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LAYOFFS AND LABOR HOARDING:

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Abstract

Layoffs account for between 25 and 50% of the increase in unemployment during U.S. recessions and thus it is no surprise that theoretical work on cyclical unemployment has concentrated on modeling layoffs, e.g., the implicit contract literature. Almost all these models have assumed that laid-off workers never leave their firm. In contrast, empirically approximately one-third of laid-off workers change jobs. This paper analyzes the impact of the positive attrition rate from the pool of laid-off workers on the employment policy of a firm. In the context of the simplest possible model that will generate layoffs we show that when the conventional assumption of a zero rate of attrition is broken, the firm's employment policy changes qualitatively. In particular we find that

(1) Labor hoarding (VMP < wage) will occur and will always occur when layoffs occur. Note that labor hoarding is a form of layoff insurance.

(2) Over some range of poor states of the world labor hoarding and employment vary inversely with the size of the firm's labor force.

The employment, layoff and labor hoarding policies of the firm are characterized and their implications for empirical estimation of employment and layoff functions noted. We then argue that sensible models of layoffs should result in the firm's labor force converging to a unique, stationary limiting distribution which is independent of initial conditions. The model in this paper is shown to fulfill this criterion.

0. Introduction

In the U.S. layoffs account for a significant percentage of those people who become unemployed in any given year. For instance, in 1982 22.4% of the unemployed were laid-off workers and these layoffs accounted for over a third of all employer-initiated separations during that year (Bednarzik, 1983). Not only do layoffs contribute significantly to the level of unemployment but, because they are very sensitive to the business cycle, they contribute disproportionately to business cycle peak-to-trough fluctuations in unemployment. This is shown in Table 1 for four selected peak-to-trough periods. The first column shows that job losses (as opposed to quits) have generated between 60% and 85% of the cyclical increases in unemployment while the second and third columns show that layoffs accounted for between 38% and 56% of these job losses. Thus, as Feldstein (1975) originally pointed out, it is crucial that we be able to explain layoffs if we are to have any hope of explaining cyclical unemployment.

Table 1. Job Losses in Peak-to-Trough Periods

Peak to-Trough	Job Losers as a Percentage of the Increase in Unemployment		
	Total	Layoffs	Permanent Separations
12/69 - 11/70	60.0	22.9	37.1
11/73 - 03/75	72.6	35.3	37.3
01/80 - 07/80	82.3	46.3	36.0
07/81 - 11/82	84.5	31.4	53.1

Source: Bednarzik, 1983, p. 7.

This empirical observation has given rise to considerable theoretical analysis of the layoff policies of firms and the development of both transaction cost based models (e.g., Feldstein, 1976, 1978; Baily, 1977) and models based upon incomplete markets and risk sharing (e.g., Azariadis, 1975; Baily, 1974; Gordon, 1974). However, both classes of models have been developed, with very few exceptions,[1] upon the maintained assumption that workers on layoff never leave the firm they have been laid off from. While very convenient for the model builder, this assumption has no empirical validity. Laid-off workers search for alternative jobs to much the same extent as the average unemployed worker (Bradshaw and Scholl, 1976; Clark and Summers, 1979) and imputed rehire rates (Parsons, 1976; Lilien, 1980) suggest that about one-third of laid-off workers find new jobs. More recently this has been confirmed directly by the BLS (Bednarzik, 1983).

This paper focuses on the overlooked attrition rate among laid-off workers and demonstrates that the conventional assumption that this rate is zero has a substantial qualitative effect on the layoff and employment policies of the firm; an effect, moreover, with significant implications for empirical work on layoff functions. To do this we take the simplest possible model of a firm that generates layoffs, namely, a firm that incurs a fixed cost of hiring new workers. To this canonical model we add a stochastic but perfectly anticipated rate of loss from the firm of workers on layoff (employed workers are assumed not to quit jobs). This small change has a large impact on the firm's employment policy. With no loss of workers on layoff, the size of the firm's labor force (employed

[1]Examples of exceptions are Baily, 1977; Peris, 1982; Pissarides, 1982; Bull, 1983; Haltiwanger, 1984.

workers plus workers on layoff) only affects the choice of the optimal level of employment via the constraint that in order to employ more workers than its labor force it must hire new workers and incur the fixed hiring cost. The firm's employment rule deviates from the usual "employ up until VMP equals wage" rule only when following that rule results in new hires. This is not the case when we allow a loss of workers on layoff. Now employing a worker not only generates output this period but also raises the probability that the worker will be available for work next period. This availability represents a valuable option to the firm, namely, the option of avoiding hiring costs. Thus employing a worker raises the value of the option to the firm and so will cause the firm in some (poor) states of the world to employ workers beyond the point where wage equals VMP, i.e., to hoard labor. Indeed, it will be shown that if the risk of losing a worker from layoff is positive then layoffs and labor hoarding will always and only occur simultaneously.

Labor hoarding represents, for the individual worker, a reduction in the risk of layoff, i.e., (incomplete) insurance. Thus without appealing to risk shifting contracts or risk aversion on the part of the worker we see that layoff insurance will be provided by the firm if there is a risk of laid-off workers leaving the firm. More precisely, if the laid-off workers have alternative market opportunities the firm is forced into paying in a bad state of the world for the option of recall in the future. However, labor hoarding behaves perversely as insurance because, as we will show, there exists some range when layoffs are occurring in which employment falls (labor hoarding decrease) as the size of the labor force, and so the probability of layoff, increases. A higher labor force for a fixed level of employment means more workers available next period for recall and hence a lower marginal option value for an employed worker and so reduced employment. In contrast, if the conventional assumption of no attrition from layoff is maintained, employment never varies inversely with labor force size.

