Group Specific Public Goods, Orchestration of Interest Groups and Free Riding [1]

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Abstract

We consider a two group contest over a group specific public good where each member of a group has a different benefit from the good. Our model can be interpreted in two ways: Each of the players has a non-linear investment cost in the contest, or alternatively, the returns to effort are decreasing as reflected in the contest success function. In the first part of the paper we show conditions under which free-riding decreases and consider the different properties of the equilibrium. In the second part of the paper we develop the properties of the optimal formation of the group and its affect on the equilibrium outcome.

Keywords: Contests, rent seeking, public good, heterogeneity, free-riding, orchestration of interest groups.

JEL Classification: D72, C72, H41.

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1. Introduction

Frequently economic policy involves a struggle between two groups: one group that defends the status-quo and another group that challenges it by fighting for an alternative policy. For example, a tax reform may involve a struggle between different industries. Existing pollution standards may be defended by the industry and challenged by an environmental interest group. A monopoly may face a customers' coalition fighting for appropriate regulations. Capital owners and a workers' union can be engaged in a contest that determines work place safety standards, and so on. The outcome of these contests depend on the stakes of the contestants and, in turn, on their exerted efforts. In many cases, these contests may involve group specific public-goods.

The purpose of this paper is to examine the equilibrium efforts invested by the individual players in each group. We consider the case of two groups. Each member of the group *derives different benefits* from his group winning the contest over the public policy/ public good. We analyze two alternative types of contests which have identical outcomes. In the first, each of the players faces an increasing marginal cost when investing in the contest. The idea behind this assumption is that one tries to affect the policy outcome at a low cost, such as by writing an e-mail, signing a petition on the internet or sending a text message by phone. In the case where an individual would like to be more involved and invest more effort in the contest, the cost for each extra unit of investment will increase. Under the second alternative, the marginal cost for all players within and between the groups equals unity; however effort faces decreasing returns to scale in the contest success function. Sending the first e-mail has a stronger effect than sending the second e-mail, signing the first petition has a stronger effect than the second petition etc. Under this scenario, the effect each individual has in a group of size N by sending one e-mail will be stronger than the effect of only one individual in the group sending N e-mails. The probability of winning, therefore, depends not only on the total amount of resources invested in the contest but also on the number of individuals investing effort in the contest. To simplify the mathematical description, we will focus on one scenario throughout the paper.

We start by deriving the Nash equilibrium of the contest, focusing the on effort invested by each individual, the probability of winning and the expected payoffs to the different individuals in the contest. Our first result is with regard to the level of free-riding in the contest. We show that the level of free riding depends on the return to investment in the contest. After which we consider different equilibrium properties of the contest.

In the second part of the paper we develop the properties of the optimal formation of the group - the orchestration of interest groups, and its affect on the equilibrium outcome. We consider the situation under which one group initiates a contest and can add different individuals and/or groups to the fight for the specific cause. The question we pose is what would be the optimal structure of the groups being added? There are many real life examples for such situations. Let us consider the following four examples (in section 4 we will present an expanded example):

1. Between two cities, "A" and "B", a contracting firm wishes to build a new highway. Opposing the construction of the new highway are members of an environmental movement who whish to leave the land free of highways. There are many alternative routes for the highway. The land owners on the route that the highway will be constructed will benefit and the size of the net benefit is a direct function of the size of land they own. On each possible route there are different land owners who own different amounts of land. In one route there are only two land owners, each holding large parcels of land and under a second route there are 7 land owners, each holding smaller parcels of land. In terms of direct profits from constructing the highway, the construction firm is indifferent between the two possibilities. However, in one case there will be two land owners joining the contest for the approval of the highway while under the other optional route there would be 7 land owners each having a smaller stake in the contest joining the firm in the contest over building of the highway. Which option would the construction firm prefer?
2. The owners of the stores and businesses in a large shopping center wish to obtain a permit to build a large underground parking lot in order to increase accessibility to the shopping center. Environmental groups oppose the construction of the parking lot since they claim it will have a negative affect on the local springs. In order for the owners of the shopping center to apply for the permit to build the underground parking lot they have to state the firm or firms that will provide security, maintenance, cleaning, etc., after the parking lot will be completed. The shopping center owners could decide to hire one specific firm that could supply all the different services, or they could

approach different firms, each providing some of the services. The firms that would provide the different services would benefit from the construction of the parking lot and would join the contest for receiving the permit. The members of the shopping center have to determine whether to hire the one comprehensive firm or the several independent firms.

3. Let us consider a variation on the second example. Assume that each of the services can only be provided by a different firm. In this case what would the owners of the stores and businesses in the shopping center prefer: 1. all the firms that providing the services would combine together with one representative determining the lobbying effort of the group or 2. all the firms join the contest separately each with its own representative?
4. Consider a municipality which is considering building a park in its jurisdiction. Opposed to building the park is a group of home construction firms that want to build a new neighborhood on this land. The park would be limited for use to the population of municipality "A". Since the population of municipality "A" is smaller than the optimal capacity of the park, municipality "A" is considering joining forces with either a big municipality in the east, municipality "B" or two smaller municipalities in the west, "C" and "D". The total population of municipalities is such that the population of "A" and "B" just fit the restricted capacity and the same for "A" plus "C" and "D". Municipality "A" must determine with which of the other municipalities should it join forcers with, "B" or "C" and "D

These examples show that the structure of the interest groups is an important influence on the winning probability and final outcome of the contest.

