NBER WORKING PAPER SERIES

INTERNATIONAL RESERVES UNDER ALTERNATIVE EXCHANGE RATE REGIMES AND ASPECTS OF THE ECONOMICS OF MANAGED FLOAT

Jacob A. Frenkel

Working Paper No. 287

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

October 1978

Previous drafts of this paper were presented at the conference on "The Economics of Flexible Exchange Rates", Institute for Advanced Studies, Vienna, Austria, March 29-31, 1978 and the Carnegie-Rochester Conference held at Carnegie-Mellon University, November 18-19, 1977. I am indebted to Craig Hakkio for helpful comments, suggestions and efficient research assistance. In revising the paper I have benefited from useful comments by Russell Boyer, Kenneth Clements, Robert Hodrick, Edi Karni and David Laidler. The present version draws heavily on Frenkel (1978); it extends the analysis by including the framework for the analysis of managed float and by modifying the definitions of the measures of variability and scale that are used in the empirical work. The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

Working Paper 287
October 1978

Summary of

INTERNATIONAL RESERVES UNDER ALTERNATIVE EXCHANGE RATE REGIMES AND ASPECTS OF THE ECONOMICS OF MANAGED FLOAT

by

Jacob A. Frenkel

This paper contains an analysis of the role of international reserves under a regime of pegged exchange rates and under a regime of managed floating. It presents evidence on the stability of the demand for reserves during the periods 1963-72 and 1973-75. It is shown that the demand for reserves by developed countries differs from that of less-developed countries and that the system underwent a structural change by the end of 1972. In view of the drastic change in the international monetary system, the extent of the structural change has not been as large as might have been expected, thus leading to the observation that economic behavior seems to be more stable than legal arrangements. From the policy perspective it follows that the problems concerning the role of the International Monetary Fund in this context are as relevant at the present as they were in the past.

The paper concludes with a sketch of a stochastic framework for the analysis of the optimal degree of managed floating. And its purpose is to suggest an additional set of variables which might be incorporated into the specification of the demand for international reserves. It is shown that the optimal degree of exchange rate flexibility depends on the stochastic nature of the shocks that the economy faces. The stochastic characteristics of the shocks include a distinction between real and monetary shocks, domestic and foreign shocks and depend on the covariances among the various shocks.

Professor Jacob A. Frenkel
Department of Economics
University of Chicago
1129 E 59th Street
Chicago, Illinois 60637

(312) 753-4516

June 1978

I. Introduction

One of the striking features characterizing the international monetary system since the early 1970's has been the continued use of international reserves even though, legally, the system has been characterized as a flexible exchange rate regime.

In this paper I analyze the interrelationship between the determinants and patterns of reserve holdings and the international monetary system. Section II contains an empirical analysis of the demand for international reserves. In this context I analyze the differences in behavioral patterns of developed and less-developed countries. Section III deals with the empirical question of the timing at which the system moved from pegged to floating rates. The interest in this question stems from the belief that timing of changes in economic behavior need not correspond to, or be associated with, the timing of changes in legal commitments. One of the conclusions emerging from the analysis is that the extent of the change in economic behavior (as far as the holdings of international reserves is concerned) has not been as large as one might have predicted. Countries have preferred to move to a system of managed floating that is intermediate between the extremes of fixed rates and of clean float. Section IV provides a sketch of an analytical framework for the analysis of a managed float regime. Some concluding remarks and suggested extensions are contained in Section V.

II.1 The Determinants of the Demand for International Reserves

Earlier studies of the demand for reserves considered the variability of international receipts and payments as an important argument in the demand function (e.g., Kenen and Yudin, 1965; Clower and Lipsey, 1968; Archibald and Richmond, 1971). In addition, it has been suggested that the demand function also depends on the propensity to import (e.g., Heller, 1966; Kelly, 1970; Clark, 1970a; Flanders, 1971; Frenkel, 1974a, 1974b, 1978; Hipple, 1974; Iyoha, 1976). These and other studies have recently been surveyed by Grubel (1971), Williamson (1973), Claassen (1974) and Cohen (1975).

The choice of a variability measure as an argument in the demand function stems directly from the role of international reserves in serving as a buffer stock accomodating fluctuations in external transactions. Consequently, it has generally been expected that the demand for reserves is positively associated with the extent of these fluctuations.

The rationale for the use of the propensity to import as an argument is more involved. It stems from an application of the Keynesian priceless model of the foreign trade multiplier. According to that model, an external disequilibrium that is induced by a decline in export earnings, could be corrected by a decline in output proportional to the multiplier. The cost of output adjustment could be saved if the monetary authorities are able to run down their stock of international reserves thereby enabling them to finance the external deficit. Since the foreign trade multiplier (and thus the required output dampening in the absence of reserves) is inversely related to the marginal propensity to import, the popular approach argues that the cost of not having reserves, and hence the demand for reserves, is <u>inversely</u> related to the marginal propensity to import (see references op cit.). In the absence of data on the marginal propensity to import, earlier empirical studies have replaced it by the ratio of imports to income, i.e., by the average propensity (typically referred to as the degree of "openness" of the economy). The coefficient of the average propensity to import frequently appeared with the "wrong" (positive) sign when used to estimate the demand for reserves.

Using an adjustment mechanism which emphasizes the role of relative prices, price level and demand for money, it was previously shown (Frenkel, 1974a) that under certain assumptions, the demand for reserves was expected to be associated positively with the average propensity to import. This association was shown to be consistent with data for the period 1963-1967 for both Developed and Less-Developed Countries (Frenkel, 1974b). A simplified derivation of the association between reserve holdings and the propensity to import is presented in Appendix A. It is shown that the relationship between these two variables is, in general, not a clear cut, although, under some assumptions this relationship is expected to be positive.

In what follows I present cross-sectional estimates of the demand for international reserves where, in addition to the above mentioned variables, I have also included a scaling variable as one of the determinants of the demand.[1]

II.2 Cross-Sectional Estimates of the Demand for International Reserves

The empirical analysis includes data from 22 Developed Countries and 32 Less-Developed Countries (LDC's) and covers the period 1963-75. Appendix B contains the list of countries and definitions of variables used

[1] An additional variable which in principle should have been included in the list of the determinants of reserve holdings is the opportunity cost of holding reserves. In practice, a large fraction of international reserves is held in the form of short term interest-bearing assets and thus, the opportunity cost is the difference between the alternative yield and the rate of return on reserves. Previous studies faced serious difficulties in estimating this cost. Clark (1970a) decided to exclude this variable from his estimation. Heller (1966) assumed that the cost was the same for all countries (5 percent). Kenen and Yudin (1965) used income per capita as a proxy, and found that it had the "wrong" sign and was not significant. Kelly (1970) used in addition to per capita income, the value of foreign assets and liabilities as proxies, but found that in all cases the latter appeared with the "wrong" sign while in some cases the former had the "wrong" sign. Courchene and Youssef (1967) used the long-term interest rate as a proxy, and found that in five out of nine cases its coefficient was not significantly negative. All these attempts taken together, provided the rationale for not including this variable in the estimating equations.

in the analysis. The classification of countries as Developed and Less-Developed is based on that of the IMF. The choice of countries and the period of analysis was determined by the availability of continuous series of dat As indicated above, the demand function was assumed to depend on three variables: (i) a measure of variability of international receipts and payments denoted by σ; the value of σ for each year was estimated by computing the standard deviation over the previous 15 years of the trend-adjusted annual changes of the level of reserves;[3] (ii) a scaling variable measuring the size of international transactions represnted by the level of GNP, Y; in the few cases where this was not available, it was replaced by Gross Domestic Products, GDP, and (iii) the average propensity to import, m = IM/Y, where IM denotes imports.

