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PRIVATE INFORMATION, WAGE BARGAINING AND EMPLOYMENT FLUCTUATIONS

John Kennan

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ABSTRACT

Shimer (2005) pointed out that although we have a satisfactory theory of why some workers are unemployed at any given time, we don?t know why the number of unemployed workers varies so much over time. The basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment, for plausible parameter values. This paper extends the Mortensen-Pissarides model to allow for informational rents. Productivity is subject to publicly observed aggregate shocks, and to idiosyncratic shocks that are seen only by the employer. It is shown that there is a unique equilibrium, provided that the idiosyncratic shocks are not too large. The main result is that small fluctuations in productivity that are privately observed by employers can give rise to a kind of wage stickiness in equilibrium, and the informational rents associated with this stickiness are sufficient to generate relatively large unemployment fluctuations.

John Kennan
Department of Economics
University of Wisconsin
1180 Observatory Drive
Madison, WI 53706
and NBER
jkennan@ssc.wisc.edu

1. Introduction

The standard view of unemployment is that it takes time for workers to find the right job, and for employers to find the right worker. Fluctuations in the productivity of jobs naturally give rise to fluctuations in the number of workers looking for jobs, and in the number of employers looking for workers. High productivity is associated with a tight labor market in which more workers have jobs and fewer workers are looking for jobs, while employers are keen to hire more workers, so vacancies are plentiful; conversely, when productivity is low, unemployment is high and there are few vacancies.

This simple description of the source of unemployment fluctuations suggests that it should be possible to measure the variability of productivity and use this to explain the variability of unemployment, to a rough approximation. The Mortensen-Pissarides (1994) model is the natural framework for such a calculation, since it gives a precise account of the relationship between productivity and search on each side of the labor market. Shimer (2005) showed that the basic Mortensen-Pissarides model in fact translates fluctuations in labor productivity into unemployment fluctuations that are very much smaller than those seen in U.S. data. Thus although we have a satisfactory theory of why some workers are unemployed at any given time, we don't know why the number of unemployed workers varies so much over time. To a substantial extent the number of unemployed workers varies because of movements into and out of the labor force, which are not included in the Mortensen-Pissarides model. But even for people who are firmly attached to the labor force, the variations are large. For example, in the U.S. over the period 1967-2006, the median annual unemployment rate of white men aged 35-39 was 3.65%; in 10 of these 40 years, the rate was 4.4% or higher, while there were 11 years with a rate of 2.6% or lower. The basic reason for unemployment in this group is that no two workers are the same, and no two jobs are the same. Given that job separation rates are relatively stable, the unemployment rate is a measure of how long it takes to match workers and jobs. The question then is why the matching process should be so much slower in some years than in others.

Hall (2005) argued that this volatility problem can be fixed if the Nash bargaining component of the Mortensen-Pissarides model is replaced by a "sticky" wage-setting process. Brügemann and Moscarini (2007) showed that the volatility of unemployment remains implausibly low for a broad class of surplus-sharing rules: the Nash bargaining rule is not an isolated case. On the other hand when there is some stickiness in wages, the employers' incentive to create vacancies is magnified when the economy improves, and this increases unemployment volatility. Brügemann and Moscarini (2007) show that unless there is enough stickiness to generate countercyclical fluctuations in the rents accruing to workers, the job creation incentive cannot vary enough to match the unemployment volatility data.

As Rotemberg (2006) points out, the basic Mortensen-Pissarides model also predicts procyclical wages, which are not seen in the data, and this problem persists in the more general model developed by Yashiv (2006). Wage stickiness helps to resolve this discrepancy as well, but of course this is useful only if we understand why wages are sticky. Hall (2005) assumed that the wage level in a previous contract establishes a "social norm" that largely determines the wage in the next contract. In the absence of a theory of social norms, this explanation is incomplete. Similarly, Gertler and Trigari (2006) showed that staggered wage contracts magnify the incentive to create vacancies, but did not try to explain why workers and employers who are interested only in the present value of income would negotiate contracts that constrain the division of the surplus in matches that have not yet been made.

This paper shows that an extension of the Mortensen-Pissarides model in which some productivity fluctuations are privately observed by employers can explain the volatility of unemployment in a more parsimonious way. The introduction of private information precludes the Nash bargaining rule; instead, the surplus is divided using Myerson's Neutral Bargaining Solution, which generalizes the Nash bargaining solution to allow for private information. There are two main results. First, the extended model has a unique equilibrium. Second, this equilibrium exhibits a kind of wage stickiness, and the informational rents associated with this stickiness are sufficient to translate small fluctuations in productivity into large unemployment fluctuations.[2]

Brügemann and Moscarini (2007) consider generalizations of the Mortensen-Pissarides model in which both employers and workers have private information about the job match surplus, and they analyze a representative selection of bargaining models that yield procyclical worker rents, and therefore insufficient unemployment volatility. But their analysis is confined to situations in which the extent of private information does not change over the cycle, so there is no scope for procyclical informational rents, which is the main focus of the analysis in this paper.

[2]Menzio (2005) develops this idea in great detail, and derives a bargaining equilibrium in which transient productivity fluctuations that are privately observed by employers are not transmitted to wages.

2. A Model of Sticky Wages with Private Information and Aggregate Shocks

A successful job match generates a surplus to be divided between the worker and the employer. The value of the worker's output is the sum of two components, $p+y$, where p is common to all matches, and y is a random idiosyncratic component whose realization ("L" for low or "H" for high) is observed privately by the employer when the match is made. The aggregate component p is a publicly observed Markov pure jump process with two states (s = 1 in the bad state and s = 2 in the good state), and exit hazards λ_1 and λ_2. The high value of y has probability τ_s, depending on the aggregate state. The flow surplus is $p + y - y_0 > 0$, where $y_0 > 0$ is the flow value of unemployment (including unemployment benefits and the value of leisure). The expectation of the surplus is assumed to be higher in the good state.

When the joint continuation value from a match falls below the joint opportunity cost, the match is destroyed. The job destruction hazard rate is a constant, δ, and there is a constant returns matching function that generates a flow of new matches $M(N_U,N_V)$ from unemployment and vacancy stocks N_U and N_V. There is an infinitely elastic supply of potential vacancies, and the actual number of vacancies posted is such that the expected profit from a vacancy is zero. Workers and employers maximize the present value of net income, using the interest rate r.

In the Mortensen-Pissarides model, the match surplus is divided according to the Nash Bargaining Solution. In the model considered here, the surplus is not common knowledge, so this solution is not applicable. What is needed is a generalization of Nash's argument to cover bargaining problems involving private information. A natural choice is the Neutral Bargaining Solution (NBS) developed by Myerson (1984). The NBS coincides with the Nash solution under complete information, and in the more general case it divides the surplus equitably while respecting the incentive compatibility constraints arising from the existence of private information. Myerson (1984) shows that a Neutral Bargaining Solution always exists, and provides a characterization that is relatively tractable in simple cases.

In Appendix A it is shown that for the bargaining problem considered in this paper, the Neutral Bargaining Solution is unique, and it can be implemented by Myerson's Random Dictator mechanism. This means that the surplus is divided in the following way. Either the employer or the worker is randomly selected to make an offer, and if this offer is rejected the match dissolves. Clearly, the employer's offer will just match the worker's reservation level, which is the value of searching for another match. The worker effectively has two choices: an offer that exhausts the low surplus, with a sure acceptance, or an offer that exhausts the high surplus, with acceptance only if the high surplus has actually been realized. It is assumed that the parameters are such that the worker always finds it optimal to demand the low surplus, conceding the difference between the high and the low surplus to the employer as an informational rent.

Match Surplus

The match surplus depends on whether the idiosyncratic component of output is high or low, and it also depends on the aggregate state. Let S^L_s be the surplus when the idiosyncratic component is low, and the aggregate state is s, and similarly when the realization of y is high; and let $y^L_s = p_s + y^L$, and $y^H_s = p_s + y^H$, with $\Delta y = y^H - y^L$.

Let U denote the state-dependent continuation value of an unmatched worker, and let G denote the joint continuation value of a matched worker-employer pair. When $y = y^L$, the joint match values are determined by the following asset pricing equations

$$\begin{aligned} rG_1^L &= y_1^L - \delta\left(G_1^L - U_1\right) + \lambda_1\left(G_2^L - G_1^L\right) \\ rG_2^L &= y_2^L - \delta\left(G_2^L - U_2\right) - \lambda_2\left(G_2^L - G_1^L\right) \end{aligned} \quad \textbf{(1)}$$

and similarly when $y = y^H$. This specification rules out two interesting alternatives. First, the flow value of a match is the same for all workers. Nagypál (2005) shows that heterogeneity in workers' (private) evaluations of nonpecuniary job characteristics can substantially increase the volatility of unemployment. Second, there is no possibility of switching from low to high output, once the match has been made. Even in the absence of informational rents, this tends to increase unemployment volatility, by strengthening the incentive to create vacancies when a high-output match is more likely because the aggregate state is good. Costain and Reiter (2005) show that this vintage productivity effect can potentially explain the volatility of unemployment, but Brügemann (2005) shows that this effect is quite weak in the model considered in this paper.

It is assumed that there is free entry of employers, so that the continuation value of an unmatched employer is zero in all states. Thus the (state-dependent) match surplus S is the difference between G and U, and the match value equations can be rewritten as

$$\begin{aligned} (r+\delta)\left(S_1^L + U_1\right) &= y_1^L + \delta U_1 + \lambda_1\left(S_2^L - S_1^L + \Delta U\right) \\ (r+\delta)\left(S_2^L + U_2\right) &= y_2^L + \delta U_2 - \lambda_2\left(S_2^L - S_1^L + \Delta U\right) \end{aligned} \quad \textbf{(2)}$$

where $\Delta U = U_2 - U_1$. This implies

$$S_2^L - S_1^L + \Delta U = \frac{y_2^L - y_1^L + \delta \Delta U}{r + \delta + \Lambda} \tag{3}$$

where $\Lambda = \lambda_1 + \lambda_2$. Substituting this in (2) gives

$$\begin{aligned} (r+\delta)S_1^L &= y_1^L - rU_1 + \frac{\lambda_1\left(y_2^L - y_1^L + \delta \Delta U\right)}{r+\delta+\Lambda} \\ (r+\delta)S_2^L &= y_2^L - rU_2 - \frac{\lambda_2\left(y_2^L - y_1^L + \delta \Delta U\right)}{r+\delta+\Lambda} \end{aligned} \tag{4}$$

Using these equations, and the analogous equations for a high-output match, the effect of the aggregate state on the match surplus is given by

$$S_2^H - S_1^H = S_2^L - S_1^L = \frac{y_2^L - y_1^L - (r+\Lambda)\Delta U}{r+\delta+\Lambda} \tag{5}$$

Thus even if an unmatched worker has better prospects when the aggregate state is good, the match surplus might be lower, for a given output draw. On the other hand there is a higher probability of drawing a high output value in the good aggregate state.

