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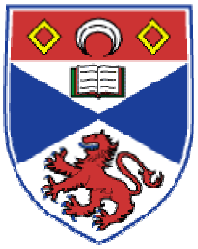

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Endogenous Price Flexibility and Optimal Monetary Policy

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ABSTRACT

Much of the literature on optimal monetary policy uses models in which the degree of nominal price flexibility is exogenous. There are, however, good reasons to suppose that the degree of price flexibility adjusts endogenously to changes in monetary conditions. This paper extends the standard New Keynesian model to incorporate an endogenous degree of price flexibility. The model shows that endogenising the degree of price flexibility tends to shift optimal monetary policy towards complete inflation stabilisation, even when shocks take the form of cost-push disturbances. This contrasts with the standard result obtained in models with exogenous price flexibility, which show that optimal monetary policy should allow some degree of inflation volatility in order to stabilise the welfare-relevant output gap.

JEL Classification: E31, E52.

Keywords: Welfare, Endogenous Price Flexibility, Optimal Monetary Policy.

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1 Introduction

Much of the recent literature on optimal monetary policy uses models in which the degree of nominal price flexibility is imposed exogenously (see for example Clarida, Gali and Gertler (1999), Woodford (2003) and Benigno and Woodford (2005)). There are, however, good theoretical and empirical reasons to suppose that the degree of price flexibility adjusts endogenously to changes in economic conditions, including changes in monetary policy. The ability of monetary policy to affect the real economy is closely linked to the degree of price flexibility, so endogenous changes in price flexibility may in turn have important effects on the design of welfare maximising monetary policy.

This paper extends the standard New Keynesian DSGE model to incorporate an endogenous degree of price flexibility and uses this model to analyse optimal monetary policy in the face of stochastic shocks. The model is based on an adaptation of the Calvo (1983) price setting structure. The key difference compared to the standard version of the Calvo model is that we allow producers to choose the average frequency of price changes. We assume that producers face costs of price adjustment which are increasing in the average frequency of price changes and we allow producers optimally to choose the degree of price flexibility based on the balance between the costs and benefits of price adjustment.

The model demonstrates the link between monetary policy and the equilibrium degree of price flexibility. Monetary policy, by determining the volatility of macro variables, affects the stochastic environment faced by producers. This has an impact on the benefits of price flexibility for individual producers and thus affects the optimal degree of price flexibility. For example, for a given cost of price adjustment, the greater is the volatility of CPI inflation, the larger will be the benefits of price flexibility for individual producers. Producers will therefore tend to choose a greater frequency of price adjustment when the volatility of CPI inflation is large. A monetary rule which allows volatility in CPI inflation will therefore tend to imply more price flexibility in equilibrium.

Having established a framework which captures the connection between monetary policy and price flexibility, we re-examine one of the main results from the standard literature on welfare maximising monetary policy. This result (analysed in detail by, for instance, Woodford (2003)) is that, in the face of cost-push shocks, it is optimal for monetary policy to allow some volatility in CPI inflation in order to stabilise the "welfare-relevant output gap". This result is obtained in models where the degree of price flexibility is exogenous. How might this result be changed when the degree of price flexibility is endogenised? Will endogenising the degree of price flexibility make it optimal for the monetary authority to raise or lower the volatility of inflation? On the one hand, an increase in the volatility of inflation will tend to increase the equilibrium degree of price flexibility. This implies that output prices can more easily respond to shocks. But it also reduces the ability of monetary policy to affect the real economy and thus reduce the effectiveness of monetary policy in tackling the distortionary effects of cost-push shocks.

Our model shows that endogenising the degree of price flexibility tends to shift the focus of optimal monetary policy towards a reduction in inflation volatility relative to the case of exogenous price flexibility. Indeed, when the degree of price flexibility is endogenous, it appears that optimal policy should completely stabilise CPI inflation in the face of cost-push shocks. This is in sharp contrast to the standard result emphasised in Woodford (2003) and Benigno and Woodford (2005). The essential point is that, lower inflation volatility tends to reduce the equilibrium degree of price flexibility. This both enhances the power of monetary policy and reduces the resource cost of price adjustment.

Besides demonstrating these results, this paper makes a technical modelling contribution by showing how to incorporate endogenous price flexibility into an otherwise standard New Keynesian model. Romer (1990), Devereux and Yetman (2002) and Yetman (2003) have previously proposed adaptations of the Calvo model which are similar in nature to the one described below. However, unlike Romer and Devereux and Yetman, we incorporate the modified approach into the standard microfounded New Keynesian model

widely used by the recent literature on optimal monetary policy. The optimal choice of price flexibility in our model is based on an approximation of individual utility functions derived directly from the microfoundations of the model. This contrasts with the Romer and Devereux and Yetman approach, which is based on largely *ad hoc* macro models and an *ad hoc* approximation of the profit function. Note also that the main issue examined by Romer (199) and Devereux and Yetman (2002) is the impact of a non-zero (but constant) rate of inflation on the equilibrium degree of price flexibility. They do not consider the impact of inflation volatility or output volatility on the equilibrium degree of price flexibility, nor do they analyse welfare maximising monetary policy in the face of stochastic shocks.

In two further related papers Devereux and Yetman (2003, 2010) analyse exchange rate pass-through using a version of the Calvo model which incorporates endogenous price flexibility. These papers are particularly notable because they are amongst the first to introduce this modification into a microfounded general equilibrium model (in this case an open economy model). In technical terms, there are close parallels with our modelling approach.[1] However, Devereux and Yetman again base their analysis on an *ad hoc* approximation of the profit function. This contrasts with our approach, which is directly based on the utility function of individual producers. Moreover, in order to solve their model, Devereux and Yetman either assume that all shocks are i.i.d. (Devereux and Yetman, 2003) or they make use of stochastic simulation techniques (Devereux and Yetman, 2010). In contrast to this, we show that, for quite general cases, it is possible to derive a closed-form representation for the relationship between producers' utility and the degree of price flexibility. This greatly facilitates the derivation of equilibrium in our

[1] The parallels with this paper are primarily technical. The research questions tackled by Devereux and Yetman (2003, 2010) are entirely focused on how price flexibility affects exchange rate pass-through in open economies. In contrast, we are analysing the implications of endogenous price flexibility for optimal monetary policy in a closed economy setting.

model.

The model we describe clearly builds on the standard Calvo (1983) approach to price stickiness and thus shares many of the benefits, as well as the drawbacks, of the Calvo model. There are a number of other approaches to modelling endogenous price flexibility, but an important advantage of our approach is that it is based on, what has become, the general workhorse model used throughout the monetary policy literature.

Other approaches to endogenising price flexibility include the models described by Calmfors and Johansson (2006), Devereux (2006), Kiley (2000), Levin and Yun (2007) and Senay and Sutherland (2006). A number of these contributions focus on the implications of endogenous price flexibility for the long run trade-off between inflation and output, while others analyse the implication of endogenous price flexibility for the propagation of monetary shocks and the causes and nature of business cycles. Only Calmfors and Johansson (2006), Devereux (2006) and Senay and Sutherland (2006) consider the interaction between endogenous price flexibility and monetary policy choices. Calmfors and Johansson (2006) analyse the stabilising properties of endogenising wage flexibility for a small open economy joining a monetary union, while Devereux (2006) analyses the implications of exchange rate policy for the flexibility of prices in an open economy stochastic general equilibrium model. In Senay and Sutherland (2006) we consider the impact of exchange rate regime choice and price elasticity of international trade on the equilibrium degree of price flexibility in an open economy general equilibrium model.[2]

[2]The degree of price flexibility is, in a sense, also endogenous in models of state-dependent pricing. In these models prices may or may not adjust following a shock, depending on the size, duration and nature of the shock, and the costs of price adjustment. This approach is considerably more difficult to implement in a general equilibrium model suitable for analysing optimal monetary policy. Examples of state-dependent pricing models include Devereux and Siu (2007), Dotsey, King and Wolman (1999), Dotsey and King (2005) and Golosov and Lucas (2007). These contributions typically focus on the implications of state dependent pricing for inflation and business cycle dynamics, as well as the propagation of monetary shocks.

This paper proceeds as follows. Section 2 describes the model. Section 3 explains how we solve for the equilibrium level of price flexibility. Section 4 describes how the equilibrium degree of price flexibility is affected by different assumptions about the costs of price adjustment and other key parameter in the model. Section 5 shows how welfare and optimal monetary policy is affected by endogenising the degree of price flexibility. Section 6 concludes the paper.

2 The Model

The model is a variation of the sticky-price general equilibrium structure which has become standard in the recent literature on monetary policy. The model world consists of a single country which is populated by yeoman-farmers indexed on the unit interval. Each yeoman-farmer consumes a basket of all goods produced in the economy and is also a monopoly producer of a single differentiated product. In what follows, we refer to individual yeoman-farmers as "producers" or "consumer-producers".

Price setting follows the Calvo (1983) structure. In any given period, producer j is allowed to change the price of good j with probability $(1-\gamma(j))$.

The timing of events is as follows. In period 0 the monetary authority makes its choice of monetary rule. Immediately following this policy decision, all producers are allowed to make a first choice of price for trade in period 1 (and possibly beyond). Simultaneously, all producers are also allowed the opportunity to make a once-and-for-all choice of Calvo-price-adjustment probability (i.e. $\gamma(j)$). In each subsequent period, beginning with period 1, stochastic shocks are realised, individual producers receive their Calvo-price-adjustment signal (which is determined by their individual choices of γ, i.e. $\gamma(j)$), those producers which are allowed to adjust their prices do so, and finally trade takes place.

The model economy is subject to stochastic shocks from three sources: labour supply, government spending and cost-push (or mark-up) shocks.

2.1 Preferences

All consumer-producers have utility functions of the same form. The utility of consumer-producer j is given by

$$U_t(j) = E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{C_s^{1-\rho}(j)}{1-\rho} - \frac{\chi K_s}{\mu}y_s^{\mu}(j)\right)\right] - A(\gamma(j)) \tag{1}$$

where C is a consumption index defined across goods, $y(j)$ is the output of good j and E_t is the expectations operator (conditional on time t information). K is a stochastic shock to labour supply preferences which evolves as follows

$$\log K_t = \delta_K \log K_{t-1} + \varepsilon_{K,t} \tag{2}$$

where $0 < \delta_K < 1$ and ε_K is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\varepsilon_K] = 0$ and $Var[\varepsilon_K] = \sigma_K^2$.

