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ABSTRACT

This paper analyzes the theoretical possibility of rational bubbles in stock prices in a model in which stockholders have infinite planning horizons and in which free disposal of equity rules out the existence of negative rational bubbles. The analysis shows that in this framework if a positive rational bubble exists, then it started on the first date of trading of the stock. Thus, the existence of a rational bubble at any date would imply that the stock has been overvalued relative to market fundamentals since the first date of trading and that prior to the first date of trading potential stockholders who anticipated the initial pricing of the stock expected that the stock would be overvalued relative to market fundamentals. The analysis also shows that any rational bubble will eventually burst and will not restart. Thus, even if a positive rational bubble exists, stockholders know that after a random, but almost surely finite, date the stock price will conform to market fundamentals forever.

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Most studies that analyze the theoretical possibility and empirical implications of rational bubbles in stock prices--see, for example, Blanchard and Watson (1982) or West (1984a, 1984b)--utilize a simple model in which the required rate of return from holding equity is constant. Quah (1985), in contrast, considers a more general model in which risk-averse asset holders with infinite planning horizons explicitly maximize expected utility. In Quah's model, the product of a stock's price and the marginal utility of consumption satisfies a first-order linear expectational difference equation that has an eigenvalue greater than unity and a stochastic forcing term that reflects the expected evolution of the stock's dividends.

A particular solution to such an expectational difference equation--referred to as the market-fundamentals component of the stock price--equates the stock price to the present value of expected future dividends, discounted at the expected marginal rate of intertemporal substitution. This discount factor depends uniquely on the process generating endowments and dividends and on the rate of time preference, both of which are exogenously given. The general solution to the expectational difference equation allows the stock price to have a rational-bubbles component in addition to the market-fundamentals component. The existence of a rational-bubbles component would reflect a self-confirming belief that the stock price depends on a variable (or a combination of variables) that is intrinsically irrelevant--that is, not part of market fundamentals--or on truly relevant variables in a way that involves parameters that are not part of market fundamentals.

The property that the eigenvalue of the expectational difference equation is greater than unity has two important consequences. First, it guarantees the existence of an economically meaningful (i.e., forward looking) market-fundamentals solution except in extreme cases of the process generating dividends. Second, it implies that rational bubbles have explosive conditional expectations. Specifically, the expected value of a rational-bubbles component of a stock price either would increase or would decrease geometrically into the infinite future.

The results of Mussa (1984) underscore the association of economically interesting market fundamentals with nonconvergent rational bubbles. Mussa shows that various examples of attempts to construct alternative models in which potential rational bubbles are convergent all preclude a forward-looking market-fundamentals solution for some relevant price variable.[1]

The fact that rational bubbles have explosive conditional expectations implies that a negative rational-bubbles component cannot exist, because, given free disposal of equity, stock holders cannot rationally expect a stock price to decrease without bound and, hence, to become negative at a finite future date. The property of explosive conditional expectations also suggests that if a positive rational bubble exists, stockholders might expect it eventually to dominate the stock price, which would then bear little relation to market fundamentals. Positive rational bubbles are empirically plausible only if, despite explosive conditional expectations, the probability is small that a rational bubble would become arbitrarily large. This observation focuses attention on processes, like one suggested by Blanchard and Watson, that apparently can generate rational bubbles that are likely to start, burst, and restart repeatedly.

The present paper explores more deeply the theoretical possibility of rational bubbles in stock prices by focusing on the circumstances of the inception of rational bubbles. The inception of a rational bubble after the first date of trading of a stock would involve an innovation in the stock price. Accordingly, any rational-bubbles component that starts after the first date of trading has an expected initial value of zero. Moreover, because free disposal rules out negative rational bubbles, this expected initial value can equal zero only if any initial realization of a rational bubble after the first date of trading equals zero with probability one.

This theoretical argument means that the impossibility of negative rational bubbles also rules out the inception of a positive rational bubble except at the first date of trading of a share. One important implication of this argument is that the process suggested by Blanchard and Watson for generating empirically interesting positive rational bubbles is inconsistent with the impossibility of negative rational bubbles. Once a positive rational bubble that began at the first date of trading has burst, it cannot restart.

