The Tax-Foundation Theory of Fiat Money

Dror Goldberg

Department of Economics

Bar Ilan University

dg@drorgoldberg.com

Abstract

Throughout modern history governments have tried to promote the general acceptance of their unbacked paper currencies. One of the most common devices has been legal tender laws that have assured the acceptance of these currencies as tax payments. Economic theory has largely ignored this mechanism, except for the static models of Ross Starr (Econometrica 1974, Economic Theory 2003). I provide the first dynamic model of this mechanism, thus showing explicitly the medium of exchange role of money, accounting for expectations about the government's survival, and enabling more realistic taxation systems. I show that whether competing with other paper moneys, commodity moneys, or checks, a stable government can promote its currency by refusing to accept the other objects in tax payments. While this mechanism has similarities to convertibility, it differs from it on a critical aspect: With this mechanism the government can often keep its favorite money in circulation even while increasing its quantity and thus causing it to decrease in value. This opens the door for a successful inflationary policy.

Keywords: Fiat money, taxes, legal tender, contract law, inflation

Codes: E42, K12

The paper is based on Chapter 1 of my Ph.D. dissertation at the University of Rochester. I thank Per Krusell for his guidance. I benefited from suggestions of Leonardo Auernheimer, Charles Goodhart, Paula Hernandez-Verme, Jessica Myers, George Selgin, Alan Stockman, Daniela Puzzello, Neil Wallace, Randall Wray, Randall Wright, and seminar participants (Rochester, U. of Georgia, MMM, SAET).

"In practice credible sovereign power—specifically, the ability to enforce the legal tender status of fiat money—is necessary to create the expectations that support a viable fiat money. (Recall that the defeat of the Confederacy rendered Confederate fiat money worthless). ... The reconciliation of theory with 'facts' about fiat money remains a central problem in monetary economics."

Herschel Grossman (1991)

1 Introduction

The circulation of inconvertible, intrinsically useless money is a fundamental puzzle in monetary theory. Standard models, such as the random matching model and the overlapping generations model, attribute it to self-fulfilling expectations. In these models there is always an equilibrium in which such money does not circulate because nobody believes that it will. This equilibrium must exist in any model that takes the micro-foundations of such money seriously. Studies have shown that this non-monetary equilibrium can only be eliminated by introducing an external entity, i.e., a government. The government can simply force agents to accept fiat money in trade (e.g., Lotz and Rocheteau 2002, Selgin 2003). Alternatively, if the government itself accepts the money in trade, it can induce agents to do the same (e.g., Aiyagari and Wallace 1997, Li and Wright 1998). However, the application of either mechanism to modern capitalistic democracies is questionable. In most of these countries the government's favorite money is not forced on spot transactions. Although some believe that legal tender laws force money on all transactions, this is clearly not true, as central banks openly admit (see Section 2). As for government sales, in many countries they amount to nothing more than sale of postal stamps.

This paper explores an alternative mechanism of government intervention. The government chooses which objects to accept in tax payments and in turn, affects the value of these objects and their potential to circulate as media of exchange. Unlike the above-mentioned mechanisms this one is highly realistic,

appearing in every modern legal tender law. It also has a distinguished past. In China it had been for a long time the only legal support of paper money (von Glahn 1996), and it was reinvented in the West by Massachusetts in 1690 (Goldberg 2009). Many governments have explicitly showed much faith in this mechanism.

In contrast, economic theorists have largely ignored the potential and significance of this mechanism. Succinctly named "the tax-foundation theory" by Ellis (1934, p. 11), it was briefly discussed by Smith (1776) and Lerner (1947).[1] Its only mathematical models are Starr (1974, 2003). Starr's models are static. The former is a Walrasian model with a cash-in-advance constraint on consumption, while the latter is a trading post model. In both models all agents are taxed with probability one.

Some may argue that these models—being static—only show that money has value but not that it is a medium of exchange. The models also cannot take into account expectations about the future viability of the government. Their strict assumptions about taxation do not account for tax deferment, tax evasion and tax exemption. Does the theory still hold if some people do not pay taxes some of the time?

In order to address these concerns I use for the first time a dynamic model to analyze the tax-foundation theory. Specifically, I use a monetary search model. In this class of models the monetary object is received in one period and later spent in shopping. Thus, I show that money not only has value but also functions as a genuine medium of exchange. I show that the theory holds as long as the government is expected not only to survive but also to maintain an effective tax-collection system. At the same time, the theory is sustainable even if taxes are not paid by everyone in every period. I also explore whether the tax-foundation theory is equivalent to convertibility or to government sales.[2] While there are strong similarities, one exceptional difference stands out: with this mechanism, the government's ability to sustain the money's circulation may be unharmed by excessive money printing. This can explain the

[1] Wray (1998) and Forstater (2006) survey the history of economic thought on this theory.

[2] Such equivalence is implied in the Introduction of Li and Wright (1998).

"success" of numerous hyperinflations—where the money lost value but kept circulating while raising seigniorage revenue.

The need to reconcile the tax-foundation theory with modern monetary search models is emphasized by Charles Goodhart's critique; he claims that economists are attached to such "nicely constructed models, whatever the facts may be" even though the tax-foundation theory "does far better in explaining and predicting historical reality" (Goodhart 1998, p. 408-9). This paper demonstrates that Goodhart's preferred theory can be well formalized in a variant of the monetary search model.

The mechanism explored here should not be confused with similar ones. It is an ingredient of Knapp's (1905) total state/chartal theory of money, according to which the government must support the money in many additional ways.[3] The tax-backing theory (Wallace 1981, Sargent 1982, Smith 1985) and the fiscal theory of the price level focus on the determination of the price level *given* that the money does circulate. In contrast, the present issue is whether any monetary equilibrium is realized *at all*, and the key determinant of that is not the tax rate or deficits, but rather which object is used to pay taxes, and what the penalty is for paying taxes with other objects. To emphasize the different focus of the tax-foundation theory, in my model public deficit can never exist. On the surface, the tax-foundation theory is similar to the legal restrictions theory (Cowen and Kroszner 1994, pp. 148-9). Both theories claim that government-issued fiat money may be valued only because of government regulation. Yet the two theories differ on an important matter as the legal restrictions theory claims that only the government is strong enough to suppress market-created money. The tax-foundation theory has occasionally been seen to imply that markets are too weak to create their own fiat money.

The paper is organized as follows. Section 2 explains in simple terms what "legal tender" really means.

[3] These include: accepting the money for any other payments such as fines, fees, payments for government-produced goods and services, and payments of banks to the central bank; using the money in its purchases; declaring it legal tender in private contractual debts; and fixing the money's exchange rate with the previous domestic money.

Section 3 presents the basic model with limited randomness in matching, exogenous prices, a unit upper bound on money holdings, and a fixed money supply. Section 4 introduces competing outside money and inside money. Section 5 allows multiple money holdings. Section 6 features endogenous prices and an increasing money supply. I then briefly sketch other variants of the model. Section 7 features complete randomness in matching, as in most search models. Section 8 introduces a competing commodity money. Section 9 relates the models to monetary history. I conclude in Section 10.