The expected costs of adjusting prices are represented by the function $A(\gamma(j))$. The form of this function is discussed in more detail below.

The consumption index C is defined as

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\phi-1}{\phi}}\,di\right]^{\frac{\phi}{\phi-1}} \tag{3}$$

where $\phi > 1$, $c(i)$ is consumption of good i. The aggregate consumer price index is

$$P_t = \left[\int_0^1 p_t(i)^{1-\phi}\,di\right]^{\frac{1}{1-\phi}} \tag{4}$$

The budget constraint of consumer-producer j is given by

$$B_t + C_t(j) = r_t B_{t-1} + p_t(j)\,y_t(j)/P_t + R_t \tag{5}$$

where B_t represents holdings of risk-free real bonds at the end of period t, r_t is the gross real rate of return on bonds and R_t is the payoff to a portfolio of state-contingent assets which are optimally chosen to hedge against income risk caused by Calvo price setting.

The intertemporal dimension of consumption choices gives rise to the familiar consumption Euler equation

$$C_t^{-\rho}(j) = \beta r_t E_t \left[C_{t+1}^{-\rho}(j)\right] \tag{6}$$

and individual demand for representative good j is given by

$$c_t(j) = C_t \left(\frac{p_t(j)}{P_t}\right)^{-\phi} \tag{7}$$

Perfect consumption risk sharing implies for all i and j

$$C_t^{-\rho}(j) = C_t^{-\rho}(i)$$

2.2 Government Spending Shocks

Government spending, G_t, is defined as a basket of individual goods with an aggregator function similar to (3). Government demand for representative good j is therefore given by

$$g_t(j) = G_t \left(\frac{p_t(j)}{P_t}\right)^{-\phi} \tag{8}$$

Total demand for good j is thus

$$y_t(j) = c_t(j) + g_t(j) = Y_t \left(\frac{p_t(j)}{P_t}\right)^{-\phi} \tag{9}$$

where aggregate output can be defined as

$$Y_t = C_t + G_t \tag{10}$$

Aggregate government spending is assumed to be a stochastic AR(1) process which evolves as follows

$$G_t - \bar{G} = \delta_G(G_{t-1} - \bar{G}) + \varepsilon_{G,t} \tag{11}$$

where $\bar{G}$ is the steady state level of government spending, $0 < \delta_G < 1$ and ε_G is symmetrically distributed over the interval $[-\varepsilon, \varepsilon]$ with $E[\varepsilon_G] = 0$ and $Var[\varepsilon_G] = \sigma_G^2$. In what follows we will assume that $\bar{G} = 0$.

2.3 Price Setting and Cost-push Shocks

In equilibrium, all producers will choose the same value of $\gamma(j)$, which will be denoted by γ. The determination of γ is discussed below. Thus, in any given period, proportion $(1-\gamma)$ of producers are allowed to reset their prices. All producers who set their price at time t choose the same price, denoted x_t. The first-order condition for the choice of prices implies the following

$$E_t\left\{\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left[\frac{x_t^{\zeta}Y_s}{C_s^{\rho}P_s^{1-\phi}}-\frac{\phi}{(\phi-1)}\frac{\chi\Lambda_s K_s Y_s^{\mu}}{P_s^{-\phi\mu}}\right]\right\}=0 \tag{12}$$

where $\zeta=1-\phi(1-\mu)$ and Λ is a stochastic shock to the mark-up which evolves as follows

$$\log\Lambda_t=\delta_{\Lambda}\log\Lambda_{t-1}+\varepsilon_{\Lambda,t} \tag{13}$$

where $0<\delta_{\Lambda}<1$ and ε_{Λ} is symmetrically distributed over the interval $[-\varepsilon,\varepsilon]$ with $E[\varepsilon_{\Lambda}]=0$ and $Var[\varepsilon_{\Lambda}]=\sigma_{\Lambda}^2$. The first order condition can be re-written as follows

$$x_t^{\zeta}=\frac{\phi\chi}{(\phi-1)}\frac{B_t}{Q_t} \tag{14}$$

where

$$B_t=E_t\left\{\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\Lambda_s K_s Y_s^{\mu}P_s^{\phi\mu}\right\}\qquad Q_t=E_t\left\{\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}Y_s C_s^{-\rho}P_s^{\phi-1}\right\}$$

It is possible to re-write the expression for the aggregate consumer price index as follows

$$P_t=\left[(1-\gamma)\sum_{s=0}^{\infty}\gamma^s x_{t-s}^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{15}$$

For the purposes of interpreting some of the results reported later, it proves useful to consider the price that an individual producer would choose if prices could be reset every period. This price is denoted p_t^o and is given by the expression

$$p_t^{o\,\zeta}=\frac{\phi\chi}{(\phi-1)}\Lambda_t K_t Y_t^{\mu-1}P_t^{\zeta}C_t^{\rho} \tag{16}$$

We refer to this price as the "target price" or "desired price".

2.4 Costs of Price Adjustment

Price flexibility is made endogenous in this model by allowing all producers to make a once-and-for-all choice of the Calvo-price-adjustment probability in period zero.[3] When making decisions with regard to price flexibility each producer acts as a Nash player. Given that all producers are infinitesimally small, the choice of individual $\gamma(j)$ is made while assuming that the aggregate choice of γ is fixed. The equilibrium γ is assumed to be the Nash equilibrium value (i.e. where the individual choice of $\gamma(j)$ coincides with the aggregate γ).

Producers make their choice of γ in order to maximise the discounted present value of expected utility. From the point of view of the individual producer, the optimal γ is the one which equates the marginal benefits of price flexibility with the marginal costs of price adjustment. The benefits of price flexibility arise because a low value of γ implies that the individual price can more closely respond to shocks. The costs of price adjustment may take the form of menu costs, information costs, decision making costs and other similar costs. These costs of price adjustment are captured by the function $A(\gamma(j))$ in equation (1). It is assumed that the cost of price adjustment is proportional to the expected number of price changes per period, i.e. proportional to $1-\gamma(j)$. Thus $A(\gamma(j))$ is of the following form

$$A(\gamma(j)) = \frac{\alpha}{1-\beta}(1-\gamma(j)) \tag{17}$$

where $\alpha > 0$ and the factor $1/(1-\beta)$ converts the per-period cost of price changes to the present discounted value at time zero. It is important to note that the cost of price flexibility is a function of the average rate of price adjustment, and is not linked to actual

[3]By restricting the choice of γ to period zero we avoid the need to track the distribution of γ's across the population of producers as the economy evolves. Given that our main objective is to investigate how the choice of γ responds to the choice of monetary rule, and given that the choice of the monetary rule is itself assumed to be a once-and-for-all decision, it is unlikely that much is lost by restricting the choice of γ in this way.

price changes.

As is standard in the literature, individual producers are assumed to have access to insurance markets which allow them to insure against the idiosyncratic income shocks implied by the Calvo pricing structure.

2.5 Monetary Policy

Monetary policy is modelled in the form of a targeting rule. The monetary authority is assumed to choose the monetary instrument (which is the nominal interest rate, $i_t = r_t P_{t-1}/P_t$) in order to ensure that the following targeting relationship holds

$$\log \frac{P_t}{P_{t-1}} + \psi \log \frac{Y_t}{Y_t^*} = 0 \tag{18}$$

where Y_t^* is monetary authority's target output level. We assume that Y_t^* is chosen to be the welfare maximising output level. This implies that Y_t^* is a function of labour supply, government spending and cost-push disturbances. The determination of Y_t^* is specified in more detail below and follows the definition used in Benigno and Woodford (2005).

Thus the monetary authority follows a state-contingent inflation targeting policy where ψ measures the degree to which inflation is allowed to vary in response to changes in the welfare-relevant output gap. The analysis below focuses on the welfare implications of the choice of ψ. A rule of this form is of particular interest because it is known to be optimal in the context of a model analogous to the model outlined above but where the degree of price flexibility is exogenously specified (see for instance Benigno and Woodford, 2005). Note that it is not necessary to specify explicitly the form of the interest rate rule which delivers the targeted outcome defined by (18).

3 Model Solution

It is not possible to derive an exact solution to the model described above. The model is therefore approximated around a non-stochastic equilibrium (defined as the solution

which results when $K = \Lambda = 1$, $G = \bar{G}$ and $\sigma_K^2 = \sigma_\Lambda^2 = \sigma_G^2 = 0$). In what follows a bar indicates the value of a variable at the non-stochastic steady state and a hat indicates the log deviation from the non-stochastic equilibrium.[4]

Our objective is to solve for the Nash equilibrium value of γ. In order to do this it is necessary to consider in detail the optimal choice of $\gamma(j)$ at the level of the individual producer. Before going into detail, however, we outline our general solution approach.

First note that, for a given value of aggregate γ it is possible to solve for the behaviour of the aggregate economy. The behaviour of the aggregate economy is, by definition, unaffected by the choices of an individual consumer-producer (because each consumer-producer is assumed to be infinitesimally small). It is therefore possible to analyse the choices of producer j while taking the behaviour of the aggregate economy as given. This allows us to solve for the expected evolution of producer j's output price as a function of aggregate γ and $\gamma(j)$. In turn, using the solutions for aggregate variables and producer j's output price, it is possible to solve for the utility of producer j for given values of γ and $\gamma(j)$.

It is thus possible to analyse producer j's utility as a function of $\gamma(j)$. In particular, the utility maximising value of $\gamma(j)$ can be identified for any given value of γ. In other words, it is possible to plot the "best response function" of producer j to aggregate γ. Using the best response function it is straightforward to identify Nash equilibria in the choice of γ. A Nash equilibrium is simply one where the utility maximising choice of $\gamma(j)$ is equal to aggregate γ.

Below we show the detailed derivation of each stage of this solution process. We start with the aggregate economy, then move on to producer j's utility and prices. We are then able to derive an explicit solution for producer j's utility as a function of γ and $\gamma(j)$. We use a grid search technique to analyse the utility function and to plot the best response function. This allows us to identify all possible Nash equilibria for each parameter

[4]Except in the case of government spending, where $\hat{G} = (G - \bar{G})/\bar{Y}$.

combination. We show the solution for aggregate prices and producer j's prices for all three types of shocks (labour supply, government spending and cost-push disturbances). However, for the sake of simplicity we show the closed form solution for producer j's utility function for the case of cost-push shocks only. The extension to the other shocks is straightforward.

