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Abstract:

In a previous article Ramsey and Rothman warned against incautious use of the Grassberger-Procaccia procedure to estimate correlation dimension with relatively small data sets and recommended an improved procedure. In this paper we apply those techniques to a series used by DeCoster and Mitchell who claimed that their dimension calculations produced evidence of chaos. We show that even with the enhanced procedures of Ramsey and Yuan, there is no evidence for a simple attractor in these data. However, dimension calculations do not provide evidence either for or against the presence of nonlinear dynamical processes that are not restricted to attracting sets.

Keywords:

Chaos; Correlation Dimension; Nonlinearity;
Small Sample Bias; Dynamical Systems.

JEL Classification:

C22, E40

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1. Introduction

Dimension is an important concept. Unfortunately, there seems to be some lack of clear understanding about the concept, its uses, and the difficulties in the empirical determination of its values. A recent article in the JBES, DeCoster and Mitchell (1991), illustrates our contention.

The remainder of this article is in three sections. The first reviews the concept and use of a commonly used measure of "dimension" and its relationship to dynamics. The second summarizes the practical difficulties in estimating the Procaccia and Grassberger (1983) concept, and recommends an alternative method that mitigates some of the problems that stem from small sample sizes. The last section re-estimates one of DeCoster and Mitchell's results and shows that the "finding of nonlinearity in Divisia M2" (see DeCoster and Mitchell [1991, p. 460]) cannot be sustained by the procedures cited. We hasten to add that our result in no way denies that nonlinearity of some form may exist in measures of money, but that there is no evidence of an attractor from the procedures used.

2. Concepts of Dimension and their Interpretation

The basic concepts involved in the application of dimension concepts to dynamical systems can be briefly summarized. A dynamical system with p degrees of freedom can be expressed in terms of a flow:

$$\dot{x}(t) = f(x(t),t),$$

where $\dot{x}(t)$ is a p dimensional vector of first derivatives of the p dimensional state vector state vector x(t) with solution orbit $x(t) = \Phi_0(t)$, given initial conditions $x(0) = x_0$. Or the system can be expressed as a mapping:

$$x_{t+1} = F(x_t, t)$$

that defines the evolution of the system from specified initial conditions $x_0 = \bar{x}_0$.

An attractor is an attracting set for the evolution of the flow $\Phi_0(t)$ or for the mapping $F(\cdot)$ in that once the state variables are contained in the attracting set under the influence of the flow or mapping they will remain there indefinitely. Attractors are the limit sets for the paths of asymptotic steady state behavior.

As has been discussed in a wide variety of sources (see, for example, Ramsey, Sayers, and Rothman [1990] and Casdagli, et. al. [1991]), the topological and some metrical properties of attracting sets can be determined from a sequence of time series observations on a single component of the p dimensional vector x(t), or even of a homeomorphism of it, where a homeomorphism is a one to one onto continuous mapping with continuous inverse.

One way to achieve this, but only one of several alternatives, is known as "delay reconstruction". Delay reconstruction at a delay of τ creates a k dimensional vector from a single series $\{y(t)\}$, by defining the k dimensional vector w(t):

$$\{w(t)\} = \{y(t+\tau),\ y(t+2\tau),\ \ldots,\ y(t+k\tau)\}.$$

For a full discussion of the justification for and an evaluation of the strengths and weaknesses of this approach, see Casdagli et. al. (1991).

Dimension is fundamentally a topological concept, although it can usefully be formulated in either measure theoretic or metric terms. Ramsey, Sayers, and Rothman (1990) contains a succinct review of some alternative definitions of dimension as well as definitions of all other dynamical terms used in this paper. The root importance of dimension in any of its various forms is that it is a topological invariant, that is, it is invariant to homeomorphisms. In some cases a stronger form of invariance, metric invariance, is required.

In principle dimension can be determined without having to specify the actual dynamical system that is involved, list all the relevant variables, or even specify an appropriate coordinate system. Dimension concepts can indicate:

(a) the amount of information needed to specify a point on an attractor, if one exists;

(b) the lower bound on the number of essential variables that are needed to model a dynamical system within an attractor;

(c) the relative density of points over an attractor.

For a discipline such as economics, these properties are extremely useful.

In the physics literature, much excitement was created in trying to establish the "fractile nature of certain observed phenomena" so that the estimation of fractional measures of dimension was important. In economics, however, our demands can be far more modest in that it is very useful for us to know if a variable is from a system of "low" dimension, or is from one that has "high" dimension. The former provides some hope that we can model the system, the latter indicates that there is little hope in finding a parsimonious description of the data besides specifying its probability distribution. We do not need to know whether a data set has a dimension of 3.6; merely to know that the dimension is "about 4" is extremely useful. Consequently, our demands on dimension calculation can be far less rigorous.

