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MODELING PRICE RIGIDITY OR PREDICTING THE QUALITY OF THE GOOD THAT CLEARS THE MARKET

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ABSTRACT

To say that the price of some good is inflexible over time has little meaning if the "good" is changing over time. In this paper we concentrate on delivery lags as being the only dimension other than price that varies. We show how one can predict the relative importance of price and delivery lag fluctuations as equilibrating mechanisms. The complications of the theory as well as the surprising results underscore the complexity of predicting price behavior when the characteristics of the good are endogenous. The empirical results provide strong support for the theory that delivery lags are an important influence on market behavior and therefore that an understanding of their influence is crucial in predicting how markets will respond to supply and demand shocks.

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I. Introduction

The idea that prices especially in manufacturing are inflexible and non-market clearing appears persistently in the economics literature (see Carlton 1979, Section II for a brief survey). Evidence often used to support the nonmarket clearing hypothesis has been the observation that large quantity adjustments, either through inventory changes or delivery lag changes, and not large price adjustments, characterize the response of many markets to supply and demand changes. This paper builds on the ideas in Carlton (1979) to develop an equilibrium model that is capable of explaining how markets will respond to changes in supply and demand.

The basic idea of the paper is a simple one. Consumers care about not only price but also quality attributes of a good. In response to supply and demand shocks, adjustment in quality of the good may be more important than adjustment in price. To say that price appears inflexible over time for some good has little meaning if the "good" is changing over time. In this paper we concentrate on delivery lags as being the only dimension of the good other than price that varies. This is of course a simplification but delivery lag is often the easiest (least costly) quality attribute to adjust and it is an attribute for which aggregate data are available. We extend the theory developed by Rosen (1974) and Zarnowitz(1962)[1] to show how one can predict for a market the relative importance of price and delivery lag fluctuations as equilibrating mechanisms.

The complications of the theory as well as the surprising results emerging from the theory underscore the complexity involved in predicting price behavior when the characteristics of the "good" may be endogenous. Simple supply equal demand models not only may be too simple, they might provide the wrong intuition.

Census data are used to illustrate the theory. Unfortunately, only aggregate census data are available to test the theory, and we are forced to make simplifying assumptions in order to empirically test the theory. The empirical results are consistent with the theory that delivery lags do indeed affect behavior and therefore that an understanding of their influence is crucial in predicting how markets will equilibrate. Data more disaggregate than census data are needed before the specific implications of the theory can be adequately tested.

II. Theory

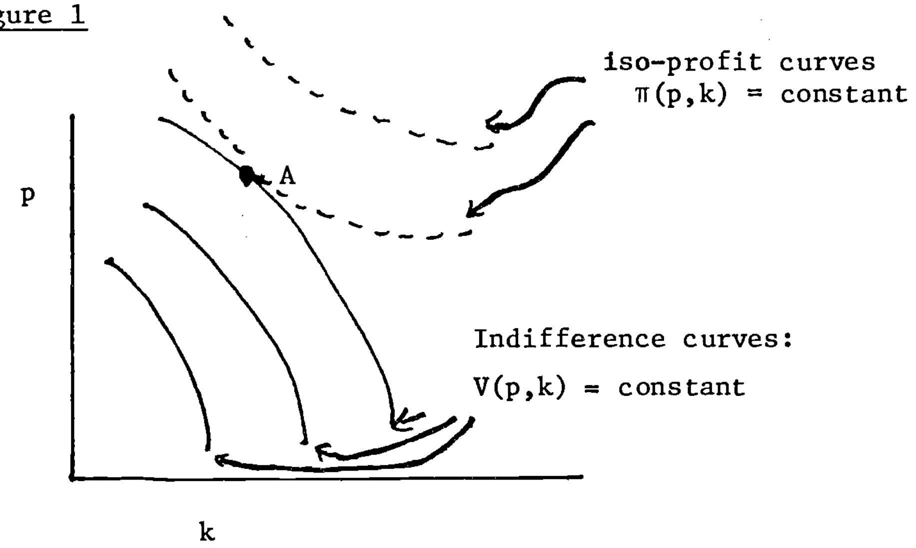
Let all firms be alike, and let all consumers be alike. Firms take orders for a homogeneous good on day 1 and decide how quickly to produce and deliver the good to the buyers. To keep the exposition simple and in view of the data limitations to be discussed later, we assume that all buyers who order on day 1 pay the same price p and obtain delivery on the same day k. (An obvious extension of the theory is to have price depend on delivery lag and allow different delivery lags for buyers.) Consumers are assumed not to be able to forecast their demand and order sufficiently far in advance to avoid any delay between the time they want the good and the time they receive the good. Consumers have an indirect utility function V(p, k, y) which depends not only on p and k but also on income y and other prices which for notational simplicity we omit from the list of arguments in V. We further assume that the marginal utility of income, V_y, is constant and equal to 1. The indirect utility function determines those (p, k) combinations that leave the consumer indifferent. Clearly, as p is raised, k must fall along any indifference curve, and utility falls as p or k increases. However, there is no reason to suppose that the amount purchased along an indifference curve in (p, k) space is constant.[2] Indeed, one way for a buyer to take advantage of a lower price and longer delivery time is to purchase more of the "good."

It is not possible a priori to characterize the shape of the indifference curve in (p, k) space, though it is possible to derive the conditions (see Appendix A) under which the indifference curves are concave and to relate the conditions to whether the quantity demanded rises or falls along an indifference curve as p rises (k falls). The consumer chooses that available (p, k) combination that yields highest utility.

