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RATIONAL EXPECTATIONS, THE EXPECTATIONS HYPOTHESIS, AND TREASURY BILL YIELDS: AN ECONOMETRIC ANALYSIS

David S. Jones

V. Vance Roley

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# ABSTRACT

This paper tests the joint hypothesis of rational expectations and the expectations model of the term structure for three- and six-month Treasury bills. Previous studies are extended in three directions. First, common efficient markets-rational expectations tests are compared, and it is shown that four of the five tests considered are asymptotically equivalent, and that the fifth is less restrictive than the other four. Second, the joint hypothesis is tested using weekly data for Treasury bills maturing in exactly 13 and 26 weeks beginning in 1970 and ending in 1979. In contrast, previous studies using comparable data have typically discarded 12/13 of the sample to form a nonoverlapping data set. Finally, a more complete set of possible determinants of time-varying term premiums is tested.

David S. Jones Board of Governors of the Federal Reserve System Washington, D.C. 20551 (202) 452-2989

V. Vance Roley Federal Reserve Bank of Kansas City 925 Grand Avenue Kansas City, Missouri 64198 (816) 881-2959

# RATIONAL EXPECTATIONS, THE EXPECTATIONS HYPOTHESIS, AND TREASURY BILL YIELDS: AN ECONOMETRIC ANALYSIS

David S. Jones and V. Vance Roley\*

The joint hypothesis consisting of rational expectations and the expectations model of the term structure has been subject to an increasing amount of scrutiny in recent years. Some of this research has followed Roll [26] and examined this joint hypothesis under the guise of the "efficient markets" hypothesis. Other researchers have taken the study by Modigliani and Shiller [18] as the starting point and tested rationality and the expectations hypothesis explicitly. Prominent examples of recent studies focusing on this model include McCulloch [13], Mishkin [14,15], Pesando [23], Sargent [27], Shiller [28,29], Friedman [6], and Singleton [31,32], who either explicitly or implicitly test this joint hypothesis. In virtually all of these studies, however, not only are different "nonoverlapping" data sets employed, but the tests themselves apparently are not uniform. For example, Mishkin [15] and Sargent [27] test cross-equation restrictions implied by this theory, while Pesando [23], Shiller [28,29], and Friedman [6] have collected data and utilized tests that enable straightforward single-equation estimation.

As a whole, the evidence surrounding the validity of this joint hypothesis is mixed. In cases where the potential roles of time-varying term premiums are not explicit (e.g., Sargent [27] and Shiller [29]), the null hypothesis associated with this model cannot be rejected. However, when time-varying term premiums dependent on the level of interest rates are included, both Shiller [28] and Friedman [6] reject the joint hypothesis. Moreover, in studies allowing other possible determinants of time-varying term premiums, Nelson [19] and Friedman [6] again reject the model. Still other researchers have substituted alternative models of market equilibrium for the expectations hypothesis and tested whether security markets are efficient (e.g., Fama [5] and Mishkin [16]). In these latter studies, the expectations hypothesis is rejected <u>a priori</u>.

The range of results from previous studies is attributable to at least two factors. First, different data sets are used, some of which are constructed more carefully than others. Second, the list of potential determinants of timevarying term premiums has varied considerably among studies. Some might also argue that a third possible reason for the divergent results is due to the apparently different formulations of the tests. As is shown in the first section of this paper, however, in tests using observed market data (as opposed to the survey data used by Friedman [6]), many efficient markets-rational expectations tests are asymptotically equivalent.

The purpose of this paper is to investigate the joint hypothesis of rational expectations and the expectations model of the term structure in the context of the Treasury bill market. This market is selected for two reasons. First, the Treasury bill market has been studied extensively, with empirical results that highlight the possible roles that data and methodology play in tests of the model (e.g., Roll [26], Hamburger and Platt [10], Fama [5], Friedman [6], and Mishkin [16]). Second, Treasury bill data are ideally suited for such an exercise because (a) Treasury bills are issued at regular and frequent intervals, (b) different maturities have homogenous tax treatment, and (c) they are pure discount securities which avoids complications related to coupons. While others have noted these attributes and investigated the behavior of Treasury bill yields as a consequence, previous studies are almost uniformly based on

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nonoverlapping quarterly samples that effectively discard 12/13 of the available data. In contrast, in this paper <u>all</u> available data are used for Treasury bills maturing in <u>exactly</u> 13 and 26 weeks in a sample spanning most of the 1970s. As Hansen and Hodrick [11] show in the context of the foreign exchange market, more powerful asymptotic tests may be obtained using all of the available weekly data than in constructing nonoverlapping samples to avoid the problem of serial correlation.

Following this introductory section, the first section compares alternative tests of the joint hypothesis of rationality and the expectations hypothesis. In the second section, the Treasury bill yield data used here as well as the data for possible determinants of time-varying term premiums are described. The estimation problems posed by the use of weekly overlapping data are also discussed in this section. In the third section, the estimation and test results are presented. The main conclusions of this paper are summarized in the final section.

# I. Tests of Rationality and the Expectations Hypothesis

In this section, five efficient markets-rational expectations tests appearing in the literature are compared. Extending the results of Abel and Mishkin [1], who show that Test I and Test II below are asymptotically equivalent, two other tests may also be shown to be asymptotically equivalent to Test I. In particular, it is demonstrated that a simple single-equation test is in fact equivalent to more complicated tests involving restrictions across equations.

In the discussion below, all derivations are in terms of the three- and six-month yields which are examined empirically in a later section.<sup>1/</sup> Further-

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more, it is assumed for analytical convenience that the data are nonoverlapping in each case.<sup>2/</sup> Under these conditions, the joint hypothesis of rational expectations and the expectations model of the term structure may be represented by the usual approximation<sup>3/</sup>

$$R_{6,t} = (1/2) \cdot R_{3,t} + (1/2) \cdot E(R_{3,t+1} | \Omega_t) + \alpha$$
 (1)

where

$$\begin{split} R_{6,t} &= \text{ yield on six-month Treasury bills at time t} \\ \alpha &= \text{ constant term premium (Hicks [12])} \\ \Omega_t &= \text{ information set used by investors at time t} \\ E(\dots | \Omega_t) &= \text{ expectation conditional on } \Omega_t, \text{ taken as the linear least squares forecast of a random variable based on information available at time t.} \end{split}$$

The above model (1) merely states that the yield to maturity on a six-month Treasury bill equals one-half of the sum of the current three-month Treasury bill yield and the expected three-month yield in period t+1 evaluated at time t, plus a constant term premium. Different permutations of the basic relationship are used to derive the tests considered immediately below.

