NBER WORKING PAPER SERIES

VARIABLE COST FUNCTIONS AND THE RATE OF RETURN TO QUASI-FIXED FACTORS: AN APPLICATION TO R & D IN THE BELL SYSTEM

M. I. Nadiri

M. A. Schankerman

Working Paper No. 597

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138

December 1980

The research reported here is part of the NBER's research program in Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #597
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ABSTRACT

We formulate a variable cost function model in which certain inputs are treated as quasi-fixed, and develop a simple statistical test of whether optimization occurs for the quasi-fixed inputs. It is shown how to retrieve characteristics of the long-run cost function from the variable cost parameters, with specific reference to the cost elasticity and the elasticities of substitution. We also present a model of the returns to R & D in the context of a regulated firm and show how to estimate the net rate of return to R & D from the variable cost function.

A translog version of the model is estimated for the Bell System for the period 1947-1976. The empirical results suggest substantial long-run economies of scale at the aggregate level. The formal envelope test indicates that the Bell System's use of capital and R & D was cost-minimizing during the post-war period, but the conclusion is seriously qualified by evidence that the power of the test in this application is low. Finally, we estimate the net rate of return to R & D in the Bell System in the range of 25-40 percent, which is somewhat higher than available estimates for manufacturing industries.

M. I. Nadiri
Department of Economics
New York University
15-19 West 4th Street
New York, N.Y. 10003

(212) 598-7042

M. A. Schankerman
Department of Economics
New York University
15-19 West 4th Street
New York, N.Y. 10003

(212) 598-7037

1. Introduction

There are several approaches used in empirical analyses of cost functions and productivity growth. The most common is the static equilibrium approach which assumes that the firm minimizes the total cost of production by fully optimizing with respect to all inputs. Each observation is interpreted as a long run, cost minimizing position. At the other extreme are disequilibrium models in which the firm encounters costs of adjusting all its inputs (Nadiri and Rosen, 1969), or some specific "quasi-fixed" inputs (White and Berndt, 1979). The special case of long run equilibrium is potentially testable within this framework. The problem is that these models are difficult to implement empirically.

The third approach is to specify a variable cost function in which some inputs are treated as "quasi-fixed." The theoretical development is found under the heading of restricted profit functions, of which they are a special case (Diewert, 1978; Lau, 1976). The level of variable cost is expressed as a function of the prices of variable inputs, the level of output, and the quantity of the quasi-fixed factor. The estimation of these models is conducted without making any assumption about how the quantity of the quasi-fixed input is determined. In fact, one of the main uses of these models has been to determine whether there is any "excess capacity" in the quasi-fixed factor (Keeler, 1974; Brown and Christensen, 1980). The procedure involves estimating the variable cost model, and then deriving the associated envelope condition evaluated with the estimated parameters.

The value of the quasi-fixed factor which solves this computed envelope condition is interpreted as the cost-minimizing level of the quasi-fixed input. The difference between the actual level and this optimal level of the quasi-fixed factor is taken as a measure of "excess capacity." One serious problem with this procedure is that it fails to provide a test of whether the computed degree of "excess capacity" is statistically significant or simply reflects sampling variation in the estimation of the variable cost function. Notice that such a test would be equivalent to a test of optimization over the quasi-fixed factor.

The first objective of this study is to suggest a static (or long run) equilibrium model in which one can test whether optimization occurs for selected inputs. In Section 2 we use a variable cost function model, on the supposition that the envelope condition for the quasi-fixed factor holds, and show that the estimated variable cost parameters fully characterize the underlying technology. In Section 3 we formulate a statistical test of the null hypothesis that the envelope condition holds.

The second objective is to use the variable cost function to estimate the (marginal) net rate of return to the quasi-fixed input. Clearly, this is not possible in a long run cost function framework since it is assumed that all inputs are optimized over and that their net rates of return are therefore equal. As an illustration, in Section 4 we present a model of the returns to R & D in a regulated-industry context.

A translog version of the model is formulated in Section 5 and applied to annual data on the Bell System for the period 1947-1976.

Sction 6 presents the empirical results, including a discussion of the envelope test and the estimates of the net rate of return to R & D in the Bell System.

2. Variable and Total Cost Functions

Any empirical test of optimal input use requires that a variable (or short run) cost function be used, since the long run function presupposes full optimization. In this section we show how to retrieve the long run cost elasticity and Allen partial elasticities of substitution from the parameters of the variable cost function.[1] This guarantees that the null hypothesis of optimization can be tested without sacrificing information about the underlying technology.

The (total) variable cost function can be written :

$$(1) \quad CV = F(P, Y; Z)$$

where P is a vector of prices of the N variable inputs, Y is the level of output and Z is the quantity of the quasi-fixed input. As usual, the variable cost function must be monotonically increasing and concave in factor prices. The corresponding (total) short run cost function is

$$(2) \quad CS = F(P, Y; Z) + P_Z Z$$

where P_Z is the service price of the quasi-fixed input. The optimal quantity of the quasi-fixed input is defined implicitly by the envelope condition

$$(3) \quad \frac{-\partial F(\cdot)}{\partial Z} = P_Z$$

provided that the variable cost function is decreasing and convex in Z. Letting $Z = \psi(P, Y, P_Z)$ denote the derived demand for Z which is implied by (3) and substituting it into (2), we obtain the (total) long run cost function

(4) $$C = F(P,\ Y,\ \psi(P,Y,P_z)) + P_z\ \psi(P,Y,P_z) \equiv G(P,Y,P_z)$$

Note for future reference that $\partial C/\partial Z = 0$ if and only if the envelope condition holds.