Attractors, or the attracting sets of dynamical systems, have the topological property of "dimension", dynamical systems as such do not; although the word "dimension" in this context is often used as a synonym for degrees of freedom. Consequently, dimension is not a relevant concept for the description of dynamical systems themselves. Not finding a low dimension for some set of data does not imply that there is no underlying dynamical system, merely that there is no low dimensional attracting set. Further, we should recall that attractors by definition represent long term steady state behavior, no matter how complicated the time path of the orbit might appear to be. Consequently, if the observed system is evolutionary, or if the system is constantly being "kicked off" the attracting set, it is unlikely that evidence of an attractor will be found.

An opposing problem is that even linear stationary stochastic processes produce orbits that yield low dimension estimates. This statement is illustrated in Figure 1, which shows the path of one realization of a univariate Gaussian fourth order stationary autoregressive process. This result is plausible in that a stationary autoregressive process is in fact a simple dissipative dynamical system whose energy level is maintained by the constant inflow of random shocks and this produces an attractor-like result.

3. Some Practical Difficulties in Estimating Correlation Dimension

The correlation integral measure developed by Grassberger and Procaccia (1983) is based on counting the average number of delay reconstructed points in k-dimensional points that are within an ϵ distance of each other. This number C_{ϵ}^{N}, where N is the sample size used, approaches under general conditions a limit, C, as $N \rightarrow \infty$ and $\epsilon \rightarrow 0$ such that:

$$C \propto \epsilon^{dc},$$

where dc is the relevant dimension.

In any dimension calculation, a first choice is to pick both the τ lag for delay reconstruction and to find the appropriate embedding dimension; neither are trivial matters. A useful insight into the role of the embedding dimension is illustrated by considering a 1 degree of freedom dynamical system represented on a two dimensional surface as a figure of eight; so that in 2—space the attractor "self-intersects," whereas if we increase the space by 1 to 3—space we can, as it were, unravel the orbit to remove the self-intersection. Adding each embedding dimension reduces the self-intersection possibilities of a "d" dimensional system by 1. This is why we need at most 2d + 1 embedding dimensions to ensure that we can represent any dynamical system by a non self-intersecting orbit. We also see that very simple orbits may only need "d" embedding dimensions; see, for example, Casdagli, et. al. (1991).

Similarly, the choice of τ is not to be made by "convention." Reconsider Figure 1, which was constructed with a τ lag of 3. A shorter lag would yield little more than a straight line and a much higher choice would yield nothing but noise. If we were observing a cyclic orbit, the phase diagrams would themselves oscillate between straight diagonal lines and ones that fully reveal the structure as the choice of the delay in the reconstruction is increased from 1. While the precise value of τ is not critical (that is, if the optimal value of τ is 5 a choice of 4 or 6 would be reasonable), the choice of the appropriate region for τ is.

Casdagli et. al. (1991) review several alternatives for choosing an optimal region for τ. But a useful first choice for limited noisy data sets and no prior information about the supposed attractor, is to pick the lowest value for which the autocorrelations have died out. While this procedure is not optimal, it is simple and in most circumstances met in practice is a very reasonable choice. In noisy systems, one wants to keep the τ lag as small as possible in order to reduce buildup of noise, but one also has to separate the points to be able to provide an informative reconstruction.

The estimation procedure that is used to estimate dimension and how one interprets the results involves many more difficulties than DeCoster and Mitchell (1991) seem to realize. The rest of this section is based on Ramsey and Yuan (1990) and Ramsey, Sayers, and Rothman (1990), to which references the interested reader is referred. We would like to emphasize that the Grassberger and Procaccia (1983) procedure used by DeCoster and Mitchell as well as many others has the following characteristics:

"(a) dimension can be estimated with substantial upward bias for attractors;

(b) dimension is always estimated with downward bias for random noise;

(c) the bias effect increases with embedding dimension, but decreases with sample size;

(d) dimension estimates are normally distributed for sample sizes larger than 1000;

(e) the rate of increase in estimated dimension to embedding dimension for random noise is an increasing function of the distribution's entropy;

(f) the actual variance of dimension estimates can be as high as 64 times larger than that estimated by the usual least squares approach;

(g) the variance of the dimension estimate decreases with sample size, but increases very rapidly with embedding dimension;" Ramsey and Yuan (1990, p. 156).

Each of these points is important for being able to interpret usefully the results of dimension calculations. At the least some healthy skepticism should be maintained about results that do not consider these comments. For example, DeCoster and Mitchell (1991) place stress on their coefficient H, which is the lowest discrete rate of change of dimension estimate to the change in embedding dimension. Unfortunately, limited data sets induce a decline in the rate of growth of dimension with embedding dimension even for random data.

An important, but neglected aspect, of dimension calculation is that delay reconstructed points within an ϵ distance of any reference point should not be included merely because they are the next points in the series, i.e., are nearby in time and only through this condition are recorded as nearby in space; see Theiler (1986). In our calculations we have allowed for such effects, which, if neglected, seriously bias the dimension estimates downwards.