Firms have a production technology whose costs depends upon how quickly the good is produced as well as how much of the good is produced. So, for example, it is cheaper to produce 1 unit in 1 week than in 1 minute. The firm has two margins to concern itself with. First, holding delivery time constant, how much should be produced? Second, what delivery time should be chosen? Firms have a restricted profit function $\pi(p, k)$ from which (p, k) combinations yielding constant profit can be constructed. Clearly, profits increase the higher is p, and the larger is k. In general, along any iso-profit curve the quantity supplied by the firm will vary.[3]

An equilibrium (p, k) combination in this market requires a) that the marginal rate of substitution for consumers between p and k equal the marginal rate of technological transformation for suppliers, and b) that the total amount demanded equal the total amount supplied. Condition a) requires that the equilibrium be a point of tangency between the indifference and iso-profit curves. This tangency is illustrated as point A in Figure 2 for concave indifference curves and convex iso-profit curves. The condition b) determines which of the many possible tangencies between indifference and iso-profit curves is equilibrium. As demand increases relative to supply, equilibrium moves to a tangency point between a higher iso-profit curve and an indifference curve of lower utility. We want to investigate the equilibrium with the number of consumers and firms fixed. We then want to investigate the equilibrium short run response when the number of buyers relative to the number of sellers randomly fluctuates. Without loss of generality we will assume that the number of competitive firms is one, and the number of consumers is N.[4]

Figure 1



The condition that the consumers' marginal rate of substitution between p and k equal the marginal rate of technological transformation of the firm can be written as (subscripts denote partial derivatives)

$$(1) \qquad -\frac{V_k}{V_p} = -\frac{\pi_k}{\pi_p}$$

The second order condition to guarantee that (1) represents an optimum for the consumer is

$$(2) \qquad \frac{d}{dk}\left[-\frac{V_k}{V_p} + \frac{\pi_k}{\pi_p}\right] < 0.$$

The condition that supply equals demand can be written as

$$N(-V_p) = \pi_p \tag{3}$$

where we have used Roy's identity that quantity demanded equals $-V_p/V_y$, the fact that V_y is assumed equal to 1, and Hotelling's Lemma that the quantity supplied equals π_p. Equation (1) and (3) are two equations in two unknowns, p and k. By dividing (3) by (1), it is possible to rewrite the equilibrium conditions as (3) and

$$N(-V_k) = \pi_k. \tag{4}$$

We now want to see how the equilibrium price, p, and delivery lag, k, will be altered if the number of demanders relative to the number of sellers, N, suddenly increases. Such a change corresponds to an unpredictable short-run increase in demand. To determine the percent fluctuations in p and k in response to percent changes in N, we perform comparative statics on (3) and (4) to obtain,

$$\begin{bmatrix} \dfrac{\partial[\ln(-V_k)-\ln \pi_k]}{\partial \ln p} & \dfrac{\partial[\ln(-V_k)-\ln \pi_k]}{\partial \ln k} \\[2ex] \dfrac{\partial[\ln(-V_p)-\ln \pi_p]}{\partial \ln p} & \dfrac{\partial[\ln(-V_p)-\ln \pi_p]}{\partial \ln k} \end{bmatrix} \begin{bmatrix} \dfrac{d \ln p}{d \ln N} \\[2ex] \dfrac{d \ln k}{d \ln N} \end{bmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}. \tag{5}$$

Equation (5) completely characterizes the equilibrium fluctuations in p and k that occur in response to short run shifts in supply and demand. We now wish to express the quantities in (5) in terms of elasticities of supply and demand.[5] Notice that since all firms are idential and all individuals are identical elasticities of market demand and supply curves will be the same as those of individual consumers and firms.

We adopt the following notation:

n_p = price elasticity of demand

n_k = delivery lag elasticity of demand

n_p^s = price elasticity of supply

n_k^s = delivery lag elasticity of supply

θ_k = demand elasticity of marginal disutility of delivery lag $(\frac{\partial \ln V_k}{\partial \ln k})$

θ_k^s = supply elasticity of marginal profit gain of delivery lag. $(\frac{\partial \ln \pi_k}{\partial \ln k})$

Lemma 1: Using the above notation, (5) can be rewritten as

$$\begin{bmatrix} \frac{p\pi_p}{kV_k}(n_k - n_k^s) & (\theta_k - \theta_k^s) \\ (n_p - n_p^s) & (n_k - n_k^s) \end{bmatrix} \begin{bmatrix} \frac{d \ln p}{d \ln N} \\ \frac{d \ln k}{d \ln N} \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \tag{6}$$

Proof: Notice that

$$\frac{\partial \ln -V_k}{\partial \ln p} = \frac{p}{-V_k} \cdot V_{kp} = \frac{p}{k}\frac{V_p}{V_k}\left[\frac{-V_{kp}}{-V_p} \cdot k\right] = \frac{pV_p}{kV_k} n_k,$$

$$\frac{\partial \ln \pi_k}{\partial \ln p} = \frac{p}{\pi_k} \pi_{kp} = \frac{p}{k}\frac{\pi_p}{\pi_k}\left[\frac{\pi_{kp}}{\pi_p} \cdot k\right] = \frac{p}{k}\frac{\pi_p}{\pi_k} n_k^s,$$

that from (2)

$$\frac{p}{k}\frac{V_p}{V_k} = \frac{p}{k}\frac{\pi_p}{\pi_k},$$

and that

$$\frac{\partial \ln -V_k}{\partial \ln k} = \frac{k}{V_k} V_{kk} \equiv \theta_k$$

$$\frac{\partial \ln \pi_k}{\partial \ln k} = \frac{k}{\pi_p} \pi_{pp} \equiv \theta_k^s$$

$$\frac{\partial \ln -V_p}{\partial \ln p} = \frac{-V_{pp}}{-V_p} p \equiv n_p$$

$$\frac{\partial \ln \pi_p}{\partial \ln p} = \frac{\pi_{pp}}{\pi_p} p \equiv n_p^s$$

$$\frac{\partial \ln -V_p}{\partial \ln k} = \frac{-V_{pk}}{-V_p} k \equiv n_k$$

$$\frac{\partial \ln \pi_p}{\partial \ln k} = \frac{\pi_{pk}}{\pi_p} k \equiv n_k^s.$$

Using the above results (6) follows immediately from (5). Q.E.D.

If delivery lag k were set arbitrarily then (3) above would determine equilibrium and only the second row of (6) would represent the comparative statics. In that situation, we would obtain the usual result that price fluctuations are smaller the larger is $|n_p|$, the absolute value of the price elasticity of demand.

To solve (6) for dlnp/dlnN and dlnk/dlnN is straightforward.

