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Journal of Agricultural and Applied Economics, 32,2(August 2000):363–372
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Variation in Marginal Response to Nitrogen Fertilizer between Locations

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ABSTRACT

A logistic growth equation with time and location varying parameters was used to model corn response to applied nitrogen. A nonlinear dummy-variable regression model provided a parsimonious representation of site and time effects on parameter values. The model was used to test for the equality of the mean marginal product of nitrogen fertilizer between locations on the coastal plain of North Carolina. Monte Carlo simulation and bootstrap simulation were used to construct finite sample covariance estimates. Results support rejection of the hypothesis that mean marginal products are equal when nitrogen is applied at 168 kg/ac. A comparison of bootstrapped errors and asymptotic errors suggests that results based on asymptotic theory are fairly reliable in this case.

Key Words: *bootstrap, corn yield, marginal product, nitrogen fertilizer, nonlinear regression.*
JEL: C200, C150, Q100

The specification and empirical implementation of agronomic response functions has attracted considerable interest in the agricultural economics literature as well as the agronomic literature. The interest by agronomists is natural: modeling experimental results is part of the process of gaining greater insight into the growth mechanism of the plant. Applied economists have a normative motivation: they seek to use the results of agronomic experiments to calculate levels of fertilizer use that are, in some sense, optimal. This tradition goes back at least as far as Heady, Pesek, and Brown. Additionally, it is hoped that agronomic experiments will provide substantive information regarding the nature of factor demands at the firm level. This latter motivation is particularly relevant to the evaluation of commodity programs and environmental policies that affect or seek to affect the use of chemical fertilizer at the economic margin. Recent technological innovations in site-specific nutrient management have created renewed interest in crop response research (Lowenberg-DeBoer and Boelje). If these new technologies are to be economically viable, the gain from varying fertilizer application between sites within a field must outweigh the costs associated with implementing the new technology. A prerequisite to economically viable site-specific nitrogen management is variation in the crop-response function between sites. Without significant variation in marginal crop response, there is no variation in optimal nitrogen application rates and hence, no gain from site-specific nitrogen management.

Recent research has focused on the comparison of alternative functional forms for crop response. In particular, there has been an interest in comparing smooth differentiable functional forms with functional forms that

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This paper is a winner of the SAEA Outstanding Graduate Paper Award.

provide an explicit growth plateau as predicted by von Liebig's law of the minimum. Cerrato and Blackmer compared five alternative functional forms using data on Iowa corn response to nitrogen fertilizer. They considered the theoretical loss incurred when an incorrect specification was used to determine the optimal fertilizer rate and found that plateau models (quadratic plus plateau) resulted in lower fertilizer recommendations and a lower loss if the model were incorrect. Frank, Beattie, and Embleton performed a similar analysis, but also considered input substitution between nitrogen and phosphorus. They found evidence of a growth plateau and nonzero substitution possibilities between nitrogen and phosphorus. Paris extended the law of the minimum to apply to smooth functions by specifying a Mitscherlich-Baule growth model for the limiting nutrient and using switching regressions to model a change in the identity of the limiting nutrient. Berck and Helfand reconcile the linear-plateau growth model with differentiable response functions by demonstrating that aggregation of linear-plateau models leads to smooth functional forms.

The studies mentioned above suggest that a single-nutrient response function should allow for self-limiting growth—i.e., a plateau. In addition, variation in the amounts of other inputs will affect response if one of these other inputs is a limiting input. This implies that a single-nutrient response model should not be so rigid in its specification as to impose a unique functional relationship on all sites and time periods. The logistic growth model is a functional form that allows for an approximation to a yield plateau while retaining the benefit of being a smooth differentiable function. Overman, Wilson, and Kamprath developed an extended logistic model that coupled nitrogen accumulation with dry matter yield in a system of two logistic growth equations. The model appeared to perform admirably when fit to mean yields at three locations in North Carolina. These means were derived from four years of observations (Kamprath) and so temporal variation in yield response is not accounted for.