The functional form of the demand function was assumed to be:

$$\ell n\ R = \alpha_0 + \alpha_1\ \ell n\ \sigma + \alpha_2\ \ell n\ Y + \alpha_3\ \ell n\ m + u \tag{1}$$

where u denotes an error term.

Tables 1 and 2 present for each year the cross-sectional ordinary-least-squares estimates of the demand for reserves by developed and less-developed countries. In all cases the coefficients have the expected positive sign and in most cases these coefficients are statistically significant at the 95 percent confidence level.

---

[2] From the list of countries for which data were available, two countries--Canada and the United States--were excluded from the analysis. Canada was excluded since it had a flexible exchange rate system during most of the period for which the variability measure was calculated. Since the discussion focuses on the behavior of countries as demanders of resérves, the exclusion of the United States as the main supplier seems justified. This relates to one of the limitations of the present approach of estimating the demand for reserves by means of a single regression equation. To the best of my knowledge, the potential difficulty of an identification problem is shared by all previous empirical studies of the demand for reserves.

[3] A limitation of this measure is that actual changes in reserves need not provide an exact measure of the disturbance, since countries may use other policies. Kenen and Yudin (1965) and Heller (1966) who have used similar measures, have assumed that its estimates are not materially affected by national policies.

TABLE 1

ESTIMATED EQUATION: $\ell n\ R = \alpha_0 + \alpha_1\ \ell n\ \sigma + \alpha_2\ \ell n\ Y + \alpha_3\ \ell n\ m + \mu$

DEVELOPED COUNTRIES, N = 22

(standard errors in parentheses)

Year	Constant	$\ell n\ \sigma$	$\ell n\ Y$	$\ell n\ m$	R^2	s.e.
1963	4.141 (.701)	.618 (.211)	.441 (.214)	.781 (.271)	.90	.436
1964	4.243 (.809)	.608 (.255)	.399 (.259)	.738 (.276)	.88	.480
1965	4.464 (.916)	.507 (.294)	.546 (.292)	.867 (.314)	.85	.538
1966	4.448 (.914)	.596 (.288)	.506 (.286)	1.042 (.329)	.86	.540
1967	4.291 (.899)	.650 (.280)	.495 (.270)	1.096 (.316)	.86	.556
1968	4.254 (1.114)	.570 (.324)	.582 (.280)	1.019 (.341)	.85	.587
1969	4.282 (.940)	.510 (.269)	.483 (.239)	.735 (.298)	.84	.551
1970	4.006 (.817)	.569 (.249)	.446 (.225)	.627 (.292)	.85	.534
1971	4.378 (.771)	.352 (.242)	.661 (.230)	.405 (.303)	.86	.539
1972	3.568 (.620)	.733 (.219)	.209 (.212)	.265 (.237)	.90	.425
1973	3.915 (.667)	.675 (.272)	.291 (.248)	.533 (.283)	.88	.475
1974	3.171 (.796)	.803 (.325)	.257 (.319)	.676 (.334)	.87	.533
1975	3.366 (.944)	.668 (.348)	.510 (.298)	1.012 (.414)	.85	.606

TABLE 2

ESTIMATED EQUATION: $\ell n\ R = \alpha_0 + \alpha_1\ \ell n\ \sigma + \alpha_2\ \ell n\ Y + \alpha_3\ \ell n\ m + \mu$

LESS-DEVELOPED COUNTRIES, N = 22

(standard errors in parentheses)

Year	Constant	$\ell n\ \sigma$	$\ell n\ Y$	$\ell n\ m$	R^2	s.e.
1963	-.766 (.867)	.285 (.170)	.825 (.209)	.985 (.253)	.78	.602
1964	-.767 (.870)	.249 (.175)	.830 (.209)	.990 (.251)	.77	.599
1965	-.715 (.764)	.228 (.147)	.851 (.176)	.998 (.214)	.80	.532
1966	-.335 (.970)	.279 (.177)	.722 (.215)	.775 (.257)	.70	.661
1967	-.679 (1.107)	.440 (.212)	.654 (.259)	.614 (.311)	.66	.797
1968	-.839 (1.072)	.341 (.195)	.749 (.232)	.754 (.278)	.69	.742
1969	-1.983 (1.045)	.210 (.200)	.973 (.235)	.905 (.270)	.77	.666
1970	-2.265 (1.165)	.244 (.216)	.957 (.256)	.767 (.288)	.75	.726
1971	-2.165 (1.160)	.159 (.228)	1.050 (.261)	1.069 (.289)	.76	.697
1972	-1.923 (.913)	.230 (.182)	.987 (.201)	.938 (.219)	.83	.586
1973	-.677 (.837)	.565 (.166)	.621 (.176)	.530 (.171)	.87	.530
1974	-.091 (1.103)	.701 (.211)	.413 (.222)	.246 (.225)	.79	.681
1975	-.512 (.921)	.556 (.137)	.537 (.156)	.416 (.167)	.80	.655

In summary it should be noted that the overall fit of the regressions reported in Tables 1 and 2 (as measured by the coefficients of determination R^2) is very satisfactory. This point is noteworthy since these results pertain to cross-sectional estimates. Of special interest is the good fit of the cross-sectional regressions for the last few years reported in Tables 1 and 2. During the latter years of the sample period, the international monetary system moved towards a greater flexibility of exchange rates. This move was expected to result in different as well as in less stable estimates of the parameters of the demand for reserves. Since, as will be shown below, the cross-sectional estimates seem to have remained stable (at least during the periods 1963-72 and 1973-75), more efficient estimates may be obtained by ppoling the time series with cross sections.

II.3 Pooled Time-Series and Cross-Sections: Pegged Versus Floating Exchange Rates

In order to examine the effect of the move to a regime of flexible exchange rates, I have divided the sample into two periods: the pegged exchange rate period (1963-72) and the flexible exchange rate period (1973-75). In Section III I provide a formal justification for this division.

To the extent that the coefficients of the cross-sectional equations remained stable within each of the periods 1963-72 and 1973-75, one may obtain more efficient estimates by pooling the time-series with the cross-sections. Table 3 contains the ordinary-least-squares estimates of the pooled regression for both periods. In all cases the coefficients are positive and significant at the 95 percent confidence level.

