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CURRENCY UNIONS

Alberto Alesina
Robert J. Barro

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ABSTRACT

What is the optimal number of currencies in the world? Common currencies affect trading costs and, thereby, the amounts of trade, output, and consumption. From the perspective of monetary policy, the adoption of another country's currency trades off the benefits of commitment to price stability against the loss of an independent stabilization policy. The nature of the tradeoff depends on co-movements of disturbances, on distance, trading costs, and on institutional arrangements such as the willingness of anchor countries to accommodate to the interests of clients.

Alberto Alesina
Department of Economics
Harvard University
Cambridge, MA 02138
and NBER
aalesina@harvard.edu

Robert J. Barro
Department of Economics
Littauer Center 218
Harvard University
Cambridge, MA 02138
and NBER
rbarro@harvard.edu

1 Introduction

Traditionally, each country had its own currency, and only one currency circulated in each country. Monetary unions were rare, and, therefore, the surge in the number of countries in the post-war period generated a large increase in the number of currencies circulating in the world. In 1947 there were 76 countries in the world, today there are 193, and, with few exceptions, each country has its own currency.[1] Unless one believes that a country is, by definition, an "optimal currency area," either there were too few currencies in 1947 or there are too many today. In fact, the increasing integration of international markets implies that the optimal number of currencies would tend to *decrease,* rather than almost triple as it has.

Only recently, however, and perhaps as a result of this proliferation of currencies, the sanctity of "one country one money" has come into question. Eleven countries in Europe have adopted the same currency, dollarization is under active consideration in many countries in Latin America and is currently being implemented in Ecuador, and a currency union is being discussed in Central America. Countries in Eastern Europe and the former Soviet Union are considering adopting unilaterally the euro. In addition, several countries have adopted currency boards, including Hong Kong and Argentina with the dollar and Estonia and Bulgaria first with the German mark and later with the euro.

[1]See Rose (2000) for a list of countries that use currencies other than their own. Two examples of currency unions are the French Franc Zone in Africa and the Caribbean currency union. Some other countries that use another nation's currency are Panama, Liechtenstein, Luxembourg, and San Marino.

Two factors have contributed to these trends. One is the increase in international trade in goods and services, expanded cross-border financial transactions, and heightened cross-country flows of technology, in one word, "globalization." The second is the increased emphasis on price stability, as opposed to active macroeconomic stabilization, as a goal for monetary policy. This switch followed two decades (the seventies and eighties) with exceptionally high inflation rates in many developing countries and double-digit inflation in several industrial ones.

Mundell (1961) pioneered the analysis of monetary union. The benefit of a common-currency area was its role in minimizing transaction costs and facilitating the flow of information about relative prices.[2] The offsetting force was that fixed exchange rates entailed the loss of independent monetary policies. Mundell stressed factor mobility and price flexibility as key elements in this tradeoff.

In this paper, we begin by investigating the role of monetary union in reducing the transaction costs for foreign trade. This benefit is greater the larger the size of the union, because money, like language, is more useful the greater the number of persons who share the same type. We then add monetary issues, emphasizing the distinction between rules and discretion as in Barro and Gordon (1983).[3] Flexible exchange rates allow monetary

[2]Several papers have investigated the effects of exchange rate stability on trade flows, reaching mixed results. See, in particular, Hooper and Kohlhagen (1978), Kenen and Rodrik (1986), and International Monetary Fund (1984). Rose (2000) argues that the effect of currency union on the volume of trade is large.

[3]There is now a large literature on the rules-versus-discretion trade off. An application of that framework that is especially related to the present paper is in Alesina and Grilli

independence, but the monetary authorities of many countries lack the ability to commit their policies to a stable and predictable rule. Policies carried out under these conditions may produce high and variable inflation. In contrast, a system of irrevocably fixed exchange rates may be useful as a discipline device to assure price stability. However, this mechanism works effectively only if the domestic authority is willing to subordinate its monetary policy to the fixing of the exchange rate. Dollarization—or, less extreme, a currency board—is attractive as a way to ensure the credibility of a fixed-rate system. However, even with a permanently fixed exchange rate, as guaranteed by full dollarization, a country would experience changes in prices relative to those of the anchoring country. These relative price movements reduce the desirability of fixed exchange rates. Therefore, countries would prefer to link to anchors with which they have small variations in relative prices.

The analysis is complicated by two factors that we take into account. First, the choice of regime tends itself to affect the variances of relative prices and the co-movements of output. Second, the anchor country's monetary policy may change as a function of which countries adopt the anchor's currency. This adjustment of policy may feature compensation schemes between "clients" and "anchors," possibly involving the amount of seignorage revenue accruing to the various governments.

After discussing the pros and cons of adopting another country's currency, we study how, given a distribution of independent countries, certain types of currency unions would emerge in equilibrium. Under a broad range of conditions, an increase in the number of countries (thus, a reduction in their

(1992).

average size) would increase the desirability of currency unions. Hence, as the number of countries increases, the number of currencies should increase less than proportionately. In fact, under certain conditions, if one moves from, say, 100 countries to 200, the total number of currencies circulating may decrease in *absolute terms.* Consequently, in a world of small and highly integrated countries, where the benefits of low and stable inflation are highly valued, one should observe a collapse of the one-country one-money identity and a move toward a world with relatively few currencies.

The paper is organized as follows. The next section presents a model that highlights the pros and cons that a country faces when considering the adoption of a foreign currency. The following section discusses the endogenous formation of currency unions given a distribution of sizes of independent countries. The last section concludes.

2 A Model of Currency Unions

2.1 Output, Trade, and Country Size

We begin with a simple model of the real economy with a role for trade and country size. The text contains a sketch of the model with the main results. The details are in the appendix.

Suppose that the world consists of W individuals or economic regions, each of which has a fixed labor endowment, L. We can view these individuals as arrayed along a line segment, starting from the origin and then having equally spaced points at the positions $r = 1, ..., W$.

Each individual produces output, Y_r, using a varieties-type production function, which was originated by Spence (1976), Dixit and Stiglitz (1977), and Ethier (1982),

$$Y_r = A \cdot \left(\sum_{v=1}^{W} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha}, \tag{1}$$

where $A > 0$ is a parameter, X_{vr} is the amount of nondurable intermediate input of type v used by individual r, and $0 < \alpha < 1$. Output, Y_r, can be used on a one-for-one basis for consumption, C_r, or to produce r-type intermediates, X_r. All consumer goods are identical, but each person produces a unique variety of intermediate. Prices of consumer goods are the same everywhere and are normalized to one. Person r is assumed to have monopoly power over the supply of his or her unique type of intermediate, X_r. The price set for this good is denoted by P_r, where $P_r > 1$ will apply. The production function in equation (1) implies that every individual will want to use all of the available intermediate goods as long as all of the prices are finite.

A country is a collection of adjacent individuals. The size of country i, measured by the number of individuals, is denoted by N_i. Within each country, there is assumed to be free trade and no transaction costs for shipping goods. The shipping of an intermediate good across country borders entails transaction costs, which can reflect trade barriers and differences in language and currency. (For simplicity, we neglect any transaction costs for shipping consumer goods.) Specifically, we assume an iceberg technology, whereby, for each unit of intermediate good shipped from one country to another, only $1 - b$ units arrive, with $0 < b < 1$. The transaction costs would generally depend on the country pairs involved—for example, on distance and on differences in language—but we neglect these heterogeneities for now.[4]

Each producer of intermediates selects a single price, P_r, which applies at the point of origin for domestic purchasers and foreigners. Since foreigners receive only $1-b$ units for each unit purchased, their effective price per unit of r-type intermediate employed in production is $P_r/(1-b)$. Thus, trade within a country faces monopoly pricing, whereas international trade faces monopoly pricing and shipping costs.

Each individual r chooses the quantity of intermediates to buy at home or abroad, X_{vr}, for $v = 1, ..., W$ $(\neq r)$; the quantity of own output to retain for use as an intermediate input, X_{rr}; and the price of its intermediate, P_r. The choice of the quantity of each type of intermediate to import takes as given the monopoly prices, P_v, set by $v \neq r$. Given the demand function for the r^{th} intermediate good, the setting of P_r determines the quantity of intermediate goods sold by r. The budget constraint determines consumption, C_r, as output, Y_r, less the amount of retained intermediates, X_{rr}, plus the net revenue from sales abroad and at home (the quantity sold multiplied by $[P_r - 1]$), less the amount paid for purchasing intermediates. The terms involving imports and exports take account of the iceberg losses on goods transported across country borders. The objective of each individual is to maximize C_r.