3.1 The Aggregate Economy

For a given value of aggregate γ, the aggregate economy behaves exactly like the standard New Keynesian model analysed by Benigno and Woodford (2005) (and many others). In this section we summarise the equations of the aggregate model and derive first-order accurate solutions for some of the key aggregate price variables. These solutions will then be used in the derivation of a second-order accurate solution for the utility of producer j.

First-order approximation of equations (14) and (16) imply that the evolution of the new contract price in period t, $\hat{x}_t$, is given by[5]

$$\hat{x}_t = (1-\beta\gamma)(\hat{p}_t^o - \hat{P}_t) + \beta\gamma E_t[\hat{x}_{t+1}] + O\left(\varepsilon^2\right)$$

In turn, a first-order approximation of (15) implies that $\hat{x}_t$ is related to consumer price inflation as follows

$$\hat{x}_t = \frac{\gamma}{1-\gamma}\pi_t + O\left(\varepsilon^2\right)$$

where $\pi_t = \hat{P}_t - \hat{P}_{t-1}$. It is therefore possible to write a version of the New Keynesian Phillips curve in the form

$$\pi_t = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}(\hat{p}_t^o - \hat{P}_t) + \beta E_t[\pi_{t+1}] + O\left(\varepsilon^2\right)$$

Using (16) the optimal price expressed in real terms, $\hat{p}_t^o - \hat{P}_t$, can be written as follows

$$\hat{p}_t^o - \hat{P}_t = \frac{1}{\zeta}[\hat{\Lambda}_t + \hat{K}_t + (\mu-1)\hat{Y}_t + \rho\hat{C}_t] + O\left(\varepsilon^2\right) \tag{19}$$

[5] All log-deviations from the non-stochastic equilibrium are of the same order as the shocks, which (by assumption) are of maximum size ε. When presenting an equation which is approximated up to order n it is therefore possible to gather all terms of order higher than n in a single term denoted $O\left(\varepsilon^{n+1}\right)$.

which, after substituting for $\hat{Y}_t$ using $\hat{Y}_t = \hat{C}_t + \hat{G}_t$ yields

$$\hat{p}_t^o - \hat{P}_t = \frac{1}{\zeta}[\hat{\Lambda}_t + \hat{K}_t - \rho\hat{G}_t + (\mu + \rho - 1)\hat{Y}_t] + O\left(\varepsilon^2\right) \tag{20}$$

It is useful to define the natural rate of output, $\hat{Y}_t^N$, to be the flexible price equilibrium output level. An expression for $\hat{Y}_t^N$ can be obtained from (16) by imposing the equilibrium condition $\hat{p}_t^o = \hat{P}_t$, thus

$$\hat{Y}_t^N = \frac{1}{(\mu + \rho - 1)}(-\hat{\Lambda}_t - \hat{K}_t + \rho\hat{G}_t) + O\left(\varepsilon^2\right) \tag{21}$$

Using (20) and (21) it is possible to write the real optimal price in terms of the "output gap"

$$\hat{p}_t^o - \hat{P}_t = \frac{(\mu + \rho - 1)}{\zeta}(\hat{Y}_t - \hat{Y}_t^N) + O\left(\varepsilon^2\right) \tag{22}$$

so the New Keynesian Phillips curve can be written in the form

$$\pi_t = \kappa(\hat{Y}_t - \hat{Y}_t^N) + \beta E_t[\pi_{t+1}] + O\left(\varepsilon^2\right) \tag{23}$$

where

$$\kappa = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma}\frac{(\mu + \rho - 1)}{\zeta}$$

The New Keynesian Phillips curve provides one of the key relationships which ties down equilibrium in the macro economy. The second key relationship is provided by the policy rule (18), which can be written in the form

$$\pi_t + \psi(\hat{Y}_t - \hat{Y}_t^*) = 0 + O\left(\varepsilon^2\right) \tag{24}$$

$\hat{Y}_t^*$ is defined to be the welfare maximising output level, which following Benigno and Woodford (2005) is given by

$$\hat{Y}_t^* = c_\Lambda\hat{\Lambda}_t + c_K\hat{K}_t + c_G\hat{G}_t$$

where

$$c_\Lambda = \frac{-\mu\Phi}{(\mu + \rho - 1)[\mu + \rho - 1 + (1-\rho)\Phi]}$$

$$c_K = -\frac{1}{(\mu+\rho-1)}$$

$$c_G = \frac{\rho[\mu+\rho-1-\rho\Phi]}{(\mu+\rho-1)[\mu+\rho-1+(1-\rho)\Phi]}$$

and where

$$\Phi = 1 - \frac{\phi-1}{\phi}$$

Using (22), (23) and (24) it is simple to show that the solution for $\hat{p}_t^o - \hat{P}_t$ can be written in the form

$$\hat{p}_t^o - \hat{P}_t = p_\Lambda \hat{\Lambda}_t + p_K \hat{K}_t + p_G \hat{G}_t + O\left(\varepsilon^2\right) \tag{25}$$

where

$$p_\Lambda = \Gamma_\Lambda \frac{1-\beta\delta_\Lambda}{\zeta}, \quad p_K = \Gamma_K \frac{1-\beta\delta_K}{\zeta}, \quad p_G = \Gamma_G \frac{1-\beta\delta_G}{\zeta}$$

and where

$$\Gamma_\Lambda = \frac{[1+c_\Lambda(\mu+\rho-1)]\,\psi}{\kappa+\psi(1-\beta\delta_\Lambda)}, \quad \Gamma_K = \frac{[1+c_K(\mu+\rho-1)]\,\psi}{\kappa+\psi(1-\beta\delta_K)}, \quad \Gamma_G = \frac{[c_G(\mu+\rho-1)-\rho]\,\psi}{\kappa+\psi(1-\beta\delta_G)} \tag{26}$$

Similarly, the solution for π_t can be written in the form

$$\pi_t = \pi_\Lambda \hat{\Lambda}_t + \pi_K \hat{K}_t + \pi_G \hat{G}_t + O\left(\varepsilon^2\right) \tag{27}$$

where

$$\pi_\Lambda = \Gamma_\Lambda \frac{\kappa}{\mu+\rho-1}, \quad \pi_K = \Gamma_K \frac{\kappa}{\mu+\rho-1}, \quad \pi_G = \Gamma_G \frac{\kappa}{\mu+\rho-1}$$

3.2 Utility of the Representative Producer

In order to derive the equilibrium value of γ it is necessary to consider the impact of the choice of γ on the utility of a representative individual producer. The utility of producer j at time zero (i.e. at the time $\gamma(j)$ is chosen) is given by

$$U_0(j) = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{C_t^{1-\rho}(j)}{1-\rho} - \frac{\chi K_t}{\mu} y_t^\mu(j)\right] - \frac{\alpha}{1-\beta}(1-\gamma(j)) \tag{28}$$

The usual approach to analysing the optimal choice of a variable is to consider the first order conditions with respect to that variable. In this case, however, the first order condition of producer j's utility maximisation problem with respect to $\gamma(j)$ involves (expectations of) derivatives which cannot be written explicitly and thus are difficult to write even in approximated form. It is therefore easier to work directly with the utility function of producer j, given in equation (28), and to approximate the utility function directly. It is then possible to derive the first-order condition with respect to $\gamma(j)$ using the approximated utility function.

Before approximating the utility function there is one further complication. This is introduced by the assumption that producers are fully insured against the consumption risk caused by Calvo pricing uncertainty. This complication can be resolved by replacing the true utility (28) function with the following expression

$$\tilde{U}_0(j) = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[C_t^{-\rho} \left(\frac{y_t(j)\, p_t(j)}{P_t} \right) - \frac{\chi \Lambda_t K_t}{\mu} y_t^{\mu}(j) \right] - \frac{\alpha}{1-\beta}(1-\gamma(j)) \tag{29}$$

where the term representing the utility of consumption has been replaced by a term based on producer j's output, $y_t(j)$, valued using the stochastic discount factor (which is common to all households). The term representing the disutility of work effort has also been modified to incorporate cost-push shocks (Λ). This expression has the same derivative with respect to $\gamma(j)$ as the true utility function but it pre-imposes the assumption that all producers have the same marginal utility with respect to consumption.

Appendix A shows that a second-order approximation of (29) can be written in the form

$$\begin{aligned} \tilde{U}_0(j) - \bar{U} \;=\; & \frac{(\phi-1)\zeta}{2} \bar{C}^{1-\rho} E_0 \sum_{t=1}^{\infty} \beta^{t-1} \Big[-(\hat{p}_t(j) - \hat{P}_t)^2 \\ & + 2(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t) \Big] \\ & + tij - \frac{\alpha}{1-\beta}(1-\gamma(j)) + O\left(\varepsilon^3\right) \end{aligned} \tag{30}$$

where tij represents terms independent of producer j.[6]

[6]Note that in deriving (30) we approximate with respect to C, y, $p(j)$, P, Λ, etc, but not with

We will evaluate and use (30) to analyse the optimal choice of $\gamma(j)$ for producer j. Before analysing this equation in more detail, it is useful to consider the general form of (30) in order to understand the underlying links between $\gamma(j)$ and producer j's utility. The choice of $\gamma(j)$ most obviously affects utility via the costs of price adjustment. However, $\gamma(j)$ also affects utility via its impact on the evolution of producer j's output price, $\hat{p}_t(j)$. Equation (30) shows that there are two routes by which the evolution of $\hat{p}_t(j)$ affects utility. The first route arises via the term $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$. This term effectively captures the variance of $\hat{p}_t(j)$ relative to the general price level. A higher variance of $\hat{p}_t(j) - \hat{P}_t$ implies lower utility. Note that the variance of $\hat{p}_t(j) - \hat{P}_t$ depends on the difference between the flexibility of $\hat{p}_t(j)$ and the flexibility of $\hat{P}_t$. This is captured by the difference between $\gamma(j)$ and aggregate γ. This term is therefore likely to create a tendency for the individual producer to prefer a value of $\gamma(j)$ close to aggregate γ. The second route by which the evolution of $\hat{p}_t(j)$ affects utility arises via the term $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$. This term is effectively the covariance between the actual price charged by producer j and the optimal price if producer j had complete freedom to choose a new price every period. Not surprisingly, an increase in the covariance has a positive effect on utility. Also not surprisingly, this covariance is negatively related to $\gamma(j)$, i.e. the more rigid is $\hat{p}_t(j)$ the less correlated it can be with $\hat{p}_t^o$.[7]

The effect of $\gamma(j)$ on utility is a combination of these three effects, i.e. it is a combination of the effect on the costs of price adjustment, the variance of $\hat{p}_t(j) - \hat{P}_t$ and the covariance between $\hat{p}_t(j)$ and $\hat{p}_t^o$. An increase in $\gamma(j)$ will reduce the costs of price

respect to $\gamma(j)$. We are thus treating $\gamma(j)$ in the same way as the parameters of the monetary policy rule are treated in the monetary policy literature where policy rules are analysed using a second-order approximation of aggregate utility.