The dimension estimation procedures recommended in Ramsey and Yuan (1990) go a long way to offset these difficulties. In addition, they provide useful graphical evidence for distinguishing attractors as well as a parametric test for choosing between noise and simple attractors. The key to understanding the benefit from the Ramsey and Yuan procedure is to recognize the joint interdependence between sample size and embedding dimension on dimension estimates and on the evaluation of the variance of the estimates. Plots of dimension estimates on both embedding dimension and sample size are most revealing about the presence or absence of attractors; see for example Ramsey, Sayers, and Rothman (1990) and Ramsey and Yuan (1990). These results are summarized in equation (5.1) and (5.2) in Ramsey and Yuan (1990) which are reproduced below.

Ramsey and Yuan demonstrated empirically that the conditional mean of the estimate of dc depends both upon the sample size and the embedding dimension in the following manner:

$$\ln(\overline{dc}) = \gamma_1 + \gamma_2 N^{\gamma_3} + \gamma_4 N^{\gamma_5} [\mathrm{Exp}(\gamma_6/ED^{\gamma_7}) - 1.0], \qquad (3.1)$$

where $\overline{dc}$ is the mean of the dimension estimator $\hat{dc}$, N is sample size, and ED is embedding dimension.

This equation is not as parameter extensive as it would appear. The expression $(\gamma_1 + \gamma_2 N^{\gamma_3})$ indicates the main effect of small sample size on the expected value of the estimator $\hat{dc}$. For random variables that scale monotonically in ED, $\gamma_1 = 0$ and both γ_2 and γ_3 are positive. The higher the entropy of the distribution for equivalent ranges of the support of the distribution, the larger the values of γ_2 and γ_3.

If, however, one has an attractor, then $\gamma_1 > 0$ and $\gamma_3 < 0$, so that as $N \to \infty$ the small sample bias provided by the term $\gamma_2 N^{\gamma_3}$ goes to zero. The asymptote as both N and $ED \to \infty$, but such that $\lim_{N\to\infty} ED/N \to 0$ is given by γ_1.

The second part of the expression in equation (3.1) summarizes the joint effect of sample size and embedding dimension on dimension estimates, especially when sample size is modest. γ_4 depends on the units of measurement chosen for ED and the relative weight of the ED effect to sample size, N. γ_5 seems to be negative for attractors and zero for random variables. For both random and attractor generated data, one expects γ_6 to be negative, but γ_7 to be positive.

The relationship expressed in equation (3.1) contains a potential test for differentiating between low dimensional processes and random phenomena. If the underlying model is a random process, then $\gamma_1 = 0$, and γ_2, $\gamma_3 > 0$. If the observations are being generated by a low dimensional process, then $\gamma_1 > 0$, and $\gamma_3 < 0$. We used this procedure to reassess the empirical results of DeCoster and Mitchell as discussed in the next section.

Ramsey and Yuan also found that the actual variance of the dimension estimates is much larger than that given by the usual least squares variance. The equation that expresses the relationship between the actual standard deviation and the design parameters, N and ED, is:

$$\ln(\text{Std}) = \alpha_1 + \alpha_2 \ln N + \alpha_3 \ln ED + \alpha_4\, ED/N \tag{3.2}$$

where Std is the actual standard deviation of the dimension estimate.

The parameters $\{\gamma_i\}$ and $\{\alpha_i\}$ must be estimated from the data.

4. Re-Analysis of the Divisia M2 Data

DeCoster and Mitchell (1991) reported correlation dimension estimates for various weekly monetary aggregates, their components, the monetary base, and the money multipliers. We focused on the estimated residuals from fitting an AR(4) model to the growth rates of the demand Divisia M2 weekly series constructed by Fayyad (1986). Our reasons for concentrating on this filtered series are twofold. First, as noted above, it is a mistake to perform dimension analysis on data that are highly autocorrelated, since such a practice can lead to spurious evidence of chaos. Second, within the set of prewhitened series considered by DeCoster and Mitchell, the AR(4) filtered Divisia M2 growth rates provided the strongest evidence of saturation in the dimension calculations.

Barnett and Chen (1988) initially asserted evidence of chaos in the demand Divisia M2 series. The sample period is weekly from January 1969 to February 1985. This weekly series was generated as follows. The Divisia procedure was applied to the components of monthly M2, as published by the Federal Reserve Board. Barnett's (1980, 1987) Divisia procedure provides second order approximations to the exact monetary aggregates of economic theory under the assumption of risk neutrality. The weekly series was defined from the monthly data by spline interpolation at an approximate period of 0.23 to represent a week's fraction of a month. Barnett and Hinich (1991, p. 12) argue that given the paucity of actual weekly data, some interpolation for construction of a weekly series is inevitable. While this is true, we note that the spline interpolation procedure used by Fayyad means that only one quarter of the total number of observations are observed data. Consequently, the splining procedure is most likely to dominate the analysis.