I turn now to a comparison of the regression coefficients between Developed and Less-Developed countries during the two periods. The first method used for this comparison was the dummy variables method as outlined by Gujarti (1970). According to this method, each and every coefficient was

TABLE 3

ESTIMATED EQUATION: $\ell n\ R = \alpha_0 + \alpha_1\ \ell n\ \sigma + \alpha_2\ \ell n\ Y + \alpha_3\ \ell n\ m + \mu$

POOLED TIME SERIES-CROSS SECTION
(standard errors in parentheses)

Year	Group	Constant	$\ell n\ \sigma$	$\ell n\ Y$	$\ell n\ m$	R^2	s.e.
1963-72	DC's NOB = 220	4.129 (.247)	.593 (.076)	.467 (.072)	.760 (.089)	.86	.504
1963-72	LDC's NOB = 320	-1.167 (.293)	.284 (.057)	.840 (.066)	.858 (.078)	.75	.642
1973-75	DC's NOB = 66	3.521 (.469)	.666 (.183)	.364 (.168)	.582 (.193)	.84	.554
1973-75	LDC's NOB = 96	-.349 (.531)	.590 (.094)	.514 (.101)	.361 (.103)	.80	.625

allowed to differ between developed and less-developed countries by including dummy variables pertaining to data for developed countries. The estimated coefficients of the dummy variables are reported in Table 4 where it is seen that for the first period the coefficients of the constant term and the variability measure are significantly higher for developed countries while the coefficients of income are lower. For the latter period, however, the two groups differ significantly only in their constant term. These estimates reveal a larger degree of economies of scale in the demand for reserves (as measured by the income elasticity) in Developed Countries than in LDC's. It is also apparent that the demand for reserves by LDC's is less sensitive to the variability measure than the demand by Developed countries. The differences in the income elasticities may reflect differences in the character of the financial systems. The financial system in Developed countries is more sophisticated than its counterpart in LDC's and thus entailing larger possibilities for economies of scale in reserve management. In addition, Developed countries have a larger access to world capital markets and greater facilities for swap agreements. The higher sensitivity of Developed countries to the variability measure may reflect larger reluctance to cope with unexpected shocks by imposing trade controls and

TABLE 4

TESTS FOR DIFFERENCE BETWEEN DEVELOPED AND LESS-DEVELOPED COUNTRIES
(standard errors in parentheses)

Year	Estimated Dummies for Developed Countries				F-statistic for Chow-Test
	Constant	$\ell n\ \sigma$	$\ell n\ Y$	$\ell n\ m$	
1963–72	5.296 (.395)	.309 (.103)	–.373 (.104)	–.098 (.126)	78.09
1973–75	3.870 (.717)	.076 (.217)	–.150 (.206)	.221 (.230)	11.21

Note: The estimated dummies measure the differential values of the constant and the slope coefficients for developed countries from the corresponding estimates for LDC's. The F-statistics for the Chow-test correspond to the null hypothesis that the regression coefficients are the same for developed and less-developed countries. The relevant degrees of freedom for the two periods are F(4, 540) and F(4, 162), respectively. The null hypothesis is rejected since the values of the F-statistics exceed the critical values at conventional confidence levels.

restrictions.[4] These differences have diminished significantly with the move towards greater flexibility of exchange rates.

The dummy variables method focuses on comparisons between individual coefficients. The second method that was employed in the comparison between Developed and Less-Developed countries was that of the Chow-test following the procedure described by Fisher (1970). The results of this test are also reported in Table 4 where the F-statistics correspond to the null-hypothesis that the OLS regressions in Table 4 do not differ from each other. The null hypothesis is rejected since the values of the F-statistics exceed the critical values at the 95 percent confidence level. Thus, also the Chow-test leads to the conclusion that Developed and Less-Developed countries reveal different behavior concerning the holdings of international reserves. It is of interest to explore in greater detail the pattern of the move to the floating rates regime, as are implied by the characteristics of reserve holdings.

III. When Did the System Move from Pegged to Floating Rates?

The analysis in the previous sections made a distinction between the periods 1963-72 and 1973-75. The presumption was that the evolution of the international monetary system from pegged exchange rates to floating exchange rates might have resulted in a structural change in the demand for international reserves. In the present section I examine formally the timing and extent of the structural change following the method proposed by Quandt (1958,1960).[5]

---

[4] For further elaboration of these and other interpretations see Frenkel (1974b).

[5] For this and other methods of analyzing switching regressions see Goldfeld and Quandt (1976, Chs. 1, 4).

Consider a situation in which a structural change occurred at year t* within the period 1,...,T and assume that the demand for reserves corresponding to the two regimes (before and after t*) can be characterized by two distinct regression equations like (2) - (3)

$$(2) \qquad y_{nt} = x'_{nt}\beta_1 + u_{1nt}, \qquad t \leq t^*$$

$$(3) \qquad y_{nt} = x'_{nt}\beta_2 + u_{2nt}, \qquad t > t^*$$

where u_{1nt} and u_{2nt} are the error terms that are assumed to be distributed as $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2)$, β_1 and β_2 are the vectors of the regression coefficients corresponding to the two regimes, and where n = 1,...,N denote the countries (for the Developed countries N = 22 and for the LDC's N = 32). The analysis of the timing of the structural change amounts to searching for the value of t*. Quandt's method of estimating t* involves the following: first, a maximization of the likelihood function (4) conditional on t*:

$$(4) \qquad L(y|t^*) = \left(\frac{1}{2\pi}\right)^{NT/2} \sigma_1^{-Nt^*} \sigma_2^{-N(T-t^*)} \exp\left\{-\frac{1}{2\sigma_1^2} \sum_{t=1}^{t^*} \sum_{n=1}^{N} (y_{nt} - x'_{nt}\beta_1)^2 - \frac{1}{2\sigma_2^2} \sum_{t=t^*+1}^{T} \sum_{n=1}^{N} (y_{nt} - x'_{nt}\beta_2)^2\right\}$$

(where N denotes the number of countries and T denotes the number of years), and second, determination of the breakpoint t* as the value which yields the highest maximum likelihood L(y|t*). The application of this procedure to determining the breakpoint in the demand for reserves, yields 1973 and 1972 as the estimates for t* for Developed and Less-Developed countries, respectively.

Based on these results, 1972 is used as the estimate of the break point for both the Developed and the Less Developed Countries, implying that the two subperiods are 1963-72 and 1973-75. This choice is based on the fact that while for LDC's 1972 seems to be an unambiguous estimate for the date of the structural change, the case for Developed Countries is less clear cut; it is therefore assumed that, by analogy with the LDC's, the break point for Developed Countries also occurred in 1972. The validity of this assumption can be tested by applying the likelihood-ratio test to the null hypothesis that no switch took place between 1972 and 1973. The likelihood ratio statistic is

$$\phi = \hat{\sigma}_1^{N\hat{t}^*} \hat{\sigma}_2^{N(T-t^*)} / \hat{\sigma}^{NT}$$

where $\hat{\sigma}^{NT}$ is the estimated standard deviation of the residuals from the single regression estimated over the entire period 1963-75.

