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THE FORECASTING ABILITY OF CORRELATIONS IMPLIED IN FOREIGN EXCHANGE OPTIONS

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ABSTRACT

This paper evaluates the forecasting accuracy of correlation derived from implied volatilities in dollar-mark, dollar-yen, and mark-yen options from January 1989 to May 1995. As a forecast of realized correlation between the dollar-mark and dollar-yen, implied correlation is compared against three alternative forecasts based on time series data: historical correlation, RiskMetrics' exponentially weighted moving average correlation, and correlation estimated using a bivariate GARCH (1,1) model. At the one-month and three-month forecast horizons, we find that implied correlation outperforms, often significantly, these alternative forecasts. In combinations, implied correlation always incrementally improves the performance of other forecasts, but not the converse; in certain cases historically based forecasts contribute no incremental information to implied forecasts. The superiority of the implied correlation forecast holds even when forecast errors are weighted by realized variances, reflecting correlation's contribution to the dollar variance of a multicurrency portfolio.

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I. Introduction

Options, as forward-looking financial indicators, have gained increasing attention in recent years as informative predictors of future asset price behavior. Several recent papers have documented that implied volatility contains information not found in the underlying time series that is useful for predicting future volatility [Day and Lewis (1992); Scott (1992); Kroner, Kneafsey, and Claessens (1993); Lamoureux and Lastrapes (1993); Jorion (1995)]. In a different approach to characterizing future asset price behavior, resent research [Shimko (1993), Rubinstein (1994), Abken (1995), Lo and Ait-Sahalia (1995), Jackwerth and Rubinstein (1996), Malz (1996a), McCauley and Melick (1996a,b), Mizrach (1996), Melick and Thomas (1997)] has used option prices to derive the implied risk-neutral density function of the underlying variable, based on the methods of Breeden and Litzenberger (1978). Other papers have shown that options anticipated the 1987 stock market crash (Bates 1991) or the 1992 ERM crisis [Campa and Chang (1996), Malz (1996b), Mizrach (1996)]---often better than did time series data or, in the case of the ERM, interest rate differentials. In brief, the recent literature has exhibited an increased appreciation as well as greater exploitation of the information content of options.

Foreign currency options in particular provide an especially rich, if not unique, context in which to examine options' information content in that they can potentially predict not only volatility but also *correlation*. Unlike other commodities, the underlying assets (foreign currencies) can trade directly against each other, permitting an "implied correlation" to be computed from the implied volatilities of options on three currency-pairs. For example, the implied volatilities of dollar-mark, dollar-yen, and mark-yen options uniquely identify an implied correlation between the dollar-mark and dollar-yen exchange rates.[1]

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[1] Technically, implied correlations also exist in various "quanto" options in which the underlying security is quoted in one currency while the payoff is made in another, as in the case of the Nikkei options traded in Chicago. The liquidity of such markets is much smaller than that of the mark-yen

The purpose of this paper is to provide a further assessment of the information content of options by evaluating the predictive power of implied correlations in forecasting realized correlation. We compare this forecast against three alternative methods of forecasting correlation based on time series data: 1) recent historical correlation; 2) an exponentially-weighted moving average of recent historical correlation, as used in JP Morgan's RiskMetrics™; and 3) correlations derived from a bivariate GARCH(1,1) model. We evaluate the predictive accuracy of these alternative forecasts by comparing their root-mean-squared prediction errors, their prediction bias, and their statistical significance in regressions of realized correlation against forecasts. We perform Wald tests to ascertain whether these time-series-based correlation forecasts contribute incremental information to implied correlation, and vice versa. We also examine other properties of correlation such as the possible comovements between correlation and volatility. Our tests are applied to the correlation between the dollar-mark and dollar-yen exchange rates from January 1989 through May 1995, nearly six and a half years, representing approximately 1600 daily observations.

As an assessment of the predictive power found in option prices, this paper is closely related to the recent literature on the relative ability of implied and historical (either simple or GARCH-based) volatility in predicting future realized volatility. In foreign exchange options, Scott (1992) finds that a combination of implied volatility and historical volatility is most effective in the prediction of realized volatility. Again in foreign exchange, Jorion (1995) finds that implied volatility significantly outperforms historical volatility or GARCH-based volatility estimates, which incrementally provide virtually no information. For individual stocks, early studies by Chiras and Manaster (1978) and Beckers (1981) indicated that implied volatilities forecast better than historical volatilities. More recently, Day and Lewis (1992) find that within sample, both implied volatilities and GARCH-based estimates of conditional volatility provide information incremental to the other forecast of realized volatility, but out-of-sample,

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options market.

neither forecast dominates in terms of superior information content. Lamoureux and Lastrapes (1993) find that a combination of implied and historical volatility is informative in forecasting future volatility, with GARCH-based estimates providing no incremental information. In contrast, Canina and Figlewski (1993) find implied volatility in S&P 100 index options to have zero predictive ability and historical volatility to have some predictive ability. For commodities, Kroner, Kneafsey, and Claessens (1993) find that forecasts combining implied volatility and GARCH-based estimates tend to perform best.

Prospects for successfully forecasting correlation should in principle be somewhat better than those for forecasting volatility, as correlation is generally believed to be more stable. Using international equity returns from 1967 to 1982, Kaplanis (1988) finds correlation to be stable over time while conditional variances are unstable. Although Longin and Solnik (1995) reject the stability of the correlation matrix on 8 international equity markets from 1960 to 1990, they find correlations to be more stable than variances.

In an applied context, correlation is important in areas such as asset management, risk management, and the pricing and hedging of certain derivative instruments dependent on correlation.[2] Accurate prediction of correlation of key international financial variables---such as interest rates, exchange rates, equity markets, commodity prices---clearly would have important benefits for portfolio managers, risk managers, and financial regulators alike. Recent academic work on correlation, dealing mainly with equity markets, has focused on whether correlation is constant over time (Kaplanis (1988), Longin and Solnik (1995)) or linked to the business cycle (Erb, Harvey, and Viskanta (1995)). In general, however, relative to the extensive literature on the predictability of returns and volatility, comparatively little has

[2] Derivative instruments dependent on correlation include "best-of-two" asset options, outperformance options, basket options, quanto options, swaptions, quanto swaps, diff swaps. Correlation enters directly ("first order effects") into the option payoffs in spread options, outperformance options, yield curve options, cross-currency caps. Correlation results in the modification of payoffs ("second order effects") in diff swaps and quanto options.

been written on the predictability of correlation.

To our knowledge, the only other paper analyzing correlation using implied correlation in options is Bodurtha and Shen (1995). These authors use data on dollar-mark, dollar-yen, and mark-yen options traded on the Philadelphia Stock Exchange (PHLX). They find that realized correlation can be explained by a combination of historical and implied correlations, though in certain cases coefficient estimates are affected by multicollinearity. Bodurtha and Shen (1995) also document a positive relation between changes in implied volatility and changes in implied correlation.

Our work will expand upon that of Bodurtha and Shen (1995) in a number of ways. First, our paper uses data on over-the-counter foreign currency options, a larger and more liquid market where the instruments traded have constant time-to-expiration and a strike price that is always at-the-money. This results in less estimation error, as will be discussed further in the data section. Second, our data will cover a time span that is roughly three times longer, with fewer missing observations, resulting in more than six times as many observations of implied correlation. Third, we consider not only implied, historical, and RiskMetrics correlations, but also conditional correlation under a bivariate GARCH specification. Finally, we use a richer econometric methodology. We not only perform regressions, but also compute forecasts' bias, compare weighted and unweighted root-mean-square errors using the methods of Diebold and Mariano (1995), and perform Wald tests to test parameter restrictions in the presence of multicollinearity.