[4]A large empirical literature has shown that political borders matter greatly for the volume of trade. That is, regions of the same country trade with each other much more than they would if they were independent. See, for example, McCallum (1995) and Helliwell (1998). More generally, the "home-bias" effect is pervasive in various aspects of international economic relationships, as discussed in a unified framework by Obstfeld and Rogoff (2000).

We show in the appendix that each producer of intermediates faces a demand curve with the constant elasticity $-1/(1-\alpha)$. This demand curve leads to the choice of the monopoly price or "markup ratio," $P_r = 1/\alpha > 1$, which is the same for all varieties of intermediate goods. The appendix also shows that the equilibrium level of output for individual r is given by

$$Y_r = \tilde{A}L \cdot \left\{1 + \alpha^{\alpha/(1-\alpha)} \cdot \left[(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)\right]\right\}, \qquad (2)$$

where $\tilde{A} \equiv A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$ and N_i is the size of the country to which r belongs. Note, inside the brackets in equation (2), that the production for own use counts as 1, the other $N_i - 1$ members of the same country count with the weight $\alpha^{\alpha/(1-\alpha)} < 1$ because of monopoly pricing of the traded intermediates, and the $W - N_i$ foreigners count with the even smaller weight $[\alpha \cdot (1-b)]^{\alpha/(1-\alpha)} < 1$ because of monopoly pricing and shipping costs. From the perspective of incentives to produce, monopoly pricing and trading costs have similar and reinforcing effects.

We show in the appendix that trades in intermediates between individuals in a country and across country borders are balanced. Hence, there are no net trades across borders in consumer goods. Trade in intermediates is partly domestic, that is, among residents of the same country, and partly foreign, that is, across country borders. The volume of trade (value of exports or imports) for region r with the other $N_i - 1$ regions of the same country is

$$\text{Value of domestic trade} = \tilde{A}L\alpha^{1/(1-\alpha)} \cdot (N_i - 1), \qquad (3)$$

and that with all of the foreign regions is

$$\text{Value of foreign trade} = \tilde{A}L\alpha^{1/(1-\alpha)} \cdot (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i). \qquad (4)$$

The country totals of domestic and foreign trade equal N_i multiplied by the expressions in Eqs. (3) and (4), respectively. The ratios of trade to output are given by

$$\text{Value of domestic trade/output} = \frac{\alpha^{1/(1-\alpha)} \cdot (N_i - 1)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]} \tag{5}$$

and

$$\text{Value of foreign trade/output} = \frac{\alpha^{1/(1-\alpha)} \cdot (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]}. \tag{6}$$

Note that the output concept given in equation (2) is gross of production of intermediates. In the case of balanced trade in intermediates, net output corresponds to consumption, which equals gross output less production of intermediates (including those that vanish due to the iceberg trading costs for international transactions). The appendix shows that the formula for consumption is

$$C_r = \tilde{A}L \cdot (1-\alpha) \cdot \left\{1 + \alpha^{\alpha/(1-\alpha)} \cdot (1+\alpha) \cdot \left[(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)\right]\right\}. \tag{7}$$

The qualitative implications of equations (2)-(7) are intuitively reasonable and generalize beyond the specific model that we have adopted. The implications include the following:

- If trading costs, b, were zero and pricing were competitive (which corresponds to $\alpha = 1$), then Y_r/L and C_r/L would be proportional to the size of the world, W. This scale benefit arises because a larger world means more varieties of intermediate inputs. In this case, the size of the country, N_i, would not matter. More generally, for given N_i, a higher W raises Y_r/L and C_r/L.

- If trading costs exist, then Y_r/L and C_r/L increase with N_i for given W. This effect arises because an increase in the size of the country expands the number of intermediate inputs for which the trading costs are nil.

- Y_r/L and C_r/L are decreasing in the international trading cost parameter, b.

- For given W, the larger the country, N_i, the smaller is the effect of trading costs, b, on Y_r/L and C_r/L. Analogously, the lower b, the smaller is the effect of country size, N_i, on Y_r/L and C_r/L.

- The ratio of foreign trade to output falls with b and N_i. The ratio of trade within a country to output rises with b and N_i.[5]

For given country sizes and trading costs, the distorting element in the model comes from the monopoly pricing of the intermediate goods. A social planner for the world would effectively price each of these goods at 1, rather than $P_r = 1/\alpha > 1$. Output, denoted by Y_r^*, would then be higher than before, corresponding to the replacement of the term $\alpha^{\alpha/(1-\alpha)}$ in equation

[5]If the production for own use is negligible, which holds, for example, if $N_i >> 1$, then these two effects are nearly offsetting. In this case, changes in trading costs, b, and country size, N_i do not have a significant effect on the ratio of total trade to output.

(2) by 1:

$$Y_r^* = \tilde{A}L \cdot [N_i + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]. \tag{8}$$

This result assumes that the social planner takes as given the sizes of countries, N_i, and must pay the costs b for inter-country trades. If country i contains many individuals, so that $N_i >> 1$, then the shortfall of production due to monopoly pricing is given from equations (2) and (8) by

$$Y_r/Y_r^* \approx \alpha^{\alpha/(1-\alpha)} < 1. \tag{9}$$

In this model, consumption per person (and, hence, the utility of the representative consumer) would be maximized if the entire world consisted of one country, because cross-border transaction costs would then be eliminated. However, this conclusion arises only because we have neglected some costs that tend to rise with the size of the country. In particular, larger political jurisdictions typically have to deal with a more heterogeneous citizenry. The growing heterogeneity makes it increasingly difficult to agree on a set of polices and institutions. In addition, diseconomies of scale in public administration tend to emerge at some level of country size.

Suppose that the per capita costs of heterogeneity are an increasing function of country size and can be represented by the function $h(N_i)$, with $h'(\cdot) > 0$. Then, in an interior equilibrium, the optimal size of a country is determined by the condition that the marginal benefit of size, emerging from equation (2), equal the marginal cost of heterogeneity.[6] Given the symmetry of the model, this condition will tend to dictate that all countries be of the

[6]This kind of tradeoff for determining country size is the one emphasized in Barro (1991), Alesina and Spolaore (1997), and Alesina, Spolaore, and Wacziarg (2000).

same size. However, if the heterogeneity costs—or the costs of trading across country borders—depend on the identity of the individuals, then we can have equilibria in which countries have different optimal sizes. In any event, we treat the country sizes, N_i, as exogenous in the present context.

2.2 Currency Unions and Trading Costs

The model just described shows how trading costs, b, influence the volume of foreign trade and, hence, the levels of production and consumption in each region. If we extend the model to allow the trading cost to depend on the country pairs, i and j, then the volume of trade (value of exports or imports) between countries i and j is given from a generalization of equation (4) by

$$\text{Value of trade between countries } i \text{ and } j = \tilde{A}L\alpha^{1/(1-\alpha)} \cdot (1-b_{ij})^{\alpha/(1-\alpha)} \cdot N_i N_j, \tag{10}$$

where b_{ij} is the trading cost between the countries. This expression for trade is the aggregate of the value of exports of interemediate goods from all of the regions of country i to all of the regions of country j. Correspondingly, the contribution of this foreign trade to the output and consumption in country i (or country j) follows from generalizations of Eqs. (2) and (7) as

$$\text{Effect on output of country } i = \tilde{A}L\alpha^{\alpha/(1-\alpha)} \cdot (1-b_{ij})^{\alpha/(1-\alpha)} \cdot N_i N_j, \tag{11}$$

$$\text{Effect on consumption of country } i = \tilde{A}L \cdot (1-\alpha^2) \cdot \alpha^{\alpha/(1-\alpha)} \cdot (1-b_{ij})^{\alpha/(1-\alpha)} \cdot N_i N_j. \tag{12}$$

One component of the trading cost, b_{ij}, consists of shipping costs, which depend on distance and available methods of transportation. Other components would involve government regulations, familiarity with foreign rules and business practices, and so on. In addition, trading costs would depend on financial considerations, including currency exchanges. We assume that the cost parameter, b_{ij}, can be expressed as the sum of two elements: the first, b_{1ij}, reflects all but the financial aspects, and the second, b_{2ij}, contains the financial terms. We assume that b_{2ij} is lower if countries share the same currency than if they use different currencies. Hence, the adoption of a common currency tends, on this count, to promote trade and, thereby, to raise output and consumption.