Our first step was to replicate the correlation dimension results reported in Table 1 of DeCoster and Mitchell (1991, p. 458) for the prewhitened Divisia M2 series. In addition, we simulated a series of the same length using the AR(4) coefficient values that were obtained by estimation from the original data. These coefficient estimates were combined with Normal independently and identically distributed deviates with the same variance as estimated in the original data to produce the simulated series. In Table 1 we have presented the same type of dimension calculations for these simulated data as were performed on the original log first differenced data for Divisia M2. Figure 1 is a phase space diagram of these simulated data at a lag of 3. There is little to choose between the dimension calculations for the actual data and the linear autoregressive simulated data. This result illustrates clearly that it is vital for the detection of an attractor by means of dimension calculations to use relatively uncorrelated data and to choose a τ lag that is greater than any residual autocorrelation.

The next step was to split this series into sub-samples of 200, 300, 400, etc. observations in order to estimate equation (3.1). For each set of sub-sets of data, we estimated dc for ED = 2,3,...14, if the data set could sustain such a high embedding dimension. Our practical decision rule was to stop increasing the embedding dimension as soon as the estimated dimension $\hat{dc}$ fell, indicating that the extreme limits of the data had been exceeded. For example, the highest sustainable dimension was 8 for the sub-sample of length 200. The final step was to regress the estimated dc on N, sample size, and ED, embedding dimension, using equation (3.1). The estimated equation was then analyzed in accordance with the discussion above in order to try to resolve the issue of whether the observed time series indicated the presence of a chaotic attractor.

A time series plot and the estimated autocorrelation function for the AR(4) filtered Divisia M2 growth rates appear in Figures 2 and 3. The dimension calculations were performed at a lag of 7, i.e., $\tau = 7$, the first lag with zero autocorrelation.

A plot of estimated dimension against embedding dimension and sample size is found in Figure 4. There is a characteristic sequence of shapes to the plots for attractors that differs significantly from the sequence of shapes of plots for noise generated data. Attractor data yield plots in which the sequence of estimated dimensions plotted against embedding dimension for higher levels of sample size decay towards the actual dimension. Such plots for random data do not show this characteristic decay.

The plot in Figure 4 clearly demonstrates the tendency for estimated dimension to increase as both sample size and embedding dimension are increased. This plot is not consistent with the results expected from an attractor and suggests essentially no evidence of an attractor in the series.

The results of estimating equation (3.1) appear in Table 2. The signs of all parameter estimates are consistent with this series being stochastic. If the series is noise, then the expected value of γ_1 is 0 and both γ_2 and γ_3 are expected to be positive. The estimated value for γ_1 is -0.761 and its t-ratio is only -0.788. The estimates for both γ_2 and γ_3 are positive. Thus, the results in Table 1 suggest that the AR(4) filtered Divisia M2 growth rates are stochastic.

In light of these results, we reestimated equation (3.1) under the restriction that $\gamma_1 = 0$. The results for this regression appear in Table 3. All parameter estimates are consistent with this series being random and the estimated t-ratios are substantial. R^2 is virtually unaffected by the restriction of $\gamma_1 = 0$.

If a series is generated by a chaotic attractor, then γ_1 is an estimate of the asymptotic value of the natural logarithm of the dimension of the data. We reestimated equation (3.1) under the restriction that $\gamma_1 = \ln(4.98)$, where 4.98 is the dimension estimate obtained by DeCoster and Mitchell at embedding dimension 14. The results appear in Table 4. Under this restriction all t-ratios are now extremely small and the R^2 value has fallen to 0.54.

A comparison of these three regressions shows clearly that the data favor the random and not an attractor model.

5. Conclusions

The results presented here indicate most clearly that there is no evidence from these calculations for the presence of an attractor. One should be careful about the interpretation of these results. A more complicated higher dimensional attractor may in fact exist for these data, but our existing tools, even given the enhanced capability of the Ramsey and Yuan procedure to discover attractors with limited data sets, are incapable of such discovery.

More importantly, the above results do not, as we indicated in the first section of this paper, provide evidence either for or against the existence of nonlinear dynamical systems in these data. Indeed, the authors have already indicated that during the Volker experiment from 1979 to 1981 there was a most significant shift in the dynamic structure of the M2 data. We have also with a number of other data sets more than convinced ourselves, if not others, that there is abundant and varied evidence for the presence of nonlinear dynamics in economic, if not yet in financial data.

But, we must carefully distinguish complicated long term steady state dynamics as captured by an attractor from evolutionary or frequently shocked systems that produce time series with relatively few observations, if any, within an attractor. Tools for detecting attractors, such as dimension calculations, are not designed for, nor effective in, discovering nonlinear dynamical paths.