According to the null hypothesis, $-2 \ln \phi$ is distributed $\chi^2(5)$. The null hypothesis is rejected for both groups of countries. For the LDC's the value of $-2 \ln \phi$ is 16.4 while the corresponding value for Developed Countries is 21.2, both exceeding 15.1--the critical value at the 99 percent confidence level.[6] The practical implication is that for the purpose of estimation, data from the period 1963-72 should not be pooled with those from the period 1973-75 and that the structural change occurred by the end of 1972.

In addition to the above tests one may also use a Chow-test to test for the equality of the regression coefficients between the two periods.[7] The resulting values of the F-statistics relevant for testing the null hypothesis

[6] It should be noted that the application of this method to the problem at hand is not without conceptual difficulties since it requires differentiating the likelihood function with respect to t^*. It should also be noted that the analysis assumes that the structural change has taken place at a given point in time. An alternative approach would allow for a gradual evolution, and would estimate regression equations with variable coefficients. For references and discussion of the properties of the distribution of $-2 \ln \phi$ see Goldfeld and Quandt (1976, Ch. 1).

[7] In implementing the Chow-test one pretends that t^* was known a priori. Goldfeld and Quandt (1976, Ch. 1) provide reference to evidence that the Chow-test yields satisfactory results.

are 4.97 for Developed Countries and 3.79 for LDC's--well above the critical values at the 99 percent confidence level. Thus, the Chow-test as well leads to the rejection of the null hypothesis. The overall inference is that the system has changed by the end of 1972. It is this conclusion which provides the rationale for the pattern of the intertemporal pooling that is employed in Tables 3-4.[8] It should be noted that even though, as a statistical matter, the system underwent a structural change, the extent of the change has not been l enough to make obsolete parameter estimates that are based on the period 1963-72. In fact, forecasting reserve holdings during the period 1973-75 based on paramete estimates from 1963-72, yield extremely good predictions (for details see Frenkel, 1978).

The interpretation of the relative stability of the patterns of reserve holdings is quite obvious: during the pegged rate regime the rate was adjustable rather than fixed and during the so-called floating rate regime the rate has beer managed rather than free. Economic behavior seems to be more stable than legal arrangements. Central banks have revealed that their choice is for neither of the extreme exchange rate systems, but have rather, preferred to have the intermediate system of managed floating. Since managing the float requires internatic reserves, it seems relevant to analyze the determinants of the degree to which countries will attempt to manage the float. In the following section I deal with some analytical aspects of managed floating.

[8]The above discussion associated the structural change with the change in the exchange rate regime; it could of course reflect other (not necessarily unrelated) phenomena which occurred in the early 1970's like the oil crisis and the commodity price boom.

IV. A Sketch of a Framework for the Analysis of Managed Float[9]

In this section I provide a sketch of an analytical framework for the analysis of the economics of managed floating. The analysis will highlight some variables which should be included as determinants of the demand for reserves under a regime of managed float.

Earlier discussions of the optimal exchange rate regime centered around comparisons between fixed and flexible exchange rate systems. More recent explorations--originating with Mundell's contributions in the early 1960's--have shifted to the determination of optimal currency areas within which exchange rates are fixed and between which exchange ares are flexible. The following analysis of the optimal managed float recognizes that the spectrum of possibilities is broader and that the rate of exchange between any pair of currencies need not be entirely fixed or flexible but rather it might be some optimal mix of the two extremes. In what follows I will sketch a simple analytical framework that highlights some of the determinants of the optimal degree of exchange rate flexibility. The analysis will emphasize the role of the stochastic structure of the various shocks which affect the economy, and its purpose is to suggest and additional set of variables which might be incorporated into the specification of the demand for international reserves under a regime of managed float.

Consider a small economy that is subject to two types of repetitive and serially uncorrelated shocks. These shocks, which are specified below in a manner similar to that of Fischer (1977), are referred to as real and monetary shocks.

---

[9] I am indebted to Michael Bazdarich for research assistance and suggestions concerning the material in this section. The material in this section draws on Frenkel (1976b).

Denote the supply of output by Y_t and let

$$(5) \qquad Y_t = ye^{\mu} \; ; \quad \mu \sim N(-\sigma_{\mu}^2/2, \; \sigma_{\mu}^2)$$

where μ is a stochastic shock (the "real" shock) with a constant variance σ_{μ}^2. The mean of the distribution of the real shock is chosen to be $-\sigma_{\mu}^2/2$ so as to assure that the expected value of output equals y (permanent income).

The second shock arises from the monetary sector. Let the demand for nominal cash balances L_t be

$$(6) \qquad L_t = kP_tY_te^{\varepsilon}; \quad \varepsilon \sim N(-\sigma_{\varepsilon}^2/2, \; \sigma_{\varepsilon}^2)$$

where k denotes the desired money-income ratio, P denotes the domestic price level and ε denotes the stochastic shock to the demand for money (the "monetary" shock).

The domestic price level is linked to the foreign price through the purchasing power parity relationship. Thus

$$(7) \qquad P_t = S_tP_t^*$$

where P_t^* denotes the foreign price level and S_t denotes the exchange rate (the price of foreign currency in terms of domestic currency). To simplify the analysis it is assumed that foreign prices are fixed, (and thus it will be denoted henceforth as P*), that purchasing power parity holds deterministically and that μ and ε are independent.[10]

[10] A more elaborate analysis would allow for disturbances which originate from the foreign sector and which are allowed to be correlated with domestic shocks. Another extension would allow for possible short run deviations from purchasing power parity in which case, in the short run changes in the exchange rate induce changes in relative prices. For a rationale for managed float that is based on this phenomenon see Mussa (1976); for recent empirical evidence on the short run relationship between exchange rates and relative prices see Dornbusch and Krugman (1977). In a recent analysis of the optimal foreign exchange intervention Boyer (1976) extends and applies Poole's framework (1970) to the problem at hand. Boyer assumes that real income is fixed and that the objective function is to minimize variability of prices.

Assume that the flow demand for money ΔM^d corresponds to a stock adjustment process such that

$$(8) \qquad \Delta M^d = \alpha(L_t - \bar{M}_t)$$

where $\bar{M}_t$ denotes the money stock at the beginning of the period.

