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TWO NOTES ON INDETERMINACY PROBLEMS

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Two Notes on Indeterminacy Problems:
A. Collapsing Exchange-Rate Regimes and the Indeterminacy Problem

ABSTRACT

In this paper we show, in an example, that the arbitrary behavior which results in an indeterminacy in the time path of a flexible exchange rate and is associated with "badly behaved" speculation has a manifestation under a regime of fixed rates in an indeterminacy in the time path of government holding of international reserves. Thus, to the extent that arbitrariness is characteristic of agents' behavior it is not resolved but only masked by the fixing of exchange rates.

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Dynamic models of exchange rate determination employing the rational expectations assumption produce multiple solutions for the exchange rate. Of these solutions only one is stable and dependent only on market fundamentals.[1/] Other solutions exhibit a degree of arbitrariness and often allow explosive instability of the exchange rate, with the explosion driven only by agents' arbitrary (but rational) beliefs.

The assumption of rational expectations has provided discipline to the discussion of arbitrary speculative behavior in foreign exchange markets. In the past, the possibility of arbitrary or "badly behaved" speculation has been martialed as an argument against flexible rates and thus in favor of fixed rates.[2/] However, to the extent that arbitrariness is a characteristic of agents' behavior it is not resolved but only masked by the fixing of exchange rates. The objective of the present paper is to demonstrate that the possibility of multiple solutions also appears under a regime of fixed rates, manifesting itself in a multiplicity of paths of government international reserve holding.

For the case of fixed rates, the indeterminacy of reserve paths arises because of the possibility of arbitrary (but rational) speculative attacks on the currency whose price is being fixed. It is in the timing and magnitude of such an attack that the identical arbitrary behavior which might cause a solution multiplicity under flexible rates manifests itself under fixed rates.

In studying a price fixing scheme for gold, Salant and Henderson (1978) first proposed a model for the timing of a rational speculative attack on government stocks. The attack causes the collapse of the fixed-price regime and a shift to a floating-price regime. Krugman (1979) applied a similar idea to the collapse of a fixed exchange rate. However, in both cases, the fixed-price system is bound to collapse due to market fundamentals. In Salant and Henderson's paper private consumption of gold ultimately forces a speculative attack while in Krugman's paper the regime collapse is forced by a steady expansion of domestic credit.

As an example to demonstrate our claim that a fixed exchange-rate regime contains the same arbitrary element as a flexible-rate regime, we construct a simplified version of Krugman's model. The model is suitable for solving explicitly for the time of a collapse in the case either that government policy alone forces the collapse or that an arbitrary speculative attack causes the collapse.

I) A Model of the Collapse of a Fixed Exchange-Rate Regime

For our example we will employ a model of a small country in a world of purchasing power parity. We will assume that agents have perfect foresight and that the assets available for domestic residents are domestic money, foreign money, and foreign bonds. The government holds a stock of foreign currency for use in fixing the exchange rate. Foreign currency, which yields no monetary services to domestic agents, will be dominated by domestic money and by foreign bonds; therefore, private domestic agents will hold no foreign currency.

The model is built around a demand function for domestic money:[3/]

$$\frac{M^d(t)}{P(t)} = a_0 - a_1\, r(t) \qquad a_0 > 0,\ a_1 > 0 \tag{1}$$

where $M^d(t)$ is the nominal quantity of domestic money demanded, $P(t)$ the domestic price level and $r(t)$ the level of the domestic interest rate. We assume both purchasing power parity and uncovered interest parity, which respectively are stated as

$$P(t) = S(t) \cdot P^* \tag{1a}$$

$$r(t) = r^* + (\dot{S}(t)/S(t)) \tag{1b}$$

where P^* is the foreign price level, $S(t)$ is the level of the exchange rate, r^* is the level of the foreign interest rate, and $(\dot{S}(t)/S(t))$ is the actual and expected percentage rate of change of $S(t)$. The last of these definitions reflects our assumption of perfect foresight. The above may be manipulated to yield

$$M^d(t) = \beta S(t) - \alpha\dot{S}(t), \quad \beta > 0, \quad \alpha > 0 \tag{2}$$

where $\beta \equiv (a_0P^* - a_1P^*r^*)$, which we assume to be postive, and $\alpha \equiv a_1P^*$. We assume r^* and P^* to be constant.

