rate $\dot{R}$ with trade differs from autarky, so that consumers with ꞊sitive assets may not be able to earn the same interest as in autarky.

autarky.

Turning to country 2, we now choose $w_2 \equiv 1$ as the numeraire. Since t rate of product development with trade is *less* than in autarky, it follows f (19) that the price of its own final good y_2 is *higher* under free trade. On other hand, country 2 has available the final good from country 1. So wheth the overall price index $\pi(p_{y1}, p_{y2})$ is greater or less than in autarky depends if the availability of y_1 more than offsets the higher price of y_2. In order make this comparison, we shall use the result:

$$\pi(p_{y1}, p_{y2}) = p_{y2}\, s_2^{(1-\beta)/\beta} \quad . \qquad (2$$

which is proved in Feenstra (1990). Thus, the price index π with trade will lower the smaller is s_2, indicating that imports from country 1 make up a greater portion of country 2 expenditure, or the lower is β, indicating that two final goods are less perfect substitutes.

Let the price of y_2 under autarky be denoted by $\tilde{p}_{y2}$, with the number o products denoted $\tilde{n}_2$. The relationship between these is shown by (19), where we set $w_2 \equiv 1$. We are then interested in comparing $\pi(p_{y1}, p_{y2})$ with $\tilde{p}_{y2}$. Making use of (23), we see that the price index with trade will be *less* than autarky if and only if,

$$s_2^{(1-\beta)/\beta} < (n_2/\tilde{n}_2)^{(1-\alpha)/\alpha} \quad . \qquad (2$$

Taking logs of (24), we can express this inequality in the equivalent form:

$$\frac{(1-\beta)}{\beta} > \frac{(1-\alpha)}{\alpha}\left[\frac{\log(n_2/\tilde{n}_2)}{\log s_2}\right] . \qquad (2$$

Consider taking the limit of (25) as $t \to \infty$. Applying L'Hospitals Rule, t ression in square brackets on the right becomes $\lim_{t\to\infty} (\mu_2 - g_2)/(\dot{s}_2/s_2) = \alpha$,

ng (14) and the fact that $\mu_2 \to 0$. Thus, as $t\to\infty$, the inequality (25) becomes $\beta)/\beta \geq (1-\alpha)$, which is certainly violated for some values of α and β. We clude that for these values of α and β, there will exist T such for $t \geq T$, the ce index $\pi(p_{y1},p_{y2})$ with trade *exceeds* the price $\tilde{p}_{y2}$ faced in autarky.

These results are illustrated in Figure 2. In the upper portion of this gram we show the region $(1-\beta)/\beta < (1-\alpha)$, labelled as B, in which (25) is lated and the price index for country 2 exceeds its autarky value for ficiently high t. As an extreme example, consider $\beta = 1$, so that the final ods of each country are perfect substitutes. Then the slower rate of product elopment in country 2 with trade will lead to a higher price for y_2, and this not offset by the availability of y_1 (since this perfect substitute does not ovide extra utility). In this extreme case the price index $\pi(p_{y1},p_{y2})$ with de exceeds the autarky price $\tilde{p}_{y2}$ for all t after trade is opened. For lower ues of β, the price index for country 2 will be higher than in autarky only a restricted range of α, and for t sufficiently high.

Next, consider the complementary region in Figure 2, in which $(1-\beta)/\beta \geq$ $\alpha)$. Can we conclude that the prices index for country 2 will be less than in arky, so that the country experiences gains from trade in this sense? It ns out that this assertion is true so long as a condition on the initial value s_2 is satisfied. Our welfare results are summarized by:

<u>oposition 3</u>

ppose Proposition 1 applies with $T = 0$. Then:

With $w_1 \equiv 1$, the price index of country 1 under free trade is less than it uld have been in autarky;

(b) With $w_2 \equiv 1$, and $\alpha < 1-[(1-\beta)/\beta]$, there will exist T such that for $t > T$ t
price index for country 2 under trade exceeds its autarky value;