One question is whether countries that naturally trade more because of a smaller trading cost b_{1ij} would be more inclined to adopt a common currency. Specifically, is the net benefit from lowering the financial part of trading costs, b_{2ij}, greater if the other part of trading costs, b_{1ij}, is larger or smaller? To address this question, we assume that the use of a common currency involves some costs, which we do not specify precisely here. The assumption, however, is that these costs are independent of b_{1ij}.

The key matter is the effect of a reduction in trading costs on consumption. If the use of a common currency reduces b_{2ij} to zero,[7] then equation (12) implies that the increase in consumption is proportional to the expression

$$\Omega \equiv (1 - b_{1ij})^{\alpha/(1-\alpha)} - (1 - b_{1ij} - b_{2ij})^{\alpha/(1-\alpha)}.$$

The effect of b_{1ij} on Ω depends on α:

$$\partial\Omega/\partial b_{ij}^{1} \lesseqgtr 0 \text{ as } \alpha \gtreqless 1/2.$$

Since a higher b_{1ij} means less trade between the countries (equation [10]), one might have expected the effect of b_{1ij} on Ω to be unambiguously negative—

[7] The results that we obtain will be the same if costs are reduced but not to zero.

that is, the benefit from adopting a common currency would be smaller for countries that naturally trade less. The offsetting force is that, if b_{1ij} is large, the trades that occur must have a high value at the margin to justify the large trading cost. Specifically, the traded intermediates must have a high marginal product. Hence, the trade that is facilitated by reducing b_{2ij} to zero has a correspondingly large effect on a country's output and consumption.

The net effect depends on elasticities, which are determined in the model by the parameter α. If $\alpha > 1/2$, then the various intermediates are relatively close substitutes, and the dominant effect is that a lowering of b_{2ij} to zero saves on the trading costs incurred (which are more important when the volume of trade is large). If $\alpha < 1/2$, then the intermediates are poor substitutes, and the dominant effect involves the high marginal product of intermediates when the trading cost is high. Thus, to get the usual result—whereby countries or regions that naturally trade a lot would particularly benefit from using a common currency—one has to assume that the underlying tradable goods are relatively close substitutes.

2.3 Monetary Policy

To discuss the interaction between currency unions and monetary policy, we have to enrich the model to allow a role for nominal prices. We use a simple setting in which the nominal price of the monopolized intermediate goods involves some stickiness, whereas the prices of the competitive final goods are flexible. We describe the model for a two-country world, with regions $r = 1, ..., N$ in country 1 and $r = N+1, ..., W$ in country 2. The generalization to many countries is straightforward.

For country 1, let p_r be the nominal price of the rth intermediate good and p the nominal price of final goods (and, hence, consumer goods), all of which sell at one price. The second country uses a different currency and denominates its prices, p_r^* and p^*, in units of that currency. If all nominal prices were flexible, then the preceding analysis would go through, with the relative prices of each intermediate good, p_r/p and p_r^*/p^*, set at the monopoly level, $1/\alpha$, in each country.

Suppose that p and p^* are determined through some stochastic processes by each country's monetary authority. (The nominal monetary aggregates, which we do not model explicitly, adjust to achieve target nominal prices of final goods in each country.) We assume that the nominal exchange rate, ϵ, is flexible and adjusts so that the standard PPP condition holds:

$$\epsilon = p/p^*. \tag{13}$$

Assume that, in country 1, the nominal price p_r for $r = 1, ..., N$ must be set one period in advance by the producer of each respective type of intermediate good. (We shall make a parallel assumption about price setting in country 2.) The nominal price of each of country 1's intermediate goods in the nominal currency unit of country 2 is given from equation (13) by $p_r/\epsilon = p_r \cdot (p^*/p)$. Hence, the relative price (after division by p^*) is p_r/p, just as in country 1. Quantity demanded of this intermediate good by producers of final product in both countries will again be a constant-elasticity function of this relative price.

To find the nominal price p_r that maximizes a country 1 producer's expected profit, the only new element that we need to know is the probability distribution of p. As a first approximation, the set price will be given by

$$p_r \approx (1/\alpha) \cdot E_{t-1}p \tag{14}$$

for $r = 1, ..., N$, where $E_{t-1}p$ is the producer's one-period-ahead expectation of p. Hence, $p_r/p = 1/\alpha$, as before, when p is known with certainty one period in advance. When p is uncertain, the entire probability distribution of p will generally matter for the optimal choice of p_r.[8] However, for present purposes, we assume that equation (14) is a satisfactory approximation. By analogy for country 2, we have

$$p_r^* \approx (1/\alpha) \cdot E_{t-1}p^* \tag{15}$$

for $r = N+1, ..., W$.

If p exceeds $E_{t-1}p$, then the relative price p_r/p falls correspondingly below the monopoly level, and the demand for country 1 intermediates in both countries rises above the monopoly level. Analogously, an excess of p^* above $E_{t-1}p^*$ raises the demand for country 2 intermediates in both countries above the monopoly level. We assume, for now, that the producers of intermediate goods in each country meet the demands that are forthcoming at these

[8]The value of p_r that maximizes expected profit is given in general by

$$p_r = (1/\alpha) \cdot \frac{\int_0^\infty p^{1/(1-\alpha)} \cdot f(p)dp}{\int_0^\infty p^{\alpha/(1-\alpha)} \cdot f(p)dp},$$

where $f(\cdot)$ is the producer's one-period-ahead probability density function for p. If $\alpha = 1/2$, then this expression simplifies to

$$p_r = \frac{E_{t-1}(p)}{\alpha} \cdot (1 + s^2),$$

where s is the coefficient of variation of p. Hence, in this case, equation (14) holds if the coefficient of variation is much less than 1.

reduced real prices. From the standpoint of output in country 1, the parameter $\alpha^{\alpha/(1-\alpha)}$ in equation (2) is then replaced by $[\alpha \cdot (p/E_{t-1}p)]^{\alpha/(1-\alpha)}$ for the $N-1$ external regions of country 1. Similarly, the term $[(\alpha \cdot (1-b)]^{\alpha/(1-\alpha)}$ is replaced by $[\alpha \cdot (1-b) \cdot (p^*/E_{t-1}p^*)]^{\alpha/(1-\alpha)}$ for the $W-N$ regions of country 2. Hence, unexpected inflation in either country tends to raise output in country 1 (and, similarly, for country 2). Moreover, because of the distortion from the monopoly pricing of the intermediate goods, unexpected inflation tends to offset the distortion and leads thereby to an efficient expansion of output. The outcomes $p/E_{t-1}p = p^*/E_{t-1}p^* = 1/\alpha > 1$ would generate the efficient levels of production in each country.

However, there are two reasons why too much unexpected inflation would be undesirable in this model. From the standpoint of producers of intermediates from country 1, if $p/E_{t-1}p > 1/\alpha$, then the real price of produced intermediates falls short of the unit cost of production. Given the constant-cost assumption, the producers lose money on each unit produced and sold. If the producers nevertheless meet the demand, then the output of country 1 intermediates is inefficiently high. Alternatively, if the producers shrink output to zero to avoid losses on each unit produced, then no intermediates are produced in country 1 and output decreases drastically (in an inefficient manner). The general lesson is that the effect of unexpected inflation on relative prices can create distortions as well as reduce existing ones.

Second, with respect to a region's sales of intermediates to other regions of the same country, some amount of unexpected inflation leads to an efficient expansion of production but also implies a transfer of income from monopolistic providers to consumers. It seems reasonable that a policymaker would ignore these domestic transfers, especially because the monopolistic providers and the consumers are the same agents in the present model. Hence, on this count, a policymaker would value unexpected inflation, though the marginal benefit would diminish as unexpected inflation became larger (and would become negative, as already indicated, if $p/E_{t-1}p$ reached $1/\alpha$).

However, with respect to sales to regions of the other country, the loss of monopoly profits is not compensated by any benefits to domestic residents. As in the case of a monopoly tariff, the monopolistic providers of intermediate goods were already optimizing from the standpoint of the home country with respect to choices of export prices and quantities. Thus, unexpected inflation at home distorts the results from the perspective of the domestic country (while simultaneously generating benefits to the foreign country). Moreover, since the net benefit from the home effect approaches zero as $p/E_{t-1}p$ tends to $1/\alpha$, the net benefit of unexpected domestic inflation to the home country must become negative before $p/E_{t-1}p$ reaches $1/\alpha$.