Differentiating (1) and (2) with respect to output and rearranging yields the relationship between the cost elasticity along the variable and short run cost functions:

(5) $$\eta^s = (1 + \pi)^{-1}\ \eta^v$$

where $\pi = P_z\ Z/CV$ is the ratio of quasi-fixed costs to variable costs evaluated at the point of comparison. Note that this relationship holds at all points along the variable cost function. However, the short and long run cost elasticities can be linked only at those points which satisfy the envelope condition, in which case equations (3) and (4) imply that they are identical ($\eta^{\ell} = \eta^s$). If the envelope does not hold, there is no unique relationship between η^{ℓ} and η^s.

The Allen elasticities of substitution (AES) for the different cost functions are also related. The variable cost AES between factors i and j is defined as (Uzawa, 1962):

(6) $$\sigma^v_{ij} = \frac{F(.)\ \partial^2 F/\partial P_i \partial P_j}{\partial F/\partial P_i \quad \partial F/\partial P_j} \quad \text{for } i,\ j \neq z$$

where subscripts denote inputs. It follows from (2) and (6) that $\sigma^s_{ij} = (1 + \pi)\sigma^v_{ij}$. The long run AES is defined:

(7) $$\sigma_{ij} = \frac{G(.)\ \partial^2 G/\partial P_i \partial P_j}{\partial G/\partial P_i \quad \partial G/\partial P_j} \quad \text{for all } i,j$$

To relate σ_{ij} and σ^{v}_{ij} note first from (4):

$$(8) \qquad \frac{\partial G}{\partial P_i} = \frac{\partial F}{\partial P_i} + \frac{\partial F}{\partial \psi}\frac{\partial \psi}{\partial P_i} + P_z \frac{\partial \psi}{\partial P_i}$$

Using the envelope conditon (3), this collapses to $\partial G/\partial P_i = \partial F/\partial P_i$, which is the derived demand for input i by Shephard's Lemma. Differentiating (8) with respect to P_j and using the envelope condition:

$$(9) \qquad \frac{\partial^2 G}{\partial P_i \, \partial P_j} = \frac{\partial^2 F}{\partial P_i \, \partial P_j} + \frac{\partial^2 F}{\partial \psi \, \partial P_j} \frac{\partial \psi}{\partial P_i}$$

Substituting (8) and (9) in (7), using (6) and rearranging, we obtain:

$$(10) \qquad \sigma_{ij} = (1 + \pi)\left[\sigma^{v}_{ij} + S_i^{-1} \frac{\partial \ln X_j}{\partial \ln Z} \frac{\partial \ln Z}{\partial \ln P_i}\right] \qquad i, j \neq z$$

where $\pi = P_z Z/CV$ and $S_i = P_i X_i/CV$ are the ratio of fixed to variable costs and the share of input i in variable cost, respectively. Equation (10) links the variable cost and long run AES for the variable factors. It is easily verified that the required condition $\sigma_{ij} = \sigma_{ji}$ is met. The long run AES involving the quasi-fixed input are retrieved from the relation $\sum_j \alpha_j \sigma_{ij} = 0$, where $\alpha_j = P_j X_j/C$ is the share of input j in long run cost. Note that the relation between the variable cost and long run AES requires that the envelope condition holds.

3. Test Procedure for the Envelope Condition

The test procedure is based on the null hypothesis H_0 that the envelope condition holds in the data, i.e., that the observed values of the quasi-fixed factor are determined according to the envelope condition. On H_0, estimation of the system of equations consisting of the variable cost function and the derived demands for variable factors yields unbiased but not efficient estimates of all parameters. The inefficiency arises from not exploiting the envelope condition in the estimation. These parameter estimates are then used to obtain unbiased predictions of the dependent variable in the envelope condition. On H_0 the prediction errors are shown to be (normally) distributed with zero mean and a computable variance.

The validity of this test does not require that the envelope condition yield a closed-form solution for the quasi-fixed factor (i.e., the derived demand for Z). The empirical application to the translog cost function in Section 6 is an example. However, if such a closed-form solution does exist, our procedure amounts to testing whether there is a statistically significant difference between the observed quantity and the cost-minimizing quantity of the quasi-fixed factor, as computed from the estimated envelope condition. Then, our procedure provides a statistical test of the significance of estimated "excess capacity", as measured by Keeler (1974) and Brown and Christensen (1980).

Consider a model implying the system of m+1 linear (in the parameters) equations

$$Y_i = X_i A_i + U_i \qquad i = 1,\ldots,m+1 \tag{11}$$

where Y_i and U_i are $n \times 1$ vectors of observations and disturbances, X_i is an $n \times k_i$ matrix of regressors and A_i is a $k_i \times 1$ vector of parameters in the i^{th} equation, and where the first equation represents the cost function, followed by $m-1$ derived demand equations, and finally the envelope condition. Then we can write $A_i = L_i A$ where L_i is a $k_i \times k$ selection matrix and A is a $k \times 1$ vector of all parameters that appear in the system.

The subset of equations excluding the envelope condition can be written

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_m \end{bmatrix} = \begin{bmatrix} X_1 & & 0 \\ & \ddots & \\ 0 & & X_m \end{bmatrix} \begin{bmatrix} L_1 \\ \vdots \\ L_m \end{bmatrix} A + \begin{bmatrix} U_1 \\ \vdots \\ U_m \end{bmatrix}$$

or in obvious notation $Y = \tilde{X}A + U$. Assume $E(\tilde{X}U) = 0$ and $U \sim N(0,\Sigma)$ where $\Sigma = \Omega \otimes I$. Define $\underline{Y} = (M \otimes I)Y$, $\underline{X} = (M \otimes I)\tilde{X}$ and $\underline{U} = (M \otimes I)U$ where $M'M = \Omega^{-1}$. Then the transformed model $\underline{Y} = \underline{X}A + \underline{U}$ has a diagonal covariance structure. Estimating the vector A from this model and using it in the envelope equation yields the prediction

$$\hat{Y}_{m+1} = \tilde{X}_{m+1}A + \tilde{X}_{m+1}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{U}$$

and the prediction error

(12) $\varepsilon \equiv Y_{m+1} - \hat{Y}_{m+1} = U_{m+1} - \tilde{X}_{m+1}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{U}.$

Under H_0, $\varepsilon \sim N(0,V(\varepsilon))$. Taking the expectation of $\varepsilon'\varepsilon$ from (12) and rearranging we have

(13) $V(\varepsilon) = V(U_{m+1}) + Tr(\tilde{X}_{m+1}C(\hat{A})\tilde{X}'_{m+1}) - 2\,Tr(C(U_{m+1}\underline{U}')\underline{X}(\underline{X}'\underline{X})^{-1}\tilde{X}'_{m+1})$

where Tr denotes a trace and $C(\cdot)$ denotes a covariance matrix.