<u>Test I</u>

The first test considered here is taken as the standard when investigating the asymptotic properties of alternative tests. This test is by far the easiest to implement, and it follows directly from equation (1). In particular, the hypothesis to be tested is that the expected quarterly holding-period yield on a six-month Treasury bill differs from the current three-month Treasury bill yield by at most a constant term premium. To conform to this test, equation (1) may be rewritten as

where

$$E(R_{6,t}^{n} | \Omega_{t}) = 2R_{6,t} - E(R_{3,t+1} | \Omega_{t})$$

$$R_{6,t}^{h} = \text{approximate holding-period yield on six-month Treasury}$$
bills in period t  $(R_{6,t}^{h} = 2R_{6,t} - R_{3,t+1})$ .

In turn, the expression for the one-period-ahead three-month Treasury bill yield may be represented as

$$R_{3,t+1} = E(R_{3,t+1} | \Omega_t) + e_{t+1}$$
(3)

where  $e_{t+1}$ , the forecast error for the three-month rate, is uncorrelated with any information in  $\Omega_t$ . The equation to be tested may therefore be estimated as

$$R_{6,t}^{h} = b_{0} + b_{1} \cdot R_{3,t} + \underline{x}_{t} \underline{b}^{*} + e_{t+1}$$
(4)

where

$$\underline{x}_{t} = 1 \times (k-1)$$
 vector of variables consisting of information which  
is costlessly available to investors at time t  $\frac{4}{2}$ 

 $b_0, b_1, \underline{b}^* = \text{coefficients to be estimated}$ and the test becomes  $\frac{5}{}$ 

$$H_0: b_1 = 1, \underline{b}^* = \underline{0}.$$
 (5)

If the null hypothesis (5) can be rejected, it may be attributable to either the rejection of rational expectations or the expectations model of the term structure, or both. On the basis of this test, as well as the alternative tests below, the precise cause of the rejection cannot be isolated. If, however, the expectations hypothesis is valid, then rejection of the null hypothesis (5) indicates that investors do not efficiently use all costlessly available information. On the other hand, if investors' form expectations "rationally," then the model of equilibrium yields may include time-varying term premiums.

In any event, the null hypothesis may be tested by estimating equation (4),

or equivalently

$$R_{6,t}^{h} - R_{3,t} = \underline{z}_{t} \underline{b} + e_{t+1}$$
 (41)

by ordinary least squares and computing the appropriate test statistic for

$$H_0: \quad \underline{b} = \underline{0} \tag{5}$$

where

$$\frac{z}{t} = \{R_{3,t}, \frac{x}{t}\}$$

$$\underline{b}^{*} = \{b_{1}^{-1}, \underline{b}^{**}\}$$

 $\hat{b} = (Z^2)^{-1}Z^2y$ 

and the constant term is assumed to be zero for simplicity. $\frac{6}{}$  In this case, the usual test statistic is

$$Q = \hat{\underline{b}} [\hat{\sigma}^{2} (Z^{2})^{-1}]^{-1} \hat{\underline{b}} = (1/\hat{\sigma}^{2}) \hat{\underline{b}}^{2} Z^{2} \hat{\underline{b}}$$
(6)

where

 $Z = N \times k \text{ matrix with row j equal to } \underline{z_j}, \underline{j=1, \ldots, N}$   $y = N \times 1 \text{ vector with element j equal to } R_{6,j}^h - R_{3,j}, \underline{j=1, \ldots, N}$   $\hat{\sigma}^2 = [1/(N-k)](\underline{y}-\underline{Z}\underline{\hat{b}})^{-}(\underline{y}-\underline{Z}\underline{\hat{b}}).$ 

As is well known, under the null hypothesis, Q has an F-distribution with (k,N-k) degrees of freedom, and is asymptotically distributed as  $\chi^2(k)$ .

#### Test II

Another test formulated by Mishkin [15,16] involves cross-equation rationality restrictions. In this case, a distinction is made between anticipated and unanticipated movements in economic variables following Barro [2,3]. The main feature of this framework is that if the economic variables in the model are precisely those comprising the information set used by the market, then the effects of unanticipated movements in variables on <u>ex post</u> holding-period yields may be estimated, as well as testing the joint hypothesis of rational expectations and the expectations model of the term structure.<sup>7/</sup> Following Mishkin [15,16], the joint hypothesis of rational expectations and the expectations model-equations (1) and (3)-may be shown to imply

$$R_{6,t}^{n} - R_{3,t} = \left[\underline{w}_{t+1} - E(\underline{w}_{t+1} | \Omega_{t})\right] \underline{\beta} + v_{t}$$
(7)

where

 $\frac{\mathbf{w}}{\mathbf{t}+1} = \frac{1 \times \mathbf{m} \text{ vector of variables relevant to the determination of Treasury bill yields}}{\underline{\beta}} = \mathbf{m} \times 1 \text{ vector of coefficients}}$  $\mathbf{v}_{t} = \text{ an error that is orthogonal to } \Omega_{t}.$ 

This model implies that unanticipated changes in the holding-period yield on six-month Treasury bills will occur only when unanticipated information is observed by investors.

To represent investors' anticipations, Mishkin specifies the vector autoregression  $\frac{8}{}$ 

$$\underline{w}_{t+1} = \underline{z}_{t}\Gamma + \underline{u}_{t+1} \tag{8}$$

where

$$\underline{\mathbf{u}}_{t+1} = 1 \times \mathbf{m} \text{ vector of errors such that } \mathbf{E}(\mathbf{z}_t \cdot \mathbf{u}_{t+j}) = 0, j \ge 0; \text{ and } \mathbf{E}(\underline{\mathbf{u}}_t \cdot \underline{\mathbf{u}}_{t-j}) = 0, j \ne 0; \mathbf{E}(\underline{\mathbf{u}}_t \cdot \underline{\mathbf{u}}_{t-j}) = \Sigma, j = 0.$$

Substituting investors' predictions of  $\underline{w}_{t+1}$  into equation (7) yields

$$R_{6,t}^{h} - R_{3,t} = (\underline{w}_{t+1} - \underline{z}_{t} \Gamma^{*})\underline{\beta} + v_{t}.$$
 (7)

The cross-equation test of rational expectations and the expectations model becomes

 $H_0: \Gamma = \Gamma^*.$  (9)

As shown by Abel and Mishkin [1], the Wald test of the null hypothesis (9) associated with the Zellner estimation of (8) and (7') is asymptotically equivalent to Test I. Following these authors' proof, this may be seen by noting that equation (7') may be rewritten as

$$R_{6,t}^{h} - R_{3,t} = (\underline{w}_{t+1} - \underline{z}_{t}\Gamma)\underline{\beta} + \underline{z}_{t}\underline{\Theta} + v_{t}$$
(7.1)

where  $\underline{\Theta} = (\Gamma - \Gamma^*) \underline{\beta}$ 

and the null hypothesis is

$$H_0: \quad \underline{\Theta} = \underline{0}. \tag{9^{-}}$$

Because the  $\Gamma$  constraint across equations (8) and (7<sup>-+</sup>) is not binding, the Zellner estimator of  $\underline{\Theta}$  may be obtained by estimating equations (8) and (7<sup>-+</sup>) recursively using OLS. In the matrix notation used earlier, the OLS estimates of equation (8) implies

$$Z\Gamma = W - \hat{U}$$
(10)

where W and U are N×m matrices corresponding to  $\underline{w}_{t+1}$  and  $\underline{u}_{t+1}$ , respectively. Substituting equation (10) into equation (7<sup>(\*)</sup>) yields

$$\underline{\mathbf{y}} = \underline{\mathbf{U}}\underline{\boldsymbol{\beta}} + \underline{\mathbf{Z}}\underline{\boldsymbol{\Theta}} + \mathbf{v}_{t} \,. \tag{7^{(7)}}$$

