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## Consumer Search and Information Intermediaries

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Cahier de recherche $\mathrm{n}^{\circ}$ IEA-07-01

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January 14, 2007


#### Abstract

In this paper we model the market for a homogeneous good and examine the role of information in determining market outcomes. Unlike in Baye and Morgan (2001) where consumers can only learn about the prices charged by different firms by subscribing to an information intermediary's service, we allow consumers to shop for price quotes. We are interested in determing the impact on market outcomes of allowing for this additional means of information acquisition. Relative to the case where consumers have no interest in searching for prices, consumers become no better off as the cost of search falls. The intermediary, in an effort to compensate for the loss of revenue that it might have earned from consumers, increases the fees that it charges to firms for the right to advertise their product through it. As a result, fewer firms advertise in equilibrium, and so, those that do post higher prices, and, in expectation, consumers pay more for the product. The price increase appropriates all of the gains in consumer surplus generated by the decrease in the cost of search.


Keywords: Search, Advertising, Information Intermediary, Price Dispersion

## 1 Introduction

Baye and Morgan (2001) examine the impact on product market outcomes of information gatekeepers on the internet. They argue that modern communication has lowered the marginal cost of acquiring and transmitting information and impacted the competitiveness of markets. In their set up, firms decide whether to advertise their price through the internet gatekeeper in exchange for a fee. Gatekeepers such as Mortgagequotes.com and Expedia.com allow consumers, should they choose to become members of their websites, easy access to a list of prices charged by different firms.

In the Baye and Morgan model, each firm is located in a geographically separate town and it is assumed that travel costs are sufficiently important that consumers living in one town will not visit the store in another. In other words, search costs are high enough that consumers who do not access the gatekeeper's website have no other way to learn about the prices charged by firms other than their local provider.

But in fact it is often the case that searching for prices is not prohibitively expensive. In cities of even modest size, many firms will have bricks and mortar outlets and so consumers may be able to visit multiple stores as they search for price quotes. Moreover, although they may choose to post their prices on the sites of internet gatekeepers, many firms also have websites of their own on which they announce the prices that they charge. And therefore search may not be that costly.

In this paper we examine the impact on market outcomes of allowing for this additional means of information acquisition. We suppose that consumers can learn about firm prices either by subscribing to the intermediary's service and viewing the advertisements of firms that post them with the intermediary, or by visiting individual stores. The intermediary can be thought of as an internet gatekeeper, an infomediary, a newspaper, or any other information clearinghouse.

As in Baye and Morgan we model the market for a homogeneous product. In a first stage, an advertising intermediary maximizes profits by choosing both a fee to charge consumers to view prices on its site and a fee to charge firms to advertise through it. In a second stage, the
firms simultaneously choose a price and whether or not to advertise this price to consumers via the intermediary. Finally, in a third stage, consumers shop. They decide whether to subscribe to the intermediary and/or whether to engage in search, and they decide from which seller to buy.

Baye and Morgan show that the intermediary sets its consumer subscription fee sufficiently low that all consumers subscribe to its service, and earns its revenue by charging the firms a positive advertising fee. There is price dispersion in the product market. There is some positive probability that firms will not advertise, in which case they will charge the maximum consumer willingness to pay. If they advertise, they draw from a price distribution.

Once search is added to the model, enticing consumers to subscribe to its service becomes more difficult for the intermediary. We show that, despite this, the intermediary will set subscription fees such that all consumers subscribe, but that, compared to Baye and Morgan, this fee must be lower. Essentially search represents an alternative means of information acquisition and so acts as competition for the intermediary. It must lower the fee it charges if it hopes to attract consumers.

Given this one would expect consumers to always be better off when they are able to search for prices. We show that this is not the case. Relative to the case where consumers have no interest in searching for prices (since search costs are high), consumers become no better off as the cost of search falls (locally). This is because the intermediary, in an effort to compensate for the loss of revenue that it might have earned from consumers, increases the fees that it charges to firms for the right to advertise their product through it. As a result, fewer firms choose to advertise in equilibrium, and so those that do advertise post higher prices, and in expectation, consumers pay a higher price for the product. Essentially, the price increase appropriates all of the gains in consumer surplus generated by the decrease in the subscription fee.

So in equilibrium, unless the cost of search is very low, consumers are no better off when they can search. The information intermediary is worse off since its gain in profits from the increased advertising fee is never sufficient to cover the loss from the lowering of the consumer
subscription fee. Firms are better off.
Our paper is closely related to Robert and Stahl (1993) in that in both models consumers are a priori uninformed as to the prices offered by firms and can learn about prices either by engaging in search, or by observing price advertisements from firms. However, in their model, the costs of sending and acquiring information are exogenous. Advertising in their model is not done through an advertising intermediary, or at least not one that behaves strategically. Firms, should they choose to advertise, send their messages directly to consumers in order to inform those viewing the ads of their price. We show that it is precisely the reaction by the intermediary to consumers' ability to search that determines the effect of search on welfare.

More generally our paper is related to the literatures on search and on price advertising. In the search literature (see for example, Stigler (1961), Rothschild (1973), Reinganum (1979), Stahl (1989)) consumers search for price information and incur a cost for each additional price quote. The price advertising literature includes papers in which consumers are targeted directly by firms (see for example Butters (1977), Grossman and Shapiro (1984), Stahl (1994)), and papers in which consumers can access price quotes through newspapers or internet gatekeepers (see for example Salop and Stiglitz (1977), Shilony (1977), Varian (1980)).

The rest of this paper proceeds as follows. In the next section we introduce the model. In Section 3 we characterize equilibrium behavior. In Section 4 we examine the effect of search on equilibrium outcomes. Finally, Section 5 concludes.

## 2 Model

The model we set up is related to Baye and Morgan (2001) and to Robert and Stahl (1993). Relative to Baye and Morgan, in our set up consumers are allowed to engage in search. Relative to Robert and Stahl, an information intermediary is added such that the costs of acquiring and disseminating information are endogenized.

A finite number, $n$, of firms sell a homogeneous good. The cost of producing a unit of the good is assumed to be constant and the same for all firms, and for simplicity the
marginal cost is normalized to be zero. There is a continuum of consumers of measure one, and each has demand $q(p)$ for the good. A consumer that pays $p$ for the good has surplus of $S(p)=\int_{p}^{\infty} q(y) d y\left(\right.$ where $\left.\frac{d S(p)}{d p}=-q(p)\right)$.

Consumers are a priori aware of firms but uninformed as to the prices of the goods they offer for sale. There are two methods for them to learn about prices. The first is through sequential search at a cost of $\varepsilon$ per firm visited. The second is by viewing a firm's advertisement which indicates its price. We assume that advertising can only be done through a single intermediary that acts as an information provider. The information intermediary can be thought of as an information clearinghouse (internet gatekeeper, newspaper, etc). Consumers can search for prices, and/or can pay a subscription fee, $\kappa$, to access the intermediary's service. Subscribing to the service allows consumers to observe the price quotes of advertising firms, and so to determine the lowest advertised price. The search cost of $\varepsilon$ should be interpreted as the cost of visiting a firm either virtually or physically. Even consumers that obtain a favorable price quote via the intermediary must pay $\varepsilon$ to visit the low-price store and acquire the good. We also assume free recall and so if, after visiting other stores, a consumer wishes to purchase from a store that he visited earlier, he can do so at no extra cost.

Firms can advertise their price through the intermediary for a fee $\phi$. We denote the profits earned from a consumer that pays price $p$ as $\pi(p)=p q(p)$. We assume that $\pi(\cdot)$ is globally concave and that there is a unique profit maximizing price $r$. We assume that $S(r) \geq \varepsilon$, so consumers are willing to visit at less one store in order to purchase a good at price $r$.

The timing of the game is as follows. In the first stage, the information intermediary selects fees, $\kappa$ and $\phi$, to charge consumers and firms respectively. In the second stage, consumers decide whether they wish to subscribe to the intermediary's network. We denote by $\mu$, the proportion of consumers that choose to subscribe. In this stage the firms also simultaneously choose their price and whether or not to advertise this price through the intermediary. In a final stage, consumers who subscribe to the intermediary observe price quotes from firms that have chosen to advertise. Consumers may choose to purchase from the low-price advertising firm, or they may at that point choose to engage in sequential search among firms until they
find a more acceptable price quote.

## 3 Equilibrium Behavior

We are interested in characterizing the (perfect Bayesian) symmetric equilibria of this game. In order to characterize equilibrium outcomes, we proceed by backward induction. First, we describe the consumers' shopping behavior including the search rules used by subscribers and non-subscribers respectively. Next, we describe the pricing and advertising strategies of firms and the subscription decision of consumers given the intermediary's fees $\phi$ and $\kappa$. Typically, firms will employ a mixed strategy in their pricing which generates a price distribution. Finally, we characterize the optimal strategy for the intermediary.

### 3.1 Consumer shopping behavior

We begin by characterizing the shopping behavior of consumers. To do so we must characterize two different price distributions. We let $F(p)$ denote the probability that a firm charges a price strictly less than $p$, and we let $H(p)$ denote the probability that a firm charges and advertises a price strictly less than $p .{ }^{1}$ We assume that both $F(\cdot)$ and $H(\cdot)$ are left-continuous. Consusmer shopping rules will depend on these two distributions.