The objective of this paper is to use a logistic growth equation to model corn grain yield response to applied nitrogen at three locations in North Carolina for each of four years. The data are from Kamprath's study of corn response to nitrogen. A parsimonious specification of the model is achieved by modeling each parameter of the model as the sum of a location effect and a time effect. In this way, parameters can vary between sites and between years and yet there remain sufficient degrees of freedom for relatively robust statistical inference. The primary focus of this research is the evaluation of the variation in the expected marginal product of nitrogen fertilizer between sites, a task that requires estimation of nonlinear functions of the parameters. Two simulation-based methods are compared: a Monte Carlo approach based on asymptotic normality of the parameter estimates and a bootstrap simulation. This paper lays a theoretical foundation for future evaluations of site-specific nutrient management programs.

Theoretical Model

Logistic Growth Model

Corn grain yields are modeled using a logistic growth function as proposed by Overman, Wilson, and Kamprath. Only grain yield response to applied nitrogen will be considered here. The form of the model is given by:

$$(1) \qquad Y = \frac{A}{(1 + \exp[B + C \cdot N])}.$$

Y is grain yield (Mg/ha) and N is applied nitrogen (kg-N/ac). The parameters are A, which is the non-negative maximum attainable yield; B, which in effect discounts the yield to the zero nitrogen level; and C, which parameterizes the response of grain yield to applied nitrogen and is typically expected to be non-positive. The logistic model allows for non-negative marginal products and an asymptotic growth plateau. It allows for the possibility of increasing returns to the input over an initial range, though this need not be the case. Thus, the model is flexible enough to accommodate an

S-shaped response function or a concave response function (Myers). The marginal product of applied nitrogen is obtained by differentiating equation (1) with respect to N:

$$\text{(2)} \quad MP(N) = \frac{\partial Y}{\partial N} = \frac{-A \cdot C \cdot \exp(B + C \cdot N)}{(1 + \exp[B + C \cdot N])^2}.$$

For $A > 0$ and $C < 0$, the marginal product will be positive. Note that the marginal product is a function of N and it includes all three parameters. Thus, there is not a dichotomy between the "level" of the response curve and the marginal return to the input as is often posited in crop response research.

The model given by equation (1) can be extended in many ways to account for variation between sites and time periods. The linear equation in the exponential can be expanded to include other inputs including higher order polynomials. Likewise, the numerator could be expanded to include variables thought to affect the yield potential of a particular site or year. A simplified approach is to specify each parameter as consisting of a time effect and a location effect. Thus the model is re-parameterized with:

$$\text{(3a)} \quad A_{s,t} = a^{(0)} + a_s^{(1)} + a_t^{(2)},$$

$$\text{(3b)} \quad B_{s,t} = b^{(0)} + b_s^{(1)} + b_t^{(2)},$$

$$\text{(3c)} \quad C_{s,t} = c^{(0)} + c_s^{(1)} + c_t^{(2)}.$$

The subscript s refers to the location and the subscript t refers to the time period. The superscripts (0), (1), and (2) refer to the mean, site-effect, and time-effect, respectively. This formulation assumes that the time-effect is the same across sites and that the site-effect is the same across time periods. With three sites and four time periods, this specification requires 18 parameters in a dummy variable model, while 36 parameters are required to model each site × time combination separately.

Economic Model

Production theory suggests that efficient input use can be obtained by equating the expected marginal product of the input with the expected input-output price ratio. Consider a producer that grows a crop at L locations; these could be different fields or a partition of a single field. The aggregate output per unit area obtained by applying nitrogen at a rate N^l at each location is:

$$\text{(4a)} \quad F(N^1, \ldots, N^L) = \sum_{l=1}^{L} \omega_l \cdot f_l(N^l);$$

and the aggregate rate of nitrogen application is:

$$\text{(4b)} \quad \bar{N} = \sum_{l=1}^{L} \omega_l \cdot N^l.$$

In equation (4), ω_l is the proportion of total area at location l (let this be 1/L for simplicity) and f_l is the response function for location l (in output per unit area). Let r be the expected value of the ratio of the input price to the output price and suppose that this is the same for all locations. Then, a profit-maximizing producer equates the expected marginal product of each site's nitrogen application to r:

$$\text{(5a)} \quad E\left[\frac{\partial f_l(N_l)}{\partial N_l}\right] = r, \quad \forall\, l.$$