¿From the standpoint of the policymaker for country i, the model rationalizes a loss function in which some amount of unexpected inflation (for prices of final product), $\pi_i - \pi_i^e$, reduces the loss. This effect diminishes with the size of $\pi_i - \pi_i^e$, eventually becomes nil, and subsequently changes sign. The amount of the initial loss reduction and the size of the interval over which unexpected inflation is beneficial depends on the extent of the existing distortion. In the model, the distortion varies inversely with the parameter α (see equation [9]). Thus, if we view this parameter as varying across countries and over time, then the policymaker of country i values unexpected inflation more when α_i is lower (that is, when the markup ratio, $1/\alpha_i$, is higher).

2.4 Independent Monetary Policy under Discretion

We assume that the objective of monetary policy in country i can be described by the minimization of the expected net costs of inflation, $\pounds_i$, which we write as a fraction of country i's GDP in a simple functional form:

$$\pounds_i = (\gamma/2)\cdot(\pi_i)^2 + (\theta/2)\cdot[\phi\cdot(\pi_i - \pi_i^e) - z - \eta_i]^2 - a\pi_i. \qquad (16)$$

The key term is the second one, $(\theta/2)\cdot[\phi\cdot(\pi_i - \pi_i^e) - z - \eta_i]^2$, where $\theta > 0$, $\phi > 0$, $z > 0$, and η_i is an error term with zero mean, serial independence, and constant variance $\sigma^2_{\eta_i}$. This term, which looks like an expectational Phillips curve, is intended to approximate the results from the preceding section. Specifically, if $\eta_i = 0$, then unexpected inflation, $\pi_i - \pi_i^e$, initially reduces the loss, $\pounds_i$. However, as in the model, the marginal benefit diminishes and eventually changes sign, when $\pi_i - \pi_i^e$ reaches z. The error term η_i corresponds in the model to movements of the markup ratio, $1/\alpha_i$, away from its mean value. A higher value of η_i (lower value of α_i) raises the initial benefit from unexpected inflation and expands the interval over which this benefit is positive.[9]

The first term in equation (16), $(\gamma/2)(\pi_i)^2$, where $\gamma > 0$, captures dead-

[9] An additional benefit of surprise inflation could reflect effects of surprise inflation on the real value of nominal obligations, for example, of government debt denominated in domestic currency. With distorting taxation, these kinds of capital levies would be valued, because they would reduce the distortions from other sources of revenue. In this case, a positive η_i would represent a situation in which this type of revenue is especially valuable, perhaps because of an emergency that motivates temporarily high levels of public spending.

weight losses from actual inflation. We do not model these costs formally but note that they could reflect costs of changing prices.

The final term in equation (16), $-a\pi_i$, where $a > 0$, represents seignorage revenue, which is taken to be linear in actual inflation.[10] Thus, the monetary authority values the seignorage revenue on a one-to-one basis. More generally, seignorage would be useful to the government because it expands the menu of taxes available. For an analysis of a currency union, this term is interesting because it may be allocated in different ways among members of the union.

Country i has the choice of conducting monetary policy on its own or anchoring to another country. On its own, the inflation rate is assumed to be determined in a discretionary manner each period to minimize $\pounds_i$, as defined in equation (16). The authority cannot make commitments about inflation, and the rational formation of expectations, π_i^e—based on information from the previous period—takes this incapacity into account. The timing is as follows: first, expectations on inflation are set, then the shock is realized and publicly observed, then the policymaker chooses inflation.

The solution for the discretionary equilibrium, which follows the approach of Barro and Gordon (1983), is

$$\hat{\pi}_i = \frac{a}{\gamma} + \frac{\theta\phi z}{\gamma} + \frac{\theta\phi\eta_i}{(\gamma + \theta\phi^2)}. \tag{17}$$

The resulting expectation of the net costs of inflation can be calculated from equations (16) and (17) as

[10] More complicated functional forms, including making seignorage a function of unexpected inflation, would not change the qualitative nature of the results.

$$E\hat{\mathcal{L}}_i = \frac{1}{2} \cdot \left[-\frac{a^2}{\gamma} + \theta z^2 + \frac{(\theta \phi z)^2}{\gamma} + \frac{\theta \gamma \sigma_{\eta_i}^2}{\gamma + \theta \phi^2} \right]. \qquad (18)$$

If the monetary authority could commit inflation at least one period ahead, then the inflation rate in equation (17) would be reduced by the inflation-bias term, $\frac{\theta \phi z}{\gamma}$. The term $\frac{(\theta \phi z)^2}{\gamma}$ in equation (18) reflects these costs from the inflation bias.

The monetary authority's reaction to the economic disturbance η_i, as shown in equation (17), is a countercyclical policy. This reaction creates unexpectedly high or low inflation—with corresponding effects on output—in response to movements of η_i. The ability of the monetary authority to tailor inflation to current economic conditions, as represented by η_i, is valuable in the model, that is, $E\hat{\mathcal{L}}_i$ is lower than it would be if this ability were absent. This effect provides the key benefit from an independent monetary policy in the model. A monetary authority that can commit to an optimal contingent rule would also have π_i responding to η_i in the manner shown in equation (17).

2.5 Outcomes under Dollarization

Consider now a potential anchor country, denoted by the subscript j. We assume that this country has the same underlying preference and cost parameters as country i, that is, the parameters in equation (16) are the same. However, the monetary authority of country j is able to commit its method for choosing inflation at least one period ahead. This authority picks an optimal contingent rule (a relation between π_j and η_j) to minimize the prior expectation of $\pounds_j$. The inflation rate in country j will be given by the form of equation (17), except that the inflation bias term is absent:

$$\pi_j^* = \frac{a}{\gamma} + \frac{\theta \phi \eta_j}{(\gamma + \theta \phi^2)}. \tag{19}$$

Note that country j's monetary authority reacts to its own economic disturbance, η_j, which is serially independent with zero mean and constant variance $\sigma^2_{\eta_j}$. However, η_j need not be independent of η_i.

Suppose that country i irrevocably fixes the exchange rate of its currency to that of country j by adopting country j's currency. In what follows we assume that the decision to "dollarize" is irrevocable. That is, even though country i cannot make a binding commitment to a policy rule, it can make an irrevocable commitment to give up its currency. This assumption rests on the idea that it is institutionally much more costly to renege on a dollarization commitment than on a monetary policy rule.[11] In the case of a fixed exchange rate, the inflation rate in country i, π_i, would equal π_j^* plus the rate of change of the price of a market basket of goods in country i expressed relative to that in country j. We assume that this rate of change of relative prices is given by an exogenous, random error term, ϵ_{ij}. This shock is serially independent with zero mean, constant variance σ^2_ϵ, and is distributed independently of the shocks to economic activity, η_i and η_j, in the two countries. Hence, under dollarization, country i's inflation rate is given by

[11] In any event, a foreign monetary authority lacks the power to erode the real value of dollar bills. The foreign government may, however, be able to depreciate the real value of dollar denominated domestic obligations by formal defaults.

$$\pi_i^j = \frac{a}{\gamma} + \frac{\theta\phi\eta_j}{(\gamma+\theta\phi^2)} + \epsilon_{ij}. \tag{20}$$

The j superscript indicates that the outcome applies for country i under anchoring to country j.

If country i no longer issues its own currency, then it loses the seignorage income, given by $a\pi_i$. The corresponding income accrues instead to country j. Country j may or may not compensate country i for this transfer of seignorage revenue. We assume, for now, that the anchor returns to country i the full amount of the seignorage obtained in country i. In this case the anchor country has no incentive to change its policy regardless of what country i chooses. We discuss below alternative arrangements. With inflation determined from equation (20), country i's expected net costs of inflation are given from equation (16) by:[12]

$$E\pounds_i^j = -\frac{a^2}{2\gamma} + \frac{\theta z^2}{2} + \frac{(\gamma+\theta\phi^2)\sigma_\epsilon^2}{2} + \frac{\theta^2\phi^2\sigma_{\eta_j}^2}{2(\gamma+\theta\phi^2)} + \frac{\theta\sigma_{\eta_i}^2}{2} - \frac{\theta^2\phi^2 COV(\eta_i,\eta_j)}{\gamma+\theta\phi^2}. \tag{21}$$

The covariance between η_i and η_j appears in equation (21) because it determines the extent to which country j's adjustments to its own disturbances, η_j, are helpful for country i. This criterion neglects any impact of dollarization on trading costs. In section 3, we combine trading costs with monetary policy effects in a general discussion of optimal currency areas.

[12]We are assuming that the form of equation (16) still applies under this regime, although the underlying model assumed a flexible exchange rate.