We test the null hypothesis that the envelope condition holds by testing that the sample prediction errors are (normally) distributed with mean zero and the variance in (13).[2] Estimates of $V(U_{m+1})$ and $C(U_{m+1}\underline{U}')$ are obtained from the residuals when the entire system (11) is estimated. The matrix $C(\hat{A})$ is obtained from the system excluding the envelope. Both are consistent on H_0.

4. Modelling the Rate of Return to R & D

One advantage of the variable cost approach is that it enables one to calculate the net (internal) rate of return to the quasi-fixed input. In this discussion we consider research and development (R & D) as the quasi-fixed input in the variable cost function. The cost-minimizing quantity of R & D is described by the envelope condition, so that analytically the envelope condition holds if and only if the net rate of return to R & D is equal to the opportunity cost of funds (and the net rate of return to other assets). Therefore, the proper procedure involves the following steps: 1) estimate the variable cost model, 2) perform the envelope test described in Section 3 and then, if the envelope condition is rejected, 3) compute the rate of return to R & D from the subset of equations excluding the envelope. In this section we present a procedure for deriving the net rate of return from the variable cost function.

In the present model investment in the stock of R & D shifts the variable cost curve downward and establishes a new long-run equilibrium price. The market price may only adjust with a lag, however. In each period the private returns to the R & D investment are the difference between total revenues and the new total variable costs, evaluated at the prevailing short-term equilibrium. These rents which accrue to

R & D arise from the firm's temporary monopoly power resulting from the cost reduction, and they depend critically on the way in which the price of output adjusts over time to the shift in the variable cost curve.

There are two separate matters which must be considered; the level of the new equilibrium price and the speed of adjustment toward it. The specific approach depends on the institutional context. For example, consider a highly competitive environment in which "leakage" of the new knowledge (in the form of a cost reduction) to other firms cannot be blocked effectively with the available devices, such as patents, secrecy, etc. If other firms in the industry succeed in learning the cost-reducing technique without payment, the price of output will fall eventually to the new level of average variable cost. In this case, the entire reward to the original R & D arises from whatever sluggishness there is in the output price adjustment process. If adjustment were instantaneous, there would be no returns at all and, consequently, no R & D would be undertaken by profit-oriented firms. More generally, the level of R & D will adjust until the expected appropriable rents generate the normal rate of return to such investment (see Pakes and Schankerman, 1980).

In this paper we analyze the case of a regulated, single-product monopoly operating under economies of scale. The firm is constrained to earn zero profits eventually; the regulator fixes (possibly with some lag) the price of output at the intersection of the demand and average total cost curves. Included in the average total costs is the remuneration paid to all capital assets (including R & D), which we assume to be the "normal" or "allowed" rate of return which reflects their opportunity cost.

Suppose that the regulated firm makes an investment in R & D which lowers both the average variable and average total cost curves. If there were no regulatory lag, the price of output would adjust immediately to the new average total cost and the R & D investment would earn only the normal rate of return which is embodied in that cost. Note that in the regulated environment instantaneous price adjustment implies a normal rate of return to R & D, whereas in the competitive environment it implies no private returns at all.

If there is any regulatory lag, however, some rents will accrue and the private returns to R & D will necessarily exceed its opportunity cost.[3] Since an equilibrium requires that the marginal R & D earn only normal returns, the question arises whether, and under what conditions, there can be an equilibrium level of R & D in the presence of regulatory lag. This can occur only if the average total cost curve is not shifted by the marginal R & D investment, since in that case the equilibrium price remains unchanged, there are no potential rents to the marginal R & D, and the effect of regulatory lag is thereby neutralized. The average total cost is unchanged if the reduction in average variable cost due to the marginal R & D just equals the price of the R & D, i.e., if the envelope condition for R & D holds. In short, in this regulated environment with regulatory lag, an equilibrium level of R & D is precisely defined by the envelope condition.

We have argued that in a regulated industry the private benefits to R & D consist of the rents due to regulatory lag in adjusting the output price, plus a normal return paid each period to R & D (as to other capital assets). The rents are the difference between the output price and the new average total cost, multiplied by the number of units of output. It is clear that the rents will increase as the market expands, but they will erode over time as the price is adjusted downward. Does the normal return paid to R & D also decline over time and, if so, at what rate? The normal return is payment for the opportunity cost of the R & D. If we add up these payments over all units in the stock of R & D, as that stock is defined in the variable cost function, we obtain the total normal return paid to "surviving" R & D which is reflected in the average total cost function. Therefore, the rate at which the normal return to a unit of R & D declines must be the same as the rate of depreciation which is appropriate for the stock of R & D in the variable cost function. But what is the rate?