We first consider the shopping strategy of non-subscribers. Since they do not see any advertisements, the only way for consumers to learn about the prices charged by particular firms is through costly search. If consumers engage in search, they draw from the distribution $F$ at a cost of $\varepsilon$ per draw. Suppose that a consumer is in a store that charges $\widehat{p}$, he can either accept $\widehat{p}$, or reject it and continue on with his search. If he does, he will suffer another search cost $\varepsilon$, and draw a new price. Consumers will continue to search if their surplus from price $\widehat{p}$ is less than their expected surplus from continuing their search:

$$
S(\widehat{p})<\int_{0}^{\widehat{p}} S(y) d F(y)+S(\widehat{p})[1-F(\widehat{p})]-\varepsilon
$$

$[1-F(\widehat{p})]$ corresponds to the probability that the new draw yields a price equal to or higher than $\widehat{p}$. Otherwise the consumer draws some new price less than $\widehat{p}$. Rearranging and inte-

[^0]grating by parts, we can rewrite the above expression as
$$
\int_{0}^{\widehat{p}}[S(y)-S(\widehat{p})] d F(y)=\int_{0}^{\widehat{p}} q(y) F(y) d y>\varepsilon .
$$

So the optimal strategy is to search until a price quote is obtained at or below the reservation price, $r_{s}$, where $r_{s}$ solves

$$
\int_{0}^{r_{s}} q(y) F(y) d y=\varepsilon
$$

This is the search rule so long as there are still stores from which the consumer still does not have a price quote. If the consumer has all price information, he simply goes to the store with the lowest price.

Note that if $\varepsilon$ is sufficiently high then $r_{s} \geq r$, and there will be no search at all. In this case our model resembles Baye and Morgan. Otherwise, the possibility of search will have an impact on consumer behavior.

Next we consider the search rule for subscribers. We let $\alpha$ denote the probability that a store advertises in equilibrium and we let $T(p)$ denote the probability that a store charges less than $p$ conditional on not advertising. We have:

$$
T(p)=\frac{F(p)-H(p)}{1-\alpha}
$$

Subscribers may receive advertisements. If they do, one must ask under what condition they will accept to spend $\varepsilon$ to visit the store advertising the best price rather than to search through non-advertising firms. Suppose a subscriber has seen an ad via the intermediary for a price of $\widehat{p}$. One option for the subscriber is to accept this quote and get surplus of $S(\widehat{p})$ minus $\varepsilon$ since he must now visit the firm. Let us denote by $\widetilde{p}$ the price such that $S(\widetilde{p})=S(\widehat{p})-\varepsilon$. In other words, $\widetilde{p}$ is the price that yields equivalent net utility to having $\widehat{p}$ without having to pay to travel to the store advertising this price. The alternative to accepting $\widetilde{p}$ is to reject it and continue on with search. If he does, he will suffer another search cost $\varepsilon$, and draw a new price. Subsribers will continue to search if their surplus from price $\widetilde{p}$ is less than their expected surplus from continuing their search:

$$
\begin{equation*}
S(\widetilde{p})<\int_{0}^{\widetilde{p}} S(y) d T(y)+S(\widetilde{p})[1-T(\widehat{p})]-\varepsilon \tag{1}
\end{equation*}
$$

We denote by $r_{m}$ the price where (1) holds with equality. That is, $r_{m}$ is the search reservation price for a subsriber. if a subscriber sees $r_{m}$ or less once at a store, he will accept rather than spend $\varepsilon$ to visit another store:

$$
S\left(r_{m}\right)=\int_{0}^{r_{m}} S(y) d T(y)+S\left(r_{m}\right)\left[1-T\left(r_{m}\right)\right]-\varepsilon
$$

It follows that a subscriber will accept to pay $\varepsilon$ to visit store that charges $p \leq r_{a}$, where $S\left(r_{a}\right)-\varepsilon=S\left(r_{m}\right)$. Because firms never charges prices above $r$, we have that $r_{a} \leq r$.

### 3.2 Firm pricing and advertising behavior

Firm behavior depends on the proportion of the population that are subscribers of the intermediary's network, $\mu$. For small values of $\mu$, the proportion of non-subscribers is sufficiently high that firms will never charge a price above the non-subscriber's reservation price, $r_{s}$. As the proportion of subscribers increases, it becomes profitable to charge higher prices. When $\mu=1$, there will be a mass point in the price distribution at the monopoly price, $r$. We begin our analysis by describing the firm's strategy when $\mu=1$, i.e. when all consumers are subscribers of the intermediary's network and have access to the price advertisements of firms. We do this, for two reasons. The first is expositional in that the description of the pricing and advertising equilibrium is relatively straightforward when $\mu=1$. Second, we show later in the paper that the symmetric equilibrium that generates the highest profit for the intermediary involves full consumer participation $(\mu=1)$. It is useful, therefore, to focus on this case.

### 3.2.1 Firm behavior when $\mu=1$

When $\mu=1$, the description of the firms' equilibrium behavior is straightforward. We have the following (i) if the advertising fee is too high, firms will never advertise and they will always charge the monopoly price; (ii) if the advertising fee is not too high, firms will sometimes advertise and there will be an equilibrium price distribution, (iii) there will be a mass point at the top of the price distribution, (iv) the mass point will necessarily be at the monopoly price $r$, and all quotes for prices less than $r$ will be advertised; (v) below $r$, the price distribution will have no mass points nor gaps. Formally, we can state the following:

Lemma 1 When $\mu=1$, if $\phi \geq \frac{(n-1) \pi(r)}{n}$, firms never advertise and always charge the monopoly price, $r$. If $\phi<\frac{(n-1) \pi(r)}{n}$, the following characterizes the unique symmetric equilibrium strategy:
(i) Firms advertise if and only if they charge a price strictly less than $r$.
(ii) The equilibrium profit accruing to each firm, $\Pi^{f}$, is given by $\frac{\phi}{(n-1)}$.
(iii) For all prices in the support of the distribution, $p \in\left[p_{\min }, r\right]$, we have:

$$
\begin{equation*}
F(p)=1-\left[\frac{n \phi}{\pi(p)(n-1)}\right]^{\frac{1}{n-1}} \tag{2}
\end{equation*}
$$

where $p_{\min }$ is such that $\pi\left(p_{\min }\right)=\frac{n \phi}{n-1}$.
(iv) There is a mass point at price $r$, and firms charge $r$ with probability $\left[\frac{n \phi}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}$.

Proof. Let $p_{\text {max }}$ be the highest price ever charged in equilibrium and let $\alpha$ denote the probability that a firm advertises. Clearly, a firm will never advertise when charging price $p_{\max }$, since it would reveal to consumers that it charges the highest possible price. Also $p_{\max } \leq r$, since it never pays to offer a price above the monopoly price. The expected profits for a firm that sets a price $p_{\max }$ and does not advertise are:

$$
\begin{equation*}
\Pi^{f}=\pi\left(p_{\max }\right)\left(\frac{[1-\alpha]^{n-1}}{n}\right) . \tag{3}
\end{equation*}
$$

Where $\Pi^{f}$ represents equilibrium profits and $[1-\alpha]^{n-1}$ denotes the probability that all other firms do not advertise. Recall that if no firm advertises, each firm gets $\frac{1}{n}$ th of the market. Now, notice that all prices charged below $p_{\max }$ must be advertised. If a firm charges less than $p_{\max }$ but does not advertise, it will attract consumers with the same probability but earn less because $\pi(p)<\pi\left(p_{\max }\right)$. It follows that $\alpha=F\left(p_{\max }\right)$. Since the only price charged by those not advertising is $p_{\max }$, we have $r_{a}=p_{\max }$. Also we must have $p_{\max }=r$. Indeed, suppose that $p_{\max }<r$, then a firm can deviate and charge a higher price $p$ that will yield higher profits (i.e. $\pi(p)>\pi\left(p_{\max }\right)$ ) and that will be acceptable to consumers (i.e. $\left.S(p)>S\left(p_{\max }\right)+\varepsilon\right)$. This establishes (i).

Now consider the expected profits for a firm that chooses to charge $p$ and to advertise it with the intermediary. The probability that this firm attracts all consumers is given by the probability that all other firms charge more than $p$. Notice that for prices below $r$, there cannot be a positive probability of a tie. If there were a positive probability of a tie, a firm could offer a slightly lower price, break the tie in its favor and make strictly more profit. Hence, we have:

$$
\begin{equation*}
\Pi^{f}=\pi(p)[1-F(p)]^{n-1}-\phi \tag{4}
\end{equation*}
$$

$[1-F(p)]^{n-1}$ is the probability that no other firm advertises a price lower than $p$ through the intermediary. If we take the limit as $p$ converges to $r$ from below, we have:

$$
\begin{equation*}
\Pi^{f}=\pi(r)\left[\frac{[1-F(r)]^{n-1}}{n}\right]=\pi(r)[1-F(r)]^{n-1}-\phi \tag{5}
\end{equation*}
$$

Rearranging, we get:

$$
\begin{equation*}
\Pi^{f}=\pi(r)\left[\frac{[1-F(r)]^{n-1}}{n}\right]=\frac{\phi}{(n-1)} \tag{6}
\end{equation*}
$$

This establishes (ii) in Lemma 1. We can now solve for the distribution of prices, $F(p)$. We have:

$$
F(p)=1-\left[\frac{n \phi}{\pi(p)(n-1)}\right]^{\frac{1}{n-1}}, \text { for } p \in\left[p_{\min }, r\right]
$$

where the minimum price, $p_{\min }$, is such that $F\left(p_{\min }, \phi\right)=0$, or $\pi\left(p_{\min }\right)=\frac{n \phi}{(n-1)}$.
Finally, for (iv) we have that the $\operatorname{Pr}(p<r)=1-F(r)$ and so $F(r)=\left[\frac{n \phi}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}$.
Notice that for firms to be willing to actually advertise and charge less than $r$ with some probability, we must have $F(r)>0$ or

$$
0<\phi<\frac{(n-1) \pi(r)}{n}
$$

So this condition is necessary to have an equilibrium in which firms advertise through the intermediary.