Because the vector autoregression (8) is estimated by OLS,  $\hat{U}$  is orthogonal to Z and the estimator of  $\Theta$  is

$$\hat{\underline{\Theta}} = (Z^{T}Z)^{-1}Z^{T}\underline{Y}$$
(11)

which is numerically identical to the estimator obtained in Test I.

The final step involves computing the Wald test statistic corresponding to the null hypothesis (9<sup>-</sup>). This statistic may be computed as

$$R = \underline{\hat{\Theta}} \left[ \hat{v} (Z^{2}Z)^{-1} \right]^{-1} \underline{\hat{\Theta}}$$

$$\hat{v} = (1/N) \left( \underline{y} - \widehat{U} \underline{\hat{\beta}} - Z \underline{\hat{\Theta}} \right) \left( \underline{y} - \widehat{U} \underline{\beta} - Z \underline{\hat{\Theta}} \right)$$
(12)

where

which is asymptotically distributed as  $\chi^2$  with k degrees of freedom. Abel and Mishkin [1] further show that the  $\hat{v}$  may be computed from the OLS estimation of equation (4<sup>-</sup>), and is equivalent, except for degrees of freedom, to  $\hat{\sigma}^2$  used in equation (12). Thus Test I—as described by equations (5<sup>°</sup>) and (6)—and Test II—as described by equations (9<sup>°</sup>) and (12)—are asymptotically equivalent.

#### Test III

Another test involving cross-equation restrictions focuses on the forward three-month Treasury bill yield and the time-series process generating the threemonth yield. In this respect, the rationality restrictions to be tested are analogous to those specified by Modigliani and Shiller [18], and tested by Pesando [21] and Friedman [7] with survey data for inflation and interest rates, respectively.

Using the expression for the expectations hypothesis (1), the implied three-month forward yield equals investors' rational expectation of the future spot rate

$$2R_{6,t} - R_{3,t} = E(R_{3,t+1}|\Omega_t)$$
(13)

where the constant term premium is again deleted without any loss of generality. In turn,  $R_{3,t+1}$  is assumed to be generated by a stochastic process including all available information at time t relevant to the determination of the equilibrium yield

$$R_{3,t+1} = \underline{z}_t \underline{\gamma} + \varepsilon_{t+1} \tag{14}$$

where  $\varepsilon_{t+1} = an$  error that is orthogonal to  $\Omega_t$ .

The cross-equation test then involves the comparison of the estimated coefficients ( $\underline{\gamma}$ ) in equation (14) with the estimated coefficients ( $\underline{\gamma}$ \*) in

$$2R_{6,t} - R_{3,t} = \underline{z}_{t} \underline{\gamma}^{*} + \eta_{t+1}$$
(15)

where  $\eta_{t+1} = an$  error that is orthogonal to  $\Omega_t$ .

That is, the null hypothesis is

$$H_0: \quad \underline{\gamma} = \underline{\gamma}^*. \tag{16}$$

Note that the right-hand side variables of equations (14) and (15) are identical, implying that the Zellner and OLS estimators for this two-equation system are equivalent.<sup>9/</sup> In matrix notation, these estimators may be written as

$$\begin{bmatrix} \hat{\mathbf{Y}} \\ \hat{\mathbf{Y}} \\ \hat{\mathbf{Y}}^{*} \end{bmatrix} = \begin{bmatrix} (\mathbf{Z}^{*}\mathbf{Z})^{-1}\mathbf{Z}^{*} & \mathbf{0} \\ \mathbf{0} & (\mathbf{Z}^{*}\mathbf{Z})^{-1}\mathbf{Z}^{*} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{1} \\ \mathbf{y}_{2} \end{bmatrix}$$
(17)

where  $\underline{y}_1 = N \times 1$  vector consisting of  $R_{3,t+1}, t=1,...,N$  $\underline{y}_2 = N \times 1$  vector consisting of 2  $R_{6,t}-R_{3,t}, t=1...,N$ .

The variance-covariance matrix of these estimators is

$$\mathbf{v}\begin{bmatrix} \hat{\mathbf{Y}}\\ \hat{\mathbf{Y}}^{\star} \end{bmatrix} = \begin{bmatrix} \hat{\sigma}_{11}(\mathbf{Z}^{\star}\mathbf{Z})^{-1}, \ \hat{\sigma}_{12}(\mathbf{Z}^{\star}\mathbf{Z})^{-1}\\ \hat{\sigma}_{12}(\mathbf{Z}^{\star}\mathbf{Z})^{-1}, \ \hat{\sigma}_{22}(\mathbf{Z}^{\star}\mathbf{Z})^{-1} \end{bmatrix}$$
(18)

where

$$\hat{\sigma}_{11}, \hat{\sigma}_{12}, \hat{\sigma}_{22} = \text{consistent estimates of } E(\varepsilon_t^2), E(\varepsilon_t \eta_t), \text{ and } E(\eta_t^2),$$
  
respectively.

In this case, it may be verified that the Wald test statistic for the null hypothesis (16) is

$$R = [1/(\hat{\sigma}_{11} + \hat{\sigma}_{22} - 2\hat{\sigma}_{12})](\hat{\underline{\gamma}} - \hat{\underline{\gamma}}^{*})^{*}(Z^{*}Z)(\hat{\underline{\gamma}} - \hat{\underline{\gamma}}^{*})$$
(19)

which is asymptotically distributed as  $\chi^2(k)$  under the null hypothesis.

Alternatively, the stochastic processes (14) and (15) may be combined to form

$${}^{2R}_{6,t} - {}^{R}_{3,t} - {}^{R}_{3,t+1} = \underline{z}_{t} \underline{\delta} + \mu_{t+1}$$
(20)

where  $\delta = k \times 1$  vector of coefficients

 $\mu_{t+1}$  = an error that is orthogonal to  $\Omega_t$ .

From equation (2), the left-hand side of equation (20) merely equals  $R_{6,t}^{h} - R_{3,t}$ . The OLS estimator of  $\delta$  may therefore be expressed as

$$\hat{\underline{\delta}} = (Z^2 Z)^{-1} Z^2 \underline{Y}$$
(21)

with variance-covariance matrix

$$\mathbf{v}(\hat{\underline{\delta}}) = \hat{\sigma}^2 (\mathbf{Z}^{\prime} \mathbf{Z})^{-1}$$
(22)

where  $\hat{\sigma}^2$ 

$$\sigma^2 = OLS$$
 estimate of  $E(\mu_{+}^2)$ .