The supply of money, $M(t)$, is the sum of domestic credit, $D(t)$, and the book value of international reserves, $R(t)$,

$$M(t) = D(t) + R(t). \tag{3}$$

We also assume that $D(t)$ grows at the constant rate μ,

$$\dot{D}(t) = \mu . \tag{4}$$

When the exchange rate is fixed, at some level $\overline{S}$, reserves adjust to keep the money market in equilibrium. With $S(t)$ fixed at $\overline{S}$ the quantity of reserves at any time t is

$$R(t) = \beta\overline{S} - D(t), \tag{5}$$

and the rate of change of reserves (the balance of payments) is obtained as the time derivative of (5), which is

$$\dot{R}(t) = -\dot{D}(t) = -\mu. \tag{6}$$

With a finite level of reserves and $\mu > 0$, the fixed-rate regime cannot last forever. The plan of our exposition is first to describe our model after the collapse of the fixed-rate regime and then to investigate the transition from fixed rates to the post-collapse flexible-rate system. After studying such a collapse due to market fundamentals ($\mu > 0$) we will describe an arbitrary collapse.

If a collapse of the fixed-rate regime takes place at some time Z then the government will have exhausted its reserve stock at time Z. In general the reserve stock is exhausted in a final speculative attack yielding a discrete downward jump in domestic money. Thus, following the attack, money market equilibrium is

$$M(Z_+) = \beta S(Z_+) - \alpha\dot{S}(Z_+), \tag{7}$$

where Z_+ is notation for the instant after the attack at Z, and $M(Z_+) = D(Z)$ as $R(Z_+) = 0$.[4/] Under this flexible rate regime, with $\dot{M}(t) = \dot{D}(t) = \mu$, we hypothesize the solution $S(t) = \lambda_0 + \lambda_1 M(t)$ and substitute this solution into the equation $M(t) = \beta S(t) - \alpha\dot{S}(t)$ finding $\lambda_0 = \alpha\mu/\beta^2$ and $\lambda_1 = 1/\beta$. Thus,

$$S(t) = \frac{\alpha\mu}{\beta^2} + \frac{M(t)}{\beta} \qquad t \geq Z. \tag{8}$$

In particular,

$$S(Z_+) = \frac{\alpha\mu}{\beta^2} + \frac{M(Z_+)}{\beta}. \tag{8a}$$

Prior to the collapse, equation (5) governs reserves and equation (5) implies

$$\bar{S} = \frac{R(Z_-) + D(Z_-)}{\beta} , \tag{9}$$

where Z_- is notation for the moment before the collapse at Z. The collapse in question is one that is forseen by agents so the exchange rate may not jump at the instant of collapse. If it were to jump capital gains at an infinite rate would also be forseen. The absence of such profits implies $\bar{S} = S(Z_+)$. Further, since D is continuous, when equation (9) is subtracted from (8a) and the result rearranged we obtain

$$R(Z_-) = \frac{\alpha\mu}{\beta} . \tag{10}$$

To find time Z we first use (6) to obtain

$$R(t) = R(0) - \mu t \quad , \quad t < Z \tag{11}$$

or

$$R(Z_-) = R(0) - \mu Z. \tag{11a}$$

Combining (10) and (11a) yields an equation determining Z

$$Z = \frac{R(0)}{\mu} - \frac{\alpha}{\beta} . \tag{12}$$

Equation (12) makes intuitive sense; an increase in reserves (R(0)) delays the collapse and an increase in μ hastens the collapse; as $\mu \to 0$ the collapse is delayed indefinitely.[5/]

The Indeterminacy Problem

The results of the previous section were predicated on the exchange rate's following market fundamentals after the collapse. In general, however, following the collapse the exchange rate obeys[6]

$$S(t) = A(Z)\exp\{\frac{\beta}{\alpha}(t-Z)\} + \frac{\alpha\mu}{\beta^2} + \frac{M(t)}{\beta}, \tag{13}$$

where A(Z) is an arbitrary constant determined at time Z, which previously we have set at zero. Allowing an arbitrary value of A(Z) we have

$$S(Z_+) = A(Z) + \frac{\alpha\mu}{\beta^2} + \frac{M(Z_+)}{\beta} \quad . \tag{14}$$

By using our previous argument about capital gains ($\bar{S} = S(Z_+)$) and the continuity of D, (14) may be combined with (9) to yield

$$R(Z_-) = \beta A(Z) + \frac{\alpha\mu}{\beta} \quad . \tag{15}$$

(11a) is then used to obtain

$$Z = (\frac{R(0)}{\mu} - \frac{\alpha}{\beta}) - \frac{\beta}{\mu} A(Z). \tag{16}$$

Equation (16) reveals that the timing of the collapse depends not only on market fundamentals $(R(0)/\mu - \alpha/\beta)$ but also on the constant A(Z). **A(Z)** captures the behavior possibly causing an indeterminacy in the path of the post-collapse floating rate.[7/] An increase in $A(Z)$ will hasten the collapse, causing it to take place at a higher value of $R(Z_-)$, and thus magnify the amplitude of the attack on the currency.