(c) With $w_2 \equiv 1$, and $\alpha \geq 1-[(1-\beta)/\beta]$, the price index for country 2 will be le
than in autarky provided that,

$$s_2(0) \leq \left(\frac{L_2 + \rho}{L_2 + \rho + [(1-\alpha)/\alpha]^2(L_1 - L_2)}\right). \qquad (2$$

It is difficult to check whether (26) is satisfied in general, since the initial market shares are determined by (21). However, lower values of the initial products $n_2(0)/n_1(0)$ in (21) tend to correspond to lower values of th initial shares $s_2(0)/s_1(0)$, as shown by the example in (22). So condition (2 is consistent with condition (i) of Proposition 2 that $n_2(0)/n_1(0) \leq N$, and t is one of the conditions that allows us to apply Proposition 1 with $T=0$.

We conclude this section by briefly examining the behavior of relative wages in the countries. From (20) we obtain,

$$\left(\frac{a_{x1}w_1}{a_{x2}w_2}\right) = \left(\frac{s_1}{s_2}\right)^{-(1-\beta)/\beta}\left(\frac{n_1}{n_2}\right)^{(1-\alpha)/\alpha}. \qquad (2$$

Differentiating this expression and making use of earlier results, we can sho that *if* $\dot{s}_1 > 0$ *for* $t \geq T$, *then* (w_2/w_1) *will be declining for* $t \geq T$.[10] This behaviour of the relative wage is in constrast to the relative product price: when $\dot{s}_1 > 0$ then the terms of trade (p_{y2}/p_{y1}) must be *increasing*, leading to the loss of market share for country 2. Thus, our model generates opposite movements in the relative wage and terms of trade across countries.

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[10] We use (A3) and (A6) from the Appendix.

ieneralization and Conclusions

We have been assuming throughout the paper that the intermediate inputs not traded internationally, but can now relax this. When these products are ed the final goods in (1) will use the inputs from both countries, so that y_1 y_2 will have identical prices and sell in identical quantity.[11] It follows the market shares for the *final* goods are fixed at 1/2. Since $y_1 = y_2$ the antaneous utility function in (3) can be written as,

$$u(y_1,y_2) = [y_1^{\beta} + y_2^{\beta}]^{1/\beta}$$

$$= 2^{1/\beta}\left(\int_0^{n_1} x_1^{\alpha}\, d\omega + \int_0^{n_2} x_2^{\alpha}\, d\omega\right)^{1/\alpha}, \qquad (3')$$

re the second line follows from $y_1 = y_2$ in (1), with the intermediate inputs ınd x_2 from *both* countries used in the production of each final good. Comng (3') with (3), we see that the utility function with intermediate inputs led is identical to our earlier case with $\alpha = \beta$ imposed there. Working ıugh the rest of our earlier analysis, we find that it continues to hold, with nterpreted as the share of world expenditure on intermediate inputs of ıtry i. We conclude that the model with *free trade in the intermediate its is formally equivalent to the case of no trade with $\alpha = \beta$ imposed.*

Thus, the qualitative results on growth rates reported in Propositions 1 2 carry over to the case of free trade in the intermediate inputs. The fare results in Proposition 3 also carry over, but now the condition that β makes a difference. In Figure 2, the restriction $\alpha = \beta$ corresponds to the

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This result depends on having the final goods costlessly assembled from the ːrmediate inputs, so that that wage in each country does not affect the final d's price.

diagonal line, which lies *outside* the region B where country 2 could face a possible welfare loss. Thus, when intermediates are traded, and condition (2 is satisfied, we conclude that country 2 will face lower prices with trade a experience gains in this sense. The reason is that the slower rate of produc development there is offset by the availability of imported intermediates, s that the final goods price p_{y2} is less than in autarky.