The remainder of this paper is organized as follows. Section II describes alternative approaches to measuring and forecasting exchange rate correlations. Section III presents our data and certain key descriptive statistics. Section IV compares the performance of the alternative forecasts of conditional correlation using three criteria: root-mean-squared errors, regressions of realized correlation on individual forecasts, and an "encompassing regression" including multiple alternative forecasts. Section V introduces a variance-weighted loss function which we then use to compare forecasts, since the economic value of accurate correlation forecasts may be greatest when variance is high. Section VI summarizes and concludes.

II. Alternative Forecasts of Exchange Rate Correlation

The underlying variables whose correlation we wish to forecast are daily changes in the dollar-yen and dollar-mark exchange rates. We let $\rho_{t,T} \equiv$ the realized correlation over a period of T days beginning at time t. This correlation equals the ratio of the covariance to the product of each exchange rate's standard deviation, all measured over the same T days beginning at t. Let $S_t \equiv$ the spot exchange rate at time t, $s_t \equiv \ln(S_t/S_{t-1})$, and $\bar{s}_{t,T} \equiv$ the mean daily return for T days beginning at t. For notational convenience, we introduce numerical subscripts 1 and 2 to refer to the dollar-mark and dollar-yen exchange rates respectively. Each exchange rate is expressed in accordance with industry convention, i.e. yen or marks per dollar. For the period of T days beginning at t, the covariance between the daily changes in the dollar-mark and dollar-yen exchange rates is expressed as $\sigma_{12,t,T}$, while the variance of each exchange rate is $\sigma^2_{1,t,T}$ and $\sigma^2_{2,t,T}$.

The variable being forecast, realized correlation of exchange rate changes over T days beginning at t, is computed as:

$$\rho_{t,T} = \frac{\sum_{i=1}^{T} [(s_{1,t+i} - \bar{s}_{1,t,T})(s_{2,t+i} - \bar{s}_{2,t,T})]}{\sqrt{\sum_{i=1}^{T} (s_{1,t+i} - \bar{s}_{1,t,T})^2}\sqrt{\sum_{i=1}^{T} (s_{2,t+i} - \bar{s}_{2,t,T})^2}} = \frac{\sigma_{12,t,T}}{\sqrt{\sigma^2_{1,t,T}}\sqrt{\sigma^2_{2,t,T}}} \tag{1}$$

Forecasts of correlation will require forecasts of the covariance of the dollar-mark and dollar-yen exchange rates, as well as forecasts of the individual variances of the dollar-mark and dollar-yen. All these forecasts are based on information at time t, and apply to the following T days. We now discuss four alternative approaches to forecasting this conditional correlation.

1) *Implied Correlation:* Our first forecast of conditional correlation is derived from options on the dollar-mark, dollar-yen, and mark-yen exchange rates. Arbitrage in the spot markets for foreign exchange assures that $s_{DEM\text{-}JPY,t} \equiv s_{2,t} - s_{1,t}$, where the mark-yen cross-rate is expressed in yen per mark. This means that the implied variances (denoted by $\tilde{\sigma}^2_{i,t,T}$) across these exchange rates at time t over a horizon of T days, are related according to:

$$\tilde{\sigma}^2_{DEM\text{-}JPY,t,T} = \tilde{\sigma}^2_{1,t,T} + \tilde{\sigma}^2_{2,t,T} - 2\tilde{\rho}_{t,T}\tilde{\sigma}_{1,t,T}\tilde{\sigma}_{2,t,T} \tag{2}$$

Thus, implied correlation ($\tilde{\rho}$) can be expressed as:

$$\tilde{\rho}_{t,T} = \frac{\tilde{\sigma}^2_{1,t,T} + \tilde{\sigma}^2_{2,t,T} - \tilde{\sigma}^2_{DEM\text{-}JPY,t,T}}{2\tilde{\sigma}_{1,t,T}\tilde{\sigma}_{2,t,T}} \tag{3}$$

This forecast of correlation is obtained directly by substituting into this equation the observed implied volatilities in option prices.

2) *Historical Correlation:* Many forecasts of conditional correlation are based on historical time series data. The simplest of these alternatives is simply the realized correlation over the T days leading up to t, i.e. $\rho_{t-T,T}$.

3) *RiskMetrics™ Correlation:* An alternative estimate proposed in JP Morgan's RiskMetrics™ is an exponentially weighted moving average of historical correlations, where the weighting coefficients decline geometrically the more distant the historical correlation used.[3] Designating the weight as λ and the number of days' historical correlations used as n, one can express the exponentially weighted moving average correlation as:

$$\rho^{EWMA}_{t,T} = \left[\frac{1}{\sum_{j=1}^{n} \lambda^j}\right] \sum_{i=1}^{n} \lambda^i \rho_{t-i-T,T} \tag{4}$$

The number of previous days' historical correlations (n) is set so that the sum of the weights is reasonably

[3] For further details, see JP Morgan (1995), pp. 83-84.

close to $1/(1-\lambda)$, the asymptotic limit as n becomes infinite. For monthly correlations, RiskMetrics sets the coefficient λ to 0.97, based on historical performance of forecasts using alternative values for λ. Given $\lambda=0.97$, we set n=151 days, implying that the omitted weights sum to less than 1%, i.e. $(1-\lambda)$ $\Sigma_{j=152}^{\infty} \lambda^j \leq 1\%$.

4) *GARCH-based Correlation:* The conditional correlation between the dollar-mark and dollar-yen exchange rates can also be modelled using a GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) approach that explicitly models the time-varying volatility in the dollar-mark and dollar-yen rates.[4] Estimates of the conditional covariance and variances derived from GARCH models can be used to form the conditional correlation.[5]

Within the GARCH framework, correlation could be estimated using either a univariate approach, where each of the three exchange rates is modelled as an independent univariate process, or a bivariate approach, in which two exchange rates and the covariance between them are explicitly modelled. Under the univariate approach, which assumes that innovations in each exchange rate process do not contribute to the conditional variance of the other processes, conditional correlations can be derived from three independently estimated conditional variances. Since the three variances are estimated separately, there is no constraint that the implied correlation fall between negative and positive one.

After performing preliminary univariate GARCH(1,1) estimations, we conducted Lagrange multiplier tests of the univariate model against an alternative specification in which the conditional variance of each exchange rate series was a function not only of its lagged innovations and conditional

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[4] We do not use asymmetric GARCH models such as an EGARCH model, since foreign exchange is not characterized by the asymmetry found in, for example, the stock market, where very large negative shocks are more likely than very large positive shocks.

[5] For volatility, it can be shown that the RiskMetrics estimate based on an exponentially weighted moving average with weight λ is, ignoring the constant term α_0, equivalent to a GARCH(1,1) with $\alpha_1=1-\lambda$ and $\beta_1=\lambda$. For correlation, this equivalence between RiskMetrics and GARCH(1,1) does not hold because of the highly nonlinear relation between conditional correlation and the elements of the variance-covariance matrix.

variance but also of lagged daily returns from the other exchange rates. We rejected the null hypothesis that a univariate representation of these GARCH processes was appropriate.