(7) Let $D = \begin{bmatrix} \frac{p\pi_p}{k\pi_k} [n_k - n_k^s] & [\theta_k - \theta_k^s] \\ [n_p - n_p^s] & [n_k - n_k^s] \end{bmatrix} = \frac{p\pi_p}{k\pi_k} [n_k - n_k^s]^2 - [\theta_k - \theta_k^s][n_p - n_p^s]$

(8) $A = \begin{bmatrix} -1 & (\theta_k - \theta_k^s) \\ -1 & n_k - n_k^s \end{bmatrix} = [\theta_k - \theta_k^s] - [n_k - n_k^s]$

$$(9)\qquad B = \begin{vmatrix} \frac{p\pi_p}{k\pi_k}[n_k - n_k^s] & -1 \\ & \\ [n_p - n_p^s] & -1 \end{vmatrix} = [n_p - n_p^s] - \frac{p\pi_p}{k\pi_k}[n_k - n_k^s].$$

It follows then that

$$(10)\qquad \frac{d\ \ln p}{d\ \ln N} = \frac{A}{D} \quad \text{and}$$

$$(11)\qquad \frac{d\ \ln k}{d\ \ln N} = \frac{B}{D}. \quad \text{Therefore}$$

$$(12)\qquad \frac{\frac{d\ \ln p}{d\ \ln N}}{\frac{d\ \ln k}{d\ \ln N}} = \frac{A}{B}.$$

Equation (12) is the ratio of the percentage fluctuations in price to those in delivery lag. To see how (12) depends on the relevant underlying elasticities, we need to sign A, B and D. It is not possible a priori to sign A, B and D without any further assumptions. We make the following assumption:

<u>Assumption 1</u>. In response to an increase in demand, N, (decrease in supply) the equilibrium price, p, and delivery lag, k, <u>both</u> increase.

In a model where the good has two attributes p and k it is possible that in response to demand increases, one attribute of the good could increase while the other decrease. However, in general, price and delivery delays are positively correlated (Zarnowitz 1973, p. 315) making Assumption 1 quite a reasonable one. Assumption 1 together with (10) and (11) implies that A, B, and D are all of the same sign. The following lemma establishes that the sign of A, B and D is positive.

Lemma 2. The second order condition that a consumer is at an optimum [see equation (3)] in conjunction with Assumption (1) and (10) and (11) establishes that A, B, and D are all positive.

Proof: The second order condition for the consumer is

$$\frac{d}{dk}\left[\frac{-V_k}{V_p} + \frac{\pi_k}{\pi_p}\right] < 0, \quad \text{or}$$

(13)
$$\frac{-V_{kk}}{V_p} + \frac{V_k}{V_p^2} V_{pk} \frac{\pi_{kk}}{\pi_p} - \frac{\pi_k \pi_{pk}}{\pi_p^2} + \frac{dp}{dk}\left[\frac{-V_{kp}}{V_p} + \frac{V_k}{V_p^2} V_{pk} + \frac{\pi_{kp}}{\pi_p} - \frac{\pi_k \pi_{pp}}{\pi_p^2}\right] < 0,$$

or since from (2) and (4), $\frac{\pi_p}{\pi_k} = \frac{V_p}{V_k}$ and $\frac{dp}{dk} = \frac{-V_k}{V_p}$, we have that (13) can be written as

$$\frac{-V_k}{kV_p}\frac{-V_{kk}}{-V_k} k + \frac{V_k}{kV_p}\frac{-V_{pk}}{-V_p} k + \frac{\pi_k}{k\pi_p}\frac{\pi_{kk}}{\pi_k} k) - \frac{\pi_k}{k\pi_p}\frac{\pi_{pk}}{\pi_p} k +$$

$$+ \frac{\pi_k}{\pi_p}\frac{1}{k} + \left(\frac{V_{kp}}{V_p} k\right) - \left(\frac{\pi_k}{\pi_p}\right)^2 \frac{1}{p}\left(\frac{V_{pp}}{V_p} k\right) - \frac{\pi_k}{\pi_{pk}}\left(\frac{\pi_{kp}}{\pi_p} k\right) + \frac{\pi_k^2}{\pi_p^2 p}\left(\frac{\pi_{pp}}{\pi_p} p\right) < 0$$

or

$$\frac{-V_k}{kV_p}\left[\theta_k - \theta_k^s\right] + \frac{V_k}{kV_p}\left[n_k - n_k^s\right] + \frac{\pi_k}{k\pi_p}\left[n_k - n_k^s\right] - \frac{\pi_k^2}{\pi_p^2 p}\left[n_p - n_p^s\right] < 0,$$

or

$$\frac{-\pi_k}{k\pi_p}\left[\left(\theta_k - \theta_k^s\right) - \left(n_k - n_k^s\right)\right] - \frac{\pi_k^2}{p\pi_p^2}\left[\left(n_p - n_p^s\right) - \frac{p\pi_p}{k\pi_k}\left(n_k - n_k^s\right)\right] < 0,$$

or

$$\frac{-\pi_k}{k\pi_p} A - \left(\frac{\pi_k}{\pi_p}\right)^2 \frac{1}{p} B < 0,$$

or

$$\alpha_1 A + \alpha_2 B > 0, \quad \text{where } \alpha_1, \alpha_2 > 0.$$

If A and B are of the same sign (Assumption 1), then they must both be positive, which from (10) and (11) and Assumption 1, implies that D is positive. Q.E.D.[6]

Equation (10)-(12) relate the equilibrium price and delivery lag fluctuations to various elasticities of supply and demand. We now investigate how the underlying elasticities influence the equilibrium fluctuations in price and delivery lag.[7] First, observe that in the expression

for D, all quantities except $\theta_k - \theta_k^s$ are signed. By Lemma 2 , $D > 0$. If D is to be positive for all possible equilibrium values of n_k, n_k^s, n_p, n_p^s across all industries, then $\theta_k - \theta_k^s$ must be positive. If $\theta_k - \theta_k^s$ is positive, then from (10) $\frac{\partial D}{\partial |n_p|} > 0$. Moreover, it follows from (8) and (9) that $\frac{\partial A}{\partial |n_p|} = 0$, and $\frac{\partial B}{\partial |n_p|} < 0$, therefore we have shown

<u>Proposition 1</u>: As the demand curve (or supply curve) becomes more price elastic and under Assumption 1 and the further implication of Assumption 1 that $\theta_k - \theta_k^s > 0$, the equilibrium fluctuations in p and k in response to short run shocks in supply and demand fall. $\left[\underline{\text{i.e.}}\ \frac{\partial}{\partial |n_p|} \left|\frac{d\ln p}{d\ln N}\right| \text{ and } \frac{\partial}{\partial |np|} \left|\frac{d\ln k}{d\ln N}\right| < 0\right]$

It is interesting to note that for any particular industry, D could be positive, yet $\theta_k - \theta_k^s$ could be negative. In such a case, the equilibrium price fluctuations increase as demand becomes more price elastic.