The effect of the output draw on the match surplus is given by

$$S_2^H - S_2^L = S_1^H - S_1^L = \frac{\Delta y}{r+\delta} \tag{6}$$

Unemployment Continuation Values

The rate at which unemployed workers find new matches is $M(N_U,N_V)/N_U = m(\theta)$, where $\theta = N_V/N_U$ represents market tightness, and $m(\theta) = M(1,\theta)$. The job-finding rate function $m(\theta)$ is assumed to be increasing, and strictly concave, with $m(0) = 0$.

When a match is made, the worker is selected to make an offer with probability ν. In this case, the worker gets the low-output surplus, and the employer gets an informational rent if the realized output value is high. If the employer is selected to make an offer, the worker gets the reservation level U and the employer gets the whole surplus. So an unmatched worker's continuation values are determined by the asset pricing equations

$$
\begin{aligned}
rU_1 &= y_0 + m(\theta_1)\nu S_1^L + \lambda_1\left(U_2 - U_1\right) \\
rU_2 &= y_0 + m(\theta_2)\nu S_2^L - \lambda_2\left(U_2 - U_1\right)
\end{aligned}
\tag{7}
$$

Thus

$$
\begin{aligned}
rU_1 &= y_0 + \frac{r+\lambda_2}{r+\Lambda} m(\theta_1)\nu S_1^L + \frac{\lambda_1}{r+\Lambda} m(\theta_2)\nu S_2^L \\
rU_2 &= y_0 + \frac{r+\lambda_1}{r+\Lambda} m(\theta_2)\nu S_2^L + \frac{\lambda_2}{r+\Lambda} m(\theta_1)\nu S_1^L
\end{aligned}
\tag{8}
$$

Vacancy Creation

Employers post new vacancies to the point where the net profit from doing so is zero. When a match is made, the employer gets an informational rent if the match value is high, and also gets a fraction 1-ν of the low-output surplus (in expectation). The value to the employer of a filled vacancy in state s is given by

$$
\begin{aligned}
J_s &= \tau_s(1-\nu)S_s^H + \tau_s\nu\left(S_s^H - S_s^L\right) + \left(1-\tau_s\right)(1-\nu)S_s^L + \left(1-\tau_s\right)\nu \times 0 \\
&= (1-\nu)S_1^L + \tau_s\left(S_s^H - S_s^L\right)
\end{aligned}
\tag{9}
$$

Thus the zero-profit conditions implied by free entry are

$$
\begin{aligned}
0 &= -c + \frac{m(\theta_1)}{\theta_1}\left((1-\nu)S_1^L + \tau_1\left(S_1^H - S_1^L\right)\right) \\
0 &= -c + \frac{m(\theta_2)}{\theta_2}\left((1-\nu)S_2^L + \tau_2\left(S_2^H - S_2^L\right)\right)
\end{aligned}
\tag{10}
$$

where c is the flow cost of maintaining a vacancy.

It is convenient to let d = θ/m(θ) denote the expected duration of a vacancy. Then the free-entry conditions can be written as

$$
\begin{aligned}
cd_1 &= (1-\nu)S_1^L + \frac{\tau_1 \Delta y}{r+\delta} \\
cd_2 &= (1-\nu)S_2^L + \frac{\tau_2 \Delta y}{r+\delta}
\end{aligned}
\qquad \textbf{(11)}
$$

Solution

The model can be solved as follows. For given values of d_1 and d_2, the free entry conditions determine the low-state surplus values:

$$
S_s^L = \frac{c\left(d_s - \rho_s\right)}{1-\nu} \qquad \textbf{(12)}
$$

where

$$
\rho_s = \frac{\tau_s \Delta y}{c(r+\delta)} \qquad \textbf{(13)}
$$

for s = 1, 2. Equation (2) can be rearranged to give U_1 and U_2 as linear functions of S_1^L and S_2^L, and U_1 and U_2 can then be expressed in terms of d_1 and d_2 as

$$
\begin{aligned}
rU_1 &= y_1^L + \frac{c(\delta+r)(d_1-\rho_1)}{1-\nu} + \frac{\lambda_1\left(y_2^L - y_1^L\right)}{\Lambda + r} + \frac{\lambda_1 \delta c\left(d_2-\rho_2-(d_1-\rho_1)\right)}{(1-\nu)(r+\delta)} \\
rU_2 &= y_2^L + \frac{c(\delta+r)(d_2-\rho_2)}{1-\nu} - \frac{\lambda_2\left(y_2^L - y_1^L\right)}{\Lambda + r} + \frac{\lambda_2 \delta c\left(d_1-\rho_1-(d_2-\rho_2)\right)}{(1-\nu)(r+\delta)}
\end{aligned}
\qquad \textbf{(14)}
$$

Next (12) can be substituted in (8), giving

$$
\begin{aligned}
rU_1 &= y_0 + \frac{r+\lambda_2}{r+\Lambda}\frac{\nu c m(\theta_1)}{1-\nu}\left(d_1-\rho_1\right) + \frac{\lambda_1}{r+\Lambda}\frac{\nu c m(\theta_2)}{1-\nu}\left(d_2-\rho_2\right) \\
rU_2 &= y_0 + \frac{r+\lambda_1}{r+\Lambda}\frac{\nu c m(\theta_2)}{1-\nu}\left(d_2-\rho_2\right) + \frac{\lambda_2}{r+\Lambda}\frac{\nu c m(\theta_1)}{1-\nu}\left(d_1-\rho_1\right)
\end{aligned}
\qquad \textbf{(15)}
$$

Since m(θ) is strictly concave, with m(0) = 0, the ratio m(θ)/θ is strictly decreasing, so the function d = θ/m(θ) has an inverse, θ = H(d). Using equations (14) and (15) to eliminate U_1 and U_2, and making the substitution $m(\theta_s) = H(d_s)/d_s$ to eliminate θ, gives the following equations determining d_1 and d_2

$$
\begin{aligned}
\psi_1(d) &= Z_1 + \left(\frac{\rho_1}{d_1} - 1\right)\nu H(d_1) - (r + \delta + \lambda_1)(d_1 - \rho_1) + \lambda_1(d_2 - \rho_2) = 0 \\
\psi_2(d) &= Z_2 + \left(\frac{\rho_2}{d_2} - 1\right)\nu H(d_2) - (r + \delta + \lambda_2)(d_2 - \rho_2) + \lambda_2(d_1 - \rho_1) = 0
\end{aligned}
\tag{16}
$$

where

$$
Z_s = \frac{(1-\nu)\left(y_s^L - y_0\right)}{c}, s = 1,2 \tag{17}
$$

3. Existence and Uniqueness of Equilibrium

It is assumed that the function θ = H(d) is convex, with H(0) = 0.[3] Under this assumption, it will be shown that an equilibrium with informational rents exists, and that it is unique.

Proposition 1

If the function θ = H(d) is convex, and if H(0) = 0, then there is a unique vector $d^* = (d_1^*, d_2^*)$ such that $\psi(d^*) = 0$.

The proof uses the following result.

Lemma 1

Suppose H is a twice differentiable function, with H(0) = 0, H'(x) > 0 and H"(x) > 0, for x > a > 0. Define the function h, on the domain [a,∞), as

---

[3]This assumption holds in the Cobb-Douglas case. The condition H(0) = 0 means that the expected vacancy duration shrinks to zero as the number of vacancies per unemployed worker shrinks to zero. Although this is a reasonable condition, it effectively rules out any constant returns CES matching technology except for the Cobb-Douglas case. Indeed if the matching function is defined by $(M/\mu)^{\varrho} = \alpha U^{\varrho} + (1-\alpha)V^{\varrho}$, then a positive value of ϱ is ruled out because it implies that matches can be made even if there are no vacancies. On the other hand a negative value of ϱ is ruled out by the condition that θ/m(θ) shrinks to zero as θ decreases to zero. This is a case in which local behavior around θ = 0 has global implications because the CES parametric family is inflexible. It is not difficult to stitch together a Cobb-Douglas and a CES with negative ϱ, so that the function is Cobb-Douglas near zero, with H(0) = 0. Then if ϱ < -1, the function H(d) is not convex.

$$h(x) = \left(\frac{a}{x} - 1\right) H(x) \tag{18}$$

Then h'(x) < 0 and h''(x) < 0.

Proof

The first and second derivatives of h are as follows

$$\begin{aligned} h'(x) &= \left(\frac{a}{x} - 1\right) H'(x) - \frac{a}{x^2} H(x) \\ h''(x) &= \left(\frac{a}{x} - 1\right) H''(x) - 2\frac{a}{x^2} H'(x) + 2\frac{a}{x^3} H(x) \\ &= \left(\frac{a}{x} - 1\right) H''(x) + 2\frac{a}{x^3}\left(H(x) - xH'(x)\right) \end{aligned} \tag{19}$$

Since $x \geq a$, and $H'(x) > 0$, it is clear that h is decreasing. Any convex (differentiable) function H that passes through the origin has the property that $xH'(x) \geq H(x)$. Thus $h''(x) \leq 0$.

Proof of Proposition 1

First it will be shown that $\psi(d^*) = 0$ implies $d^* > \rho$. Indeed if $d_1 \leq \rho_1$ and $d_2 \geq \rho_2$ then $\psi_1(d) > 0$; and if $d_1 \geq \rho_1$ and $d_2 \leq \rho_2$ then $\psi_2(d) > 0$. If $d \leq \rho$, write $\psi(d)$ as

$$\begin{aligned} \psi_1(d) &= Z_1 + (\rho_1 - d_1)\,\nu\frac{H(d_1)}{d_1} - (r+\delta)(d_1 - \rho_1) + \lambda_1\left[(d_2 - \rho_2) - (d_1 - \rho_1)\right] \\ \psi_2(d) &= Z_2 + (\rho_2 - d_2)\,\nu\frac{H(d_2)}{d_2} - (r+\delta)(d_2 - \rho_2) - \lambda_2\left[(d_2 - \rho_2) - (d_1 - \rho_1)\right] \end{aligned} \tag{20}$$

These equations show that either $\psi_1(d)$ or $\psi_2(d)$ is a sum of four positive terms: the first three terms are positive in both equations, and if the last term is negative in one equation, it is positive in the other. Thus $\psi(d) \neq 0$ if $d \leq \rho$.

Next it will be shown that a solution exists. Note that $\psi(\rho) = Z > 0$. Define b as the solution of the linear equations obtained by setting H = 0. Then

$$b_s = \rho_s + \frac{\overline{Z}_s}{r+\delta} \quad , s = 1{,}2 \tag{21}$$

where

$$\begin{aligned} \overline{Z}_1 &= \frac{(r+\delta+\lambda_2)Z_1 + \lambda_1 Z_2}{r+\delta+\lambda_1+\lambda_2} \\ \overline{Z}_2 &= \frac{(r+\delta+\lambda_1)Z_2 + \lambda_2 Z_1}{r+\delta+\lambda_1+\lambda_2} \end{aligned} \tag{22}$$

Thus $b > \rho$ and $\psi(b) < 0$.