Given the empirical rejection of the univariate approach, we model the dollar-mark and dollar-yen exchange rates as a bivariate GARCH process. The general model for a bivariate GARCH(p,q) estimation is as follows:

$$\begin{aligned} s_t &= \bar{s}+\epsilon_t \\ vech(H_t) &= c+\sum_{i=1}^{q} A_i vech(\epsilon_{t-i}\epsilon'_{t-i})+\sum_{j=1}^{p} B_j vech(H_{t-j}) \\ \epsilon_t|\Psi_{t-1} &\sim N(0,H_t) \end{aligned} \tag{5}$$

where s_t is a 2x1 vector denoting the daily return on the dollar-mark and dollar-yen exchange rate on day t, $\bar{s}$ is a 2x1 vector of constants representing the mean daily return, ϵ_t is a 2x1 innovation vector, H_t is a 2x2 variance-covariance matrix, Ψ_{t-1} represents information at time t-1, c is a 3x1 vector of constants, and A_i ($i= 1,\ldots,q$) and B_i ($i=1,\ldots,p$) are 3x3 matrices representing the dependence of current conditional variance-covariance terms on q past innovations and p past conditional variance-covariance terms respectively. The vech($\cdot$) operator denotes the column-stacking operator applied to the lower portion of a symmetric matrix. Among others, Baillie and Bollerslev (1989) and Hsieh (1989) show that daily exchange rate returns can be quite successfully represented by a GARCH(1,1) process. Hence, we will set $p=1$ and $q=1$.

The conditional log likelihood function for this problem at a single time period t can be expressed as:

$$L_t(\theta)=-\frac{N}{2}\log 2\pi-\frac{1}{2}Log|H_t(\theta)|-\frac{1}{2}\epsilon_t(\theta)'H_t^{-1}(\theta)\epsilon_t(\theta) \tag{6}$$

where all the parameters have been combined into the vector θ.

To facilitate estimation of $L(\theta)$, we restrict the form of the conditional variance-covariance matrix following Bollerslev, Engle and Wooldridge (1988). In particular, we assume that each covariance element depends only on its own past values and innovations, i.e. the matrices A and B (subscripts dropped since p=q=1) are diagonal. The diagonality assumption for A and B reduces the number of parameters to be estimated from 23 [$\bar{s}$ (2 elements), c (3 elements), A (9 elements), B (9 elements)] to 11 (A and B are now three elements each). This specification implies that shocks in dollar-mark returns, for example, do not directly influence dollar-yen conditional variance or mark-yen conditional covariance. Nonetheless, all shocks still contribute to the correlation estimate, which on any day is based on $h_{12,t}/[\sqrt{h_{11,t}} \cdot \sqrt{h_{12,t}}]$.

Therefore, the estimated model will be a bivariate GARCH(1,1) of the following form:

$$\begin{aligned} s_t &= \bar{s}+\epsilon_t \\ h_{ij,t} &= c_{ij}+\alpha_{ij}\epsilon_{i,t-1}\epsilon_{j,t-1}+\beta_{ij}h_{ij,t-1} \\ \epsilon_t|\psi_{t-1} &\sim N(0,H_t) \end{aligned} \tag{7}$$

where subscript i refers to a vector's ith element, and subscript ij to a matrix's ijth element. The α and β terms are elements of matrices A and B respectively.

Estimating these equations on a daily basis leads to one-day ahead-forecasts (estimates of conditional variance-covariance elements denoted by $\bar{h}$) of the elements in the daily variance-covariance matrix. These forecasts can be expressed as:

$$\bar{h}_{ij,t+1}=\hat{c}_{ij}+\hat{\alpha}_{ij}\epsilon_{i,t}\epsilon_{j,t}+\hat{\beta}_{ij}\bar{h}_{ij,t} \tag{8}$$

where $\bar{h}_{ij,\tau}$ denotes the daily estimate of conditional variance or covariance for day τ, and $\hat{c}_{ij},\hat{\alpha}_{ij},\beta_{ij}$ represent the estimated values for c_{ij}, α_{ij}, and β_{ij} respectively. Since we will typically be interested in horizons of one month or more, one can forecast ahead recursively, using the expected value $\bar{h}_{ij,t+n}$ for the unknown realizations of $\epsilon^2_{ij,t+n}$. Thus, expectations at time t of daily variance-covariance elements at time t+n would be:

$$\bar{h}_{ij,t+n}=\hat{c}_{ij}+(\hat{\alpha}_{ij}+\hat{\beta}_{ij})\bar{h}_{ij,t+n-1}$$

Under the GARCH approach, the forecasts of conditional variances or covariance elements at time t over the next T days would be $\Sigma_n^{T=1}\ \bar{h}_{ij,t+n}$, since variances are additive in time. The forecast of conditional correlation over the next T days equals the ratio of the conditional covariance to the product of conditional standard deviations over the same horizon.

In GARCH-based forecasts estimated using the full data sample, coefficients used at any instant in time incorporate knowledge of the future behavior of exchange rates. If the system is stable, the performance of GARCH-based forecasts will be improved given the advantage of full-sample coefficients not available in "real time" to the forecaster. As a more realistic alternative replicable in real time, one can perform a "rolling GARCH" estimation in which one uses a sliding window of data up to time t, or an "updating GARCH" in which one uses all data up to time t, to derive coefficients for the forecast at time t. Rolling GARCH would be superior to updating GARCH and full-sample GARCH forecasts if for any reason coefficients are not stable over time, as might be true if there are shifts in policy or the formation of expectations. In forecasting conditional variance in exchange rates, West and Cho (1995), for example, find that a rolling GARCH slightly outperforms an updating GARCH. Therefore, to compute our GARCH-based forecasts of correlation, we use a rolling GARCH approach. As will be discussed later, we find that the predictive performance of GARCH-based forecasts relative to other forecasts is virtually unaffected by our choice of whether to use rolling GARCH, updating GARCH, or full-sample GARCH.

III. Data and Descriptive Statistics

Our options data consist of 1600 daily observations of over-the-counter price quotes on dollar-mark, dollar-yen, and mark-yen options from 3 January 1989 through 23 May 1995, a period of nearly six and a half years.[6] In this market, which is primarily traded interbank, the prices of these options are

[6] In addition to dollar-denominated options, there also exist data on certain major cross-rates within the European Monetary System---e.g. pound-mark, mark-French franc, mark-lira. These could in principle be used to calculate implied correlations of two European currencies vs. the dollar. These rates, however, are or were "managed" by the respective central banks within the context of the Exchange Rate Mechanism (ERM). Thus, a more appropriate analytical framework would be a target zone model with

quoted directly in terms of implied volatility, which traders then substitute into a Garman-Kohlhagen (Black-Scholes, with modifications for the foreign interest rate) option pricing formula. Note that this does not mean that traders necessarily believe that Black-Scholes assumptions hold; the Black-Scholes formula simply serves as a convenient mapping between implied volatilities and option premia. These prices refer to interbank market's most commonly traded instrument: at-the-money forward straddles, i.e. one call plus one put whose common strike price equals the forward rate. Both the call and put are European-type options; there is no possibility of early exercise. Daily data on spot rates are the average of day-end bid and ask quotes provided by commercial banks in London and New York. Time periods for which we have multiple sources of data for certain series indicate practically no difference in quotes across banks.