2.6 The Choice of Whether to Dollarize

We assess here the choice of currency regime based on a comparison of the monetary and inflation policies that result. The difference between $E\hat{\pounds}_i$ from equation (18) and $E\pounds_i^j$ from equation (21) is given by

$$\Delta\pounds^{ij} \equiv E\hat{\pounds}_i - E\pounds_i^j = \frac{(\theta\phi z)^2}{2\gamma} - \frac{1}{2}\cdot\left[(\gamma+\theta\phi^2)\cdot\sigma_\epsilon^2 + (\frac{\theta^2\phi^2}{\gamma+\theta\phi^2})\cdot VAR(\eta_i-\eta_j)\right]. \tag{22}$$

A positive value for $\Delta\pounds^{ij}$ indicates that the independent regime is more costly for country i than the system with anchoring to country j. Hence, anything that raises the terms on the right-hand side of the equation favors dollarization.

The first term, $\frac{(\theta\phi z)^2}{2\gamma}$, is the cost associated with the inflation bias under a discretionary regime in country i. The linkage to the committed country j avoids these costs and thereby favors dollarization. The second term, which involves σ_ϵ^2, derives from the random shifts in relative prices between countries i and j. Since country i receives country j's inflation rate only up to the random error, ϵ_{ij}, a higher value for σ_ϵ^2 makes dollarization less attractive. The third term, which contains $VAR(\eta_i-\eta_j)$, reflects the benefits from an independent monetary policy, in the sense that π_i can react to η_i in the autonomous regime. The extent of this benefit depends on how closely η_j moves with η_i. Equation (22) shows that the variance of $\eta_i - \eta_j$ is what matters for the comparison between the regimes.

Note from equation (22) that there are two senses in which greater co-movement between countries i and j favors dollarization. One relates to the variance of relative prices, σ_ϵ^2. This effect arises even if the monetary authorities do not conduct countercyclical policies. An effect of σ_ϵ^2 applies in equation (22) even if inflation surprises do not affect output ($\phi = 0$), as long as costs, $\pounds_i$, depend on actual inflation ($\gamma > 0$). Second, a greater variance of relative economic disturbances, $\eta_i - \eta_j$, makes dollarization less attractive.

2.7 Extensions

2.7.1 Dollarization affects the shocks

It is often argued that a common-currency link affects co-movements among countries, for example, by promoting trade and factor mobility. If we allow for an effect of the monetary system on the distributions of the shocks, then the criterion for dollarization is modified from equation (22) to

$$\begin{aligned} \Delta\pounds^{ij} &= \frac{(\theta\phi z)^2}{2\gamma} - \frac{1}{2}\cdot\left[(\gamma+\theta\phi^2)\cdot\sigma_\epsilon^2 + (\frac{\theta^2\phi^2}{\gamma+\theta\phi^2})\cdot VAR(\eta_i-\eta_j)\right] \quad (23) \\ &\quad + \frac{\theta\gamma(\tilde{\sigma}_{\eta_i}^2 - \sigma_{\eta_i}^2)}{2(\gamma+\theta\phi^2)}, \end{aligned}$$

where $\tilde{\sigma}_{\eta_i}^2$ is the variance of η_i in the autonomous regime, and the unmarked variances refer to the dollarized system. The last term indicates that dollarization would be favored if this linkage reduces the variance of disturbances in country i, that is, if $\tilde{\sigma}_{\eta_i}^2 > \sigma_{\eta_i}^2$. This effect would be predicted if the currency linkage buffers the disturbances that impinge on country i (because of the easier adjustments of trade and factor flows). Dollarization is also more attractive the lower σ_ϵ^2 and $VAR(\eta_i - \eta_j)$—these values are the ones applicable in the dollarized setting. Hence, if linkage reduces these variances, then dollarization looks more favorable.

2.7.2 Simple rules

The analysis of dollarization has assumed that country j commits to the contingent rule for π_j that minimizes the prior expectation of $\pounds_j$. However, one may argue that commitment is difficult to verify and, hence, maintain when it involves these sorts of contingent reactions of π_j to η_j.[13] In our model, the contingent rule is easy to implement and verify, but matters become much more complicated if shocks are not immediately and universally observable.

The nature of the issue can be illustrated by assuming that country j can follow discretion or commit to a simple rule that precludes feedback from η_j to π_j. In this case π_j would be set to the constant a/γ.[14] If the anchor follows the simple rule, the next to last term in equation (23) becomes

$$-\frac{\theta^2\phi^2\sigma_{\eta_i}^2}{2(\gamma+\theta\phi^2)}.$$

This term is smaller in magnitude than the corresponding term in equation (23) if

$$\sigma_{\eta_j} > 2\rho_{ij}\sigma_{\eta_i},$$

where ρ_{ij} is the correlation (under the dollarized regime) between η_i and η_j. Thus, if $\sigma_{\eta_i} = \sigma_{\eta_j}$, then if $\rho_{ij} > 1/2$ country j is more attractive as an anchor for country i if country j follows an optimal contingent rule where π_j responds to η_j. If $\rho_{ij} < 1/2$, then country j is a more attractive anchor if it follows the simple rule in which π_j is constant. In other words, active countercyclical

[13]See, for example, the symposium on central bank independence in the 1995 *NBER Macroeconomic Annual.*

[14]In this situation, country j might prefer discretion to the simple rule. Discretion allows for flexible responses of π_j to η_j, whereas the simple rule precludes these reactions.

policy by the anchor country is attractive to linking countries only if their disturbances (η_i) are—under the dollarized system—highly correlated with those of the anchor (η_j). Thus, for some potential clients, the inability of the anchor to follow a contingent first-best rule is a plus.

2.7.3 The anchor keeps the seignorage

If country j's objective is to minimize the expectation of $\pounds_j$ less the seignorage revenue obtained from country i (with no allowance for the costs of inflation borne by country i), then the only difference from equation (19) is in the choice of intercept. The new coefficient is

$$\frac{a}{\gamma} \cdot \frac{1}{(1-\tau_i)},$$

where $\tau_i \equiv Y_i/(Y_j + Y_i)$ is the shared of country i in the combined GDPs. Hence, the seignorage obtainable from country i motivates country j to select higher inflation than otherwise. The greater is τ_i the more inflation is raised above its previous level, a/γ. Thus, if the anchor country values the seignorage obtainable from clients but does not consider the costs that inflation imposes on these clients, then dollarization can be inflationary. The results are different, as discussed below, if the anchor takes account of the costs imposed on clients.

2.7.4 Adjustments by the anchor country with compensation

Another issue is whether the anchor country would be motivated to alter its policies to consider the interests of the linking countries, in effect, the clients of the anchor. We explore whether a system of transfers can make an adjustment of the anchor's policy mutually beneficial.[15]

The net cost of inflation, $\pounds_i$, from equation (16) applies as a fraction of country i's GDP, Y_i. If we take the universe as the anchor country j plus one linking country i, then the total net cost due to inflation, expressed as a share of the combined GDPs, $Y_j + Y_i$, is

$$\pounds = \tau_j \pounds_j + \tau_i \pounds_i, \tag{24}$$

where $\tau_j \equiv Y_j/(Y_j + Y_i)$ and $\tau_i \equiv Y_i/(Y_j + Y_i)$. One possibility is that the anchor country determines its policy rule to minimize the prior expectation of $\pounds$, rather than $\pounds_j$, as assumed before. The $\pounds$ objective weighs foreigners' net costs equally with those of domestic residents. Such an objective need not reflect global altruism by the anchor nation. Rather, this objective would emerge in equilibrium from competition among anchor countries, assuming that clients effectively compensate the anchor for deviating from policies that are otherwise best for the anchor's domestic residents. One way that this compensation could occur, as part of a competitive equilibrium, is for each anchor country to retain the amount of seignorage that just compensates for the worsening of policy from a domestic perspective. If there is not enough seignorage revenue to compensate, then some other mechanism would have to be devised to allow international payments for monetary services.