In the existing literature, Brock (1982) and Tirole (1982) already have constructed arguments against the existence of positive rational bubbles in stock prices.[2] Brock's argument is based on a transversality condition implied by the optimizing behavior of asset holders with infinite planning horizons. The existence of a rational bubble would violate this transversality condition. Specifically, as Gray (1984) explains, the existence of a positive rational bubble would imply that stockholders expect to gain utility from selling the stock now and never buying it back.

Tirole's argument assumes that, even if stockholders have infinite planning horizons, they would not plan to hold an overvalued asset--that is, one with a positive rational bubbles component--forever. Instead, each stockholder would want to realize the capital gain associated with a positive rational bubble at some date in the finite future. Consequently, if the number of potential asset holders is finite, a finite future date would exist beyond which no one would plan to hold overvalued shares. Under these conditions, a backward unraveling argument precludes the existence of positive rational bubbles.

The arguments of Brock and Tirole do not apply to a model of an infinite succession of overlapping generations of asset holders with finite planning horizons. In addition, Quah claims that, even if planning horizons are infinite, Brock's argument does not rule out positive rational bubbles that almost surely burst at a date in the finite future. The idea seems to be that, even if such rational bubbles can restart repeatedly, the probability that stockholders will gain utility from a strategy that involves selling shares now and never buying them back is zero. Accordingly, the transversality condition would not rule out such rational bubbles if stockholders ignore zero probability events.

An analogous objection would apply to Tirole's argument if stockholders cannot identify a finite future date by which demand for shares whose price contains a positive rational-bubbles component will have vanished with probability one--say, because the total number of potential stockholders or the length of some stockholders' holding periods are not known. In this case, each stockholder could only infer that a positive rational bubble must, with probability one, eventually burst. This inference would rule out the possibility of a rational bubble that, with nonzero probability, lasts forever but not the possibility of one that almost surely bursts.

The argument developed in the present paper applies to all forms of rational bubbles, including those that apparently can burst and restart repeatedly. Moreover, unlike the analyses of Brock and Tirole, the present argument does not exploit the properties of either infinite planning horizons or a finite number of potential stockholders. Accordingly, although we formalize the analysis within the model of infinite planning horizons studied by Quah, we presumably could develop an analogous argument that would apply to the inception of rational bubbles in an overlapping-generations framework.

In this regard, note that the results derived below are directly applicable to a model in which the required rate of return on equity is constant. This model can represent a special case, which arises under risk neutrality, of either the model of infinite planning horizons or the model of overlapping generations. (Analysis of a more general overlapping-generations model would present an additional conceptual problem because in such a model the marginal rate of intertemporal substitution would depend on asset prices in addition to the rate of time preference and the process generating endowments and dividends. Consequently, the overlapping-generations framework generally does not yield a unique relation between the stock price and expected future dividends that defines market fundamentals.)

In what follows, section 1 reviews the basic properties of rational bubbles in stock prices. Section 2 derives the result that if a rational bubble exists, it must have started on the first date of trading. Section 3 derives the further result that rational bubbles cannot burst and simultaneously restart. In light of these results, section 4 discusses the possible forms that interesting rational bubbles in stock prices could take if stockholders have finite planning horizons. Section 5 offers concluding remarks.

1. Properties of Rational Bubbles

Assume that a representative household maximizes expected utility over an infinite horizon,

$$(1) \qquad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau), \quad 0 < \beta < 1,$$

where $\{c_\tau\}$ is a stochastic process representing consumption of a single perishable good, and β is a discount factor for future consumption. Positive time preference implies that β is less than unity. The utility function, $u(\cdot)$, is strictly concave, increasing, and continuously differentiable. The conditional expectations operator E_t is based on an information set that contains, at least, current and past values of all the variables entering the model.

Each period, the household receives an endowment, y_τ, of the consumption good. The household can attempt to smooth consumption by acquiring shares, s_τ, at the price of p_τ (units of the consumption good) per share. Each share pays a dividend of d_τ units of the consumption good per period. The budget constraint faced by the household at date τ is

(2) $$c_\tau + p_\tau(s_{\tau+1} - s_\tau) \leqq y_\tau + d_\tau s_\tau.$$

The stochastic process $\{d_\tau, y_\tau\}$ is exogenous to the model and assumed stationary.