[7]Note that Romer (1990) and Devereux and Yetman (2002) use an *ad hoc* profit function of the form $K\left[\hat{p}_t(j) - \hat{p}_t^o\right]^2$. This can be expanded to yield $K\left[\hat{p}_t^2(j) - 2\hat{p}_t^o\hat{p}_t(j) + \hat{p}_t^{o2}\right]$. If it is noted that $\hat{p}_t^{o2}$ is independent of the actions of producer j this can be written as $K\left[\hat{p}_t^2(j) - 2\hat{p}_t^o\hat{p}_t(j)\right] + tij$ which has the same form as the first term in (30).

adjustment and will thus increase utility, but it will also tend to reduce the covariance between $\hat{p}_t(j)$ and $\hat{p}_t^o$ which tends to reduce utility. The effect of an increase in $\gamma(j)$ on the variance of $\hat{p}_t(j) - \hat{P}_t$ will depend on whether $\gamma(j)$ is greater than or less than aggregate γ. The optimal $\gamma(j)$ will obviously depend on the net outcome of the interaction of all these three factors.

3.3 Producer j Prices

In order to analyse equation (30) in more detail it is necessary to derive equations which describe the evolution of producers j's price, i.e. $\hat{p}_t(j)$. In particular it is necessary to derive second-order accurate solutions for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$. This requires first-order accurate solutions for the behaviour of $\hat{p}_t(j)$.

Since $\hat{p}_t(j)$ depends on $\hat{x}_t(j)$ (i.e. the optimal price chosen when producer j is allowed to reset the price of good j) it is first necessary to solve for the first-order accurate behaviour of $\hat{x}_t(j)$. Appendix B shows that the first-order condition for producer j's pricing decision implies

$$\hat{x}_t(j) - \hat{P}_t = (1 - \beta\gamma(j))(\hat{p}_t^o - \hat{P}_t) + \beta\gamma(j)E_t[\hat{x}_{t+1}(j) - \hat{P}_{t+1}] + \beta\gamma(j)E_t[\pi_{t+1}] + O\left(\varepsilon^2\right) \quad (31)$$

By combining this with (25) it is possible to show that

$$\hat{x}_t(j) - \hat{P}_t = x_\Lambda \hat{\Lambda}_t + x_K \hat{K}_t + x_G \hat{G}_t + O\left(\varepsilon^2\right) \quad (32)$$

where

$$x_\Lambda = \Gamma_\Lambda \frac{(1 - \beta\delta_\Lambda)(1 - \beta\gamma(j))(\mu + \rho - 1) + \kappa\zeta\beta\delta_\Lambda\gamma(j)}{(1 - \beta\delta_\Lambda\gamma(j))(\mu + \rho - 1)\zeta}$$

$$x_K = \Gamma_K \frac{(1 - \beta\delta_K)(1 - \beta\gamma(j))(\mu + \rho - 1) + \kappa\zeta\beta\delta_K\gamma(j)}{(1 - \beta\delta_K\gamma(j))(\mu + \rho - 1)\zeta}$$

$$x_G = \Gamma_G \frac{(1 - \beta\delta_G)(1 - \beta\gamma(j))(\mu + \rho - 1) + \kappa\zeta\beta\delta_G\gamma(j)}{(1 - \beta\delta_G\gamma(j))(\mu + \rho - 1)\zeta}$$

Equation (32) provides one of the required components in solving for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$.

To complete the solution for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$ and $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$ it is useful to decompose the expectations operator E_0 into E_0^D, expectations across aggregate disturbances, and E_0^C, expectations across the Calvo pricing signal for producer j, where $E_0[.] = E_0^D[E_0^C[.]]$. Since aggregate disturbances and aggregate variables are independent from the Calvo pricing signal for producer j, we may write

$$E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)] = E_0^D[(\hat{p}_t^o - \hat{P}_t)E_0^C[\hat{p}_t(j) - \hat{P}_t]]$$

This makes clear that it is necessary to obtain a solution for the first-order accurate behaviour of $E_0^C[\hat{p}_t(j) - \hat{P}_t]$.[8] Appendix B shows that the first-order accurate behaviour of $E_0^C[\hat{p}_t(j) - \hat{P}_t]$ is governed by the following

$$E_0^C[\hat{p}_t(j) - \hat{P}_t] = \gamma(j)E_0^C[\hat{p}_{t-1}(j) - \hat{P}_{t-1}] + (1 - \gamma(j))(\hat{x}_t(j) - \hat{P}_t) - \gamma(j)\pi_t + O\left(\varepsilon^2\right) \quad (33)$$

This can be solved and combined with (25) and (32) to yield a second-order accurate expression for $E_0[(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)]$.

In a similar way Appendix B shows that the second-order behaviour of $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$ is governed by the following

$$\begin{aligned} E_0[(\hat{p}_t(j) - \hat{P}_t)^2] &= \gamma(j)E_0[(\hat{p}_{t-1}(j) - \hat{P}_{t-1})^2] + \gamma(j)E_0[\pi_t^2] \\ &\quad -2\gamma(j)E_0^D[\pi_t E_0^C(\hat{p}_{t-1}(j) - \hat{P}_{t-1})] \\ &\quad +(1 - \gamma(j))E_0[(\hat{x}_t(j) - \hat{P}_t)^2] + O\left(\varepsilon^3\right) \end{aligned} \quad (34)$$

which can be solved in combination with (25) and (32) to yield a second-order accurate expression for $E_0[(\hat{p}_t(j) - \hat{P}_t)^2]$.

3.4 Closed Form Solution for Utility

Using equations (25), (27), (32), (33) and (34) we can derive a closed-form solution for the utility of producer j. For the sake of simplicity we focus on the case where the only

[8] To be clear, $E_0^C[\hat{p}_t(j) - \hat{P}_t]$ is the expected value of $\hat{p}_t(j) - \hat{P}_t$ conditional on particular realised values of the aggregate disturbances, $\hat{\Lambda}_t$, $\hat{K}_t$ and $\hat{G}_t$.

source of shocks is the cost-push disturbance, Λ. Extending the expression to incorporate the other two shocks is straightforward. The resulting expression is

$$\begin{aligned}\tilde{U}_0(j) - \bar{U} &= -\frac{\zeta(\phi-1)\bar{C}^{1-\rho}\Delta\sigma_\Lambda^2}{2(1-\beta)(1-\beta\delta^2)(1-\beta\delta\gamma(j))(1-\beta\gamma(j))} \\ &\quad +tij - \frac{\alpha}{1-\beta}(1-\gamma(j)) + O\left(\varepsilon^3\right) \end{aligned} \tag{35}$$

where

$$\begin{aligned}\Delta &= (1-\beta\delta_\Lambda\gamma(j))(1-\gamma(j))x_\Lambda^2 + (1+\beta\delta_\Lambda\gamma(j))\gamma(j)\pi_\Lambda^2 \\ &\quad -2\beta\delta_\Lambda\gamma(j)(1-\gamma(j))x_\Lambda\pi_\Lambda - 2(1-\beta\gamma(j))[(1-\gamma(j))x_\Lambda p_\Lambda - \gamma(j)\pi_\Lambda p_\Lambda]\end{aligned}$$

Equation (35) shows the dependence of producer j's utility on aggregate γ and $\gamma(j)$. This expression therefore allows us to identify producer j's optimal choice of $\gamma(j)$ for any given value of γ.

In principle it would be possible to analyse the optimal choice of $\gamma(j)$ by examining the derivatives of (35). The resulting expressions are however very complex. Furthermore, because the choice of $\gamma(j)$ must lie in the $[0,1]$ range (since it is a probability) the optimal choice of $\gamma(j)$ may be a corner solution rather than an interior solution. It is therefore easier to analyse (35) by means of a numerical grid search technique. This is the most reliable way to identify the global maximising value of $\gamma(j)$ within the $[0,1]$ range for each γ in the $[0,1]$ range. We use this grid search technique to plot the best response function for each parameter combination. In turn, this allows us to identify all possible Nash equilibria for each parameter combination.[9]

[9]Devereux and Yetman (2003) derive a closed-form solution for their model which plays a similar role to (35). However, their expression is only valid for the case of i.i.d. shocks. Our expression (35) is valid for the general case of autoregressive shocks. But note that our model differs from the Devereux and Yetman model in many respects, so our expression (35) does not encompass the equivalent expression in Devereux and Yetman (2003).

Discount factor	$\beta = 0.99$
Elasticity of substitution for individual goods	$\phi = 8.00$
Work effort preference parameter	$\mu = 1.50$
Elasticity of intertemporal substitution	$\rho = 1$
Price adjustment costs	$\alpha = 0.00055$
Cost-push shocks	$\delta_\Lambda = 0.9$, $\sigma_\Lambda = 0.01$
Policy parameter	$\psi = 0.05$

Table 1: Parameter Values

3.5 Calibration

We analyse the model using a wide range of parameter values, but as a benchmark we choose the set of values shown in Table 1. Most of the values chosen are quite standard and require no explanation. The only parameter which requires some discussion is α, i.e. the coefficient in the function determining the costs of price adjustment. The function $A(\gamma(j))$ in principle captures a wide range of costs associated with price adjustment. Not all these costs are directly measurable, so there is no simple empirical basis on which to select a value for α. As a starting point, for the purposes of illustration, the value of α is set at 0.00055 in the benchmark case. In equilibrium this implies prices are adjusted at an average rate of once every four quarters (i.e. $\gamma = 0.75$) so aggregate price adjustment costs are 0.01375 per cent of GDP in equilibrium. This total aggregate cost is not implausibly high, given the potentially wide range of costs incorporated in $A(\gamma(j))$, but it is acknowledged that a more satisfactory basis needs to be found for calibrating α. In order to test the sensitivity of the main results we consider a wide range of alternative values for α.