A special case of interest involves $\mu = 0$, which is a situation where the fixed-rate regime need never collapse in the absence of arbitrary behavior. For this case, equation (15) becomes $R(Z_-) = \beta A(Z)$. If $\mu = 0$ then $R(t)$ is some constant $R(0)$. Thus, the collapse would take place at any arbitrary date Z when agents choose to make $A(Z) = R(0)/\beta$.[8/]

Conclusion

The analysis in this paper has shown that the behavioral indeterminacy which arises for exchange-rate paths under flexible exchange rates manifests itself in an indeterminacy in the timing of a speculative attack in a fixed-rate regime. Such a behavioral indeterminacy, if present, is an economic force which is masked, not purged, by the fixing of exchange rates.

Footnotes

1/ In particular, we have in mind the models of Dornbusch (1976), Frenkel (1976) and Mussa (1976).

2/ Perhaps the best known discussion of "badly behaved" speculation is in Friedman (1953). More recently such problems have been studied by Britton (1970) and Driskill and McCafferty (1980).

3/ We view equation (1) as being the linear terms of a Taylor Series expansion of some non-linear function $M^d(t)/P(t) = \ell(r)$. Our linearization is appropriate for values of a_0, a_1, and $r(t)$ such that $a_0 - a_1 r(t) > 0$. We have adopted the present linearization rather than the more standard semi-log linearization to exploit the inherent linearity of our money supply definition, presented below.

4/ $D(t)$ is a continuous variable so $D(Z_+) = D(Z) = D(Z_-)$.

5/ A transition from fixed rates to flexible rates due to a collapse of the fixed-rate regime implies that the process governing reserves is not stationary. Such nonstationarity, if present in a transition, would make inappropriate a technique like that of Girton and Roper (1977), which was used to wed data from a fixed-rate regime with data from a flexible-rate regime.

6/ Our general solution is specific to the process $\dot{D}(t) = \mu$. For an arbitrary path of $D(t)$ and thus $M(t)$ we would have,

$$S(t) = A(Z)\exp\{\frac{\beta}{\alpha}(t-Z)\} + \exp\{\frac{\beta}{\alpha}t\} \int_t^\infty \frac{M(\tau)}{\alpha} \exp\{\frac{-\beta}{\alpha}\tau\}d\tau .$$

7/ Notice that if a collapse takes place due to the arbitrary element $A(Z)$ then the post-collapse exchange rate must be expected to follow what Flood and Garber (1980) have called a bubble. Such a bubble could be distinguished in the data using tests like those in Flood and Garber (1980).

8/ In Flood and Garber (1981) we refer to such a collapse as being generated entirely by mass hysteria.

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Two Notes on Indeterminacy Problems:
B. Further Evidence on Price Level Bubbles

ABSTRACT

In this note we study means of testing for price-level bubbles in inflationary situations. Our results contradict the outcomes of some tests reported in Flood and Garber's (1980) paper on bubbles in that the hypothesis of price-level bubbles cannot be rejected using likelihood ratio tests.

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Price solutions in dynamic models which assume rational expectations always contain an arbitrary, time-dependent element in additon to the exogenous forcing variables of the model. The arbitrary element, which enters the solution through self-generating expectations, is often explosive; and the number of such possible arbitrary elements in a solution is infinite. Many attempts have been made to preclude the market selection of such solutions through a priori reasoning.[1] However, Flood and Garber (1980), hereafter F+G, showed that the question of the existence of these "bubble" solutions is an empirical problem, subject to the usual methods of hypothesis testing. Using data from the German hyperinflation, they accepted the hypothesis that bubbles of the type generated in a particular rational expectations model were not a factor in the determination of the German price level.