Katz, Nitzan and Rosenberg (1990), Ursprung (1990), Baik (1993), Riaz, Jason and Stanley (1995), Baik, Kim and Na (2001), Konrad (2007, and references within), Esteban and Ray (2001) and Baik (2007) study contests with group-specific public-good prizes. Among them, Esteban and Ray (2001) and Baik (2007) are the closest to our paper. Esteban and Ray (2001) consider a collective action that has three features: it is undertaken in order to counter similar action by competing groups, marginal individuals effort is increasingly costly, and collective prizes are permitted to have mixed public-private characteristics. All individuals in a group are assumed to have the same benefit from the pubic good, while from the private good is a given total benefit for the group. Thus increasing the size of the group decreases the benefit to each of its members. Esteban and Ray (2001) show that there exist conditions under which increasing the size of the group will increase the probability of winning, even though the benefit per member has decreased. Our setting differs from theirs in the following way: there is a public good and each player receives *a different benefit* from it. The benefit for each player is independent of the size of the group. Baik (2007), on the other hand, presents a model in which *n* groups compete to win a group-specific public-good prize, the individual players choose their effort levels simultaneously and independently, and the probability of winning depends on the groups' effort levels. The main dissimilarities between our paper and Baik (2007) are: 1. in this paper, the costs are non-linear (Baik (2007) assumes linear costs); 2. in Baik (2007) investment can be aggregated, in the present model we can only aggregate them after a using non-linear transformation. In terms of the e-mail example, under Baik (2007) one person sending ten e-mails is identical to ten different individuals sending one e-mail each. Our model assumes that the two cases are not equivalent. Moreover, while Baik (2007) shows that full free-riding will exist and only the player with the highest valuation in the contest will invest, we show that all players will invest and free-riding is decreased.

In the next section we present the basic model and the equilibrium outcomes. In section 3 we present comparative-statics and in section 4 we present the orchestration of an interest group.

2. The Model

Consider a contest with two groups competing for a prize. As in Epstein and Nitzan (2004) suppose that a status-quo policy is challenged by one interest group and defended by the other. This policy can be the price of a regulated monopoly, the maximal degree of pollution the government allows or the existing tax structure. The defender of the status-quo policy (henceforth, the defending interest group) prefers the status-quo policy to any alternative policy. The challenger of the status-quo policy (the challenging group) prefers the alternative strategy. For example, in the contest over monopoly regulation studied in Baik (1999), Ellingsen (1991), Epstein and Nitzan (2003, 2007) and Schmidt (1992), the monopoly firm defends the status-quo by lobbying for the profit-maximizing monopoly price (and against any price regulation), while consumers challenge the status-quo lobbying effort preferring a competitive price. In the *challenging* group there are N players and in the *defending* group there are M players.

In the *challenging* group player i receives a benefit of n_i $(i = 1,..., N)$ from winning the contest and in the defending group player j receives a benefit of m_j $(j = 1,..., M)$ from winning the contest. Player i $(i = 1,..., N)$ invests x_i resources to change the status-quo and player j $(j = 1,..., M)$ from the defending group, invests y_j units in the contest.

We consider two alternative identical (mathematical) problems. In the first, the probability that the new policy will be accepted and the status-quo changed, $\Pr_c$, is given by the generalized logit contest success function as:

(1)
$$\Pr_c = \frac{\sum_1^N x_i^{\alpha}}{\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}} \quad \textit{with} \ 0 < \alpha < 1.$$

We restrict our analysis to the case for which $0 < \alpha < 1$.[2]
The expected payoff of each of the players in the challenging group will thus equal:

(2)
$$E(U_i) = \frac{\sum_1^N x_i^{\alpha}}{\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}} n_i - x_i \quad \forall \ i = 1,..., N,$$

and for each of the players in the defending group:

(3)
$$E(U_j) = \frac{\sum_1^M y_j^{\alpha}}{\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}} m_j - y_j \quad \forall \ j = 1,..., M.$$

[2] For $\alpha = 1$, Baik (2007) applies.

In the second alternative, we aggregate the efforts of the contestants in a linear way but with non-linear costs, and obtain the following contest success function: $\Pr_c = \frac{\sum_1^N X_i}{\sum_1^N X_i + \sum_1^M Y_j}$. The expected payoff becomes:

$$E(U_i) = \frac{\sum_1^N X_i}{\sum_1^N X_i + \sum_1^M Y_j} n_i - X_i^{\beta} \quad \forall \; i = 1,\ldots,N \; with \; \beta = \frac{1}{\alpha} > 1. \tag{2'}$$

As we can see, the relationship between both alternatives is given by $x_i^{\alpha} = X_i$ and $y_j^{\alpha} = Y_j$. This second interpretation states investment has increasing marginal cost. Today with new technologies the easiest way to try and affect a policy decision is by sending an e-mail or signing a petition through the internet. If one wants to increase the investment beyond this, costs increase rapidly. To simplify our presentation, we portray the model using the first option presented above. Any interpretation regarding x_i or y_j may be transformed to $x_i = X_i^{\beta} = X_i^{\frac{1}{\alpha}}$ and $y_i = Y_i^{\beta} = Y_i^{\frac{1}{\alpha}}$.