Using equations (5) and (7) we may express the demand for money (6) as

$$(9) \qquad L_t = kS_t P^* y e^{\mu+\varepsilon}.$$

When the exchange rate is flexible, it will adjust so as to eliminate any stock disequilibrium in the money market, and thus ensuring that $L_t - \bar{M}_t = 0$. Using (9) it follows that when the exchange rate is flexible,

$$(10) \qquad kS_t P^* y e^{\mu+\varepsilon} - \bar{M}_t = 0.$$

The equilibrium exchange rate can be written as

$$(11) \qquad = \quad S_t = (\bar{M}_t / kP^* y) e^{-(\mu+\varepsilon)}$$

and the percentage change thereof is

$$(12) \qquad \ell n\ S_t - \ell n\ S_{t-1} = \ell n\ (\bar{M}_t / kS_{t-1} P^* y) - (\mu + \varepsilon).$$

The other extreme system is the fixed exchange rate system for which

$$(13) \qquad \ell n\ S_t - \ell n\ S_{t-1} = 0.$$

Using (12) and (13) we may define an index γ such that $0 \leq \gamma \leq 1$:

$$(14) \qquad \gamma = (\ell n\ S_t - \ell n\ S_{t-1}) / [\ell n\ (\bar{M}_t / kS_{t-1} P^* y) - (\mu + \varepsilon)].$$

The coefficient γ characterizes the whole spectrum of exchange rate regimes. When $\gamma = 0$, the system is that of a fixed exchange rate, and when $\gamma = 1$ the system is that of a freely flexible exchange rate. An intermediate value of γ indicates an intermediate degree of exchange rate flexibility. We will refer to the coefficient γ as the coefficient of managed float, or the intervention index. From the policy perspective, the basic question is to determine the optimal value of γ, and thus to determine the optimal degree of exchange rate flexibility.

In principle, if shocks were identifiable, the optimal policy would be to allow the exchange rate to correct monetary disturbances but not real disturbances.[11] We will assume, however, that information is incomplete and that during a given period only the joint outcome of the shocks $\mu + \varepsilon$ is known but not the separate values of μ and ε. It is this lack of complete information which necessitates a second-best policy of managed float.[12]

Assume that the objective is to minimize the losses due to imperfect information and that the policymaker wishes to minimize the quadratic loss function H:

$$\text{(15)} \qquad \text{Minimize } H = E[c_t - E(Y_t)]^2$$

where c_t denotes the rate of consumption which, from the budget constraint, equals the rate of income minus the real value of additions to real cash balances

$$\text{(16)} \qquad c_t = Y_t - \frac{\Delta M}{P_t} .$$

[11] For an analysis of this proposition see Fischer (1977).

[12] The assumption of the lack of complete information has been used by Gray (1976) to analyze partial wage indexation.

The previous relationships imply that

$$(17)\qquad [c_t - E(Y_t)] = y(e^{\mu} - 1) - \alpha ky[e^{\mu+\varepsilon} - (\bar{M}_t/kS_{t-1}P^*y)^{1-\gamma}e^{\gamma(\mu+\varepsilon)}],$$

and using (17) in (15) yields the loss function which is to be minimized with respect to the intervention index γ.

The above analysis has not specified the value of the money stock $\bar{M}_t$ at the beginning of the period. It is assumed, following Fischer (1977), that at the beginning of each period the monetary authority changes the money supply so as to compensate for past disturbances according to

$$(18)\qquad \bar{M}_t = kS_{t-1}P^*y.$$

Substituting (18) in (17) and minimizing the loss function with respect to γ yield the following implications concerning the choice of the optimal degree of exchange rate flexibility:

(i) In general, the optimal value of γ will be within the range (0, 1), so that the optimal exchange rate regime will correspond to neither of the extremes of a completely fixed or of a completely flexible rate regime.

(ii) The intermediate solution is more likely the higher is αk--the propensity to save out of transitory income.

(iii) When the only shocks to the system are real ($\sigma_\varepsilon = 0$), the optimal solution is that of fixed rates ($\gamma = 0$); and conversely, when the only source of shocks is monetary ($\sigma_\mu = 0$), the optimal solution is that of freely floating rates ($\gamma = 1$).

(iv) The higher the variance of the monetary shock, the larger will be the optimal value of γ while the higher is the variance of the real shock the lower will be the optimal value of γ. That is, high variance of real shocks, ceteris paribus, tends to raise the desirability of greater fixity of exchange rates and conversley for high variance of monetary shocks.

Table 5 contains the results of illustrative computations of the optimal degree of the intervention index γ for alternative values of αk--the propensity to save out of transitory income--and for alternative assumptions concerning the magnitudes of the various shocks. These results illustrate the above propositions.

The analysis in this section provides a simple framework for determining the optimal degree of exchange rate flexibility. It highlights the unique role of the stochastic structure of the economy and suggests, therefore, that the magnitude of and the relationship between these variables are among the determinants of the demand for reserves and should be incorporated

TABLE 5

OPTIMAL MANAGED FLOAT FOR ALTERNATIVE VALUES OF REAL AND MONETARY DISTURBANCES AND SAVING PROPENSITIES

σ_μ \ σ_ε	$\alpha k = .5$					$\alpha k = 1$				
	.01	.03	.05	.07	.09	.01	.03	.05	.07	.09
.01	.0	.80	.92	.96	.98	.50	.90	.96	.98	.99
.03	.0	.47	.69	.80	.87	.10	.50	.74	.85	.90
.05	.0	.0	.0	.33	.53	.04	.27	.50	.67	.77
.07	.0	.0	.0	.0	.25	.02	.16	.34	.50	.63
.09	.0	.0	.0	.0	.0	.01	.10	.24	.38	.51

into the analysis of the demand for international reserves under a regime of managed float.

As a general comment on the analysis in this section it should be noted that monetary policy and foreign exchange intervention are treated as close substitutes. In fact, as a first approximation, these two policies are non-distinguishable. It is believed that this feature of the model is much closer to reality than would be the other extreme in which monetary policy and foreign exchange policies are viewed as two independent policy instruments.

The special role of the exchange rate should also be noted. In the above framework the exchange rate (and thereby the price level) is determined by considerations of asset market equilibrium. This characteristic is in accord with the recent developments of the asset market approach to the analysis of exchange rates.[13] An important limitation of the model is the absence of an explicit incorporation of an integrated world capital market which would relfect itself in the capital account of the balance of payments.

Finally, it should be noted that the present specification of the nature of the shocks is somewhat biased in favor of government intervention since the shocks have been presumed to originate from the instability of the private sector rather than from the actions of government policies. Furthermore, the concept of the optimal managed float and thereby the derived demand for international reserves were developed as a policy prescription for the monetary authorities. This was motivated by realism and could be rationalized in terms of the assumption that the monetary authorities possess superior information concerning its own actions as well as concerning the official holdings of international reserves. In principle, however, much of the optimal mix could also be performed by the private sector.

[13]See for example Dornbusch (1976), Frenkel (1976a), Kouri (1976), Mussa (1976), Bilson (1978) and Frenkel and Johnson (1978).

V. Concluding Remarks

This paper contained an analysis of the role of international reserves under a regime of pegged exchange rates and under a regime of managed float. It presented evidence on the stability of the demand for reserves during the periods 1963-72 and 1973-75. It was shown that the demand for reserves by developed countries differs from that of less-developed countries and that the system underwent a structural change by the end of 1972. In view of the drastic changes in the international monetary system, the extent of the structural change (in particular with reference to the behavior of developed countries) has not been as large as one might have expected.[14] This finding led to the observation that economic behavior seems to be more stable than legal arrangements. The evidence show that countries have continued to hold and use international reserves. Likewise, countries have chosen to manage their exchange rates rather than let it float freely. From the policy perspective, it follows that the problems concerning the provision of international reserves and the discussions concerning the role of the IMF in this context are as relevant at the present as they were in the past.[15] The paper concluded with a sketch of a stochastic framework for the analysis of the optimal degree of managed floating.