### 3.2.2 Firm behavior when $\mu<1$

When $\mu<1$, the complete characterization of the firms' equilibrium strategies is quite cumbersome. For expositional purposes, we simply provide an overview of what the strategies look like and we present the different possible equilibrium configurations. We do this below, but first we state some general results for the cases where $\mu<1$.

Lemma 2 Suppose that $\mu<1$, then we have the following:
(i) Firms always advertise prices tendered below $r_{s}$ and $H\left(r_{s}\right)=F\left(r_{s}\right)$.
(ii) There are no mass point above or below $r_{s}$.
(iii) The probability that a firm charging $p$ sells to a subscriber is given by:

$$
\left\{\begin{array}{c}
{[1-H(p)]^{n-1}, \text { if } p \text { is advertised and } p \leq r_{a}} \\
\frac{[1-\alpha]^{n-1}}{n}, \text { if } p \text { is not advertised }
\end{array}\right.
$$

(iv) Let $\beta$ be the probability that a store charges $r_{s}$ or less, then the probability that a firm charging $p$ sells to a non-subscriber is given by:

$$
\left\{\begin{array}{c}
{[1-F(p)]^{n-1}, \text { if } p>r_{s}} \\
\sum_{i=0}^{n-1} \frac{[1-\beta]^{i}}{n}, \text { if } p \leq r_{s}
\end{array}\right.
$$

(v) If there is a mass point at $r_{s}$, then price $r_{s}$ is not advertised in equilibrium.
(vi) There is always a range of prices $\left(r_{s}, r_{0}\right)$ where $r_{s}<r_{0}$, which are never charged in equilibrium.
(vii) Firms will never advertise prices above $r_{a}$, or offer prices above $r_{m} \leq r$.
(viii) If $r_{a}>r_{0}$, then all prices between $r_{0}$ and $r_{a}$ are advertised with some probability.

Proof. (i) Suppose that a firm charges a price less than $r_{s}$ but does not advertise this price. It will not attract more non-subscribers than if it instead charged $r_{s}$ (non-subscribers would accept $r_{s}$ anyway) or more subscribers (since it does not advertise). Therefore,
the firm makes strictly less profit than it would by charging $r_{s}$ and not advertising. Hence we have $H\left(r_{s}\right)=F\left(r_{s}\right)$.
(ii) If a firm charges some price $p>r_{s}$, it will sell to the non-subscribers only if all other stores charge $p$ or more. In this case, non-subscribers will visit all the stores and will in the end purchase from the store offering the lowest price. If there is a mass point in the symmetric price distribution above $r_{s}$, there will be a strict probability of a tie at that price and firms will have incentive to deviate and offer a marginally smaller price in order to break the tie. Similarly, because all prices below are advertised, a mass point below $r_{s}$ will lead to a strict probability of a tie among firms seeking to attract subscribers. Hence, a mass point cannot exist.
(iii) If a seller advertises some price $p \leq r_{a}$, it will attract all subscribers if and only if none of the other firms advertise a lower price. This occurs with probability $[1-\alpha]^{n-1}$. If a seller does not advertise, it will attract $\frac{1}{n}$ th of subscribers if and only if all the other sellers do not advertise a price less than $r_{a}$.
(iv) If a seller charges $p>r_{s}$, it will sell to the non-subscribers only if all other firms charge $p$ or greater. This occurs with probability $[1-F(p)]^{n-1}$. Now suppose that the store charges $p<r_{s}$ and it is the $i$ th store in a particular consumer's search itinerary. The store will sell to that consumer with probability $[1-\beta]^{i-1}$, i.e. only if all stores visited earlier according to the consumer search itinerary charge strictly more than $r_{s}$. Because there is probability of $\frac{1}{n}$ of being the $i$ th store in a consumer's itinerary for all $i$, the probability of selling to a non-subscriber when charging $p<r_{s}$ is indeed given by the expression in the Lemma.
(v) If price $r_{s}$ is advertised with positive probability and there is a mass point at $r_{s}$, there is a strictly positive probability of a tie. This cannot be the case.
(vi) Consider a price $p$ marginally above $r_{s}$. Because non-subscribers do not immediately accept $p$ but do accept $r_{s}$, the probability of selling at price $p$ is strictly less than of selling at $r_{s}$. So there are values of $p$ sufficiently close to $r_{s}$ for which profits
are strictly less than those earned from charging $r_{s}$. These prices are not charged in equilibrium. At $r_{0}$, we have $\beta=F\left(r_{0}\right)$. Further $r_{0}$ is given by:

$$
\begin{align*}
& \pi\left(r_{s}\right)\left[\mu\left[1-F\left(r_{s}\right)\right]^{n-1}+(1-\mu) \sum_{i=0}^{n-1} \frac{\left[1-F\left(r_{0}\right)\right]^{i}}{n}\right]-\phi  \tag{7}\\
= & \max \left\{\begin{array}{c}
\pi\left(r_{0}\right)\left[\mu_{\frac{(1-\alpha]}{n}_{n-1}^{n}}+(1-\mu)\left[1-F\left(r_{0}\right)\right]^{n-1}\right] \\
\pi\left(r_{0}\right)\left[\mu\left[1-H\left(r_{0}\right)\right]^{n-1}+(1-\mu)\left[1-F\left(r_{0}\right)\right]^{n-1}\right]-\phi
\end{array}\right\}
\end{align*}
$$

Note that this notation allows for a mass point at $r_{s}$ which is expressed by probability $F\left(r_{0}\right)-F\left(r_{s}\right) \geq 0$.
(vi) Consumers prefer to go to a non-advertising firm than to go to a seller charging $p>r_{a}$ and advertising. Hence, it never pays to advertise prices above $r_{a}$. Suppose that $p>r_{m}$ is the highest price charged in equilibrium. Because there is no mass point above $r_{s}$ and consumers will never accept to pay a price above $r_{m}$ unless they have searched all stores, the probability that the store sells is zero. $r_{m}$ is the highest price charged in equilibrium.
(vii) Suppose that $r_{0}<r_{a}$ but that the highest (supremum) price advertised in equilibrium is some price $p<r_{a}$. We have $H(p)=\alpha$. Also at price $p$ a seller will be indifferent between advertising and not advertising, hence we have:

$$
\pi(p) \mu{\frac{[1-\alpha]^{n-1}}{n}=\pi(p) \mu[1-\alpha]^{n-1}-\phi . . . . . .}
$$

But if this is true for some price $p<r_{a}$, then for all $p^{\prime} \in\left(p, r_{a}\right)$, the seller will strictly prefer to advertise its price. This contradicts the assumption that no price above $p$ is advertised. This completes our proof.

When $\mu<1$, the equilibrium is defined by a vector of threshold prices $r_{s}<r_{0}$ and $r_{1} \leq$ $r_{a}<r_{m} \leq r$. The value $r_{0}$ is the lowest price above $r_{s}$ that yields the same profit as charging $r_{s}$ assuming that no prices are charged between $r_{s}$ and $r_{0}$ (i.e. $\Pi^{f}=\pi\left(r_{0}\right)\left[\mu \frac{\left[1-F\left(r_{s}\right)\right]^{n-1}}{n}\right]$ ). The threshold prices $r_{a}$ and $r_{m}$ are given by the subscribers' shopping rules. Finally $r_{1}$ is the
highest value for which $H\left(r_{1}\right)=F\left(r_{1}\right)$. The structure of the equilibrium will depend on the value of $r_{0}$ relative to $r_{1}, r_{a}$ and $r_{m}$. There will be four different cases:

Case A: If $r_{0} \geq r_{m}$, in equilibrium the highest price charged is $r_{s}$ and non-subscribers never engage in search. We have $F\left(r_{0}\right)=1$, and there will be a mass point at price $r_{s}$. Only prices below $r_{s}$ will be advertised. We have:

$$
\begin{align*}
& \Pi^{f}=\pi\left(r_{s}\right)\left[\mu \frac{\left[1-F\left(r_{s}\right)\right]^{n-1}}{n}+\frac{(1-\mu)}{n}\right]  \tag{8}\\
& \Pi^{f}=\pi(p)\left[\mu[1-F(p)]^{n-1}+\frac{(1-\mu)}{n}\right]-\phi \text { for } p<r_{s} \tag{9}
\end{align*}
$$

For this to constitute an equilibrium, we must verify that firms cannot gain by charging more than $r_{s}$. Suppose that a store charges a price $p>r_{s}$. Conditional on not observing an ad, subscribers will believe that all stores charge $r_{s}$. Hence, so long as $p \leq r_{m}$, we have $S(p) \geq S\left(r_{s}\right)-\varepsilon$ and subscribers that have not observed an advertisement and have arrived in the store through search will accept to pay price $p$. So long as $r_{0} \geq r_{m}$, there will exist no $p \leq r_{m}$ that will generate higher profits for the firms.

The condition $r_{0} \geq r_{m}$ implies that in Case A the proportion $\mu$ is sufficiently small that firms prefer not to charge above $r_{s}$ and attract only subscribers.