The key to analyzing this problem is to distinguish between the quantity and the value of the R & D stock.[4] R & D is expenditure on different inputs which are used to "produce" variable cost reductions. The quantity of R & D, Q, is defined as the productive capacity of these inputs measured in terms of the maximal variable cost reductions which they can yield (given the available "knowledge production function"). We call the rate at which this productive capacity (quantity) of R & D inputs declines over time the "rate of deterioration." The value of the R & D stock, V, is defined as the discounted stream of expected (gross) earnings which will accrue to that stock. This value can be expressed as the quantity of R & D, as defined above, multiplied by its price, which is exacly the rent it earns ($V = PQ$). The rate of depreciation of the value of R & D, $\hat{V} = \hat{P} + \hat{Q}$ (hats denoting rates of change), is the sum of the (physical) deterioration rate and the rate at which the rent to R & D declines. By our earlier argument, the latter is the difference between the rate at which the output price adjusts and the rate of growth of the market.

The question is, which concept of the R & D stock belongs in the variable cost function? As long as factor prices are assumed to be determined exogenously, the cost function (like its dual, the production function) is a technological relationship, and as such the quantities (not values) of inputs belong. It is the productive capacity of the R & D inputs, not their market value, which contribute to the determination of the level of variable cost.

To summarize the three important conclusions: 1) the stock of R & D appropriate for estimation of the variable cost function should be constructed as a cumulation of R & D flows using their rate of deterioration; 2) the normal returns to a unit of R & D decline at the rate of deterioration; and 3) the quasi-rents which accrue to R & D decline at a rate which reflects output price adjustment, market growth, and deterioration.

To formalize the derivation of the rate of return to R & D, we begin with the average total cost function:

$$(14) \qquad \bar{C} = \bar{F}(P,Y;Z) + P_z(r)\bar{Z}$$

where a "bar" denotes an average over output, $F(\cdot)$ refers to variable (non-R & D) costs, Z is the stock of R & D, and P_z is the rental price as a function of the interest rate, r. Note that the derivative of $\bar{C}$ with respect to Z may be either positive or negative, and it will equal zero only if the envelope condition holds.

The marginal net (internal) rate of return, r^*, is defined by the equation:

$$(15) \qquad e^{\theta r^*} = \int_0^\infty B(t)e^{-r^*t}\, dt$$

where θ is the R & D gestation period (assumed to be a fixed rather than distributed lag, for simplicity), and $B(t)$ represents the private returns accruing to the marginal unit of R & D. We assume that the firm expects demand to grow at constant rate g. Letting δ denote the rate of adjustment of output price and ϕ the rate of deterioration, the private returns in year t of a unit investment in R & D in year 0 are:

$$(16) \qquad B(t) = Y(0)\frac{\partial \overline{C}}{\partial Z} e^{(g-\delta-\phi)T} + P_z(r)e^{-\delta t}$$

The first term is the depreciated unit rent (reduction in average total cost) multiplied by the market size in year t; the second is the normal return in year t. Using (14) and (16) in (15), and performing the integration, we write the internal rate of return to R & D implicitly as:

$$(17) \qquad e^{\theta r^*}(r^* + \delta + \phi - g) = -\frac{\partial F}{\partial Z} + P_z(r)\left[\frac{\delta - g}{r^* + \phi}\right]$$

Two points should be noted. First, equation (17) represents a generalization of the formula given in Pakes and Schankerman (1978). In that work the discussion centers on a competitive environment where output price adjusts to average variable cost, and the second term in (17) does not appear. Second, we know from the earlier discussion that only the normal rate of return is earned when the envelope condition holds (the average total cost curve is not shifted by R & D). This condition, $r^* = r$ when $\partial\overline{C}/\partial Z = 0$, can be used in (17) to identify the service price of R & D, $P_z(r)$. We obtain

$$(18) \qquad P_z(r) = e^{\theta r}(r + \phi)$$

Equation (18) defines the appropriate service price of R & D to use in the empirical work when the envelope condition is imposed. Substituting it back into (17), we obtain

$$e^{\theta r^*}(r^* + \delta + \phi - g) - e^{\theta r}(\delta - g)\left[\frac{r + \phi}{r^* + \phi}\right] + \frac{\partial F}{\partial Z} = 0 \qquad (19)$$

Equation (19) is a non-linear function which implicitly defines the private net rate of return to R & D, r^*, given the values for the other parameters. The term $\partial F/\partial Z$ can be computed from the estimated variable cost function.

5. Translog Formulation

For the empirical work we use the translog as a flexible functional form to approximate the variable cost function. Writing logarithms as lower case letters, the translog approximation is

$$\begin{aligned} cv = {} & \beta_0 + \beta_y y + \sum_i \beta_i p_i + \tfrac{1}{2}\gamma_{yy} y^2 + \sum_i\sum_j \gamma_{ij} p_i p_j \\ & + \sum_i \gamma_{iy} p_i y + \lambda_z z + \tfrac{1}{2}\lambda_{zz} z^2 + \sum_i \lambda_i p_i z + \lambda_y yz \end{aligned} \qquad (20)$$

where $i,j = 1,\ldots,N$ index the N variable inputs and all variables are defined around some expansion point.[5]

Applying Shephard's Lemma to (20) yields the set of share equations:

$$S_i = \beta_i + \sum_j \gamma_{ij} p_j + \gamma_{ij} y + \lambda_i z \qquad (21)$$

where $S_i = P_i X_i/CV$ is the share of input i in variable cost. If the firm also optimizes over the quasi-fixed factor, then the envelope condition holds. Applying it to (20) yields:

$$-\pi = \lambda_z + \lambda_{zz} z + \sum \lambda_i p_i + \lambda_y y \qquad (22)$$

The variable cost function must be linearly homogeneous in variable input prices at all points, which implies the restrictions:

$$(23) \qquad \sum_i \beta_i = 1; \quad \sum_j \gamma_{ij} = \sum_i \gamma_{iy} = \sum_i \lambda_i = 0$$

The symmetry restrictions $\gamma_{ij} = \gamma_{ji}$ are also imposed in the empirical work.