The analysis in this paper would benefit from several extensions. First, it was assumed that, during the sample period, countries were "on" their long-run demand functions. An extension would allow for a distinction between short-run and long-run demand functions and would examine the determinants of the speed of adjustment along the lines of Clark (1970b). Second,

---

[14] For analyses of the effects of the move to floating exchange rates on the demand for and the optimal provision of international reserves see Grubel (1976), Makin (1974) and Williamson (1976). For evidence on reserve use during the two regimes see Suss (1976) and for a comparison of the estimated coefficients in the Canadian case see Saidi and Barro (1977).

[15] On this issue see, however, Haberler (1976).

an examination of the residuals from the estimated equations revelas the existence of persistent negative residuals for some countries (e.g., U.K. and New Zealand) and positive residuals for other (e.g., Switzerland). An extension would refine the grouping of countries by allowing for country-specific factors. In this connection it would be of interest to test the hypothesis that countries behave so as to trade-off large stocks of reserves for high speeds of adjustment; some preliminary results in Bilson and Frenkel (1978) are consistent with this hypothesis.

Third, the specification of the demand for reserves was assumed to remain unchanged as between the two exchange rate regimes. In principle, however, the specification of the demand for reserves during a regime of managed float should be derived from the analysis of the determinants of countries' decisions concerning the degree of optimal foreign exchange intervention (i.e., the optimal managed float). In Section IV it was shown that the optimal degree of managed float depends on the details of the stochastic structure of the various shocks which affect the economy. It suggests, therefore, that these stochastic characteristics should be incorporated into the specification of the demand for reserves under a regime of managed float.[16]

Finally, the analytical framework underlying the literature on the demand for international reserves needs to be tied up with the framework underlying the literature on the monetary approach to the balance of payments. The monetary approach to the balance of payments emphasizes the considerations of monetary equilibrium and highlights the fact that when the monetary authorities peg the rate of exchange, they lose control over the nominal monetary stock. In many of the simplified versions of the monetary approach to the balance of payments, credit policies are viewed as the exogenous variable which, along with the path of the demand for money, determine (as a residual) the path of

---

[16] On the relationship between the stochastic structure of the economy and the estimated coefficients see Lucas (1976).

international reserves.[17] In this framework little attention has been paid to incorporating into the analysis the possibility that the monetary authorities may have preferences concerning the composition of their assets.[18] These preferences are emphasized in the analytical framework underlying the demand for international reserves. In that literature, however, little attention has been paid to the conditions of monetary equilibrium. A useful extension would combine these two strands of analysis. In such an extension, credit policies would not be viewed as exogenous but rather, international reserves and domestic credit would be determined jointly so as to satisfy simultaneously equilibrium in the money market as well as equilibrium in the composition of central banks' assets.

---

[17] See for example the various studies in Frenkel and Johnson (1976).

[18] For an exception which incorporates the monetary authorities' reaction function into the analytical framework of the monetary approach to the balance of payments see Ujiie (1978).

APPENDIX A

Reserve Holdings and the Average Propensity to Import

In this appendix I derive the relationship between long run holdings of international reserves and the average propensity to import. It is shown that in contrast with the implications of the simple foreign trade multiplier, the relationship is not a clear cut one. Furthermore, it is shown that according to a specific interpretation of the "small country assumption," there exists a positive long run relationship between openness (the average propensity to import) and reserve holdings.

Consider the following simple two-goods model of a fully employed economy. The economy produces and consumes the quantities $[X_1, X_2]$ and $[x_1, x_2]$ respectively, and exports the first commodity. Nominal income (Y) is:

$$Y = P_1X_1 + P_2X_2 \tag{A.1}$$

where P_1 and P_2 are the money prices of the two goods. Denoting a percentage change in a variable by a circumflex (e.g., $\hat{X} = dX/X$), and using the fact that for small changes $P_1dX_1 + P_2dX_2 = 0$ (the envelope theorem), one may write the percentage change in nominal income as:

$$\hat{Y} = \theta_1\hat{P}_1 + \theta_2\hat{P}_2 \tag{A.2}$$

where

$$\theta_1 = \frac{P_1X_1}{Y}; \quad \theta_2 = \frac{P_2X_2}{Y}$$

and $\theta_1 + \theta_2 = 1.$

In equation (A.2), θ_i denotes the relative share in <u>production</u> of good i (i = 1, 2). Let consumer price level (P) be a linear homogenous function of the money prices of goods with constant elasticities that are equal to the relative shares of expenditures on these goods in total expenditures. Thus:

$$\text{(A.3)} \qquad \hat{P} = \beta_1\hat{P}_1 + \beta_2\hat{P}_2$$

where

$$\beta_1 = \frac{P_1x_1}{Y}\ ; \quad \beta_2 = \frac{P_2x_2}{Y}$$

and

$$\beta_1 + \beta_2 = 1.$$

In equation (A.3), β_i denotes the average propensity to consume good i (i = 1, 2). The following analysis is confined to the long run when expenditures are equal to income.

The real value of income in terms of the consumer basket is $y = Y/P$;

$$\text{(A.4)} \qquad y = \frac{P_1X_1 + P_2X_2}{P}$$

using (A.2) - (A.3) in (A.4), the percentage change in real income is

$$\text{(A.5)} \qquad \hat{y} = (\beta_2 - \theta_2)(\hat{P}_1 - \hat{P}_2).$$

Equation (A.5) emphasizes the fact that a change in the terms of trade (P_1/P_2) raises real income if the relative share in production of the good whose relative price has risen exceeds its relative share in consumption. In particular, since the second commodity is the imported good, $\beta_2 > \theta_2$ and thus when $\hat{P}_1 - \hat{P}_2 > 0$ real income rises. Using the above notations, the average propensity to import is $(\beta_2 - \theta_2)$.

The stock demand for nominal cash balances (M^d) is assumed to depend on the price level and on real income. This demand function is homogenous of degree one in all prices:

$$(A.6) \qquad M^d = PL(y).$$

The long run holdings of international reserves by the monetary authorities is assumed to be an increasing function of the central bank's assets. For simplicity, it is assumed that in the long run the money supply is proportional to the stock of reserves as it is when the money multiplier is fixed and when the operations of the monetary authorities are confined to pegging the exchange rate. By an appropriate choice of units, the proportionality factor is set to equal unity. It should be noted, however, that the following qualitative results do not depend on this simplification.

At equilibrium the existing stock of cash balances (reserves) equals the desired stock:

$$(A.7) \qquad R = PL(y)$$

where R denotes the stock of reserves.