Case B: If $r_{m}>r_{0}>r_{a}$, the prices between $r_{m}$ and $r_{0}$ will be chosen by firms in equilibrium, but they will not be advertised. There must a mass point at $r_{s}$, and $r_{s}$ is not advertised in equilibrium. We have $\alpha=F\left(r_{s}\right)=1-\left(\frac{n \phi}{(n-1) \mu \pi\left(r_{s}\right)}\right)^{\frac{1}{n-1}}<F\left(r_{0}\right)<F\left(r_{m}\right)=1$. Equilibrium profits are given by:

$$
\begin{aligned}
& \Pi^{f}=\pi(p)\left[\mu \frac{\left[1-F\left(r_{s}\right)\right]^{n-1}}{n}+(1-\mu)[1-F(p)]^{n-1}\right], \text { for } p \in\left[r_{0}, r_{m}\right] \\
& \Pi^{f}=\pi\left(r_{s}\right)\left[\mu \frac{\left[1-F\left(r_{s}\right)\right]^{n-1}}{n}+(1-\mu)\left(\sum_{i=0}^{n-1} \frac{\left[1-F\left(r_{0}\right)\right]^{i}}{n}\right)\right], \text { for } p=r_{s} \\
& \Pi^{f}=\pi(p)\left[\mu[1-F(p)]^{n-1}+(1-\mu)\left(\sum_{i=0}^{n-1} \frac{\left[1-F\left(r_{0}\right)\right]^{i}}{n}\right)\right]-\phi \text { for } p<r_{s}
\end{aligned}
$$

Case B occurs when $r_{0}>r_{a}$, which holds when the following condition holds:

$$
\begin{align*}
S\left(r_{m}\right) & =\int_{r_{0}}^{r_{m}} S(y) d\left[\frac{F(y)-F\left(r_{s}\right)}{1-F\left(r_{s}\right)}\right]+S\left(r_{s}\right)\left[\frac{F\left(r_{0}\right)-F\left(r_{s}\right)}{1-F\left(r_{s}\right)}\right]-\varepsilon \\
& >S\left(r_{0}\right)-\varepsilon \tag{10}
\end{align*}
$$

Case C: If $r_{m}>r_{a}>r_{0}>r_{1}$, the prices between $r_{m}$ and $r_{a}$ will be charged in equilibrium but not advertised. The prices between $r_{0}$ and $r_{a}$, will only sometimes be advertised. Finally there will be a mass point $r_{s}$, and $r_{s}$ is not advertised in equilibrium. We have:

$$
\begin{gather*}
\Pi^{f}=\pi\left(r_{m}\right)\left[\mu \frac{\left[1-H\left(r_{a}\right)\right]^{n-1}}{n}\right], \text { for } p=r_{m}  \tag{11}\\
\Pi^{f}=\pi(p)\left[\mu \frac{\left[1-H\left(r_{a}\right)\right]^{n-1}}{n}+(1-\mu)[1-F(p)]^{n-1}\right], \text { for } p \in\left[r_{a}, r_{m}\right] \\
\Pi^{f}=\left\{\begin{array}{c}
\pi(p)\left[\mu \frac{\left[1-H\left(r_{a}\right)\right]^{n-1}}{n}+(1-\mu)[1-F(p)]^{n-1}\right] \\
\pi(p)\left[\mu[1-H(p)]^{n-1}+(1-\mu)[1-F(p)]^{n-1}\right]-\phi
\end{array}, \text { for } p \in\left[r_{0}, r_{a}\right]\right. \tag{12}
\end{gather*} \Pi_{\Pi^{f}=\pi\left(r_{0}\right)\left[\mu \frac{\left[1-H\left(r_{a}\right)\right]^{n-1}}{n}+(1-\mu)\left(\sum_{i=0}^{n-1} \frac{\left[1-F\left(r_{0}\right)\right]^{n-1}}{n}\right)\right], \text { for } p=r_{s}}^{\Pi^{f}=\pi(p)\left[\mu[1-F(p)]^{n-1}+(1-\mu)\left(\sum_{i=0}^{n-1} \frac{\left[1-F\left(r_{0}\right)\right]^{n-1}}{n}\right)\right]-\phi, \text { for } p<r_{s}} .
$$

Because there can be no mass point at the top of the distribution, we have $F\left(r_{m}\right)=1$. So from above we can obtain the following condition on the distributions $H(p)$ and $F(p)$. From (12) evaluated at $r_{a}$, we obtain:

$$
\begin{equation*}
\left[1-H\left(r_{a}\right)\right]^{n-1}=\frac{n \phi}{(n-1) \mu \pi\left(r_{a}\right)} . \tag{13}
\end{equation*}
$$

This, together with (11), yields an expression for $\Pi^{f}$ :

$$
\begin{equation*}
\Pi^{f}=\left[\frac{\pi\left(r_{m}\right) \phi}{\pi\left(r_{a}\right)(n-1)}\right] \tag{14}
\end{equation*}
$$

Using this, we can compute the distributions $H($.$) and F($.$) :$

$$
\begin{align*}
{[1-H(p)]^{n-1} } & =\frac{\phi}{\mu \pi(p)}+\frac{\phi}{\mu(n-1) \pi\left(r_{a}\right)}  \tag{15}\\
{[1-F(p)]^{n-1} } & =\frac{1}{(1-\mu)}\left[\frac{\phi \pi\left(r_{m}\right)}{\pi(p)(n-1) \pi\left(r_{a}\right)}-\frac{\phi}{(n-1) \pi\left(r_{a}\right)}\right] \tag{16}
\end{align*}
$$

One can verify that $F(p)>H(p)$ if and only if $p>r_{1}$, where $r_{1}$ denotes the price such that:

$$
\begin{equation*}
(1-\mu)(n-1) \pi\left(r_{a}\right)+\pi\left(r_{1}\right)=\mu \pi\left(r_{m}\right) . \tag{17}
\end{equation*}
$$

Hence, if $r_{0}>r_{1}$, we have $F\left(r_{0}\right)>H\left(r_{0}\right)$, and there must be a mass point a $r_{s}$.
Case D: If $r_{m}>r_{a}>r_{1}>r_{0}$, prices above $r_{a}$ are never advertised, prices between $r_{a}$ and $r_{1}$ are advertised with some probability and all prices below $r_{1}$ are always advertised. We have $F\left(r_{1}\right)=H\left(r_{1}\right)$ and there is no mass point at $r_{s}\left(F\left(r_{s}\right)=F\left(r_{0}\right)\right)$.

When $\mu$ converges to 1 , we are in case D. One can verify from (17) that the threshold prices $r_{1}$ and $r_{a}$ converge to $r$ and there is no mass point at $r_{s}$. Also from (7), one can verify that $r_{0}$ converges to $r_{s}$.

To summarize, the description and computation of the equilibrium when $\mu<1$ is complex. For some values of $\mu<1$, we have active search by non-subscribers, and a mass point in the price distribution.

### 3.3 The consumer subscription decision

The consumer's subscription decision will depend on the comparison between $\kappa$, the subscription fee charged by the information intermediary, and the benefit to a consumer of becoming a subscriber. Let $U_{N S u b}(\mu, \phi)$ and $U_{S u b}(\mu, \phi)$ denote the expected utility of a non-subscriber and subscriber respectively when there is a proportion $\mu$ of consumers subscribed and the advertising fee is $\phi$.

Lemma 3 Given $\kappa$ and $\phi$ chosen by the intermediary, $\mu^{*}$ is an equilibrium proportion of subscribers if and only if:
(i) $\mu^{*}=1$ and $\kappa \leq U_{\text {Sub }}(1, \phi)-U_{N S u b}(1, \phi)$.
(ii) $\mu^{*}=0$ and $\kappa \geq U_{S u b}(0, \phi)-U_{N S u b}(0, \phi)$.
(iii) $\mu^{*} \in(0,1)$ and $\kappa=U_{S u b}\left(\mu^{*}, \phi\right)-U_{N S u b}\left(\mu^{*}, \phi\right)$.

The result is immediate and a formal proof is omitted. Note that there is always an equilibrium with $\mu^{*}=0$ when $\kappa \geq 0$. Indeed, when $\mu^{*}=0$, firms never advertise and they always charge $r$ and there is no benefit of becoming a subscriber and $U_{S u b}(0, \phi)-U_{N S u b}(0, \phi)=$ 0 . When $\mu^{*} \in(0,1)$, consumers must be indifferent between becoming subscribers or not.

We are interested in an equilibrium with advertising and so in order to better understand the impact of $\phi$ and $\mu$ on consumer welfare, we need to compute more precisely $U_{N S u b}(\mu, \phi)$, the expected utility of a non-subscriber and $U_{S u b}(\mu, \phi)$, the expected utility of a subscriber. The follow lemma (proven in the appendix) summarizes the main result of this subsection.

Lemma 4 Let $F(., \mu, \phi)$ denote the equilibrium distribution of prices for given $\mu$ and $\phi$, and let $r_{s}(\mu, \phi)$ be the optimal reserve price for the non-subscribers. Then, we have:

$$
\begin{align*}
U_{N S u b}(\mu, \phi) & = \begin{cases}S(r)+\left(\int_{r_{s}(\mu, \phi)}^{r} q(y)\left(1-[1-F(y, \mu, \phi)]^{n}\right) d y\right), & \text { if } r_{s}(\mu, \phi)<r \\
S(r)+\int_{0}^{r} q(y) F(y, \mu, \phi) d y-\varepsilon, & \text { if } r_{s}(\mu, \phi)=r\end{cases}  \tag{18}\\
U_{S u b}(\mu, \phi) & \leq S(r)+\left(\int_{0}^{r} q(y)\left(1-[1-F(y, \mu, \phi)]^{n}\right) d y\right)-\varepsilon \tag{19}
\end{align*}
$$

And eq. 19 holds with equality when $\mu=1$.