We characterize the employment, layoff and labor hoarding policies of the firm in Section 1 where we also draw out the implications of the model for empirical estimation of layoff and employment functions. In Section 2 we propose a basic test which must be passed if a model of layoffs is to be sensible, namely, that if the firm is faced by a stationary environment its optimal employment policy should induce its labor force to converge to a unique, stationary distribution independently of initial conditions. Stationarity is required if we are to rule out models in which the firm is expected to disappear or grow without bound. Uniqueness and independence of initial conditions is a minimal condition if potential new hires are to be able to value correctly the firm's offer to them at the time of hiring.

While the model of the next section is extremely simple it does demonstrate clearly what is lost by the usual assumption of no loss of workers on layoff. Ways in which the model could be improved upon are outlined in Section 3.

1. Layoff and Labor Hoarding Policies

A firm employs a number of workers $e_t \geq 0$ in period t who generate a perishable output according to the production function $f: R_+ \rightarrow R_+$ which is C^1, strictly concave and satisfies $\lim_{e \to 0} f'(e) = \infty$ and $\lim_{e \to \infty} f'(e) = 0$. The firm sells this output in a competitive market at a stochastic price x_t which lies in the closed interval $[\underline{x}, \bar{x}]$, $0 < \underline{x} < \bar{x} < \infty$. Every worker employed is paid a fixed wage $w > 0$. In order to confine ourselves to the properties of the simplest possible model of layoffs we assume that the wage paid is exogenously given and both time and state invariant.

In order to generate temporary rather than permanent layoffs, the firm must be given both some incentive to store or inventory labor out of employment and some incentive to rehire previously laid-off workers before new hires. The former incentive is most simply provided by assuming that the firm pays nothing to laid-off workers. In the nonunion sector there are no direct payments by firms to laid-off workers[2] though most firms make indirect payments via the taxes levied on them to finance state unemployment insurance. However, these payments are typically substantially below the wages paid to employed workers.[3] While the lack of payments to laid-off workers explains why the firm will inventory workers outside employment, a lump-sum hiring cost $k > 0$ for all new hires gives the firm an incentive to rehire previously laid-off workers before making new hires. This cost can be thought of as the differentially higher bureaucratic and training costs of new hires relative to rehires.

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[2]Many union contracts require direct payment by the firm of Supplementary Unemployment Benefits.

[3]Allowing direct payments at a rate less than w would not alter qualitatively any of the results of this paper.

The existence of the cost means that the firm will distinguish between the perfectly elastic supply of new workers it is assumed to face at w and the set of previously hired workers available for employment at t. We will call this set of workers its labor force and denote it by s_t.

The above assumptions would generate an optimal labor force together with the usual wage- (or, if hiring, wage-plus-hiring-cost-) equals-value-marginal-product employment rule. This condition for productive efficiency would then, together with the size of the labor force, generate a layoff or labor inventory policy. Moreover, there would be no labor hoarding in the sense of employing workers beyond the level required for productive efficiency. However, these conventional results stem directly from the omission of one very important empirical fact, namely, that laid-off workers do not simply wait around for recall. They are not, even in the union sector, bound by contract to respond to a recall notice and so they choose optimally whether or not to search for a new job and whether or not to accept any offers they might receive (or go into home production). This is supported by the Clark and Summers (1979) finding that workers on layoff search almost as much as unemployed workers in general. Empirically, approximately 60% of persons on layoff search for work in a given month and of those on layoff who move back into employment about one-third will have changed jobs (Bednarzik, 1983, pp. 8-9).[4] Given these figures it is clear that there is a cost to the firm in laying off a worker, namely, that that worker, and the hiring costs

[4]These figures may overstate the problem as there is some evidence that some permanent separations are incorrectly classified in the data as layoffs. The BLS does, however, over-sample large establishments that tend to have low job turnover rates (Hall and Lilien, 1978).

invested in him or her, may be lost prior to recall.

We assume that a stochastic proportion, $0 \leq \gamma_t < 1$, of workers on layoff are lost to the firm each period. Thus if $e_{t-1} \geq s_{t-1}$, $s_t = e_{t-1}$ but if there are layoffs, i.e., $e_{t-1} < s_{t-1}$, $s_t = e_{t-1} + (1 - \gamma_{t-1})(s_{t-1} - e_{t-1})$ or

$$s_t = (1 - \gamma_{t-1})s_{t-1} + \gamma_{t-1}e_{t-1} \tag{1}$$

Finally, it is assumed that (x_t, γ_t) is i.i.d. with a c.d.f. $G(x,\gamma)$ and $dG(x,\gamma) > 0$ for all $x \in [\underline{x}, \bar{x}]$ and $\gamma \in [0,1)$. Furthermore we assume that (x_t, γ_t) is realized and revealed to the firm before it chooses e_t.

The firm's one period profit function $\pi(e_t; x_t, s_t)$ can be written, defining $H_t = \max(e_t - s_t, 0)$ as new hires, as

$$\pi(e_t; x_t, s_t) = x_t f(e_t) - we_t - kH_t \tag{2}$$

Although (2) is continuous and strictly concave it is not, unfortunately, differentiable at $e_t = s_t$. The value function for the firm is then $(0<\beta<1)$

$$V(x_t, \gamma_t, s_t) = \sup_{e_t} \pi(e_t; x_t, s_t) + \beta \int V(x, \gamma, s_{t+1}) dG(x, \gamma) \tag{3}$$

While it can readily be shown that such a value function exists (Harris, 1982, p. 22) and is continuous and concave, it can also be shown that the function is not differentiable in s which will complicate somewhat the task of characterizing the optimal layoff and labor hoarding policies. Before turning to these policies we establish the following lemma.

<u>Lemma 1</u>: $V(x_t, \gamma_t, s_t)$ is monotonically increasing in x_t and s_t and non-increasing in γ_t.

Proof:

(i) x_t: From the Inada-type conditions on $f(\cdot)$, the optimally chosen e_t, $\hat{e}_t > 0$. For all $e_t > 0$, $\pi(\cdot;x_t,\cdot)$ is monotonically increasing in x_t and so for any fixed employment policy, $V(x_t,\gamma_t,s_t)$ is strictly increasing in x_t.