Let the anchor's policy rule be designated by

$$\pi_j = \mu + \nu_j \eta_j + \nu_i \eta_i + \nu_\epsilon \epsilon_{ij}, \tag{25}$$

[15]A complex political game may be involved in the fixing and implementation of these schemes. This game is not modeled here.

where $(\mu,\ \nu_j,\ \nu_i,\ \nu_\epsilon)$ are the feedback coefficients chosen by the monetary authority. Equation (19) is the special case of equation (25) that arises when $\pounds$ depends only on $\pounds_j$. The inclusion of $\pounds_i$ as part of the revised objective will affect the choice of some of the coefficients in equation (25), but the linear form will still be optimal in the present model.[16]

If country j's objective is to minimize the prior expectation of $\pounds$, then the optimal values of the coefficients that appear in equation (25) turn out to be

$$\begin{aligned} \mu &= a/\gamma, \\ \nu_j &= \tau_j \cdot \frac{\theta\phi}{(\gamma+\theta\phi^2)}, \\ \nu_i &= \tau_i \cdot \frac{\theta\phi}{(\gamma+\theta\phi^2)}, \\ \nu_\epsilon &= -\tau_i. \end{aligned} \tag{26}$$

The constant term, $\mu = a/\gamma$, is the same as before. That is, the consideration of the broader universe that encompasses country i does not change the average inflation rate chosen by country j. Hence, dollarization is not inflationary when the anchor takes account of costs imposed on clients. Country j's response, ν_j, to its own economic disturbance, η_j, is the same as before, except that the coefficient is attenuated by multiplication by the GDP share, τ_j. Correspondingly, the anchor's choice of inflation, π_j, now reacts in accordance with the coefficient ν_i to country i's economic disturbance, η_i. This

[16]Note that we have returned to the setting in which country j can commit to a contingent rule in the sense of committing to the coefficients shown in equation (25). We also neglect here, for simplicity, any effect of dollarization on the distribution of the disturbances, as explored before.

response depends on country i's GDP share, τ_i. The coefficient $\nu_\epsilon = -\tau_i$ means that country j's monetary authority partly offsets an increase in relative prices in country i by lowering π_j. The extent of the offset is given by τ_i, the share of country i's GDP.

¿From the perspective of minimizing the expectation of its own net costs, $\pounds_j$, country j's reactions of π_j to η_i and ϵ_{ij} and the insufficient reaction of π_j to η_j are, *per se*, unattractive. That is why this behavior by country j hinges on some sort of compensating payment from country i to country j. As already mentioned, one possibility is that country j retain part of the seignorage income associated with country i's use of country j's money.

On its own, country j chooses the inflation rate π_j^* given in equation (19). With the accommodation to country i, country j chooses the inflation rate π_j given by equations (25) and (26). The amount that country j loses from the accommodation can be calculated by looking at the difference in expected costs, $\pounds_j$, associated with the two choices of inflation. The result is

$$\begin{aligned} &\text{Cost of accommodation} \\ = \; &\frac{1}{2}(\tau_i)^2 \cdot \left\{ (\gamma + \theta\phi^2) \cdot \sigma_\epsilon^2 + (\frac{\theta^2\phi^2}{\gamma + \theta\phi^2}) \cdot VAR(\eta_i - \eta_j) \right\}. \end{aligned} \tag{27}$$

Thus, the cost to country j depends on the relative size of country i, τ_i, on the variance of the relative price shocks, σ_ϵ^2, and on the variance of the difference in the economic disturbances, $\eta_i - \eta_j$. If there were no relative price shocks and no differences in economic disturbances, then it would be costless for country j to accommodate its inflation choice to country i.

Suppose now that country i can choose whether to link to country j, that country j accommodates its inflation choice to the presence of country i (as implied by equations [25] and [26]), and that country i pays the compensation corresponding to equation (27).[17] The criterion for country i to dollarize is then modified from equation (22) to

$$\Delta \pounds^{ij} = \frac{(\theta\phi z)^2}{2\gamma} - \frac{\tau_j}{2} \cdot \left\{ (\gamma + \theta\phi^2) \cdot \sigma_\epsilon^2 + (\frac{\theta^2\phi^2}{\gamma + \theta\phi^2}) \cdot VAR(\eta_i - \eta_j) \right\}. \quad (28)$$

The new element in equation (28) is that the terms involving σ_ϵ^2 and $VAR(\eta_i - \eta_j)$ are smaller in magnitude than before because they are multiplied by τ_j, which is less than one. These terms are smaller because country j's partial adjustment of π_j for country i's disturbances makes these disturbances less costly for country i (even after considering the compensation that country i pays to country j). Thus, overall, the choice of dollarization looks more favorable because of the anchor country's willingness to accommodate its clients.

Another result from equation (28) is that a smaller value for τ_j makes dollarization more attractive. The reason is that a smaller τ_j reduces the compensation that country i must pay to country j for its accommodations. In this model, the attraction of dollarization is that it buys a committed monetary policy. A small anchor country is, in this respect, as good as a large one, because the commitment technology is assumed to work as well in either case. However, for the large anchor country, the costs of accommodating to country i are greater (because the term in equation [27] applies over a larger scale, Y_j). Thus, for given values of σ_ϵ^2 and $VAR(\eta_i - \eta_j)$, the small country is preferred as an anchor.

The conclusion about the desirable size of the anchor country may change if the capacity to maintain a commitment depends on the relative economic

[17]The level of compensation is the amount shown in equation (27) multiplied by Y_j.

sizes of the anchor country and its customers. For example, consider a large country, such as Russia, using a small one, say Latvia, as an anchor. This arrangement may not work because ex-post pressure from Russia to create "unanticipated" inflation could be too much for Latvia to bear. In other words, anchors that are larger (in relation to their clients) may be more solid because they can better withstand pressures to be time inconsistent.

3 Number of countries and of currencies

3.1 The setup

We now combine issues of trade and monetary policy to investigate the equilibrium number of currency unions in a world composed of an exogenous number of independent countries. To keep things simple, we return to the case of no compensation from clients to anchors, and we neglect any effect of dollarization on the variances of shocks.

In this situation, equation (22) implies that the criterion for country i to prefer linkage to country j over autonomy is given by

$$\Delta \pounds^{ij} = \frac{(\theta\phi z)^2}{2\gamma} - \frac{1}{2}\left\{(\gamma+\theta\phi^2)\cdot\sigma_\epsilon^2 + (\frac{\theta^2\phi^2}{\gamma+\theta\phi^2})\cdot VAR(\eta_i-\eta_j)\right\} > 0. \tag{29}$$

Recall that this criterion assumes that country j follows a committed policy, whereas country i would, on its own, follow a discretionary policy. Hence, the first element in the choice about currency unions is whether a country can make a commitment to a rule for monetary policy. We assume that there are two types of countries in this respect. The indicator β_i takes the value one if country i can make binding commitments and zero if it cannot. We treat this commitment ability as exogenous and do not allow for intermediate cases in which some form of partial commitment is feasible.

The second element concerns the distribution parameters for the disturbances in equation (29). Linkage is more attractive if σ_ϵ^2 and $VAR(\eta_i - \eta_j)$ are low under the dollarized system. We focus here on a key factor that would influence these distribution parameters—the extent to which countries i and j are linked by trade.[18]

Let T'_{i_j} be the value of the bilateral trade between countries i and j. Equation (10) implies that this trading volume depends inversely on trading costs, represented by the parameter $b_{ij} = b_{1ij} + b_{2ij}$, and positively on country sizes, N_i and N_j:

$$T'_{i_j} = \tilde{A}L\alpha^{1/(1-\alpha)} \cdot (1 - b_{ij})^{\alpha/(1-\alpha)} \cdot N_i N_j.$$

We posit that b_{1ij} increases with the distance between the countries. In the empirical gravity literature, the concept of distance captures physical distance and other factors, such as language, colonial history, sharing a border, being an island, etc. In our formalization, we assume that a country's position along the line segment that describes the world captures all these aspects of distance. Formally, if D_{ij} is the distance between the mid-points of countries i and j, then b_{1ij} is increasing in D_{ij}. Hence, T'_{i_j} is decreasing in D_{ij}.

[18]See Imbs (2000) for a review of the literature on how trade affects co-movements of output. Engel and Rose (2000) investigate determinants of the variances of relative prices, as measured by real exchange rates.

The trade volume is increasing in the size of each country. However, the correlation between the shocks of the two economies will be related to the volume of trade scaled in some manner by country sizes. If T_{i_j} is the trade volume scaled by size, then we assume that the larger T_{ij} the lower are σ_ϵ^2 and $VAR(\eta_i - \eta_j)$. Thus, smaller D_{ij} and, hence, higher T_{i_j} raise $\Delta \pounds^{ij}$ in equation (29).