2 Legal Tender

The legal foundation of the monetary system is the law of legal tender. Monetary theorists have recently started labeling the money they model as "legal tender." The meaning of this concept changes from paper to paper: it is the only money that sellers are allowed to accept;[4] it is the only money that buyers are allowed to offer;[5] buyers can force sellers to accept it (but they can agree on another medium of payment)[6]; sellers can force buyers to use it;[7] money is accepted if and only if it is legal tender;[8] it must be used by default if the buyer and seller do not agree on the medium of payment during negotiations.[9]

All these papers are wrong in relating the money they model to the legal tender concept. Thus, applying their results to real legal tender currencies can be misleading. Some textbooks are also in error.[10] In fact, the legal tender concept has nothing to do with the spot transactions that dominate

[4] Lotz (2004), p. 967.

[5] This is implied in the cash-in-advance constraints of Sargent and Velde (2002), p. 368, and the money-in-the-utility function of Sussman and Zeira (2003), p. 1777.

[6] Selgin (2003), p. 160; Shy and Tarkka (2002), p. 303.

[7] Shy and Tarkka (2002), p. 308.

[8] Lotz and Rocheteau (2002), p. 568.

[9] Shy and Tarkka (2002), p. 308.

[10] See Lipsey, Courant and Ragan (1999), p. 581. Barro (1993), p. 96, mentions "legal tender" but does not explain what

economic theory. Its practical importance as the legal foundation of the monetary system, if there is one, comes from its implication on tax payments.

2.1 What is Legal Tender?

In law, a commercial contract is born when the parties agree on some necessary terms. In a contract to sell goods the quantity is a necessary term. For example, an agreement to "sell apples at the price of one dollar per pound, tomorrow, in my store," is not a contract and cannot be enforced in court, because the quantity is undetermined. Legislatures worldwide resolved long ago that specifying the medium of payment is *not* a necessary term. Thus, if the above example is modified by adding the quantity term "ten pounds of apples," then it is a valid contract, even though the medium of payment (as opposed to the unit of account) is undetermined.

However, this raises a potential problem. A contract has been created, and each side now has an obligation. How should the buyer's obligation to pay ten dollars be discharged? Actually, almost anything on which the parties mutually agree is acceptable.[11] Examples include: Ten one-dollar bills, a check, peso bills according to some exchange rate, or a watch which the seller estimates as worth at least ten dollars. This is just one aspect of the freedom of contracts, which is a fundamental building block of capitalism. Legislatures have outlawed very few media of payment, such as gold (in post Great Depression legislation), or illegal drugs (which could conflict with the public interest). It does not matter if the agreement regarding the medium of payment is part of the contract, or made separately after the contract is created.

---

it means. Case and Fair (2003), p. 481, exclude tax payments from the legal tender law. Mankiw (2000), p. 156, only mentions an unspecified "government decree" (also see Auerbach and Kotlikoff, 1998, p. 175).

[11] Bank of England (2008), Bank of Canada (2008), Williston (2003), vol. 28, pp. 752-3, 778.

The main goal of contract law is to solve disagreements after a contract is created (for instance, where the terms are vague and give rise to a dispute). Suppose that the buyer in my example, where no medium of payment was specified, offers to pay in a ten dollar bill, but the seller rejects it because he wants pesos. Given that a contract was formed and payment was tendered but rejected, can the seller sue the buyer in court for breach of contract due to this non-payment? What if the buyer offers one thousand one-cent coins, or a ten-dollar watch?

"Legal tender" is an object that confers a right on the payer. If the buyer in my example offers the correct quantity of anything that has been declared by law to be legal tender, then the seller's lawsuit fails. The buyer may be asked to deliver the proffered payment to court, which the court would offer to the seller. The buyer is then off the hook, having fully performed his contractual obligation of making payment.[12] On the other hand, any object that is not legal tender will not give the buyer such peace of mind. Judgment will be entered against the buyer for breach of contract if the seller delivered the goods and rejected a proffered payment from the buyer that did not constitute legal tender. For this very practical purpose, every country specifies which objects are considered legal tender for debts that are subject to its contract law. Typically, the government gives this status to currency it issues itself, but this is not necessary.

Since legal tender laws protect buyers, sellers may want to protect themselves from these laws. Usually, it is remarkably easy to do so. Before the necessary details of the contract are finalized (that is, before contract formation), the seller can specify the medium of payment. If the parties agree to a specific medium of payment, then this term will become part of their contract.[13] If that medium of payment is not outlawed by other laws (for example, voided as a matter of public policy, as in the illegal drug example above), then legal tender laws will not apply. If, on the other hand, there is disagreement about

[12] Williston (2003), vol. 28, pp. 746, 805-14, Bank of England (2008), Reserve Bank of Australia (2008).

[13] Board of Governors of the Federal Reserve System (2008), Reserve Bank of Australia (2008).

the medium of payment, then a contract fails to come into existence. Going back to my example, suppose that before agreeing on the quantity of apples to be delivered, the seller states (e.g., by posting a sign near the cash register) that he must be paid in pesos. If the buyer refuses and this medium of payment is not acceptable to both parties, then a contract is not formed, and nobody has any contractual obligation at all.

Another easy way to avoid legal tender laws is to use a different unit of account. The legal tender law of the United States, which gives a legal tender status to dollars in the form of coins and bills, cannot apply to contracts that specify payments in pesos or potatoes.

The conclusion is that sellers are not really forced to accept legal tender money if they are slightly cautious. They only need to state in advance that they want to be paid in a different object, or use a different unit of account. The websites of some central banks are honest about this limited legal status of their money.[14] The role of the state, after declaring what is legal tender, can be described as passive and negative: To dismiss a creditor's lawsuit if the debtor offers the right quantity of legal tender. A legal tender law never results in the state affirmatively prosecuting a buyer or a seller for using another currency or for rejecting the legal tender in a spot transaction. Other laws might do that, but they are rare in Western democracies.

Certain monetary obligations are created not by contract, but by statutory or common law. These obligations invoke some of the practical issues of contractual obligations. If a would-be taxpayer delivers her used car (valued at the outstanding amount of taxes owed) to the Internal Revenue Service as her tax payment, what can the IRS do? Must it accept the car, or can it sue the taxpayer for not paying the tax?[15] What about paying a parking ticket with foreign currency, or vengefully paying alimony with

[14] E.g., Board of Governors of the Federal Reserve System (2008), Bank of England (2008), Bank of Canada (2008), Reserve Bank of Australia (2008).

[15] This actually happened (United States, 2003, 31 § 5103, p. 27, note 17).

small change?

For this reason, although the legal tender concept originates in contract law, it has been universally extended to include all non-contractual obligations as well.[16] As with contracts, the legal tender law is irrelevant if the tax authority and the taxpayer agree on another medium of payment, such as a check or a credit card.