Descriptive statistics for the implied volatilities are shown in Table 1 for dollar-mark, dollar-yen, and mark-yen options with times to expiration of one and three months. As the standard deviations and minimum-maximum ranges indicate, one-month volatility is more variable than three-month volatility. Since short-dated implied variance in foreign exchange is typically mean-reverting, long-dated implied variance (which can be interpreted as average short-dated variance expected over the lifetime of the option) is less variable than short-dated implied variance. Properties of this term structure of implied volatilities in over-the-counter foreign exchange options are discussed further in Campa and Chang (1995).

It is also worth mentioning certain institutional differences between over-the-counter (OTC) foreign exchange option markets and foreign exchange option trading on the Philadelphia Stock Exchange. First, the OTC markets are far more liquid, especially for cross-rate options. A survey conducted in April 1995 by the Bank for International Settlements (1996) reports both outstanding notional amounts

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realignment risk. Observed correlation of two ERM currencies vs. the dollar would tend to be close to one when the ERM band holds, and zero or negative during a realignment. See Campa and Chang (1996) for a further discussion of ERM cross-rate options.

and daily turnover on dollar-mark, dollar-yen, and mark-yen options. At end-March 1995, outstanding notional amounts in all OTC options were $ 518.720 billion, $ 625.163 billion, and $ 93.991 billion in dollar-mark, dollar-yen, and mark-yen options respectively. Corresponding figures on exchange-traded markets were $ 36.395 billion, $ 21.749 billion, and $ 230 million respectively. In outstanding notional amounts, OTC option markets are from 14 to 408 times the size of exchange-traded option markets. In daily turnover, OTC options averaged $ 10.241 billion, $ 13.266 billion, and $ 1.936 billion in dollar-mark, dollar-yen, and mark-yen respectively---compared with $ 1.223 billion, $ 842 *million*, and $ 16 *million* on exchanges. Again, the ratio favoring OTC markets ranges from 8 to 121. More important, the dominance of OTC markets, by a factor of at least 100, is most pronounced in mark-yen options, the focus of this study.[7] To the degree that illiquidity may introduce measurement error, the superiority of the OTC data may lead to more accurate economic inference.

A second difference between OTC and PHLX series is consistency in the options' time-to-expiration. For each time series of OTC options prices, the time to expiration is constant (one and three months), whereas the PHLX options expire only four times per year, resulting in variable times to expiration. Given the significant empirical variation in the term structure in implied volatilities in foreign exchange options (documented by Xu and Taylor (1994) for PHLX options, and Campa and Chang (1995) for OTC options), combining options with varying times to expiration makes it unclear over what horizon implied correlation is estimated, since the horizon is constantly varying.[8] Third, OTC options are always at-the-money-forward, thereby avoiding "smile" effects in which implied volatility varies with

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[7] On the dates used by Bodurtha and Shen (1995), for example, PHLX mark-yen options in the appropriate maturity range traded on only 46% (261 out of 565) of the trading days in their sample. At the same time, PHLX data represent actual trades, whereas our OTC data are average bid-ask closing price quotes, not necessarily corresponding to actual trades.

[8] Bodurtha and Shen (1995) choose to estimate correlation over the 30-day horizon using options with time-to-expiration ranging from 2 weeks to 2 months. This choice seems sensible given the constraints of the PHLX data. Nonetheless, the OTC data permit a sharper distinction of the horizon over which correlation is being estimated.

the strike price. For exchange-traded options, discreteness in strike prices means that usually, even options nearest-the-money are only approximately *at*-the-money.

If realized correlation is positively correlated with realized volatility, then precisely when diversification is most important, i.e. volatility is high, then the benefits of diversification, i.e. low correlation, are reduced. Table 2 reports the correlation between volatility and correlation (implied in first panel, realized in second panel). Realized correlation is positively correlated with one-month and three-month dollar-yen realized volatility and one-month dollar-mark realized volatility, but negatively correlated with three-month dollar-mark volatility. This suggests that higher correlation occurs during periods of higher volatility, as found in Longin and Solnik (1995) for international stock markets.

IV. Performance of Correlation Forecasts

We evaluate in three ways the relative performance of the four alternative forecasts introduced in Section III. First, we compute the root-mean-squared-error for the alternative forecasts. Second, we run regressions of realized correlation individually against each of the alternative forecasts. We compare R^2 to determine which forecast best explains variations in realized correlation. Finally, we run "encompassing regressions" in which two or more alternative forecasts are included as regressors. For these specifications, we perform Wald tests to ascertain whether the predictive performance of implied correlation is improved by adding alternative forecasts based on time series, and vice versa.[9]

The correlation forecasts based on the bivariate GARCH(1,1) specification require estimation of the system of equations (7). We use the Berndt et. al (1974) algorithm, as well as numerical first-order

[9] Dollar-mark/dollar-yen correlation is presumably influenced by macroeconomic factors not directly reflected in exchange rates or options, such as shifts in monetary policy, trade balance announcements, or foreign exchange intervention. We do not include these variables in our regressions because our purpose is not to identify what fundamental variables may affect correlation, but to evaluate four alternative "benchmark" forecasts based on financial data.

derivatives. As discussed earlier, we use a rolling GARCH rather than full-sample GARCH in order to replicate real-time forecasting, as well as to allow for the possibility of parameters changing over time. We choose a window of 1000 daily observations, just under four years, resulting in the use of daily exchange rate data going back to early 1985 to compute correlation forecasts in January 1989.

Table 3 reports our parameter estimates for the first day of the rolling GARCH, as well as the parameter range over all our rolling GARCH estimations. With the exception of $\bar{s}_1$, the trend in the dollar-mark, all parameters in this diagonalized GARCH specification are significant on the first day of the rolling GARCH. Robust standard error are computed using the methods of Bollerslev and Wooldridge (1992). The negative $\bar{s}$ elements indicate a trend depreciation of the dollar against the mark and yen. We have verified for each window of the rolling GARCH that each element (i,j) of the variance-covariance matrix H_t is stationary since A and B are by construction diagonal, and $\alpha_{ij} + \beta_{ij}$ sum to less than one for all i,j.

Figures 1a-1f show the time series of realized correlation versus implied, RiskMetrics, and GARCH-based correlation forecasts over one-month and three-month horizons. Certain consistent patterns emerge from these graphs. Realized correlation during this period is almost always positive, with mean values (and standard deviation adjusted for overlapping observations using the techniques of Newey and West (1987)) of 0.575 (0.199) and 0.558 (0.169) for the one and three-month horizons respectively. Partial autocorrelations on the time series of realized correlations suggest that the series follow an AR1 process, where the daily autocorrelation coefficient equals 0.967 and 0.992 over the one and three-month horizons respectively, thereby indicating mean-reversion in these series. Realized correlation is more variable than any of the forecasts, so forecasts tend to underpredict when correlation is high, and overpredict when correlation is low. Among the forecasts, the forecast derived from a bivariate GARCH(1,1) model is the least variable, being largely confined to the 0.5-0.7 range when in fact realized correlation is often above or below this range. Implied and RiskMetrics correlation forecasts exhibit considerably greater variation over time than GARCH-based forecasts, but less than realized correlation.