The variable cost elasticity is obtained by differentiating (20) with repsect to output:

$$(24) \qquad \eta^v = \beta_y + \gamma_{yy} y + \sum_i \gamma_{iy} p_i + \lambda_y z$$

and the long run cost elasticity is $\eta^\ell = \eta^v (1 + \pi)^{-1}$. In general, both η^ℓ and η^v vary with factor prices, output, and the quantity of quasi-fixed factor. The reciprocal of the cost elasticity is an appropriate measure of scale economies (Hanoch, 1975).

The variable cost AES between inputs i and j can be computed directly as:

$$\sigma^v_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad i,j \neq z$$

Using (10), (20) and (21), the variable cost and long run AES are related in terms of the translog as follows.

$$(25) \qquad \sigma_{ij} = (1 + \pi) \left[\sigma^v_{ij} - \frac{(\lambda_i - S_i \pi)(\lambda_j - S_j \pi)}{S_i S_j \pi} \right] \qquad i,j \neq z$$

Using the relation $\sum_{\forall j} \alpha_j \sigma_{ij} = 0$, the restrictions in (23), and (25), one can show that for the translog:

$$(26) \qquad \sigma_{iz} = (1 + \pi) \left[1 - \frac{\lambda_i}{\pi S_i} \right]$$

The long run elasticites of factor demand are given by

$\varepsilon_{ii} = \alpha_i \sigma_{ii}$.

Empirical test of homotheticity and homogeneity of both the variable and the underlying long run cost functions can be derived from the variable cost parameters. Homotheticity requires separability in factor prices and output, which implies that the optimal input mix (and hence relative shares) is independent of the level of output. Using (21) and (23), variable cost homogeneity imposes the N constraints $\gamma_{iy} = 0$, N-1 of which are independent. For long run homotheticity, relative long run input shares must be independent of output both directly and indirectly (through the quasi-fixed input). It can be shown that this imposes the 2N+2 restrictions $\gamma_{iy} = \lambda_i = \lambda_y = \lambda_{zz} = 0$, 2N of which are independent.[6]

Homogeneity implies that the cost elasticity is constant. Using (24), variable cost homogeneity imposes the N+1 restrictions $\gamma_{iy} = \gamma_{yy} = 0$, N of which are independent. From the relation $\eta^{\ell} = (1 + \pi)^{-1}\eta^{v}$, it follows that long run homogeneity requires that both η^{v} and π be constant. Using (22) and (24) this imposes the 2N+3 restrictions $\gamma_{iy} = \gamma_{yy} = \lambda_i = \lambda_y = \lambda_{zz} = 0$, 2N+1 of which are independent.

6. An Empirical Application

In this section the translog variable cost model is applied to data on the Bell System during the period 1974-76.[7] Four factors of production are included: labor, materials, (physical) capital, and a stock of R & D. Aggregate output is measured by the sum of deflated operating revenues for local service, intrastate toll, interstate toll, and a small miscellaneous category. Labor input is the number of man hours actually worked in twenty-two categories of labor, weighted by their (fixed) relative wages. An implicit price index for labor is obtained as the ratio of total employee compensation to the quantity of labor input. The materials input consists of six categories of materials, rents and supplies, each separately deflated. An implicit price index for the materials input is used.

The stock of capital is the sum of net tangible plant, cash, net accounts receivable, and inventories. Tangible plant is constructed from twenty-three different types of capital, each identified by vintage and separately depreciated. The service price of capital is constructed as the sum of the cost of investment funds and the rate of depreciation, multiplied by the investment goods deflator and adjusted by tax parameters in the standard way.[8]

The stock of R & D is constructed as a geometrically weighted sum of deflated non-military R & D expenditures by the Bell System, lagged four years. The lag is designed to reflect the gestation period during which R & D has no effect on variable costs. R & D flows are deflated by the implicit GNP price index. Following the argument in Section 3, the cumulation of R & D should be based on its rate of (physical) deterioration, which we assume to be 0.05.[9] The service price of R & D is given by (18) in Section 3, multiplied by the implicit GNP deflator.

The system of equations consists of the variable cost function (20), the variable share equations (21), and the envelope condition (22). Because the variable shares sum to unity, one of them is redundant and is dropped in the estimation procedure. We follow the literature in specifying additive disturbances in each equation which are assumed to have a joint normal distribution with contemporaneous correlation among equations. The system is estimated using Zellner's procedure (1962) iterated to convergence, which ensures that the parameter estimates are invariant to which share equation is deleted (Barten, 1969). A correlation is made for first-order serial correlation, in accordance with the requirements of a singular system of equations (Berndt and Savin, 1975). Two versions are estimated, one with R & D and another with capital as the quasi-fixed factor. In the discussion of the results we focus on three items:[10] 1) tests of parameter restrictions, 2) long run cost elasticities, and 3) long run AES.

Table 1 presents the likelihood ratio tests of the homotheticity (HT) and homogeneity (HG) constraints. Since the tests are nested (see table), the overall significance level is set at .05 and is divided equally between the two tests in each sequence. Turning to the results, short run HT is rejected, while short run HG is accepted (conditional on HT) when R & D is the quasi-fixed factor. When capital is the quasi-fixed input, short run HT is accepted and short run HG is marginally rejected. Since variable costs are defined differently in the two versions, these findings are not contradictory. However, the implied characteristics of the long run cost function should not depend on which input is treated as quasi-fixed (if the envelope condition is imposed). We find that both versions of the variable cost model strongly reject long run HT, which is consistent with earlier findings based on direct estimation of the long run cost function (Nadiri and Schankerman, 1979). The evidence is mixed on long run HG (conditional on HT).

Table 1. Test Statistics For Parameter Restrictions

	Homotheticity		Homogeneity	
	Short Run	Long Run[a]	Short Run[a]	Long Run[b]
χ^2, R & D Envelope	11.2	82.2	0.2	0.8
χ^2, Capital Envelope	1.2	77.2	5.4	7.2
Critical $\chi^2_{.025}$	7.4	11.1	5.0	5.0
No. of Independent Restrictions	2	4	1	1

[a]Conditional on short run homotheticity.