From the above Lemma we can set a bound on the difference between subscriber and non-subscriber welfare and so for the value of the subscription fee that the information intermediary can charge consumers

$$
\begin{align*}
\kappa(\mu, \phi) & =U_{\text {Sub }}(\mu, \phi)-U_{N S u b}(\mu, \phi) \\
& \leq \int_{0}^{r_{s}(\mu, \phi)} q(y)\left(1-F(y, \mu, \phi)-\left([1-F(y, \mu, \phi)]^{n}\right) d y\right. \\
& =\int_{0}^{r_{s}(\mu, \phi)} q(y)\left(\sum_{i=1}^{n-1}[1-F(y, \mu, \phi)]^{i}\right) F(y, \mu, \phi) d y \tag{20}
\end{align*}
$$

### 3.4 Intermediary behavior

Revenue for the intermediary comes from two sources: the advertising fees charged to firms and the subscription fees charged to consumers. The probability that a firm advertises is given by $H(r, \phi, \mu)$ and so the revenues from advertising fees are $n \phi H(r, \phi, \mu)$. The subscription fee is set so that it equals $U_{S u b}(\phi, \mu)-U_{N S u b}(\phi, \mu)$, i.e. consumers are indifferent between
subscribing or not. The intermediary's profits are given by

$$
\begin{equation*}
\Pi^{I}(\phi, \mu)=n \phi H(r, \phi, \mu)+\mu\left[U_{S u b}(\phi, \mu)-U_{N S u b}(\phi, \mu)\right] . \tag{21}
\end{equation*}
$$

Proposition 1 Among the possible symmetric equilibria, the profit of the intermediary is maximized when the following holds:
(i) The subscription fee is such that all consumers subscribe to the intermediary's service, i.e. $\mu=1$.
(ii) The subscription fee is given by

$$
\begin{equation*}
\kappa(\phi)=\left(\int_{p_{\min }}^{r_{s}} q(y)\left(\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}-\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{n}{n-1}}\right) d y\right) \tag{22}
\end{equation*}
$$

where

$$
\begin{align*}
& \int_{p_{\min }}^{r_{s}} q(y)\left(1-\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}\right) d y  \tag{23}\\
= & \min \left[\varepsilon ; \int_{p_{\min }}^{r} q(y)\left(1-\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}\right) d y\right]
\end{align*}
$$

(iii) The advertising fee, $\phi$, maximizes:

$$
\max _{\phi} \Pi^{I}=n \phi\left(1-\left[\frac{n \phi}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}\right)+\kappa(\phi)
$$

subject to 23.

The proof of part (i) of this proposition is cumbersome and therefore delegated to the appendix. The main idea is that for all $\mu<1$ there exists a $\phi^{*}$, such that for any $\phi$, $\Pi^{I}(\mu, \phi)<\Pi^{I}\left(1, \phi^{*}\right)$. Therefore, it is never optimal to set $\mu<1$. The rest of the proposition follows immediately from the previous discussion. We have replaced $F($.$) by the equation$ given by (??).

## 4 Effect of Search on Equilibrium Behavior

Now that we have described equilibrium behavior we would like to determine the effect of allowing consumers to search on equilibrium outcomes. To do so we first characterize the
search cost that is such that it is not worthwhile for consumers to search. This occurs when the reserve price of consumers is equal to the monopoly price.

Proposition 2 There exists a search cost, $\varepsilon_{n s}$, such that for all $\varepsilon>\varepsilon_{n s}$, the optimal advertising fee is $\phi_{n s}$, and $r_{s}\left(\phi_{n s}, \varepsilon\right)=r$ so that search is not worthwhile for consumers.

That is, $\varepsilon^{n s}=\left(\int_{p_{\min }\left(\phi_{n s}\right)}^{r} q(y) F\left(y, \phi^{n s}\right) d y\right)$ is such that $\varepsilon^{n s}$ is the lowest value of $\varepsilon$ such that the reserve price of consumers is equal to $r$. Moreover, for all search costs $\varepsilon>\varepsilon^{n s}$ the increase in the cost of search (over $\varepsilon^{n s}$ ) does not directly affect the intermediary profit function and so $\phi^{n s}$ will always be the optimal advertising fee.

We next characterize what happens to the optimal advertising fee as the cost of search converges to zero. At $\phi_{n s}$ and for $\varepsilon>\varepsilon^{n s}$, revenue from advertising is increasing in $\phi$, while revenue from consumer subscription is decreasing in $\phi$. At the optimum, there is an arbitrage between increasing advertising revenues and increasing revenue from subscription. When the cost of search converges to zero, this arbitrage disappears. Intuitively, when search costs are low, the intermediary will decrease the subscription fees for consumers in order to guarantee that all of them decide to use its service. In order to be compensated for the loss incurred from these lower subscription fees, it increases the advertising fee. In the limit, when search is costless, the reserve price for searchers, $r_{s}$, is equal to the minimum price ( $p_{\min }$ ) and so, from eq. (22), the optimal subscription fee is equal to zero. In this case, the intermediary charges the advertising fee, $\phi_{0}$, that maximizes advertising revenues.

Proposition 3 As $\varepsilon$ converges to zero, the optimal advertising fee converges to

$$
\phi_{0}=\left(\frac{n-1}{n}\right)^{n} \pi(r),
$$

which is greater than $\phi_{n s}$.

Although we do not show that $\phi$ increases for all small decreases in the cost of search (this would require more restrictions on the shape of the demand function), Proposition 3 implies that, globally, as the cost of search falls towards 0 , the advertising fee charged by
the intermediary increases. Increasing $\phi$ has a number of important effects on equilibrium outcomes, which we summarize in the following proposition.

Proposition 4 In equilibrium, as $\varepsilon$ decreases and $\phi$ increases,

1. the probability that a firm advertises, $F(r, \phi)$, decreases.
2. the probability that a subscriber observes an advertisement, $\left(1-[1-F(r, \phi)]^{n}\right) d e-$ creases.
3. expected firm profits, $\Pi^{f}$, increase.
4. the price range becomes more narrow (so $p_{\min }$ is larger) and firms charge higher prices (stochastically).
5. the expected intermediary profits, $\Pi^{I}$, decrease.

By decreasing the cost of search, the intermediary increases the advertising fee for firms which lowers their propensity to advertise through it. Consequently the probability that a consumer observes an advertisement also falls.

The increase in the cost of search (and associated increase in the advertising fee) has two opposing effects on firm profits. On the one hand, the cost of advertising is now higher. On the other, the benefit of advertising is also greater. Competition is reduced since fewer firms will choose to advertise through the intermediary, and consequently those that do choose to advertise can post higher prices and have a higher probability of being the lowest advertised price. The second effect dominates and so firms' profits increase with the advertising fee.

For the intermediary, expected profits fall as the cost of search decreases. The increase in the advertising fee it charges to firms in order to advertise will compensate it only partially for the loss it incurs from decreasing $\kappa$ in order to encourage consumers to subscribe to its service. This is because the intermediary cannot afford to significantly increase advertising fees for fear of alienating firms.

Now that we have shown what happens globally, we can investigate what happens as the cost of search decreases below $\varepsilon_{n s}$. That is, we can determine what happens locally as search
becomes interesting for consumers. We show in the following proposition that if the cost of search decreases in a neighborhood below $\varepsilon_{n s}$, consumers will be no better off than they were at $\varepsilon_{n s}$. The intuition for this surprising result is that all of the benefits from search are appropriated by the firms through higher prices. If the intermediary were to hold the advertising fee, $\phi$, constant, the decrease in $\varepsilon$ would make the outside option for consumers more attractive and force the intermediary to make subscription more affordable (by lowering $\kappa)$. In order to avoid this, the intermediary can increase $\phi$. This shifts the price distribution (first order stochastic dominance) and reduces the incentive to search. The net result is that the increase in $\phi$ will offset the reduction in the cost of search and so despite the lower search cost, non-members will still not want engage in search.

Proposition 5 There exists an $\widehat{\varepsilon}<\varepsilon_{n s}$ such that for all $\varepsilon \in\left[\widehat{\varepsilon}, \varepsilon_{n s}\right]$, the optimal advertising fee, $\phi^{*}$ satisfies

$$
\varepsilon=\left(\int_{p_{\min \left(\phi^{*}\right)}^{r}}^{r} q(y) F\left(y, \phi^{*}\right) d y\right)
$$

Furthermore, for all $\varepsilon \in\left[\widehat{\varepsilon}, \varepsilon_{n s}\right]$, a decrease in the cost of search is offset by an increase in $\phi^{*}$ and therefore in expected prices, and so the ability to search does not make consumers better off in equilibrium.

For all $\varepsilon \in\left[\widehat{\varepsilon}, \varepsilon_{n s}\right]$, consumers are not affected by the existence of search possibility since their expected profit is always equal to $S(r)$. Indeed we have:

$$
U_{S u b}\left(\phi^{*}\right)-\kappa\left(\phi^{*}\right)=U_{N S u b}\left(\phi^{*}\right)=S(r)+\left(\int_{p_{\min \left(\phi^{*}\right)}^{r}}^{r} q(y) F\left(y, \phi^{*}\right) d y\right)-\varepsilon=S(r)
$$

When $\varepsilon \in\left[\widehat{\varepsilon}, \varepsilon_{n s}\right]$, the intermediary's optimal strategy is to make non-subscribers just indifferent between search and not searching at the monopoly price. The benefit from this alternative source of price information is captured by the intermediary and by the firms since the intermediary will maintain this equilibrium by increasing advertising fees and expected prices.