4 The Equilibrium Degree of Price Flexibility

This section presents numerical results which illustrate the general nature and properties of equilibrium in the model described above. We also discuss the implications for equilibrium γ of variations in the main parameters of the model.

Figure 1 illustrates some of the main features of equilibrium in the benchmark case (i.e. the parameter set in Table 1) and a number of simple variations around that benchmark case. Figure 1 plots the optimal value of $\gamma(j)$ against values of aggregate γ. In other words it shows the "best response function" of producer j against aggregate γ. The benchmark case is illustrated with the plot marked with circles. This plot is nearly horizontal, implying that the optimal value of $\gamma(j)$ is little affected by aggregate γ, and it intersects the 45^0 just once. This point of intersection represents a single Nash equilibrium where $\gamma(j) = \gamma \approx 0.75$.

The position of the best response function, and its shape, are contingent on the values of other parameters of the model. A change in the value of any parameter implies a change in the best response function and a change in the equilibrium value of γ. Figure 1 shows four other examples of best response function - each based on a different value of α, the parameter which determines price adjustment costs. The general pattern that emerges from the cases illustrated in Figure 1 is that an increase in α shifts the best response function upwards, and thus leads to an increase in the equilibrium value of γ, while a reduction in α shifts the best response function downwards and thus leads to a reduction in the equilibrium value of γ. The intuition for this result is simple - an increase in the cost of price adjustment implies that it is optimal to reduce the degree of price flexibility (i.e. increase γ), while a reduction in the cost of price adjustment has the opposite effect.

Figure 1 also illustrates a number of other potentially important features of the best response function. In one of the cases shown (where $\alpha = 0.005$) the best response function does not intersect with the 45^0 line within the zero-one interval. In this case the Nash

equilibrium is a corner solution at $\gamma = 1$, i.e. completely fixed prices. In other words, when the costs of price adjustment are sufficiently high, producers optimally choose to avoid price adjustment entirely. The next lower plot shown in Figure 1, where $\alpha = 0.0015$, illustrates another possibility. In this case the best response function intersects the 45^0 line in two places. There are thus two Nash equilibria. At the other extreme, when α is very low, it is possible to find an equilibrium where prices are completely, or almost completely flexible, i.e. equilibrium γ is close to zero.[10]

Figure 2 illustrates the relationship between equilibrium γ and the costs of price adjustment, α, in more detail. This figure shows that equilibrium γ rises rapidly at low levels of α, but the rate of increase then falls and equilibrium γ is relatively insensitive to α for values of α greater than 0.0005. Notice that, for values of α greater than (approximately) 0.00022 there is no Nash equilibrium within the zero-one interval. For this range of α the Nash equilibrium is a corner solution at $\gamma = 1$.

Figures 3 - 8 illustrate the effects of varying ϕ, the elasticity of substitution between goods, μ, the elasticity of utility with respect to work effort, and ρ, the intertemporal elasticity of substitution. In each case we show the impact of varying each parameter on the best response function, and we also plot the resulting equilibrium value of γ. These plots show that, at least for the range of parameters used here, the equilibrium γ is relatively insensitive to variations in ϕ, μ and ρ.

Finally, Figures 9 and 10 show the effects of varying the parameter of the monetary policy rule, ψ. This parameter measures the degree to which policy is aimed at stabilising inflation relative to stabilising the welfare-relevant output gap, $\hat{Y} - \hat{Y}^*$. The higher is ψ the more monetary policy is aimed at stabilising the output gap, while a lower value of

[10]For some extreme parameter combinations it is possible to find knife-edge cases where the utility function of producer j has two peaks yielding equal utility levels. In these knife-edge cases there appears to be no Nash equilibrium where all producers choose the same $\gamma(j)$. There may be equilibria where the set of producers divides into two groups, each group setting a different value of γ (corresponding to the two utility maxima). We do not investigate these equilibria in this paper.

ψ implies that monetary policy is aimed at stabilising inflation. $\psi = 0$ implies complete inflation stabilisation. Figures 9 and 10 show that the best response function of producer j and the equilibrium γ are quite sensitive to the choice of ψ. A higher value of ψ shifts the best response function downwards and therefore reduces the equilibrium value of γ. In other words, the more policy focuses on stabilisation of the output gap, the greater is equilibrium price flexibility. Conversely, the more monetary policy focuses on inflation stabilisation, the greater will be price stickiness.

The intuition behind this result is relatively straightforward. If the monetary authority is stabilising aggregate inflation it is by definition stabilising the desired price, $\hat{p}_t^o$. This can be seen very clearly from equations (25), (26) and (27) which show the impact of shocks on inflation and the desired price. These equations show that the lower is the value of ψ, the more stable is $\hat{p}_t^o$. If the desired price is very stable then the incentive to incur the costs of price flexibility are much reduced, hence producers choose a high value of γ, i.e. more price stickiness. In the extreme case where $\psi = 0$ (i.e. perfect inflation stabilisation) the equilibrium value of γ is unity, implying perfect price rigidity.

Equations (25), (26) and (27) also show that a high value of ψ (i.e. a monetary rule which allows fluctuations in inflation in order to achieve some stabilisation of the output gap) will necessarily cause fluctuations in the desired price, $\hat{p}_t^o$. This raises the incentive for producers to incur the costs of price flexibility and therefore choose a lower value of γ. Figure 10 shows that, beyond a certain value of ψ (in this case approximately 0.43) the equilibrium value of γ is zero, implying perfectly flexible prices.

5 Welfare and Optimal Policy

We now consider the welfare implications of endogenous price flexibility. In particular we consider the implications for the welfare maximising choice of the policy parameter, ψ. For the purposes of this exercise aggregate welfare in period 0 (i.e. at the time the

monetary policy parameter, ψ, is set) is defined as

$$\Omega = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \int_0^1 \left(\frac{C_t^{1-\rho}(j)}{1-\rho} - \frac{K_t}{\mu} y_t^{\mu}(j) \right) dj \right\} - \frac{\alpha}{1-\beta}(1-\gamma) \tag{36}$$

and a second-order approximation of Ω can be written as follows

$$\begin{aligned} \Omega - \bar{\Omega} = \bar{C}^{1-\rho} E_0 \sum_{t=1}^{\infty} \beta^{t-1} \Bigg\{ & \hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 \\ & - \frac{\phi - 1}{\phi} \left[\hat{Y}_t + \frac{1}{2}\mu(\hat{Y}_t + \frac{1}{\mu}\hat{K}_t)^2 + \frac{1}{2}\frac{\phi\zeta\gamma}{(1-\gamma)(1-\beta\gamma)}\pi_t^2 \right] \Bigg\} \\ & - \frac{\alpha}{1-\beta}(\bar{\gamma} - \gamma) + O\left(\varepsilon^3\right) \end{aligned} \tag{37}$$

Figure 11 plots welfare against a range of values for ψ. This figure shows the behaviour of welfare for the case of endogenous price flexibility. As a point of comparison, it also shows a plot of welfare for the case where price flexibility is exogenously fixed at a given level. In this example we set $\gamma = 0.75$. The exogenous γ case corresponds to the standard model widely analysed in the literature on optimal monetary policy (see e.g. Benigno and Woodford (2005)) and it is therefore a natural point of comparison.

The welfare plot for the exogenous γ case shows that the optimal value of ψ is approximately 0.012. This implies that it is optimal for monetary policy to allow some volatility in inflation in order to achieve some stabilisation of the welfare-relevant output gap. This corresponds to the result emphasised by Benigno and Woodford (2005). This result is quite standard and is analysed and explained in detail by Benigno and Woodford (2005). In essence, cost-push shocks of the type assumed in this model, are distortionary and imply that the flexible price equilibrium is sub-optimal. In a sticky price environment it is therefore *not* optimal for monetary policy to reproduce the flexible price equilibrium. Sticky prices give monetary policy some degree of leverage which can be used to stabilise output around the welfare maximising level. This requires some volatility in inflation. In terms of the policy rule of the form assumed in this model, it is optimal for ψ to be positive, as implied by the welfare plot in Figure 11.

The second plot in Figure 11 shows welfare when price flexibility is endogenous. The parameter values and the relationship between ψ and the equilibrium value of γ are as shown in Figure 10 (although the range of values of ψ shown in Figure 11 is narrower than in Figure 10). It is immediately clear from Figure 11 that, in the endogenous price flexibility case, welfare appears to increase monotonically as ψ decreases towards zero. In other words, when price flexibility is endogenous, it appears to be optimal to engage in strict inflation stabilisation. This is in complete contrast to the case of exogenous price flexibility, as analysed by Woodford (2003) and Benigno and Woodford (2005), where it is optimal to allow some variability in inflation.

The contrast between the two cases arises for two reasons. Firstly, as ψ is reduced, the rise in the equilibrium value of γ reduces the resource cost of price flexibility (i.e. fewer price changes imply lower costs). Secondly, the rise in the equilibrium value of γ implies that monetary policy becomes a more effective policy tool (because the real effects of monetary policy depend on the presence of nominal rigidities and these become more significant as γ increases). Therefore, as γ increases, monetary policy becomes more effective in dealing with the distortions caused by cost-push shocks. This is reflected in a higher level of welfare as ψ decreases and γ increases.

This result represents a significant departure compared to the literature based on exogenous price flexibility. That literature has emphasised that complete inflation stabilisation is *not* optimal in the face of cost-push shocks. The comparison shown in Figure 11 implies that this result may be overturned when price flexibility is endogenised.

Of course, Figure 11 is based on just one set of parameter values. Nevertheless, experiments (not reported) testing the sensitivity of this basic qualitative result indicate that it is robust across a wide range of values for key parameters, such as ϕ, μ and ρ.

The results described above focus entirely on the implications of cost-push shocks. Woodford (2003) and Benigno and Woodford (2005), using models with exogenous price flexibility, show that government spending shocks are very similar to cost-push shocks in

terms of their implications for optimal monetary policy. This similarity carries over into our model, so extending our results to consider shocks to G add no significant new insights. Extending our analysis to consider labour supply shocks (i.e. shocks to K in our model) also adds little. Woodford (2003) and Benigno and Woodford (2005), using models of exogenous price flexibility, show that optimal monetary policy should completely stabilise inflation in the face of labour supply shocks (i.e. policy should replicate the flexible price equilibrium). In that case, our model of endogenous price flexibility simply predicts that prices will be completely rigid. This has no impact on the nature of optimal monetary policy in the face of labour supply shocks.