Table 1

Estimated Dimensions for Simulated Gaussian AR(4) Process as Compared to the DeCoster and Mitchell Results for Divisia M2 Growth Rates

Embedding Dimension	$\hat{dc}$ [a]	$\hat{dc}$ [b]
4	2.49	2.07
6	2.68	2.53
8	3.00	2.88
10	3.24	3.11
12	3.40	3.21
14	3.54	3.24
	H = 0.07 [a c]	H = = 0.02 [b c]

[a] Results for simulated Gaussian AR(4) process.

[b] Results cited by DeCoster and Mitchell (1991, p. 458), Table 1, Row 2.

[c] H = lowest value of Δ(estimated dimension)/Δ(embedding dimension) in this column, as defined in DeCoster and Mitchell (1991).

Table 2

Regression Results from Fitting

$$\ln(\overline{dc}) = \gamma_1 + \gamma_2 N^{\gamma_3} + \gamma_4 N^{\gamma_5}[\mathrm{Exp}(\gamma_6 ED^{\gamma_7})) - 1]$$

to AR(4) Residuals of Barnett Divisia M2 Growth Rates

Restrictions	γ_i	Estimated Coefficient	Estimated t-ratio	R^2 Value
None	1	-0.761	-0.788	0.992
	2	0.216	2.391	69 DOF
	3	2.496	1.370	
	4	10.431	2.741	
	5	0.058	2.026	
	6	-0.352	-9.427	
	7	0.685	3.134	

Table 3

Regression Results from Fitting

$$\ln(\overline{dc}) = \gamma_1 + \gamma_2 N^{\gamma_3} + \gamma_4 N^{\gamma_5}[\mathrm{Exp}(\gamma_6 ED^{\gamma_7})) - 1]$$

to AR(4) Residuals of Barnett Divisia M2 Growth Rates

Restrictions	γ_i	Estimated Coefficient	Estimated t-ratio	R^2 Value
$\gamma_1 = 0$	2	0.222	3.206	0.991
	3	2.301	2.129	70 DOF
	4	7.391	20.627	
	5	0.087	3.023	
	6	-0.337	-11.486	
	7	0.963	19.249	

Table 4

Regression Results from Fitting

$$\ln\ (\overline{dc}) = \gamma_1 + \gamma_2\ N^{\gamma_3} + \gamma_4\ N^{\gamma_5}[\text{Exp}\ (\gamma_6 ED^{\gamma_7})) - 1]$$

to AR(4) Residuals of Barnett Divisia M2 Growth Rates

Restrictions	γ_i	Estimated Coefficient	Estimated t-ratio	R^2 Value
$\gamma_1 = \ln(4.98)$ [1]	2	-8.090	-0.008	0.538
	3	0.102	0.056	70 DOF
	4	26.905	0.009	
	5	0.117	0.300	
	6	-0.145	-0.010	
	7	0.323	0.043	

[1] 4.98 = dimension estimate obtained by DeCoster and Mitchell (1991) at embedding dimension 14 through the Grassberger-Procaccia algorithm.