[b]Conditional on long run homotheticity.

Contrary to expectation, it depends on which input is considered quasi-fixed.

Both versions of the model yield similar long run cost elasticities which suggest substantial scale economies in the Bell System at the aggregate level. At the sample means, the estimated long run cost elasticity (standard error) varies from .50(.055) to .62(.019) with R & D as the quasi-fixed factor, and from .59(.014) to .67(.016) with capital as the quasi-fixed input, depending on the parameter restrictions. These estimates are similar to those obtained directly from long run cost functions in Nadiri and Schankerman (1979) and Christensen, Cummings and Schoech (1980). Note that none of the confidence intervals includes unity; constant returns at the aggregate level can be rejected by these data.

Table 2 presents the long run AES and factor price elasticities implied by the (short run) homothetic variable cost function. (The results are robust to parameter restrictions). For comparison we include the results from Nadiri and Schankerman (1979) which are based on direct estimation of the long run cost function. Estimation of the variable cost function was conducted on the supposition that the envelope condition holds. If this supposition is correct, both variable cost models should produce the same set of implied long run AES and factor price elasticities.

The results may be summarized briefly. When R & D is treated as the quasi-fixed factor, the estimates indicate that all inputs are substitutes and that factor demands are price inelastic, capital being the most inelastic. The specific parameters are actually quite similar to those from Nadiri and Schankerman (1979). There are some differences

Table 2. Long Run Elasticities of Substitution and Factor Price Elasticities[a,b]

Parameter \ Model	Variable Cost, R & D Envelope	Variable Cost, Capital Envelope	Long Run Cost
$\sigma_{\ell k}$	.60* (.09)	2.33* (.04)	.48* (.10)
$\sigma_{\ell m}$	1.54* (.27)	-.30 (.22)	2.32* (.31)
σ_{km}	.37 (.22)	1.92* (.13)	.46* (.15)
$\sigma_{\ell r}$	.48 (.26)	-.74* (.24)	-.45 (.28)
σ_{kr}	.52* (.15)	.85* (.14)	.52* (.14)
σ_{mr}	4.67* (.83)	2.50* (1.18)	1.67* (.70)
$\varepsilon_{\ell\ell}$	-.53* (.05)	-1.21* (.02)	-.55* (.06)
ε_{kk}	-.25* (.04)	-1.93* (.01)	-.26* (.04)
ε_{mm}	-.78* (.14)	-.98* (.13)	-1.12* (.13)
ε_{rr}	-.99* (.01)	-.52* (.14)	-.31* (.03)

[a]An asterisk denotes statistical significance at the .05 level.

[b]Columns 1 and 2 are computed from equation (25) using the parameters from the short-run homothetic versions of the models. The reported values are for 1967, but they are stable over the sample. The third column is taken from Tables 3 and 4 in Nadiri and Schankerman (1979).

in the point estimates, but all except one (ε_{rr}) of the confidence intervals overlap. However, the results are quite different when capital is treated as the quasi-fixed input. Of the ten long run parameters, only four (σ_{kr}, σ_{mr}, ε_{mm} and ε_{rr}) are statistically equivalent to those from the model with R & D as the quasi-fixed input. The results are also less credible. In particular, the substitution elasticities involving capital and the factor price elasticities for labor and capital are implausibly large.

Whatever their plausibility, the important point is that the two sets of estimates from the variable cost models differ from one another. This would appear to contradict the supposition underlying their estimation, namely, that the envelope condition holds and hence that all inputs are chosen optimally. The fact that the implied long run parameters appear to depend on whether R & D or capital is treated as quasi-fixed constitutes indirect evidence that the envelope does not hold for one or both of those inputs.

In view of this finding, it is especially interesting to implement the formal test of the envelope condition outlined in Section 3. The procedure involves estimating the system of equations without the envelope and using the estimated parameters to predict the dependent variable in the envelope condition--in the translog context, to predict the ratio of quasi-fixed to variable costs (filtered for serial correlation). On the null hypothesis, the prediction errors distribute normally with zero mean. We apply the test to the (short-run) homothetic versions of the model, but the findings are robust to different specifications.

Let v denote the ratio of the mean prediction error to its standard deviation. Applying this test to the model with capital as the quasi-fixed input, we obtain $\hat{v} = .05/.37 = .14$. The critical value of v at the .05 level is about two, so we cannot reject the null hypothesis that the envelope condition holds for capital. This test on capital is conditioned on the assumption that the firm chooses R & D optimally. Alternatively, we can treat R & D as the quasi-fixed factor and test the envelope condition for it, conditioned on the supposition that the quantity of capital is chosen optimally. This test yields $\hat{v} = .03/.15 = .2$ and again we cannot reject the null hypothesis.

The implication of these results is that the Bell System use of capital and R & D was cost-minimizing during the post-war period. This conclusion appears to contradict the indirect evidence, i.e., that the implied long run parameters depend on which input is treated as quasi-fixed in the variable cost function. However, even though the null hypothesis of the envelope test is not rejected formally, the mean prediction error is very large relative to the predicted variable, about 30 and 200 percent with capital and R & D as quasi-fixed factors,respectively. The failure to reject the envelope condition is due mainly to a large variance in the prediction error rather than to a small absolute prediction error, especially with R & D as the quasi-fixed factor. The envelope test may exhibit greater power to detect departures from cost minimization in other empirical applications, but in the present context the formal finding of cost minimization should be heavily qualified.