2.2 Frequently Asked Questions in the United States

1. What is the legal tender law?

Answer: United States Code 31 § 5103, "United States coins and currency (including Federal Reserve notes and circulating notes of Federal Reserve Banks and national banks) are legal tender for all debts, public charges, taxes, and dues."

2. How about banks?

Answer: Those notes of Federal Reserve Banks and national banks are too rare today to be seriously considered. On the other hand, national banks and members of the Federal Reserve System must accept Federal Reserve notes in all transactions. This special obligation, which is not imposed on anyone else, is balanced by special privileges that only banks have.

3. The writing on Federal Reserve notes mentions only "debts" but not taxes. Which is the correct one?

Answer: The writing on the notes is a relic from a time when courts interpreted "debt" as any obligation.[17] Nobody bothered fixing it, but it is the United States Code that legally matters.

4. Federal tax forms order taxpayers: "Do not send cash." Doesn't this contradict the legal tender

[16] The word "debt" in most legal tender laws includes any tax, while "creditor" includes any tax-collecting agency. See Nussbaum (1950), p. 49, 58, 139, Mann (1982), p. 52, 80-100, European Union (1998, 2005), especially Articles 1, 8.

[17] United States (2003), 31 § 5103, p. 21.

law?

Answer: It obviously does, although virtually all taxpayers obey this request anyway for their own benefit (if you send your tax payment in cash, the mailman might steal it without a trace). For the IRS, this order is a weakly dominant strategy: That is, it benefits the IRS if taxpayers obey, and in the unlikely case that a taxpayer does send cash, the IRS can simply accept it upon arrival, with no harm done.

5. Can I pay a large debt in small change?

Answer: That is what the United States Code implies. Legal tender laws of some other countries allow creditors to reject such payments.[18]

6. How can the legal status of money in the U.S. be summarized in one paragraph?

Answer: All Federal Reserve notes and U.S. coins are legal tender for all dollar-denominated obligations. This means that contractual creditors who do not specify another medium of payment in their contracts, as well as all tax authorities and courts (federal, state and local), cannot reject a payment made using these objects. In addition, many banks (national banks and members of the Federal Reserve System) must accept Federal Reserve notes in all transactions. Anyone else can reject these notes and coins. Practically nothing else is legal tender, and thus anything else can be rejected by anyone in any transaction. These notes and coins are redeemable by their issuers only for other notes and token coins, possibly of different denominations.

2.3 Implications

This paper focuses on the tax aspect of legal tender laws. It is stronger than the contractual aspect, because the latter can be so easily avoided. Although he did not use the label "legal tender," Freeman (1996) modeled legal tender for contractual debts. If nobody chooses to have a debt, then the law is not

[18] European Union (1998) United Kingdom Ministry of Justice (2008), Reserve Bank of Australia (2008).

effective. While denominating a contract in foreign currency makes the legal tender law irrelevant for your contract, receiving your entire income in foreign currency will not exempt you from paying your taxes in dollars.

Contractual debts can create a strong demand for a new currency like the one created by taxes only by applying a legal tender law retroactively to pre-existing contracts. This happened with the Civil War greenbacks, and invoked a constitutional firestorm. Creditors who did not expect this first U.S. paper money failed to specify "gold" in their contracts. Debtors were happy to pay debts with paper greenbacks instead of gold, and their demand for the greenbacks gave the greenbacks value.

3 The Basic Model

The goal of this paper is to model the tax-foundation theory in a monetary search model. Such models typically exhibit complete randomness of all meetings between agents; therefore they are also known as random matching models. However, a salient feature of real-life taxation is some lack of randomness. People know when they are going to be taxed and they usually know the terms: how much, where, how and in what medium of payment. They also have enough time to prepare for a tax payment. Some taxes can be completely, and legally, avoided by eschewing certain activities. It is thus useful to have a model in which taxation is not completely random. The alternative of complete randomness is featured in Section 7.

As for the trade meetings, I maintain the same randomness as in random matching models. I do this for the sake of comparison with random matching models of competing mechanisms. In particular, the model is designed to be as close as possible to Li and Wright (1998), who model sales of government goods for money as a way to support that money.

3.1 Environment

Time is continuous. A continuum $[0,1]$ of infinitely-lived agents are randomly matched in pairs according to a Poisson arrival rate α. A fraction $\gamma \in [0,1)$ of them are potential government buyers and the rest are private agents. Each private agent derives utility $U > 0$ from the consumption of one indivisible unit of some goods. It can produce only one type of good and it does not consume it. Production of one unit of any (perishable) good is instantaneous and involves disutility $C \in (0,U)$. Production is independent of previous consumption, but agents can produce if and only if they do not hold money.[19] The probability that any private agent consumes the good of its trading partner is $x > 0$, and the conditional probability that the converse holds too is $y \geq 0$. The discount rate is $r > 0$.

At present, the only durable object is fiat money called a dollar bill, which has a storage cost $c > 0$ and a fixed supply $M_{\$} \in [0,\gamma]$.[20] A fraction m_p of private agents are endowed with one dollar each and are called *private buyers*. The other private agents hold nothing and are called *sellers*. Variables that relate to sellers and real goods have a subscript 0.

The potential government buyers participate in the matching process described above. They have the potential to consume all goods, cannot produce anything, and cannot store any real good. These assumptions, starkly different from Aiyagari and Wallace (1997) and Li and Wright (1998), emphasize that the tax-foundation mechanism, unlike convertibility, can work even for a government which is completely parasitic and cannot credibly promise convertibility into real goods. Among the potential government buyers, I focus on the fraction m_g of them who hold money (one dollar each). These are called *government buyers*. When a government buyer is randomly matched with a seller, the former receives a taste shock.

[19] This assumption differs from Li and Wright (1998). It allows agents who were just taxed to resume market activities, while keeping one unit of money as an effective upper bound on their holdings.

[20] As in Aiyagari and Wallace (1997), this upper bound on money simplifies the analysis because then all the money is held by the government when private agents reject it.

With probability x it wants to consume the good that the seller can produce. The government buyer then offers its money for the seller's good. It *does not* force the seller to trade. If the money is accepted, the government buyer consumes and becomes a potential government buyer. Being moneyless, it can do nothing until it receives a new dollar, as described below.

In addition to all these randomly matched agents, there is another class of government agents, called *tax collectors*. They operate outside of the matching process described above. They are idle during trade meetings and operate right after them (as in Shi 2005). The tax collectors are capable of identifying agents who had just produced, say because these sellers show signs of exhaustion or because production is noisy. Tax law in the economy states the following:

1. Only current income is taxed.
2. An agent who just sold is taxed with probability $\tau \in (0, 1)$.
3. The size of the tax payment is the entire income just earned.
4. Tax collectors cannot reject tax payments in dollar bills.
5. Tax collectors are allowed to choose whether to accept a tax payment in a real good.
6. Agents whose payment is rejected face a non-monetary punishment $P > 0$.[21]

After taxes are collected all the proceeds are transferred to the potential government buyers, with each buyer getting one dollar at most. Then trade meetings resume.