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Figure 1

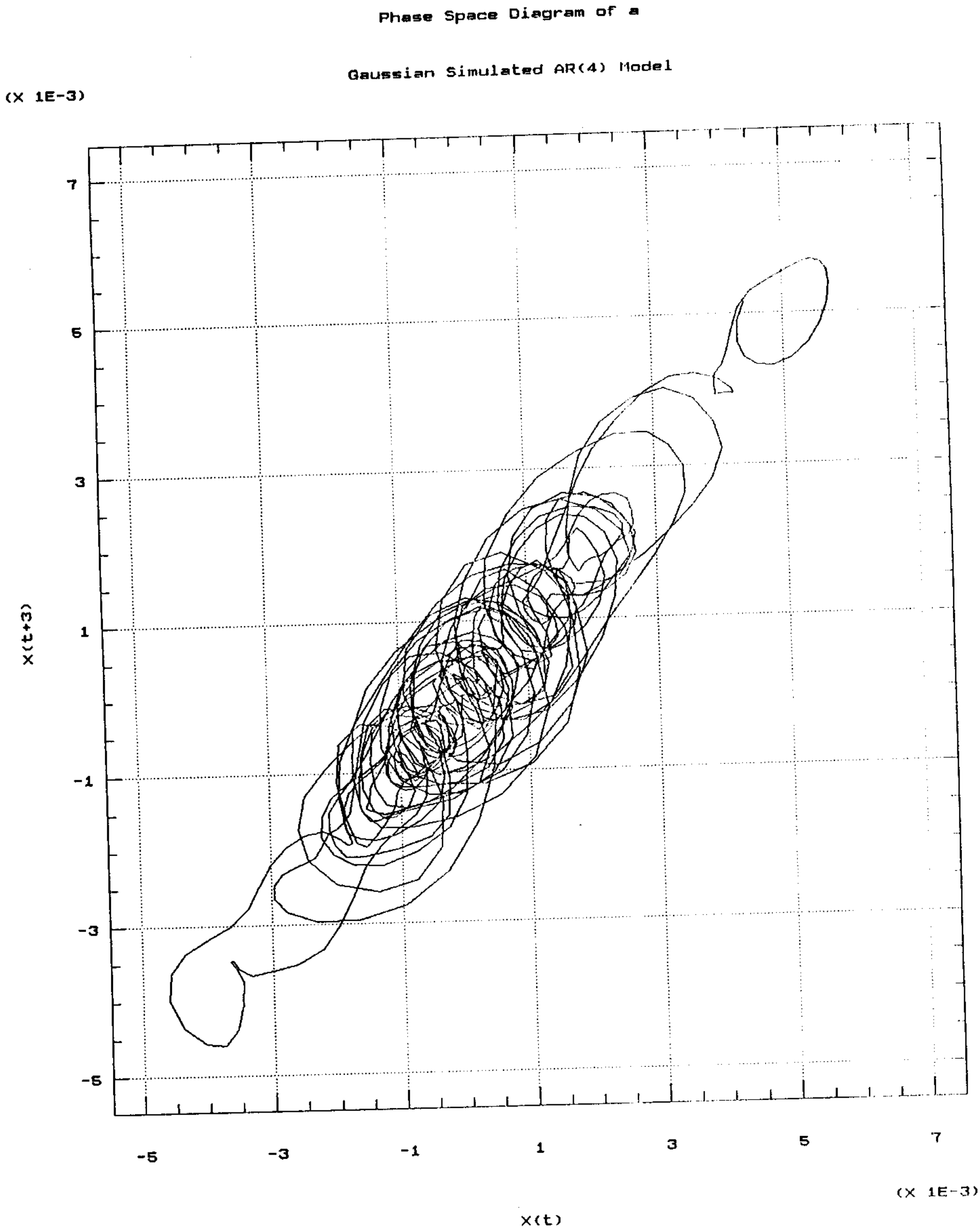