However, the German episode is only one example; to establish confidence that bubbles are merely technical artifacts of rational expectations models, it is necessary to test for their existence in other cases. In this note, we extend in a number of ways the methods used by F+G to test for bubbles.[2] First, we examine data from a number of other hyperinflationary experiences in the same manner as F+G. Second, we use likelihood ratio tests in addition to the t-tests originally employed. Finally, we test for whether a bubble simultaneously existed across the countries which experienced hyperinflation in the early 1920's; this test is a means of implementing the conceptual experiment proposed by F+G in deriving asymptotic distributions for their test statistics. The results which we report are mixed: the t-tests lead to the acceptance of the hypothesis that bubbles did not exist while in most cases the likelihood ratio tests lead to the rejection of the hypothesis.

In section I we report our results from tests for individual countries. In section II we describe our test for the simultaneous existence of a bubble across countries.

I) Individual Country Tests

The model used here to test for the presence of a bubble is identical to that used by F+G. A Cagan-type money demand function is combined with an exogenous process determining money growth rates to produce a solution for the inflation rate. The parameters of this solution and of the money demand function are then estimated simultaneously with non-linear, cross-equation restrictions. Explicitly, the system of equations to be estimated is

$$(1) \quad m_t - p_t = \gamma + \theta \cdot t + \alpha \pi_t^e + \varepsilon_t \qquad \alpha < 0 \qquad \text{[Money Demand]}$$

$$(2) \quad \pi_t = \delta + \beta_1 \mu_{t-1} + \beta_2 \mu_{t-2} + \ldots + \beta_k \mu_{t-k} + A_o \Psi^t + v_t \qquad \text{[Inflation Solution]}$$

where m_t is the logarithm of the money stock, p_t is the logarithm of the price level, $\pi_t \equiv p_{t+1} - p_t$ is the actual inflation rate between time t and time t+1, and $\mu_{t-1} \equiv m_t - m_{t-1}$ is the percentage growth in the money stock between time t-1 and t. π_t^e is the expected inflation rate between t and t+1 based on time t information. All variables realized at or before time t are included in the time t information set. Equation (1) contains a time trend term, $\theta \cdot t$. Equation (2) is a rational expectations solution for the inflation rate if it is assumed that the growth rate of money is a kth order autoregressive process. The random disturbances are ε_t, assumed to be a random walk, and v_t, a white noise disturbance. Finally, for (2) to be a rational expectations solution it is necessary that the constant root $\Psi \equiv \frac{\alpha-1}{\alpha} > 1$. The term $A_o \Psi^t$ in equation (2) is the arbitrary term associated with rational expectations solutions; if $A_o = 0$ then there is no bubble in the solution. Since $\pi_t^e = \delta + \beta_1\mu_{t-1} + \beta_2\mu_{t-2} + \ldots + \beta_k\mu_{t-k} + A_o\Psi^t$, we have a system of two equations with non-linear cross-equation restrictions. If we assume that v_t and the random disturbance of the first-differenced version of equation 1 are normally distributed, we can estimate the model's parameters with maximum likelihood methods.

The episodes which we examined were the Hungarian, Polish and German cases.[3] The sample periods were July, 1921-February, 1924 for Hungary, July, 1921-November, 1923 for Poland, and July, 1920-June, 1923 for Germany.[4] For each country, the parameters of money demand and of the inflation solution were estimated first with A_o restricted to equal zero and then with $A_o \neq 0$. Four lags on the growth rate of money were used in equation (2). The results for these two types of estimates are reported in Tables I and II, respectively.[5] From Table II, the t-statistics constructed from ratios of the estimates of A_o to their standard errors indicate that the null hypothesis that $A_o = 0$ would be accepted for each country at standard significance levels. However, if likelihood ratio tests are used this result is reversed. The values of $-2\cdot\log\lambda$, where λ is the ratio of the maximized restricted likelihood function to the maximized unrestricted likelihood function, are 7.59, 1.37, and 3.87 for the German, Polish and Hungarian cases, respectively. For the German and Hungarian cases, this leads to the rejection of the hypothesis that $A_o = 0$ at the 95% significance level.[6] In the Polish case, the hypothesis can still be accepted for standard significance levels.