(ii) s_t: There are two exhaustive cases to consider. If $s_t < \hat{e}_t$, then raising s_t for a fixed $\hat{e}_t$ will raise $\pi(\cdot;\cdot,s_t)$, leave s_{t+1} unaffected and so raises $V(x_t,\gamma_t,s_t)$. Alternatively, if $s_t \geq \hat{e}_t$, then for fixed $\hat{e}_t$ raising s leaves $\pi(\cdot;\cdot,\cdot)$ unaffected and raises s_{t+1} by $(1 - \gamma_t)ds_t > 0$. If for some set of positive measure of realizations of (x,γ) in $t + 1$, $\hat{e}_{t+1} > s_{t+1} + (1 - \gamma_t)ds_t$, then $\int V(x,\gamma,s_{t+1})dG(x,\gamma)$ is raised as is $V(x_t,\gamma_t,s_t)$. If no such set of (x,γ) exists, then ds_t will raise s_{t+2} by (in expectation) $\int(1 - \gamma)(1 - \gamma_t)ds_t dG(x,\gamma) > 0$ and the argument above can be repeated.

(iii) γ_t: If $s_t > \hat{e}_t$, then for fixed $\hat{e}_t$ raising γ_t lower s_{t+1} and, by (ii), lowers $\int V(x,\gamma,s_{t+1})dG(x,\gamma)$ and so $V(x_t,\gamma_t,s_t)$. If $s_t \leq \hat{e}_t$, then for fixed $\hat{e}_t$, changing γ_t leaves s_{t+1} unaffected. Q.E.D.

We are now in a position to derive the layoff and labor hoarding policies of the firm. These can be derived directly from the employment policy of the firm which is defined as the (Borel measurable) function $\hat{e}_t = \eta(x_t,\gamma_t,s_t)$,[5] $\eta: [\underline{x},\bar{x}] \times [0,1) \times R_+ \to R_+$. Given this definition, the layoff policy of the firm $\ell(x_t,\gamma_t,s_t)$ is simply $\max\{s_t - \eta(x_t,\gamma_t,s_t),0\}$.[6]

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[5]It is straightforward to show that the optimal policy that can be obtained from (3) is a function of only the current values of x, γ and s.

[6]This assumes that new hiring and layoffs never occur simultaneously. This can be derived as a result easily as follows. Holding $\hat{e}_t > s_t$ constant, an additional new hire will lower π_t by k and raise s_{t+1} by $(1 - \gamma_t)$. This can be dominated by not making the additional hire at t and hiring $(1 - \gamma_t)$ extra workers at $t + 1$.

While the definitions of these two policies are conventional, labor hoarding is not a term that is usually formally defined. Here what we mean by labor hoarding is employment in excess of the level required to equate value marginal product with the wage. Denote the latter unique level of employment by $\bar{e}_t = \bar{e}(x_t) = f'^{-1}(w \cdot x_t^{-1})$, $d\bar{e}_t/dx_t > 0$. The level of labor hoarding, h_t, at t would then be $h_t = \hat{e}_t - \bar{e}_t$ and so the labor hoarding policy of the firm can be defined as follows:

Definition: The labor hoarding policy of the firm is the (Borel measurable) function $h(x_t,\gamma_t,s_t)$, $h\colon [\underline{x},\bar{x}] \times [0,1] \times R_+ \to R_+$, constructed as follows: $h(x_t,\gamma_t,s_t) \equiv \max\{\eta(x_t,\gamma_t,s_t) - f'^{-1}(w \cdot x_t^{-1}),0\}$.

We turn now to characterizing these policies of the firm beginning with the existence of labor hoarding.

Proposition 1: Given the assumptions above $h(x_t,\gamma_t,s_t) > 0$ (strictly positive labor hoarding) if, and only if, both $\gamma_t > 0$ and $\bar{e}(x_t) < s_t$.

Proof: "if". At $\bar{e}(x_t) < s_t$ raising employment by an arbitrarily small amount de will change $\pi(e_t;x_t,s_t)$ by $de[x_t f'(\bar{e}(x_t)) - w] = 0$. Thus the firm will choose $de > 0$ if raising e_t by de strictly raises $\int V(x,\gamma,s_{t+1})dG(x,\gamma)$. If $\gamma_t > 0$, then the increment in s_{t+1}, $\gamma_t de > 0$. By Lemma 1 $V(x,\gamma,s_{t+1})$ is increasing in s_{t+1}. Thus $\int V(x,\gamma,s_{t+1})dG(x,\gamma)$ will be raised.

"only if". If $\gamma_t = 0$, then choosing $de > 0$ leaves s_{t+1} and so $V(x,\gamma,s_{t+1})dG(x,\gamma)$ unaltered. If $\bar{e}(x_t) \geq s_t$, then increasing e_t by de will lower current period profits by $de[w + k - f'(\bar{e}(x_t))] = kde$ and raise s_{t+1} by de. Consider the alternative choice of not raising e_t and hiring de in period t + 1. This would save the firm $(1 - \beta)kde$ in discounted profits and leave it with the same stock of labor in t + 1. Thus if $\bar{e}(x_t) \geq s_t$, $\eta(x_t,\gamma_t,s_t) \leq \bar{e}(x_t)$ and so $h(x_t,\gamma_t,s_t) \leq 0$. Q.E.D.

Remark: Notice that while $k > 0$ is necessary to generate layoffs we require in addition a positive probability of laid-off workers not being available for recall to generate labor hoarding. $k > 0$ provides the firm with an incentive to employ laid-off workers before new hires, i.e., to recall but only $\gamma > 0$ encourages it to ensure availability for recall by raising employment beyond wage = VMP.