Proposition 5 implies that consumers may not be better off when they are allowed to search. In fact, if we distinguish between the cost of visit the first store (call this cost $\varepsilon^{f}$ )
and the cost of visiting subsequent stores (call this cost $\varepsilon^{s}$ ), consumers may actually be worse off when the cost of search falls far enough that search becomes interesting for consumers. One reason to distinguish between these two costs is that in some cases, visiting the first story may indeed be more costly than visiting subsequent stores. There may be a fixed cost associated with going to the shopping mall or with opening one's computer and browser. More importantly, from a strategic point of view, these two costs are quite different. Visiting the first store represents a sunk cost for firms. That is, in order to acquire a product, the consumer must show up at a store. Although it appears in the welfare function of consumers, the cost of visiting the first store has no strategic impact on consumers or on the intermediary. In contrast, the cost of visiting subsequent stores does have a strategic impact. It is this cost, $\varepsilon^{s}$, that will determine the search behavior of consumers and, by extension, their membership fee. So if we make the distinction between $\varepsilon^{f}$ and $\varepsilon^{s}$, Proposition 5 implies for all $\varepsilon^{s} \in\left[\widehat{\varepsilon}, \varepsilon_{n s}\right]$, we have:

$$
U_{S u b}\left(\phi^{*}\right)-\kappa\left(\phi^{*}\right)=U_{N S u b}\left(\phi^{*}\right)=S(r)+\varepsilon^{s}-\varepsilon^{f}
$$

In this case, if $\varepsilon^{s}$ decreases more than $\varepsilon^{f}$, the consumers' welfare actually decreases with a reduction of search costs.

## 5 Conclusion

In this paper we have examined the market for a homogeneous good and considered the role of information in determining market outcomes. In contrast with Baye and Morgan (2001) we allow consumers to learn about price quotes by visiting the individual stores - be they virtual or physical - of firms as well by subscribing to the services of an information intermediary.

We show that despite the fact that an alternative source of price information exists for consumers, allowing for search may not increase their welfare. When consumers are able to search for price quotes, the intermediary's profit decreases since it must lower its consumer subscription fee. In an effort to maintain revenue it increases the fee it charges firms to advertise, but can never increase it enough to compensate for the loss of revenue from subscription
fees. Firms make up for higher advertising fees by increasing their prices, and their expected profits increase. So consumers pay a lower subscription fee, but firms appropriate all of this increase in surplus by increasing their prices.

This paper extends Robert and Stahl (1993) by making the advertising fee endogenous. By doing so, we obtain a surprising result. The idea that lower search costs will necessary benefit consumers and lead to lower expected prices does not necessarily hold. The effect of search on consumer welfare depends on how the information provider reacts to search. In our model, the information intermediary has strong market power and uses its power to extract maximal rents. This may lead to a higher advertising fee and less competition when consumers' search costs decreases. If the market for information intermediation were more competitive, the results might be different. For instance suppose that the intermediation market is opened up to competition and that there exists a fixed cost to enter. With this set up equilibrium intermediation profits should always equal the entry cost. In this case a reduction in consumer search costs may simply cause some intermediaries to exit the market resulting in little or no effect on the advertising fee. The goal of our paper has been to introduce search into the Baye and Morgan framework. Extending the model to allow for competing intermediaries is a topic for future research.

## 6 Appendix

## Proof of Lemma 4

We need to consider two cases: One where $r_{s}=r$, ie. $\left(\int_{0}^{r} q(y) F(y, \mu, \phi) d y \leq \varepsilon\right)$, and one where $r_{s}<r$. In the former, since non-subscribers never search, they will pay whatever price
they find in the first store, while subscribers pay the minimum price. We have:

$$
\begin{align*}
U_{N S u b}(\mu, \phi) & =\int_{0}^{r} S(y) d F(y, \mu, \phi)+S(r)[1-F(r, \mu, \phi)]-\varepsilon \\
& =S(r)+\int_{0}^{r} q(y) F(y, \mu, \phi) d y-\varepsilon \\
U_{S u b}(\mu, \phi) & =\int_{0}^{r} S(y) d\left(1-[1-F(y, \mu, \phi)]^{n}\right)+S(r)\left(1-\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{n}\right)-\varepsilon \\
& =S(r)+\int_{0}^{r} q(y)\left(1-[1-F(y, \mu, \phi)]^{n}\right) d y-\varepsilon \tag{24}
\end{align*}
$$

Now, consider the case where non-subscribers will visit stores until they find one that offers a price less than or equal to $r_{s}<r$. If no firm offers a price less than or equal to $r_{s}$, the consumer will continue searching until he has visited all firms and will purchase from the one offering the lowest price. Let $\beta$ denote the probability that a firm offers a price less or equal $r_{s}$. Because there can be a mass point at $r_{s}$, we have to $\beta \geq F\left(r_{s}, \mu, \phi\right)$. The expected surplus of a non-subscriber is given by:

$$
\begin{align*}
& U_{\text {NSub }}(\mu, \phi)  \tag{25}\\
& =\max _{r_{s}} \sum_{i=1}^{n}[1-\beta]^{i-1} \beta\left(\int_{0}^{r_{s}} S(y) \frac{d F(y, \mu, \phi)}{\beta}+S\left(r_{s}\right) \frac{\beta-F\left(r_{s}, \mu, \phi\right)}{\beta}-i \varepsilon\right) \\
& \\
& +[1-\beta]^{n}\left(\int_{r_{s}}^{r} S(y) d \frac{\left(1-[1-F(y, \mu, \phi)]^{n}\right)}{[1-\beta]^{n}}+S(r) \frac{\left(1-[1-F(r, \mu, \phi)]^{n}\right)}{[1-\beta]^{n}}-n \varepsilon\right) .
\end{align*}
$$

The first part of this expression is the welfare from receiving an offer below $r_{s}$ at the $i^{\text {th }}$ firm visited. The second part represents the welfare from never receiving an offer below $r_{s}$ and buying at the lowest price. One can verify that

$$
\varepsilon\left(\sum_{i=1}^{n}[1-\beta]^{i-1} \beta i+[1-\beta]^{n} n\right)=\sum_{i=1}^{n}[1-\beta]^{i-1}=\left(\frac{1-[1-\beta]^{n}}{\beta}\right) .
$$

And so after some manipulation (integration by parts and simplification) eq.(25) can be rewritten as

$$
\begin{align*}
& \max _{r_{s} \leq r} S(r)+\sum_{i=1}^{n}[1-\beta(\mu, \phi)]^{i-1}\left(\int_{0}^{r_{s}} q(y) F(y, \mu, \phi) d y-\varepsilon\right) \\
& +\left(\int_{r_{s}}^{r} q(y)\left(1-[1-F(y, \mu, \phi)]^{n}\right) d y\right) \tag{26}
\end{align*}
$$

One can verify that the optimal search rule implies that the second term is equal to 0 , hence we have:

$$
U_{N S u b}(\mu, \phi)=S(r)+\left(\int_{r_{s}(\mu, \phi)}^{r} q(y)\left(1-[1-F(y, \mu, \phi)]^{n}\right) d y\right) .
$$

Similarly, we can set an upper bound on the welfare of subscribers. At best, subscribers will be able to observe all prices charged by firms and they will be able to purchase from the store offering the lowest price in the market. in particular, this will occur when $\mu=1$. Indeed if $\mu=1$, firms will advertise if and only if they charge less than $r$ and subscribers will be able to observe any price less than $r$. Hence, eq. (19) corresponds to an upper bound for $U_{\text {Sub }}(\mu, \phi)$ which holds with equality when $\mu=1$.

## Proof of Theorem 1(i):

We wish to prove that for all $\mu<1$ and $\phi$, there exists a $\phi^{*}$, such that $\Pi^{I}(\mu, \phi)<\Pi^{I}\left(1, \phi^{*}\right)$ and hence, it is never optimal to set $\mu<1$. Let $\Pi^{f}(\mu, \phi)$ denote the equilibrium profit of the firm for some $\mu$ and $\phi$ and let $\alpha(\mu, \phi)$ denote the probability that a firm advertises for some $\mu$ and $\phi$. From the discussion in Section 3, we have the following:

|  | Firms' profits, $\Pi^{f}(\mu, \phi)$ | Probability of advertising, $\alpha(\mu, \phi)$ |
| :---: | :---: | :---: |
| Case A | $\left[\frac{n \phi}{(n-1)}+\pi\left(r_{s}\right) \frac{(1-\mu)}{n}\right]$ | $\left(1-\left[\frac{n \phi}{(n-1) \mu \pi\left(r_{s}\right)}\right]^{\frac{1}{n-1}}\right)$ |
| Case B | $\left[\frac{\pi\left(r_{m}\right) \phi}{(n-1) \pi\left(r_{s}\right)}\right]$ | $\left(1-\left[\frac{n \phi}{(n-1) \mu \pi\left(r_{s}\right)}\right]^{\frac{1}{n-1}}\right)$ |
| Case C | $\left[\frac{\pi\left(r_{m}\right) \phi}{(n-1) \pi\left(r_{a}\right)}\right]$ | $\left(1-\left[\frac{n \phi}{(n-1) \mu \pi\left(r_{a}\right)}\right]^{\frac{1}{n-1}}\right)$ |
| Case D | $\left[\frac{\pi\left(r_{m}\right) \phi}{(n-1) \pi\left(r_{a}\right)}\right]$ | $\left(1-\left[\frac{n \phi}{(n-1) \mu \pi\left(r_{a}\right)}\right]^{\frac{1}{n-1}}\right)$ |
|  | Firms' profits, $\Pi^{f}\left(1, \phi^{*}\right)$ | Probability of advertising, $\alpha\left(1, \phi^{*}\right)$ |
| $\mu=1$ | $\left[\frac{\phi^{*}}{(n-1)}\right]$ | $\left(1-\left[\frac{n \phi^{*}}{(n-1) \pi(r)}\right]^{\frac{1}{n-1}}\right)$ |

For Case A ( $r_{s}$ is the highest price charged in equilibrium), we select some advertising fee $\phi^{*}=\frac{\phi}{\mu}>\phi$. Note that $\alpha(\mu, \phi) \leq \alpha\left(1, \phi^{*}\right)$. It follows that $n \phi^{*} \alpha\left(1, \phi^{*}\right) \geq n \phi \alpha(\mu, \phi)$.