If the adoption of a common currency reduces trading costs, then we noted before that the currency linkage also has a direct positive effect on trade, output, and consumption. Equation (12) shows how the trading cost, b_{ij}, relates to consumption:

Effect on consumption of country $i = \tilde{A}L \cdot (1-\alpha^2) \cdot \alpha^{\alpha/(1-\alpha)} \cdot (1-b_{ij})^{\alpha/(1-\alpha)} \cdot N_i N_j$.

Let $\Delta(1-b_{ij})^{\alpha/(1-\alpha)} > 0$ represent the effect from the reduction in b_{2ij} caused by the adoption of a currency union. The effect of union on country i's consumption—expressed as a ratio to N_i—is then given by

$$\Delta C^{ij} = \tilde{A}L \cdot (1-\alpha^2) \cdot \alpha^{\alpha/(1-\alpha)} N_j \cdot \Delta(1-b_{ij})^{\alpha/(1-\alpha)} > 0. \qquad (30)$$

The consumption gain is increasing in N_j and in the term, $\Delta(1-b_{ij})^{\alpha/(1-\alpha)}$, that reflects the reduction in trading costs. As discussed in section 2.3, if $\alpha > 1/2$, then lower b_{1ij}—caused, say, by smaller D_{ij}—raises the effect of a reduction in b_{2ij} on ΔC^{ij}. Hence, if $\alpha > 1/2$, the trade effect provides another reason for smaller D_{ij} to favor dollarization.

Country i will now choose to link to country j depending on whether the total benefit, given by $\Delta \pounds^{ij} + \Delta C^{ij}$, is positive. The country therefore cares about the expression

$$\Delta \mathcal{L}^{ij} + \Delta C^{ij} = \Gamma(\beta_j - \beta_i, T_{ij}, N_j), \tag{31}$$

where $\Gamma(\cdot)$ increases with $\beta_j - \beta_i$ and N_j and falls with D_{ij} (because of the reduction of T_{ij}).

We are interested in an equilibrium defined as follows:

Definition: *An equilibrium is a configuration of currency unions in which no country belonging to a union would like to leave the union to have its own currency or to join another union. In addition, no country not belonging to a union would like to join one.*

We begin by imposing some structure on the problem.

3.2 The case of equal country sizes

Assume first that the world consists of M countries of equal size $N = 1/M$. Obviously, countries for which $\beta = 1$ have a comparative advantage at providing the currencies used in multi-country currency unions. One can easily show that the largest D_{ij} for which country i would adopt the currency of country j is larger if $\beta_j = 1$ than if $\beta_j = 0$. Suppose that there are M countries, numbered from 1 to M from left to right. Assume that $\beta_k = \beta_h = 1$, with $1 \leq k < h \leq M$ and $\beta_i = 0$ for $i \neq k, h$. In the following, we let N^u represent the size of the currency union that a country is considering joining. The possible configurations of equilibria are as follows:

Configuration of equilibria: *If countries i and $i+2$ belong to the same currency union, so does country $i+1$. If* $\Gamma(0, T_{ij}, N^u) < 0$ for all i, j and any N^u, *then the possible configurations are: 1) M currencies in the world, no currency unions; 2) two currencies in the world, those of country k and country h; if $k-1 = M-h$, then the two currency unions include an equal number of countries, $m = M/2$; 3) two multi-country currency unions adopting currencies k and h, composed respectively of m_k and m_h countries. The remaining $(M - m_h - m_k)$ countries all have their own currency. If $k - 1 = M - h$, then $m_k = m_h$.*

If $\Gamma(0, T_{ij}, N^u) \gtrless 0$, depending on i, j and N^u, *then the additional possible configurations are as follows: 4) all the countries adopt one currency, either that of country k or country h; 5) $\zeta > 2$ multi-country currency unions that include a total of $M' \leq M$ countries.*

The first statement implies that currency unions are formed by adjacent countries. This result depends on all the countries having the same size. The sufficient condition that isolates the first three cases implies that the only countries that would want to adopt a currency other than their own are $\beta = 0$ countries, which may adopt the currency of a committed anchor. This condition tends to be satisfied if the main reason to enter a currency union is to obtain the policy commitment of the anchor. That is, the first term on the left side of equation (31) is dominant. Also, if the benefits from trade arising from sharing the same currency are relatively low, then not much is gained by $\beta = 0$ countries (or $\beta = 1$ countries) in giving up an independent monetary policy. A third factor that would work in favor of satisfying this condition is a high value of $VAR(\eta_i - \eta_j)$ or σ_ε^2, for given trade shares. Case 2 is a situation in which all the countries belong to one of two currency unions. This outcome tends to emerge when country shocks are similar or the trade benefits from belonging to a union are high. In case 3, some of the countries with $\beta = 0$ are too far from countries k and h and their currency unions to join either union.

If $\Gamma(0, T_{ij}, N^u) > 0$ for some countries, then some countries may want to form a union even without the benefit of commitment. This outcome arises if the trade gains are sufficient to compensate for the loss of monetary autonomy. In this situation two or more non-committed countries may form a union, because they are too far from a $\beta = 1$ country. For instance, consider two countries with $\beta = 0$ bordering each other but far from any country with $\beta = 1$. These countries may form a currency union if the trade benefits are sufficiently high and the benefit of commitment comes at too high a price because of the great distance of the closest $\beta = 1$ country. An analogous argument applies to countries with $\beta = 1$. Thus, two additional possibilities emerge. In case 4, all the countries adopt the same currency, either of country k or h. In case 5, some of the countries that do not belong to the currency unions of k or h in case 3 form their own multi-country currency union. A natural example is one in which countries k and h are close to the extremes of the line segment, so that a large range of countries in the middle of the line segment is far from a committed anchor country. A set of countries in the middle may then find it beneficial to form a currency union even without the benefits of commitment.[19]

[19] An interesting example is the discussion about a monetary union in Central America, as an alternative to dollarization.

3.3 Many countries and few currencies

As the number of countries increases, the equilibrium number of currencies may go up less than proportionally with the number of countries or may even decrease. Consider the following example with 3 countries of equal size—thus of size 1/3—numbered from 1 to 3 from left to right. Suppose that $\beta_1 = \beta_3 = 1$ and $\beta_2 = 0$ and that each country has its own currency. This configuration means that country 2 prefers autonomy, which implies, from equation (31), that[20]

$$\Gamma(1, T_{21}, 1/3) < 0 \text{ and } \Gamma(1, T_{23}, 1/3) < 0. \tag{32}$$

Suppose now that country 2 splits exogenously into two equal-sized countries, labeled from left to right by $2a$ and $2b$. In the new situation, countries $2a$ and $2b$ may find it attractive to adopt the currencies of countries 1 and 3, respectively. Consider, for instance, country $2a$. This country prefers to use the currency of country 1 if

$$\Gamma(1, T_{2a,1}, 1/3) > 0. \tag{33}$$

Note, since $D_{1,2a} < D_{12}$, $T_{2a,1} > T_{21}$. Therefore, conditions (32) and (33) can both be satisfied. Furthermore, country $2a$ does not want to adopt the currency of $2b$ instead of that of 1 if

$$\Gamma(1, T_{2a,1}, 1/3) > \Gamma(0, T_{2a,2b}, 1/6). \tag{34}$$

[20] It follows immediately, if this condition holds, that it is not in the interest of countries 1 and 3 to form a currency union without country 2. A three-country currency union is also not an equilibrium.

This condition can be satisfied together with the previous two, but it is not satisfied for all parameter values, because $D_{2a,2b} < D_{2a,1}$. Analogous considerations apply to country $2b$ and its decision to adopt the currency of country 3.

In summary, the example shows that a configuration of 3 countries/3 currencies can be an equilibrium and one with 4 countries/2 currencies can also be an equilibrium. Hence, as the number of countries increases, the number of currencies may fall. Two forces underlie this result. One is that smaller countries benefit more from currency unions because a larger fraction of their economy relies on foreign trade. The second is that a new country can be closer to an anchor than the original larger country to which the new one originally belonged.

By the same logic, consider the case of an initial 4 countries/4 currencies equilibrium. The two middle countries (2 and 3) are those with $\beta = 0$. Suppose that the two middle countries split in half, becoming $2a$ and $2b$ and $3a$ and $3b$, respectively. It is easy to verify that countries $2a$ and $3b$ may want to adopt the currencies of country 1 and 4, respectively. The other countries $2b$ and $3a$ may not adopt these anchor currencies because they are further away from the respective anchors. Hence, the equilibrium can move from 4 countries/4 currencies to 6 countries/4 currencies. It is also possible that countries $2b$ and $3a$ may want to form a currency union of their own even without a committed monetary policy. In this case, the new equilibrium would have 6 countries/3 currencies.