Table I

Restricted Individual Country Estimates: $A_o = 0$*

Parameter		Germany	Poland	Hungary
θ		-.0467 (.0312)	-.0225 (.0346)	-.0493 (.0230)
α		-.9162 (.3777)	-.6955 (.3754)	-.8778 (.5508)
δ		.0736 (.0552)	.0324 (.0767)	.1580 (.0462)
β_1		.9875 (.4119)	.9278 (.3550)	.5815 (.2932)
β_2		-.5788 (.4775)	-.0362 (.4399)	-.0705 (.1962)
β_3		.1314 (.3570)	.3771 (.4458)	-.1700 (.1706)
β_4		.7230 (.3851)	-.0708 (.4306)	-.0195 (.1517)
log likelihood		113.8165	92.0969	121.1312
Eq. 1	DW	1.914	1.932	1.783
	R^2	.1657	.0903	.1803
	σ^2	.0332	.0301	.01615
Eq. 2	DW	1.230	1.696	1.391
	R^2	.3051	.457	.1077
	σ^2	.07265	.0516	.0262
Sample Period		July '20-June '23	July '21-Nov '23	July '21-Feb '24

*Standard errors in parentheses.

Table II

Unrestricted Individual Country Estimates: $A_o \neq 0$*

Parameter	Germany	Poland	Hungary
θ	-.0364 (.0317)	-.0380 (.0355)	-.0379 (.0223)
α	-.7218 (.2471)	-.1887 (.3562)	-1.408 (.8371)
δ	.0788 (.0479)	-.0666 (.0824)	.1551 (.0425)
β_1	.8769 (.3662)	.7931 (.4503)	.4088 (.2344)
β_2	-.1600 (.4689)	.4168 (.6288)	-.1115 (.1263)
β_3	.0440 (.4230)	-.0385 (.640)	-.1356 (.1199)
β_4	.0765 (.4729)	.7675 (.5705)	-.0524 (.1006)
A_o	1.2586×10^{-16} (1.049×10^{-15})	-4.647×10^{-29} 2.59×10^{-27}	3.451×10^{-10} (3.166×10^{-9})
log likelihood	117.613	92.783	123.065
Eq. 1 DW	1.721	1.844	2.112
Eq. 1 R^2	.1919	-.0133	.303
Eq. 1 σ^2	.0322	.0335	.0137
Eq. 2 DW	1.396	1.536	1.432
Eq. 2 R^2	.5003	.5535	.1094
Eq. 2 σ^2	.0522	.0424	.0262

*Standard errors in parentheses.

II) A Test for a Simultaneous Bubble

In this section we describe a test for whether a bubble existed simultaneously in Germany, Poland, and Hungary.[7] We perform such a test in order to implement the conceptual experiment of observing parallel bubbles suggested by F+G in their footnote 18. A large sample of such parallel bubbles is sufficient to produce well-behaved asymptotic distributions for our test statistics, whereas the usual conceptual experiment of letting time go to infinity produces degenerate asymptotic distributions due to the explosive term in (2).

Associated with each country in our sample is a first differenced version of equation (1) and an inflation equation (2). The parameters of these equations are allowed to differ across countries. The hypothesis is that at the same time (t=0) a bubble of the same magnitude arose in the inflation solutions of each country. Since the α parameter is not restricted to be equal across countries, the exploding term, $A_o\Psi^t$, may be different across countries for $t \neq 0$. Hence, the magnitude of the "mass hysteria" which produced these explosive terms is assumed to be the same across countries only at one moment of time. Since the earliest observations are from the German case, we set $t = 0$ on July, 1920. Thus, the hypothesis is that on July, 1920 an arbitrary element of equal magnitude, A_o, entered the inflation rate solutions for Germany, Poland and Hungary and continued to explode throughout the remainder of the data set.

We estimate six equations simultaneously with maximum likelihood methods. We assume for purposes of this estimation that the random disturbances in equations (1) and (2) have a non-zero contemporaneous covariance for a given country but have zero covariances across countries and across time. Then the only gain in efficiency to the simultaneous estimation method arises through the restrictions which make the parameter A_o common to each country.

We report our results in Table III. Again, using a t-statistic we accept the hypothesis that there was no bubble common to the three countries. Based on our assumptions on the disturbances' covariances, the logarithm of the maximized likelihood function estimated with A_o restricted to zero is the sum of the logarithms of the restricted likelihood functions for the individual countries reported in Table I. The logarithm of the restricted likelihood function is 327.045 while that for the unrestricted likelihood is 332.687. Since in this case $-2 \cdot \log\lambda = 11.28$, we reject at standard significance levels the hypothesis that there was no bubble common to these episodes.