Since ψ_1 is increasing in d_2 and decreasing in d_1, the equation $\psi_1(d) = 0$ can be solved to obtain d_2 as an increasing function of d_1. Write this as $d_2 = \Upsilon_1(d_1)$. Since ψ_2 is increasing in d_1 and decreasing in d_2, the equation $\psi_2(d) = 0$ can also be solved to obtain d_2 as an increasing function of d_1. Write this as $d_2 = \Upsilon_2(d_1)$. Define the function $\Upsilon_0(x) = \Upsilon_2(x) - \Upsilon_1(x)$. Since $\psi_1(\rho_1, \Upsilon_1(\rho_1)) = 0$, and $\psi_1(\rho_1, \rho_2) > 0$, and ψ_1 is increasing in d_2, it follows that $\Upsilon_1(\rho_1) < \rho_2$. Also, since $\psi_2(\rho_1, \Upsilon_2(\rho_1)) = 0$, and $\psi_2(\rho_1, \rho_2) > 0$, and ψ_2 is decreasing in d_2, it follows that $\Upsilon_2(\rho_1) > \rho_2$. Therefore $\Upsilon_0(\rho_1)$ is positive. By a similar argument, $\Upsilon_0(b_1)$ is negative. Also, Υ_0 is continuous (since ψ_1 is linear in d_2 and ψ_2 is linear in d_1). So by the intermediate value theorem $\Upsilon_2(x) = \Upsilon_1(x)$ for some $x \in (\rho_1, b_1)$. This means that $\psi(x, \Upsilon_1(x)) = 0$, showing that a solution $d^* = (x, \Upsilon_1(x))$ exists (with $d^* > \rho$).

To show uniqueness, define the function $g(z) = \psi(\rho + z)$. Then $g(0) > 0$, g_1 is increasing in z_2 and g_2 is increasing in z_1, and both g_1 and g_2 are concave by Lemma 1. Therefore, by Theorem 1 in Kennan (2001), g has at most one positive root, meaning that ψ has at most one root above ρ. Since it has already been shown that ψ does have a root above ρ, and no roots anywhere else, the proof is complete.

Optimality of Pooling Offers

It has been assumed that when a match is made in the good aggregate state, and the worker is selected to make an offer, it is optimal to demand the low surplus, rather than demand the high surplus at the risk of destroying the match. Thus the equilibrium surplus values must satisfy the following no-screening conditions

$$
\begin{aligned}
S_1^L &\geq \tau_1 S_1^H = \tau_1\left(S_1^L + \frac{\Delta y}{r+\delta}\right) \\
S_2^L &\geq \tau_2 S_2^H = \tau_2\left(S_2^L + \frac{\Delta y}{r+\delta}\right)
\end{aligned} \tag{23}
$$

which can be written as

$$
S_s^L \geq \frac{\tau_s}{1-\tau_s}\frac{\Delta y}{r+\delta} \tag{24}
$$

for s = 1,2. Using the free entry conditions, this reduces to

$$
d_s \geq \bar{\rho}_s \tag{25}
$$

where

$$
\bar{\rho}_s = \left(1 + \frac{1-\nu}{1-\tau_s}\right)\rho_s \tag{26}
$$

Since $\rho_s = 0$ for $\tau_s = 0$, Proposition 1 implies that a unique equilibrium satisfying the no-screening conditions exists if τ_1 and τ_2 are small enough. Conversely, the no-screening condition fails as τ_s approaches 1 (as of course it should). Also, the condition necessarily holds if the expected cost of filling a vacancy is high enough. The choice between pooling and screening depends on how big the difference is between the good idiosyncratic draw and the bad draw, relative to the surplus associated with the bad draw. A screening offer risks losing the low surplus, in exchange for a chance of getting the high surplus. When the expected cost of filling a vacancy is high, the free entry condition implies that the surplus associated with a filled vacancy is high. The effect of this is to increase the opportunity cost of screening, without changing the benefit, so pooling becomes more attractive.

The main theoretical result is Theorem 1, which characterizes a set of parameter values for which an equilibrium exists, and shows that if the parameters lie in this set, the equilibrium is unique.

Theorem 1

If H(d) is a convex function, with H(0) = 0, and if $\psi(\bar{\rho}) \geq 0$, then a unique equilibrium exists.

Proof

By Proposition 1, there is a unique vector d^* such that $\psi(d^*) = 0$. Since $\psi(\bar{\rho}) \geq 0$ and $\psi(b) < 0$, the argument in the proof of Proposition 1 can be used to show that ψ has a root in the rectangle $[\bar{\rho}, A]$, and since there is only one root above ρ, this root is d^*. The no-screening conditions are satisfied because $d^* \geq \bar{\rho}$. Therefore d^* is the unique equilibrium.

4. **The Effects of Informational Rents**

Suppose that there are no transitions, and that the wage rate is fixed, as in Hall (2005). Then the free entry condition is

$$cd = \frac{y^L - w + \tau\Delta y}{r + \delta} \tag{27}$$

The right side of this equation is the capital gain from a filled vacancy, and the left side is the flow cost of maintaining the vacancy, multiplied by the expected vacancy duration. A higher productivity level, with a fixed wage, is offset in equilibrium by an increase in duration. If the profit flow is small (because the wage is high), a small productivity change implies a large proportional change in profits, and therefore a large proportional change in the rate at which vacancies are filled, which implies a large change in the unemployment rate.

The following lemma characterizes the wage as a nested weighted average of the productivity levels while employed (y) and while unemployed (y_0). This characterization applies to the standard Mortensen-Pissarides model, and in the informational rents model, the wage is determined in exactly the same way, assuming the low realization of the productivity shock (y^L). Let $\phi = m(\theta)$ be the job-finding rate, and define w_0 as the wage such that the worker's share of the flow surplus is ν:

$$w_0 = \nu y^L + (1 - \nu) y_0 \tag{28}$$

Lemma 2

If the aggregate state is permanent, the equilibrium wage is given by

$$w = \Omega y^L + (1-\Omega) w_0 \tag{29}$$

where

$$\Omega = \frac{\phi}{\phi + A} \quad ; \quad A = \frac{r+\delta}{\nu} \tag{30}$$

Proof:

From equation (7), the continuation value of an unemployed worker is

$$rU = y_0 + \phi \nu S^L \tag{31}$$

Equation (4) gives the surplus as

$$(r+\delta) S^L = y^L - rU \tag{32}$$

Solving these equations for U and S^L yields

$$\nu S^L = \frac{y^L - y_0}{A + \phi} \tag{33}$$

and

$$rU = \Omega y^L + (1-\Omega) y_0 \tag{34}$$

The continuation value of an employed worker is $E = U + \nu S^L$. The wage that delivers this value satisfies the asset pricing equation

$$rE = w - \delta(E - U) \tag{35}$$

Thus the wage is given by

$$
\begin{aligned}
w &= (r+\delta)E - \delta U \\
&= rU + (r+\delta)\nu S^L \\
&= \Omega y^L + (1-\Omega)y_0 + (1-\Omega)\nu\left(y^L - y_0\right) \\
&= \Omega y^L + (1-\Omega)w_0
\end{aligned}
\tag{36}
$$

which proves the result.

Using Lemma 2, the free entry condition can be written as

$$
cd = \frac{1}{\phi + A}\frac{1-\nu}{\nu}\left(y^L - y_0\right) + \frac{\tau\Delta y}{r+\delta}
\tag{37}
$$

The result for the standard model (with $\tau\Delta y = 0$) differs from the fixed wage result in two respects. First, if the job-finding rate is held constant, a large proportional change in d requires a large proportional change in the flow surplus from employment (rather than in the flow profit). This means that small productivity shocks do not cause large unemployment movements unless the flow surplus is small, as in Hagedorn and Manovskii (2006). Second, this exaggerates the relationship between productivity and unemployment, because the job-finding rate does not in fact stay constant when d increases. An increase in d implies an increase in ϕ, and this dampens the relationship between productivity and unemployment: workers receive a larger share of the flow surplus when an increase in the job-finding rate increases the continuation value of being unemployed, and this diminishes the incentive to create vacancies.

Informational rents affect unemployment in much the same way as fixed wages, because small productivity changes that are observed privately by employers do not affect wages. The wage is close to the low productivity level, for standard parameter values, so the profit flow y^L - w is small in equation (27). Since $\tau\Delta y$ is also small (in the sense that the no-screening condition is satisfied), changes in $\tau\Delta y$ therefore give rise to large proportional changes in profits, and in the unemployment rate.

The Cobb-Douglas Case

The equilibrium relationships between productivity, informational rents and the unemployment rate can be characterized more explicitly in the case of a constant-returns Cobb-Douglas matching function, $M = \mu U^{\alpha}V^{1-\alpha}$, with $m(\theta) = \mu\theta^{1-\alpha}$. In this case the equilibrium conditions (16) can be stated as

$$\begin{aligned}\zeta_1 &= (\phi_1 + A)\xi_1 + \frac{\lambda_1}{\nu}(\xi_1 - \xi_2)\\ \zeta_2 &= (\phi_2 + A)\xi_2 + \frac{\lambda_2}{\nu}(\xi_2 - \xi_1)\end{aligned} \tag{38}$$

where $\phi_s = m(\theta_s)$, and

$$\begin{aligned}\xi_s &= (\phi_s)^{\frac{\alpha}{1-\alpha}} - \frac{\mu^{\frac{1}{1-\alpha}}}{c}\frac{\tau_s \Delta y}{r+\delta}\\ \zeta_s &= \frac{\mu^{\frac{1}{1-\alpha}}}{c}\frac{1-\nu}{\nu}\left(y_s^L - y_0\right)\end{aligned} \tag{39}$$

Thus, as Shimer (2005) noted, the parameters c and μ enter only through the ratio $\mu_0 = \frac{\mu^{\frac{1}{1-\alpha}}}{c}$.

If the aggregate state is permanent, equation (38) reduces to (two copies of) the following equation:

$$\zeta = \left(\phi^{\frac{\alpha}{1-\alpha}} - R\right)(\phi + A) \tag{40}$$

where

$$R = \frac{\mu_0 \tau \Delta y}{r+\delta},\ \zeta = \frac{1-\nu}{\nu}\mu_0\left(y^L - y_0\right) \tag{41}$$

The effect of productivity variation with a square-root matching function ($\alpha = ½$) and no informational rents is illustrated in Figure 1, which plots the quadratic function on the right side of equation (40) against the constant on the left side, with R set to zero. Productivity changes move the horizontal line up and down in this figure, and the equilibrium job-finding rate adjusts along the quadratic curve. For standard parameter values, this curve is steep at the baseline equilibrium, and small productivity differences therefore have little effect on the job-finding rate.[4]

[4]In this figure, μ_0 is chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1 below, with $\Delta y = 0$ and $y_2^L = 1.03$, setting $\mu_0 = 1360/21$ implies $\phi_2 = 6$. The horizontal lines are drawn for $y_1^L = 1$ and $y_2^L = 1.03$.