3.4 Countries of different size

Suppose now that countries come in two sizes, large and small, denoted by n and N, respectively. We can have four types of countries in terms of size and commitment ability: 1) Size N, $\beta = 1$; 2) Size N, $\beta = 0$; 3) Size n, $\beta = 1$; 4) Size n, $\beta = 0$.

Consider now the configuration of equilibria. A trivial case is one in which there are only countries of types 1 and 4, that is, the committed countries are also the large countries. The results of section 3.2 generalize immediately. A more interesting case is one in which all four types of countries exist. In this case, an important difference from before is that currency unions are not necessarily formed by countries adjacent to each other. For instance, suppose country j is of type 3 (small but committed), country $j+1$ is of type 2 (large but not committed), and country $j+2$ is of type 4 (small and not committed). It is possible that $\Gamma(1, T_{j+1,j}, n) < 0 < \Gamma(1, T_{j+2,j}, n)$. That is, it may be in the interest of a small but relatively far country $(j+2)$ to adopt the currency of an anchor (j), although a closer but larger country $(j+1)$ may opt out. For example, it may be in the interest of Panama and El Salvador to adopt the dollar, although it may not be in the interest of Mexico; or it may be in the interest of Latvia and Estonia to link to the euro, although it may not be worthwhile for Poland. The intuition is clear: the small country may have a higher trade share with the anchor even though it is farther away, precisely because it is small.

Another dimension in which countries differ is in their location. A country at the extreme of the line segment is relatively far from more countries than a country located in the middle. *Ceteris paribus*, a country in the middle is a more likely anchor than a country at the extremes.[21] Therefore, a small uncommitted country at the "borders" of the world is the least likely anchor, whereas a large committed country in the middle is the most likely anchor. Obviously, the real world is not a line segment and these observations have to be interpreted *cum grano salis*, but the point is that New Zealand may be a less likely anchor than Switzerland, not only because of the different inflationary histories of the two countries but also because of their geographical locations.

4 Conclusions

Currency unions have several real and monetary effects. To the extent that trade costs are lowered by a common currency, the latter leads to real output and consumption gains. The loss of monetary flexibility has costs and benefits. On the one hand, a country giving up its currency loses a stabilization device targeted to domestic shocks; on the other hand, it may gain credibility and thereby reduce undesired inflation. We have shown how the determination of optimal currency areas depends on a complex web of variables and interactions, including the size of countries, their "distance," the levels of trading costs, the correlations between shocks, and on institutional arrangements that determine how the seignorage is allocated and whether transfers between members of a union are feasible. The type of country with the strongest incentive to give up its own currency is a small country with

[21]Note that the literature on the gravity model (e.g. Rose [2000]) accounts for the "remoteness" of a country with an appropriate empirical specification.

a history of high inflation that is close (in a variety of different ways) to a large and monetarily stable country.

As the number of countries increases, their average size decreases and the volume of international transactions rises. As a result, more and more countries will find it profitable to give up their independent currency. We have shown that it is possible that as the number of countries increases, the number of currencies may not only increase less than proportionally but may even fall.

5 Appendix: The Model of Output, Trade, and Country Size

Consumption for individual or region r satisfies the budget constraint

$$\begin{aligned} C_r &= A \cdot \left(\sum_{v=1}^{W} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha} - X_{rr} + (P_r - 1) \cdot (X_r - X_{rr}) \qquad \text{(A1)} \\ &\quad - \sum_{v=1 \neq r}^{N_i} P_v X_{vr} - \left(\frac{1}{1-b}\right) \cdot \sum_{v=N_i+1}^{W} P_v X_{vr}, \end{aligned}$$

where r belongs to country i that contains individuals $v = 1, ..., N_i$; X_r is the total of intermediates produced by r; and we used the expression for output from equation (1):

$$Y_r = A \cdot \left(\sum_{v=1}^{W} X_{vr}^{\alpha} \right) \cdot L^{1-\alpha}. \qquad \text{(A2)}$$

The first-order conditions for maximizing C_r relate the quantities of intermediate inputs employed by individual r, X_{vr}, to the price, P_v, in accordance with

$$\begin{aligned} A\alpha L^{1-\alpha} X_{rr}^{\alpha-1} &= 1, \\ A\alpha L^{1-\alpha} X_{vr}^{\alpha-1} &= P_v,\ v = 1, ..., N_i\ (\neq r), \\ A\alpha L^{1-\alpha} X_{vr}^{\alpha-1} &= (\frac{P_v}{1-b}),\ v = N_{i+1}, ..., W. \end{aligned} \tag{A3}$$

The first-order condition for choosing P_r to maximize C_r is

$$\frac{(P_r - 1)}{P_r} \cdot \epsilon_{(X_r - X_{rr}), P_r} = -1, \tag{A4}$$

where the ϵ term denotes the elasticity of demand for exports, $X_r - X_{rr}$, with respect to P_r.

Conditions of the form of equation (A3) determine the demand, X_{rv}, from the other producers for r's intermediate good. Each of these demands and (since the relative weights are fixed) the overall demand have constant price elasticities equal to $-1/(1-\alpha)$. Substitution of this result into equation (A4) determines the monopoly price of intermediates to be the constant

$$P_r = 1/\alpha. \tag{A5}$$

This price is the same for all intermediate goods.

Substituting $P_v = 1/\alpha$ into equation (A3) determines the quantities of intermediates:

$$\begin{aligned} X_{rr} &= (A\alpha)^{1/(1-\alpha)} \cdot L, \\ X_{vr} &= (A\alpha^2)^{1/(1-\alpha)} \cdot L,\ v = 1, ..., N_i\ (\neq r), \\ X_{vr} &= [A\alpha^2 \cdot (1-b)]^{1/(1-\alpha)} \cdot L,\ v = N_{i+1}, ..., W. \end{aligned} \tag{A6}$$

Substitution of the results from equation (A6) into equation (A2) leads to the expression for output in equation (2):

$$Y_r = \tilde{A}L \cdot \left\{1 + \alpha^{\alpha/(1-\alpha)} \cdot \left[(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)\right]\right\}, \quad \text{(A7)}$$

where $\tilde{A} \equiv A^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}$. The result for consumption can be obtained by substituting from equations (A5)-(A7) into equation (A1) to get equation (7):

$$C_r = \tilde{A}L \cdot (1-\alpha) \cdot \left\{1 + \alpha^{\alpha/(1-\alpha)} \cdot (1+\alpha) \cdot \left[(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)\right]\right\}. \quad \text{(A8)}$$

An individual's total value of purchases of intermediates can be determined by multiplying the quantities X_{vr} for $v \neq r$ from equation (A6) by the monopoly price, $P_v = 1/\alpha$, as

$$\text{Value of purchases } = \tilde{A}\alpha^{1/(1-\alpha)}L \cdot [N_i - 1 + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]. \quad \text{(A9)}$$

This expression is gross of the losses from the iceberg transaction costs. The first term inside the brackets, $N_i - 1$, corresponds to purchases from individuals of the same country (equation [3]), whereas the second, $(1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)$, corresponds to foreign imports (equation [4]). Equation (A6) can also be used to show that an individual's sales of intermediates—to persons in the same country and to foreigners—equals the value of purchases.

The ratio of the value of trade to output is given from equations (A7) and (A9) by

$$\text{Value of trade/output } = \frac{\alpha^{1/(1-\alpha)} \cdot [N_i - 1 + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1-b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]}. \quad \text{(A10)}$$

If $\alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1 - b)^{\alpha/(1-\alpha)} \cdot (W - N_i)] >> 1$, then this ratio is approximately equal to the constant α and is therefore roughly independent of N_i and b.

The total trade ratio breaks down into two parts:

$$\text{Value of domestic trade/output} = \frac{\alpha^{1/(1-\alpha)} \cdot (N_i - 1)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1 - b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]} \tag{A11}$$

and

$$\text{Value of foreign trade/output} = \frac{\alpha^{1/(1-\alpha)} \cdot (1 - b)^{\alpha/(1-\alpha)} \cdot (W - N_i)}{1 + \alpha^{\alpha/(1-\alpha)} \cdot [(N_i - 1) + (1 - b)^{\alpha/(1-\alpha)} \cdot (W - N_i)]}. \tag{A12}$$

Hence, the domestic trade ratio in equation (A11) rises with N_i and b, whereas the foreign trade ratio in equation (A12) falls with N_i and b.

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