Our result that the possibility of a welfare loss for country 2 occurs mainly in the *absence* of trade in the intermediate inputs is consistent with the literature on trade and distortions (Bhagwati, 1983). In this case, openi trade in the final goods can worsen the distortion caused by no trade in the intermediates, leading to the possibility of a higher price index in country 2 with trade. From a policy perspective, our results indicate the importance free trade in intermediate products.

Two other generalizations can be readily discussed. First, suppose tha the knowledge in one country does spillover to the other, but at a slow rate. Grossman and Helpman (1989b) examine the case where knowledge disseminat within and across countries with exponential lags. In order for a balanced growth solution with both countries growing at the same rate to exist, they argue that the lags within and across countries should be similar. We have considered the extreme case of instantaneous dissemination within a country but zero spillover across countries, so that the nations grow at uneven rates Their discussion suggests that a similar result might be obtained with limi but small, spillover of knowledge across countries.

Second, suppose that a country can either create new products, or imi the technology of the other country (at some fixed cost). This approach is taken by Grossman and Helpman (1989c). Significantly, they assume that the technical knowledge in the smaller country (the South) depends only on the ovation and imitation which has occurred there, so that there is no direct llover of knowledge from the larger country (the North). While this is the ıe assumption we have made, they obtain the quite different result that *both* ntries can grow at a quicker rate with trade than in autarky. However, this ult depends on the South being not too small: it must have enough resources imitate at the same rate as Northern innovation.[12] For smaller Southern ıntries, the balanced growth equilibrium they have described does not occur. would conjecture that in this case the countries will grow at uneven rates. we have analysed here.

In their Figure 1, SS cannot lie below NN.

Appendix

Proof of Proposition 1

We first solve for the limiting values of s_1 and μ_i, and then determine thei values for large t in part (d).

(a) For any $0<\sigma<1$, we need to show that there exists T such that $s_1(t)>$ for $t>T$. Suppose not. Then for some $0<\sigma<1$, we must have either $s_1(t)<\sigma$ $t>T$, or $s_1(t_i)=\sigma$ for an increasing, unbounded sequence of times $t_i=t_1,t_2,t_3,\ldots$ In the former case we can take the limit of (21) as $t\to\infty$, and find that the side is less than $\sigma/(1-\sigma)$ but the right side approaches infinity, which is a contradiction. So consider the latter case.

Evaluating (21) at the times t_i, we see that the left side is constant $s_1(t_i)/s_2(t_i) = \sigma/(1-\sigma)$. The term $e^{[(1-\alpha)/\alpha]^2(L_1-L_2)t_i}$ on the right approaches infinity as $t_i\to\infty$, so the bracketed expression on the far right of (21) must approach zero. It follows that,

$$\lim_{t_i\to\infty} \int_{t_i}^{\infty} e^{(L_2+\rho)(t_i-\tau)} s_2 \, d\tau = 0 \, . \qquad \text{(A}$$

Since $s_2(t_i)=\sigma$ in the numerator of (18), it follows that $\mu_2(t_i)$ must becom negative for large t_i. This contradicts our assumption that $\mu_i>0$, $i=1,2$. I follows that no such sequence exists, which proves that $s_1\to1$.

(b) Using L'Hospitals Rule, take the limit of (18) to obtain:

$$\lim_{t\to\infty} \mu_i = g_i + \lim_{t\to\infty} \alpha\left(\frac{\dot{s}_i}{s_i}\right) . \qquad \text{(A}$$

For i=1, $\lim_{t\to\infty} s_1 = 1$ implies that $\lim_{t\to\infty} \dot{s}_1 = 0$, so that $\lim_{t\to\infty} \mu_1 = g_1$.