Characterization of $h(x_t,\gamma_t,s_t)$ is most easily done after establishing some properties of the firm's employment policy, $\eta(x_t,\gamma_t,s_t)$. First we give two unsurprising properties of this policy.

Proposition 2: The firm's optimal choice of employment, $\eta(x_t,\gamma_t,s_t)$, is nondecreasing in γ_t.

Proof: Consider two exhaustive cases:

(i) $\eta(x_t,\gamma_t,s_t) \geq s_t$. In this case layoffs are zero and so any small change in γ_t will leave both the current period's profits and s_{t+1} unaltered. Hence $\eta(x_t,\gamma_t,s_t)$ will be constant in γ_t.

(ii) $\eta(x_t,\gamma_t,s_t) < s_t$. By the optimality of $\eta(\cdot,\cdot,\cdot)$, for $de > 0$,

$$\int_{\hat{e}_t-de}^{\hat{e}_t} [x_t f'(\varepsilon) - w]d\varepsilon \geq \beta\int\{V(x,\gamma,(1 - \gamma_t)s_t + \gamma_t\hat{e} - \gamma_t de) - V(x,\gamma,(1 - \gamma_t)s_t + \gamma_t\hat{e}_t)\}dG(x,\gamma) \tag{4}$$

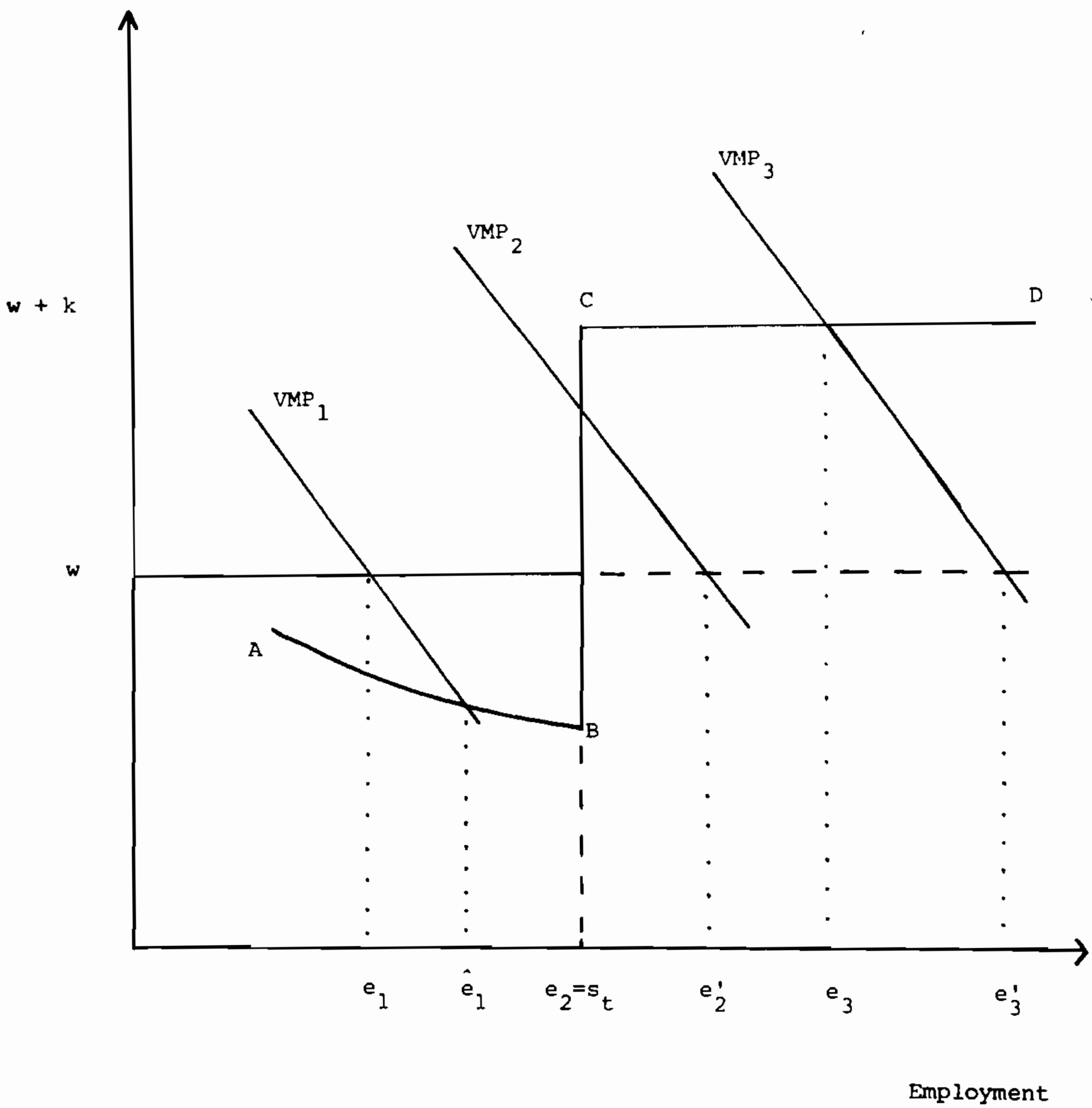
As $\int V(x,\gamma,s_{t+1})dG(x,\gamma)$ is concave in s_{t+1}, any increase in γ_t will lower the right hand side of the above expression and so the weak inequality, and thus the suboptimality of choosing $de > 0$, will continue to hold. **Q.E.D.**

Proposition 3: The firm's optimal choice of employment, $\eta(x_t,\gamma_t,s_t)$ is nondecreasing in x_t.

Proof: From the optimality condition (4), a rise in x_t will raise the left hand side of the inequality while leaving the right hand side constant. The suboptimality of choosing $de > 0$ will, therefore, be maintained. Q.E.D.