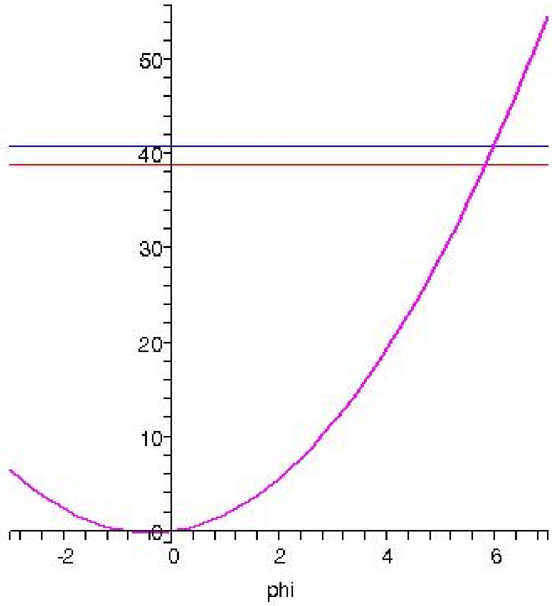


Figure 1

The elasticity of the job-finding rate with respect to productivity with no informational rents is

$$\frac{\partial \log(\phi)}{\partial \log(y)} = \frac{\dfrac{\partial \log(\zeta)}{\partial \log(y)}}{\dfrac{\partial \log(\zeta)}{\partial \log(\phi)}} = \frac{y}{y - y_0} \frac{1}{\dfrac{\alpha}{1-\alpha} + \dfrac{\phi}{\phi + A}} \tag{42}$$

This elasticity is not large unless the match surplus is small.

The effect of informational rents is shown in Figure 2. When R is positive, the quadratic curve shifts to the right (in the relevant region), and a comparison of the two curves shows that a small informational rent has a large effect on the equilibrium job-finding rate. On the other hand, the effect of (publicly observed) productivity movements remains small.[5]

[5]Here μ_0 is again chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1, with $p_2\Delta y = 3/100$ and $y_2^L = 1$, setting $\mu_0 = 1360/37$ implies $\phi_2 = 6$..

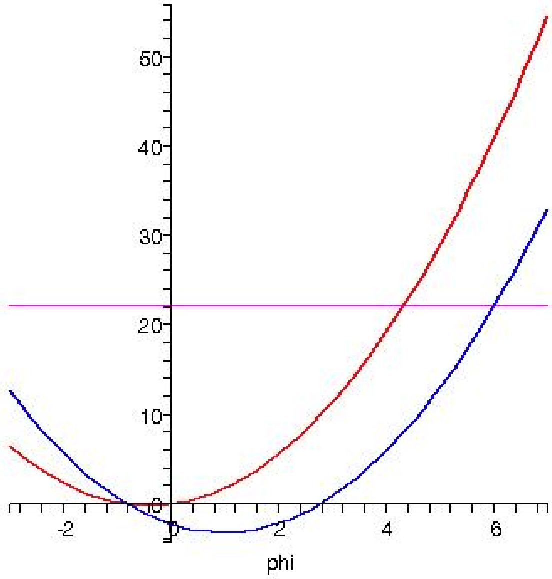


Figure 2

5. Wages

Pissarides (2007) has recently argued that wage stickiness is not the answer to the unemployment volatility puzzle, simply because wages are not in fact sticky: the wages of job-changers vary procyclically, and there is also evidence (due to Beaudry and DiNardo, 1991) that wages in continuing matches depend on labor market conditions at the time the match was made. In this paper wages are sticky in the sense that they do not respond directly to improvements in the distribution of the idiosyncratic component of match productivity, and the informational rent associated with this stickiness provides an incentive to create vacancies. But this does not imply that wages are acyclical: the informational rent has an indirect effect on wages by increasing the job-finding rate, and thereby increasing the continuation value of an unemployed worker. When the aggregate state is permanent ($\lambda_1 = \lambda_2 = 0$) the magnitude of this effect is given by Lemma 2. More generally, the cyclical variation of wages can be analyzed using a matrix version of Lemma 2.

Suppose $\tilde{U}$ is the continuation value of an unemployed worker, starting from the end of the job (that is, assuming that wages are zero on the present job from now on). Then

$$\begin{pmatrix} r+\delta+\lambda_1 & -\lambda_1 \\ -\lambda_2 & r+\delta+\lambda_2 \end{pmatrix}\begin{pmatrix} \tilde{U}_1 \\ \tilde{U}_2 \end{pmatrix} = \delta\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

This can be written in matrix form as $\Gamma\tilde{U} = \delta U$.

A job that starts in state s is worth $U_s + \nu S^L_s$ to the worker. This is delivered in two parts:

$$U + \nu S^L = W + \tilde{U}$$

Thus the present value of wages is

$$\begin{aligned} W &= \nu S^L + U - \tilde{U} \\ &= \nu S^L + \left(I - \delta\Gamma^{-1}\right)U \end{aligned}$$

where W is the vector $(W_1\ W_2)'$.

The surplus in a match with a low idiosyncratic component is given by equation (2):

$$\Gamma\left(S^L + U\right) = y^L + \delta U$$

The continuation value of an unemployed worker is given by equation (7):

$$\left(\Gamma - \delta I\right)U = y_0 + \nu\Phi S^L$$

where Φ is a diagonal matrix with elements ϕ_1 and ϕ_2. Combining these equations gives

$$\begin{aligned} S^L &= \left(\Gamma + \nu\Phi\right)^{-1}\left(y^L - y_0\right) \\ U &= \left(\Gamma - \delta I\right)^{-1}\left(y_0 + \nu\Phi\left(\Gamma + \nu\Phi\right)^{-1}\left(y^L - y_0\right)\right) \end{aligned}$$

Thus the present value of wages is

$$\begin{aligned} W &= \nu S^L + \left(I - \delta\Gamma^{-1}\right)\left(\Gamma - \delta I\right)^{-1}\left(y_0 + \nu\Phi S^L\right) \\ &= \Gamma^{-1}y_0 + \left(I + \Gamma^{-1}\Phi\right)\nu S^L \\ &= \Gamma^{-1}y_0 + \nu\left(I + \Gamma^{-1}\Phi\right)\left(\Gamma + \nu\Phi\right)^{-1}\left(y^L - y_0\right) \end{aligned}$$

The matrix analog of Lemma 2 is

$$\begin{aligned} \Gamma W &= y_0 + \nu\left(\Gamma + \Phi\right)\left(\Gamma + \nu\Phi\right)^{-1}\left(y^L - y_0\right) \\ &= \nu\left(\Gamma + \Phi\right)\left(\Gamma + \nu\Phi\right)^{-1}y^L + \Gamma\left(\Gamma + \nu\Phi\right)^{-1}\left(1 - \nu\right)y_0 \end{aligned}$$

Since $w_0 = \nu y^L + (1-\nu) y_0$, this gives

$$\begin{aligned}\Gamma W &= \nu(\Gamma+\Phi)(\Gamma+\nu\Phi)^{-1} y^L + \Gamma(\Gamma+\nu\Phi)^{-1}(w_0 - \nu y^L) \\ &= \left((\Gamma+\Phi)(\Gamma+\nu\Phi)^{-1} - \Gamma(\Gamma+\nu\Phi)^{-1}\right)\nu y^L + \Gamma(\Gamma+\nu\Phi)^{-1}(w_0 - \nu y^L) \\ &= \Phi(\Gamma+\nu\Phi)^{-1}\nu y^L + \Gamma(\Gamma+\nu\Phi)^{-1} w_0\end{aligned}$$

Thus

$$\Gamma W = \Omega y^L + (I - \Omega) w_0$$

where the weighting matrix Ω is defined by

$$\Omega^{-1} = I + \frac{1}{\nu}\Gamma\Phi^{-1}$$

In the one-dimensional case, this reduces to $\Omega^{-1} = 1 + (r+\delta)/(\nu\phi)$, as in Lemma 2, while $\Gamma W = (r+\delta)W$ when $\lambda_1 = \lambda_2 = 0$, so that this represents the flow value of wages.

There are of course many ways to specify a wage flow that adds up to the required present value of wages. The simplest way is make the wage constant for the duration of the job, in which case the flow wage is $w_s = (r+\delta)W_s$. Alternatively, the wage might be state-dependent, meaning that when the aggregate state changes, the wage changes to match the wage paid to new hires. In that case, the flow wage is given by $w = \Gamma W$, so it is a weighted average of y^L and y_0, as was just shown. This has the rather implausible implication that the wage in continuing matches falls when there is a transition to the bad aggregate state. The simplest way to avoid this is to specify a wage that is constant for the life of the match if the match begins in the good aggregate state, with a lower wage initially for matches that begin in the bad state, followed by a wage increase when there is a transition to the good state. In this case the flow wages are given by

$$\begin{aligned}w_2 &= (r+\delta)W_2 \\ w_1 &= w_2 - (r+\delta+\lambda_1)(W_2 - W_1)\end{aligned} \qquad \textbf{(50)}$$

The quantitative implications of these results are illustrated below (in Table 2).

6. **Unemployment Volatility**

The volatility of unemployment can be analyzed by comparing the steady-state levels of unemployment associated with each aggregate state (rather than measuring standard deviations in simulated data). Although this ignores movements along the transition paths from one steady state to the other, these transitions occur very rapidly, since the job-finding rate in the data is about 50% per month.

Standard parameter values are used as far as possible, following Shimer (2005) and Hall (2005). The interest rate is set at 5% per annum, and the job destruction rate δ is set at .35 per annum, so that the monthly rate is about 3%. The flow value of nonemployment is set initially at 40% of the flow value of employment. The matching function is Cobb-Douglas. The exit rate from unemployment is about 50% per month in the data, so μ_0 is chosen to solve the equilibrium equations with $\phi_1\lambda_2 + \phi_2\lambda_1 = 6\,(\lambda_2 + \lambda_1)$, meaning that the average job-finding rate is 6 per annum, the average being taken with respect to the invariant distribution of the Markov process. The expected cost of filling a vacancy in state s is given by $cd_s = \dfrac{(\phi_s)^{\frac{\alpha}{1-\alpha}}}{\mu_0}$.

In the NBER postwar data, the average duration of a recession is about a year, and the average duration of an expansion is about 5 years. This implies that the exit hazards are $\lambda_2 = 1/5$ and $\lambda_1 = 1$. Shimer (2005) reports summary statistics for detrended labor productivity (output per person), using an HP filter with smoothing parameter 100,000: the standard deviation is .02 log points. Since the model in this paper assumes that productivity is a two-state process, it is perhaps better to measure volatility as the difference between the average levels of productivity during recessions and expansions. Using the same detrended productivity series, this difference is .028 log points. Letting Y_1 and Y_2 denote aggregate state-contingent productivity levels, this implies that Y_2 should be about 3% above Y_1, so Y_2 is set to 1.03, with Y_1 normalized at one.[6]

The variation in the informational rent is chosen so as to match the fluctuations in productivity. A simple way to do this is to set $(\tau_2 - \tau_1)\Delta y = .03$, with $y_1^L = y_2^L$, so that the common component of the surplus does not depend on the aggregate state, but the probability of drawing the high idiosyncratic component is higher in the good state. For example, if there is no informational rent in the bad state

[6] Productivity could alternatively be measured as output per hour, and smaller smoothing parameters could also be justified. Since output per hour varies less than output per person, and smaller smoothing parameters (like the conventional choice of 1,600) attribute more of the variance to the trend component, these alternatives would give smaller volatility estimates. The point is that by any reasonable measure, labor productivity is not very volatile.

($\tau_1 = 0$), the rent in the good state is enough to account for the observed variation in aggregate productivity levels.

The parameter values are summarized in Table 1.