As the delivery lag elasticity of demand increases, we see that D unambiguously increases, but both A and B also increase. It is possible to prove using the assumptions above that

Proposition 2: As the delivery lag elasticity of demand becomes a larger negative number, and under the same assumptions as Proposition 1, the equilibrium price fluctuation falls [i.e., $\frac{\partial}{\partial |n_k|} \frac{d\ \ln p}{d\ \ln N} < 0$] and the equilibrium delivery lag fluctuation falls [i.e., $\frac{\partial}{\partial |n_k|} \frac{d\ \ln k}{d\ \ln N} < 0$].

Proof: See Appendix B.[8]

Propositions 1 and 2 accord well with intuition gained from markets where quality is exogenous and price alone clears the market. In those markets too, as demand or supply becomes more price elastic, the equilibrium fluctuations in price in response to supply and demand shifts diminishes. Intuition (mine at least) would also suggest that the relative price fluctuations versus relative delivery lag fluctuations would depend negatively on the absolute value of price elasticity of demand and positively on the absolute value of the delivery lag elasticity of demand. The reasoning behind this intuition is straightforward. If the demand curve is very price elastic, holding delivery time fixed, and very delivery lag inelastic, holding price fixed, one would expect price not to change much but delivery lag to change a lot in response to shifts in supply and demand. In fact, this intuition is incorrect, as shown in Proposition 3 below.

Before proving Proposition 3, it will be useful to explain why the intuition, just discussed, is incorrect. The basic reason is that elasticities of supply and demand do not determine the trade-offs consumers and firms are willing to make between price and delivery lag, and these trade-offs play an important role in determining equilibrium.

The distinguishing feature of markets with endogenous quality attributes is that the relative p-k fluctuations are determined not only by price and delivery lag elasticities of supply and demand, but by the relative shapes of indifference curves and iso-profit curves in (p, k) space. The trade-offs that consumers and producers are willing to make between p and k are not in one to one correspondence with demand and supply elasticities. For example, let $V(p, k) = k^{-1} - \ln p$, then the demand curve for a good of delivery lag k is $+ \frac{1}{p}$ and is independent of k. Knowledge of a demand curve is not sufficient to determine the shape of indifference curves which express the trade offs consumers are willing to make between p and k. The relative equilibrium variation of p and k to demand and supply shocks will depend on the shapes of the indifference and iso-profit curves. Surprisingly, the elasticities interact with the relative shapes of the indifference and iso-profit curves in such a way as to contradict one's (my) simple intuition about the determinants of the ratio of relative price and relative delivery lag changes.

Proposition 3. The ratio of relative price to relative delivery lag fluctuation depends positively on the absolute value of price elasticity and negatively on the absolute value of delivery lag demand elasticities.[9]

Proof: From (12), we know that

$$R \equiv \frac{\dfrac{d\ \ln p}{d\ \ln N}}{\dfrac{d\ \ln k}{d\ \ln N}} = \frac{A}{B} = \frac{(\theta_k - \theta_k^s)-(n_k - n_k^s)}{(n_p - n_p^s) - \dfrac{p\pi_p}{k\pi_k}(n_k - n_k^s)}.$$

Clearly,

$$\frac{\partial R}{\partial\ n_p} < 0 \quad \text{or} \quad \frac{\partial R}{\partial |n_p|} > 0.$$

Also,

$$\frac{\partial A/B}{\partial n_k} = \frac{-1}{B} + \frac{A\dfrac{p\pi_p}{k\pi_k}}{B^2}$$

$$= \frac{-(n_p - n_p^s) + \dfrac{p\pi_p}{k\pi_k}(n_k - n_k^s) + \dfrac{p\pi_p}{k\pi_k}(\theta_k - \theta_k^s) - \dfrac{p\pi_p}{k\pi_k}(n_k - n_k^s)}{B^2}$$

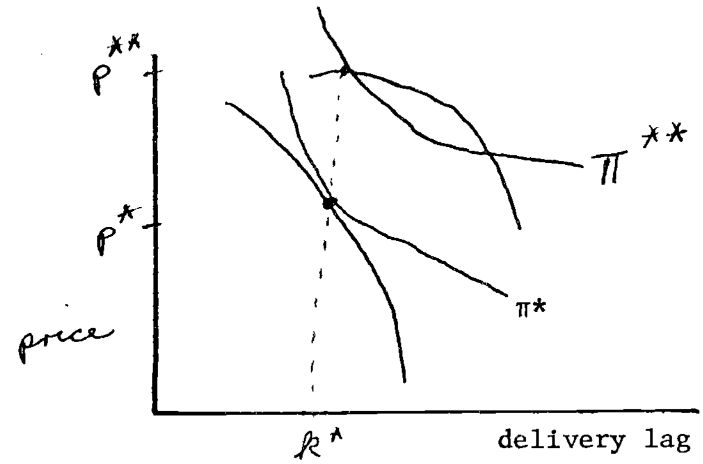
$$= \frac{-(n_p - n_p^s) + \dfrac{p\pi_p}{k\pi_k}(\theta_k - \theta_k^s)}{B^2}$$

$$> 0,$$

provided $\theta_k - \theta_k^s > 0$ as argued earlier.[10] Therefore, $\dfrac{\partial\ A/B}{\partial |n_k|} < 0$. Q.E.D.