Note that  $\underline{y} = \underline{y}_2 - \underline{y}_1$ , which implies that equation (14) and (15) may be substituted into (21) to yield

$$\hat{\underline{\delta}} = (z^2 z)^{-1} z^2 \underline{y}_2 - (z^2 z)^{-1} z^2 \underline{y}_1$$

$$= \hat{\underline{\gamma}} - \hat{\underline{\gamma}}^*.$$
(23)

A similar substitution further implies that

$$\hat{\sigma}^2 = \hat{\sigma}^*_{11} + \hat{\sigma}^*_{22} - \hat{2\sigma}^*_{12}$$

where the  $\hat{\sigma}_{ij}^{*}$  differ from  $\hat{\sigma}_{ij}$  only with respect to degrees of freedom.

Under the null hypothesis analogous to that in Test I (5<sup>-</sup>), the relevant test statistic for equation (20) is

$$R^{\prime} = [1/\hat{\sigma}^{2}] \underline{\hat{\delta}}^{\prime} (Z^{\prime}Z) \underline{\hat{\delta}}$$
$$= [1/(\hat{\sigma}_{11}^{*} + \hat{\sigma}_{22}^{*} - \hat{2\sigma}_{12}^{*})] (\hat{\underline{\gamma}} - \hat{\underline{\gamma}}^{*})^{\prime} (Z^{\prime}Z) (\hat{\underline{\gamma}} - \hat{\underline{\gamma}}^{*})$$

which is asymptotically equivalent to the Wald test statistic (19). The test statistic for the cross-equation restrictions is, therefore, asymptotically equivalent to that of the single-equation test described in Test I.

## Test IV

In addition to using volatility measures to assess the joint hypothesis of rational expectations and the expectations model of the term structure, Shiller [29] also conducts formal econometric tests using a single-equation estimation approach.  $\frac{10}{}$  In terms of the three- and six-month yields examined here, Shiller's [29] single-equation approach involves estimating an equation of the form

$$R_{3,t+1} - R_{6,t} = b_0 + b_1(R_{6,t} - R_{3,t}) + \psi_{t+1}$$
(24)

where  $\psi_{t+1} = an \text{ error that is orthogonal to } \Omega_t$ and testing the null hypothesis  $\frac{11}{}$ 

$$H_0: b_1 = 1.$$
 (25)

To motivate this test, notice that from (1) investors' rational expectation of the future three-month yield may be expressed in terms of available yield data and a constant term  $premium \frac{12}{}$ 

$$E(R_{3,t+1}|\Omega_t) = 2 \cdot R_{6,t} - R_{3,t} - \alpha.$$
 (26)

Substituting expression (26) into (3) and then subtracting  $R_{6,t}$  from both sides yields the expression

$$R_{3,t+1} - R_{6,t} = -\alpha + R_{6,t} - R_{3,t} + e_{t+1}$$
 (27)

From the above, it is apparent that a slightly more general representation of this same test may be formed by rewriting equation (24) as

$${}^{R}_{3,t} + {}^{R}_{3,t+1} - {}^{2} {}^{R}_{6,t} = {}^{b*}_{0} + {}^{b*}_{1} {}^{R}_{6,t} + {}^{b*}_{2} {}^{R}_{3,t} + {}^{\psi}_{t+1}$$
(28)

and testing

$$H_0: b_1^* = b_2^* = 0.$$
 (25')

Further transformation of equation (28), using the definition of  $R_{6,t}^{h}$  provided

earlier, results in

$$R_{6,t}^{h} - R_{3,t} = -b_{0}^{*} - b_{1}^{*} R_{6,t} - b_{2}^{*} R_{3,t} - \psi_{t+1}.$$
(28')

This latter equation, together with the null hypothesis (25<sup>-</sup>), is equivalent to specialized version of Test I in which  $\frac{z}{-t}$  consists of only  $R_{6,t}$  and  $R_{3,t}$ . Test V

The final case considered here is the test of cross-equation rationality restrictions presented by Sargent [27]. $\frac{13}{}$  In this test, cross-equation restrictions are tested for the vector autoregression. $\frac{14}{}$ 

$$R_{3,t} = \sum_{i=1}^{m} a_{i} \cdot R_{3,t-i} + \sum_{i=1}^{m} b_{i} \cdot R_{6,t-i} + u_{t}$$

$$R_{6,t} = \sum_{i=1}^{m} c_{i} \cdot R_{3,t-i} + \sum_{i=1}^{m} d_{i} \cdot R_{6,t-i} + v_{t}$$
(29)

where  $a_i, b_i, c_i, d_i$  = coefficients to be estimated

 $u_t, v_t = \text{errors uncorrelated with } R_{3,t-i} \text{ and } R_{6,t-i}, \text{ for } i \ge 1,$ but possibly contemporaneously correlated.

The cross-equation restrictions are again derived from the approximation used to represent the joint hypothesis of rational expectations and the expectations model of the term structure, but in this case the model is specified in terms of information known at time t-l

$$E(R_{6,t}|\Omega_{t-1}) = (1/2) \cdot E(R_{3,t}|\Omega_{t-1}) + (1/2) \cdot E(R_{3,t+1}|\Omega_{t-1}) + \alpha. \quad (30)$$

As shown by Sargent [27], nonlinear restrictions on the vector autoregression (29) are implied by this model (30).

As both Sargent [27] and Shiller [29] note, however, such complicated nonlinear restrictions are not necessary if the data allow the computation of forward rates. In particular, with the three- and six-month yields used here, the expected three-month spot rate in period t may be expressed as

where

$$E(R_{3,t}|_{t-1}) = 2 \cdot R_{6,t-1} - R_{3,t-1} - \alpha_{t-1}$$
(31)

where  $\alpha_{t-1} = \alpha$  under the null hypothesis. In turn, under the hypothesis of rational expectations, the realizations of  $R_{3,t+1}$  and  $R_{6,t}$  may be represented as

$$R_{3,t+1} = E(R_{3,t+1}|\Omega_{t-1}) + \varepsilon_{t+1}$$

$$R_{6,t} = E(R_{6,t}|\Omega_{t-1}) + \eta_{t}$$

$$\eta_{t} = \text{an error that is orthogonal to } \Omega_{t-1}$$
(32)

$$\varepsilon_{t+1} = \text{first-order moving-average error process, with } \varepsilon_{t+1}$$
  
orthogonal to  $\Omega_{t-1}$ .