6 Conclusion

This paper takes a standard sticky-price general equilibrium model and incorporates a simple mechanism which endogenises the degree of nominal price flexibility. We show how the degree of price flexibility is determined within the model and analyse the effects of key parameter values on the equilibrium degree of price flexibility. The analysis shows that the equilibrium degree of price flexibility is sensitive to changes in monetary policy. The more weight monetary policy places on the stabilisation of the output-gap, the more flexible prices become. Conversely, the more weight monetary policy places on inflation stabilisation, the more inflexible prices become. The paper also analyses welfare maximising monetary policy when the degree of price flexibility is endogenous. This is compared to welfare maximising monetary policy in the case of exogenous price flexibility. We show that endogenising the degree of price flexibility tends to shift optimal monetary policy towards complete inflation stabilisation, even when shocks take the form of cost-push disturbances. This contrasts with the standard result obtained in models with exogenous price flexibility, which show that optimal monetary policy should allow some degree of inflation volatility in order to stabilise the welfare-relevant output gap.

Appendix A: Approximation of Producer j's Utility

A second-order approximation of (29) is derived as follows. First substitute for $y_t(j)$ to yield

$$\begin{aligned}\tilde{U}_0(j) &= E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[C_t^{-\rho} Y_t \left(\frac{p_t(j)}{P_t} \right)^{1-\phi} - \frac{\chi \Lambda_t K_t}{\mu} Y_t^{\mu} \left(\frac{p_t(j)}{P_t} \right)^{-\phi\mu} \right] \\ &\quad - \frac{\alpha}{1-\beta}(1-\gamma(j)) \end{aligned} \tag{38}$$

which can be written as

$$\begin{aligned}\tilde{U}_0(j) &= E_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\alpha_{1,t} \left(\frac{p_t(j)}{P_t} \right)^{1-\phi} - \alpha_{2,t} \left(\frac{p_t(j)}{P_t} \right)^{-\phi\mu} \right] \\ &\quad - \frac{\alpha}{1-\beta}(1-\gamma(j)) \end{aligned} \tag{39}$$

where

$$\alpha_{1,t} = C_t^{-\rho} Y_t \text{ and } \alpha_{2,t} = \frac{\chi \Lambda_t K_t Y_t^{\mu}}{\mu}$$

This form of the utility function isolates terms which depend on $\gamma(j)$. A second order approximation of (39) implies

$$\begin{aligned}\tilde{U}_0(j) - \bar{U} &= \bar{C}^{1-\rho} E_0 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\hat{\alpha}_{1,t} - \frac{\phi-1}{\phi\mu} \hat{\alpha}_{2,t} \\ &\quad - \frac{(\phi-1)\zeta}{2} (\hat{p}_t(j) - \hat{P}_t)^2 + \frac{1}{2} \hat{\alpha}_{1,t}^2 - \frac{\phi-1}{2\phi\mu} \hat{\alpha}_{2,t}^2 \\ &\quad - (\phi-1)\hat{\alpha}_{1,t}(\hat{p}_t(j) - \hat{P}_t) + (\phi-1)\hat{\alpha}_{2,t}(\hat{p}_t(j) - \hat{P}_t) \Big] \\ &\quad - \frac{\alpha}{1-\beta}(1-\gamma(j)) + O\left(\varepsilon^3\right) \end{aligned} \tag{40}$$

which can be simplified to

$$\begin{aligned}\tilde{U}_0(j) - \bar{U} &= -\frac{(\phi-1)}{2} \bar{C}^{1-\rho} E_0 \sum_{t=1}^{\infty} \beta^{t-1} \Big[\zeta (\hat{p}_t(j) - \hat{P}_t)^2 \\ &\quad - 2(\hat{\alpha}_{2,t} - \hat{\alpha}_{1,t})(\hat{p}_t(j) - \hat{P}_t) \Big] \\ &\quad + tij - \frac{\alpha}{1-\beta}(1-\gamma(j)) + O\left(\varepsilon^3\right) \end{aligned} \tag{41}$$

where tij represents terms independent of producer j. Note that

$$\hat{\alpha}_{2,t} - \hat{\alpha}_{1,t} = \hat{\Lambda}_t + \hat{K}_t + \mu\hat{Y}_t + \rho\hat{C}_t - \hat{Y}_t + O\left(\varepsilon^2\right)$$

which, from (19) implies

$$\hat{\alpha}_{2,t} - \hat{\alpha}_{1,t} = \zeta(\hat{p}_t^o - \hat{P}_t) + O\left(\varepsilon^2\right)$$

so

$$\begin{aligned}\tilde{U}_0(j) - \bar{U} &= \frac{(\phi-1)\zeta}{2}\bar{C}^{1-\rho}E_0\sum_{t=1}^{\infty}\beta^{t-1}\Big[-(\hat{p}_t(j) - \hat{P}_t)^2 \\ &\quad +2(\hat{p}_t^o - \hat{P}_t)(\hat{p}_t(j) - \hat{P}_t)\Big] \\ &\quad +tij - \frac{\alpha}{1-\beta}(\bar{\gamma} - \gamma(j)) + O\left(\varepsilon^3\right)\end{aligned} \tag{42}$$

which is equation (30) in the main text.

Appendix B: Expected Dynamics of Producer j's Price

The first-order condition for price setting for producer j has the same form as (14) and therefore implies

$$\hat{x}_t(j) = (1 - \beta\gamma(j))\hat{p}_t^o + \beta\gamma(j)E_t[\hat{x}_{t+1}(j)] + O\left(\varepsilon^2\right)$$

or

$$\hat{x}_t(j) - \hat{P}_t = (1 - \beta\gamma(j))\hat{p}_t^o - \hat{P}_t + \beta\gamma(j)E_t[\hat{x}_{t+1}(j) - \hat{P}_t] + O\left(\varepsilon^2\right) \tag{43}$$

Using the relationship

$$E_t[\hat{x}_{t+1}(j) - \hat{P}_t] = E_t[\hat{x}_{t+1}(j) - \hat{P}_{t+1}] + E_t[\pi_{t+1}]$$

equation (43) can be written as

$$\hat{x}_t(j) - \hat{P}_t = (1 - \beta\gamma(j))(\hat{p}_t^o - \hat{P}_t) + \beta\gamma(j)E_t[\hat{x}_{t+1}(j) - \hat{P}_{t+1}] + \beta\gamma(j)E_t[\pi_{t+1}] + O\left(\varepsilon^2\right)$$

which is equation (31) in the main text.

The Calvo pricing process implies that $E_0^C[\hat{p}_t(j)-\hat{P}_t]$ evolves according to the following equation

$$E_0^C[\hat{p}_t(j)-\hat{P}_t]=\gamma(j)E_0^C[\hat{p}_{t-1}(j)-\hat{P}_t]+(1-\gamma(j))(\hat{x}_t(j)-\hat{P}_t)+O\left(\varepsilon^2\right) \qquad (44)$$

Using the relationship

$$E_0^C[\hat{p}_{t-1}(j)-\hat{P}_t]=E_0^C[\hat{p}_{t-1}(j)-\hat{P}_{t-1}]-\pi_t$$

equation (44) can be written as

$$E_0^C[\hat{p}_t(j)-\hat{P}_t]=\gamma(j)E_0^C[\hat{p}_{t-1}(j)-\hat{P}_{t-1}]+(1-\gamma(j))(\hat{x}_t(j)-\hat{P}_t)-\gamma(j)\pi_t+O\left(\varepsilon^2\right)$$

which is equation (33) in the main text.

An equation for the evolution of $E_0[(\hat{p}_t(j)-\hat{P}_t)^2]$ can be derived in a similar way. First note that the Calvo pricing process implies that

$$E_0[(\hat{p}_t(j)-\hat{P}_t)^2]=\gamma(j)E_0[(\hat{p}_{t-1}(j)-\hat{P}_t)^2]+(1-\gamma(j))E_0[(\hat{x}_t(j)-\hat{P}_t)^2]+O\left(\varepsilon^3\right) \quad (45)$$

Using the relationships

$$E_0[(\hat{p}_{t-1}(j)-\hat{P}_t)^2]=E_0[(\hat{p}_{t-1}(j)-\hat{P}_{t-1})^2]+E_0[\pi_t^2]-2E_0[\pi_t(\hat{p}_{t-1}(j)-\hat{P}_{t-1})]$$

and

$$E_0[\pi_t(\hat{p}_{t-1}(j)-\hat{P}_{t-1})]=E_0^D[\pi_t E_0^C(\hat{p}_{t-1}(j)-\hat{P}_{t-1})]$$

equation (45) can be written as

$$\begin{aligned}E_0[(\hat{p}_t(j)-\hat{P}_t)^2] &= \gamma(j)E_0[(\hat{p}_{t-1}(j)-\hat{P}_{t-1})^2]+\gamma(j)E_0[\pi_t^2]\\ &\quad -2\gamma(j)E_0^D[\pi_t E_0^C(\hat{p}_{t-1}(j)-\hat{P}_{t-1})]\\ &\quad +(1-\gamma(j))E_0[(\hat{x}_t(j)-\hat{P}_t)^2]+O\left(\varepsilon^3\right)\end{aligned}$$

which is equation (34) in the main text.