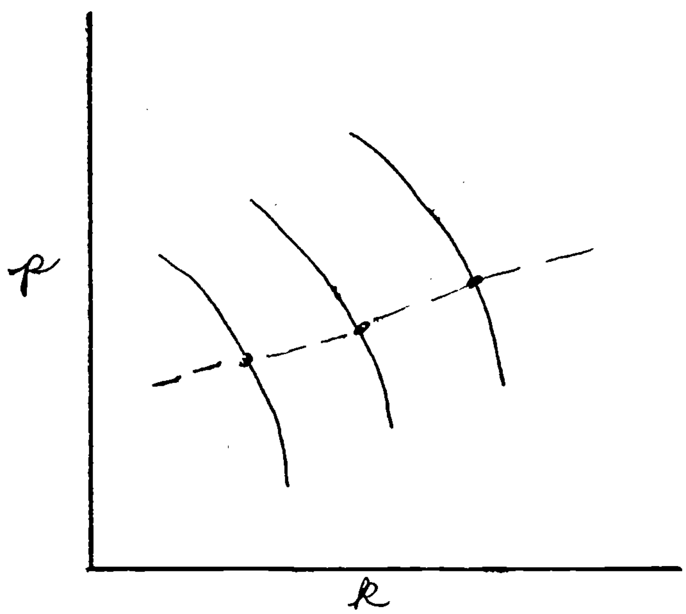
Let me now attempt to heuristically explain the puzzling result in Proposition 3 that R (ratio of relative price to relative delivery lag fluctuations) rises as $|n_p|$ increases (i.e., as the demand curve becomes more price elastic). Suppose that initially there is an equilibrium at p^*, k^* as illustrated in the diagram below.

Fig. 2

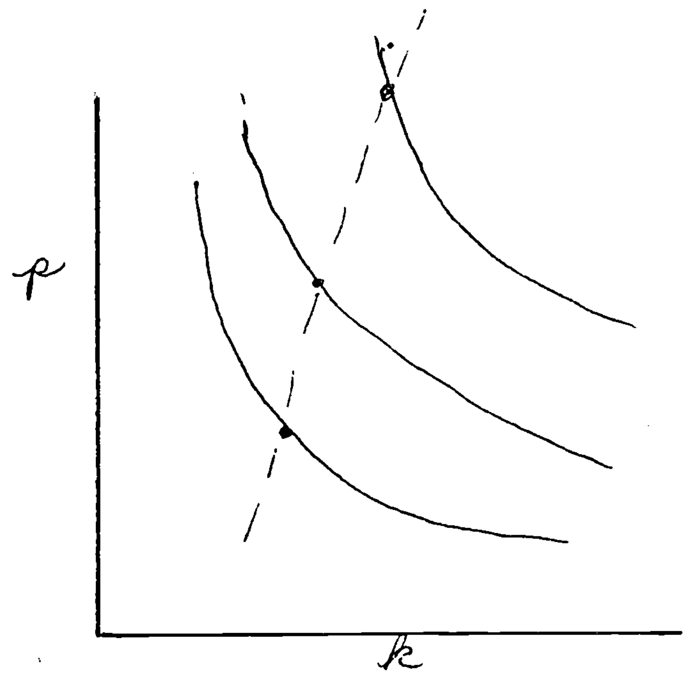


Let demand (N) increase, so that now the new equilibrium is on a higher isoprofit curve, and lower indifference curve. Let π^{**} be the new equilibrium profit level. Hold k at k^*, and raise p until it hits the $\pi(p, k) = \pi^{**}$ curve at $p = p^{**}$. The distance in p space $p^{**}-p^*$, will be the maximum that the equilibrium p can move (otherwise k would have to fall from k^* and that would violate Assumption 1). The amount of price decline from p^{**} and delivery lag increase beyond k^* that will occur in the new equilibrium will be positively related to the discrepancy in the slopes of the isoprofit and indifference curves at (p^{**}, k^*). It can be shown that this discrepancy in slope decreases as $|n_p|$ increases (or as $|nk|$ decreases). In other words, increases in $|n_p|$ (decreases in $|n_k|$) affect the curvature of indifference surfaces so as to favor price over delivery lag fluctuations. A numerical example in Appendix C illustrates the counterintuitive result $\frac{\partial R}{\partial |n_p|} > 0$ of Proposition 4, even when $n_k - n_k^s$, $\theta_k - \theta_k^s$ are not held constant.

There is no reason in Figures 1 and 2 why the curves must have the particular shapes drawn. All that is required is that consumers are maximizing utility and producers profits. As demand increases it seems reasonable to expect that in comparing the new equilibrium to the old one, that the p-k trade off is steeper in the new equilibrium. The more that is produced, the more valuable is it to delay delivery. One can see immediately then (as Zarnowitz 1973 also pointed out earlier) that whether the indifference curves are concave or convex will have a lot to do with relative price versus delivery lag fluctuations. The diagrams below suggest how price fluctuations are likely to be less important relative to delivery lag fluctuations when the indifference curve is concave (see Appendix A for a derivation of the conditions guaranteeing concave indifference curves).



line through equilibrium points tends to be flat



line through equilibrium points tends to be steep

III. Empirical Testing

The theory just presented illustrated how important it is to take delivery lags into account when delivery lags influence either consumer welfare or firm profits. Ideally, one would like to have sufficiently detailed data to enable estimation of all the parameters in (6). Such an effort would require data at each point in time on the price that would be charged for

<u>different</u> delivery lags. Unfortunately such data are unavailable. All that are available are census data from which aggregate quantities and average delivery lags can be calculated. It is this data that we use. (Had the more refined data been available, the theory would have had to be modified to allow for different consumers consuming at different delivery lags.) To establish the point that delivery lags matter in the determination of equilibrium it suffices to show that either the structural supply or demand equations have non-zero delivery lag elasticities.[11] I present results below only for structural demand equations. (I chose to estimate demand and not supply curves because it is much easier to find good instruments for the demand curves.) After presenting the estimates, I briefly discuss whether any of the implications of the previous section, especially those of Propositions 1-3, seem to be consistent with the data.

III.1. Data Description

Data were taken from census and BLS sources. Data are usually available monthly and non-seasonally adjusted. The price series, kindly provided me by John Geweke, are quarterly and were constructed by Al-Samarrie, Kraft, and Roberts (1977), from BLS sources in an effort to regroup BLS data along Census SIC code definitions. This quarterly price series was transformed into a monthly series by assuming that prices during each quarter are unchanged. The delivery lag variable was constructed as follows. At any given time, there is a stock of unfilled orders and new orders. By asking how long would it have taken for the first new order to be filled[12] (using data on subsequent shipment rates) we can calculate a delivery lag, k1. By asking how long it would have taken to fill the last new order (using data on subsequent shipment rates) we can calculate a delivery lag, k2. We used both k1 and k2 in the estimation to make sure that the results were not sensitive to the definition of delivery lag.