The remaining task is to use the procedure described in Section 4 to calculate the net rate of return to R & D. We estimate the variable cost model without the envelope condition, using R & D as the quasi-fixed factor, and calculate the marginal reduction in variable cost due to an increment to R & D, $\frac{\partial F}{\partial Z}$. At the sample means, the estimates range from -2.1 to -2.8. The (internal) net rate of return, r*, is then computed as the solution to the nonlinear equation /see (19)/:

$$e^{\theta r^*}(r^*+\delta+\phi-g) - e^{\theta r}(\delta-g)\left[\frac{r+\phi}{r^*+\phi}\right]+\frac{\partial F}{\partial Z} = 0$$

where θ is the R & D gestation lag, g is the expected growth rate in output, r is the opportunity cost of funds, ϕ is the rate of deterioration of R & D, and δ is the rate of output price adjustment. We let g = .07, r = .055, ϕ = .05 and perform the calculation for δ = .10, .15, .20 and θ = 2, 4, 6.[11] Table 3 presents the results.

The estimated net rate of return is not very sensitive to δ within the plausible range $.1 \le \delta \le .2$, but it is quite sensitive to the value of θ. Pakes and Schankerman (1978) conclude that $\theta \simeq 2$ for manufacturing industries, but this probably understates the lag in telecommunications R & D. Hence, we conclude from Table 2 that the net rate of return to R & D in the Bell System is about 25-40 percent. This is somewhat higher than the available estimates for manufacturing industries, which indicate a gross rate of return of about 30-50 percent. (See Griliches, 1973, for a review).

Table 3. Net Rates of Return to R & D

	$\theta = 2$	$\theta = 4$	$\theta = 6$
$\frac{\partial F}{\partial Z} = -2.1$			
$\delta = .10$	.58	.38	.29
$\delta = .15$	.56	.36	.28
$\delta = .20$	.54	.35	.26
$\frac{\partial F}{\partial Z} = -2.8$			
$\delta = .10$	.66	.43	.31
$\delta = .15$	.65	.41	.30
$\delta = .20$	.63	.40	.26

Conclusion

In this paper we provide a long run equilibrium model in which one can test whether optimization (cost-minimization) occurs for specific inputs. The method is based on a variable cost function in which certain inputs are treated as quasi-fixed. The question of optimization over the quasi-fixed factors amounts to asking whether the envelope condition associated with those inputs holds empirically in the sample. A simple procedure is developed to test the null hypothesis that the envelope condition holds. The model and test procedure are developed for the case of a single quasi-fixed input, but the generalization is straightforward.

We demonstrate how empirical estimates of the variable cost function can be used to obtain a characterization of the underlying long run cost function. Specifically, we derive the relationship between the variable cost and long run cost elasticities, and the variable cost and long run elasticities of substitution. It is also shown that the variable cost function can be used to estimate the net rate of return to the quasi-fixed factor, and that this rate of return equals the opportunity cost of the quasi-fixed input only if the envelope condition holds. The discussion focuses on the rate of return to R & D, and a model of the returns to R & D in the context of a regulated firm is presented.

The model is applied to aggregate data for the Bell System for the period 1947-1976. Two versions are estimated, one with R & D and another with capital as the quasi-fixed input. In both cases, the empirical results indicate substantial long run economies of scale at the aggregate level. The formal envelope test suggests that the Bell System's use of capital and R & D was cost-minimizing during the post-war period, but the conclusion is seriously qualified by evidence that the power of the test in this application is low. Finally, our empirical results place the net rate of return to R & D in the Bell System in the range of 25-40 percent, which is somewhat higher than available estimates for manufacturing industries.

Notes

*We would like to thank William Baumol and Roman Frydman for their constructive comments, and James Ramsey for a suggestion which led to the development of the envelope test. S. Puglia and P. Kaplan provided very able research assistance. This research has been supported by the National Science Foundation, Grant PRS-7727048, and by an unrestricted grant from the American Telephone and Telegraph Company.

[1] Mundlak (1968) shows how to retrieve the short run elasticities of substitution from the long run values. Our concern is the reverse and the approach we use is different. Also, after a draft of this paper was written (April, 1980), we discovered that Brown and Christensen (1980) independently derive the main results in this section.

[2] For a special case of the envelope test, see Courville (1974). He tests the null hypothesis that there is no Averch-Johnson bias in electric utilities by examining the difference between the marginal rate of substitution between capital and labor(estimated from a production function) and the relative factor prices.

[3] This statement assumes that the marginal R & D lowers the average total cost. This will be true if less than the cost-minimizing amount of R & D is being performed (see eq.(14) and discussion). However, if more than the cost-minimizing amount of R & D is done, the marginal R & D will raise the average total cost even though it reduces the average variable cost. Then any regulatory lag in adjusting the price of output will induce negative rents and the private returns to R & D will fall short of its opportunity costs. The conclusion which follows holds for either case.

[4] For an excellent discussion of the concepts of the quantity and value of (traditional) capital and their appropriate uses, see Griliches (1963).

[5] We do not include time trends in the specification to represent technical change. Shifts in the function are captured by the quasi-fixed factor and its interaction with variable input prices and output. In the model with the envelope condition, the main effect of introducing neutral time trends is to increase the estimated standard errors (especially on the output variables). The trends themselves are not statistically significant. When the envelope condition is not imposed, the coefficients of output and of the quasi-fixed factor are more sensitive to inclusion of time trends.

[6] First, consider the long run shares α_i/α_j for $i, j \neq z$. Since $\alpha_i/\alpha_j = S_i/S_j$, from (21) this ratio is directly independent of output only if $\gamma_{iy} = 0$. In order for relative shares to be indirectly independent of output, either the quantity of the quasi-fixed input must not vary with output or the variable input shares must be independent of the quasi-fixed input. The former condition is excluded if all inputs are normal, so the required restrictions are $\lambda_i = 0$ for $i \neq z$. Second, long run homotheticity requires that S_i/π be independent of output. By analogous reasoning, direct independance imposes $\gamma_{iy} = \lambda_y = 0$, and indirect invariance requires $\lambda_{zz} = 0$.