If the joint hypothesis is true, equations (31) and (32) may be substituted into equation (30) to yield

$$(2 \cdot R_{6,t} - R_{3,t+1}) = (2 \cdot R_{6,t-1} - R_{3,t-1}) + (\varepsilon_{t+1} - 2 \cdot n_t)$$
(33)

where the left-hand side equals  $R_{6,t}^{h}$ , and the right-hand side consists of the forward three-month rate corresponding to  $R_{3,t}$  and an error term. <u>16</u>/ The joint hypothesis of rationality and the expectations theory of the term structure may therefore be tested by estimating the equation <u>17</u>/

$$R_{6,t}^{h} = b \cdot (2 \cdot R_{6,t-1}^{-R} + R_{3,t-1}) + \frac{z}{t-1} + (\varepsilon_{t+1}^{-2} + \eta_{t})$$
(33<sup>-</sup>)

where  $\underline{z}_{t-1} \in \Omega_{t-1}$  and  $\underline{c}$  is a vector of coefficients. The null hypothesis in this case corresponds to the parameter restrictions

$$H_0: b = 1, c = 0,$$
 (34)

which are a subset of the restrictions tested in the previous four tests.

To demonstrate that the restrictions given by the null hypothesis of this test (34) are a subset of those corresponding to the null hypothesis of Test I (5), equations (4), (31), and (32) together may be used to represent  $R_{6.t}^{h}$  as

(assuming for simplicity that the term premium is constant)

$$R_{6,t}^{h} = b_{0} + b_{1} \cdot (2 \cdot R_{6,t-1} - R_{3,t-1}) + b_{2} \cdot e_{t} + e_{t+1}.$$
(35)

In terms of this expression, the null hypothesis of Test I becomes

$$H_0: b_1 = b_2 = 1.$$
 (5<sup>--</sup>)

Because  $e_t$  is orthogonal to  $(2 \cdot R_{6,t-1} - R_{3,t-1})$ , this same equation may be estimated to test the null hypothesis of Test V, i.e.,

$$H_0: b_1 = 1.$$
 (34<sup>-</sup>)

Thus, in Test V restrictions are only placed on the information set available at time t-1, while in Test I additional restrictions are placed on the innovations in variables between time t-1 and time t. The usefulness of Test V is therefore limited.

## II. Data and Estimation Techniques

The disparity among the data sets in previous studies may in part account for the range of empirical findings. Moreover, inadequate yield data for both dependent and independent variables in equations such as (4) may have reduced the asymptotic power of tests—in cases where the dependent variable is measured with error—or caused biased coefficient estimates—in cases where the independent variables are measured with error. This paper seeks to improve on previous studies by measuring yields to maturity and holding-period yields precisely, and by using all available data within a given time period. Following the descriptions of the data, the problems posed by an overlapping weekly sample and the estimation procedure used to obtain consistent estimates for both coefficients and their variances are discussed.

# Data

In addition to Treasury bill yields, possible nonyield determinants of

time-varying term premiums are also examined empirically. Both the yield and nonyield data are described below, with brief rationales for the nonyield data also provided. It should also be emphasized that the possible determinants of time-varying term premiums were selected <u>a priori</u>, and were not subject to any "pretesting."  $\frac{18}{}$ 

U.S. Treasury Bill Yields. The Treasury bill yield data are taken from "Composite Closing Quotations for U.S. Government Securities," which is a letter published daily by the Federal Reserve Bank of New York. The data were collected for weekly intervals from Friday's letter, beginning on January 2, 1970, and ending September 13, 1979. During this period, the Federal Reserve focused on the control of one or more monetary aggregates, although perhaps to varying degrees, while maintaining an important role for "money market conditions." Extending the sample further would have encompassed the October 1979 policy change, which de-emphasized money market conditions in favor of a reserve aggregate control procedure.

During the entire sample period, a previously announced amount of threeand six-month Treasury bills were auctioned every week (usually on Monday), and made available to investors on Thursday.  $\frac{19}{}$  The data used here correspond to the average of the closing bid and ask quotations of these newly issued threeand six-month bills. As such, a six-month (26-week) bill in this sample matures on exactly the same day as two successive three-month (13-week) bills. The yield data are also expressed as coupon-equivalent yields (in percent), or  $\frac{20}{}$ 

$$R_{3,t} = [(100/P_{3,t}) - 1] \cdot (365/90)$$
  

$$R_{6,t} = [(100/P_{6,t}) - 1] \cdot (365/180)$$

where

P ,, P = prices of three- and six-month Treasury bills, respectively. 3,t'6,t In addition, the quarterly holding-period yield on a six-month bill is computed as  $R_{6,t}^{h} = \{ [1+(180/365) \cdot R_{6,t}]/[1+(90/365) \cdot R_{3,t+1}] - 1 \} \cdot (365/90).$ 

U.S. Treasury Bill Supplies  $(S3_t \text{ and } S6_t)$ . Researchers have for some time tested the statistical significance of security supply variables as determinants of relative yields.<sup>21/</sup> While most of these efforts were unsuccessful in isolating significant economic effects in a single-equation context, Roley [24,25] has recently found such effects using a disaggregated structural model. In this latter model, the reduced-form for security yields implies that the effects of Treasury security supplies vary depending on investors' wealth flows.<sup>22/</sup> Unfortunately, investors' wealth flows are not available on a weekly basis, making the use of more traditional specifications a necessity.

The Treasury bill supply data are collected to correspond exactly to the yields. These data consist of the amount of three- and six-month bills auctioned each week (in billions of dollars), as reported in the <u>Treasury Bulletin</u>. These weekly supply figures are easily accessible to investors before Thursday, the day corresponding to the yield data. For three-month bills, the supply data represent weekly flows, since some bills issued previously also mature in 13 weeks. For six-month bills, this is often not the case, implying that the data represent both stocks and flows of bills maturing in 26 weeks.

<u>Unemployment Rate (RU</u>). Following Nelson [19], researchers have occasionally tested the significance of the unemployment rate—as well as other cyclical macroeconomic variables—as a determinant of time-varying term premiums.  $\frac{23}{}$  The data series for this variable differs from those of the others included in this study in that it is a monthly series. Thus, in some cases the latest figure has been known for one week, while at other times it has been available for two, three, four, or five weeks. The unrevised unemployment rate figure (in percent), as announced initially by the Bureau of Labor Statistics (Department of Labor), is used here. To take account of the different lengths of time that the most recent figure was known, five separate variables were originally entered into the estimated equations. In the estimated equations reported in the next section, however, a single unemployment rate variable is reported. The  $\chi^2$  statistic used to test the hypothesis that these five variables have the same coefficient had a value of less than .0001.