Root-mean squared errors (RMSE) of alternative correlation forecasts, presented in Table 4, indicate that over both the one-month and three-month horizon, implied correlation outperforms any of the forecasts based on historical data. Over the one-month horizon, implied correlation forecasted best with a RMSE of 0.1702, while historical correlation performed worst, with a RMSE of 0.2215. RMSE's fall for all four approaches when the forecast horizon is lengthened from one month to three months, consistent with predictable mean-reversion in correlation. For three-month forecasts, implied correlation performs best with RMSE of 0.1377, while the GARCH forecast generates the highest RMSE, 0.1691.

We compare the predictive accuracy of alternative forecasts using the methodology outlined in Diebold and Mariano (1995), which develops an asymptotic test of the null hypothesis of no difference in the accuracy of two competing forecasts. Given any loss function, in this case a squared error loss function, one generates a time series of the difference in loss function between any two forecasts. Diebold and Mariano (1995) provide consistent estimates under the null for the asymptotic standard deviation of that difference in loss function, correcting for serial correlation.

The results of tests comparing the implied correlation against the three alternative forecasts are reported in the last column of Table 4. Implied correlation, which had the lowest RMSE, significantly outperforms all three other forecasts over the one-month horizon. Over the three-month horizon, implied correlation's superior performance is significant relative to the RiskMetrics and GARCH forecasts of correlation, but not relative to historical correlation.

We also compute the bias of the four alternative forecasts by regressing each forecast's error on a constant term, correcting the standard errors for overlapping observations according to the procedure outlined in Newey and West (1987). We find that none of the forecasts have significant (at the 5% level) bias at either the one-month or three-month horizon.

The RMSE criterion as a basis for comparing forecasts can have certain limitations, as discussed in Fair and Shiller (1990). In particular, even a forecast with a higher RMSE may contain information not present in the forecast with a lower RMSE. Further insights into the incremental informational contributions of the different forecasts can be revealed in a regression-based approach to forecast evaluation.

We next compare alternative forecasts by running regressions of realized correlation on a constant and each of the four forecasts individually, then on combinations of forecasts, as shown in Table 5.[10] Since observations are daily while the forecast horizon is always one month or greater, point estimates in these regressions will be consistent but standard errors will be understated because of overlapping observations. Therefore, all standard errors are corrected using the approach described in Newey and West (1987).

A rational forecast of realized correlation should generate regression coefficients of 0 and 1 on the constant term and the forecast respectively. Point estimates for the coefficient on each of these forecasts, however, are always significantly less than one. Nonetheless, with the exception of the GARCH (1,1) forecast at the three-month horizon, coefficients on all four forecasts at both horizons are significantly different from zero, indicating that each forecast reflects some information useful for predicting future correlation. In terms of overall goodness-of-fit, implied correlation results in a higher adjusted R^2 than any other forecast at the one-month horizon. At the three-month horizon, adjusted R^2 is very slightly higher using historical correlation, but implied correlation easily outperforms GARCH(1,1) and RiskMetrics forecasts.

One can further compare the informational contribution of forecasts by including combinations of multiple forecasts on the regression's right-hand-side, as outlined in Fair and Shiller (1990). The regression is sometimes run in first-differences if the variable being estimated is perceived as

[10] This approach is taken by Day and Lewis (1992).

nonstationary, e.g. GNP in Fair and Shiller (1990). Since nonstationarity does not present a problem in the estimation of correlation, these regressions are run in levels. Because of potential multicollinearity, we perform Wald tests (distributed as χ^2 with degrees of freedom equal to the number of restrictions) on two possible restrictions: first, that the coefficient on implied correlation equals zero; second, that the coefficient on the other coefficient(s) equals zero.

Results for the one-month forecast horizon are shown in the first panel of Table 5. Adding implied correlation to the right-hand side causes coefficients on all other forecasts to lose their significance. From our Wald tests, we cannot reject a zero coefficient on historical or GARCH(1,1) correlation. Yet, we do reject a zero coefficient on the RiskMetrics-based forecast, suggesting an incremental improvement over implied correlation alone. When all four forecasts are included, only implied correlation is significant, but one can reject the joint hypothesis of zero value for the three coefficients on the forecasts other than implied correlation. One very strong result emerges from these tests: that using implied correlation improves the performance of any of the three other forecasts, as well as a combination of the three other forecasts.

The value of implied correlation in forecasting is evident at the three-month horizon as well. In all specifications, the regression coefficient on implied correlation is significant, and the Wald test rejects a zero coefficient on implied correlation. Adding implied correlation to the right-hand side causes coefficients on GARCH(1,1) and RiskMetrics forecast to lost significance, although the Wald test rejects a zero coefficient on the GARCH(1,1) forecast. When all four forecasts are used, implied, historical, and RiskMetrics forecasts are all significant, and one cannot reject the joint hypothesis that coefficients on all forecasts but implied are zero. The overall message is similar to that of the one-month forecasts: implied correlation consistently adds value to the other forecasts, although the evidence is mixed as to which of the historically based forecasts (pure historical, GARCH, or RiskMetrics) provides the greatest incremental information to implied correlation.[11]

V. Variance-Weighted Forecast Accuracy

In practice, the economic value of accurate forecasts of correlation may vary over time, depending on the contemporaneous behavior of other variables. For example, Longin and Solnik (1995) find that correlation among international stock markets rises during periods of high volatility, precisely those periods in which an accurate forecast of correlation may be most important. Erb, Harvey and Viskanta (1995) find that international equity correlations are higher during recessions than during growth periods and lower when the two business cycles are out of phase. Because correlation is in fact related to other financial variables, accurate forecasts of correlation may be more important during certain periods than others.

As an application of this principle, we take the perspective of a hypothetical dollar-based portfolio manager with long positions in both the mark and yen. Portfolio variance σ_P^2 will be:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2 \tag{10}$$

where w_i represents the portfolio share in asset i. Correlation enters into total portfolio variance through the covariance term, i.e. $\rho_{t,T}\sigma_{1,t,T}\sigma_{2,t,T}$. Thus, for any given portfolio weights, errors in forecasting correlation will generate greater shocks to portfolio variance when the variance is high. It should therefore be of interest to know whether certain forecasts of correlation perform systematically better when forecasting accuracy is of greatest value, e.g. when $\sigma_{1,t,T}\sigma_{2,t,T}$ is high. We therefore construct a time-varying loss function for errors in forecasting correlation. In particular, we define the following

[11] These results are consistent with those for the 30-day horizon using PHLX data in Bodurtha and Shen (1995), in which both historical and implied correlation prove significant. The R^2 in our regressions is significantly larger, possibly reflecting the smaller measurement error in the OTC market.

loss function[12]:

$$g(t,T)=\left[\left(\rho_{t,T}^{realized}-\rho_{t,T}^{forecast}\right)\sigma_{1,t,T}\sigma_{2,t,T}\right]^2 \tag{11}$$

In effect, this loss function represents the shocks to portfolio variance due solely to errors in correlation forecasting, assuming unchanged portfolio weights and accurate prediction of volatility.

In Table 6, we report the root-mean-value of our loss function g(t,T) under each forecast. We find that, weighting squared forecast errors by contemporaneous realized variance in the two underlying variables (as formalized above), implied correlation again performs best over both the one-month and three-month horizons. We again test the null hypothesis of no difference in loss function under two alternative forecasts, as outlined in Diebold and Mariano (1995). Our results indicate that over the one-month horizon, the loss function using implied correlation is significantly lower than that using historical or GARCH-based correlation. The performance of implied correlation, however, is not statistically different from that using the RiskMetrics forecast. For the same loss function over the three-month horizon, implied correlation performs best, but is not statistically different from any of the three time series-based forecasts of correlation.