The percentage change of (A.7) is

$$(A.8) \qquad \hat{R} = \hat{P} + \eta\hat{y}; \quad \eta = (\partial L/\partial y)y/L$$

where η denotes the income elasticity of the demand for real cash balances. Using (A.3) and (A.5) in (A.8) yields

$$(A.9) \qquad \hat{R} = \beta_1\hat{P}_1 + \beta_2\hat{P}_2 + \eta(\beta_2 - \theta_2)(\hat{P}_1 - \hat{P}_2).$$

Equation (A.9) expresses the proportional change in reserves as a function of the proportional change in prices. The first two terms describe the effects of the change in the price level due to changes in its components, and the third term describes the effects of the change in real income due to a change in relative prices.

As can be seen from (A.9), for any given price level (so that the sum of the first two terms in (A.9) vanishes), the effect of a given change in the terms of trade on reserve holdings increases with $(\beta_2 - \theta_2)$. On these grounds alone one may expect to find a positive relationship between reserves and the average propensity to import. In general, however, the price level will also change and therefore, in addition to the value of the parameters, the relationship depends on whether the changes are due to variations in the price of exportables or in the price of importables. It is in this sense that the effect of openness on reserve holdings is not a clear cut one. However, if we adopt some variant of the "small country assumption" and assume that the price of importables (P_2) is given by the rest of the world, then equation (A.9) can be written as:

(A.10) $$\hat{R}/\hat{P}_1 = 1 - \beta_2 + \eta(\beta_2 - \theta_2)$$

which describes the effects of a given change in price on the holdings of international reserves.[19]

The effects of the magnitude of the average propensity to import on reserve holdings can be ascertained by differentiating (A.10) with respect to $(\beta_2 - \theta_2)$. An increase in the average propensity to import could result from either a decline in the production of importables and/or from an increase

[19] Clearly, this variant of the "small country assumption" does not imply that the terms of trade are fixed since the price of exportables is determined endogenously.

in the consumption of importables. Thus, from (A.10):

(A.11) $$\left.\frac{\partial(\hat{R}/\hat{P}_1)}{\partial(\beta_2 - \theta_2)}\right|_{\beta_2 = \text{constant}} = \eta$$

(A.12) $$\left.\frac{\partial(\hat{R}/\hat{P}_1)}{\partial(\beta_2 - \theta_2)}\right|_{\theta_2 = \text{constant}} = \eta - 1$$

Equation (A.11) states that for given consumption conditions the induced changes in international reserves (due to a given change in export price) are positively related to the average propensity to import. The smaller the production of importables, the larger will be the resulting change and thus the holdings of reserves. Equation (A.12) states that for given production conditions, the induced changes in international reserves are also positively related to the average propensity to import if the income elasticity of the demand for money does not fall short of unity.[20] Cross-sectional differences in the average propensity to import reflect differences in both production and consumption shares. Since empirical studies on the demand for money suggest that, in general, η does not fall short of unity, one can expect reserve holdings to depend positively on the average propensity to import. Further, if the main differences among countries are in production

[20] It was previously shown (Frenkel, 1974a) that in this case the positive association between reserve holdings and the average propensity to import will be maintained as long as η exceeds some critical number which is smaller than unity. The difference arises from the simplification adopted in the present analysis: the present analysis takes the change in export prices as being the exogenous disturbance while the previous analysis allowed export prices to be endogenously determined while taking changes in foreign demand as the exogenous factor. In the latter case, the average propensity to import will also determine (via the Marshall-Lerner condition) the effect of a given change in foreign demand on the price of exportables.

patterns rather than in consumption patterns (as in the Heckscher-Ohlin model of international trade), equation (A.11) implies that the positive association between reserves and the average propensity to import will be maintained as long as the income elasticity of the demand for money is positive. It should, however, be emphasized that the above conclusions have been derived for a specific interpretation of the "small country assumption" and for a specific assumption concerning the nature and origin of disturbances. In general the relationship between reserve holdings and the average propensity to import depends on the nature and origin of disturbances and is not clear cut. Furthermore, the formulation in this appendix assumes that under a pegged exchange rate system, the operation of the monetary authorities are confined to pegging the rate and thus yielding a "passive" path of reserves along the lines described by the monetary approach to the balance of payments. Consequently, the terms used were "reserve holdings" rather than the "demand for reserves." An extension would modify the assumption concerning the operations of the monetary authorities and would allow for credit policies that are aimed at achieving the desired stock of international reserves.

APPENDIX B

List of Countries and Definitions of Variables

1. List of Countries[a]

Developed Countries

United Kingdom
Austria
Belgium
Denmark
France
Germany
Italy
Netherlands
Norway
Sweden
Switzerland
Japan
Finland
Greece
Iceland
Ireland
Portugal
Spain
Turkey
Australia
New Zealand
South Africa

Less-Developed Countries

Argentina
Brazil
Chile
Columbia
Costa Rica
Dom. Republic
Ecuador
El Salvador
Guatemala
Honduras
Mexico
Nicaragua
Panama
Paraguay
Peru
Venezuala
Jamaica
Israel
Jordan
Egypt
Burma
Sri Lanka
China
India
Korea
Malaysia
Pakistan
Philippines
Thailand
Ghana
Sudan
Tunisia

[a] Classification based on the International Monetary Fund.

2. Definitions of Variables

All data sources are from the IFS tape obtained from the International Monetary Fund.

R - International reserves are measured in end-of-period 10^6 U.S. dollars. Reserves are defined as the sum of gold, SDR's, foreign exchange and reserve position at the Fund. When reserves are reported in local currency, they were converted to U.S. dollars using the end-of-period exchange rate.

IM - Imports are reported as cif in local currency units. These figures were then converted to U.S. dollars using the period average exchange rate. The figures used are measured in 10^9 U.S. dollars.

GNP - GNP and GDP are reported in local currency units. These figures were converted to U.S. dollars using the period average exchange rate. The figures are measured in 10^9 U.S. dollars.

m - The average propensity to import was defined as the ratio of imports to GNP. When the latter was unavailable, GDP was used instead.

σ - The variability measure. To calculate the value of σ_T^2 for year T for a given country, the following regression was first run: $R_t = \alpha + \beta_T t + u$ over $t = T - 15, \ldots, T-1$, and then using the estimated trend $\hat{\beta}_T$, σ_T^2 was defined

$$\sigma_T^2 = \sum_{t=T-15}^{T-1} (R_t - R_{t-1} - \hat{\beta}_T)^2/14$$

(except for 1963 for which, due to lack of data, σ_T^2 is based on the previous 14 observations). Thus, σ_T^2 is defined as the variance of the trend-adjusted changes in the stock of international reserves. A plot of the time-series of reserves revealed that the assumption of a linear trend seems more appropriate than that of an exponential trend.

REFERENCES

Archibald, G. C., and Richmond, J. "On the Theory of Foreign Exchange Reserve Requirements." The Review of Economic Studies XXXVIII (April 1971): 245-263.