(c) For i=2, we need to evaluate $\lim_{t\to\infty}\left(\frac{\dot{s}_2}{s_2}\right)$ and then apply (A2). Differenting the log of (21) and making use of (18), we can show that,

$$\frac{\dot{s}_1}{s_1 s_2} = \frac{-\dot{s}_2}{s_1 s_2} = \gamma(L_1 - L_2) - \left(\frac{\beta}{1-\beta}\right)(\mu_1 - \mu_2) \ . \quad \text{(A3)}$$

ıng our results in (a) and (b), it follows that,

$$\lim_{t\to\infty}\left(\frac{\dot{s}_2}{s_2}\right) = \left(\frac{\beta}{1-\beta}\right)\left(g_1 - \lim_{t\to\infty}\mu_2\right) - \gamma(L_1 - L_2) \ . \quad \text{(A4)}$$

mbining (A4) with (A2) for i=2, the limiting value of μ_2 can be computed.

l Substituting the limit of μ_2 back into (A4), we obtain,

$$\lim_{t\to\infty}\left(\frac{\dot{s}_2}{s_2}\right) = -\left(\frac{1-\alpha}{\alpha}\right)^2\left[\frac{(1-\beta)}{\alpha\beta} + 1\right]^{-1}(L_1 - L_2) < 0 \ . \quad \text{(A5)}$$

ıce (A5) is less than zero, it follows that there exists T (depending on the rameters of (21)), such that $\dot{s}_2 < 0$ and $\dot{s}_1 > 0$ for $t \geq T$. To derive the operties of μ_i for $t \geq T$, integrating by parts to obtain the formula,

$$\int_t^\infty s_i \, e^{-(L_i+\rho)\tau} d\tau = (L_i+\rho)^{-1}\left(s_i e^{-(L_i+\rho)t} + \int_t^\infty \dot{s}_i e^{-(L_i+\rho)\tau} d\tau\right).$$

bstituting this into the numerator of (18), we can write the rates of product velopment as,

$$\mu_i = g_i + \left(\alpha \int_t^\infty \dot{s}_i e^{-(L_i+\rho)\tau} d\tau \Big/ \int_t^\infty s_i e^{-(L_i+\rho)\tau} d\tau\right). \quad \text{(A6)}$$

en $\dot{s}_1 > 0$ for all $t \geq T$ implies that $\mu_1 > g_1$ and $\mu_2 < g_2$. To compute the lower und on μ_2, use $\dot{s}_1 > 0$ in (A3) to obtain:

$$\mu_2 > \mu_1 - \left(\frac{1-\alpha}{\alpha}\right)(L_1-L_2) > g_1 - \left(\frac{1-\alpha}{\alpha}\right)(L_1-L_2) = g_2 - \frac{(1-\alpha)^2}{\alpha}(L_1-L_2) \ .$$

Proof of <u>Proposition 2</u>

Once we establish that $\dot{s}_1 > 0$ for $t \geq 0$, it is immediate from our discussion just above in (d) that Proposition 1 applies for $T=0$.

<u>(i)</u> For any value of $n_2(0)/n_1(0)$, and other parameters of (21), let $s_i(t)$ denote the market shares and T denote the time at which $\dot{s}_1 > 0$ for $t \geq T$. Then define $N \equiv [n_2(0)/n_1(0)]e^{(L_2-L_1)T}$. Consider the new starting values $\tilde{n}_2(0)/\tilde{n}_1(0) = N$, and let the resulting market shares be denoted by $\tilde{s}_i(t)$. Then by inspection we see that $\tilde{s}_i(t) \equiv s_i(t+T)$ will satisfy (21). Since $\dot{s}_1 > 0$ for $t \geq T$, it follows $\frac{d}{dt}\tilde{s}_1 > 0$ for $t \geq 0$. By the same logic, this result will hold for all initial values $\tilde{n}_2(0)/\tilde{n}_1(0) \leq N$.