This tax law is different from totally random taxation. Agents know that only a sale will make them eligible for paying the tax. They can choose never to be exposed to the tax. They can also choose to make a sale only if it leaves them well prepared for the tax collector's visit (say, sell for dollars but avoid barter). Nevertheless, some randomness ($\tau < 1$) must remain for the sake of analytical tractability.[22]

[21] The punishment can be thought of as beating. It is possible to model it as a fine paid in real goods produced by the offender, as in Soller-Curtis and Waller (2000).

[22] If $\tau = 1$ was allowed, the only way to keep production incentive-compatible would have been to tax only a fraction of

A good by-product of this limited randomness is that it approximates tax evasion, tax exemption, tax deferment and the fact that income taxes are not paid after every single sale. Note that the only existing models of the tax-foundation theory already have $\tau = 1$ (Starr 1974, 2003). Let G denote the subjective probability that private agents assign to the existence of a taxing government in the next period. Denote the expected probability of being taxed as $t \equiv \tau G$.

Sections 4 and 5 of the tax law specified above are a real legal tender law. Explicitly, the law imposes an obligation only on tax collectors, and it says nothing about objects other than dollar bills. However, its silence regarding other objects means that tax collectors have full discretion whether to accept such objects. For example, in the U.S., the tax authority chooses to accept checks (to be modeled below) but generally refuses to accept real goods. According to basic legal principles, it is the silence of the U.S. legal tender law regarding real goods that empowers the tax authority to make such a decision. You cannot force the I.R.S. to accept your used car as a tax payment.

Finally, note that the government always has a balanced budget in the sense that taxation precedes consumption for all government buyers. With no government borrowing, the controversial fiscal theory of the price level is irrelevant to the current discussion.

3.2 Strategies

Trade meetings allow sellers to barter in some cases and sell for money in other cases. Let Π_0 be the probability that a random seller agrees to barter. Let $\Pi_\$$ be the probability that a random seller accepts money. Let the probabilities π_0 and $\pi_\$$ be the best responses of a maximizing seller who is offered a good it consumes, and a dollar, respectively. Let $s \equiv (1-\gamma)(1-m_p)$ be the proportion of sellers. Let T_0 indicate whether tax collectors accept real goods ($T_0 = 1$) or not ($T_0 = 0$). If V_0 and $V_\$$ are the value

one's income. This would have necessitated an increase in the upper bound on the indivisible money holdings and would have precluded closed-form solutions.

functions of sellers and private buyers respectively, then

$$rV_0 = \alpha sxy\Pi_0 \cdot \max_{\pi_0} \pi_0\{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + \alpha M_\$ x \cdot \max_{\pi_\$} \pi_\$[-C + (1-t)(V_\$ - V_0)], \quad (1)$$

$$rV_\$ = -c + \alpha sx\Pi_\$(U + V_0 - V_\$). \quad (2)$$

In (1) a seller has two interesting matching possibilities. It may have double coincidence of wants with another seller. In this model the choice of barter is not trivial. Barter, which directly yields $U - C$, also makes both agents eligible for paying income tax. With probability t a bartering agent meets a tax collector. If the tax collector agrees to accept the agent's produce then the agent produces. Otherwise, it faces the punishment P. If the seller meets any buyer (public or private), it chooses whether to accept a dollar. If it does, then it produces and gets \$1. Again, this makes the seller eligible for taxation. With probability t it is taxed of all its money so it remains a seller. With probability $1 - t$ it is not taxed so it becomes a buyer. Note that because agents can choose $\pi_0 = \pi_\$ = 0$, we have $V_0 \geq 0$. No matter how large P is, agents can choose not to be exposed to the punishment.

In (2) a buyer pays the storage cost of money and it can buy a good if the seller it meets accepts money. Following Li and Wright (1998), αx is henceforth normalized to 1. Assume that agents want to trade if and only if the trade would strictly increase their lifetime utility.

3.3 Equilibrium

The analysis is restricted to symmetric, pure strategy, stationary, *non-autarkic* equilibria.

Definition 1. A pure monetary equilibrium is an equilibrium in which all sellers accept fiat money but do not barter.

Definition 2. A monetary equilibrium is an equilibrium in which all sellers accept fiat money and barter.

Definition 3. A non-monetary equilibrium is an equilibrium in which all sellers barter but reject fiat money.

At the heart of the model stand two decisions. First, the tax collectors decide which objects are to be received in tax payments in addition to dollar bills. Second, the legislature determines the punishment for those whose tax payment is rejected.

Definition 4. An object i, $i \in \{0, \$\}$, is *tax-receivable* iff $T_i = 1$.

The tax law already stated that $T_\$ = 1$.

It is easy to discourage barter (and production in general), by simply setting a high enough probability of taxation. The point of this paper, however, is not to describe such a mechanism, but to relate to the discrimination regarding tax-receivable objects. The focus here is not on the probability of payment but rather the *medium* of payment. Assumption 1 below ensures that without such discrimination, barter—and therefore the non-monetary equilibrium—can exist.

Assumption 1. $t < U/C - 1$.

The first result is that if all objects are tax-receivable, money may not circulate at all.

Proposition 1. If all objects are tax-receivable, then: (i) the non-monetary equilibrium exists; (ii) the monetary equilibrium exists for some parameter values; (iii) a pure monetary equilibrium does not exist.

Proof. (i) Set $T_0 = \Pi_0 = 1$ and $\Pi_\$ = 0$ in (1) and (2). Clearly, $\pi_\$ = 0$, while Assumption 1 implies $\pi_0 = 1$. With barter and rejection of fiat money, we have the non-monetary equilibrium. (ii) Set $T_0 = \Pi_0 = \Pi_\$ = 1$. Then $-C+(1-t)(V_\$-V_0) > 0$ (so $\pi_\$ = 1$) iff $c < c_\exists$, where $c_\exists \equiv s(1-y)U+[sy(1+t)-\frac{r+s}{1-t}]C$, and $s = 1 - \gamma - M_\$(1-t)$. (iii) Assumption 1 implies $\pi_0 = 1$. □

As usual, fiat money's circulation depends on both its intrinsic properties and agents' beliefs.

3.4 Policy

The government may be able to affect existence of equilibria by discriminating between various objects. Specifically, the government can make the dollar bill the *only* tax-receivable object.

Theorem 1. The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium iff $t > 0$ and the money's storage cost is small enough.