The first-order condition for the household's utility maximization problem is

(3) $$E_t[u'(c_{t+1})p_{t+1}] - \beta^{-1}[u'(c_t)p_t] = E_t[u'(c_{t+1})d_{t+1}].$$

We can normalize the number of outstanding shares to unity and impose $s_\tau = 1$ as the stock market equilibrium condition. The representative household's consumption, c_τ, in equilibrium must then equal the total supply, $y_\tau + d_\tau$. Equation (3), combined with the equilibrium conditions, implies

(4) $$E_t q_{t+1} - \beta^{-1} q_t = E_t[u'(y_{t+1} + d_{t+1})d_{t+1}],$$

where

$$q_t \equiv u'(y_t + d_t)p_t.$$

Equation (4) is a first-order expectational difference equation. Because the eigenvalue, β^{-1}, is greater than unity, the forward-looking solution for q_t involves a convergent sum, as long as $E_t[u'(y_{t+j} + d_{t+j})d_{t+j}]$ does not grow with j at a geometric rate equal to or greater than β^{-1}. The forward-looking solution, denoted by F_t and referred to as the market-fundamentals component of q_t, is

$$(5) \qquad F_t = \sum_{j=1}^{\infty} \beta^j E_t[u'(y_{t+j} + d_{t+j})d_{t+j}].$$

This market-fundamentals solution to equation (4) sets the current product of the stock price and the marginal utility of consumption equal to the present value of expected future products of dividends and the marginal utility of consumption. If the representative household is risk neutral, equation (5) reduces to the simpler specification of market fundamentals, which equates the stock price to the present value of expected future dividends.

The general solution to equation (4) is the sum of the market-fundamentals component, F_t and the rational-bubbles component, B_t--that is,

$$(6) \qquad q_t = B_t + F_t,$$

where B_t is the solution to the homogeneous expectational difference equation,

$$(7) \qquad E_t\, B_{t+1} - \beta^{-1}B_t = 0.$$

A nonzero value of B_t would reflect the existence of a rational bubble at date t--that is, a self-confirming belief that q_t does not conform to the market-fundamentals component, F_t.

The assumption of rational expectations implies that in forming E_tB_{t+j}, for all $j > 0$, potential asset holders behave as if they know that any rational-bubbles component would conform to equation (7) in all future periods. Accordingly, any solution to equation (7) would have the property

$$(8) \qquad E_tB_{t+j} = \beta^{-j}B_t \quad \text{for all} \quad j > 0.$$

Equation (8) says that the existence of a nonzero rational-bubbles component at date t would imply that the expected value of the rational-bubbles component at date $t+j$ either increases or decreases with j at the geometric rate β^{-1}. Therefore, because the eigenvalue β^{-1} exceeds unity, the existence of a rational bubble would imply that $\{E_t q_{t+j}\}_{j=1}^{\infty}$ either increases or decreases without bound.

In particular, the existence of a negative rational-bubbles component at date t would imply that $E_t q_{t+j}$ becomes negative for some finite j. But, given free disposal of shares, stockholders cannot rationally expect a stock price to become negative at a finite future date. Therefore, a negative rational-bubbles component would be a contradiction and, hence, cannot exist.

Solutions to equation (7) satisfy the stochastic difference equation

$$(9) \qquad B_{t+1} - \beta^{-1} B_t = z_{t+1},$$

where z_{t+1} is a random variable (or combination of random variables) generated by a stochastic process that satisfies

$$(10) \qquad E_{t-j} z_{t+1} = 0 \quad \text{for all} \quad j \geq 0.$$

The key to the relevance of equation (9) for the general solution of q_t is that equation (7) relates B_t to $E_t B_{t+1}$, rather than to B_{t+1} itself as would be the case in a perfect-foresight model.

The random variable z_{t+1} is an innovation, comprising new information available at date $t+1$. This information can be intrinsically irrelevant--that is, unrelated to F_{t+1}--or it can be related to truly relevant variables, like d_{t+1}, through parameters that are not present in F_{t+1}. The critical property of z_{t+1}, given by equation (10), is that its expected future values are always zero.