We investigated all those 2 digit SIC code industries (SIC 331, 34, 35, 36) for which delivery lags seemed important and two industries (SIC 22 and 26) for which delivery lags seemed relatively unimportant. The time period is 1958-1972. Table 1 illustrates the average level of delivery lags, prices, and standard deviations in delivery lags and prices for each industry studied. The second and fifth columns of Table 1 convert the means of logs into unlogged values. If lnp and lnk are normally distributed then the unlogged values in the second and fifth columns can be interpreted as medians.

Table 1 shows that the industries vary considerably in the variability and level of delivery lag and price. For example, SIC 26 has a delivery delay of only .46 months, while SIC 36 typically has a delivery delay of 3.9 months. The measure of variability of delivery goes from a low of .08 for SIC 26 to a high of .25 for SIC 331 and SIC 35. However, for all the industries the fluctuations in delivery lags exceeds the fluctuations in price.

TABLE 1

SIC Code	Industry	Avg. ln p	Median p	St. Dev. ln p	Avg. ln kl	Median kl	St. Dev. of ln kl
22	Textile Mill Products	.03	1.03	.06	.23	1.26	.17
26	Paper and Allied Products	.22	1.25	.05	-.78	.46	.08
331	Steel	.02	1.02	.03	.67	1.95	.25
34	Fabricated Metals	.00	1.00	.03	1.12	3.06	.18
35	Non-electrical Machinery	-.09	.91	.04	1.29	3.63	.25
36	Electrical Machinery	-.15	.86	.05	1.35	3.86	.10

Note: p = real price index
kl = delivery lag (months).

III.2. Empirical Results

The demand curve for each industry is estimated by two stage least squares with a correction for serial correlation (i.e., Fair's method) for the period 1958-1972. The dependent variable is quantity demanded, measured as net new orders divided by price. Price and delivery delay are regarded as endogenous in accordance with the theory just presented. (See the Steuer, Ball and Eaton (1966) study of machine tools for results that ignore the simultaneity of price and delivery delay.) We postulate that the quantity demanded depends negatively on real price[13] and delivery lag and positively on the exogenous demand indicator, the FRB real index of manufacturing production.[14] We also include a measure of the current stock[15] of output to capture the fact that a large stock of output will exert a negative influence on the amount of net new orders since the demand for the good can be satisfied by existing goods. Instrumental variables included wages, interest rates, BLS cost indices, and in industry cost index·compiled by Al-Samarrie, Kraft and Roberts (1977). All variables except time are in logs. Table 2 lists the endogenous, exogenous and instrumental variables used. It is worth pointing out that the estimation procedure will yield consistent results even if the price and delivery lag variables are measured with error. Table 3 below

TABLE 2- Variables Used

	Definition and Source
Endogenous	
Quantity Demanded	Dependent variable constructed by dividing Net New Orders (in Current Industrial Reports M3-1.7, "Manufacturers, Shipments, Inventories, and Orders: 1958-1977," U.S. Dept. of Commerce, Bureau of Census) by output price.
Output Price	Constructed by Al-Samarrie, Kraft and Roberts (1977) from BLS data for each SIC code used in this study
Delivery Lag	Constructed for each SIC code from data on new orders, unfilled orders, shipments in Current Industrial Reports M3-1.7 ""Manufacturers' Shipments, Inventories, and Orders: 1958-1977," U.S. Department of Commerce, Bureau of Census (see text for construction technique)
Exogenous	
FRB	Federal Reserve Board Production Index from Federal Researve Board Publication G12.3 Business Indexes
WPI	Wholesale price index for industrial commodities; BLS. Used to deflate all price variables, including wages.
Instruments	
p1	BLS price index of intermediate materials
p2	BLS price index of crude materials (less certain foodstuffs)
p3	BLS price index of machinery and equipment*
Input Prices	Index of input costs for each SIC code constructed by Al-Samarrie, Kraft, and Roberts (1977) using BLS data and the 1958 input-output table
w	Average hourly earnings in the SIC code, from Employment and Earnings
r	Nominal interest rate on 90 day treasury bills, publication G.14 of Federal Reserve, U.S. Government Security Yields and Prices

*Results were estimated with and without the instrument p3 for SIC's 35 and 36 with little difference in results.

presents estimates of structural demand equations for two different equation specifications. As discussed earlier, each equation was estimated with two different definitions of delivery lags. Except where indicated, the main qualitative results are fairly insensitive to choice of delivery lag definition and so only equations with delivery lag k1 are reported.

In general, the results of the estimation appear quite good. The magnitudes and signs of most of the coefficients seem plausible and most of the coefficients are highly statistically significant.[16] The estimated results are least good for those industries with the shortest delivery lags (SIC 22 and 26) and best for those industries with the longest delivery lags (SIC 34, 35 and 36). None of the coefficients in Table 3 with the incorrect signs are statistically significant at the 5% level. The income elasticity ranges between 1 and 2, with the exception of the statistically insignificant and negative income elasticity for SIC 331. The price

TABLE 3

	Demand Equations							
Industry	Delivery Lag	Price	FRB Index of Demand	Capital Stock	Time	Constant	ρ	SEE
SIC 22*	-.16	-.42	1.06	-		7.50		
	(-1.66)	(-1.55)	(11.82)			(236.97)	.41	.07
	.16	-6.88	2.01	-18.15	.05	210.3		
	(.93)	(-3.37)	(6.67)	(-.257)	(2.08)	(2.69)	.97	.08
SIC 26*,**	-.06	-1.54	.77			7.46	.15	.03
	(-1.28)	(-10.24)	(32.18)			(152.91)		
	-.40	-1.37	1.54	-.71	-.002	14.21	.34	.04
	(-3.66)	(-7.86)	(10.74)	(-3.43)	(-.92)	(7.36)		
SIC 331	-.53	.26	.30			7.86	.83	.14
	(-2.62)	(.14)	(1.30)			(50.71)		
	-.78	-14.36	-.93	-15.49	.05	185.2	.91	.16
	(-2.96)	(-2.76)	(-.169)	(3.47)	(3.69)	(3.61)		
SIC 34	-.22	- 3.09	1.47			8.41	.35	.06
	(-2.74)	(- 7.39)	(18.31)			(84.78)		
	-.30	- 1.75	1.78	.54	-.004	2.38	.32	.06
	(-3.56)	(- 1.75)	(10.27)	(1.67)	(2.68)	(.63)		
SIC 35	-.53	- 1.32	1.84			9.05	.66	.07
	(-3.50)	(-2.96)	(11. 1)			(42.44)		
	-.35	- 3.5	1.86	-2.15	.007	34.56	.49	.06
	(-3.45)	(-5.39)	(11.61)	(-4.12)	(3.11)	(5.56)		
SIC 36	-.48	- .05	1.48			8.99	.17	.09
	(-4.14)	(.07)	(9.14)			(35.16)		
	-.64	-1.60	1.58	1.36	-.009	-6.79	.11	.09
	(-3.34)	(-2.15)	(9.37)	(1.89)	(-2.41)	(- .82)		

Notes: Variable definitions: see Table 2 for more detail
ρ = serial correlation coefficient
SEE = standard error of estimate of transformed regression
t ratios beneath each coefficient.