The first order conditions for maximization is given by

$$\frac{\partial E(U_i)}{\partial x_i} = \frac{n_i \alpha x_i^{\alpha-1} \sum_1^M y_j^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha} \right)^2} - 1 = 0 \qquad \forall \; i = 1,\ldots,N \,,$$

(4) and

$$\frac{\partial E(U_j)}{\partial y_j} = \frac{m_j \alpha y_j^{\alpha-1} \sum_1^N x_i^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha} \right)^2} - 1 = 0 \qquad \forall \; j = 1,\ldots,M \,.$$

Define $A = \left(\sum_1^N n_i^{\frac{\alpha}{1-\alpha}} \right)$ and $B = \left(\sum_1^M m_j^{\frac{\alpha}{1-\alpha}} \right)$. Solving the first order conditions, we obtain that the Nash equilibrium investment of the different players in the each group equals:[3]

(5)
$$x_i^* = \frac{\alpha n_i^{\frac{1}{1-\alpha}} B}{(AB)^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right)^2} \quad and \quad y_j^* = \frac{\alpha m_j^{\frac{1}{1-\alpha}} A}{(AB)^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right)^2}$$

$\forall \; i = 1,..., N$ and $j = 1,..., M$.

The equilibrium probability of the policy being accepted and the status-quo being changed equals:

(6)
$$\Pr_c^* = \frac{\sum_1^N x_i^{*\alpha}}{\sum_1^N x_i^{*\alpha} + \sum_1^M y_j^{*\alpha}} = \frac{A^{1-\alpha}}{A^{1-\alpha} + B^{1-\alpha}} .$$

The equilibrium expected payoff of the different players in the different groups equals:

$$E(U_i^*) = \frac{n_i AB^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right) - \alpha n_i^{\frac{1}{1-\alpha}} B}{(AB)^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right)^2} \quad \forall \; i = 1,..., N ,$$

(7) and

$$E(U_j^*) = \frac{m_j BA^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right) - \alpha m_j^{\frac{1}{1-\alpha}} A}{(AB)^\alpha \left(A^{1-\alpha} + B^{1-\alpha}\right)^2} \quad \forall \; j = 1,..., M .$$

[3] The calculations of the equilibrium investments (equation 5) and the second order conditions of a unique equilibrium are presented in appendix A.

3. Comparative-statics

3.1. ***Free-riding:*** It can easily be verified that the expected payoffs to the players in the different groups are positive and that each of the players invests a positive amount of effort in the contest. Therefore, we do not encounter a *full* free-rider problem. However, for $\alpha = 1$ $(\beta = 1)$, we would have found, as in Baik (2007), full free-riding under which only the player with the highest valuation invests. Now consider the relationship between the investments of two different players in the same group: $\frac{x_k^*}{x_l^*} = \left(\frac{n_k}{n_l}\right)^{\frac{1}{1-\alpha}}$. This ratio will be positive for any two players. Thus each player will make a positive investment, while in the model presented by Baik (2007), only the player with the highest valuation will invest. Note that for any two players that satisfy $n_k > n_l$ it holds that $\frac{\partial\left(\frac{x_k^*}{x_l^*}\right)}{\partial\alpha} = (1-\alpha)^{-2}\left(\frac{n_k}{n_l}\right)^{\frac{1}{1-\alpha}} ln\left(\frac{n_k}{n_l}\right) > 0$. As α decreases, the ratio $\frac{x_k^*}{x_l^*}$ also decreases. Thus, for $\alpha \to 0$, the ratio converges to $\frac{n_k}{n_l}$.

Thus for $n_k > n_l$ the investment of player k is higher than that of player l by more than the ratio of the stakes (this is true for every value of α such that $0 < \alpha < 1$). As α increases the ratio $\frac{x_k^*}{x_l^*}$ increases and the free-riding phenomenon gets stronger. Free- riding is at its highest level at $\alpha = 1$ (for the case of $\alpha = 1$ *see* Baik, 2007).

3.2. ***Winning probability***: In the case where $\alpha \to 0$ $(\beta \to \infty)$, namely the investment value of each of the players in the winning probability converges to zero (or in an alternative explanation the cost of investing effort increases rapidly), we obtain that the investment of all players converges to zero: $x_i \to 0$, $y_j \to 0$ ($X_i \to 1$, $Y_j \to 1$); however $x_i^{\alpha} \to 1$, $y_j^{\alpha} \to 1$.

This result can be explained as follows: the effect each player has on the winning probability, x_i^{α} and y_j^{α}, depends, on the one hand, on the investment of each player and, on the other hand, on increasing returns to scale. In equilibrium we obtain that the dominating effect is the increasing returns to scale, even though the investment of each of the players is very small. Thus, for $\alpha \to 0$ $(\beta \to \infty)$, ${x_i}^{\alpha} \to 1, {y_j}^{\alpha} \to 1$, we obtain that the probability that the new policy will be accepted converges to $\frac{N}{N+M}$.