This implies

$$\text{(5b)} \quad E\left[\frac{\partial f_l(N_l)}{\partial N_l}\right] = E\left[\frac{\partial f_k(N_k)}{\partial N_k}\right], \quad \forall\, l \neq k.$$

Thus, optimality requires that the expected value of the marginal products be equal between all locations. If (5b) did not hold, it would be possible to reallocate a fixed amount of nitrogen among the sites and realize a gain in expected output at no additional cost. Implicitly, the expected value is taken with respect to an information set that includes all information available up to the time of nitrogen application.

Now consider equation (5) in the context of the logistic growth model. Equation (5) can be written as:

$$(6)\quad E\left[\frac{-A_l \cdot C_l \cdot \exp(B_l + C_l \cdot N_l)}{(1 + \exp[B_l + C_l \cdot N_l])^2}\right] = E\left[\frac{-A_k \cdot C_k \cdot \exp(B_k + C_k \cdot N_k)}{(1 + \exp[B_k + C_k \cdot N_k])^2}\right], \quad \forall\, l \neq k.$$

One may be interested in determining when equation (6) will hold for a regime of uniform nitrogen application (i.e., $N^1 = N^2 = \ldots = N^L$). Clearly, if the parameters are identical between sites, then a uniform nitrogen level will satisfy equation (6). In contrast to linear models of crop growth, it is not particularly obvious that equality of certain parameters is a necessary condition for equation (6) to hold under uniform nitrogen application. In fact the expected marginal products will have to be compared directly.

If a uniform nitrogen rate will not satisfy equation (5b), or (6) in the present case, there is a potential gain to be derived from site-specific nitrogen application. Of course this gain must offset the cost associated with applying nitrogen at a site-specific level. Nevertheless, an evaluation of marginal products allows for the estimation of the potential gain—i.e., an upper limit on the net gain from site-specific management. Specifically, if the marginal conditions are satisfied by a uniform rate of nitrogen application, no gain is to be realized by site-specific application of nitrogen. The problem with this approach is that marginal product must be estimated for each location. This is not a problem when the results of a designed experiment provide the necessary data. In a production environment the estimation of site-specific marginal products may be very difficult. Each location is likely to receive only one level of fertilizer treatment per year and so many years of data may be required to disentangle the effects of random shocks that vary from year to year from the nitrogen effect. This is a matter that requires further research. In industrial applications, response surface designs have proved fruitful. For now the results of a designed experiment will be used to illustrate the procedure.

Suppose that the producer wishes to determine the optimal rate of nitrogen application subject to the constraint that a uniform rate be employed. Equation (4) can be rewritten as:

$$(7)\quad F(N^0) = \sum_{l=1}^{L} \omega_l \cdot f_l(N^0).$$

And so the first-order equation for expected-profit maximization condition becomes:

$$(8)\quad E\left[\frac{\partial F(N^0)}{\partial N^0}\right] = \sum_{l=1}^{L} \omega_l \cdot E\left[\frac{\partial f_l(N^0)}{\partial N^0}\right] = r.$$

Equation (8) simply states that the weighted-average of expected marginal products equals the expected price ratio. Note that the individual site-specific marginal products need not be equal in this case. Once the optimal uniform rate is determined it is fairly simple to substitute this quantity into equation (5b) and see if the condition is satisfied. If not, then the constraint that requires uniform application is binding and uniform application is sub-optimal relative to site-specific application, *ceteris paribus*.