Table III

Joint Parameter Estimates: A_o Restricted to be Equal Across Countries*

	Parameter	Germany	Poland	Hungary
	θ	-.0360 (.0312)	-.0104 (.0334)	-.039 (.0217)
	α	-.7351 (.1814)	-1.071 (.2663)	-1.251 (.303)
	δ	.0785 (.0479)	.1009 (.0906)	.1540 (.0429)
	β_1	.8771 (.3480)	.7175 (.2814)	.4511 (.1676)
	β_2	-.1673 (.4562)	-.0667 (.3287)	-.1287 (.1246)
	β_3	.0452 (.4166)	.2697 (.3196)	-.1472 (.1171)
	β_4	.0858 (.4667)	-.2424 (.3326)	-.0534 (.1098)
	A_o	5.613×10^{-9} (1.748×10^{-8})	-	-
	log likelihood	332.6869	-	-
Eq. 1	DW	1.723	1.976	2.097
	R^2	.1916	.1286	.3047
	σ^2	.0322	.0288	.0137
Eq. 2	DW	1.392	1.605	1.448
	R^2	.4991	.4287	.106
	σ^2	.0524	.0544	.0263

*Standard errors in parentheses.

III) Concluding Remarks

Although our results are mixed, we interpret them more as a contradiction than as a confirmation of the results in F + G (1980). Thus, the technical issue of indeterminacy in rational expectations models remains in an unresolved state. Many theoretical maximizing models of money demand, particularly those of the overlapping generations variety, are fully consistent with price-level bubbles. Empirical work designed to evaluate the hypothesis of no bubbles is inconclusive. Yet, virtually all current research involving macroeconomic rational expectations models invokes the attractive assumption of the absence of speculative bubbles. We conclude that the empirical foundation for this assumption is not yet firmly laid.[8]

Notes

[1] See for instance Brock (1973), Brock and Scheinkman (1980), Lucas (1980), or Starr (1980).

[2] Burmeister and Wall (1980) have extended the F + G exercise for Germany to the case of a stochastic bubble.

[3] Attempts to estimate the model for Russia and Austria were unsuccessful. When the value of A_0 is restricted to equal zero, the estimate of α for Russia is a positive value between zero and one and the estimate for Austria is a large negative number that is not statistically significant. For Austria, the likelihood function is relatively flat with respect to α, as observed previously by Salemi and Sargent (1978). For these reasons the results are not reported.

[4] The sources for German money and price data are the same as those listed in F + G (1980). For the Hungarian data, we used Young (Vol. II, p. 322 and p. 321) for money and prices. Polish data are also taken from Young (Vol. II, p. 349 and p. 353). For Russia, we employed data reported in Katzenellenbaum (1925, pp. 57-8). Finally, we used Walres de Bordes (1924, pp. 48-50 and p.88) for Austrian data.

[5] The maximum likelihood estimates were computed by minimizing the negative of the concentrated likelihood function, using the Davidon-Fletcher-Powell method. For a discussion of methods of non-linear optimization, see Judge, Griffiths, Hill, and Lee (1980, pp. 727-45). The Davidon-Fletcher-Powell method uses analytical first partial derivatives and computes an approximation to the inverse of the Hessian matrix in order to locate the minimum of a function. The non-linear optimization routine in TSP uses numerical approximations for first partial derivatives. Optimization methods that use analytical first partial derivatives generally outperform methods that do not. The variances of the parameter estimates were computed by inverting the information matrix evaluated at the parameter estimates. The α-estimate for Germany in Table II differs from the corresponding α-estimate of -1.615 in F + G (1980). This difference is the result of different optimization techniques and different convergence criteria. To compute the maximum likelihood estimates, we experimented with different starting values and different convergence criteria, and we found that the Davidon-Fletcher-Powell routine converged prematurely in some cases.

[6] Maddala (1977, pp. 179-81) discusses hypothesis testing with maximum likelihood estimation and notes that the different test statistics can produce contradictory results in actual practice. For an example, see Berndt and Savin (1977).

[7] We excluded Russia and Austria for the same reasons cited in footnote 2. We added Russia and Austria separately to the simultaneous bubble model with Germany, Hungary, and Poland; but the Davidon-Fletcher-Powell routine never converged.

Notes

8 We are currently extending our analysis of the German case by relaxing our assumption of exogenous money. Our preliminary results strongly fail to reject the hypthesis of absence of speculative bubbles.

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