While the two properties of the employment policy described in Propositions 2 and 3 are hardly surprising, that contained in Proposition 4 below is rather counterintuitive. One is tempted to think of s_t and k as representing an additional constraint on the profit maximization of the firm, i.e., the firm faces a marginal cost of labor function that is a step function with the step located at s_t and of height k. For certain values of x_t and s_t this step can prevent the firm from expanding employment to the point where value marginal product equals wage. The firm gets stuck with $w < x_t f'(e_t)$. This is shown in Figure 1 where e_1' denotes the employment level in the absence of k. In view of this, relaxing the constraint by raising s_t should raise $\hat{e}_t$ in cases where the constraint is binding but leave $\hat{e}_t$ unaffected elsewhere. Put differently, we would expect $\eta(x_t,\gamma_t,s_t)$ to be nondecreasing in s_t and increasing where $s_t = \eta(x_t,\gamma_t,s_t)$. In fact while it is true that for states in which $s_t = \eta(x_t,\gamma_t,s_t)$, $\eta(x_t,\gamma_t,s_t)$ is increasing in s_t, it is <u>not</u> the case the $\eta(x_t,\gamma_t,s_t)$ is nondecreasing elsewhere. Indeed, it is decreasing in s_t for $s_t > \eta(x_t,\gamma_t,s_t)$. The reason for the failure of the intuitive

Figure 1



argument is that the effect of $\gamma > 0$ has been forgotten. Carrying a labor force into the future has an option value in that one can avoid some hiring costs. However, because $\gamma > 0$, at the margin, if $s_t > \eta(x_t,\gamma_t,s_t)$, the firm pays for this option by employing workers at a loss. This is shown in Figure 1 by the "marginal cost of labor" curve ABCD and $\hat{e}_1$. As s_t is raised, for any given e_t, s_{t+1} is raised which depresses the "marginal" option value of carrying an additional worker through to t + 1 and so the firm cuts back on employment. This can be seen in Figure 1 by shifting s_t and the marginal cost curves right. With the step function e_1 remains unchanged while with ABCD, $\hat{e}_1$ moves left.

Proposition 4: The employment policy of the firm, $\eta(x_t,\gamma_t,s_t)$, is nonmonotonic in s_t. In particular,

(i) If $\eta(x_t,\gamma_t,s_t) > s_t$, $\eta(\cdot)$ is constant in s_t.

(ii) If $\eta(x_t,\gamma_t,s_t) = s_t$, $\eta(\cdot)$ is non-decreasing in s_t and, for some open interval of values of x_t, is strictly increasing in s_t.

(iii) If $\eta(x_t,\gamma_t,s_t) < s_t$, $\eta(\cdot)$ is non-increasing in s_t and in some range strictly decreasing in s_t.

Proof: (i) If $\eta(x_t,\gamma_t,s_t) > s_t$, then the firm hires to the point at which $x_t f(e_t) = w + k$, which is independent of s_t.

(ii) If $\eta(x_t,\gamma_t,s_t) = s_t$, then $w \leq x_t f'(s_t) \leq w + k$ and so any increase in s_t will not result in a decrease in the optimally chosen level of employment. For values of x_t such that $w < x_t f'(s_t) < w+k$, employment will strictly increase with s_t.

(iii) By the optimality of $\eta(\cdot,\cdot,\cdot)$, for de > 0,

$$\int_{\hat{e}_t}^{\hat{e}_t+de} [x_t f'(\varepsilon) - w]d\varepsilon \leq \beta\int\{V(\mathbf{x},\gamma,(1 - \gamma_t)s_t + \gamma_t\hat{e}_t) - V(\mathbf{x},\gamma,(1 - \gamma_t)s_t + \gamma_t\hat{e}_t + \gamma_t de)\}dG(\mathbf{x},\gamma) \quad (5)$$

By the concavity of $V(\cdot,\cdot,s_t)$ in s, raising s_t increases the right hand side of (5) while leaving the left hand side unaltered and so maintaining the suboptimality of raising e_t above $\hat{e}_t$.

To prove that for some values of $s_t < \eta(x_t,\gamma_t,s_t)$ employment is decreasing consider equation (4). Given that $\hat{e}_t$ is not increasing in s_t and the concavity of the value function, by raising s_t the right hand side of (4) can be pushed arbitrarily close to zero. The left hand side of (4) is negative by Proposition 1 and so a high enough s_t can be found to cause (4) to be violated. Q.E.D.

With these results we can characterize the firm's labor hoarding policy when hoarding is positive.

Proposition 5: For values of (x_t,γ_t,s_t) such that $h(x_t,\gamma_t,s_t) > 0$ the following are true:

(i) $h(x_t,\gamma_t,s_t)$ is nondecreasing in γ_t.

(ii) $h(x_t,\gamma_t,s_t)$ is nonincreasing in s_t.

(iii) $h(x_t,\gamma_t,s_t)$ is monotonically decreasing in x_t if $f'(\cdot)/f''(\cdot)$ is nonincreasing in e.

Proof: (i) From Proposition 2, $\hat{e}_t$ is nondecreasing in γ_t while $\bar{e}_t$ is independent of γ_t.

(ii) From Proposition 4, for $s_t > \hat{e}_t$ and so, by Proposition 1, $h(\cdot,\cdot,\cdot) > 0$, $\hat{e}_t$ is nonincreasing in s_t while $\bar{e}_t$ is independent of s_t.