Proof. Set $T_0 = 0$ and $P = (U - C)/t$. This implies $\pi_0 = 0$ so there is no barter. This rules out the non-monetary and monetary equilibria for all parameter values. Setting $\Pi_\$ = 1$ implies that $\pi_\$ = 1$ iff $c < c_u$, where $c_u \equiv sU - \frac{r+s}{1-t}C$. $\square$

Without the policy, money may circulate only if $c < c_\exists$. With policy, money circulates for sure iff $c < c_u$. It is always the case that $c_\exists < c_u$, which means that in some cases policy enables money to circulate when it otherwise would not. Looking at P in the proof, it is clear that the lower the probability of meeting tax collectors in the future, the higher the minimal punishment needs to be.[23]

It is important to compare the tax-foundation mechanism to convertibility. The latter is a commitment of the issuer to convert paper money into gold or any other good or service. In the tax-foundation mechanism the government does not give anything useful for paper money, but it does give something *harmful* if one does *not* have paper money when it is time to pay taxes. One might say that here paper money buys immunity from punishment, or that it is implicitly convertible into immunity. The mechanism is therefore somewhat analogous to convertibility. However, it is not an equivalent. It involves the government by definition, because only the government taxes. Convertibility, on the other hand, has been practiced by both governments and a wide variety of private entities. Close to convertibility are

[23] I vary only the punishment and keep the tax rate fixed, since tax rates today are usually determined by the fiscal needs of the government, rather than by the need to support monetary equilibria. It is the insistence that these taxes be paid in fiat money and the associated punishment, that can serve monetary equilibria as a positive externality. However, in colonial America many tax rates were determined so as to deliberately support monetary equilibria (Brock [1975]).

the models of the government as a seller of goods (Aiyagari and Wallace 1997 and Li and Wright 1998). In both these models and the current one, more interaction with the government implies that more private agents face an exogenous pro-money behavior, which inspires general circulation of money. The difference is that here the government's crucial role is modeled as a tax collector. In a modern economy the government sells very few goods in the marketplace, but its taxation is considerable.

In fact, this model does not critically depend on any government involvement in, or monitoring of, trade: the government does not convert money into real goods, it does not directly force agents to trade with each other with fiat money (barter in itself is legal), and it does not force sellers to accept fiat money from government buyers. Here, the only role that the government has in trade is a technical one: to return the money that it collects to the economy through its buyers, as real-life governments do. I could have assumed instead that the government destroys the collected money and then injects new money by helicopter drops.

Is the policy optimal?

Proposition 2. (i) The pure monetary equilibrium has lower welfare than the monetary equilibrium. (ii) If y is small enough, the pure monetary equilibrium is better than the non-monetary equilibrium.

The proof is trivial. The result is a trade-off, very similar to the one in Aiyagari and Wallace (1997). In that model, the government supports money by refusing some barter opportunities. This means that some welfare is lost, compared to a monetary equilibrium without such policy. However, the resulting monetary equilibrium may be better than the non-monetary equilibrium that could have existed if the policy had not existed. The only difference is that the foregone barter opportunities in the current model are always between private agents. The policy induces them to give up those trades, but guarantees the use of money. Monetary trade is more likely to be the optimal form of trade if direct barter is difficult (i.e., y is small).

One more policy should be considered, because it relates to the historical discussion below. Suppose

that the tax is denominated in goods rather than money. In particular, it is set at one good per taxpayer. In addition to accepting goods, tax collectors also accept money, but only according to its market value. If an agent pays in money and the money's market value is zero ($\Pi_{\pounds} = 0$) then the tax collector forces this agent to produce one more good as payment. A seller's value function is now

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0[U - C(1+t)] + M_{\pounds} \cdot \max_{\pi_{\pounds}} \pi_{\pounds}[-C - t(1-\Pi_{\pounds})C + (1-t)(V_{\pounds} - V_0)]. \qquad (3)$$

Proposition 3. A policy of accepting money for taxes at market value is ineffective.

Proof. $\pi_0 = 1$ because there is no punishment. For there to be any equilibrium at all, set $\Pi_0 = 1$ as well. Setting $\Pi_{\pounds} = 0$ replicates the qualitative outcome of Proposition 1(i), while $\Pi_{\pounds} = 1$ exactly replicates Proposition 1(ii). Proposition 1(iii) holds here too. □

The key point with regards to government intervention is that it introduces exogenous behaviour, which is unrelated to endogenous market expectations. Once the policy itself succumbs to these expectations and accommodates them, it has no hope of any real impact.

4 Competing Currencies

In the real world dollar bills compete not only with barter but also with other currencies. I will now turn to examine another kind of outside money (euro, denoted e) and inside money (banknote, denoted b). Let M_i, V_i, T_i, π_i, and Π_i, $i \in \{0, \$, e, b\}$, be the obvious generalization of the above notation. I consider these competing currencies one at a time. Competition with commodity money requires a very different model so it is featured in Section 8.

4.1 Another Fiat Money

Suppose that euro bills have the same physical properties of dollar bills, including the storage cost c. To simplify the analysis, I assume that currency trading of dollars for euros is impossible. Due to the symmetry between agents and between currencies, such trading could not be mutually beneficial in equilibrium, and thus would not occur in any case. The value functions are now determined as follows.

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0\{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + M_\$ \cdot \max_{\pi_\$} \pi_\$[-C + (1 - t)(V_\$ - V_0)] + \quad (4)$$

$$M_e \cdot \max_{\pi_e} \pi_e[-C + t(1 - T_e)(-P) + (1 - t)(V_e - V_0)],$$

$$rV_i = -c + s\Pi_i(U + V_0 - V_i), \quad (5)$$

for $i = \$, e$.

Proposition 4. If all objects are tax-receivable and $c < c_\exists$ there are four equilibria coexisting: a non-monetary equilibrium; a monetary equilibrium with the dollar as the unique money; a monetary equilibrium with the euro as the unique money; and a monetary equilibrium with both dollars and euros as money. There is no pure monetary equilibrium.

Proof. Essentially identical to the proof of Proposition 1.

Suppose that the government wants to promote the use of dollars as money.

Theorem 2. The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium with dollars iff $t > 0$ and the money's storage cost is small enough.

Proof. Set in (3) $T_0 = T_e = 0$. Setting $P = (U - C)/t$ is sufficient to rule out barter (and therefore all non-monetary and monetary equilibria). It is also sufficient for ruling out the pure monetary equilibrium

in which the euro is the unique money. A possibly higher punishment ($P = \frac{(1-t)(sU-c)-C(r+s)}{t[r+s+(1-t)M_\$]}$) is needed to rule out the pure monetary equilibrium in which both dollars and euros are money. This leaves only the pure monetary equilibrium with the dollar as the unique money. □

In general, if there are n types of intrinsically useless objects with the same low storage cost, then without government intervention there are $\sum_{x=1}^{n} \binom{n}{x}$ pure monetary equilibria and the same number of monetary equilibria. The non-monetary equilibrium also exists if the government does not intervene, so overall there are $2^{n+1} - 1$ equilibria. The government can make any of the n objects the unique money in a unique pure monetary equilibrium.

4.2 Inside Money

Instead of euros let us now assume that there is another entity in the economy called a bank. It has a fixed location which is costlessly accessible to all agents between trading and taxation. The bank is monitored by the government and can therefore make commitments. It has a unique technology which enables it to produce banknotes, which are durable, indivisible and are not consumed. Their advantage over dollar bills is that they have no storage cost. This advantage represents the convenience of banknotes compared with gold coins in the past, and the convenience of checks and electronic money compared with dollar bills today.