This result states that for a *sufficiently* small value of α the probability of the new policy being adopted is "almost" independent of the value the different players assign to its approval or rejection and depends on the number of players in each group.[4]

3.3. ***A change in the stakes of the game***: Now let us consider how a change in the valuation of the prize affects the players.

(8) $$\frac{\partial x_i}{\partial n_i} = \frac{\alpha n_i^{\frac{\alpha}{1-\alpha}}\left(A^{1-\alpha}+B^{1-\alpha}\right)\left\{B^{1+\alpha}\left[A-\alpha(2-\alpha)n_i^{\frac{\alpha}{1-\alpha}}\right]+A^{\alpha-1}B^2\left(A-\alpha^2 n_i^{\frac{\alpha}{1-\alpha}}\right)\right\}}{(1-\alpha)\left[(AB)^{\alpha}\left(A^{1-\alpha}+B^{1-\alpha}\right)^2\right]^2},$$

since $\alpha<1$ then $\alpha(2-\alpha)<1$ and $\alpha^2<1$ thus $A-\alpha(2-\alpha)n_i^{\frac{\alpha}{1-\alpha}} > A-n_i^{\frac{\alpha}{1-\alpha}} \geq 0$ and $A-\alpha^2 n_i^{\frac{\alpha}{1-\alpha}} > A-n_i^{\frac{\alpha}{1-\alpha}} \geq 0$. Therefore $\frac{\partial x_i}{\partial n_i}>0$ *and* $\frac{\partial x_i}{\partial n_k}<0$. Moreover, a change in m_j will have an ambiguous effect on the investment of player *i*:

(9) $$\frac{\partial x_i}{\partial m_j} = \frac{\alpha^2 n_i^{\frac{1}{1-\alpha}} m_j^{\frac{2\alpha-1}{1-\alpha}}\left(A^{1-\alpha}-B^{1-\alpha}\right)}{(AB)^{\alpha}\left(A^{1-\alpha}+B^{1-\alpha}\right)^3},$$

$\frac{\partial x_i}{\partial m_j} \gtrless 0$ *if and only if* $A^{1-\alpha} \gtrless B^{1-\alpha}$ *namely if* $A \gtrless B$.

A and *B* are the sum of the transformations of the players' benefits (stakes) in each of the groups and thus represent their strength. The condition shows that if a player is in

[4] When α is not sufficiently small the probability depends on all the different variables of the players.

the stronger group then increasing the rival's valuation will increase its rival's investment in the contest. At the same time if the increase in the valuation is in the stronger group this will make this group even stronger and will force its rivals to decrease their investment in the contest.[5] [6]

4. *Orchestration of an interest group*

In the following section we wish to consider the case of the orchestrating of an interest group. The idea behind our analysis is that there exists a leading interest group that wishes to increase its size by adding another group or groups. In the introduction we described four examples.[7]

To clarify our point let us add one more example: consider a municipality that wishes to undertake sewage water purification project. All the citizens of the municipality will benefit from the purification. Let us consider two different scenarios: 1. In the first scenario, the municipality is the leading interest group. The task of water purification can be given to a few firms or done by one firm. Assume that the benefit from water quality and value is identical whether purification is carried out by one or more firms. Since each firm that will be chosen by the municipality will join the lobbying efforts for the approval of the project, the municipality must determine with how many firms it wants to lobby for approval of this project. 2. In the second scenario there is a firm that is lobbying to have this project approved. What would be optimal for the firm: that each of the citizens invests effort to get the project approved *or* should the citizens divide into groups, (for example by neighborhoods they live in), where each group has a representative that will be active in the representation of his (her) group (neighborhood) in the lobbying process?

[5] It can be verified (see appendix B) that there exists a sufficiently small $\overline{\alpha}$ (a sufficiently large $\overline{\beta}$), such that increasing α (β) in the range $\left(0,\overline{\alpha}\right)$ $\left(\left(\overline{\beta},\infty\right)\right)$ will increase the investments made by the players $\frac{\partial x_i}{\partial \alpha} > 0$).

[6] This result has similarities to the one presented by Epstein and Nitzan (2007) under which they consider the case where a change in both the size of the prizes and in their distribution affect the investments of the contestants.

[7] An alternative way to look at this is question would be in what way would it be best to add the new individuals to the contest. Should each individual added separately or should they be added as groups with a representative that determines their investment. If the latter is the answer then the next question is: what would be the optimal size of the groups?

To answer this question we develop our theoretical framework in two stages. In section 4.1 we start by considering the case where an interest group can add one of two different groups to compete for the provision of the public good. In one group there is one member and in the other there are two members. In section 4.2 we generalize our results by considering the case of a large number of individuals that wish to join the contest to fight for the cause presented by the challenging group. In this section we analyze what is the best form for individuals to join the challenging group (in groups, single hand, etc).