VI. Conclusion

This paper provides a further test of the information content of options by assessing the predictive power of implied correlation derived from dollar-mark, dollar-yen, and mark-yen option prices. The paper compares the accuracy of this implied correlation forecast against three alternative forecasts based on time series data: historical correlation, RiskMetrics' exponentially weighted moving average

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[12] A more general approach would examine how portfolio weights are affected by alternative correlation estimates in a mean-variance framework. Out-of-sample performance of differing correlation forecasts would be reflected in differing mean-variance realizations. This approach would require forecasting mean and variance, issues already addressed at great length in the existing literature. Instead, we focus entirely on correlation, holding all else constant.

correlation, and correlation estimated using a bivariate GARCH(1,1) model.

Using daily data from 3 January 1989 to 23 May 1995, forecasts based on implied correlation over one- and three-month horizons result in lower root mean squared errors and lower forecast bias than the three alternative forecasts, often significantly. Among alternative single forecasts for realized correlation, implied correlation outperforms the historically based measures.

Although each individual forecast has some predictive power, at least at the one-month horizon, implied correlation is the only forecast that consistently adds value to any one or combination of the other three forecasts. While implied correlation always improves the performance of any of the historically based forecasts, the converse is never true. None of the forecasts based on time series data is consistently capable of providing additional information to the implied correlation forecast.

Overall, these results reinforce earlier research emphasizing the usefulness of options in forecasting future asset price behavior. As forward-looking financial instruments, options provide information not found in time series data. In many instances, forecasts based on historical data can provide no incremental information to those based on options.

On a stand-alone basis, implied correlation forecasts continue to provide the most accurate forecast when accuracy is evaluated by weighting the forecast errors by the product of realized standard deviations. This measure of performance is especially appropriate for portfolio management applications, since correlation's contribution to the overall portfolio variance depends on the realized standard deviations of the two underlying variables. The superior forecasting ability of implied correlation could be further tested by applying the same methodology with a potentially different loss function in other financial contexts where correlation is important.

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Table 1

DESCRIPTIVE STATISTICS ON IMPLIED VOLATILITY

Daily implied volatility quotes (in percent per year) of at-the-money forward straddles with one and three months to expiration on the USD-DEM, USD-JPY, and DEM-JPY exchange rates. The sample period is from 3 January 1989 to 23 May 1995 (N=1600).

	Mean	Std.Dev.	Skewness	Kurtosis	Min.	Max.
			One-Month Options			
$\sigma_{USD\text{-}DEM}$	11.961	2.177	1.021	5.392	7.500	24.00
$\sigma_{USD\text{-}JPY}$	10.427	2.202	0.759	3.086	6.600	18.45
$\sigma_{DEM\text{-}JPY}$	10.376	2.560	-0.076	3.053	4.650	20.00
			Three-Month Options			
$\sigma_{USD\text{-}DEM}$	11.965	1.475	0.640	4.224	8.350	19.00
$\sigma_{USD\text{-}JPY}$	10.412	1.539	0.679	3.154	7.200	16.50
$\sigma_{DEM\text{-}JPY}$	10.243	2.148	-0.66	2.968	4.730	16.00

Table 2

CORRELATION BETWEEN VOLATILITY AND CORRELATION

A) IMPLIED VOLATILITY AND IMPLIED CORRELATIONS

	$\tilde{\sigma}_{usd\text{-}dem}$	$\tilde{\sigma}_{usd\text{-}jpy}$	$\tilde{\sigma}_{dem\text{-}jpy}$	$\tilde{\rho}_{usd\text{-}dem,usd\text{-}jpy}$
$\tilde{\sigma}_{usd\text{-}dem}$	1.000 *1.000*			
$\tilde{\sigma}_{usd\text{-}jpy}$	0.363 *0.267*	1.000 *1.000*		
$\tilde{\sigma}_{dem\text{-}jpy}$	0.515 *0.483*	0.431 *0.369*	1.000 *1.000*	
$\tilde{\rho}_{usd\text{-}dem,usd\text{-}jpy}$	0.116 *-0.021*	-0.037 *-0.092*	-0.740 *-0.856*	1.000 *1.000*

B) REALIZED VOLATILITY AND REALIZED CORRELATIONS

	$\sigma_{usd\text{-}dem}$	$\sigma_{usd\text{-}jpy}$	$\sigma_{dem\text{-}jpy}$	$\rho_{usd\text{-}dem,usd\text{-}jpy}$
$\sigma_{usd\text{-}dem}$	1.000 *1.000*			
$\sigma_{usd\text{-}jpy}$	0.338 *0.280*	1.000 *1.000*		
$\sigma_{dem\text{-}jpy}$	0.542 *0.456*	0.597 *0.575*	1.000 *1.000*	
$\rho_{usd\text{-}dem,usd\text{-}jpy}$	0.342 *-0.125*	0.089 *0.187*	-0.340 *-0.071*	1.000 *1.000*

Top number refers to one-month horizon, bottom number *(in italics)* to three-month horizon.

Table 3

"ROLLING" BIVARIATE GARCH(1,1) PARAMETER ESTIMATES

5 January 1985 to 23 May 1995

$$s_t = \bar{s}_t + \epsilon_t$$
$$h_{ij,t} = c_{ij,t} + \alpha_{ij,t}\epsilon_{i,t-1}\epsilon_{j,t-1} + \beta_{ij,t}h_{ij,t-1}$$
$$\epsilon_t | \Psi_{t-1} \sim N(0, H_t)$$

where s_t is a 2x1 vector of daily differences in the log of the USD-DEM and USD-JPY exchange rates, $h_{ij,t}$ is the estimate of the (row i, column j) element of the variance-covariance matrix of the two exchange rates at time t, ϵ_t is a 2x1 vector of error terms and $\bar{s}_t$, $c_{ij,t}$, $\alpha_{ij,t}$, and $\beta_{ij,t}$ are parameters to be estimated.

Estimation is performed on 1000 daily observations of log-differences of the USD-DEM and USD-JPY exchange rates.

	GARCH Estimates for 3 January 1989		Range of Parameter Estimates for the 1600 "Rolling" GARCH	
	Parameter Value	Standard Error[a]	Maximum Value	Minimum Value
$\bar{s}_1$	-0.036	0.022	0.007	-0.056
$\bar{s}_2$	-0.042*	0.019	0.007	-0.069
$C_{1,1}$	0.030*	0.008	0.044	0.005
$C_{1,2}$	0.027*	0.007	0.036	0.001
$C_{2,2}$	0.049*	0.014	0.065	0.010
$\alpha_{1,1}$	0.119*	0.028	0.131	0.031
$\alpha_{1,2}$	0.080*	0.018	0.081	0.021
$\alpha_{2,2}$	0.143*	0.029	0.145	0.023
$\beta_{1,1}$	0.836*	0.028	0.961	0.823
$\beta_{1,2}$	0.847*	0.024	0.973	0.841
$\beta_{2,2}$	0.765*	0.034	0.954	0.758

[*] Significantly different from zero at the one percent level.
[a] Robust Standard Errors.