The general solution to equation (9), for any date t, $t \geq 0$, is

$$B_t = \beta^{-t} B_0 + \sum_{\tau=1}^{t} \beta^{\tau-t} z_\tau , \tag{11}$$

where date zero denotes the first date of trading of the stock. Equation (11) expresses the rational-bubbles component at date t as composed of two terms. The first term is the product of the eigenvalue raised to the power t and the value of the rational-bubbles component at date zero. The second term is a weighted sum of realizations of z_τ from $\tau = 1$ to $\tau = t$. The weights are powers of the eigenvalue such that the contribution of z_τ to B_t increases exponentially with the difference between t and τ. For example, a past realization z_τ, $1 \leq \tau < t$, contributed only the amount z_τ to B_τ, but contributes $\beta^{\tau-t} z_\tau$ to B_t.

2. The Inception of Rational Bubbles

The fact that a negative rational-bubbles component cannot exist means that, in addition to satisfying equation (9), the rational-bubbles component of a stock price at date $t+1$ satisfies $B_{t+1} \geq 0$. Taken together, equation (9) and this nonnegativity condition imply that realizations of z_{t+1} must satisfy

$$z_{t+1} \geq - \beta^{-1} B_t \quad \text{for all} \quad t \geq 0. \tag{12}$$

Equation (12) says that the realization z_{t+1} must be large enough to insure that equation (9) implies a nonnegative value for B_{t+1}.

Suppose that B_t equals zero. In that case, equation (12) implies that z_{t+1} must be nonnegative. But, equation (10) says that the expected value of z_{t+1} is zero. Thus, if B_t equals zero, then z_{t+1} equals zero with probability one.

This result says that if a rational bubble does not exist at date t, $t \geq 0$, a rational bubble cannot get started at date $t+1$, nor, by extension, at any subsequent date. Therefore, if a rational bubble exists at present, it must have started at date zero, the first date of trading of the stock, and, hence, this stock must have been overvalued relative to market fundamentals at every past date. The essential idea underlying this line of argument is that, because the inception of a rational bubble at any date after the first date of trading would involve an innovation in the stock price, the expected initial values of a positive rational bubble and a negative rational bubble would have to be equal. Accordingly, because free disposal rules out a negative rational-bubbles component, a positive rational-bubbles component also cannot start after the first date of trading.

Suppose that, prior to the first date of trading, potential stockholders anticipate the introduction of trading and they form an expectation about the initial stock price. Suppose further that this expectation coincides with market fundamentals--that is,

$$(13) \qquad E_{-1}B_0 = E_{-1}q_0 - E_{-1}F_0 = 0.$$

Equation (13) would imply that B_0 is a random variable with mean zero. Accordingly, given the nonnegativity condition $B_0 \geq 0$, B_0 would equal zero with probability one. This observation implies that if a positive rational bubble exists, potential stockholders who, prior to the first date of trading, anticipated the initial pricing of this stock expected it to be overvalued relative to market fundamentals.

3. Can Positive Rational Bubbles Burst and Restart?

As mentioned above, the existence of a positive rational-bubbles component is empirically plausible only if, despite explosive conditional expectations, the probability is small that the rational bubble component will ever become large enough to dominate the stock price. This observation suggests the following model of the innovation z_{t+1}:

(14) $$z_{t+1} = (\theta_{t+1} - \beta^{-1})B_t + \varepsilon_{t+1},$$

where θ_{t+1} and ε_{t+1} are mutually and serially independent random variables. If the processes generating θ_{t+1} and ε_{t+1} satisfy

(15) $$E_{t-j}\theta_{t+1} = \beta^{-1} \quad \text{for all} \quad j \geq 0 \quad \text{and}$$

(16) $$E_{t-j}\varepsilon_{t+1} = 0 \quad \text{for all} \quad j \geq 0,$$

then z_{t+1} as given by equation (14) satisfies equation (10).

Substituting for z_{t+1} in equation (9) from equation (14) gives

(17) $$B_{t+1} = \theta_{t+1}B_t + \varepsilon_{t+1}.$$

Equation (17) says that, with z_{t+1} given by equation (14), an existing rational-bubbles component, B_t, will burst next period if the event $\theta_{t+1} = 0$ occurs. If this event has positive probability, then any rational-bubbles component would burst at a random, but almost surely finite, future date. Specifically, if the probability associated with $\theta_{t+1} = 0$ is Π, $0 < \Pi < 1$, then the expected duration of a rational-bubbles component is Π^{-1} periods and the probability that B_t will not burst by date T $(T > t)$ is $(1-\Pi)^{T-t}$, which tends to zero as T approaches infinity.