Table 1: Parameter Values			
Parameter	**Notation**	**Value**	**Comments**
matching function	$m(\theta)$	$\mu\theta^{1-\alpha}$	see text
recession exit hazard	λ_1	1	recession duration (1year)
expansion exit hazard	λ_2	1/5	expansion duration (5 years)
unmatched flow payoff	y_0	0.4	Shimer
low output	$y_1^L = y_2^L$	1	
informational rent	$\tau_2 \Delta y$	0.030	volatility of labor productivity ($\tau_1 = 0$)
separation rate	δ	.35	Shimer
interest rate	r	.05	

The steady-state unemployment levels are determined in the usual way as

$$u_s^* = \frac{1}{1 + \dfrac{m(\theta_s)}{\delta}}$$

In the case of a (Cobb-Douglas) matching function that is symmetric in unemployment and vacancies ($\alpha = ½$), the equilibrium values of ϕ_1 and ϕ_2 for the parameters in Table 1 can be obtained from the following equations:

$$\begin{aligned} 9\mu_0 &= 20\phi_1^2 + 56\phi_1 - 40\phi_2 \\ 138\mu_0 &= 200\phi_2^2 + 240\phi_2 - 80\phi_1 - 15\mu_0\phi_2 \end{aligned}$$

When μ_0 is chosen so as to give an average job-finding rate of 6, the solution is (ϕ_1 = 4.295536223, ϕ_2 =6.340892756, μ_0 = 39.54966078). In this example, ρ and $\bar{\alpha}$ are given by

$$
\begin{aligned}
\bar{\rho}_1 &= \rho_1 = 0 \\
\rho_2 &= \frac{3}{40c} \\
\bar{\rho}_2 &= \left(1 + \frac{1}{2(1-\tau_2)}\right)\rho_2
\end{aligned}
\qquad \textbf{(51)}
$$

In the bad state there is no informational rent, so the no-screening condition is irrelevant. In the good state the condition holds if $d_2 = \phi_2/\mu^2 \geq \bar{\rho}_2$. The equilibrium depends on τ_s only through the effect of τ_s on ρ_s (provided that the no-screening condition holds), and with $\tau_1 = 0$, ρ_2 depends on τ_2 only through the product $\tau_2 \Delta y$, which is set to 0.03. The no-screening condition then holds provided that $\tau_2 \leq 0.5605$.

Table 2 shows that informational rents can generate realistic variations in the unemployment rate. Even though the informational rent is only 3% of the productivity level, it moves the unemployment rate by about 40%. To put this in context, the table also shows the unemployment rates for a baseline parameter set that matches the variance of aggregate productivity by letting the match surplus depend on the aggregate state, with no idiosyncratic variation. These baseline parameter values are as in Table 1, but with $y_1^L = p_1 + y^L = 1$, $y_2^L = p_2 + y^L = 1.03$, and $y^L = y^H = 0$. In this case, the unemployment rate is virtually constant. The table includes results for a symmetric Cobb-Douglas matching function, with $\nu = ½$, and also for the labor share and matching elasticity parameters used by Shimer ($\alpha = \nu = 0.72$). Although these parameters affect the level of unemployment, they have little effect on volatility.

Table 2: Unemployment and Wage Volatility					
		Baseline Model		Informational Rent Model	
Productivity Variation	y_2^L	1.03		1.0	
	$\tau_2 \Delta y$	0		.03	
	$\nu = \alpha$	0.50	0.72	0.50	0.72
Unemployment Rates (Steady State)	u_1^*	5.61%	5.56%	**7.53%**	7.32%
	u_2^*	5.49%	5.50%	**5.23%**	5.25%
Wages: flat rates	w_1	0.983	1.004	0.957	0.982
	w_2	0.989	1.011	0.966	0.986
	$\Delta w\%$	0.7%	0.7%	1.0%	0.4%
Wages: non-decreasing rates	w_1	0.966	0.987	0.933	0.973
	w_2	0.989	1.011	0.966	0.986
	$\Delta w\%$	2.4%	2.5%	3.5%	1.4%

Note:

The "flat rate" wage is given by $w_s = (r+\delta)W_s$, where W_s is the present value of wages.

The "nondecreasing rate" is given by $w_1 = (r+\delta)W_1 - \lambda_1(W_2 - W_1)$, as explained in Section 5.

Table 2 also shows that even though wages are sticky with respect to cyclical changes in the distribution of the idiosyncratic component of productivity, there is nevertheless substantial cyclical wage variation. Thus although ad hoc sticky wage models have been strongly criticized by Pissarides (2007) on the grounds that they generate too little wage volatility, this criticism does not apply to the informational rents model. The present value of wages is about 1% higher in the good aggregate state, and if the wage contract delivers this present value by specifying a single constant wage for all matches when the aggregate state is good, and a temporary initial wage for matches made in the bad state, then the wage rate (in new matches) is about 3.5% higher in the good state.

Hagedorn and Manovskii (2006) have argued that the Mortensen-Pissarides model can generate realistic unemployment fluctuations if the value of the worker's outside option is close to the value of production. In the model considered here, this means setting y_0 near 1. Hagedorn and Manovskii calibrated y_0 as .955, with $\nu = .052$. Table 3 explores the implications of these parameter values, in the model with no informational rents.

Table 3: Unemployment Volatility (no informational rent)						
		Baseline	High y_0	Low ν	High y_0 low ν	Higher y_0 low ν
Variant		$y_0 = .40$ $\nu = .5$	$y_0 = .955$ $\nu = .5$	$y_0 = .40$ $\nu = .052$	$y_0 = .955$ $\nu = .052$	$y_0 = y_1^L = 1$ $\nu = .052$
Steady State Unemployment Rates	u_1^*	5.61%	6.56%	5.58%	6.19%	8.08%
	u_2^*	5.49%	5.34%	5.50%	5.39%	5.18%

When the workers' outside opportunities are almost as good as their market production opportunities, unemployment is indeed more volatile. Mortensen and Nagypál (2007) argue that this is quite unrealistic, since it implies that the *average* worker has little to gain from employment. Moreover, as Costain and Reiter (2006) and Hornstein, Krusell and Violante (2005) point out, it also implies implausibly large changes in unemployment rates in response to small changes in unemployment benefits. And even the rather extreme value of y_0 advocated by Hagedorn and Manovskii (2006) generates only about a 20% difference between the unemployment rates in the two states. The last column of the table shows that volatility increases sharply as y_0 approaches 1. It might seem that everyone should be unemployed in the bad state if $y_0 = 1$, since this means that jobs produce no surplus, and in order to move workers into jobs, it is necessary to expend resources on vacancy costs. But in fact the bad state is not expected to last very long, and jobs generate a (small) surplus in the good state. Moving some workers into jobs in the bad state reduces congestion in the matching process when the economy switches to the good state. From the employer's point of view, it is worthwhile to create vacancies in the bad state in anticipation of a transition to the good state, because when that transition occurs the aggregate component of productivity will increase. If the transition to the good state is unlikely, the unemployment rate in the bad state will be high. But in the data, recessions are relatively short-lived, so the Hagedorn and Manovskii calibration yields a relatively small difference between the unemployment rates in the good and bad states.

Table 4 shows that in a comparison of steady states with no transitions, the Hagedorn and Manovskii calibration gives much more volatility.[7] But this is largely beside the point, since the volatility in the data is generated by a single economy with transitions between states, while Table 4 compares the steady states of two different economies. This is illustrated by considering the effects of the very low value for the labor share parameter used by Hagedorn and Manovskii. Although this generates additional volatility in the comparison of two unrelated economies shown in Table 4, it actually reduces volatility in the more relevant comparison of steady states of a single stochastic economy, as shown in Table 3. Again, the reason for this is evidently that when the employer gets most of the surplus there is a stronger tendency to create vacancies in the bad state in anticipation of a transition to the good state, and this effect is absent in the model without transitions.

Table 4: Unemployment Volatility with no transitions ($\lambda_1 = \lambda_2 = 0$)

		Baseline	High y_0	Low ν	High y_0 low ν	Informational Rent
Variant		y_0 =.40 ν = .5	y_0 =.955 ν = .5	y_0 =.40 ν = .052	y_0 =.955 ν = .052	y_0 =.40 ν = .5
Steady State Unemployment Rates	u_1^*	5.63%	6.85%	5.66%	7.37%	7.24%
	u_2^*	5.49%	5.30%	5.48%	5.25%	5.26%

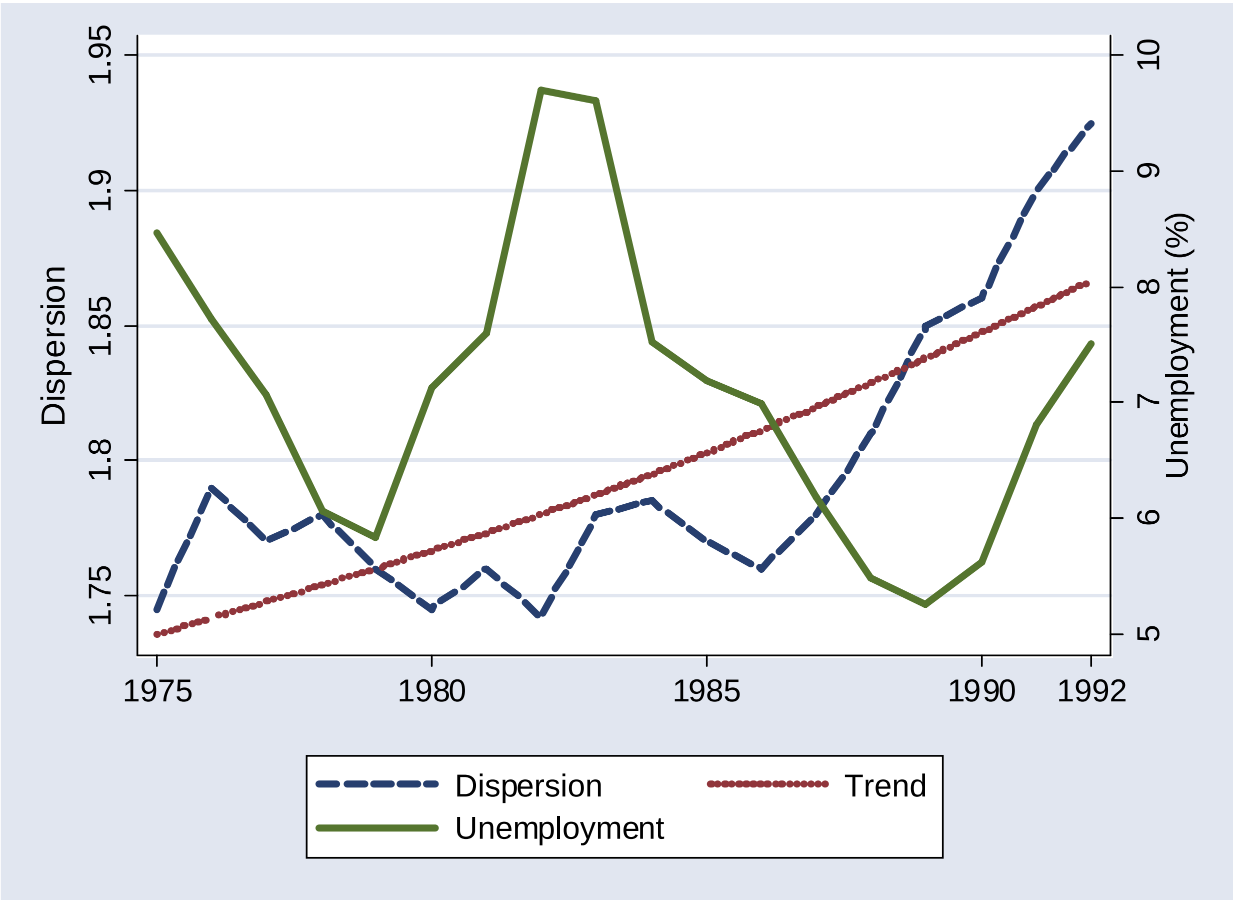
Cyclical Movements in the Dispersion of Productivity