(iii) Here there are two cases to consider corresponding to whether the optimality condition (5) holds with strict inequality or equality. If it holds with strict inequality $\hat{e}_t$ will be locally constant for a small increase in x_t. However, $\bar{e}_t$ is monotonically increasing in x_t and so $h(x_t,\cdot,\cdot)$ will decrease. Alternatively, if (5) holds with equality, noting that both sides are concave in de and that $\pi(e_t;x_t,s_t)$ is differentiable at $e_t < s_t$, we can use Lemma 1 in Benveniste and Scheinkman (1979) to show that $\beta\int V(x,\gamma,(1-\gamma_t)s_t + \gamma_t\hat{e}_t)$ is differentiable with respect to $\hat{e}_t$. The optimality condition then becomes,

$$x_t f'(\hat{e}_t) - w_t + \beta\int \gamma_t V_3(x,\gamma,(1-\gamma_t)s_t + \gamma_t\hat{e}_t)dG(x,\gamma) = 0$$

and so

$$\frac{de_t}{dx_t} = -f'(\hat{e}_t)\{x_t f''(\hat{e}_t) + \beta\int \gamma_t V_{33}(x,\gamma,(1-\gamma_t)s_t + \gamma_t\hat{e}_t)dG(x,\gamma)\}^{-1} > 0$$

Differentiating $\bar{e}_t$ gives $\frac{d\bar{e}_t}{dx_t} = -f'(\bar{e}_t)[x_t f''(\bar{e}_t)]^{-1}$. Thus if $f(e)$ is such that $f'(e)/f''(e)$ is nonincreasing in e, for example if the elasticity of output is constant, then $d\bar{e}_t/dx_t > d\hat{e}_t/dx_t > 0$ and so $h(x_t,\cdot,\cdot)$ is decreasing. Q.E.D.

Once we recognize the option value to the firm of the labor force it carries into future periods, the characteristics of its labor hoarding policy make intuitive sense. Raising γ_t, the proportion of laid-off workers who leave the firm, raises the relative cost of storing workers out of employment and so raises the incentive to store them in employment. Raising the current labor force reduces the marginal option value of a worker and so reduces the incentive to hoard labor. Finally, given sufficient curvature in the production function, raising x_t will have a larger impact on $\bar{e}_t$ than the larger $\hat{e}_t$ because at this larger $\hat{e}_t$ marginal physical product is dropping faster than at $\bar{e}_t$. Notice that Proposition 5(iii) gives a sufficient condition for labor hoarding to fall with a rise in product price. From the proof we can see that if the marginal option value of an additional worker is declining fast enough the curvature condition on the production function can be relaxed somewhat while maintaining the result.

From the point of view of the existing literature on layoffs perhaps the most interesting aspect of this labor hoarding policy is its role as insurance for the worker against income fluctuations. Consider a firm for which $\gamma = 0$ and so that followed a wage-equals-VMP employment rule. Compared with such a firm, for any given level of s_t, a low realization of the product price will result in fewer layoffs at a firm using the above labor hoarding policy. If workers are chosen at random for layoff then this lowers the risk of layoff for a worker at the labor hoarding firm. Notice that a firm will only choose to hoard labor if $\gamma_t > 0$. As $\gamma > 0$ represents the existence of labor market (or home production) alternatives that are preferred, given the probability of recall, to remaining on layoff, we can see that the firm will only provide insurance via labor hoarding if the labor market is providing some such insurance, i.e., $\gamma > 0$.[7] Phrased more accurately, only if the market offers attractive alternatives to laid-off workers is the firm forced to pay for the option of recalling workers in the future.

[7] For a discussion of the insurance provided by a competitive but noncontractual labor market see Rosen (1984).

This insurance provided by labor hoarding behaves a little perversely in the sense that the amount of insurance provided (insurance paid out) by the firm is less in, for the workers, worse states of the world. Thus when market opportunities are poor, γ_t low, the firm will hoard less labor, and when the probability of layoff is high (and recall after layoff low), s_t high, the firm will hoard less labor. This perverse behavior comes from the fact that the firm is paying the workers for the option of recall by raising employment. In the poor states of the world, however, the value to the firm of the option drops and along with it labor hoarding.

Finally, we characterize the firm's layoff policy $\ell(x_t,\gamma_t,s_t)$. As these results follow directly from the characteristics of $\ell(x_t,\gamma_t,s_t)$ they are stated without proof.

<u>Proposition 6</u>: For values of (x_t,γ_t,s_t) such that $\hat{e}_t < s_t$, the following is true:

(i) $\ell(\cdot,\cdot,s_t)$ is monotonically increasing in s_t.

(ii) $\ell(\cdot,\gamma_t,\cdot)$ is nonincreasing in γ_t.

(iii) $\ell(x_t,\cdot,\cdot)$ is nonincreasing in x_t.

Although they come from an extremely simple model, the employment and layoff policies derived have strong implications for empirical work. In particular, the model requires the inclusion of s_t and γ_t in any function trying to explain employment or layoffs. Moreover, although the model deals with an individual firm there is no reason why aggregation to the industry level at which data is available should reverse this conclusion. No empirical study to our knowledge (e.g., Lilien, 1980, 1982;

Medoff, 1979; Blau and Kahn, 1981; Topel, 1982) has included either variable. This is particularly notable because the BLS provides data at the industry level on the labor force (s_t). Furthermore, the model suggests that the labor force should enter the employment function in a very particular non-linear way (Proposition 4) which holds out the prospect of a powerful test of the model.

The omission of γ_t, the rate at which workers on layoff change jobs, is quite understandable as data are not available on this variable. However, its omission may be particularly pernicious because of its probable correlation with x_t, or more generally the location of the demand curve for the industry's output. Presumably the size of γ_t is a function of the demand for labor in other industries. Thus if x_t is low because of an economy-wide recession, γ_t will tend to be low whereas if x_t is low for reasons peculiar to that specific industry one would expect γ_t to be relatively high. If we assume that firms can determine, at least imperfectly, whether the reduction in the demand they face is idiosyncratic or macroeconomic at the time of the layoff decision then we can expect their decision to differ in the two situations. Empirically, this means that not conditioning for the so-called local or global nature of the fluctuations in industry demand will result in estimates of layoff and employment functions that are misspecified and that could lead to erroneous conclusions if they were used to, for instance, simulate the reduction in layoffs to be expected from a macroeconomic recovery. We are currently pursuing empirical research to establish the empirical significance of this and more generally to test the model.