<u>(ii)</u> We will assume that $\beta/(1+\beta) < \alpha \leq 0.5$, and deal with $\alpha = \beta/(1+\beta)$ in the text. Note that these inequalities on α imply,

$$\left(\frac{1-\alpha}{\alpha}\right)^2 \left[\frac{(1-\beta)}{\alpha\beta} + 1\right]^{-1} > \frac{\beta(1-2\alpha)}{\alpha(1-\beta)} \geq 0 \ . \qquad \text{(A7)}$$

Suppose that $\dot{s}_1(t_0)=0$. From (A5), $\lim_{t\to\infty}\left(\frac{\dot{s}_1}{s_1 s_2}\right) = \left(\frac{1-\alpha}{\alpha}\right)^2 \left[\frac{(1-\beta)}{\alpha\beta} + 1\right]^{-1}(L_1-L_2)$, so using (A7) there must exist $t_1 \geq t_0$ such that,

$$\left(\frac{\dot{s}_1}{s_1 s_2}\right) = \frac{\beta(1-2\alpha)}{\alpha(1-\beta)}(L_1-L_2) \text{ at } t_1. \qquad \text{(A8)}$$

Then substituting this into (A3), we obtain $(\mu_1-\mu_2) = (L_1-L_2)$ at t_1. The differential equations (14) can then be written as,

$$\frac{\alpha\dot{\mu}_1}{(L_1-\mu_1)} - \frac{\alpha\dot{\mu}_2}{(L_2-\mu_2)} = (\mu_1-\mu_2) - (g_1-g_2) - \alpha(\dot{s}_1/s_1 s_2)$$

$$= (L_1-L_2)\,\frac{[\alpha(1+\beta)-\beta]}{(1-\beta)} > 0\,, \qquad (A9)$$

-e the second line follows using $(\mu_1-\mu_2) = (L_1-L_2)$, (A8) and $\alpha > \beta/(1+\beta)$.

Since $(L_1-\mu_1) = (L_2-\mu_2)$ at t_1, it is evident from (A9) that $(\dot{\mu}_1-\dot{\mu}_2) > 0$ $_1$. Taking the derivative of (A3), it follows that for t slightly greater ; t_1 we must have $(\dot{s}_1/s_1s_2)$ *less than* $[\beta(1-2\alpha)/\alpha(1-\beta)](L_1-L_2)$. However, implies that $(\dot{s}_1/s_1s_2)$ *can never exceed* $[\beta(1-2\alpha)/\alpha(1-\beta)](L_1-L_2)$, since never it reaches this value (from below) it then becomes less. But this :radicts that fact that $\lim_{t\to\infty} (\dot{s}_1/s_1s_2) > [\beta(1-2\alpha)/\alpha(1-\beta)](L_1-L_2)$ from (A7). ollows that $\dot{s}_1(t_0)=0$ cannot occur, and since $s_1\to 1$, then $\dot{s}_1 > 0$ for all $t \geq 0$.

ɔf of Proposition 3

and (b) These are proved in the text.

Note that $\alpha \geq 1-[(1-\beta)/\beta]$ implies $\gamma \leq 1/\alpha$, since $\gamma = (1-\alpha)\beta/(1-\beta)\alpha$. n $(n_2/\tilde{n}_2)^{\gamma} \geq (n_2/\tilde{n}_2)^{1/\alpha}$. We will show that,

$$(n_2/\tilde{n}_2)^{1/\alpha} > [s_2/s_2(0)]\left(\frac{L_2+\rho}{L_2+\rho+[(1-\alpha)/\alpha]^2(L_1-L_2)}\right). \qquad (A10)$$

using (26) this implies $(n_2/\tilde{n}_2)^{\gamma} \geq (n_2/\tilde{n}_2)^{1/\alpha} > s_2$, which proves (24).