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Figure 1: Price Adjustment Costs and the Best Response Function

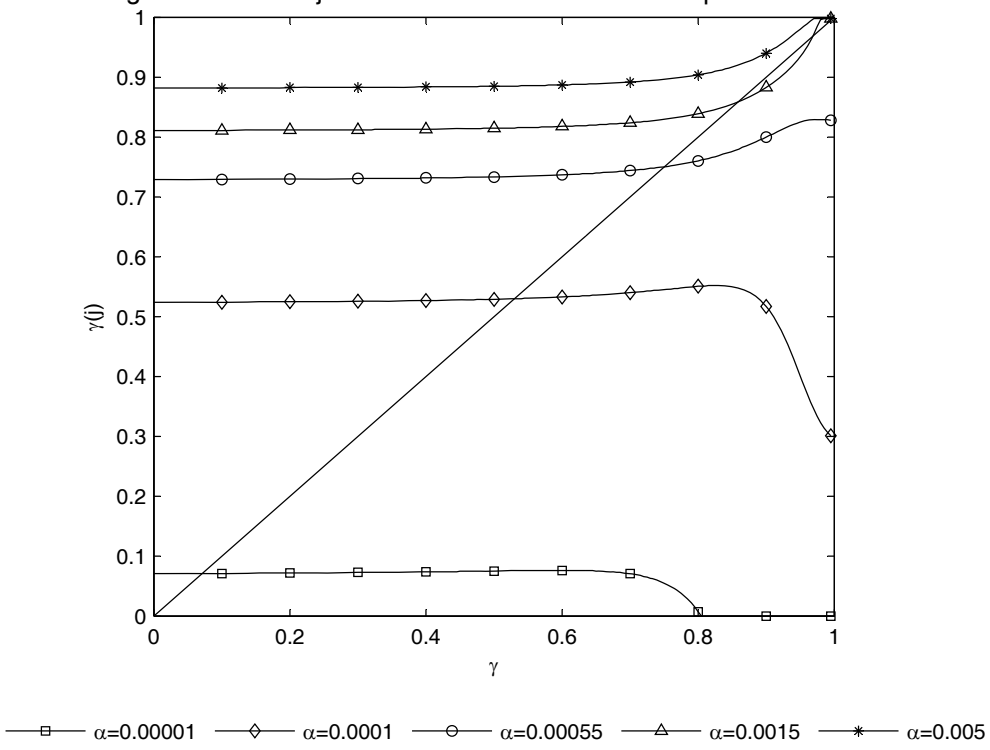


Figure 2: Price Adjustment Costs and Equilibrium γ

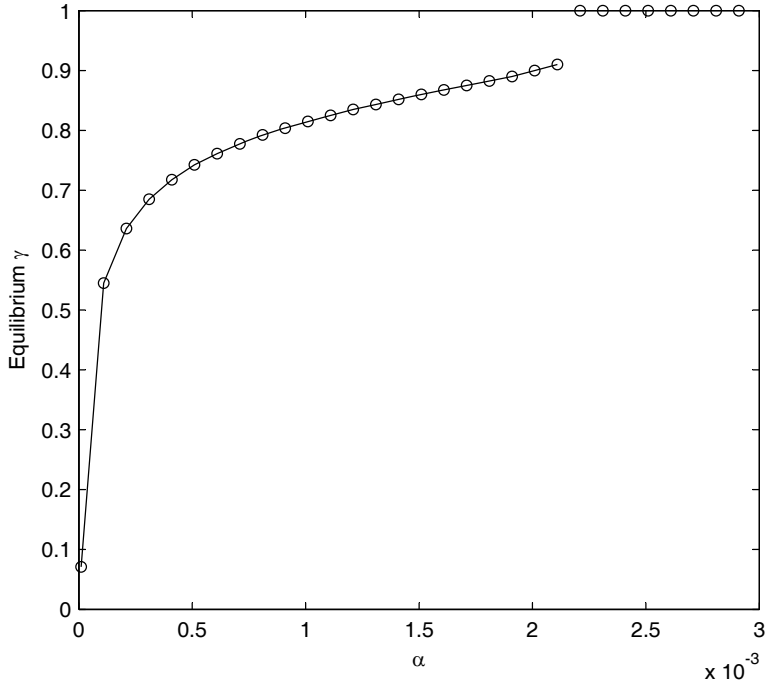


Figure 3: ϕ and the Best Response Function

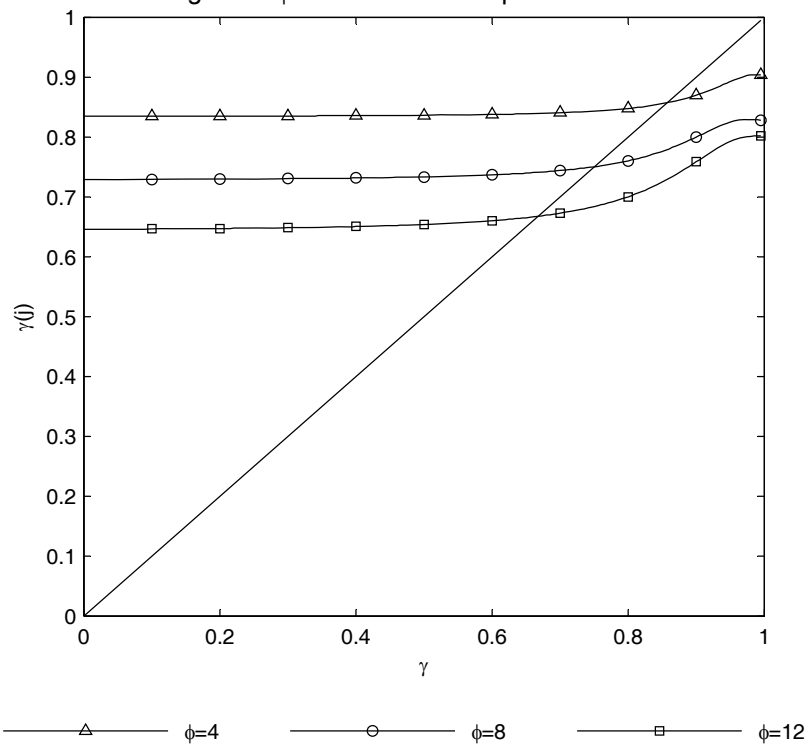


Figure 4: ϕ and Equilibrium γ

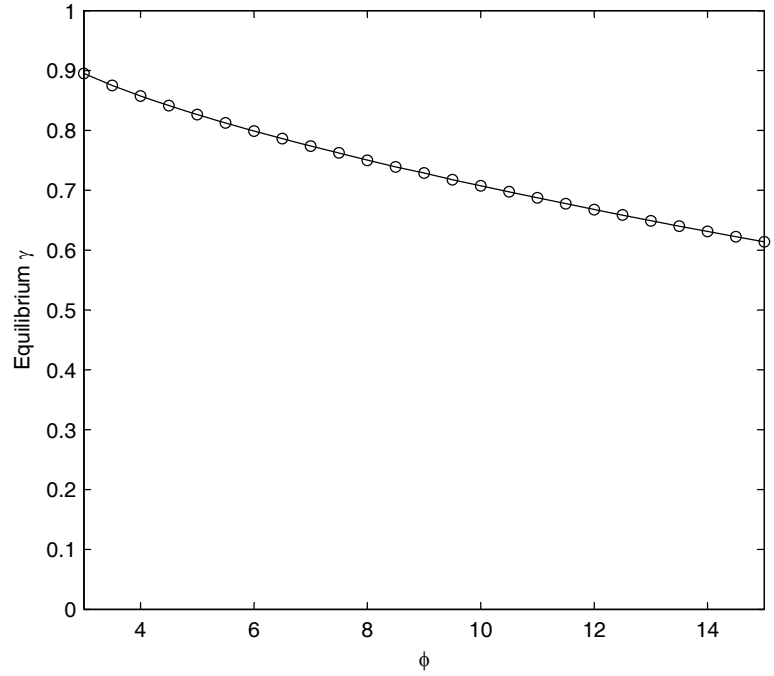


Figure 5: μ and the Best Response Function

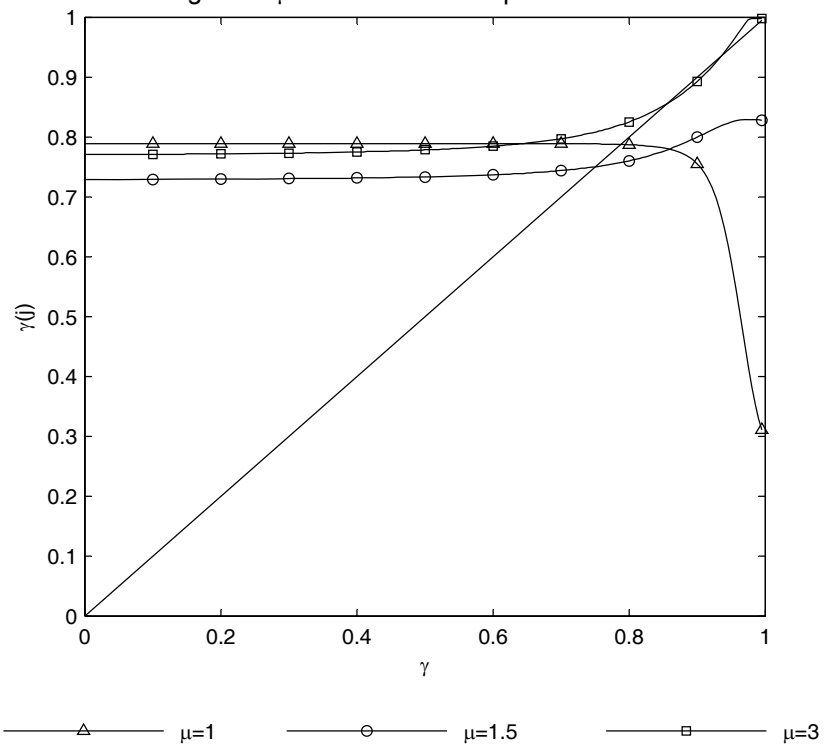