Empirical Application

Experimental Data

Kamprath conducted nitrogen studies on corn at three locations on the coastal plain of North Carolina from 1981–1984: Central Crops Research Station, Clayton NC (Dothan loamy sand); Lower Coastal Plain Tobacco Research Station, Kinston, NC (Goldsboro sandy loam); and Tidewater Research Station, Plymouth, NC (Portsmouth very fine sandy loam). Each location represented a different soil type and, of course, each year was marked by variations in climatic conditions. An experiment was conducted at each site each year. The experiments were based on a randomized complete block design with four blocks. Five fertilizer levels were replicated in each block: 0, 56, 112, 168, and 224 kg/ac. At each site a different field was used each year. Thus, differences between the outcomes of two experiments at the same site may be due to different climatic conditions between the years or they may be due to differences between fields at the same site. A complete record of the replicates was not available, so treatment means were used in the analysis that follows.

The randomized complete block design is used with the assumption that there is no block × treatment interaction; this is a prerequisite to using the design and it can not be formally tested (Lentner and Bishop). This implies that using the treatment means should not affect the evaluation of the marginal product of nitrogen provided that the marginal effect is independent of the level. Recall that in the logistic model (see equations (1) and (2)) the level of the response curve and the curvature of the response curve are not necessarily independent. Indeed, the randomized complete block design is predicated on the specification of a linear statistical model. All that can be done in the present case is to assume that the use of treatment means will not unduly influence the results. The randomized complete block design suggests one additional caveat. Since blocks are specifically chosen so that different level effects within the field are accounted for in the experimental results then the level for the entire experiment may not be representative of what would be found in an actual field in the region. The blocks receive equal weights when treatment means are calculated, but these block effects may not be distributed with equal probability in any given field. Since the focus of the present study is on the marginal effect of nitrogen fertilizer, this is not really a problem (subject to the initial caveat about the use of a nonlinear model). These potentially important theoretical issues will be overlooked in the present illustrative analysis. However, they do raise some important questions regarding the extrapolation of results from agronomic experiments to crop response models used for regional economic analysis.

In total, Kamprath's study provides 60 observations: (3 sites) × (4 years) × (5 nitrogen levels). Phosphorus and potassium were applied to the fields so that a positive response to nitrogen fertilizer was anticipated. Nitrogen was applied as ammonium nitrate; application was split between planting and two weeks after planting. Reported grain yields were converted to Mg/ha for the present analysis. Additional information on soil characteristics, other measures of nitrogen utilization, and a more detailed discussion of the experimental method can be found in Kamprath.

Statistical Model

Equation (1) with varying parameters defined by (3a), (3b), and (3c) was formulated as a univariate nonlinear regression model using dummy variables to account for site effects and year effects. For nitrogen level i applied at site s at time t, the model is written as:

$$(9) \qquad Y_{i,s,t} = \frac{A_{s,t}}{(1 + \exp[B_{s,t} + C_{s,t}\cdot N_i])} + \epsilon_{i,s,t}.$$

The parameters are further specified as:

$$(10a) \quad A_{s,t} = a^{(0)} + a_1^{(1)}\cdot\delta_1 + a_2^{(1)}\cdot\delta_2 + a_1^{(2)}\cdot\tau_1 + a_2^{(2)}\cdot\tau_2 + a_3^{(2)}\cdot\tau_3,$$

$$(10b) \quad B_{s,t} = b^{(0)} + b_1^{(1)}\cdot\delta_1 + b_2^{(1)}\cdot\delta_2 + b_1^{(2)}\cdot\tau_1 + b_2^{(2)}\cdot\tau_2 + b_3^{(2)}\cdot\tau_3,$$

$$(10c) \quad C_{s,t} = c^{(0)} + c_1^{(1)}\cdot\delta_1 + c_2^{(1)}\cdot\delta_2 + c_1^{(2)}\cdot\tau_1 + c_2^{(2)}\cdot\tau_2 + c_3^{(2)}\cdot\tau_3,$$

$\delta_s = 1$ if the observation is from site s and 0 otherwise, and $\tau_t = 1$ if the observation is from year t and 0 otherwise. Note that the restrictions $\delta_3 = 0$ and $\tau_4 = 0$ are imposed to attain identification. The error terms are assumed to be independently and identically distributed normal random variates with zero expectation and finite variance. The error term reflects random deviations that are specific to a particular treatment in a particular field in a particular year. The model is nonlinear in the parameters.