To establish (A10), use (15) and (17) to write,

$$(\tilde{n}_2/n_2)^{1/\alpha} = [(L_2-\mu_2)\int_0^\infty s_2 e^{-(L_2+\rho)\tau} d\tau]/\alpha s_2\,. \qquad (A11)$$

ce $\dot{s}_1 > 0$ for all t, then $\mu_2 > g_2 - [(1-\alpha)^2/\alpha)](L_1-L_2)$ from Proposition 1(c), I it follows that $(L_2-\mu_2) < \alpha(L_2+\rho) + [(1-\alpha)^2/\alpha)](L_1-L_2)$. Substitute this o (A11), and use $s_2(0) > s_2$ to replace s_2 in the integral. Then evaluating the egral, we obtain (A10).

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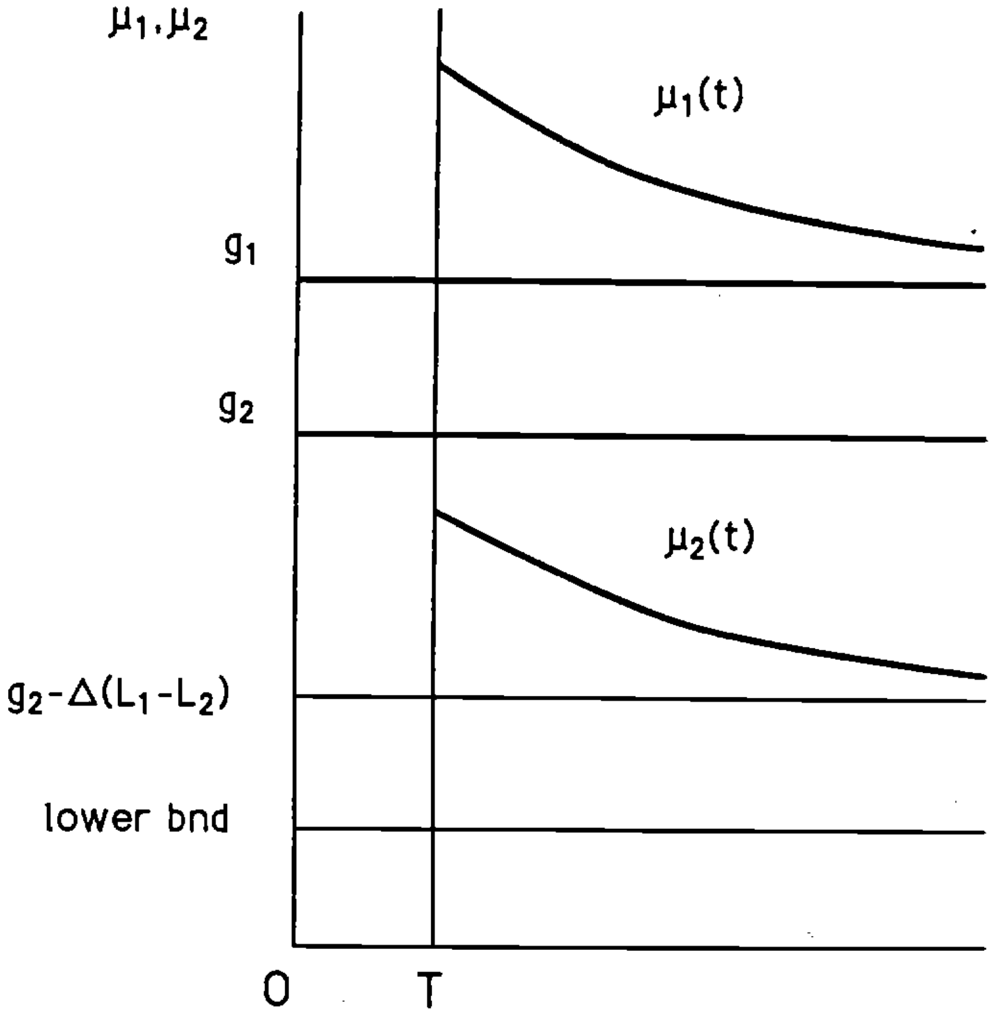