Figure 6: μ and Equilibrium γ

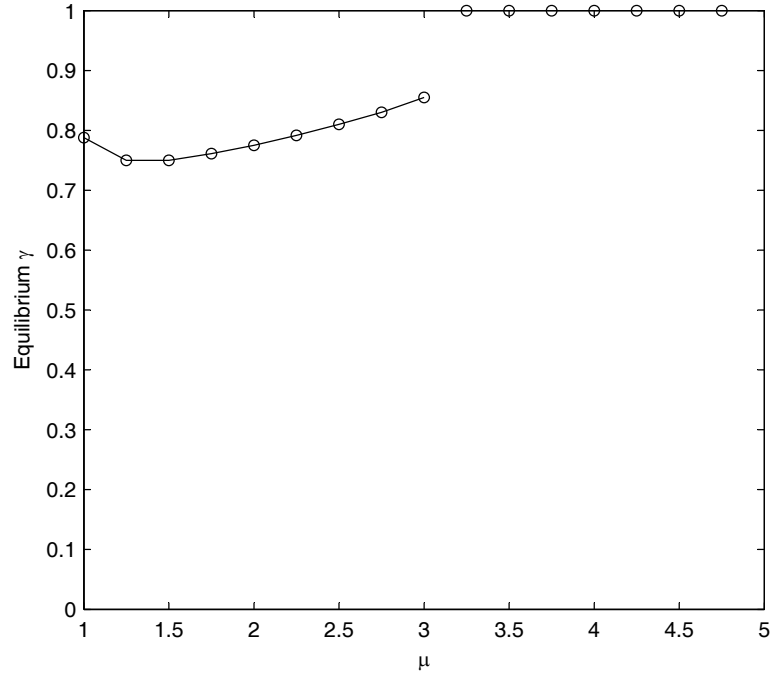


Figure 7: ρ and the Best Response Function

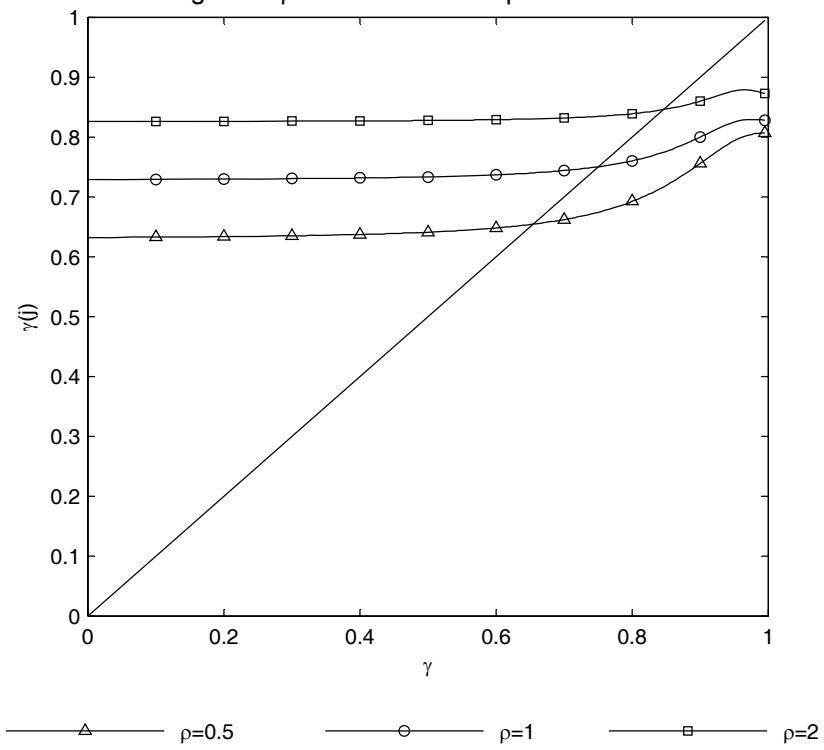


Figure 8: ρ and Equilibrium γ

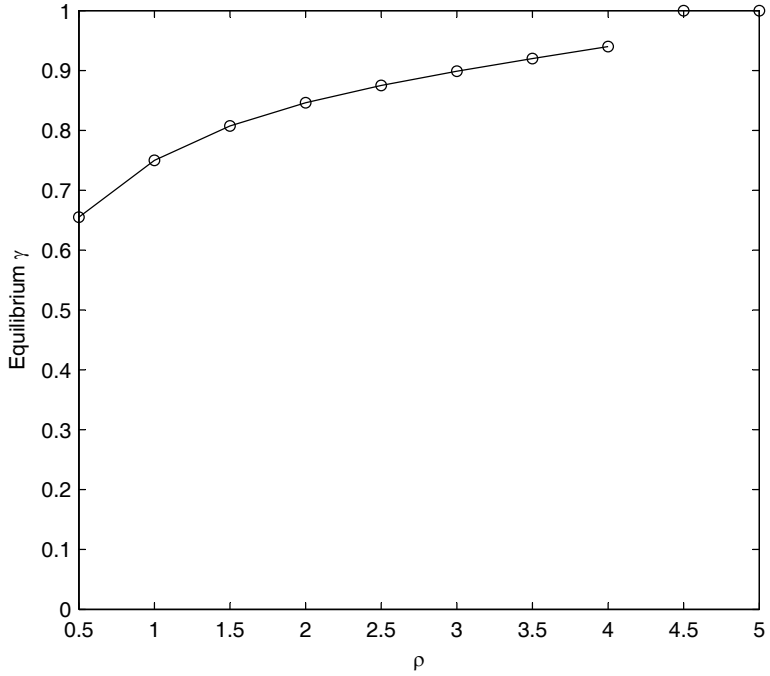


Figure 9: ψ and the Best Response Function

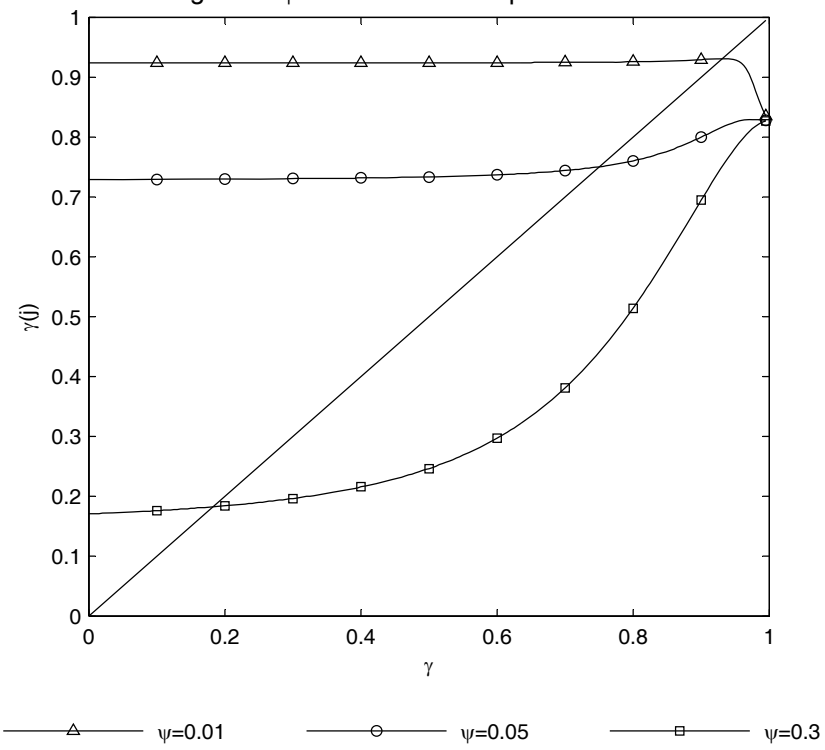


Figure 10: ψ and Equilibrium γ

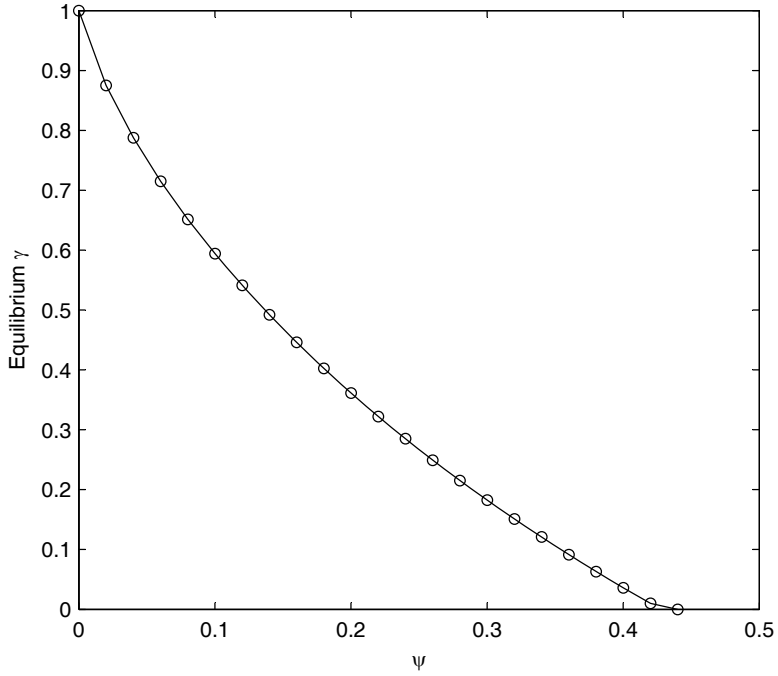
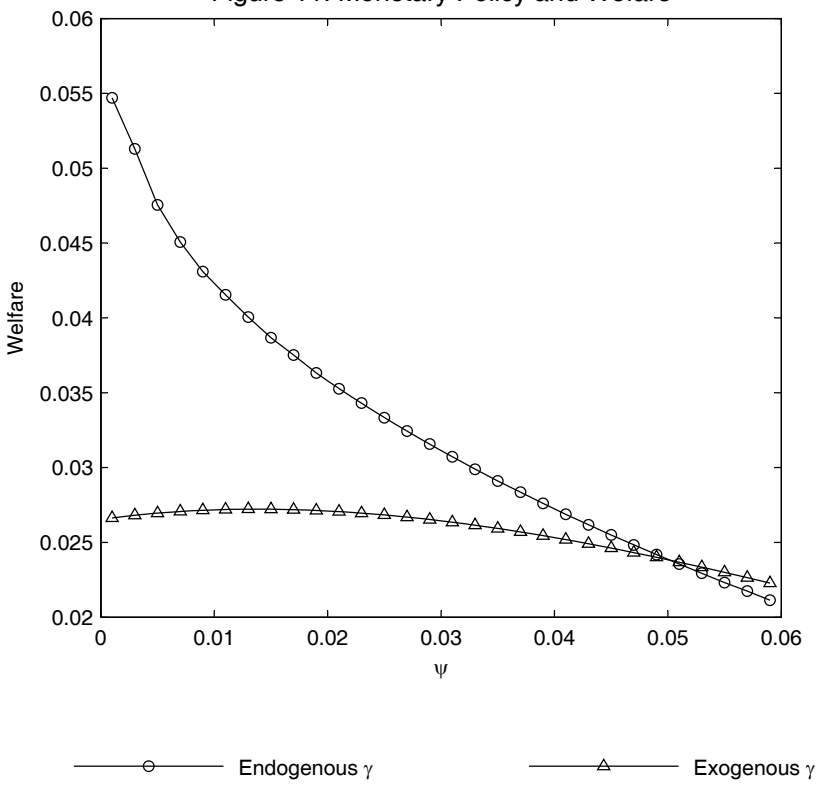


Figure 11: Monetary Policy and Wefare



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