<u>Risk</u>. Following Fama [5] and Mishkin [16], a risk variable is also included as a possible determinant of a time-varying term premium. This measure is included by these researchers in an attempt to account for the increased risk of capital loss usually associated with greater interest rate volatility. Following Mishkin [16], this variable is represented as

$$RISK_{t} = (1/8) \cdot \sum_{i=0}^{\prime} |R_{3,t-13i} - R_{3,t-13(i+1)}|$$

where t is now defined in terms of weekly intervals.  $\frac{24}{}$ 

Foreign Holdings of U.S. Treasury Securities (FH<sub>t</sub>). Throughout the late 1970's it was often alleged that foreign central banks had "preferred habitats" in terms of their purchases of U.S. Treasury securities. To examine whether this behavior ultimately affected relative yields by changing net supplies available to domestic investors, a variable representing foreign holdings of U.S. Treasury securities is included. The data series used here corresponds to marketable U.S. Government securities (in tens of billions of dollars) held in custody by the Federal Reserve System for foreign official and international accounts, and is reported each Friday in the Federal Reserve's H.4.1 release. With respect to the yield data, the previous Friday's foreign holdings figure is used in the empirical work.

## Estimation Techniques

Because four of the five tests were shown to be asymptotically equivalent and Test V was shown to be less restrictive than the others, only one basic specification is empirically examined. This specification corresponds to equation (4) which was derived in conjunction with Test I. With a nonoverlapping data set including three-month Treasury bills spaced 13 weeks apart, consistent estimates of the vector of coefficients in equation (4) and their variance-covariance matrix may be obtained using OLS estimation.<sup>25/</sup> However, with the weekly data employed here, the errors in equation (4) are described by a 12th-order moving-average process

$$e_{t+1} = v_t + \sum_{i=1}^{12} \delta_i \cdot v_{t-i}$$
(36)

where  $v_t$  = serially uncorrelated error process. This representation of the error process follows, for example, if the null hypothesis (5) also holds for Treasury bills with one week to maturity, in which case the  $v_{t-i}(i=0,1,...,12)$  in (36) represent successive weekly innovations in the one-week yield.

As discussed by Hansen and Hodrick [11], OLS estimation of equation (4) with the moving-average error process (36) results in consistent coefficient estimates, but the usual estimate of the variance-covariance matrix of the estimated coefficients is not consistent. Generalized least squares (GLS) estimation appears to be a logical alternative to OLS in this case. However, as these authors again show, in rational expectations models such as equation (4) GLS techniques do not lead to consistent coefficient estimates.  $\frac{26}{}$ 

As an alternative to the above estimation techniques, Hansen and Hodrick [11] propose estimating equations such as (4) by OLS, and then computing a modified variance-covariance matrix of the estimated coefficients using the moving-average process (36). Because the OLS estimates of the coefficients are already consistent, the procedure only involves computing a consistent estimator of the asymptotic variance-covariance matrix. In this respect, Hansen and Hodrick demonstrate that a consistent estimator of the asymptotic variancecovariance matrix of  $N^{1/2}(\hat{\underline{b}}-\underline{b})$  is

$$N(z^{2}z)^{1}z^{2}s^{2}z^{2}z^{-1}$$
 (37)

where

 $\hat{S}$  = estimated N×N variance-covariance matrix of  $e_{t+1}(t=1,...,N)$ , with individual elements computed as  $\hat{s}_{j} = (\sum_{t=j}^{N} \hat{e}_{t-j}) \cdot [1/(N-j)]$ , j=0,1,...,12

and  $\underline{b}$ ,  $\underline{b}$ , and Z are defined as before in equations (4), (4'), and (6). These authors further prove that the asymptotic variance-covariance matrix (37) obtained using overlapping data is more efficient than that estimated by OLS with a nonoverlapping data set. This latter feature provides motivation for using all of the available data. The relevant test statistic obtained from this estimation procedure is

$$(\underline{\hat{b}}-\underline{b})^{\dagger}[(Z^{\dagger}Z)^{-1}Z^{\dagger}SZ(Z^{\dagger}Z)^{-1}]^{-1}(\underline{\hat{b}}-\underline{b})$$
(38)

which is asymptotically distributed as  $\chi^2$  with k degrees of freedom.

III. Empirical Results

To test the joint hypothesis of rational expectations and the expectations model of the term structure, equation (4) from Test I is estimated. Again, of the five tests considered in the first section, Tests II, III, and

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IV were shown to be asymptotically equivalent to Test I. The equations are estimated with weekly data, starting on January 2, 1970, and ending on September 13, 1979, a total of 507 observations. In some instances lagged values of yields appear as right-hand side variables, which somewhat shortens the estimation period. Consistent coefficient estimates and test statistics are obtained using the Hansen-Hodrick procedure outlined in the previous section.

The basic notion behind this test is that if investors form their expectations rationally, and the expectations hypothesis accurately represents equilibrium yields, then the excess quarterly return on a six-month Treasury bill is uncorrelated with any previously available costless information. This test is represented by equations (4) and (5) of which the former may be written to conform to the variables introduced above as

$$R_{6,t}^{n} = b_{0} + b_{1} \cdot R_{3,t} + b_{2} \cdot R_{6,t} + b_{3} \cdot RU_{t} + b_{4} \cdot FH_{t}$$

$$+ b_{5} \cdot S3_{t} + b_{6} \cdot S6_{t} + b_{7} \cdot RISK_{t} + e_{t+1}.$$
(39)

The null hypothesis to be tested is

$$H_0: b_1 = 1, b_2 = b_3 = b_4 = b_5 = b_6 = b_7 = 0.$$
 (40)

The estimation and test results of this equation as well as seven other subcases are reported in Table 1. The first equation (1.1) merely investigates the hypothesis that  $b_1=1$ , when all other information is excluded. As is apparent from the last two columns in Table 1, this hypothesis cannot be rejected at any reasonable level of significance.

In the second row, the entire information set is included, as in equation (39), and the null hypothesis can be rejected at less than the 0.05 percent

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Table l

ESTIMATION AND TEST RESULTS (Weekly from January 2, 1970 through September 13, 1979)<u>a</u>/

 $R_{6,t}^{h} = b_0 + b_1 \cdot R_{3,t} + b_2 \cdot R_{6,t} + b_3 \cdot RU_{t} + b_4 \cdot FH_{t} + b_5 \cdot S_3 + b_6 \cdot S_6 + b_7 \cdot RISK_{t} + e_{t+1}$ 