To begin our analysis let us first consider how a change in A $\left(A=\sum_{1}^{N} n_i^{\frac{\alpha}{1-\alpha}}\right)$ affects the expected payoff of each of the players in each group (the change in A is a result of a change in the number of players and not as a result of a change in the stakes of the contest):

$$\text{(10)} \quad \frac{\partial E\left(U_i^*\right)}{\partial A}=\frac{(1-\alpha) n_i A^{-\alpha} B^{1-\alpha}}{\left(A^{1-\alpha}+B^{1-\alpha}\right)^2}+\frac{\alpha n_i^{\frac{1}{1-\alpha}} B^{1-\alpha}\left[\alpha A^{\alpha-1}\left(A^{1-\alpha}+B^{1-\alpha}\right)+2(1-\alpha)\right]}{A^{2\alpha}\left(A^{1-\alpha}+B^{1-\alpha}\right)^3}>0,$$

and

$$\text{(11)} \quad \frac{\partial E\left(U_j^*\right)}{\partial A}=\frac{\alpha-1}{B^{\alpha}\left(A^{1-\alpha}+B^{1-\alpha}\right)^3}\begin{bmatrix} m_j A^{1-2\alpha}\left(B-\alpha m_j^{\frac{\alpha}{1-\alpha}}\right) \\ +m_j A^{-\alpha} B^{2-\alpha}+\alpha m_j^{\frac{1}{1-\alpha}} A^{-\alpha} B^{1-\alpha} \end{bmatrix}<0.$$

Notice that increasing the number of members increases the value of A. By (10), increasing A increases the expected payoff of the challenging group.

4.1. Adding one group with one or two additional members

4.1.1 Identical valuations

The challenging group (the group of size N) is considering adding additional groups to its coalition. Assume that there are two possible groups it can add ($R=\{1,2\}$). In group 1, $(1 \in R)$ there is one member and in group 2 $(2 \in R)$ there are two members.

The total value of the members in each group is (F_1, F_2). Thus if we denote the stake/benefit of the player in the one player group by n_1^1 and the stakes of the two players in the two player group by (n_1^2, n_2^2) we will obtain that $F_1 = n_1^1$ and $F_2 = n_1^2 + n_2^2$.

If the challenging group adopts the two groups (which now will include 3 members), then by (10) the increase in members will increase the total value of *A* and the expected payoff of the group. But what would happen if it can only adopt one of the groups as in the examples presented above? The challenging group has to determine which of the groups it should adopt.

Let us start by assuming that both groups have the same total value: $F = F_1 = F_2$ and later relax this assumption. What would the challenging group prefer, one player with a valuation of $n_1^1 = F$ or two players while one player would have a valuation of $n_1^2 = \gamma_1 F$ and the second a valuation of $n_2^2 = (1-\gamma_1)F$?

As we have seen above, increasing *A* increases the payoff of the group. We are considering the optimal formation of the group which will maximize the valuation *A*. This enables us to compare different types of group structures. If we increase the challenging group by 1 player or by 2 players such that the sum of the valuations of the players equals *F*, the new value of *A* becomes, in both cases:

(13) $$A_{L=1} = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} \quad or \quad A_{L=2} = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}}\left[\gamma_1^{\frac{\alpha}{1-\alpha}} + (1-\gamma_1)^{\frac{\alpha}{1-\alpha}}\right].$$

Therefore, $A_{L=2} \underset{>}{\overset{<}{=}} A_{L=1}$ *if and only if* $\gamma_1^{\frac{\alpha}{1-\alpha}} + (1-\gamma_1)^{\frac{\alpha}{1-\alpha}} \underset{>}{\overset{<}{=}} 1$. We may conclude that,

1. $A_{L=2} < A_{L=1}$ if and only if $0.5 < \alpha < 1$ $(2 > \beta > 1)$ *;*
2. $A_{L=2} = A_{L=1}$ if and only if $\alpha = 0.5$ $(\beta = 2)$*;*
3. $A_{L=2} > A_{L=1}$ if and only if $0 < \alpha < 0.5$ $(\infty > \beta > 2)$.

In other words, for high (low) values of α (β) it is preferable to have one player. If $\alpha = 0.5$ $(\beta = 2)$ then the group is indifferent and if α (β) is sufficiently small (high) the group will prefer two players.

In order to understand these three situations let us look at two opposite effects increasing the number of players has on A (and thus on the benefits of each of the players in the challenging group): On the one hand by (5) the investments of each additional player, x_i, is affected with an increasing n_i, and on the other hand, it has a decreasing marginal effect on the probability of winning. Given that the two groups have the same valuation $F_1 = F_2$ then adding two players to the challenging group will give a smaller effect on A (compared to adding one player) via the stake effect, and will have a stronger effect on A via the probability effect. In the case where $0.5 < \alpha < 1$ the "stake effect" is stronger and thus it is optimal to add only one player. For $\alpha = 0.5$ both effects are identical and cancel each other out. For $0 < \alpha < 0.5$ the second effect is dominating, thus it is optimal to add two players.

4.1.2 Different valuations

From the above analysis we can also derive conclusions with regard to the situation under which the valuations of the groups are not identical, $F_1 \neq F_2$:

4. For $0.5 < \alpha < 1$, if $F_1 > F_2$ the challenging group will choose the group with one member and if $F_1 < F_2$ any of the two groups may be chosen. Namely it is not clear that the group with the two members will be chosen, even though it has more members and higher valuation.
5. For $\alpha = 0.5$ then the group with the higher valuation will be chosen.
6. For $0 < \alpha < 0.5$, if $F_1 < F_2$ the group with two players will be chosen and if $F_1 > F_2$ any of the two groups may be chosen.