Given that realizations of θ_{t+1} and ε_{t+1} are mutually and serially independent and also independent of B_0, then ε_{t+1} is independent of B_t for all $t \geq 0$. In this case, if the event $\theta_{t+1} = 0$ were by chance to coincide with a positive realization of ε_{t+1}, then, according to equation (17), as an existing rational-bubbles component bursts, a new rational-bubbles component, which is independent of all existing and past rational-bubbles components, would simultaneously start.

Quah suggests this model as a generalization of a model of rational bubbles that could burst and restart proposed by Blanchard and Watson (1982). Quah argues that the property that any existing rational-bubbles component will almost surely burst at a date in the finite future, in addition to implying a small probability that the rational-bubbles component would become large enough to dominate the stock price, also makes these models of rational bubbles immune to Brock's argument that a transversality condition precludes rational bubbles. Quah's presumption is that stockholders ignore the possibility, which has zero probability, that the rational-bubbles component will never burst. As mentioned above, a rational-bubbles component that will almost surely burst also would seem to be immune to Tirole's argument that stockholders would not plan to hold an overvalued asset forever.

The result derived in section 2 that, given the impossibility of a negative rational-bubbles component, a rational-bubbles component can start only on the first date of trading directly implies that a rational-bubbles component that burst could not restart at a later date. The essential property that a negative rational-bubbles component cannot exist follows directly from equation (8) and, hence, obtains whatever the process or combination of processes that generate the innovation z_{t+1}. This property means that in the present model, in addition to satisfying equation (17), the rational-bubbles component satisfies $B_{t+1} \geq 0$. Therefore, the event $\theta_{t+1} = 0$ cannot coincide with a negative realization of ε_{t+1}. Accordingly, given that the event $\theta_{t+1} = 0$ has positive probability and that the random variables ε_{t+1} and θ_{t+1} are independent, ε_{t+1}

must be nonnegative. But, equation (16) says that the expected value of ε_{t+1} is zero. Therefore, ε_{t+1} equals zero with probability one and the chance coincidence of $\theta_{t+1} = 0$ and $\varepsilon_{t+1} > 0$ has zero probability.

This result says that the impossibility of a negative rational-bubbles component also precludes the possibility that a new independent positive rational-bubbles component simultaneously starts when an existing positive rational-bubbles component bursts. In sum, the analysis in sections 2 and 3 has shown that, if a positive rational-bubbles component exists, then it must have started on the first date of trading of the stock, it has not yet burst, and it will not restart if it bursts. Together with the assumption that the event $\theta_{t+1} = 0$ has positive probability, these properties correspond to Blanchard's (1979) specification of a rational-bubbles component that exists from the first date of trading, eventually bursts, and does not restart.

A minor variation on Blanchard's specification is possible if stockholders ignore events that have small probability. Suppose that ε_{t+1} equals zero with probability one for all $t \geq 0$ except for a finite prespecified collection of dates. In this case, if the probability of the event $\theta_{t+1} = 0$ is large enough and all possible realizations of ε_{t+1} are close enough to zero, the probability that a given negative rational-bubbles component conforming to equation (17) violates the nonnegativity constraint on the stock price can be arbitrarily small. Accordingly, the probability that all of the finitely many independent negative rational-bubbles components that start eventually burst before they violate the nonnegativity constraint can be arbitrarily close to unity. If, contrary to the strict interpretation of rational expectations, stockholders ignore such possible violations because they have small probability, then this model would allow a near-rational-bubbles component that can burst and restart and can be either positive or negative. This model, however, would imply, like Blanchard's model, that after some random, but almost surely finite, date the stock price will conform to market fundamentals forever.

4. Finite Planning Horizons

The probability that a positive rational-bubbles component will ever become large enough to dominate the stock price can be small even if the rational-bubbles component never bursts. As an alternative, a rational-bubbles component, which began on the first date of trading, can exist forever as long as it will shrink periodically. If, however, stockholders have infinite planning horizons, such a model would be subject to the arguments of Brock and Tirole against the existence of rational bubbles that never burst.