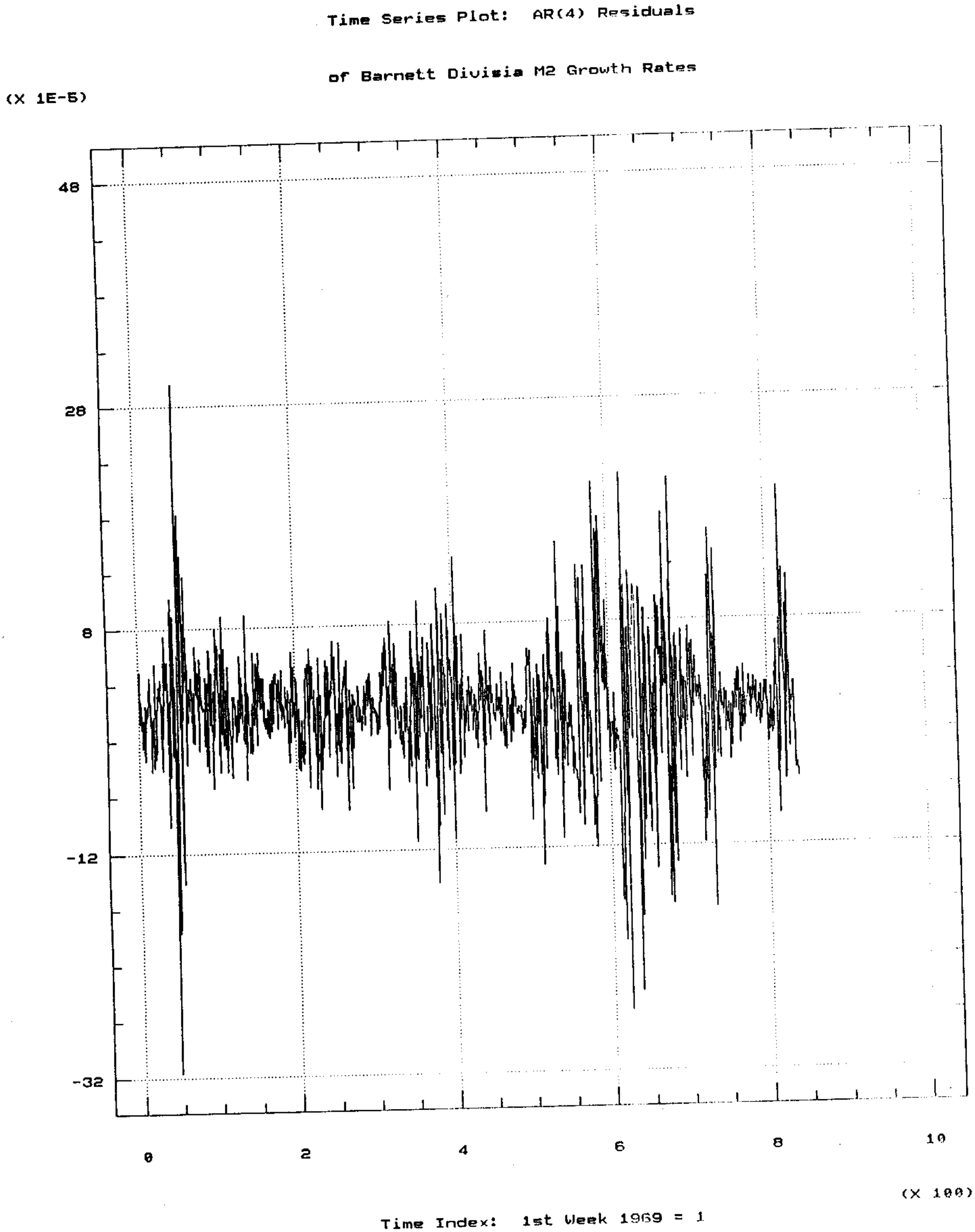
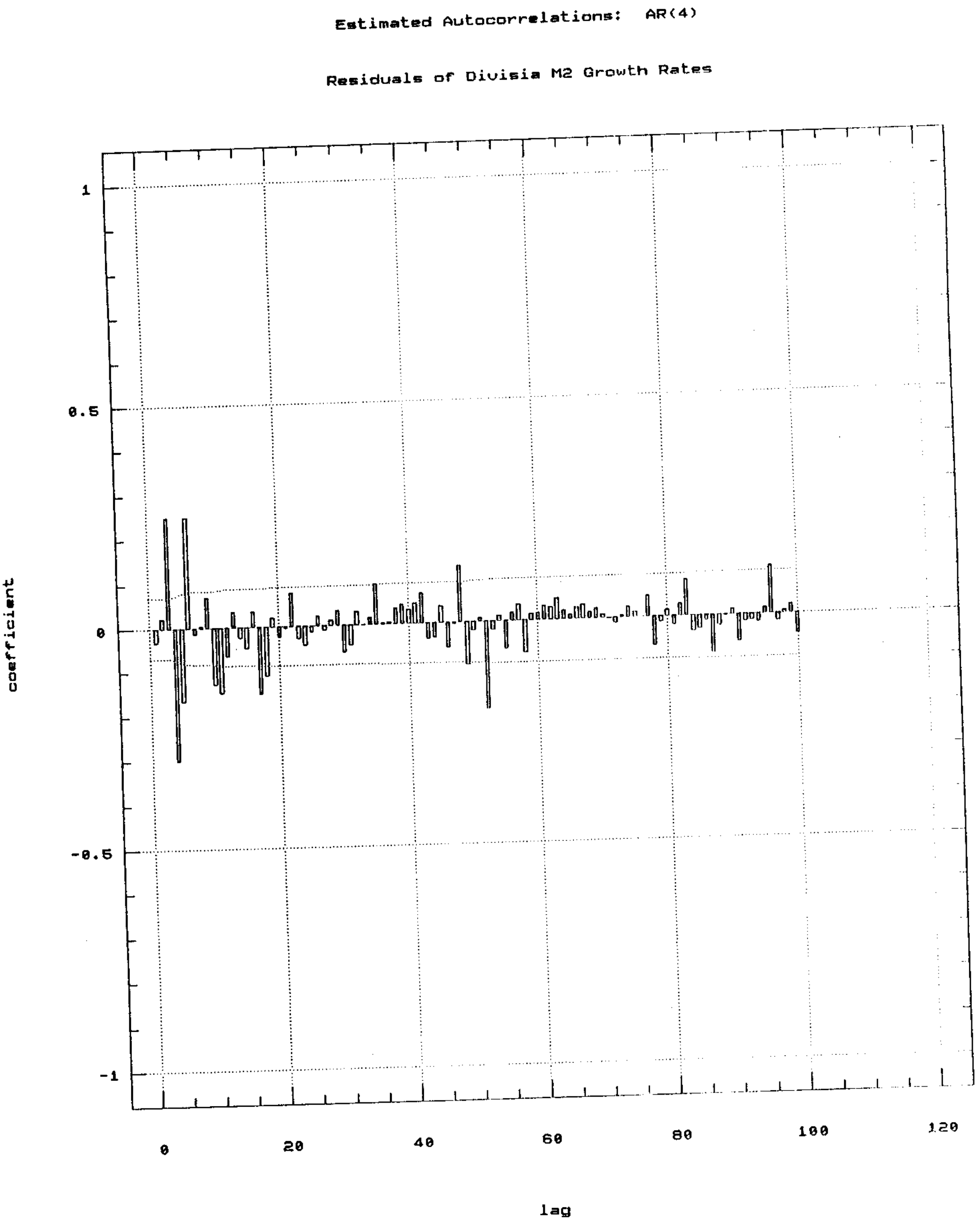