2. Existence of a Unique, Limiting, Stationary Distribution of Employment and Layoffs

There are many possible models of layoffs that can be constructed and so some criteria by which to choose between them are useful. We propose that such a model should generate an optimal employment policy that will lead the labor force of a firm facing a stationary distribution on its demand curve to converge on a unique, limiting, stationary distribution. This criterion has two merits. Firstly, stationarity rules out employment policies that will lead the firm to, in expectation, disappear or grow without bound. Its second advantage is informational. Ultimately any layoff model must be extended to allow explicitly for optimal decisions by workers to join the firm (e.g., Burdett and Mortensen, 1980; Baily, 1977) and to leave it while on layoff (Pissarides, 1982). However, these decisions by the worker hinge crucially upon his or her valuation of the job which in turn depends on the anticipated distribution of future employment. In modeling these expectations it is attractive to use a rational expectations assumption to the effect that the potential new hire or laid-off worker knows the true probability distribution over future employment levels. This distribution depends on four things, namely, the distribution of the exogenous variables here (x,γ), the current values of s_t and e_t, and the employment policy of the firm. While $G(x,\gamma)$ and the firm's profit function (and so employment policy) may be public information the worker still needs to know s_t and e_t in order to value the job correctly. However, except for unionized settings s_t and e_t are private information of the firm. As this information will affect its cost of hiring or retaining workers the firm has every incentive not to reveal the true values of s_t and e_t.

It would appear, then, that using a rational expectations assumption is not possible in this context and this would certainly be a barrier to further analysis. The situation would not be so bleak if we knew that the employment distribution generated by the firm's employment policy and $G(x,\gamma)$ went, in the limit, to a unique stationary distribution. If this were true, and the worker's discount rate was low, we could approximate closely the true value of the job by using the limiting distribution of employment and the means of e_t and s_t in place of the true values of s_t and e_t and the true distribution of employment. Thus establishing the existence of a unique limiting distribution of employment, and so layoffs, provides a useful, albeit weak, check on the analytical usefulness of a layoff model. Fortunately, Razin and Yahav (1979) provided a simple way of conducting this check.

Given that the firm chooses its employment policy optimally its labor force evolves according to

$$s_{t+1} = \eta(x_t,\gamma_t,s_t) \quad , \quad s_t \leq \eta(x_t,\gamma_t,s_t) \qquad \text{(i)}$$

$$s_{t+1} = (1-\gamma_t)s_t + \gamma_t\eta(x_t,\gamma_t,s_t) \quad , \quad s_t > \eta(x_t,\gamma_t,s_t) \qquad \text{(ii)} \qquad (6)$$

In order to prove the result we want we need to establish three properties of the transition equation (6). The first is that for all values of x_t and γ_t, s_{t+1} must lie in some finite closed interval $[\underline{s},\bar{s}]$. This is clearly true as $s_t \geq 0$ and $\eta(x_t,\gamma_t,s_t) \leq \bar{\eta}$ where $\bar{\eta}$ is defined by $\bar{x}f'(\bar{\eta}) = w$. Thus $s_t \leq \bar{s} \equiv \bar{\eta}$ and so equation (6) maps $[0,\bar{s}]$ into $[0,\bar{s}]$. The second property required is that (6) be nondecreasing in s_t. Part (i) of (6) is constant in s_t by Proposition 4(i). (6ii) is more complicated as from Proposition 4 we know that employment is decreasing in s_t in some parts of this range. However, intuitively while one would expect a firm with an exogenously increased s_t to spend some of this windfall on reducing employment (remember that w > VMP in this range) one would not expect it to reduce employment so far that s_{t+1} would fall and the marginal option value of a worker rise. This intuition is correct.

These two properties are summarized in the following lemma.

Lemma 2:

(i) s_t, for all t, lies in $[0,\bar{s}]$ where $\bar{s}$ is defined by $\bar{x}f'(\bar{s}) = w$.

(ii) Equation (6) is nondecreasing in s_t for all values of (x,γ).

Proof: (i) Immediate.

(ii) For $s_t \leq \eta(x_t,\gamma_t,s_t)$ we know from Propositions 4(i) and 4(ii) that $\eta(x_t,\gamma_t,s_t)$, and so s_{t+1} is nondecreasing in s_t. For $s_t > \eta(x_t,\gamma_t,s_t)$,

$$\frac{ds_{t+1}}{ds_t} = (1 - \gamma_t) + \gamma_t \frac{d\eta_t}{ds_t}$$

Moreover, from the proof of Proposition 5(iii) at such points the value function is differentiable and so

$$x_t f'(\hat{e}_t) - w + \beta\gamma_t \int V_3 dG(x,\gamma) = 0.$$

Differentiating totally and defining $A = \beta\gamma_t^2 \int V_{33} dG(x,\gamma) \leq 0$,

$$\frac{d\eta_t}{ds_t} = \frac{-\beta\gamma_t \int V_{33} dG(x,\gamma) + A}{xf'' + A}$$

or $$\gamma_t \frac{d\eta_t}{ds_t} = \frac{(\gamma_t - 1)}{xf'' + A} \leq 0$$

Thus $$\frac{ds_{t+1}}{ds_t} = \frac{(1 - \gamma_t)xf''}{xf'' + A} \geq 0 .$$

Q.E.D.