Although the formal analysis developed above assumed infinite planning horizons, equation (7) also describes the potential rational-bubbles component of a stock price for a model of an infinite succession of overlapping generations of risk-neutral stockholders with finite planning horizons. In this case, the required rate of return from holding equity would be the constant, $\beta^{-1} - 1$. In this case the arguments of Brock and Tirole do not apply, and a positive rational-bubbles component that will never burst can exist if, as pointed out by Tirole (1985), the required rate of return is less than the growth rate of the economy.

An example of such a positive rational-bubbles component that is consistent with the results derived in the preceding sections is

$$(18)\quad B_{t+1} = \begin{cases} \delta B_t + \varepsilon_{t+1}, & \text{with probability } \Pi \\ (1-\Pi)^{-1}(\beta^{-1}-\delta\Pi)B_t + \varepsilon_{t+1}, & \text{with probability } (1-\Pi) \end{cases}$$

where δ is a small positive constant and where $E_t \varepsilon_{t+1} = 0$ and $B_0 > 0$. This specification corresponds to setting θ_{t+1} in equation (17) equal to δ with probability Π and equal to $(1-\Pi)^{-1}(\beta^{-1} - \delta\Pi)$ with probability $(1-\Pi)$ and allowing ε_{t+1} to depend on B_t and θ_{t+1} in such a way that B_{t+1} remains nonnegative with probability one. In particular, given $\theta_{t+1} = \delta$, realizations of ε_{t+1} must satisfy $\varepsilon_{t+1} \geq -\delta B_t$. Equation (18) specifies a positive rational-bubbles component that starts on the first date of trading, that collapses with probability Π in any period, but that, given δ greater than zero and the appropriate restriction on the realizations of ε_{t+1}, always remains positive.

5. Concluding Remarks

This paper analyzed the theoretical possibility of rational bubbles in stock prices in a model in which stockholders have infinite planning horizons and in which free disposal of equity rules out the existence of negative rational bubbles. The analysis showed that in this framework if a positive rational bubble exists, then it started on the first date of trading of the stock. Thus, the existence of a rational bubble at any date would imply that the stock has been overvalued relative to market fundamentals since the first date of trading and that prior to the first date of trading potential stockholders who anticipated the initial pricing of the stock must have expected that the stock would be overvalued relative to market fundamentals. The analysis also showed that any rational bubble will eventually burst and will not restart. Thus, even if a positive rational bubble exists, stockholders know that after a random, but almost surely finite, date the stock price will conform to market fundamentals forever. We also observed that, if stockholders have finite planning horizons and if the growth rate of the economy is larger than the required rate of return on equity, a positive rational bubble that began at the first date of trading could go on forever.

In permitting the inception of a rational bubble only at the first date of trading, the rational expectations model of equity with free disposal is like a perfect foresight model and unlike the general linear rational-expectations model analyzed, for example, by Shiller (1978). As in the case of perfect foresight, a single initial condition stating that the stock price conforms to market fundamentals at the first date of trading would guarantee that both the rational expectations of the stock price and the actual realizations of the stock price conform to market fundamentals at all dates.

The analysis in this paper focused on an asset (equity) that pays a real dividend. The case of a real asset that directly yields utility--for example, gold or tulips--is identical. The case of a pure fiat money, however, is different in that free disposal does not necessarily rule out negative (that is, inflationary) rational bubbles--see, for example, Flood and Garber (1980), and Obstfeld and Rogoff (1983). We can, however, rule out rational deflationary bubbles by appealing to the arguments of Brock or Tirole against the possibility of positive rational bubbles or by assuming in the overlapping generations setting that the relevant interest rate exceeds the growth rate of the economy. In either case, an argument analogous to that of the present paper, with some technical modifications having to do with the nonlinear structure of the model involving a fiat money, would limit the possible inception of rational inflationary bubbles--see, Diba and Grossman (1986).

NOTES

1. Quah (1985) develops an example in which the stock price coincides with the expected present value of future dividends, and yet rational bubbles are convergent for certain parameter values. In this example, however, the market-fundamentals solution is essentially backward looking, because dividends depend on a set of state variables that have no apparent relation to currently available information about current and future earnings and other potentially relevant variables.

2. Lucas (1978) presents another argument for uniqueness of rational expectations equilibrium based on contraction mappings. Brock (1982) points out that this argument rules out multiple stationary equilibria but does not preclude the nonstationary price paths associated with rational bubbles.

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