Table 4

ROOT MEAN SQUARED ERRORS OF REALIZED CORRELATION VS. CORRELATION FORECASTS

The one(and three)-month correlations are computed for overlapping 23(and 69)-day periods. Data cover 3 January 1989 to 23 May 1995. Forecasts end 23(and 69) days prior to the end of the data sample.

Realized correlation is compared against the following forecasts:

1) Historical correlation, using the correlation over the 23(69) days prior to the forecast;
2) Implied correlation, based on implied volatility quotes;
3) Correlations from bivariate "Rolling" GARCH(1,1) estimation;
4) RiskMetrics correlation, using exponentially declining (with 151 lags and an exponential coefficient of 0.97) weights on historical correlation.

Forecast Method:	RMSE	Forecast Bias[a]	Forecast Different from Implied[b]
1-Month Correlation			
Historical	0.2215	0.001 (0.019)	5.42*
Implied	0.1702	0.004 (0.016)	--
RiskMetrics	0.1820	0.002 (0.019)	2.47**
GARCH(1,1)	0.2082	0.012 (0.022)	2.54**
3-Month Correlation			
Historical	0.1472	0.001 (0.026)	1.33
Implied	0.1377	-0.012 (0.025)	--
RiskMetrics	0.1639	0.002 (0.032)	2.17**
GARCH(1,1)	0.1691	0.001 (0.033)	3.66*

[a] Point estimates and Newey-West (1987) standard errors (in parentheses) of a regression of the forecast error on a constant.

[b] Test statistic on null hypothesis of no difference in the accuracy of this forecast and the implied correlation forecast (Diebold and Mariano 1995).

* Significant at the 1 percent level, 2-tailed test.

** Significant at the 5 percent level, 2-tailed test.

Table 5

OLS REGRESSIONS OF REALIZED CORRELATION ON ALTERNATIVE CORRELATION FORECASTS

$$\rho_{t,T} = a + b\hat{\rho}_{t,T} + \varepsilon_{t,T}$$

where $\rho_{t,T}$, the realized correlation from time t to T, is regressed against these alternative correlation forecasts ($\hat{\rho}_{t,T}$) at time t:

1) Implied correlation, based on implied volatility quotes;
2) Historical correlation, using the correlation over the 23(and 69) days prior to the forecast;
3) Correlations from bivariate "rolling" GARCH(1,1) estimation;
4) RiskMetrics correlation, using exponentially declining (with 151 lags and an exponential coefficient of 0.97) weights on historical correlation.

Daily data cover 3 January 1989 to 23 May 1995. Forecasts end 23(and 69) days prior to the end of the data sample. Standard errors (in parentheses below parameter estimates) corrected for overlapping observations using Newey and West (1987).

For each multiple regression, we report the Wald test (distributed as χ^2) on the restrictions that the coefficient on implied correlation equals zero, or that the coefficients on the other forecasts equal zero.

A) One-Month Correlations

Constant	Implied	Historical	GARCH(1,1)	Risk-Metrics	R^2	χ^2 Imp.=0	χ^2 Other=0
0.151*	0.741*				0.29		
(0.060)	(0.090)						
0.361*		0.369*			0.13		
(0.056)		(0.089)					
0.402*			0.302*		0.08		
(0.129)			(0.052)				
0.206*				0.643*	0.21		
(0.076)				(0.122)			
0.152**	0.762*	-0.023			0.29	337.9*	0.55
(0.060)	(0.115)	(0.099)					
0.252**	0.823*		-0.263		0.31	616.2*	28.69
(0.106)	(0.090)		(0.193)				
0.136**	0.650*			0.117	0.29	178.9*	5.53**
(0.068)	(0.117)			(0.148)			
0.235**	0.749*	-0.052	-0.268	0.161	0.31	204.5*	38.50*
(0.112)	(0.138)	(0.104)	(0.197)	(0.158)			

B) Three-month Correlations

Constant	Implied	Historical	GARCH(1,1)	Risk-Metrics	R^2	χ^2 Imp.=0	χ^2 Other=0
0.174** (0.082)	0.678* (0.123)				0.30		
0.250* (0.076)		0.560* (0.127)			0.31		
0.415** (0.187)			0.268 (0.331)		0.03		
0.318* (0.103)				0.440** (0.182)	0.15		
0.168** (0.080)	0.369* (0.126)	0.327** (0.152)			0.34	82.3*	96.5*
0.278 (0.145)	0.800* (0.161)		-0.309 (0.324)		0.32	669.0*	54.5*
0.177** (0.090)	0.710* (0.194)			-0.039 (0.253)	0.30	316.7*	1.2
0.293** (0.116)	0.551* (0.151)	0.805* (0.217)	-0.221 (0.246)	-0.666** (0.305)	0.43	179.4*	345.2*

* Significantly different from zero at the 1 percent level.

** Significantly different from zero at the 5 percent level.

Table 6

ROOT MEAN SQUARED ERRORS OF REALIZED CORRELATION VS. CORRELATION FORECASTS WEIGHTED BY REALIZED VARIANCES

This table reports the root mean square of the product of the correlation forecast error and the standard deviations over one month (23 days) and three-month (69 days) periods

$$WRMSE=\sqrt{\frac{\sum_{t=1}^{n-T}\left(\rho_{t,T}^{realized}-\rho_{t,T}^{forecast}\right)\sigma_{1,t,T}\sigma_{2,t,T}}{n-T}}$$

Correlation forecasts at time t are:

1) Implied correlation, based on implied volatility quotes;
2) Historical correlation, using the correlation over the 23(and 69) days prior to the forecast;
3) Correlations from bivariate "rolling" GARCH(1,1) estimation;
4) RiskMetrics correlation, using exponentially declining (with 151 lags and an exponential coefficient of 0.97) weights on historical correlation.

Daily data cover 3 January 1989 to 23 May 1995. Forecasts end 23(and 69) days prior to the end of the data sample.

Forecast Method:	WRMSE[a]	Forecast Different from Implied[b]
	1-Month Correlation	
Historical	0.0983	3.093*
Implied	0.0841	--
RiskMetrics	0.0850	0.287
GARCH(1,1)	0.0983	2.148**
	3-Month Correlation	
Historical	0.0668	0.160
Implied	0.0660	--
RiskMetrics	0.0758	1.429
GARCH(1,1)	0.0757	1.331

[a] The variances have been multiplied by 10^4.

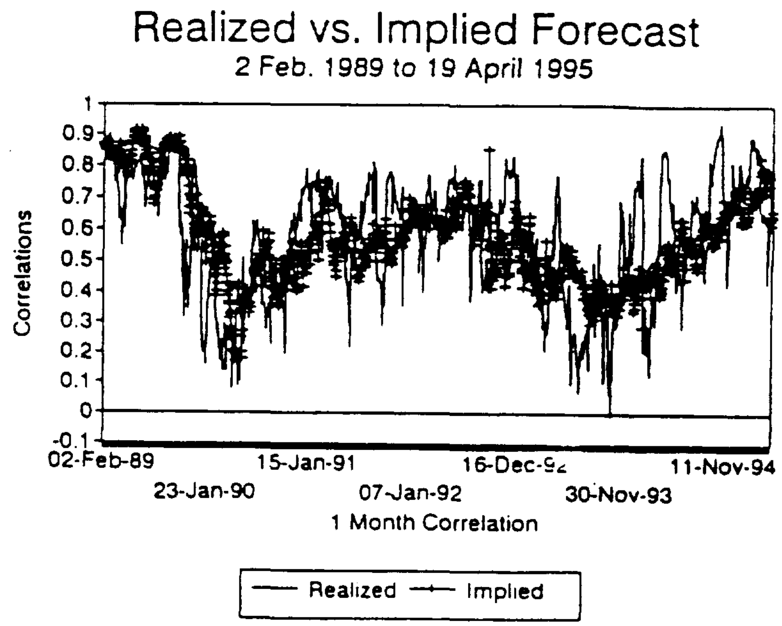
[b] Test statistic on null hypothesis of no difference in the accuracy of this forecast and the implied correlation forecast (Diebold and Mariano 1995).