There are 18 regression parameters and 60 observations, and so 42 degrees of freedom remain for the error. While (10a)–(10c) imposes considerable structure on the nature of the variation in response between sites and years, it seems superior to a specification of independent parameters for each experiment (i.e., site·time combination). Such a model would require the estimation of 36 parameters leaving only 2 error degrees of freedom per experiment.

Estimation Procedure

Equation (9), with the parameters specified by (10a)–(10c), was estimated via nonlinear least squares. Under the assumption that the errors are independent and identically distributed normal random variables, the least-squares estimator is equivalent to the maximum likelihood estimator. In either case, the parameter estimates are consistent and asymptotically normal. The purpose of this study is inference on the marginal product. Specifically, the difference between marginal products is the quantity of interest. Because marginal product is a nonlinear function of the parameters (see equation (2)), substitution of the parameter estimates into the expression for marginal product does not, in general, yield an unbiased estimate. However, if the parameter estimates are maximum likelihood estimates, then this approach yields consistent and asymptotically normal estimates for the marginal product (Greene, p.133). In finite samples or in the absence of the normality assumption, one must rely on the bias not being too large. While not considered here, this is an interesting issue for future simulation studies.

Of interest in this paper is the null hypothesis that the expected marginal products are equal at all sites. Here, the expected marginal product for a site will be estimated as the mean over the four years. The hypothesis can be stated formally as H_0: $MP_s(N^0) - MP_{s'}(N^0) = 0$, for locations, $s \neq s'$ where MP_s is the mean marginal product for site s. Since marginal product is a function of the nitrogen level N, the hypothesis must be evaluated for a specific level of N. In what follows, $N^0 = 168$ kg/ac will be used for illustrative purposes. In general, N^0 would be the optimal uniform rate of nitrogen fertilization, and one would probably be interested in looking at a range of possible values to assess the sensitivity of the result. The hypothesis H_0 will be tested in three ways. A Wald statistic based on the estimated asymptotic covariance matrix is a Chi-square statistic in the limit (Greene, p.488). Second, if the distribution of the parameter estimate is near normal, then one can sample from the appropriate multivariate normal distribution and build a simulated approximation to the distribution of the function of interest. This is a simple Monte Carlo approach that relies on two assumptions: (i) the parameter estimates are approximately normal, and (ii) the estimates of the differences in marginal products do not exhibit a significant bias. The third alternative does not rely on the normality assumption, but instead uses the empirical distribution of the error terms to construct a bootstrap approximation to the sampling distribution of the function of interest (Efron, p.4). Again, one is relying on the bias being small enough so as not to significantly affect inference. Krinskey and Robb cite studies wherein these two simulation procedures are used to construct approximations to the sampling distribution of elasticities that are nonlinear in the underlying parameters. Furthermore, they note (p.199): "if it is fair to assume that the parameters are distributed approximately multivariate normal, one would expect similar results from the two methods." The bootstrap simulation results are used to compute a finite-sample covariance matrix for use in the construction of a simulation-based Wald statistic. One can also construct the marginal distribution of the differences in marginal product and check to see if particular differences appear to be different from zero.

Results

Table 1 provides the parameter estimates for equation (9) with the individual parameters further specified by (10a)–(10c). The computed R^2 is 0.95, though this measure should be interpreted with some caution in the context of a nonlinear model. Nevertheless, it indicates a fairly high correlation between actual and predicted values of corn yield. Maximum attainable yields very significantly between sites as indicated by the significance of the location effects associated with the parameter A. The highest yield potential was at Clayton, followed by Plymouth and then Kinston. The location effect associated with the parameter

Table 1. Least-Squares Parameter Estimates of the Logistic Growth Model[1,2]

Parameter			Mean			
a(0)		11.02			(0.724)	
b(0)		0.513			(0.181)	
c(0)		—			(0.002)	
Site Effects		Clayton			Kinston	
a(1)	3.789	(0.649)		−2.551	(0.530)	
b(1)	0.518	(0.172)		0.424	(0.247)	
c(1)	0.000	(0.001)		−0.016	(0.005)	
Year Effects	1981		1982		1983	
a(2)	−0.007	(0.587)	−01.97	(0.591)	−4.937	(0.605)
b(2)	−0.633	(0.185)	−0.619	(0.191)	0.120	(0.290)
c(2)	0.004	(0.290)	−0.003	(0.002)	−0.014	(0.005)

[1] Asymptotic standard errors appear in parentheses.
[2] The year effect for 1984 and the site effect for Plymouth are restricted to zero.