4.2 The possibility of adding any number of players to the challenging group.

4.2.1 Identical valuations

Assume that there are r groups, $R = \{1,2,...,r\}$, and their objective is identical to that of the challenging group. In group 1 $(1 \in R)$ there is one member, in group 2 $(2 \in R)$ there are two members and so on till group r $(r \in R)$ which has r members. The total value of the members in each group is $(F_1,...,F_r)$. If the challenging group adopts all of the groups, then the increase in members will increase the value of A and the expected payoff of the group. But what will happen if the challenging group can only choose one of the groups? One could generalize the question asking what would be the optimal number of players the group would like to add given that the total valuation of the players is *F*?.

If we increase the challenging group by *L* players such that the sum of the valuations of the players equals *F*, the new value of *A* would equal:

(12) $$A_L = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + \sum_1^L (\gamma_l F)^{\frac{\alpha}{1-\alpha}} = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} \sum_1^L \gamma_l^{\frac{\alpha}{1-\alpha}} .$$

Therefore:

1. For $0.5 < \alpha < 1$ $(2 > \beta > 1)$, then $A_{L>1} = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} \sum_1^L \gamma_l^{\frac{\alpha}{1-\alpha}} < \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} = A_{L=1}$, namely the challenging group would prefer only one player to be added with a valuation of *F*.
2. For $\alpha = 0.5$ $(\beta = 2)$ the challenging group is indifferent with regard to the different situations.
3. For $0 < \alpha < 0.5$ $(\infty > \beta > 2)$, then $A_{L>1} = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} \sum_1^L \gamma_l^{\frac{\alpha}{1-\alpha}} > \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} = A_{L=1}$, namely the group with only one player will not be chosen.

In case 3 it is optimal to add a group with more than one player. The question that now comes to mind is what would the challenging group choose if it could select both the size of the group (from the *r* groups) and the way the value of *F* is divided amongst the members? In such a case the best choice would be the largest group, *r*, with a value of $\frac{F}{r}$ per member. In the example presented above where a municipality wishes to undertake a project purifying sewage water and it can divide the project into as many sub projects as it wishes (but not more than r), then if the total valuation for the firms is F regardless of the division, the municipality will choose the largest group, *r*, with a value of $\frac{F}{r}$ per member.

4.2.2 Different valuations

We now consider the case in which the groups may not have the same valuations. In the example of the shopping center discussed in the introduction, the stake in the contest when one firm provides all services may not be identical the sum of the stakes in the case where the each service is provided by a different firm.

4. For $0.5 < \alpha < 1$, if $F_{L=1} > F_{L>1}$ (for any $L > 1$), the group with one player will be chosen and for $F_{L=1} < F_{L>1}$ (for at least one group for $L > 1$) the group with one player or any group who satisfies the last condition, can be chosen depending on the size of *F* and how *F* is divided among the players.
5. For $\alpha = 0.5$ the group which will be chosen is the one with the highest valuation (the highest *F*).
6. For $0 < \alpha < 0.5$ then if $F_{L=1} < F_{L>1}$ (for at least one group for $L > 1$) the group with only one player will **not be chosen**.

To understand this, let us start by asking what is the best structure of A with L players. Note that A_L is given by :

$A_L = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}}\left[\gamma_1^{\frac{\alpha}{1-\alpha}} + ...\gamma_{L-1}^{\frac{\alpha}{1-\alpha}} + \underbrace{(1-\gamma_1 - ... - \gamma_{L-1})}_{\gamma_L}^{\frac{\alpha}{1-\alpha}}\right]$. If we maximize A_L with respect to γ_l $(l = (1,...,L-1))$ we obtain that the first order conditions are given by $\frac{\partial A_L}{\partial \gamma_l} = \frac{\alpha}{1-\alpha} F^{\frac{\alpha}{1-\alpha}}\left[\gamma_l^{\frac{2\alpha-1}{1-\alpha}} - \underbrace{(1-\gamma_1 - ... - \gamma_{L-1})}_{\gamma_L}^{\frac{2\alpha-1}{1-\alpha}}\right] = 0\ \forall l$.[8] We obtain $\gamma_1 = \gamma_2 = ... = \gamma_{L-1} = \gamma_L = \frac{1}{L}$. The optimal A_L becomes $A_L = \sum_1^N n_i^{\frac{\alpha}{1-\alpha}} + F^{\frac{\alpha}{1-\alpha}} L \cdot \left(\frac{1}{L}\right)^{\frac{\alpha}{1-\alpha}}$. For $0 < \alpha < 0.5$ $(\infty > \beta > 2)$ it holds that $\frac{\partial A_L}{\partial L} > 0$.

---

[8] Since $0 < \alpha < 0.5$ the second order condition for maximization holds; $\frac{\partial^2 A_L}{\partial \gamma_l{}^2} = \frac{\alpha}{1-\alpha} F^{\frac{\alpha}{1-\alpha}}\left(\frac{2\alpha-1}{1-\alpha}\right)\left[\gamma_l^{\frac{3\alpha-2}{1-\alpha}} + \underbrace{(1-\gamma_1 - ... - \gamma_{L-1})}_{\gamma_L}^{\frac{3\alpha-2}{1-\alpha}}\right] < 0$.