An agent who holds a dollar bill after a trade round can go to the bank, deposit the dollar bill, and receive a lighter banknote instead. In return for this service the agent has to produce for the bank.[24] The bank consumes all the real goods. Any agent holding a banknote can go to the bank after a trade round and try to convert it into a dollar bill. There is no cost to such conversion. The bank keeps a 100% reserve ratio but might vanish with probability $1 - R$, where $R \in [0, 1)$. Therefore, conversion succeeds with probability R.

[24] The seller can produce only after it gives the dollar to the bank and before receiving the banknote.

Let π^S_{ij}, $i,j \in \{\$, b\}$, be the strategy of a seller who just earned some money i (and thus can be taxed) and chooses whether to convert it at the bank into money $j \neq i$. Let π^B_{ij} be the similar strategy of a buyer who had money i before the last trade round and failed to spend it. Due to the increased complexity of the model, it is useful to let $y = 0$ here, and to assume that buyers cannot swap a dollar and a banknote among themselves. These simplifications do not affect the results. The value functions are now

$$rV_0 = (M_\$ - M_b) \cdot \max_{\pi_\$, \pi^S_{\$b}} \pi_\$\{-C + (1-t)(V_\$ - V_0) + \pi^S_{\$b}[-C + t(1-T_b)(-P) + (1-t)(V_b - V_\$)]\} + \quad (6)$$

$$M_b \cdot \max_{\pi_b, \pi^S_{b\$}} \pi_b\{-C + \pi^S_{b\$}R(1-t)(V_\$ - V_0) + (1 - \pi^S_{b\$}R)[t(1-T_b)(-P) + (1-t)(V_b - V_0)]\},$$

$$rV_\$ = -c + s\Pi_\$(U + V_0 - V_\$) + (1 - s\Pi_\$) \cdot \max_{\pi^B_{\$b}} \pi^B_{\$b}(-C + V_b - V_\$), \quad (7)$$

$$rV_b = s\Pi_b(U + V_0 - V_b) + (1 - s\Pi_b) \cdot \max_{\pi^B_{b\$}} \pi^B_{b\$}R(V_\$ - V_b). \quad (8)$$

In (5) the probability of meeting a dollar holder depends on how many dollars are stored at the bank (i.e., how many banknotes are outstanding). A seller who just earned a dollar can exchange it for a banknote by producing for the bank. If it is later taxed and the tax collector refuses to accept banknotes then it is punished. If it accepts a banknote in trade (second line of (5)) it can try to redeem it at the bank for a dollar bill. The motivation for this is the possibility that banknotes are not tax-receivable. Not only a recent seller can deposit a dollar at the bank: in (6) a buyer who fails to buy with a dollar can deposit it. In (7), a buyer holding a banknote can try to convert it into a dollar. Since barter is ruled out by assumption, only pure monetary equilibria are possible.

Proposition 5. If all objects are tax-receivable, then: (i) the pure monetary equilibrium with dollars exists for some parameter values. (ii) for another set of parameter values there is a pure monetary equilibrium in which banknotes circulate and are used in tax payments, while all the dollar bills—although not rejected in trade—are actually always at the bank. (iii) if dollars are rejected in trade the pure monetary equilibrium with banknotes may or may not exist.

Proof. (i) Set in (5)-(7) $T_b = \Pi_\$ = 1$, $\Pi_b = 0$. Banknotes are rejected in trade ($\pi_b = 0$) iff $c \leq sU - \frac{(R+r)[r+s+(1-t)M_\$]M_\$ C}{[r+(1-t)M_\$][(1-t)(1-R)M_\$ - R(1+r)]}$ and $R < \frac{(1-t)M_\$}{(1-t)M_\$ + 1 + r}$. It is optimal to convert banknotes into dollars ($\pi^S_{b\$} = \pi^B_{b\$} = 1$) iff $c < sU - \frac{sM_\$ C}{r+(1-t)M_\$}$. This condition is sufficient to prevent depositing of dollars ($\pi^S_{\$b} = \pi^B_{\$b} = 0$). The rest is exactly as in Proposition 1(ii). (ii) Set $T_b = \Pi_\$ = \Pi_b = 1$. Banknotes are accepted in trade iff $sU > \frac{r+s}{1-t}C$.[25] Dollars are deposited by sellers iff $c > \left(\frac{r+t}{1-t} - s\right)C$. This condition is sufficient for a buyer with a dollar to deposit it. It is not optimal to convert banknotes into dollars. (iii) With dollars rejected in trade, equations (5) and (7) are essentially the same as (1) and (2), only that there is no barter, there is no storage cost, and the notation "b" replaces "\$" everywhere. Thus Proposition 1 applies. □

Theorem 3. The government can guarantee the existence and uniqueness of the pure monetary equilibrium with dollars iff $t > 0$ and the money's storage cost is small enough.

Proof. Set $T_b = 0$ and $\Pi_\$ = 1$. It is easy to see in (5) that regardless of the value of Π_b, a high enough P results in rejection of banknotes in trade and no depositing of dollars. The rest is the same as in Theorems 1 and 2. □

As with any piece of paper, the circulation of a banknote or a check depends on agents' beliefs. The tax law gives tax collectors full discretion whether to accept banknotes. If they choose to reject them, they can drive the banknotes out of circulation, but they can also choose to accept them and not disrupt

[25] This is the same condition from Proposition 1(ii), only here $y = 0$ and banknotes have no storage cost.

this efficient use of a lighter medium of exchange.[26]

5 Multiple Money Holdings

An obvious shortcoming of the model is that agents never hold more than one object at a time. One might suspect that allowing more flexibility would allow agents to diversify their portfolios or at least be flexible about what they accept in payment. Assume then that agents can produce only if they hold up to one object of any type. This effectively increases the upper bound on money holdings from one unit to two units. Prices are still fixed for now at 1.

The point can be made by considering the economy with barter and only dollar bills as potential money. Let m_i, $i \in \{0, \$, \$2\}$, be the fraction of private agents holding i. For $i, j \in \{0, \$\}$, let Π_{ij} be the probability that a random seller holding i accepts j, and let π_{ij} be the best response of an agent who holds i and is offered j. Let $V_{\$2}$ be the value function of an agent holding \$2. The value functions are

$$rV_0 = (1-\gamma)(m_0\Pi_{00} + m_\$\Pi_{\$0})y \cdot \max_{\pi_{00}} \pi_{00}\{U - C + t[T_0(-C) + (1-T_0)(-P)]\} + \tag{9}$$

$$[M_\$ - (1-\gamma)m_{\$2}] \cdot \max_{\pi_{0\$}} \pi_{0\$}[-C + (1-t)(V_\$ - V_0)],$$

$$rV_\$ = -c + (1-\gamma)(m_0\Pi_{00} + m_\$\Pi_{\$0})y \cdot \max_{\pi_{\$0}} \pi_{\$0}[U - C + t(V_0 - V_\$)] + \tag{10}$$

$$[M_\$ - (1-\gamma)m_{\$2}] \cdot \max_{\pi_{\$\$}} \pi_{\$\$}[-C + (1-t)(V_{\$2} - V_\$)] +$$