			ć	400101990	<del>q</del> ootaaa	/			Sun	mary c/	Tests: b <sub>l</sub> =1,b <sub>1</sub>	−0(1−2,,7) <u>4</u> /
	٩	p1 P1	b2	p <sup>3</sup>	p <sup>4</sup>	• •	° P	р <u>7</u>	R <sup>2</sup>	SE	Test Statistic	Marginal Significance
1.1	0.6688 (0.5803)	0.9728* (0.0883)							.75	0.9857	$\chi^{2}(1) = 0.0951$	.7578
1.2	-1.6510 (1.4378)	-0.6333 (0.4389)	1.8459* (0.4518)	0.0721 (0.2341)	-0.2536* (0.1150)	-0.7267 (0.6098)	0.4478 (0.4082)	0.6512 (0.4447)	.84	0.7989	χ <sup>2</sup> (7) = 28.1044	.0002
1.3	0.2134 (0.6337)	0.0893 (0.5369)	1.0724 (0.5400)						.76	0.9615	χ <sup>2</sup> (2) = 4.1264	.1270
1.4	0.0351 (1.2180)	0.9939* (0.0945)		0.0808 (0.1374)					.75	0.9824	χ <sup>2</sup> (2) = .4419	.8018
1.5	0.7380 (0.5559)	1.0421* (0.0919)			-0.1198 (0.0667)				ιι.	0.9488	$\chi^2(2) = 3.3052$	.1916
1.6	0.8958 (1.1107)	0.9810* (0.0945)				-0.1154 (0.4788)			.75	0.9860	χ <sup>2</sup> (2) = .1515	.9271
1.7	0.9455 (0.7076)	0.9786* (0.0881)					-0.1296		.75	0,9808	χ <sup>2</sup> (2) = .5555	. 7575
1.8	-1.0096 (0.8696)	0,9709* (0,0802)						0.7593* (0.3115)	61.	0.9021	χ <sup>2</sup> (2) = 6.1820	.0455

 $\frac{1}{b}$ ,\*Significantly different from zero at the five percent level.  $\frac{1}{b}$ , Because of the data necessary to compute the RISK variable, the samples for equations (1.2) and (1.8) begin on December 30, 1971.  $\frac{1}{b}$ , Numbers in parentheses are standard errors of the coefficients.  $\frac{1}{c}$ , The  $\frac{1}{R}^2$  and SE statistics are based on the "untransformed" regression residuals.

level of significance. In this equation, the level of the six-month yieldsimilar to Shiller [28] and Friedman [6]—and the foreign holdings variable are statistically significant at the 5 percent level. The sign of the coefficient on this latter variable implies that the higher the level of foreign holdings of Treasury securities, the lower the term premium. Because foreign purchases are comprised mainly of three-month bills, one possible explanation of this result is that when investors observe high foreign holdings, they expect further purchases of three-month bills in the next period (t+1). In turn, continued foreign purchases reduce the net supply of three-month bills in period t+1, which may be expected to lower  $R_{3,t+1}$  and hence increase  $R_{6,t}^h$ . Thus, the risk of capital loss is reduced, which lowers the required term premium. $\frac{27}{2}$ 

Two further details concerning the estimation results of equation (1.2) also deserve comment. First, the large sample size resulting from the use of overlapping data may make it desirable to evaluate the test statistics at somewhat lower significance levels than usual. However, with the marginal significance level of 0.0002 percent reported in the table, only a drastic reduction in the significance level would alter the outcome of the test. Second, the statistical significance of the time-varying term premium does not, of course, guarantee its economic significance. In an attempt to evaluate its economic significance, the implied term premium was calculated and found to account for about 35 percent of the variance of  $R_{6,t}^{h} - R_{3,t}$ . Alternatively, the standard deviation of the implied term premium is about 60 basis points.<sup>28/</sup>

The remaining rows of Table 1 include each of the information variables separately. In these equations the null hypothesis can be rejected at the 5

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percent level in only one instance, when the risk variable is included. Nevertheless, the most meaningful test involves equation (1.2) in Table 1, which includes the entire information set.

Equation (39) is also subjected to two types of specification tests—a Chow test and a Goldfeld-Quandt test. In part, these tests are motivated by the somewhat different empirical results obtained by Mishkin [14] and Shiller [28], where the specifications only apparently differ in terms of assumptions about heteroscedasticity. To avoid problems posed by the 12th-order movingaverage error process embodied in the weekly data used here, the tests were conducted for nonoverlapping samples with observations spaced 13 weeks apart. For the first such subsample—beginning on December 30, 1971 and ending on June 21, 1979—the values of the Chow and Goldfeld-Quandt test statistics are 1.7393 and 0.1693, respectively, with marginal significance levels of 0.5750and  $0.7208.\frac{29}{}$  Thus, the results do not indicate any problems regarding structural shifts and heteroscedasticity.

IV. Summary of Conclusions

In the investigation of the joint hypothesis of rational expectations and the expectations model of the term-structure presented in this paper, previous studies were extended in three main areas. First, common efficient marketsrational expectations tests were compared, and it was shown that four of the five tests considered are asymptotically equivalent, and the fifth is less restrictive than the other four. Second, all available data for Treasury bills maturing in exactly 13 and 26 weeks beginning in 1970 and ending in 1979 were used in testing the joint hypothesis. In contrast, previous studies typically discarded 12/13 of the sample to form a nonoverlapping data set. Finally, a

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more complete set of possible determinants of time-varying term premiums was tested.

The empirical results indicated that the null hypothesis that investors form their expectations rationally and the expectations model of the term structure accurately represents equilibrium yields could be rejected at an extremely low significance level. This result most noticeably differs from those obtained recently by Sargent [27] and Shiller [29], who also used Treasury security yields and could not reject this same joint hypothesis. Because a joint hypothesis was tested, however, the precise cause of rejection cannot be determined. The results instead indicate that either investors do not form expectations rationally, or equilibrium yields contain time-varying term premiums which depend on costlessly available information.

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#### Footnotes

\*The authors are visiting professor, Board of Governors of the Federal Reserve System, on leave from Northwestern University, and assistant vice president and economist, Federal Reserve Bank of Kansas City, respectively. They are grateful to Rick Troll for research assistance, and to Douglas K. Pearce and Rick Troll for helpful discussions. The views expressed here are solely those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System. This paper is a part of the Financial Markets and Monetary Economics Program of the National Bureau of Economic Research.

- 1. These results may be generalized very easily and applied to studies involving pairs of securities such as Treasury bills and long-term bonds.
- 2. Again, overlapping data are actually used here, but this presents estimation, not analytical, problems. Nonoverlapping data may be considered without any loss of generality.
- 3. While this approximation is particularly convenient for the analytical discussion in this section, it is not employed in the empirical work. Instead, exact yields to maturity and holding-period yields are employed. For a more accurate approximation for long-term securities, see Modigliani and Shiller [18].
- 4. Similar to Friedman [7], only costlessly available data are used in the <u>x</u> vector. In the efficient markets literature, this test corresponds to a semistrong-form test. For discussions of different concepts of market efficiency, see Fama [4] and Throop [33].
- 5. Alternatively, equation (4) may be written as

$$R_{6,t}^{h} - R_{3,t} = b_{0} + \underline{x}_{t} \underline{b}^{*} + e_{t+1}$$

where  $\underline{x}_{t}$  includes the current three-month bill rate. For recent examples of this approach, see Mishkin [14], Shiller [28], and Friedman [7]. Friedman's test differs from the others in that market survey data are used to represent  $E(R_{3,t+1}|\Omega_t)$ .