Therefore the larger L is, the better off each player. If the challenging group can choose both the number of the groups and the division of F amongst the groups, the choice will be for groups with the same value, each group having a value of $\frac{F}{r}$.

Concluding Remarks

In this paper we have considered the case of a two interest groups competing for a specific public good. Each group includes a number of individuals who can invest effort in order to increase their chances of winning the contest. The uniqueness of this model is in the way we relate the investments made by the individuals in the groups. In one alternative explanation, the cost of investing increases with effort. This may well be the case with new technology under which, with a very low cost, an individual can participate and affect outcomes using the internet. However, if he wishes to increase his efforts, he must use more costly alternatives. In the second alternative, two individuals investing one unit have a stronger effect than one individual investing two units in the contest.

We have defined the contest over group specific public good or public policy and considered the equilibrium outcome. As α increases, the ratio of the investments increase and the free riding phenomenon gets stronger. Free riding is at its highest level when $\alpha = 1$. Thus, we have shown that free-riding will be relatively lower compared to what was described by Baik (2007). Moreover, if the benefit from each unit invested by the individuals is sufficiently small (the cost of investment is high) the probability of winning is "almost" independent of the stakes of the contest and depends on the number of players in each group. Increasing one’s stake will increase the group’s efforts invested in the contest, while it may or may not affect its opponent’s investment. Finally, we considered the effect the structure of the group (and, in some sense, the optimal size of the group) has on the outcome. We find conditions for having a small or a large group size competing for the provision of the specific public good. Our results show the importance of the return to, and cost of, investment in the contest in determining the optimal size of the group.

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Appendix A

Optimal investments and second order condition for a unique equilibrium

Optimal investment:

From (4) we obtain that

(A1)
$$x_i = \left[\frac{\alpha \sum_1^M y_j^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}\right)^2}\right]^{\frac{1}{1-\alpha}} n_i^{\frac{1}{1-\alpha}}$$

and

(A2)
$$y_j = \left[\frac{\alpha \sum_1^N x_i^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}\right)^2}\right]^{\frac{1}{1-\alpha}} m_j^{\frac{1}{1-\alpha}}.$$

Lets us first calculate $\sum_1^M y_j^{\alpha}$ and $\sum_1^N x_i^{\alpha}$ in equilibrium. Taking (A1) and (A2) to the power of α and summarizing both sides of the equation over all *i* and *j* accordingly. We obtain

(A3)
$$\sum_1^N x_i^{\alpha} = \left[\frac{\alpha \sum_1^M y_j^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}\right)^2}\right]^{\frac{\alpha}{1-\alpha}} \sum_1^N n_i^{\frac{\alpha}{1-\alpha}},$$

and

(A4)
$$\sum_1^M y_j^{\alpha} = \left[\frac{\alpha \sum_1^N x_i^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha}\right)^2}\right]^{\frac{\alpha}{1-\alpha}} \sum_1^M m_j^{\frac{\alpha}{1-\alpha}}.$$

Since $A = \left(\sum_1^N n_i^{\frac{\alpha}{1-\alpha}}\right)$ and $B = \left(\sum_1^M m_j^{\frac{\alpha}{1-\alpha}}\right)$ we obtain

(A5) $$\sum_1^N x_i^{\alpha} = \left[\frac{\alpha \sum_1^M y_j^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha} \right)^2} \right]^{\frac{\alpha}{1-\alpha}} A,$$

and

(A6) $$\sum_1^M y_j^{\alpha} = \left[\frac{\alpha \sum_1^N x_i^{\alpha}}{\left(\sum_1^N x_i^{\alpha} + \sum_1^M y_j^{\alpha} \right)^2} \right]^{\frac{\alpha}{1-\alpha}} B.$$

From (A5) and (A6) we can calculate $\sum_1^N x_i^{\alpha}$ and $\sum_1^M y_j^{\alpha}$:

(A7) $$\sum_1^N x_i^{\alpha} = \frac{\alpha^{\alpha} A^{1-\alpha} (AB)^{\alpha(1-\alpha)}}{\left(A^{1-\alpha} + B^{1-\alpha}\right)^{2\alpha}} \text{ and } \sum_1^M y_j^{\alpha} = \frac{\alpha^{\alpha} B^{1-\alpha} (AB)^{\alpha(1-\alpha)}}{\left(A^{1-\alpha} + B^{1-\alpha}\right)^{2\alpha}}.$$

Using (A7) together with (1) we obtain that

$$x_i = \left[\frac{\alpha \frac{\alpha^{\alpha} B^{1-\alpha} (AB)^{\alpha(1-\alpha)}}{\left(A^{1-\alpha} + B^{1-\alpha}\right)^{2\alpha}}}{\left(\frac{\alpha^{\alpha} A^{1-\alpha} (AB)^{\alpha(1-\alpha)}}{\left(A^{1-\alpha} + B^{1-\alpha}\right)^{2\alpha}} + \frac{\alpha^{\alpha} B^{1-\alpha} (AB)^{\alpha(1-\alpha)}}{\left(A^{1-\alpha} + B^{1-\alpha}\right)^{2\alpha}} \right)^2} \right]^{\frac{1}{1-\alpha}} n_i^{\frac{1}{1-\alpha}}$$