[26] The assumption $R < 1$ is critical. If the costless redemption is also riskless, then agents never reject banknotes even if banknotes are not tax-receivable. The reason is timing: agents can go to the bank between trade and taxation and convert a banknote into a tax-receivable dollar bill.

$$(1-\gamma)(m_0\Pi_{0\$} + m_\$\Pi_{\$\$})(U + V_0 - V_\$),$$

$$rV_{\$2} = -2c + (1-\gamma)(m_0\Pi_{0\$} + m_\$\Pi_{\$\$})(U + V_\$ - V_{\$2}). \tag{11}$$

In the first line of (8) a seller meets a barter partner who has either \$0 or \$1. The second line is a meeting with a buyer who has at least \$1. In (9) an agent with \$1 has three interesting matches. First, it may meet a barter partner. In this case, if it makes a sale it may be taxed and then it ends up with no money. Second, it may make a monetary sale and accumulate another dollar. Third, it may spend the money in shopping. In (10) the agent suffers the storage cost twice because it holds two dollar bills. It can only buy.

Proposition 6. (i) The government can guarantee that the non-monetary equilibrium does not exist iff $t > 0$. (ii) For some parameter values the government can guarantee the existence of a monetary equilibrium. (iii) The government cannot guarantee the existence of a pure monetary equilibrium.

Proof. (i) Set $T_0 = 0$. By setting $P = (U-C)/t$ as before, moneyless agents will not barter ($\pi_{00} = 0$). (ii) Also set $\Pi_{00} = 0$, $\Pi_{\$0} = \Pi_{0\$} = \Pi_{\$\$} = 1$. There is no closed-form solution because the distribution of money is too complicated. However, it can be verified that indeed $\pi_{\$0} = \pi_{0\$} = \pi_{\$\$} = 1$ for the following parameter values: $U = 5$, $C = 1$, $M_\$ = .5$, $r = .01$, $\gamma = .1$, $t = .25$, $y = .3$, $c = .01$. This means that agents always accept money, but they barter only if they already have money. (iii) No matter how high the punishment is, an agent who already has \$1 may still barter. The reason is apparent from (9): such an agent barters ($\pi_{\$0} = 1$) iff $U - C + t(V_0 - V_\$) > 0$. It cannot be punished. Although it does not earn money during this barter sale, it already has a dollar bill to begin with, so it can use that bill to pay the tax. □

These results obviously generalize to any larger upper bound on money holdings. For the same reason, the government cannot guarantee equilibria without other outside or inside moneys. An agent

who already holds a dollar bill may trade in any other way and pay that bill as a tax. On the other hand, the government can at least make sure that the moneyless agents accept dollar bills in sales. The result is that the government's favorite money still circulates, but not exclusively.

6 Endogenous Prices

Following Li and Wright (1998), I proceed by making goods divisible, while keeping everything else as in Section 3. Along with checking the robustness of the previous results, this also serves to show the robustness of the tax-foundation mechanism to money printing.

6.1 Robustness of Results

As in Li and Wright (1998) and similar papers, a private agent now derives utility $u(q) \geq 0$ from consuming a quantity q of one of its preferred goods, and the cost of producing a quantity q is q. Also $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all $q > 0$. In meetings between sellers the bargaining power is equal, so both sides produce the efficient q^* which satisfies $u'(q^*) = 1$. In meetings between buyers and sellers, the buyers (whether private or public) make take-it-or-leave-it offers. The quantity produced in all other matches is denoted Q. The only new notation, compared with the literature, is q^t, which is the quantity that may be produced for the tax collector after a barter meeting. Multiple money holdings is allowed only in the next subsection. The value functions are now

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0 \{u(q^*) - q^* + t[T_0(-q^t) + (1 - T_0)(-P)]\} + M_\$ \cdot \max_{\pi_\$} \pi_\$[-q + (1 - t)(V_\$ - V_0)], \quad (12)$$

$$rV_\$ = -c + s\Pi_\$[u(Q) + V_0 - V_\$]. \quad (13)$$

Assume that $q^t = V_\$ - V_0$. This makes the utility loss from tax identical for all taxpayers, regardless of how they pay.

Proposition 7. If all objects are tax-receivable, then: (i) the non-monetary equilibrium always exists. (ii) there are two monetary equilibria, which exist only for some parameter values. (iii) there is no pure monetary equilibrium.

Proof. (i) and (iii) are as in Proposition 1. (ii) Set $T_0 = \Pi_0 = \Pi_\$ = 1$. The bargaining rule implies $q = (1-t)(V_\$ - V_0)$, which is strictly positive iff $c < s\{u(Q) - y[u(q^*) - q^*]\}$. A buyer's offer is $q(Q) = \max\left[(1-t)\frac{s\{u(Q)-y[u(q^*)-q^*]\}-c}{r+s(1-yt)},\ 0\right]$. □

These equilibria are shown in Figure 1, where an equilibrium is any intersection of the offer with the 45 degrees line (i.e., $q(Q) = Q$).

The uniqueness result of Theorem 1 is replaced by something weaker.

Proposition 8. The government can guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff $t > 0$ and the money's storage cost is small enough.

Proof. Setting $P = [u(q^*) - q^*]/t$ and $T_0 = 0$ eliminates barter. Setting $\Pi_\$ = 1$ results in equilibrium iff $c < su(Q)$. A buyer's offer is $q(Q) = \max\left[(1-t)\frac{su(Q)-c}{r+s},\ 0\right]$. □

The offer has the same shape as in Figure 1.

6.2 Money Printing

I will now show that the government can maintain the circulation of its favorite money while increasing its supply and decreasing its value. Suppose that the government makes an unannounced, once-and-for-all money injection just before a trading round. The money supply doubles and is given in proportional transfers, so that all agents, including government buyers, either have \$0 or \$2.[27]

[27] Recall that throughout the paper agents are not physically prevented from holding more than \$1 at a time. Only a restriction on their production capabilities prevents such accumulation in most of the paper. I maintain the assumption

For $i \in \{\$, \$2\}$, let n_i be the probability that a seller meets a buyer who offers i (note that the buyer might hold \$2 and yet offer only \$1), let q_i be the quantity demanded by such a buyer, let Q_i be the quantity produced in all other matches for i, let Π_i be the probability that a random seller accepts a payment i, and let π_i be the best response of a seller who is offered a payment i. The value functions are