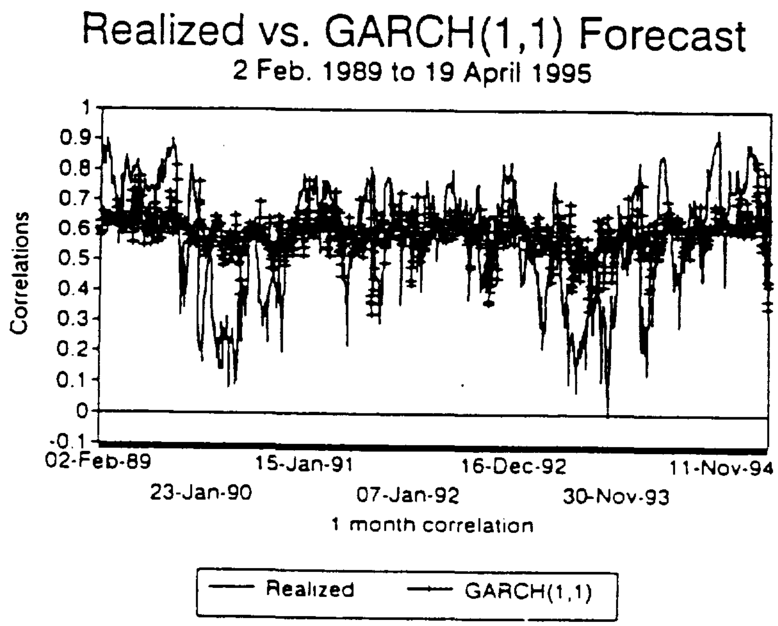
* Significantly different from zero at the 1 percent level.

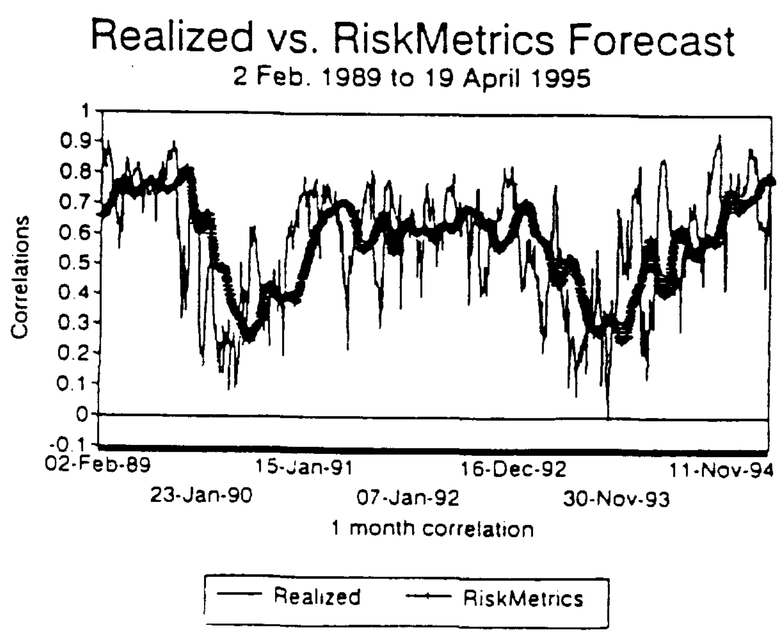
** Significant at the 5 percent level, 2-tailed test.

Figures 1a-1f

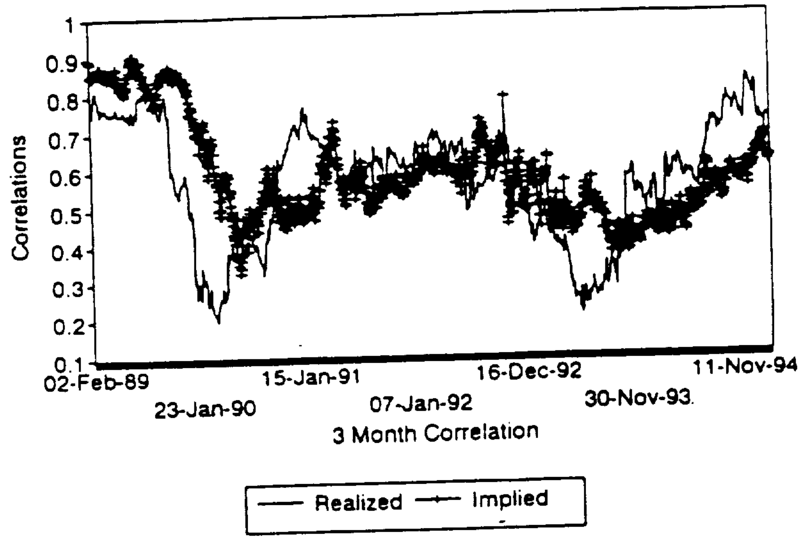
PLOTS OF REALIZED CORRELATION VS. CORRELATION FORECASTS



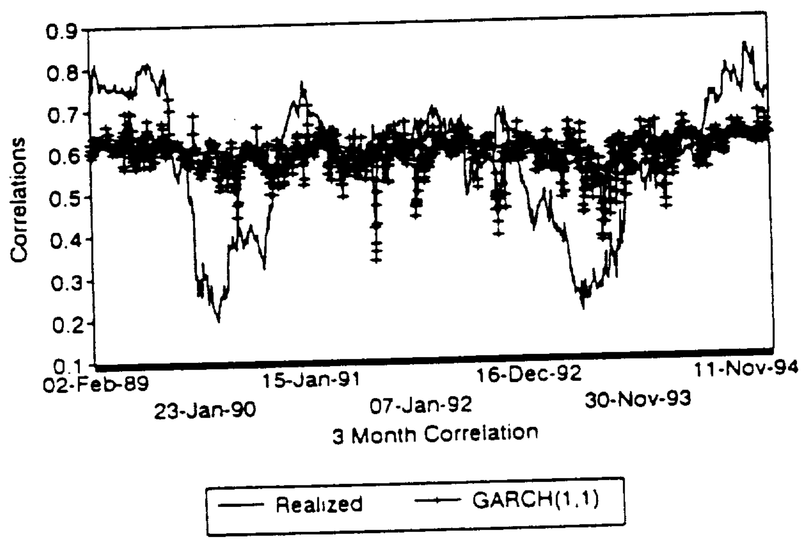




Realized vs. Implied Forecast
2 Feb. 1989 to 19 April 1995



Realized vs. GARCH(1,1) Forecast
2 Feb. 1989 to 19 April 1995



Realized vs. RiskMetrics Forecast
2 Feb. 1989 to 19 April 1995