For the Cases $\mathrm{B}, \mathrm{C}$ and D (i.e. when there is a price $r_{m}>r_{s}$, which is charged in equilibrium and which is not advertised), we choose some advertising fee $\phi^{*}$ such that $\Pi^{f}\left(1, \phi^{*}\right)+\phi^{*}=\left(\frac{n \phi^{*}}{n-1}\right)=\Pi^{f}(\mu, \phi)+\phi . \quad$ In Case B, we set $\phi^{*}=\frac{\left[\pi\left(r_{m}\right)+(n-1) \pi\left(r_{s}\right)\right]}{n \pi\left(r_{s}\right)} \phi \geq \phi$,
in Cases C and D, we set $\phi^{*}=\frac{\left[\pi\left(r_{m}\right)+(n-1) \pi\left(r_{a}\right)\right]}{n \pi\left(r_{a}\right)} \phi \geq \phi$. Again we can show that $\alpha(\mu, \phi) \leq$ $\alpha\left(1, \phi^{*}\right)$. Indeed, for Case B we have:

$$
\begin{aligned}
\alpha\left(1, \phi^{*}\right) & =\left(1-\left[\frac{n \phi^{*}}{(n-1) \pi(r)}\right]^{\frac{1}{n-1}}\right) \\
& =\left(1-\left[\frac{n \phi}{(n-1) \pi\left(r_{s}\right)} \frac{\left[\pi\left(r_{m}\right)+(n-1) \pi\left(r_{s}\right)\right]}{n \pi(r)}\right]^{\frac{1}{n-1}}\right) \\
& \geq\left(1-\left[\frac{n \phi}{(n-1) \mu \pi\left(r_{s}\right)}\right]^{\frac{1}{n-1}}\right)=\alpha(\mu, \phi)
\end{aligned}
$$

A similar argument applies for Cases C and D. Again we have $n \phi^{*} \alpha\left(1, \phi^{*}\right) \geq n \phi \alpha(\mu, \phi)$.
We now show that revenue from subscription is also higher with $\mu=1$, i.e. $\mu \kappa(\mu, \phi)<$ $\kappa\left(1, \phi^{*}\right)$. Before doing so, we first define:

$$
\begin{align*}
\Psi(\mu, \phi)= & \min _{r_{s} \leq r}\left(\int_{0}^{r_{s}} q(y)\left(\sum_{i=1}^{n-1} \mu[1-F(y, \mu, \phi)]^{i}\right) F(y, \mu, \phi) d y\right)  \tag{27}\\
& -\left(\sum_{i=1}^{n-1} \mu\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\left(\int_{0}^{r_{s}} q(y) F(y, \mu, \phi) d y-\min \left[\varepsilon ; \int_{0}^{r} q(y) F(y, \mu, \phi) d y\right]\right) .
\end{align*}
$$

Recall that

$$
\begin{aligned}
\left(\sum_{i=1}^{n-1}[1-F(y,)]^{i}\right) F(y) & =1-F(y)-[1-F(y)]^{n} \\
& =[1-F(y)]\left(1-[1-F(y)]^{n-1}\right) .
\end{aligned}
$$

Hence one can verify that when $r_{s}$ satisfies the optimal search rule, we have :

$$
\Psi(\mu, \phi)=\left(\int_{0}^{r_{s}(\mu, \phi)} q(y)\left(\sum_{i=1}^{n-1} \mu[1-F(y, \mu, \phi)]^{i}\right) F(y, \mu, \phi) d y\right) \geq \mu \kappa(\mu, \phi)
$$

We now state the following lemma.

Lemma 5 Suppose that $\left[1-F\left(p, 1, \phi^{*}\right)\right]^{n-1} \geq \mu[1-F(p, \mu, \phi)]^{n-1}$ for all $p<r_{s}$, then $\kappa(\mu, \phi) \leq \Psi(\mu, \phi) \leq \Psi\left(1, \phi^{*}\right)=\kappa\left(1, \phi^{*}\right)$.

Proof. Suppose that the optimal reservation price is given by $r_{s}\left(1, \phi^{*}\right)$. We first consider the (hardest) case where $r_{s}\left(1, \phi^{*}\right)<r$. Let $S \leq \subseteq\left[0, r_{s}\left(1, \phi^{*}\right)\right]$ denote the subset of prices for which $F\left(y, 1, \phi^{*}\right) \leq F(y, \mu, \phi)$ and let $S^{>} \subseteq\left[0, r_{s}\left(1, \phi^{*}\right)\right]$ denote the subset of prices for which $F\left(y, 1, \phi^{*}\right)>F(y, \mu, \phi)$. We have:

$$
\begin{aligned}
& \kappa\left(1, \phi^{*}\right) \\
= & \int_{S \leq} q(y)\left[1-F\left(y, 1, \phi^{*}\right)\right]\left(1-\left[1-F\left(y, 1, \phi^{*}\right)\right]^{n-1}\right) d y \\
& +\int_{S^{>}} q(y)\left[\left(\sum_{i=1}^{n-1}\left[1-F\left(y, 1, \phi^{*}\right)\right]^{i}\right)-\mu\left(\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\right] F\left(y, 1, \phi^{*}\right) d y \\
& -\mu\left(\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\left(\int_{S \leq} q(y) F\left(y, 1, \phi^{*}\right) d y-\varepsilon\right) \\
\geq & \int_{S \leq} q(y)[1-F(y, \mu, \phi)]\left(\mu-\mu[1-F(y, \mu, \phi)]^{n-1}\right) d y \\
& +\int_{S^{>}} q(y)\left[\left(\sum_{i=1}^{n-1} \mu[1-F(y, \mu, \phi)]^{i}\right)-\mu\left(\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\right] F(y, \mu, \phi) d y \\
& -\left(\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\left(\int_{S \leq} q(y) F(y, \mu, \phi) d y-\min \left[\varepsilon, \int_{0}^{r} q(y) F(y, \mu, \phi) d y\right]\right) \\
= & \left(\int_{0}^{r_{s}\left(1, \phi^{*}\right)} q(y)\left(\sum_{i=1}^{n-1} \mu[1-F(y, \mu, \phi)]^{i}\right) F(y, \mu, \phi) d y\right) \\
& -\left(\sum_{i=1}^{n-1} \mu\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)\left(\int_{0}^{r_{s}\left(1, \phi^{*}\right)} q(y) F(y, \mu, \phi) d y-\min \left[\varepsilon ; \int_{0}^{r} q(y) F(y, \mu, \phi) d y\right]\right) \\
\geq & \Psi(\mu, \phi) \geq \mu \kappa(\mu, \phi)
\end{aligned}
$$

The first inequality follows from the fact that: (i) for all $y \in S \leq$, we have $\left[1-F\left(y, 1, \phi^{*}\right)\right] \geq$ $[1-F(y, \mu, \phi)]$ and $\left[1-F\left(y, 1, \phi^{*}\right)\right]^{n-1}-\mu[1-F(y, \mu, \phi)]^{n-1} \leq(1-\mu)[1-F(y, \mu, \phi)]^{n-1} \leq$ $(1-\mu)$; (ii) for all $y \in S^{>}, F\left(y, 1, \phi^{*}\right) \geq F(y, \mu, \phi)$; (iii) since $\left[1-F\left(p, 1, \phi^{*}\right)\right]^{n-1} \geq$ $\mu[1-F(p, \mu, \phi)]^{n-1}$, we must also have: $\left[1-F\left(p, 1, \phi^{*}\right)\right]^{i} \geq \mu^{\frac{i}{n}}[1-F(p, \mu, \phi)]^{i} \geq \mu[1-F(p, \mu, \phi)]^{i}$ for all $i \leq(n-1)$ and $\left(\sum_{i=1}^{n-1}\left[1-F\left(y, 1, \phi^{*}\right)\right]^{i}\right) \geq\left(\sum_{i=1}^{n-1}[1-F(y, \mu, \phi)]^{i}\right) \geq\left(\sum_{i=1}^{n-1} \mu\left[1-F\left(r_{s}, \mu, \phi\right)\right]^{i}\right)$. The second inequality follows from the fact that $r_{s}\left(1, \phi^{*}\right)$ is not the argument that minimizes the problem $\Psi(\mu, \phi)$. The case where $r_{s}\left(1, \phi^{*}\right)=r$ is left since it is a straightforward simplification of the above argument.

In order to complete our proof we need to show that the conditions of Lemma 5 hold for all cases.

Let $D_{n}\left(r_{s}, \mu, \phi\right)<1$, denote the proportion of non-members that buys from a firm that charges $r_{s}$ or less. We have for all $p<r_{s}$

$$
\begin{aligned}
\Pi^{f}(\mu, \phi) & =\pi(p)\left[\mu[1-F(p, \mu, \phi)]^{n-1}+(1-\mu) D_{n}\left(r_{s}, \mu, \phi\right)\right]-\phi \\
\mu[1-F(p, \mu, \phi)]^{n-1} & =\left[\frac{\Pi^{f}(\mu, \phi)+\phi}{\pi(p)}-(1-\mu) D_{n}\left(r_{s}, \mu, \phi\right)\right]
\end{aligned}
$$

For Cases $\mathrm{B}, \mathrm{C}$ and D , where $\Pi^{f}(\mu, \phi)+\phi=\Pi^{f}\left(1, \phi^{*}\right)+\phi^{*}$, we have

$$
\left[1-F\left(p, 1, \phi^{*}\right)\right]^{n-1}-\mu[1-F(p, \mu, \phi)]^{n-1}=(1-\mu) D_{n}\left(r_{s}, \mu, \phi\right) \geq 0
$$

In Case A, where $\mu \phi^{*}=\phi$ we have

$$
\left[1-F\left(p, 1, \phi^{*}\right)\right]^{n-1}-\mu[1-F(p, \mu, \phi)]^{n-1}=(1-\mu)\left[\frac{n \phi^{*}}{(n-1) \pi(p)}+\frac{\pi\left(r_{s}\right)-\pi(p)}{n \pi(p)}\right] \geq 0
$$

This completes our proof.