It has been shown that small variations in informational rents generate large variations in the unemployment rate. It is of course difficult to obtain evidence that such variations actually occur, precisely because they are due to private information. But it seems reasonable to suppose that procyclical variations in informational rents would be associated with procyclical variations in the variability of labor productivity across employers. Figure 3 shows some evidence in favor of such variations. Dunne, Foster, Haltiwanger and Troske (2004) analyzed productivity dispersion across manufacturing plants over the period 1975-1992. As Dunne *et al* point out, dispersion increased over this period, especially from 1986

[7]Here μ_0 cannot be chosen so as to equate the average job-finding rate in the model with the empirical value, because each realization of the aggregate state is permanent, so there is no invariant distribution that can be used to take an average. Instead, μ_0 is chosen to solve the equilibrium equations with $\phi_1 + \phi_2 (\lambda_1/\lambda_2) = 6\,(1 + (\lambda_1/\lambda_2))$, where $\lambda_1/\lambda_2 = 5$, as in the baseline model. This corresponds to taking a limit as the transition rates become small, while their ratio stays fixed.

to 1992. Figure 3 shows the Dunne *et al* productivity dispersion series, overlaid on a Hodrick-Prescott estimate of the trend, in relation to the U.S. unemployment rate. Productivity dispersion is clearly procyclical over this (admittedly short) period.

Figure 3: Productivity Dispersion in U.S. Manufacturing



7. **Are Informational Rents Bigger when More Information is Private?**

A key feature of the model is that the dispersion of the privately observed component of the match surplus increases in the good aggregate state, and this increased dispersion gives rise to an increase in informational rent. An important theoretical issue is then whether the increase in informational rent arises merely from the assumption that the privately observed portion of the surplus takes just two possible values, with the worker optimally choosing to demand one of these values or the other. Given just two possible realizations, a small increase in the probability of the high surplus is not enough to induce the worker to switch the optimal demand from the low value to the high value, but in the case of a more general distribution an improvement in the distribution would induce the worker to make a more aggressive demand, and it is not clear that this would leave a larger informational rent for the employer.

One way to analyze this is to consider an alternative situation in which the surplus is distributed continuously over an interval. Without loss of generality, the lowest possible surplus can be taken to be 0. Thus consider a worker making a take-it-or-leave-it demand to an employer in a match where the surplus is a + s, with a > 0 and s distributed over the interval [0,b]. Let y = s/b, distributed on [0,1] according to the distribution function F. The worker's payoff from a demand a + bx is zero if this demand exceeds the actual surplus, which is the case if x > y. Thus the worker's expected payoff is (a+bx)(1-F(x)).

A simple way to formulate the question is to ask whether an increase in b leads to an increase in the informational rent. An increase in b, with F fixed, magnifies the surplus in every state of the world. This is known to the worker, and it is assumed that the employer's only options are to say yes or no to a single offer made by the worker. The question is whether the employer's expected payoff increases when the distribution improves in this way, even though the change in the distribution is common knowledge, and the worker has all of the bargaining power. In the absence of private information, the employer would get no surplus in this situation. So any positive payoff for the employer arises solely from the employer's informational advantage, and is thus an informational rent.

The informational rent when the worker demands x is R = b[E max(y,x) - x], which can be written as

$$R = b\int_{x}^{1}\left[1 - F(t)\right]dt \qquad \textbf{(52)}$$

The worker's problem can be stated as

$$\max_{x\in[0,1]} (c + x)\left(1 - F(x)\right) \qquad \textbf{(53)}$$

where c = a/b.

First, it is clear that an increase in b implies an increase in the worker's optimal demand. Indeed suppose that b increases from b_0 to b_1, meaning that c decreases from c_0 to c_1 (with a fixed), and let the corresponding optimal choices of x be x_0 and x_1. Then optimality implies

$$\begin{aligned}(c_0 + x_0)(1 - F(x_0)) &\geq (c_0 + x_1)(1 - F(x_1)) \\ (c_1 + x_1)(1 - F(x_1)) &\geq (c_1 + x_0)(1 - F(x_0))\end{aligned} \quad \textbf{(54)}$$

Since the expected payoff is positive for x = 0, and zero for F(x) = 1, it follows that 1-F(x) is positive at the optimum. Thus the above revealed preference inequalities can be written as

$$\frac{c_0 + x_0}{c_0 + x_1} \geq \frac{1 - F(x_1)}{1 - F(x_0)} \geq \frac{c_1 + x_0}{c_1 + x_1} \quad \textbf{(55)}$$

which implies $\Delta x \Delta c \leq 0$. Thus an increase in b implies an increase in x.

Next, the pooling solution (x = 0) is optimal if the opportunity cost of screening is high relative to the expected gains. Indeed pooling is optimal if and only if

$$\max_{x \in [0,1]} x\left(\frac{1}{F(x)} - 1\right) \leq c \quad \textbf{(56)}$$

For example, if F is the uniform distribution, pooling is optimal if $c \geq 1$. On the other hand, if F is differentiable at 0, then $F(x) \approx xF'(0)$, for x near 0. Thus if F has zero density at the origin, then pooling cannot be optimal for any value of c.

Obviously, if pooling is optimal, and remains so after an increase in b, then the informational rent increases with b, just as in the case of the two-point distribution analyzed above. The issue considered here is whether this result is limited to the case where pooling is optimal. In fact, it is not: it is shown in Appendix B that it holds for a reasonably broad class of distributions.

If pooling is not optimal, and if F has a density f, the optimal choice of x must satisfy the first-order condition

$$(1 - F(x)) - f(x)(c + x) = 0 \quad \textbf{(57)}$$

This can be written as

$$\frac{1}{h(x)} = \frac{a}{b} + x \tag{58}$$

where h is the hazard function.

The simplest example is the uniform distribution. In that case if b < a, pooling is optimal, so a (small) increase in b necessarily gives an increase in the informational rent. For some distributions, when b increases to the point where screening becomes optimal, there is then a downward jump in the rent. The two-point distribution has this property: when the probability of the good state or the size of the surplus in the good state rise beyond a certain point, the informational rent disappears. But it will be shown that in the case of a uniform distribution there is no downward jump, and that further increases in b lead to increases in R, even though the worker screens more aggressively.

For a uniform distribution with b > a, the optimal screening price is

$$x = \frac{1}{2}\left(1 - \frac{a}{b}\right) \tag{59}$$

Thus the rent is given by

$$\begin{aligned} R &= b\int_x^1 [1-t]\,dt = b\left(\frac{1}{2} - x + \frac{1}{2}x^2\right) \\ &= \frac{a}{8}\left(2 + \frac{b}{a} + \frac{a}{b}\right) \end{aligned} \tag{60}$$

which is increasing in b (when b > a). Thus when the surplus is magnified, the informational rent increases, regardless of whether pooling or screening is optimal.[8]

[8]More general versions of this result for a class of beta distributions are given in Appendix B. Thus the result for the case of a two-point distribution of the private information variable is reasonably robust. But it should be noted that the result certainly does not hold for all distributions, or even for all beta distributions. For example, if $F(y) = y^{3/4}$, with a = 1, screening becomes optimal when b = 2.1165. There is a downward jump in the informational rent at this point, and after this the rent continues to fall until b = 2.2684; beyond this point the rent is increasing in b.

8. **Alternative Explanations of Unemployment Volatility**

Starting with Shimer (2005) and Costain and Reiter (2005), the literature on unemployment volatility has developed very rapidly. The main developments have recently been reviewed by Mortensen and Nagypál (2007), and by Pissarides (2007). The focus has been largely on modifications of the wage-setting mechanism that increase the elasticity of the job-finding rate with respect to productivity. The main point is that if the profit margin associated with a filled job is very small, then small changes in productivity can have large effects on profits, and therefore on vacancy creation.

As a point of reference, consider a simple wage-setting mechanism in which the wage is set one day at a time. Each day, either the worker or the employer is selected to make a take-it-or-leave-it offer setting the wage for that day. Wages and profits are then given by

$$w_0 = \nu y + (1-\nu) y_0$$
$$\pi_0 = y - w_0 = (1-\nu)\left(y - y_0\right)$$

where ν is the probability that the worker makes the offer. Thus w_0 is a weighted average of the worker's productivity inside and outside of the match. It was shown in Lemma 2 above that the Nash bargaining mechanism used in the M-P model yields a wage that is an average of y and w_0, with weights that depend on the job-finding rate. Thus the wage is a nested weighted average of y and y_0, and since Φ is close to 1 for standard parameter values, the Nash wage is heavily influenced by the worker's job-finding prospects. The worker gets a bigger share of the surplus when the job-finding rate increases, but even in a recession the worker gets a very large share.

The elasticity of the job-finding rate with respect to productivity is governed by the free entry condition. Not much is lost by assuming that the matching function is Cobb-Douglas, and that it is symmetric in the number of vacancies and the number of unemployed workers, so that $\phi = \mu\sqrt{\theta}$. In this case the free entry condition can be written as

$$\frac{c}{\mu^2}\phi = \frac{y-w}{r+\delta}$$

Thus the volatility of the job-finding rate is the same as the volatility of the profit flow (with δ fixed). Using Lemma 2, the elasticity of ϕ is given by

$$\frac{\partial \log(\phi)}{\partial \log(y)} = \frac{y}{y - y_0} \frac{1}{1 + \Phi}$$

This elasticity is not large unless an unmatched worker is almost as productive as a matched worker, as in Hagedorn-Manovskii (2006).

If the wage is fixed, then

$$\frac{d \log(\phi)}{d \log(y)} = \frac{y}{y - w}$$

Thus, as Hall (2005) pointed out, if the wage is sticky the response of the job finding rate to productivity may be very elastic; but this is true only if the wage is set at a level that is close to the productivity level.

If the wage is negotiated every day, then $y\text{-}w = (1\text{-}\nu)(y\text{-}y_0)$, so

$$\frac{d \log(\phi)}{d \log(y)} = \frac{y}{y - y_0}$$

This is closely related to the point made by Hall and Milgrom (2007): if the worker's option to find another match is regarded as being irrelevant, then there is no link between ϕ and w, and so the elasticity of the job finding rate is increased. But again, the elasticity is not large unless the value of the worker's outside option is almost as good as the value of working.[9]

Nagypál (2005) and Mortensen and Nagypál (2007) develop an extended version of the Mortensen-Pissarides model that is capable of matching both the volatility of unemployment and the observed negative correlation of unemployment and vacancies. They introduce four modifications of the basic model. First, exogenous job destruction shocks provide an additional source of unemployment movements, (without being so large as to overturn the negative U-V correlation generated by productivity shocks). Second, there are substantial job-to-job flows. Although the job separation rate is relatively constant, as was argued by Hall (2006), the flow from employment to unemployment increases in recessions, as was shown by Elsby, Michaels and Solon (2007), because the job-to-job flow decreases.