*The coefficient of delivery lag was sensitive to whether k1 or k2 was used as delivery lag variable.

**The coefficients of this equation are not sensitive to the definition of capital stock. See fn. 14.

elasticity is usually above 1, while the delivery lag elasticity is always below 1. The delivery lag elasticity is, with one exception (SIC 22), always negative and statistically significant at the 5% level for the expanded equation specification. The results of Table 3 provide strong support for the view that delivery lags in addition to price are an important influence on market behavior.

Given that delivery lag elasticities are important in influencing market behavior, can we see if any of the implications of the analysis of Section II are borne out? With only six industries that differ greatly amongst each other, it is very hard to think of being able to reliably test the implications of the Propositions of the previous section.

One straightforward implication of the previous section is that the fluctuations in p and k will depend on the variability of demand. From the last two columns in Table 3[17] we see that SIC 331 has the largest variability of demand and SIC 26 the lowest. Table 1 confirms that by any reasonable criteria the combined price and delivery lag movement in SIC 331 is one of the largest of all the other industries while the combined movement for SIC 26 is one of the smallest.

Proposition 3 related R, the ratio of price variability to delivery lag variability, to the price and delivery lag elasticities. Is there any evidence of this pattern as we compare the results of Tables 1 and 3 across industries? (It is wise to reemphasize the caveat that with only six industries with widely varying characteristics, such an across-industry comparison is not the correct way to test Proposition 3 .) Using disaggregate data to estimate the parameters of (6) would be the correct way.) The answer is that it is hard to tell, but probably no. If one computes the value of R and compares it to the ratio of price to delivery lag elasticities, then it appears that there is a negative and not a positive correlation as predicted by Proposition 3. However, proper tests of Proposition 3 will have to await the use of more disaggregate data.

V. Summary

This paper has investigated how markets will reach equilibrium when both price and quality of the good are endogenous. It predicts by how much delivery lags and price will change in response to short run supply and demand shocks. The complications of the theory as well as some surprising results flowing from the theory illustrate how some intuitions based on simple supply equal demand models can be misleading in trying to understand price behavior. Understanding price fluctuations is a very complicated task. The empirical testing of the theory strongly supported the view that delivery lags do play an important role in influencing market demand and therefore must be taken into account when one tries to understand how markets will respond to short run supply or demand fluctuations. The data was not sufficiently rich to allow adequate testing of some specific implications of the theory, though very crude across-industry comparisons failed to reveal supporting evidence for some of these specific implications. Given the importance of delivery lags in a theory attempting to explain price movements and given the widespread use of delivery lags in U.S. industry and their important empirical influence on demand, further development and testing of the theory with better data definitely seem warranted.

APPENDIX A

Let V(p, k, y) be the indirect utility function where k is delivery lag, p price and y income. If V_y is a constant and equal to 1, then indifference curves in (p, k) space are concave, provided $\theta_k > 2\,n_k \;\frac{-kV_k}{pV_p}\, n_p$.

<u>Proof</u>: Along an indifference surface

$$\frac{dp}{dk} = \frac{-V_k}{V_p}, \quad \text{and}$$

$$\frac{d^2p}{dk^2} = \frac{-V_{kk}}{V_p} + \frac{V_k V_{pk}}{V_p^2} - \frac{V_k}{V_p}\left[\frac{-V_{kp}}{V_p} + \frac{V_k}{V_p^2} V_{pp}\right]$$

$$= -\theta_k \frac{V_k}{kV_p} + \frac{V_k}{kV_p} n_k^D + \frac{V_k}{V_p^2}\left[V_{pk} - \frac{V_k}{V_p} V_{pp}\right]$$

$$= (-\theta_k + n_k)\frac{V_k}{kV_p} + \frac{V_k}{kV_p}\left[n^k - \frac{kV_k}{PV_p} n_p\right]$$

$$= \frac{V_k}{kV_p}\left[-\theta_k + n_k + n^k - \frac{kV_k}{pV_p} n_p\right].$$

The condition stated above on θ_k follows immediately. Q.E.D.

Notice that if Q = quantity demanded, then

$$\frac{dQ}{dk} = -V_{pk} + \frac{V_k}{V_p} V_{pp} = \frac{-V_p}{k} n^k + \frac{V_k}{p} n_p$$

$$= \frac{-pV_p}{kV_k} n^k + n_p = \frac{-pV_p}{kV_k}\,[n_k - \frac{kV_k}{pV_p} n_p]$$

so that if quantity demanded falls as p rise (k falls) along an indifference curve, i.e., $\frac{dQ}{dk} > 0$, the indifference curves are more likely to be concave.