Figure 1

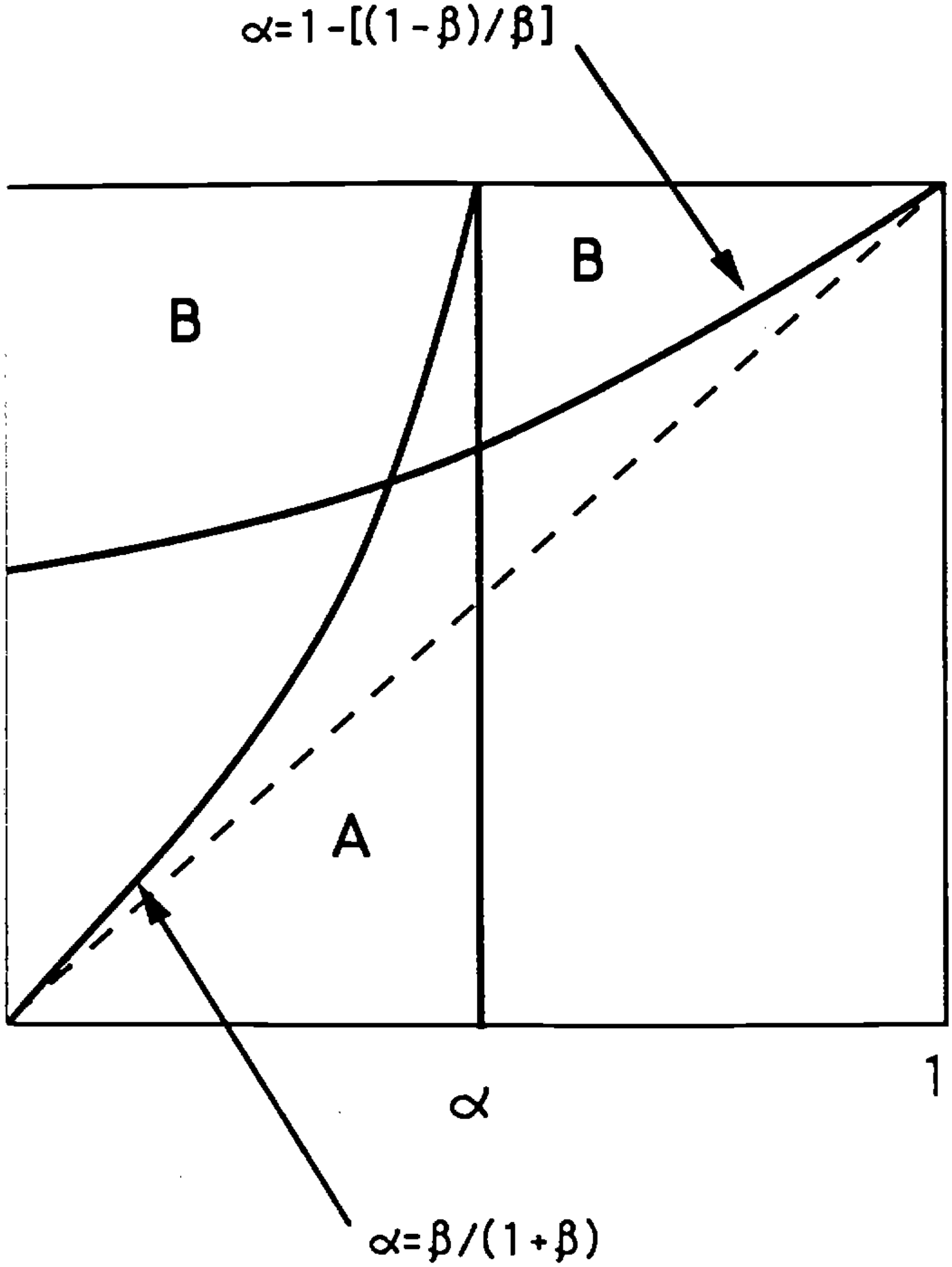

Figure 2