B was significant and positive for Clayton. This suggests that there is a greater relative difference between the maximum yield potential and the check yield (N = 0) at Clayton than at the other sites. The parameter C can be interpreted as the increase in the yield relative to the maximum yield as function of applied nitrogen. In this respect Kinston demonstrated the largest relative response to nitrogen fertilizer as indicated by the larger absolute value for C. In terms of the year effects, it is worth noting that 1983 showed a significant drop in the maximum attainable yield. Kamprath (p.7) notes that "Severe moisture stress during silking in 1983 resulted in very little response to N fertilization." It is interesting to note that this effect showed up primarily in the estimates of the maximum yield potential.

Diagnostic plots of the residuals are not provided here; however, a brief summary will be provided. Lilliefor's test for normality of the residuals resulted in a failure to reject normality at the 0.05 level of significance and a Q–Q plot supported the assumption of normality in this case. However, a plot of the residuals versus the rate of nitrogen application suggested that the model underfits the observed yields in the N = 56 kg/ac to N = 112 kg/ac range. Future work will have to determine if this result is due to an inappropriate choice of functional form for the underlying yield response function, or if the linear effects specification of the parameters is overly restrictive.

Simulation-based approximations to the

Table 2. A Comparison of Estimated Standard Errors[1]

Parameter	Asymptotic Estimate	Monte Carlo Estimate	Bootstrap Estimate
a(0)	0.724	0.723	0.624
b(0)	0.181	0.181	0.151
c(0)	0.002	0.002	0.001
a(1)—Clayton	0.649	0.639	0.554
b(1)—Clayton	0.172	0.172	0.143
c(1)—Clayton	0.001	0.0015	0.001
a(1)—Kinston	0.530	0.514	0.450
b(1)—Kinston	0.247	0.248	0.200
c(1)—Kinston	0.005	0.005	0.004
a(2)—1981	0.587	0.578	0.492
b(2)—1981	0.185	0.191	0.148
c(2)—1981	0.001	0.001	0.001
a(2)—1982	0.591	0.595	0.510
b(2)—1982	0.191	0.192	0.155
c(2)—1982	0.002	0.002	0.002
a(2)—1983	0.605	0.611	0.507
b(2)—1983	0.290	0.293	0.234
c(2)—1983	0.005	0.005	0.004

[1] Monte Carlo and bootstrap estimates based on 2500 iterations.

Table 3. Estimated Absolute Differences in Mean Marginal Product with N = 168 kg/ac[1,2]

		Bootstrap	
Monte Carlo	Clayton	Kinston	Plymouth
Clayton	na	0.0082 (0.0016)	0.0062 (0.0025)
Kinston	0.0081 (0.0020)	na	0.0045 (0.0015)
Plymouth	0.0061 (0.0029)	0.0045 (0.0019)	na

[1] Bootstrap estimates appear above the diagonal, Monte Carlo estimates below.
[2] Estimated standard errors in parentheses.

standard errors of the parameter estimates are reported in Table 2. The simple Monte Carlo method and the Monte Carlo approximation to the bootstrap distribution were each used to construct pseudo-samples of 2500 observations. The reported standard deviations from the Monte Carlo simulation are very close to the asymptotic standard errors, as would be expected. The bootstrap standard errors tend to be a little smaller, but they are of the same general magnitude as the asymptotic and Monte Carlo standard errors. This suggests that the normality assumption underlying the simple Monte Carlo approach may be quite reasonable.