APPENDIX B

Proof of Proposition 2

From (10), we know that dlnp/dlnN = A/D, and from (7(-(9) we know that

$$\frac{A}{D} = \frac{(\theta_k - \theta_k^s) - [n_k - n_k^s]}{\frac{p\pi_p}{k\pi_k}(n_k - n_k^s)^2 - (\theta_k - \theta_k^s)(n_p - n_p^s)} .$$

Therefore,

$$\frac{\partial A/D}{\partial n_k} = \frac{-1}{\frac{p\pi_p}{k\pi_k}(n_k - n_k^s)^2 - (\theta_k - \theta_k^s)(n_p - n_p^s)} - \frac{[(\theta_k - \theta_k^s) - (n_k - n_k^s)]\ \frac{2p\pi_p}{k\pi_k}(n_k - \ {}^s_k)}{\left[\frac{p\pi_p}{k\pi_k}(n_k - n_k^s)^2 - (\theta_k - \theta_k^s)(n_p - n_p^s)\right]}$$

$$= \frac{\frac{-p\pi_p}{k\pi_k}(n_k - n_k^s)^2 + (\theta_k - \theta_k^s)(n_p - n_p^s) - \frac{2p\pi_p}{k\pi_k}(\theta_k - \theta_k^s)(n_k - n_k^s) + \frac{2p\pi_p}{k\pi_k}(n_k - n_k^s)^2}{D^2}$$

$$= \frac{\frac{+p\pi_p}{k\pi_k}(n_k - n_k^s)^2 - \frac{p\pi_p}{k\pi_k}(\theta_k - \theta_k^s)(n_k - n_k^s) + (\theta_k - \theta_k^s)[(n_p - n_p^s) - \frac{p\pi_p}{k\pi_k}(n_k - n_k^s)]}{D^2}$$

$$= \frac{\frac{p\pi_p}{k\pi_k}(-)(n_k - n_k^s)[\theta_k - \theta_k^s) - (n_k - n_k^s)] + (\theta_k - \theta_k^s)[(n_p - n_p^s) - \frac{p\pi_p}{k\pi_k}(n_k - n_k^s)]}{D^2} .$$

Let

$$\alpha_3 = \frac{p\pi_p}{k\pi_k} (-)(n_k-n_k^s)/D^2 > 0$$

$$\alpha_4 = (\theta_k-\theta_k^s)/D^2 > 0, \quad \text{then}$$

$$\frac{\partial A/D}{\partial\, n_k} = \alpha_1 A + \alpha_2 B > 0, \quad \text{so } \frac{\partial A/D}{\partial |n_k|} < 0.$$

From (11), we know that $\frac{d\ln k}{d\ln N} = B/D$, and from (7)-(9) that

$$\frac{B}{D} = \frac{[n_p-n_p^s] - \frac{p\pi_p}{k\pi_k}[n_k-n_k^s]}{\frac{p\pi_p}{k\pi_k}(n_k-n_k^s)^2 - (\theta_k-\theta_k^s)(n_p-n_p^s)}.$$

Therefore,

$$\frac{\partial B/D}{\partial n_k} = \frac{\frac{-p\;\pi_p}{k\;\pi_k}}{\frac{p\pi_p}{k\pi_k}(n_k-n_k)^2-(\theta_k-\theta_k^s)(n_p-n_p^s)} - \frac{[(n_p-n_p^s) - \frac{p\pi_p}{k\pi_k}(n_k-n_k^s)]^2\,\frac{p\pi_p}{k\pi_k}(n_k-n_k^s)}{\frac{p\pi_p}{k\pi_k}(n_k-n_k^s)^2-(\theta_k-\theta_k^s)(n_p-n_p^s)]^2}$$

$$= \frac{p\pi_p}{k\pi_k}\left[\frac{\frac{-p\pi_p}{k\pi_k}[n_k-n_k^s]^2 + (\theta_k-\theta_k^s)(n_p-n_p^s) - (n_p-n_p^s)(n_k-n_k^s)^2 + \frac{p\pi_p}{k\pi_k}(n_k-n_k^s)^2{}_2}{D^2}\right]$$

$$= \frac{p\pi_p}{k\pi_k}\frac{1}{D^2}\left[-(n_k-n_k^s)[(n_p-n_p^s) - \frac{p\pi_p}{k\pi_k}(n_k-n_k^s)] - [n_p-n_p^s][(\theta_k-\theta_k^s)-[n_k-n_k^s]]\right].$$

Let $\alpha_5 = \frac{p\pi_p}{k\pi_k}\frac{1}{D^2} - [n_k-n_k^s] > 0$ and $\alpha_6 = \frac{p\pi_p}{k\pi_k}\frac{1}{D^2}(-1)[n_p-n_p^s] > 0$, then

$$\frac{\partial B/D}{\partial n_k} = \alpha_5\; B + \alpha_6\; A > 0, \quad \text{so}$$

$$\frac{\partial B/D}{\partial |n_k|} < 0.$$

indifference and one isoprofit curve, B = 0. It then follows from (11) that under Zarnowitz's assumptions to a first approximation, all equilibration takes place through p.

[7]Because it is possible to change V and π so as to change independently the elasticities, it makes sense to see how the comparative statics of the system changes as a function of these elasticities. In the discussion below, we present proofs only for demand elasticities. Proofs for supply elasticities are straightforward adaptations of those proofs.

[8]If we drop the assumption that $\theta_k - \theta_k^s > 0$, then it is still true that $\frac{\partial}{\partial|n_k|} \frac{d\ln k}{d\ln N} < 0$ but it is no longer true that $\frac{\partial}{\partial|n_k|} \frac{d\ln p}{d\ln N} < 0$.

[9]The reader may wonder why the symmetry present in (3) and (4) doesn't guarantee symmetry in Proposition 4. The reason is that n_p and n_k are not symmetric expressions, n_p and θ_k are.

[10]Again, we note that if we dropped the assumption that $\theta_k - \theta_k^s > 0$, then it would be possible that the ratio of relative price to relative delivery lag variability increases as $|n_k|$ increases.

[11]This is a sufficient condition. Both supply (π_p) and demand ($-N\ V_p$) could be independent of k, yet k could still matter (i.e., π_k and $V_k \neq 0$) in the determination of equilibrium.

[12]Value of shipments, new orders and unfilled orders are converted into number of units by dividing by price.

[13]Note that with constant after tax real rates and constant depreciation, an equation with ln p will produce the same elasticity estimates as one with ln (user cost).

[14]Attempts to include variables measuring the deviations between current values and expected values for price and delivery lags were unsuccessful.

[15]The capital stock was constructed using the depreciation rate of .1428, the figure reported in Hall and Jorgenson (1967). The initial value of the capital stock was calculated by assuming that 1957 was a year in which the capital stock was in equilibrium. SIC 22 was handled by a different method because of the obviously more rapid depreciation of non-durable goods. There we used the amount shipped during the past year as a measure of the size of the existing capital stock.

[16]The standard errors are conditional on the estimated ρ.

[17]The variability in demand is $\frac{1}{1-\rho^2}\sigma^2$ where σ is the figure reported in the last column of Table 3 while ρ is reported in the next to last column.

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