The properties described in Lemma 2 together with the i.i.d. distribution of (x,γ) ensure that if the firm at $t = 0$ started with $s_0 = 0$, the probability distribution of the labor force at t, $F(s_t \leq v | s_0 = 0)$ would approach monotonically a limiting distribution $H(s \leq v | s_0 = 0)$, i.e., $F(s_t \leq v | s_0 = 0) \downarrow H(s \leq v | s_0 = 0)$, $v \in [0,\bar{s}]$. Similarly, $F(s_t \leq v | s_0 = \bar{s}) \uparrow H(s \leq v | s_0 = \bar{s})$, $v \in [0,\bar{s}]$. If, then, s_t is to have a unique stationary distribution in the limit it must be the case that $H(s \leq v | s_0 = \bar{s}) \equiv H(s \leq v | s_0 = 0)$ for all $v \in [0,\bar{s}]$. Razin and Yahav (1979, Theorem 1) show that this will be the case if there exists a $t_1 < \infty$, $t_2 < \infty$ and a c, $c \in [0,\bar{s}]$ such that $F(s_{t_1} \leq c | s_0 = 0) < 1$ and $F(s_{t_2} \leq c | s_0 = \bar{s}) > 0$. In order to prove this in our case we require, in addition to the assumptions of the previous section, one further assumption, namely,

<u>A1</u>: Define the unique pair of numbers $\bar{e}, \underline{e}$ as follows:

$$\bar{x}f'(\bar{e}) = w + k \quad \text{and} \quad \underline{x}f'(\underline{e}) = w - \beta k.$$

Assume that the parameters $\underline{x}$, $\bar{x}$, w, k, β are such that $\underline{e} > 0$ and $\bar{e} > \underline{e}$.

To understand the necessity for this assumption notice first that $\bar{e}$ is the highest employment level and so labor force that a firm starting with $s_0 \leq \bar{e}$ would ever reach (n.b. $\bar{e} < \bar{s}$). Conversely, a firm starting with $s_0 > \underline{e}$ would never reduce its employment and labor force below $\underline{e}$. This is because the upper bound on the option value of employing a worker, for any realization of γ, is βk, i.e., the discounted cost of hiring a new worker in the next period. Thus unless $\bar{e} > \underline{e}$ the distributions of the labor force reached from initial conditions $s_0 = 0$ and $s_0 = \bar{s}$ could never be the same because their domains would be nonintersecting.

<u>Proposition 7</u>: Given the assumptions of the previous section together with A1, the stochastic process describing the firm's labor force, s_t, has a unique, stationary, limiting distribution as does the firm's level of employment.

<u>Proof</u>: (i) Let a be an arbitrarily small positive number and define $e^a < \bar{e}$ by

$$(\bar{x} - a)f(e^a) = w + k$$

which is independent of γ. As $G(\bar{x} - a, \cdot) < 1$, then there is a positive probability that $s_1 \geq e^a$ given $s_0 = 0$.

(ii) Let b be an arbitrarily small positive number and define $e^b > \underline{e}$ by

$$(\underline{x} + b)f(e^b) = w - \beta k$$

If $s_t > e^b$, then irrespective of the realization of γ, if x_t is realized in $[\underline{x}, \underline{x} + b]$, $\eta(x_t, \gamma_t, s_t) < e^b$ and so $s_t - s_{t+1} > \gamma_t(s_t - e^b)$.

Now define $\tilde{s} \equiv (e^a - e^b)/2$. From (i) we see that there is a positive probability that $s_1 \geq \tilde{s}$ given $s_0 = 0$. From (ii) note that if $\gamma_t > 0$ and $x_t < \underline{x} + b$, for $s_t > e^b$, $s_{t+1} < s_t$ and that as $t \to \infty$, $s_t \to e^b$. Thus s_t crosses $\tilde{s}$ in finite time. Let $0 < c < 1$. As

$$\int_{\underline{x}}^{\underline{x}+b} \int_{c}^{1} dG(x,\gamma)\, d\gamma\, dx > 0$$

such a crossing will occur with positive probability and so by Theorem 1 of Rasin and Yahav (1979), $\{s_t\}$ has a unique, stationary, limiting distribution. Given that the firm's employment policy is itself unique and stationary, employment has a unique, stationary, limiting distribution as well.

3. Conclusions

The model described in this paper, despite its extreme simplicity, has shown the importance of recognizing that workers on layoff search for alternative jobs and that some proportion of them will move to other firms and so will not be available for recall. This causes labor force size to affect the firm's optimal employment policy even when employment is less than the labor force, i.e., there are positive layoffs, and results in labor hoarding. A direct implication of this is that empirical estimates of layoff functions, which have so far omitted labor force size, have been misspecified. Moreover, labor force size should enter in a very particular, and so readily testable, nonlinear fashion. An indirect implication of the model is that the optimal employment and layoff policies of the firm will contain as arguments the state of labor demand in alternative markets as this will in part determine the attrition rate among laid-off workers. Again, this suggests a way in which existing empirical work can be improved. Indeed, the authors are currently engaged in an empirical study designed to exploit these insights and test the model presented in this paper.

The simplicity of the model contained here, though useful for highlighting the effects of interest, is its major drawback. Some more realism can be generated quite easily, for instance, by allowing the bivariate process (x_t,γ_t) to be autocorrelated. There are, however, two particular areas in which the model should be improved but that are nontrivial areas of research in their own right. One is to endogenize the wage and allow the workers to respond optimally to the firm's wage, layoff and recall policies. These latter two policies will affect both the wage the firm must offer to attract new hires and the search policies of laid-off workers.[8] The second improvement that needs to be made is to move to a market equilibrium setting. This has been carried out, for instance by Topel (1982), for the product market but not simultaneously for product and labor markets.[9] Although difficult, the move to a market equilibrium setting is crucial as the data available for empirical testing concerns industry-wide aggregates. Moreover, any welfare, and so policy, analysis of these markets requires a market equilibrium model.

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[8] In a very interesting paper Pissarides (1982) analyzes the firm's optimal choice of recall (but not layoff) policy given the optimal search behavior of laid-off workers. In this analysis, though, the wage is exogenously given as is the probability of layoff even though some of the recall policies considered affect the latter. Haltiwanger (1984) endogenizes layoff and recall policies. However, at optimal choices of these policies temporarily laid-off workers do not search and so do not change jobs in contradiction to the data.

[9] Burdett and Mortensen (1980) look at a labor market equilibrium in the context of a search model with layoffs. However, these layoffs are in fact permanent separations.

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