[7] The data cover the operating telephone companies and Long Lines, but exclude Western Electric and Bell Laboratories. The raw data were provided by the Bell System and inquiries regarding them should be addressed to Peter B. Linhart, Director of Regulatory Research, AT&T. See Nadiri and Schankerman (1979) for a fuller description of the data.

[8] The service price of capital is taken as:

$$C_k = P_I(1 - uz - w) \left(\frac{r + d}{1 - u}\right) + \tau$$

where P_I is the investment good deflator, u is the corporate income tax rate, w and z are the effective rate of investment tax credit and the present value of depreciation allowances, T is the indirect tax rate, r is a weighted average of debt and equity costs, and d is the depreciation rate. These parameters are constructed from Bell System data whenever possible.

[9] Note two points: 1) the empirical results for the model (with the envelope condition imposed) are not sensitive to moderate variations in the gestation lag and the deterioration rate; 2) adjustments have been made to ensure that the R & D expenditures are not double counted in the available measures of traditional inputs (see Nadiri and Schankerman, 1979).

[10] For brevity we do not present the raw parameter estimates, but they are available on request. One point should be noted. The variable cost function must be monotonically increasing and concave in factor prices, and convex in the quasi-fixed input. All the estimated models satisfy monotonicity and concavity in prices at each sample point. Convexity is violated at some sample points in the model with R & D as the quasi-fixed input and few parameter restrictions. However, the parameter estimates are similar to those versions which do satisfy convexity.

[11] Note two points. First, $g = .07$ and $r = .055$ correspond to the average values for the Bell System during the post-war period. Second, the parameter δ has a simple interpretation. Let x be the fraction of the reduction in average total cost due to R & D which has <u>not</u> yet been reflected in a reduced price of output after T years. Then if the output price is adjusted at constant rate δ, it is easy to show that $\delta = (-\log x)/T$. For example, with $T = 10$, the range $.1 \leq \delta \leq .2$ in the text corresponds to $.15 \leq x \leq .40$.

References

Barten, A.P. Maximum Likelihood Estimation of a Complete System of Demand Equations." European Economic Review (Fall, 1969), 7-73.

Berndt, D. and N.E. Savin. "Estimation and Hypothesis Testing in Singular Equation Systems with Auto-Regressive Disturbances." Econometrica, Vol. 43, Nos. 5-6 (1975), 937-57.

Brown, R. and L. Christensen. "Estimating Elasticities of Substitution in a Model of Partial Static Equilibrium: An Application to U.S. Agriculture, 1947-1974." Social Systems Research Institute, Discussion Paper #8007, University of Wisconsin (May 1980).

Christensen, L., D. Cummings and P. Schoech. "Econometric Estimation of Scale Economies in Telecommunications." Social Systems Research Institute, Discussion Paper #8013, University of Wisconsin (August 1980).

Courville, L. "Regulation and Efficiency in the Electric Utility Industry." Bell Journal of Economics and Management Science, Vol. 5, No. 1 (Spring 1974), 53-73.

Diewert, W.E. "Duality Approaches to Microeconomic Theory." Institute for Mathematical Studies in the Social Sciences, Technical Report No. 281, Stanford University (October 1978).

Griliches, Z. "Capital Stock in Investment Functions: Some Problems of Concept and Measurement." In C. Christ (ed.), Concept and Measurement in Economics, Essays in Honor of Yehuda Grunfeld. Stanford: 1963, 115-36.

---------. "Research Expenditures and Growth Accounting." In B.R. Williams, (ed.), Science and Technology in Economic Growth. MacMillan Press, 1973, 59-83.

---------. "Returns to Research and Development in the Private Sector." In J. Kendrick and B. Vaccara (eds.), New Developments in the Measurement of Productivity. NBER: Studies in Income and Wealth, 1980, 419-454.

Hanoch, G. "The Elasticity of Scale and the Shape of Average Costs." American Economic Review 65 (December 1975), 956-65.

Keeler, T.E. "Railroad Costs, Returns to Scale, and Excess Capacity." The Review of Economics and Statistics, Vol. 56, No. 2 (May 1974), 201-08.

Lau, L.J. "A Characterization of the Normalized Restricted Profit Function." Journal of Economic Theory, Vol. 12 (February 1976), 131-163.

Mundlak, Y. "Elasticities of Substitution and Theory of Derived Demand." The Review of Economic Studies, Vol. 35, No. 102 (April 1968), 225-35.

Nadiri, M.I. and S. Rosen. "Interrelated Factor Demand Functions." American Economic Review, Vol. 59 (September 1969), 457-71.

------ and M.A. Schankerman. "The Structure of Production, Technological Change and the Rate of Growth of Total Factor Productivity in the Bell System," 1979. Forthcoming in T. Cowing and R. Stevenson (eds.), Productivity Measurement in Regulated Industries. Academic Press, 1981.

Pakes, A. and M.A. Schankerman. "An Exploration into the Determinants of Research Intensity." Working Paper No. 438, National Bureau of Economic Research, Inc. (1980).

----- and M.A. Schankerman. "The Rate of Obsolescence of Knowledge, Research Gestation Lags, and the Private Rate of Return to Research Resources." Discussion Paper No. 659, Harvard Institute of Economic Research (1978).

Uzawa, H. "Production Functions with Constant Elasticities of Substitution." Review of Economic Studies, Vol. 29 (1962), 291-299.

Vinod, H.D. "Application of New Ridge Regression Methods to a Study of Bell System Scale Economies." Journal of the American Statistical Association, Vol. 71, No. 356 (December 1976), 835-41.

White, C.J. and E.R. Berndt. "Short Run Labor Productivity in a Dynamic Model." Unpublished paper, Department of Economics, University of British Columbia (November 1979).

Zellner, A. "An Efficient Method for Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias." Journal of the American Statistical Association, Vol. 57 (June 1962), 348-68.