[9]In the Hall-Milgrom model, the employer pays a bargaining cost γ each day until a wage agreement is reached. Thus on a day when the worker is selected to make the offer, the wage is $y+\gamma$ (leaving the employer indifferent between saying yes or no), and when the employer makes the offer the wage is y_0. The average profit flow is then $(1\text{-}\nu)(y\text{-}y_0\text{-}\gamma)$. Thus the elasticity ϕ can be made large even though y_0 is not close to y, by choosing a suitable value of γ.

Third, in order to post a vacancy, the employer must pay a lump sum hiring cost (in addition to the flow cost of maintaining the vacancy). Fourth, the wage bargaining is day-to-day (as described above), so the wage is not affected by the job-finding rate. Taken together, these four modifications lead to a model that can match the data if the lump sum hiring cost is sufficiently large (about nine months worth of profits). But the extended model is unwieldy, and the empirical plausibility of the required hiring costs is questionable. Moreover, as Pissarides (2007) points out, if the observed fluctuations in the job destruction rate are interpreted in the context of the original Mortensen-Pissarides model (rather than being treated as exogenous changes in the rate at which matches are destroyed), they do not generate much volatility in unemployment, because they are not associated with changes in the job creation rate. The informational rents model developed here introduces just one modification of the standard model, and thereby explains the same facts in a more parsimonious way.

9. **Conclusion**

Rent is a powerful economic force, and private information is a pervasive rent source, so it is plausible that private information can help to explain features of the economy that are otherwise difficult to understand. It has been shown here that the introduction of private information in an otherwise standard model of unemployment fluctuations provides a reasonable explanation for the volatility of unemployment. In the standard Mortensen-Pissarides model, unemployment fluctuations are driven by labor productivity shocks. In the data, these shocks are small, and the implied fluctuations in unemployment are also small, and much smaller than the fluctuations in the data. But if the productivity realizations are privately observed by employers, the implications for unemployment fluctuations are quite different. Small productivity shocks generate informational rents for employers, and small rents are a powerful job creation force. Thus privately observed productivity shocks of the magnitude seen in the data can generate realistic unemployment fluctuations.

Appendix A

The Neutral Bargaining Solution: One-sided Private Information with 2 types

Myerson (1984) proposed the Neutral Bargaining Solution as a generalization of the Nash bargaining solution suitable for a broad class of two-person bargaining problems with private information. There is a finite set of decisions, D. A direct revelation mechanism μ specifies the probabilities of the various decisions, conditional on information reported by the players. The payoff for player i, $u_i(d,t)$, depends on the decision d, and on the vector of reported types, t. Thus the expected payoff of player i of type t_i is

$$U_i\left(\mu|t_i\right) = \sum_{t\in T}\sum_{d\in D} p\left(t|t_i\right)\mu\left(d,t\right)u_i\left(d,t\right)$$

where p gives the probability of the type vector t, conditional on t_i, and μ(d,t) is the probability that the decision d is chosen, given that types are reported as t.

Consider a two-player bargaining problem in which the surplus to be divided is either S^H or S^L, with probabilities τ, and 1-τ, where the realization of S is known to player 1, but not to player 2. There are three decisions, D = $\{d_0,d_1,d_2\}$, where d_0 is the conflict outcome in which each player gets zero, d_1 means that player 1 gets S, where S is the realized value of the surplus (and player 2 gets zero), and d_2 means that player 2 gets S^H, and player 1 gets $S-S^H$. Thus the payoffs depend on whether player 1 is of type H or type L, as follows

(u_1,u_2)	d_0	d_1	d_2
1H	(0,0)	$(S^H,0)$	$(0,S^H)$
1L	(0,0)	$(S^L,0)$	$(-\Delta S,S^H)$

where $\Delta S = S^H - S^L$.

It is not difficult to show that there are incentive-efficient mechanisms that select d_0 with positive probability if and only if $\tau \geq S^L/S^H$ (see Kennan (1986)). This condition does not hold in the model considered in this paper. Incentive compatibility then requires that the probability of choosing d_1 is the same in both states, since player 1's payoff is increasing in this probability, regardless of the state. Let α be the multiplier associated with this constraint, and let $\mu(d_i)$ be the probability of choosing decision i, for $i \in \{1,2\}$.

Myerson's Theorem 5 gives a set of conditions which must hold for every mechanism μ in the set of neutral bargaining solutions. It will be shown that there is only one mechanism that satisfies these conditions. Since the NBS set is not empty (according to Myerson's Theorem 2), this is a neutral bargaining solution, and it is unique.

The conditions are as follows. The mechanism μ is incentive-efficient, and there exist $\lambda \in R^3$, $\omega \in R^3$ and $\alpha \geq 0$, such that

[(8.6)] $\lambda \geq 0$, $\omega \geq 0$ and $(\lambda,\alpha) \neq 0$

[(8.7)] μ is an optimal solution of the primal problem for λ. That is, there is some $\alpha \geq 0$ such that μ maximizes the lagrangean

$$\ell = \lambda_2 U_2(\mu) + \lambda_{1L} U_{1L}(\mu) + \lambda_{1H} U_{1H}(\mu) + \alpha\big(U_{1H}(\mu) - U^*_{1H}(\mu, L)\big)$$

with $\alpha = 0$ if $U > U^*_{1H}(\mu,L)$, where $U^*_{1H}(\mu,L)$ is player 1's payoff when falsely reporting type L. This lagrangean can be rewritten in terms of "virtual evaluations" V as

$$\ell = \sum_{d \in D} \mu_H(d)\big(V_{1H}(d) + V_{2H}(d)\big) + \sum_{d \in D} \mu_L(d)\big(V_{1L}(d) + V_{2L}(d)\big)$$

where

$$\begin{aligned} V_{1H}(d) &= \big(\lambda_{1H} + \alpha\big)u_{1H}(d) \\ V_{2H}(d) &= \lambda_2 \tau u_{2H}(d) \\ V_{1L}(d) &= \lambda_{1L} u_{1L}(d) - \alpha u_{1H}(d) \\ V_{2L}(d) &= \lambda_2 (1 - \tau) u_{2L}(d) \end{aligned}$$

[(8.8)] α is an optimal solution of the dual problem for λ. That is, α solves

$$\min_{\alpha \geq 0} \sum_{t \in T} \max_{d \in D} \sum_i V_i\big(d, t, \lambda, \alpha\big)$$

[(8.9)] The virtual evaluations satisfy the following ("warrant") equations

$$\begin{aligned} \lambda_{1L}\omega_{1L} - \alpha_{1H}\omega_{1H} &= \frac{1}{2}\max_{d \in D}\big(V_{1L}(d) + V_{2L}(d)\big) \\ \big(\lambda_{1H} + \alpha_H\big)\omega_{1H} &= \frac{1}{2}\max_{d \in D}\big(V_{1H}(d) + V_{2H}(d)\big) \\ \lambda_2\omega_2 &= \lambda_{1L}\omega_{1L} + \lambda_{1H}\omega_{1H} \end{aligned}$$

[(8.10)] Each type of each agent gets at least the warranted expected utility

$$\omega_2 \leq U_2(\mu) = \left(\tau\mu_H(d_2) + (1-\tau)\mu_L(d_2)\right)S^H$$
$$\omega_{1H} \leq U_{1H}(\mu) = \mu_H(d_1)S^H$$
$$\omega_{1L} \leq U_{1L}(\mu) = S^L - \mu_L(d_2)S^H$$

[(8.5)] The warranted and actual expected utilities satisfy the following complementary slackness conditions

$$\omega_2 = U_2(\mu) \text{ or } \lambda_2 = 0$$
$$\omega_{1H} = U_{1H}(\mu) \text{ or } \lambda_{1H} = 0$$
$$\omega_{1L} = U_{1L}(\mu) \text{ or } \lambda_{1L} = 0$$

The first step is to maximize the Lagrangean. This is done by choosing a decision in each state that maximizes the sum of the virtual evaluations, the relevant sums being given by

$$V_{1H}(d_1) + V_{2H}(d_1) = (\lambda_{1H} + \alpha)S^H$$
$$V_{1H}(d_2) + V_{2H}(d_2) = \lambda_2 \tau S^H$$
$$V_{1L}(d_1) + V_{2L}(d_1) = \lambda_{1L}S^L - \alpha S^H$$
$$V_{1L}(d_2) + V_{2L}(d_2) = \lambda_{1L}S^L + \left(\lambda_2(1-\tau) - \lambda_{1L}\right)S^H$$

The dual problem is

$$\min_{\alpha \geq 0}\left(\max_d\left(V_{1H}(d) + V_{2H}(d)\right) + \max_d\left(V_{1L}(d_1) + V_{2L}(d_1)\right)\right)$$
$$= \min_{\alpha \geq 0}\left(\max(\lambda_{1H} + \alpha, \lambda_2\tau)S^H + \lambda_{1L}S^L + \max\left(\lambda_2(1-\tau) - \lambda_{1L}, -\alpha\right)S^H\right)$$

In the low state, it must be optimal to choose d_1, since otherwise the mechanism would choose d_2 with probability 1 in the low state, and then U_L would be negative, so the mechanism would not be feasible. It must also be optimal to choose d_1 in the high state. Otherwise $\mu_H(d_1) = 0$, and then the incentive compatibility constraint implies $\mu_L(d_2) = 1$, which is impossible, as was just shown.

The dual problem can therefore be written as

$$\min_{\alpha\geq 0}\left(V_{1H}\left(d_1\right)+V_{2H}\left(d_1\right)+V_{1L}\left(d_1\right)+V_{2L}\left(d_1\right)\right)=\lambda_{1H}S^H+\lambda_{1L}S^L$$

The optimality of d_1 also implies that the warrant equations can be written as

$$\begin{aligned}\lambda_{1L}\omega_{1L}-\alpha\omega_{1H}&=\frac{1}{2}\lambda_{1L}S^L-\frac{1}{2}\alpha S^H\\ \left(\lambda_{1H}+\alpha\right)\omega_{1H}&=\frac{1}{2}\left(\lambda_{1H}+\alpha\right)S^H\\ \lambda_2\omega_2&=\frac{1}{2}\lambda_{1L}S^L+\frac{1}{2}\lambda_{1H}S^H\end{aligned}$$

If $\lambda_{1L}=0$, then optimality of d_1 in the low state implies $\alpha=0$, and $\lambda_2=0$, and then the third warrant equation implies $\lambda_{1H}=0$, which is a contradiction (since the theorem requires $(\lambda,\alpha)\neq 0$). Thus λ_{1L} is strictly positive, which implies $\omega_{1L}=U_{1L}$. And because λ_{1L} is strictly positive, the third warrant equation implies that λ_2 and ω_2 are also strictly positive, and so $\omega_2=U_2$.