Table 3 reports the mean differences between simulated mean marginal products. Below the diagonal are reported the means and estimated standard errors based on 2500 Monte Carlo simulation trials. The results from 2500 bootstrap runs appear above the diagonal. Based on an inspection of this matrix, one would doubt the validity of the hypothesis of equal marginal products between locations. A more formal approach based on the Wald statistic (Greene, pp.487–488) allows one to test the hypothesis that equality of the mean marginal products holds between all sites. The Wald statistic is asymptotically distributed as a chi-square random variable with degrees of freedom equal to the rank of the gradient of the vector of restrictions imposed under the null hypothesis. This gives 2 degrees of freedom in this case. The asymptotically valid Wald test results in a p-value of 0.043, while the bootstrap Wald statistic yields a p-value of 0.012. In either case, the hypothesis of equal mean marginal products is rejected at the 0.05 level of significance, but not at the 0.01 level of significance.

In summary, asymptotic theory and numerical approximation yield similar results in terms of the estimated standard errors and in terms of the hypothesis test. One caveat is required here: these results assume that the model is correctly specified. Diagnostics cast some doubt on this assumption and suggest that specification will need to be more rigorously addressed in future work.

Comments and Conclusions

This paper presented the results of a "varying-coefficients" approach to specifying and estimating a logistic growth model of corn response to applied nitrogen using a data from the coastal plain of North Carolina. The hypothesis that the marginal product of nitrogen is constant across sites was rejected by an asymptotic test and by the test constructed on the basis of the bootstrapped covariance estimate. Three issues warrant further consideration: model specification, the simulation methodologies, and the extension of the results to economic analysis.

The evidence of specification problems was not overwhelming, but nevertheless suggests that further work is needed here, particularly if one is interested in the performing inference for lower nitrogen levels where the specification problem appears to be most severe. One possible source of specification error is the way in which parameter variation was modeled. This approach was, admittedly, a blunt instrument used to capture likely sources of variation in the nature of the response function. However, year effects are most likely related to weather and these effects are probably not the same for locations that are separated by a considerable distance, as was the case in this study. This problem may not appear in future applications using data at the within-field level. Also, in this study, *location* referred to the location of the experiment station and not to a specific field. Thus, the linear location effect did not capture variation that may have been due to differences between fields at the same experimental location. One possible way of correcting these deficiencies in the model is to use a random-coefficients model based on a superpopulation characterization of the coefficients. A preliminary study using this approach shows promise. The other potential source of specification error is the logistic growth equation. Future work will consider alternative functional forms to see if the results are sensitive to the choice of functional form.

The two simulation methodologies appeared to perform well for the problem considered in this paper. The fact that bootstrapping produced results that were similar to the Monte Carlo results and to the results based on asymptotic theory suggests that the normal approximation worked well. This also provides one with some confidence that the maximum likelihood estimator for the marginal product is reasonable. A simulation study that uses a model and pseudo-data similar to the one being used for applied analysis would provide one with a better indication of how well the numerical approximation methods work. It would also provide an indication of the degree of bias that may exist in the estimates of the marginal product. Another issue is convergence of the simulated distribution with the target distribution. In this paper, the number of simulation trials was constrained to 2500. Convergence of the simulation was assessed informally in this case and the number of simulation trials was limited by time constraints. A formal test for convergence of the simulated distribution is desirable. Additional simulation studies on the behavior of the approximations should provide guidance as well.

Economic significance and statistical significance are not necessarily the same concept. One might question whether or not the significant differences found in this study would translate into significant differences in marginal profit. This issue would probably best be addressed by estimating the difference between net revenue at the uniform rate of nitrogen application and the net revenue that would be earned using site-specific nitrogen management. This quantity could then be compared to the cost of implementing site-specific technology. The problem here is that estimation of a yield-response function as performed in this paper requires observations of yield for different levels of nitrogen—data that may not be available for a production field unless site-specific management has already been implemented. This issue raises a question concerning the usefulness of single nutrient response functions for predicting the profitability of site-specific management at an arbitrary site. Clearly, the variability in marginal product must be linked to other observable variables that characterize the specific production site.

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