Bilson, John F. O. "Rational Expectations and the Exchange Rate." In Frenkel, J. A. and H. G. Johnson (eds.) The Economics of Exchange Rates: Selected Studies. Reading, Mass.: Addison-Wesley Publishing Co., 1978.

Bilson, John. F. O. and Frenkel, Jacob A. "International Reserves and the Dynamics of Adjustment." Unpublished manuscript, University of Chicago, 1978.

Boyer, Russell S. "Optimal Foreign Exchange Market Intervention." Unpublished manuscript, University of Western Ontario, 1976.

Claassen, Emil M. "The Optimizing Approach to the Demand for International Reserves." Weltwirtschaftliches Archiv 3, (1974): 353-398.

Clark, Peter B. "Demand for International Reserves: A Cross-Country Analysis." Canadian Journal of Economics III (November 1970): 577-594.

Clower, Robert and Lipsey, Richard. "The Present State of International Liquidity Theory." American Economic Review LVII (May 1968): 586-594.

Cohen, Benjamin J. "International Reserves and Liquidity: A Survey." In Peter B. Kenen (ed.) International Trade and Finance. Cambridge: Cambridge University Press, 1975. Pp. 411-451.

Courchene, Thomas J. and Youseff, G. M. "The Demand for International Reserves." Journal of Political Economy 75, No. 4, Part I (August 1967): 404-13.

Dornbusch, Rudiger. "The Theory of Flexible Exchange Rate Regimes and Macroeconomic Policy." Scandinavian Journal of Economics 78, No. 2 (May 1976): 255-75.

Dornbusch, Rudiger and Krugman, Paul. "Flexible Exchange Rates in the Short Run." Brookings Papers on Economic Activity, No. 3 (1976): 537-75.

Fischer, Stanley. "Stability and Exchange Rate Systems in a Monetarist Model of the Balance of Payments." In R. Z. Aliber (ed.) The Political Economy of Monetary Reform. Cambridge: Cambridge University Press, 1976. Pp. 59-73.

Fisher, Franklin M. "Tests of Equality between Sets of Coefficients in Two Linear Regressions: An Expository Note." Econometrica XXXVIII (1970): 361-366.

Flanders, M. June. "The Demand for International Reserves." Princeton Studies in International Finance 27 (1971): 1-50.

Frenkel, Jacob A. "Openness and the Demand for International Reserves." In R. Z. Aliber (ed.) National Monetary Policies and the International Financial System. Chicago, Ill.: University of Chicago Press, 1974a, Pp. 289-298.

__________. "The Demand for International Reserves by Developed and Less-Developed Countries." Economica 41, No. 161 (February 1974b): 14-24.

__________. "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence." Scandinavian Journal of Economics 78, No. 2 (May 1976a): 200-24.

__________. "Comments on Exchange Rate Flexibility and Currency Areas." Presented at a conference on Money and International Monetary Problems, Martinique, 1976b. Forthcoming in the Proceedings edited by E. Claassen and P. Salin

__________. "International Reserves: Pegged Exchange Rates and Managed Float." In Brunner, K. and Meltzer, A. H. (eds.) Economic Policies in Open Economies Vol. 9 of the Carnegie-Rochester Conference Series on Public Policy, a Supplementary Series to the Journal of Monetary Economics, July 1978.

Frenkel, Jacob A. and Harry G. Johnson (eds.) The Monetary Approach to the Balance of Payments. London: Allen and Unwin and Toronto: University of Toronto Press, 1976.

__________. The Economics of Exchange Rates: Selected Studies. Reading, Mass.: Addison-Wesley, 1978.

Goldfeld, Stephen M. and Quandt, Richard E.(eds.) Studies in Nonlinear Estimation. Cambridge: Ballinger, 1976.

Gray, JoAnna. "Wage Indexation: A Macroeconomic Approach." Journal of Monetary Economics 2, No. 2 (April 1976): 231-46.

Grubel, Herbet G. "The Demand for International Reserves: A Critical Review of the Literature." Journal of Economic Literature 9 (December 1971): 1148-1166.

__________. "The Optimum Supply of International Reserves in a World of Managed Floating." Presented at the Marcus Fleming Memorial Conference on The New International Monetary System, Washington, D. C., November, 1976.

Gujarti, Damodar. "Use of Dummy Variables in Testing for Equality between Sets of Coefficients in Linear Regressions: A Generalization." The American Statistician (December 1970): 18-22.

Haberler, Gottfried. "How Important is Control over International Reserves?" Presented at the Marcus Fleming Memorial Conference on The New International Monetary System, Washington, D.C., November, 1976.

Heller, Robert H. "Optimal International Reserves." Economic Journal LXXVI (June 1966): 296-311.

Hipple, F. Steb. "The Disturbances Approach to the Demand for International Reserves." Princeton Studies in International Finance No. 35 (May 1974).

Iyoha, Milton A. "Demand for International Reserves in Less Developed Countries: A Distributed Lag Specification." The Review of Economics and Statistics 58, No. 3 (August 1976): 351-55.

Kelly, Michael G. "The Demand for International Reserves." American Economic Review 60 (September 1970): 655-667.

Kenen, Peter B. and Yudin, Elinor B. "The Demand for International Reserves." Review of Economics and Statistics 47 (August 1965): 242-250.

Kouri, Pentti J.K. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach." Scandinavian Journal of Economics 78, No. 2 (May 1976): 280-304.

Makin, John H. "Exchange Rate Flexibility and the Demand for International Reserves." Weltwirtschaftliches Archiv 110, No. 2 (1974): 229-43.

Mundell, Robert A. International Economics. Macmillan: New York, 1968.

Mussa, Michael. "The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy under a Regime of Controlled Floating." Scandinavian Journal of Economics 78, No. 2 (May 1976): 229-49.

Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro-Model." Quarterly Journal of Economics 84 (May 1970): 197-216.

Quandt, Richard E. "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes." Journal of the American Statistical Association 53 (Sept., 1958): 873-80.

__________. "Tests of the Hypothesis that a Linear Regression System Obeys Two Separate Regimes." Journal of the American Statistical Association 55 (June, 1960): 324-30.

Saidi, Nasser and Robert J. Barro. "Unanticipated Money Growth and Unemployment in Canada." Unpublished manuscript, University of Rochester, 1977.

Suss, Esther C. "A Note on Reserve Use under Alternative Exchange Rate Regimes." IMF Staff Papers 23, no. 2 (July 1976): 387-94.

Ujiie, Junichi. "A Stock Adjustment Approach to Monetary Policy and the Balance of Payments." In Frenkel, J. A. and H. G. Johnson (eds.) The Economics of Exchange Rates: Selected Studies. Reading, Mass.: Addison-Wesley, 1978.

Williamson, John. "International Liquidity: A Survey." Economic Journal 83 (September 1973): 685-746.

__________. "Generalized Floating and the Reserve Needs of Developing Countries." In D. M. Leipziger (ed.) The International Monetary System and Developing Nations. Washington, D. C.: Agency for International Development, 1976. Pp. 75-86.