Since both decisions are optimal in both states, α must satisfy the equations

$$\begin{aligned}\alpha&=\lambda_{1L}-\lambda_2(1-\tau)\\ &=\lambda_2\tau-\lambda_{1H}\end{aligned}$$

Thus

$$\lambda_2=\lambda_{1L}+\lambda_{1H}>0$$

The complementary slackness conditions require either $\lambda_{1H}=0$, or $\omega_{1H}=U_H$. Suppose $\lambda_{1H}>0$. Then $\omega_{1H}=U_H=\mu_1S^H$ and the warrant equations can be written as

$$\begin{aligned}\lambda_{1L}U_{1L}-\alpha U_{1H}&=\frac{1}{2}\lambda_{1L}S^L-\frac{1}{2}\alpha S^H\\ \left(\lambda_{1H}+\alpha\right)U_{1H}&=\frac{1}{2}\left(\lambda_{1H}+\alpha\right)S^H\\ \lambda_2U_2&=\frac{1}{2}\lambda_{1L}S^L+\frac{1}{2}\lambda_{1H}S^H\end{aligned}$$

Adding the first two equations here gives

$$\lambda_{1L}\mu_1 S^L + \lambda_{1H}\mu_1 S^H = \frac{1}{2}\lambda_{1L}S^L + \frac{1}{2}\lambda_{1H}S^H$$

and since $\lambda_{1L} > 0$ and $\lambda_{1H} > 0$ this implies $\mu_1 = ½$. But then the third warrant equation gives

$$\left(\lambda_{1L} + \lambda_{1H}\right)\frac{1}{2}S^H = \frac{1}{2}\lambda_{1L}S^L + \frac{1}{2}\lambda_{1H}S^H$$

which is a contradiction, since $S^H > S^L$.

Therefore $\lambda_{1H} = 0$, which implies $\lambda_{1L} = \lambda_2$ and $\alpha = \tau\lambda_2$, and the warrant equations reduce to

$$\lambda_{1L}\omega_{1L} - \alpha\omega_{1H} = \frac{1}{2}\lambda_{1L}S^L - \frac{1}{2}\alpha S^H$$

$$\alpha\omega_{1H} = \frac{1}{2}\alpha S^H$$

$$\lambda_{1L}\omega_2 = \frac{1}{2}\lambda_{1L}S^L$$

Since $\lambda_{1L} > 0$, $U_L = \omega_{1L}$, and the first two equations here imply $\omega_{1L} = ½S^L$, and this implies

$$\mu_1 = 1 - \frac{1}{2}\frac{S^L}{S^H}$$

Finally, the second warrant equation implies $\omega_{1H} = ½S^H$, while $U_H = \mu_1 S^H = S^H - ½S^L$. So $\omega_{1H} < U_H$, and all of the conditions of the theorem are satisfied.

Thus it has been shown that the Neutral Bargaining Solution is unique, and that it gives player 2 half of the low surplus, with player 1 getting the residual. Since $\tau S^H < S^L$, the optimal mechanism for player two is a pooling demand, with $\mu_1 = 1\text{-}S^L/S^H$, while the optimal mechanism for player 1 sets $\mu_1 = 1$. So the NBS is implemented by the random dictator mechanism (i.e. by randomly selecting the optimal mechanism for one player or the other, with equal probabilities).

Appendix B

Are Informational Rents Bigger when More Information is Private?

When an uninformed seller makes offers to a buyer whose valuation is drawn privately from some known distribution, the optimal offer for the seller may be a pooling offer that concedes an informational rent to the buyer. In that case a small improvement in the distribution of the buyer's valuations implies an increase in the informational rent. More generally, one might expect that when the buyer's informational advantage increases, the informational rent increases as well. But this is not always true. For example, a large improvement in the distribution may cause the seller to switch from a pooling to a screening offer, and in some cases this completely eliminates the informational rent. And in the case where partial pooling is initially optimal, even a small improvement induces the seller to screen more aggressively, with the possibility that the informational rent is reduced. The purpose of this Appendix is to examine this possibility, for a limited but interesting class of distributions.

Consider a seller making a take-it-or-leave-it demand to a buyer whose valuation is a + s, with a $\geq$ 0 and s distributed over the interval [0,b]. Let y = s/b, distributed on [0,1] according to the distribution function F. The seller's payoff from a demand a + bx is zero if this demand exceeds the actual surplus, which is the case if x > y. Thus the seller's expected payoff is (a+bx)(1-F(x)). The question is whether an increase in b leads to an increase in the informational rent.

Suppose that F is a beta distribution, with density

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad \textbf{(79)}$$

where Γ is the gamma function, with $\alpha > 0$, and $\beta > 0$. Two subsets of this class of distributions will be analyzed, one (Case A) defined by taking $\beta = 1$ with $\alpha > 1$, so that x has a power distribution with an increasing density, and the other (Case B) defined by taking $\alpha = 1$ (with no restriction on β), so that 1-x has a power distribution. The uniform distribution is included in class B (and it is on the boundary of case A).

The following lemma will be used in the analysis of Case A.

Lemma

The function

$$f(\alpha) = \frac{1 - z^{\alpha}}{\alpha} \tag{80}$$

is decreasing in α, for z > 0.

Proof:

The derivative of f is

$$f'(\alpha) = \frac{-\left(1 - z^{\alpha}\right) - \alpha z^{\alpha}\log(z)}{\alpha^{2}} \tag{81}$$

Let y = -αlog(z). Then

$$\alpha^{2} f'(\alpha) = e^{-y}\left(1 + y\right) - 1 \leq 0 \tag{82}$$

where the inequality holds because $e^{y} \geq 1+y$ (with equality when y = 0, which means z = 1).

Case A

Suppose F is a beta distribution with β = 1 and α > 1. In this case pooling is never optimal, since f(0) = 0. Let z be the optimal screening price. The seller's objective function is strictly concave, and z is the unique solution of the first-order condition

$$\left(1 - z^{\alpha}\right) - \alpha z^{\alpha - 1}\left(\frac{a}{b} + z\right) = 0 \tag{83}$$

This equation can be rewritten as

$$b = \frac{a\alpha}{z^{1-\alpha} - (1+\alpha)z} \tag{84}$$

This gives b as a strictly increasing function of z, meaning that the optimal screening price is strictly increasing in b. That being so, the informational rent R is increasing in b if and only if R is increasing in z. Using equation (84), the relationship between R and z is given by

$$\begin{aligned} R &= \frac{b}{1+\alpha}\left(\alpha - (1+\alpha)z + z^{1+\alpha}\right) \\ &= \frac{a\alpha}{1+\alpha} \frac{\alpha - (1+\alpha)z + z^{1+\alpha}}{z^{1-\alpha} - (1+\alpha)z} \end{aligned} \tag{85}$$

Taking derivatives and simplifying the result gives

$$\frac{1+\alpha}{a\alpha}\frac{dR}{dz} = \frac{\alpha - 1 - (1+\alpha)\left(z - z^{\alpha} + z^{1+2\alpha}\right) + 2z^{1+\alpha}}{\left(1 - (1+\alpha)z^{\alpha}\right)^2} \tag{86}$$

Thus R is increasing in z (and therefore in b) if and only if numerator of the expression on the right side of this equation is nonnegative.

The first-order condition determining the screening price implies that

$$1 - (1+\alpha)z^{\alpha} \geq 0 \tag{87}$$

Thus

$$z^{1+\alpha} - (1+\alpha)z^{1+2\alpha} \geq 0 \tag{88}$$

So it is enough to show that

$$\psi(z) = \alpha - 1 - (1+\alpha)\left(z - z^{\alpha}\right) + z^{1+\alpha} \geq 0 \tag{89}$$

The derivative of ψ is given by

$$\psi'(z) = (1+\alpha)\left(z^{\alpha} + \alpha z^{\alpha-1} - 1\right) \tag{90}$$

Since $\psi'(0) < 0 < \psi'(1)$ and $\psi''(z) > 0$, the function ψ has a unique minimum, say at z_0. So it is enough to show that $\psi(z_0) \geq 0$. Note that

$$\psi(z_0) = \psi(z_0) + z_0\psi'(z_0) = \alpha - 1 - \alpha z_0 + z_0^{\alpha} \tag{91}$$

Thus $\psi(z_0) \geq 0$ if and only if

$$\alpha \geq \frac{1 - z_0^{\alpha}}{1 - z_0} \tag{92}$$

This is an immediate implication of Lemma A, because $f(1) \geq f(\alpha)$ for $\alpha > 1$.

Thus it has been shown that although pooling is never optimal for a power distribution with exponent greater than one, the informational rent nevertheless increases when the extent of the employer's private information is magnified.

Case B

Suppose F is a beta distribution with $\alpha = 1$. The simplest example is the uniform distribution (with $\beta = 1$). The informational rent is

$$R = b\int_x^1 (1-t)^{\beta} dt = \frac{b(1-x)^{\beta+1}}{\beta+1} \tag{93}$$

The derivative of the worker's objective function is given by

$$
\begin{aligned}
H'(x) &= (1-x)^{\beta} - \beta(1-x)^{\beta-1}(c+x) \\
&= (1-x)^{\beta-1}(1-\beta c-(1-\beta)x)
\end{aligned}
\quad \textbf{(94)}
$$

where c = a/b. If $\beta c \geq 1$ then $H'(x) \leq 0$ for all x, so pooling is optimal. If $\beta c \leq 1$ then $H'(x) > 0$ for all $x < x^*$, and $H'(x) < 0$ for all $x > x^*$, where x^* is given by

$$
x^* = \frac{1-\beta c}{1-\beta} \quad \textbf{(95)}
$$

Also, $H(x^*) = 0$, so screening at x^* is optimal. Thus the mapping from b to R is given by

$$
R(b) = \begin{cases} \dfrac{b}{\beta+1} & b \leq a\beta \\ \dfrac{b}{\beta+1}\left(\dfrac{\beta}{\beta+1}\right)^{\beta+1}\left(1+\dfrac{a}{b}\right)^{\beta+1} & b \geq a\beta \end{cases}
$$

The function R(b) is obviously increasing in the pooling region. To show that it is increasing in the screening region, it is enough to show that

$$
b\left(1+\frac{a}{b}\right)^{\beta+1}
$$

is increasing in b, for $b \geq a\beta$, or equivalently that the function

$$
\omega(z) = z\left(1+\frac{1}{z}\right)^{\beta+1}
$$

is increasing in z, for $z \geq \beta > 0$. The derivative of this function is

$$\omega'(z) = \left(1+\frac{1}{z}\right)^{\beta+1} - (\beta+1)\left(1+\frac{1}{z}\right)^{\beta}\frac{1}{z}$$
$$= \left(1+\frac{1}{z}\right)^{\beta}\left(1-\frac{\beta}{z}\right)$$

So the function is increasing.

Thus it has been shown that for any beta distribution with α = 1, the informational rent increases when the extent of the employer's private information is magnified, even though this induces the worker to switch from a pooling price to a screening price.[10]

[10]Note that the rent function R(b) is continuous (but not differentiable) at the point where screening becomes optimal.

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