6. There appears to be confusion in the literature concerning the asymptotic efficiency of OLS in situations like (4) where the residual  $e_{t+1}$  is orthogonal to all information available at time t,  $\Omega_t$ , but in which  $e_{t+1}$  is likely correlated with elements of  $\Omega_{t+1}$  not included in  $\Omega_t$ . In such cases, the  $\{e_t\}$  will be serially uncorrelated provided that  $e_{t+1} \in \Omega_{t+1}$  and  $\Omega_t = \Omega_t + 1$ , prompting some researchers to conclude that (under normality) OLS and FIML are equivalent and, hence. OLS is asymptotically efficient. (See, for example, Abel and Mishkin [1].) Such a conclusion, however, is not generally valid because in these situations correlations between the contemporaneous error term and future regressors (i.e., elements of  $\Omega_{t+i}$  not

in  $\Omega_t$ , for i>0) may permit more efficient estimation of the regression parameters than is possible with OLS. For example, consider the following model:

 $y_t = \beta \cdot x_t + e_{t+1}$ 

where the distributions of  $x_t$  and  $e_t$  exhibit the properties:  $E(x_t)=E(e_{t+1})=0$ ;  $E(x_{t-1}\cdot e_{t+1})=0$ , for  $i\geq 0$ ;  $E(x_{t+1}\cdot e_{t+1})=0$ , for i>1;  $Var(e_t)=\sigma^2$ ;  $Var(x_t)=1$ ; and the correlation between  $x_{t+1}$  and  $e_{t+1}$  is  $\rho$ . If n is the sample size, then it is straightforward to verify that the asymptotic distribution of the OLS estimator of  $\beta$ ,  $\hat{\beta}_{OLS}$ , is

$$n^{1/2}(\hat{\beta}_{OLS}-\beta) \sim N(0,\sigma^2).$$

Next, consider an alternative estimator of  $\beta$ ,  $\beta$ , given by the OLS estimator of  $\beta$  in the regression

$$y_{+} = \beta \cdot x_{+} + \rho \cdot x_{++1} + u_{t+1}$$

It may be shown that  $\beta$  is an asymptotically efficient estimator of  $\beta$ , with asymptotic distribution

$$n^{1/2}(\hat{\beta}-\beta) \sim N(0,(1-\rho^2)\sigma^2)$$

which clearly has a lower variance than  $\beta_{OLS}$  provided  $\rho \neq 0$ . In fact, if  $\rho=1$ , then  $\beta$  estimates  $\beta$  precisely! This special case is hardly surprising since if  $x_{t+1}$  and  $e_{t+1}$  are perfectly correlated (i.e.,  $\rho=1$ ), then three consecutive observations on the  $y_t$  and  $x_t$  suffice to determine two of the corresponding residuals ( $e_t$ ) precisely, thereby eliminating all uncertainty from the regression.

- 7. As Mishkin [15,16] notes, even if the entire information set is not included in the model, the joint hypothesis may nevertheless be tested. Mishkin also observes that causality is subject to the interpretation of the researcher in this framework.
- 8. Note that  $z_t$  is solely comprised of lagged values of w in this case.
- 9. Note that the estimators in (17) are not FIML nor need they be efficient for reasons similar to those discussed in footnote 6.
- For discussions of volatility measures and their implications, see Shiller [29,30]. Formal tests concerning the excess volatility of long-term yields are conducted by Singleton [32].
- 11. Shiller [29] actually tests the null hypothesis (25) against a specific alternative hypothesis, but this is not considered here.

- 12. A slightly more general representation of the expected future spot yield is used here in comparison to Shiller [29], who assumes that the term premium equals zero.
- 13. This test has also been applied recently by Hakkio [9].
- 14. Sargent [27] specifies the vector autoregression in first differences, which, as Shiller [29] notes, implies that the variances of the yields are infinite.
- 15. The basic reason that the error term  $\varepsilon_{t+1}$  is described by a first-order moving-average process is that it consists of innovations from time t-1 to time t+1. This feature is discussed in the next section in the context of the overlapping data used in the empirical work.
- 16. Note that the term premiums in equations (30) and (31) cancel.
- 17. As Hansen and Hodrick [11] demonstrate in a similar context, using a firstorder moving-average correction in the estimation of equation (33<sup>-</sup>) results in inconsistent estimates. This topic is discussed in more detail in the next section.
- 18. As discussed later in this section, foreign purchases of Treasury bills are included in the data set. A foreign exchange rate was employed initially (and was statistically significant), but it was replaced in the early stages of this project because the rationale for its inclusion depended ultimately on foreign purchases. The other possible determinants of time-varying term premiums examined here have all appeared in previous studies. Thus, it may be argued that the data have in fact been subjected to some pretesting. Nevertheless, variables such as security supplies have been included for theoretical reasons, not because of their statistical significance (in this case, lack of statistical significance) in previous studies studies using single-equation estimation procedures.
- 19. The usual proviso concerning holidays applies here.
- 20. Instead of 90 and 180, actual days to maturity ranging from 90 to 92 days for three-month bills, for example, were used.
- 21. See, for example, Okun [20], Modigliani and Sutch [17], and more recently, Friedman [6].
- 22. For an explicit representation of this reduced-form expression, along with other details of structural models of interest rate determination, see Friedman and Roley [8].
- 23. See, for example, Pesando [22] and Friedman [6].
- 24. Fama's [5] measure only differs from that in the text in that it is computed using monthly, instead of quarterly, data.

- 25. Again, OLS estimates may not be efficient in this case for reasons discussed in footnote 6.
- 26. GLS estimation requires that the right-hand side variables are strictly exogenous. In other words, future values of the right-hand side variables should be useless in forming optimal forecasts of  $\mathbb{R}^h$ , a property that is clearly violated.
- 27. To examine this explanation further, the following equation was estimated over the sample of 507 observations:

 $\Delta FH_{t} = \begin{array}{c} 0.0131 + 0.1154 \cdot \Delta FH_{t-13} + 0.2394 \cdot e_{t-1} + e_{t} \\ (0.0043) (0.0472) \end{array}$ 

 $\bar{R}^2 = 0.01$  SE = 0.0718 DW = 2.04

where standard errors are in parentheses. These estimates do, in fact, indicate a positive and statistically significant relationship between net changes in foreign holdings from time t-13 to time t.

- 28. The estimated variance of the term premium was computed as the difference in the estimated residual variances of the constrained version (i.e.,  $b_1=1$ ) of equation (1.1) and equation (1.2).
- 29. The degrees of freedom for these F-statistics are (8,15) and (5,5), respectively. The middle five observations were deleted in computing the Goldfeld-Quandt statistic. These two tests were also performed for the other 12 nonoverlapping subsamples. In only one case was a F-statistic significant at the 5 percent level. However, due to the error process (36) inherent in the data, these tests are not independent, making their interpretation as a group difficult.

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