Phase Space Diagram of a
Gaussian Simulated AR(4) Model
(X 1E-3)
7
5
3
1
-1
-3
-5
X(t+3)
-5
-3
-1
1
3
5
7
(X 1E-3)
X(t)

Figure 2



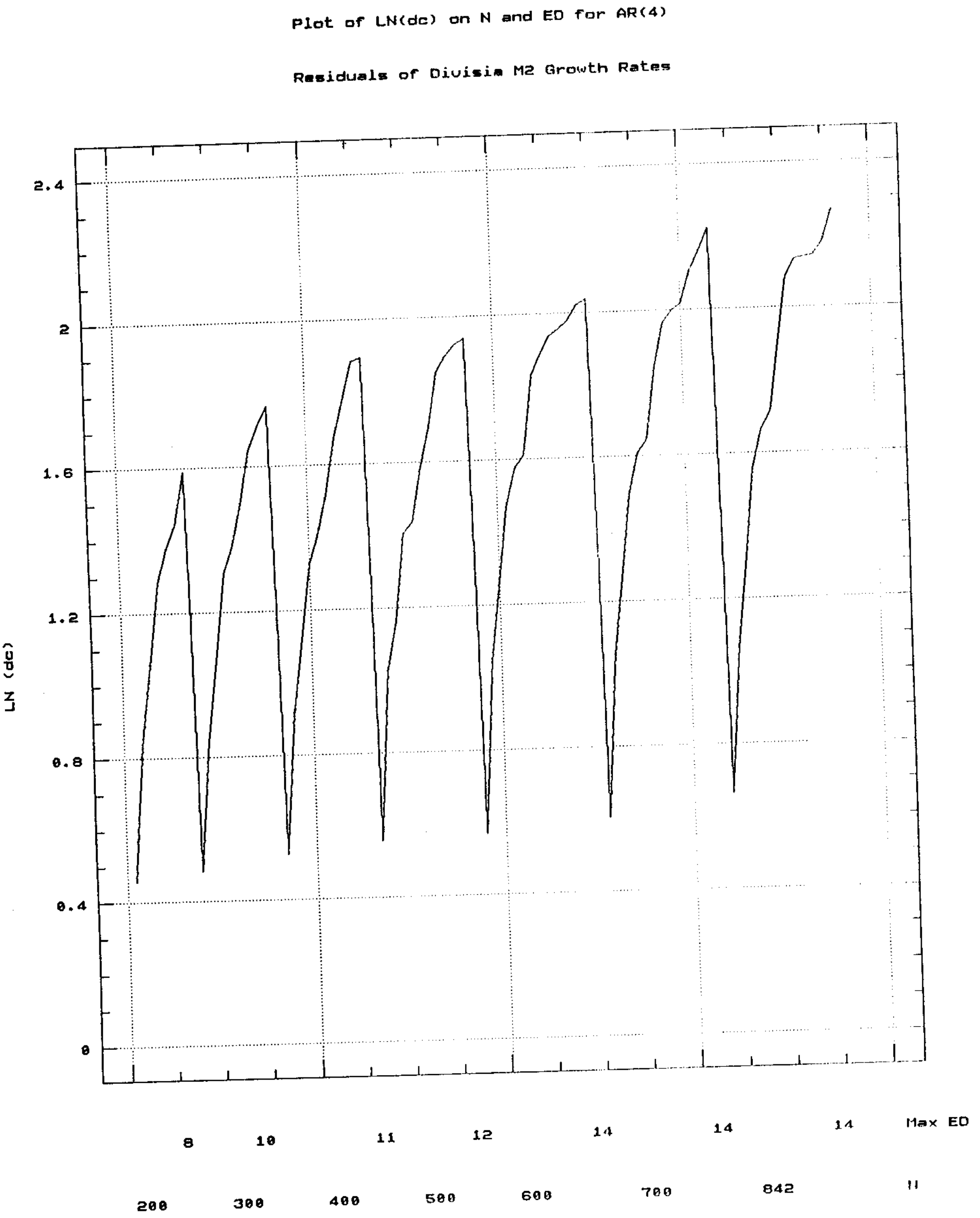
Time Series Plot: AR(4) Residuals
of Barnett Divisia M2 Growth Rates
(X 1E-5)
48
28
8
-12
-32
0
2
4
6
8
10
(X 100)
Time Index: 1st Week 1969 = 1

Figure 3



Estimated Autocorrelations: AR(4)
Residuals of Divisia M2 Growth Rates
1
0.5
0
-0.5
-1
coefficient
0
20
40
60
80
100
120
lag

Figure 4



Plot of LN(dc) on N and ED for AR(4)
Residuals of Divisia M2 Growth Rates
LN (dc)
2.4
2
1.6
1.2
0.8
0.4
0
8
10
11
12
14
14
14
Max ED
200
300
400
500
600
700
842
N