, thus rewriting this equation

$x_i = \dfrac{\alpha n_i^{\frac{1}{1-\alpha}} B}{(AB)^{\alpha} \left(A^{1-\alpha} + B^{1-\alpha}\right)^2}$. In a similar way we can calculate the optimal level of y_j:

$$y_j = \frac{\alpha m_j^{\frac{1}{1-\alpha}} A}{(AB)^{\alpha} \left(A^{1-\alpha} + B^{1-\alpha}\right)^2}$$

<u>Second order conditions</u>

It can be verified that

(A8) $$\frac{\partial^2 E(U_i)}{\partial x_i^{\ 2}} = \frac{\begin{bmatrix} n_i\alpha(\alpha-1)x_i^{\alpha-2}\sum_1^M y_j^{\ \alpha}\left(\sum_1^N x_i^{\ \alpha}+\sum_1^M y_j^{\ \alpha}\right)^2 \\ -2\alpha x_i^{\alpha-1}\left(\sum_1^N x_i^{\ \alpha}+\sum_1^M y_j^{\ \alpha}\right)n_i\alpha x_i^{\alpha-1}\sum_1^M y_j^{\ \alpha} \end{bmatrix}}{\left(\sum_1^N x_i^{\ \alpha}+\sum_1^M y_j^{\ \alpha}\right)^4},$$

which is identical to

(A9) $$\frac{\partial^2 E(U_i)}{\partial x_i^{\ 2}} = \frac{\alpha n_i\sum_1^M y_j^{\ \alpha}x_i^{\alpha-2}\left[(\alpha-1)\left(\sum_1^N x_i^{\ \alpha}+\sum_1^M y_j^{\ \alpha}\right)-2\alpha x_i^{\ \alpha}\right]}{\left(\sum_1^N x_i^{\ \alpha}+\sum_1^M y_j^{\ \alpha}\right)^3}.$$

Since $0<\alpha<1$ it is clear that $\frac{\partial^2 E(U_i)}{\partial x_i^{\ 2}}<0$ and the second order conditions are satisfied. In the same way it can be calculated for the second group.

Appendix B

$$\frac{\partial x_i}{\partial \alpha} = \frac{\alpha n_i^{\frac{1}{1-\alpha}}B^{1-\alpha}\left\{\begin{array}{l} \frac{1}{\alpha}+\frac{\ln n_i}{(1-\alpha)^2}+\frac{\sum_1^M\left(m_j^{\frac{\alpha}{1-\alpha}}\ln m_j\right)}{(1-\alpha)B} \\ -\ln(AB)-\frac{\alpha\sum_1^N\left(n_i^{\frac{\alpha}{1-\alpha}}\ln n_i\right)}{(1-\alpha)^2 A} \\ -\frac{2\sum_1^N\left(n_i^{\frac{\alpha}{1-\alpha}}\ln n_i\right)}{(1-\alpha)A^\alpha\left(A^{1-\alpha}+B^{1-\alpha}\right)}+\frac{2A^{1-\alpha}\ln A}{\left(A^{1-\alpha}+B^{1-\alpha}\right)} \\ -\frac{2\sum_1^M\left(m_j^{\frac{\alpha}{1-\alpha}}\ln m_j\right)}{(1-\alpha)B^\alpha\left(A^{1-\alpha}+B^{1-\alpha}\right)}+\frac{2B^{1-\alpha}\ln B}{\left(A^{1-\alpha}+B^{1-\alpha}\right)} \end{array}\right\}}{A^\alpha\left(A^{1-\alpha}+B^{1-\alpha}\right)^2}.$$

Since $\ln n_i$, $\ln A$, $\ln B$, $\sum_1^N\left(n_i^{\frac{\alpha}{1-\alpha}}\ln n_i\right)$ and $\sum_1^M\left(m_j^{\frac{\alpha}{1-\alpha}}\ln m_j\right)$ can be either positive, negative or zero, we can get for α close to a unit any values. However for α sufficiently small the sign will be positive. The reason for this is as follows:

For $\alpha \to 0$ it holds that $n_i^{\frac{\alpha}{1-\alpha}} \to 1$ thus $\left(\sum_1^N n_i^{\frac{\alpha}{1-\alpha}}\right) = A \to N$, $A^{1-\alpha} \to N$ and $A^{\alpha} \to 1$. Also $m_j^{\frac{\alpha}{1-\alpha}} \to 1$ thus $\left(\sum_1^M m_j^{\frac{\alpha}{1-\alpha}}\right) = B \to M$, $B^{1-\alpha} \to M$, $B^{\alpha} \to 1$. We conclude that $\sum_1^N \left(n_i^{\frac{\alpha}{1-\alpha}} \ln n_i\right) \to \sum_1^N (\ln n_i)$ and $\sum_1^M \left(m_j^{\frac{\alpha}{1-\alpha}} \ln m_j\right) \to \sum_1^M (\ln m_j)$. Since for $\alpha \to 0$ it holds that $\frac{1}{\alpha} \to \infty$, thus for α sufficiently small the expression in the bracket $\{\bullet\}$ is positive. This means that there exists $\overline{\alpha}$ such that for any α that satisfies $\overline{\alpha} > \alpha$ the following holds: $\frac{\partial x_i}{\partial \alpha} > 0$.

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