## Proof of proposition 2:

When the search condition in non-binding, the intermediary problem consists of solving the following

$$
\begin{aligned}
\max _{\phi} \Pi^{I}(\phi)= & n \phi\left(1-\left[\frac{n \phi}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}\right) \\
& +\int_{p_{\min }}^{r} q(y)\left[\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}-\left[\frac{n \phi}{\pi(y)(n-1)}\right]^{\frac{n}{n-1}}\right] d y
\end{aligned}
$$

Note that the function $\Pi^{I}(\phi)$ is strictly concave in $\phi$. So the optimal advertising fee when the search constraint is not binding, $\phi^{n s}$, is given by the following first-order condition:

$$
\begin{aligned}
0= & n\left(1-\frac{n}{n-1}\left[\frac{n \phi^{n s}}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}\right) \\
& +\int_{p_{\min }}^{r} q(y)\left[\frac{1}{\phi(n-1)}\left[\frac{n \phi^{n s}}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}-\frac{n}{\phi(n-1)}\left[\frac{n \phi^{n s}}{\pi(y)(n-1)}\right]^{\frac{n}{n-1}}\right] d y
\end{aligned}
$$

Let $\varepsilon^{n s}$ be such that:

$$
\varepsilon^{n s}=\int_{p_{\min }}^{r} q(y)\left(1-\left[\frac{n \phi^{n s}}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}\right) d y=\int_{0}^{r} q(y) F\left(y, \phi^{n s}\right) d y
$$

Then for all $\varepsilon \geq \varepsilon^{n s}$, the search constraint will not be binding and $\phi^{n s}$ will be the optimal advertising fee.

## Proof of Proposition 3:

For all $\phi$ such that $F(r, \phi)<1$, as the search cost converges to 0 , the reserve price of non-subscribers, $r_{s}$, convergence to $p_{\text {min }}$. In this case, we have:

$$
\lim _{\varepsilon \rightarrow 0} \frac{d \Pi^{I}(\phi)}{d \phi}=n\left(1-\frac{n}{n-1}\left[\frac{n \phi}{\pi(r)(n-1)}\right]^{\frac{1}{n-1}}\right)
$$

Note that when $\phi=\phi^{0}$, we have $\lim _{\varepsilon \rightarrow 0} \frac{d \Pi^{I}(\phi)}{d \phi}=0$. So as $\varepsilon \rightarrow 0$, the optimal $\phi$ convergences to $\phi^{0}=\pi(r)\left(\frac{n-1}{n}\right)^{n}$. In order to complete our proof, we need to show that $\phi^{0}>\phi^{n s}$. In order to do this we need to verify that $\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{\phi=\phi^{0}}<\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{\phi=\phi^{n s}}=0$, which implies by the concavity of $\Pi^{I}(\phi)$ that $\phi^{0}>\phi^{n s}$. Indeed, we have:

$$
\begin{aligned}
\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{\phi=\phi^{0}} & =\int_{p_{\min }}^{r} q(y)\left[\frac{1}{\phi^{0}(n-1)}\left[\frac{n \phi^{0}}{\pi(y)(n-1)}\right]^{\frac{1}{n-1}}-\frac{n}{\phi^{0}(n-1)}\left[\frac{n \phi^{0}}{\pi(y)(n-1)}\right]^{\frac{n}{n-1}}\right] d y \\
& =\frac{1}{\phi^{0}(n-1)} \int_{p_{\min }}^{r} q(y)\left(\left[\frac{n}{(n-1)}\right]\left(\frac{\pi(r)}{\pi(y)}\right)^{\frac{1}{n-1}}-n\left[\frac{n}{(n-1)}\right]^{n}\left(\frac{\pi(r)}{\pi(y)}\right)^{\frac{n}{n-1}}\right) d y<0
\end{aligned}
$$

The inequality follows from the fact that $\left[\frac{n}{(n-1)}\right]-n\left[\frac{n}{(n-1)}\right]^{n} \leq 0$, for all $n \leq 2$.

## Proof of Proposition 4:

Claims 1. to 4 . are obvious, we show in the following the results about intermediary profit and the expected utility of consumers. The optimal intermediary profit is

$$
\begin{aligned}
\Pi^{I}(\varepsilon) & =\max _{\phi}\left[n \phi F(r, \phi)+U_{S u b}(\phi)-\max _{r_{s}} U_{N S u b}\left(\phi, r_{s}\right)\right] \\
& =\max _{\phi} \min _{r_{s}}\left[\begin{array}{c}
n \phi F(r, \phi)+\left(\int_{0}^{r_{s}} q(y)\left(1-F(y, \phi)-[1-F(y, \phi)]^{n}\right) d y\right) \\
-\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \phi\right)\right]^{i}\left(\int_{0}^{r_{s}} q(y) F(y, \phi) d y-\varepsilon\right)
\end{array}\right]
\end{aligned}
$$

Using the envelop Theorem we have:

$$
\frac{\partial \Pi(\varepsilon)}{\partial \varepsilon}=\left(\sum_{i=1}^{n-1}\left[1-F\left(r_{s}, \phi\right)\right]^{i}-1\right)=\frac{\left[1-F\left(r_{s}, \phi\right)\right]-\left[1-F\left(r_{s}, \phi\right)\right]^{n}}{F\left(r_{s}, \phi\right)} \geq 0
$$

which is clearly positive.

## Proof of Proposition 5;

When the reserve price of non-subscribers is less than $r$, the change in $\phi$ has an effect on the intermediary's profit that does not exist when search does not matter. An increase of $\phi$, for a given $\varepsilon$, increases $r_{s}$, which in turn increase the $\kappa(\phi)$. So the marginal benefit of increasing $\phi$ is greater when $r_{s}<r$. Formally, we have:

$$
\begin{align*}
\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{r_{s}<r}= & n F(r, \phi)-\frac{n}{n-1}[1-F(r, \phi)]  \tag{28}\\
& -\left(\int_{p_{\min }}^{r_{s}} q(y)\left[n[1-F(y, \phi)]^{n-1} \frac{[1-F(y, \phi)]}{\phi(n-1)}\right] d y\right) \\
& +\left(\sum_{i=0}^{n-1}\left[1-F\left(r_{s}, \phi\right)\right]^{i}\right)\left(\int_{p_{\min }}^{r_{s}} q(y)\left[\frac{[1-F(y, \phi)]}{\phi(n-1)}\right] d y\right)
\end{align*}
$$

Now let $r_{s}$ be arbitrarily close to $r$, then we have:

$$
\begin{align*}
& \left(\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{r_{s}<r}-\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{r_{s}=r}\right) \\
= & \left(\sum_{i=0}^{n-1}[1-F(r, \phi)]^{i}\right)\left(\int_{p_{\min }}^{r} q(y)\left[\frac{[1-F(y, \phi)]}{\phi(n-1)}\right] d y\right)  \tag{29}\\
= & \frac{1}{(n-1)}\left(\sum_{i=0}^{n-1} \phi^{\frac{i-n+1}{n-1}}\left[\frac{n}{\pi(r)(n-1)}\right]^{\frac{i}{n-1}}\right)\left(S\left(p_{\min }(\phi)\right)-S(r)-\varepsilon\right) \tag{30}
\end{align*}
$$

In this case, the difference between $\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{r_{s}<r}$ and $\left.\frac{d \Pi^{I}(\phi)}{d \phi}\right|_{r_{s}=r}$ is positive and decreasing in $\phi$ as all the terms in (30) are decreasing in $\phi$. Let $\hat{\phi}$ be the value which $\left.\frac{d \Pi^{I}(\hat{\phi})}{d \phi}\right|_{r_{s}<r}=0$ when $r_{s}$ is just equal to $r$, then we have $\hat{\phi}>\phi^{n s}$. So let $\hat{\varepsilon}$ be given by:

$$
\hat{\varepsilon}=\int_{0}^{r} q(y) F(y, \hat{\phi}) d y
$$

Then for any $\varepsilon \in\left(\varepsilon^{n s}, \hat{\varepsilon}\right)$, and value $\phi^{*} \in\left(\phi^{n s}, \hat{\phi}\right)$, such that $\varepsilon=\int_{0}^{r} q(y) F\left(y, \phi^{*}\right) d y$, we have:

$$
\left.\frac{d \Pi^{I}\left(\phi^{*}\right)}{d \phi}\right|_{r_{s}<r}>0>\left.\frac{d \Pi^{I}\left(\phi^{*}\right)}{d \phi}\right|_{r_{s}=r}
$$

Around $\phi^{*}$ an increase of the advertising fee will not change the reserve price of nonmembers and profits of the intermediary will decrease. Conversely, a decrease of the advertising fee below $\phi^{*}$ will decrease $r_{s}$ and lower profits. Hence $\phi^{*}$ is truly optimal when the
search cost is $\varepsilon^{*}$. But this true for all $\varepsilon^{*} \in\left(\varepsilon^{n s}, \hat{\varepsilon}\right)$. In all these cases, we have:

$$
U_{N S u b}(\mu, \phi)=S(r)+\int_{0}^{r} q(y) F\left(y, \mu, \phi^{*}\right) d y-\varepsilon^{*}=S(r)
$$

The change in search cost is offset by an increase in advertising fee and hence average prices.

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[^0]:    ${ }^{1}$ We use 'strictly less than' rather than the conventional 'less than or equal to' in order to avoid problems arising due to mass points at the search reservation prices defined below.