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0 \{u(q^*) - q^* + t[T_0(-q^t) + (1 - T_0)(-P)]\} + \tag{14}$$

$$n_\$ \cdot \max_{\pi_\$} \pi_\$[-q_\$ + (1-t)(V_\$ - V_0)] + n_{\$2} \cdot \max_{\pi_{\$2}} \pi_{\$2}[-q_{\$2} + (1-t)(V_{\$2} - V_0)],$$

$$rV_\$ = -c + s\Pi_\$[u(Q_\$) + V_0 - V_\$], \tag{15}$$

$$rV_{\$2} = -2c + s\{(1 - \Pi_\$)\Pi_{\$2}[u(Q_{\$2}) + V_0 - V_{\$2}] + \tag{16}$$

$$\Pi_\$(1 - \Pi_{\$2})[u(Q_\$) + V_\$ - V_{\$2}] + \Pi_\$\Pi_{\$2} \cdot \max[u(Q_{\$2}) + V_0 - V_{\$2},\ u(Q_\$) + V_\$ - V_{\$2}]\}.$$

In (13), in addition to the usual possibility of barter, there is also the possibility of meeting a buyer who offers \$1 for $q_\$$, and a buyer who offers \$2 for $q_{\$2}$. Meeting with a tax collector results in a total loss of that monetary income, whether it is \$1 or \$2 (in accordance with the tax law). In (15), the buyer might meet a seller who accepts only \$2, a seller who accepts only \$1, or a seller who accepts either \$1 or \$2. In the latter case, the buyer chooses how much to pay.

Theorem 4. The government can still guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff $t > 0$ and the money's storage cost is small enough. All payments are of

(relaxed in the previous section) that only moneyless agents can produce.

$2. The threshold storage cost is smaller than in Proposition 8. The price level is almost exactly twice as it was before the increase in the quantity of money.

Proof. Setting $P = [u(q^*) - q^*]/t$ and $T_0 = 0$ eliminates barter. Setting $\Pi_\$ = 0$ and $\Pi_{\$2} = 1$ results in equilibrium iff $c < su(Q)/2$. A buyer's offer is $q(Q) = \max\left[(1-t)\frac{su(Q)-2c}{r+s},\ 0\right]$. □

The only change is that after the increase in the quantity of money all buyers carry twice as much money. Every trade involves a payment of $2. Dollar bills need to be even lighter now to sustain the pure monetary equilibria. The added cost also affects the bargaining outcome and thus the curve is a bit lower than in Figure 1. Depending on which equilibrium is realized, the output per meeting can be slightly higher or lower than before. Given that almost the same quantity of real goods is traded for twice as much money, money is almost neutral. Of course, even these minor changes in allocations can be avoided if the government orders that every two dollar bills must be converted into one "new dollar" bill.

The important point is that guaranteeing the circulation of money is almost as easy with more money than with less money. Put differently, the transfer is neutral not only in terms of allocations, as could be expected, but also in terms of the strength of the monetary equilibria and the policy's viability.

Section 3 shows that the tax-foundation mechanism is similar to convertibility in its ability to guarantee *circulation*. However, it cannot guarantee the *value* of fiat money (Woodward 1995, p. 929); because unlike convertibility, the tax-foundation mechanism enables the money supply to be entirely discretionary. The government can print money at will, causing higher prices, higher nominal incomes—and thus higher *nominal* tax obligations. The price of immunity from the tax collectors' punishment increases proportionately, and therefore the money's acceptability is sustained.

While some see this "indeterminacy" of the price level as a fatal flaw for a theory of money (e.g., Ellis 1934, Klein 1974), abusive governments have realized that this "indeterminacy" is their ultimate victory: they can guarantee that their money is acceptable (as with convertibility), even though they can

print more of it at will. The tax-foundation mechanism allows a government to have its cake and eat it too. Therefore, "indeterminacy" of the price level is not grounds to ignore the tax-foundation theory, but rather a crucial reason for economists to understand it.

7 Complete Randomness

This section briefly outlines a model which is even closer to Li and Wright (1998). Here all matching is completely random, even with tax collectors. Taxation is therefore entirely random. This variant of the model is thus less realistic but is closer to standard monetary search models. It turns out that there is no change in the results.

The only difference from the environment of Section 3 is that there is no distinct class of tax collectors. Every government agent can be either a buyer or a tax collector. As before, there are m_g government buyers. At any period any one of the $1 - m_g$ money-less government agents becomes a tax collector with probability $\tau/(1 - m_g)$ and remains idle otherwise. This continues until he collects money. Then he becomes a buyer, and when he buys a good he consumes it and becomes money-less. Thus, at any period a fraction τ of government agents hold nothing and are tax collectors. The remaining government agents, a fraction $1 - \tau - m_g$, are idle. Here the probability that an agent one meets turns out to be a tax collector is $t \equiv \gamma\tau$.

The basic value functions, those analogous to (1) and (2), are now

$$rV_0 = \alpha sxy(U - C) + \alpha M_\$ x \cdot \max_{\pi_\$} \pi_\$(V_\$ - V_0 - C) + \alpha t[T_0(-C) + (1 - T_0)(-P)], \tag{17}$$

$$rV_\$ = -c + \alpha sx\Pi_\$(U + V_0 - V_\$) + \alpha t(V_0 - V_\$). \tag{18}$$

In (16) there are three interesting matching possibilities. A barter opportunity is always exploited because taxation is totally random and is not conditional on generating current income. Avoiding barter will not make a seller any more or less prone to taxation. A second possibility is meeting a buyer (private or public). A last possibility is meeting a tax collector. Having no money, this results, as usual, in either production or punishment. In (17) a buyer may be taxed because taxation is random and does not require current income.

Proposition 9. If all objects are tax-receivable, then: (i) the non-monetary equilibrium exists; (ii) the monetary equilibrium exists for some parameter values; (iii) a pure monetary equilibrium does not exist.

Proof. (i) Set $T_0 = 1$ and $\Pi_\$ = 0$. Then $\pi_\$ = 0$ iff $c + (1-\gamma)y(U-C) + rC \geq 0$, which always holds. (ii) Set $T_0 = \Pi_\$ = 1$. Then $\pi_\$ = 1$ iff $c < c_\exists$, where $c_\exists < (1-\gamma)(1-m_p^*)(1-y)(U-C) - rC$, and m_p^* solves $(1-\gamma)m_p^2 - (1-\gamma+\alpha t+M)m_p + M = 0$. (iii) As discussed above, barter is always conducted because it does not affect taxation. □

Theorem 5. The government can guarantee the *existence* and *uniqueness* of the monetary equilibrium iff $t > 0$.

Proof. Set $T_0 = 0$, $P = \frac{c+(1-\gamma)y(U-C)+rC}{\alpha t}$, and $\Pi_\$ = 1$.

For more details, see Goldberg (2006).

8 Commodity Money

In the sections above fiat money competed with barter, inside money and outside money. However, in quite a few cases fiat money had to compete with commodity money, which is quite a different challenge. Unlike the other alternatives, this one is both intrinsically valuable and can function as money. It is also quite a challenge to model commodity money. In fact, it requires a very different model from

the ones above. Here I briefly sketch a variant of a model which builds on the first monetary search model—Kiyotaki and Wright (1989).

Time is discrete. There are indivisible durable goods 1, 2, and 3. A fraction γ of the agents are government agents and the rest of the population has equal proportions of private agents 1, 2, and 3. As in Kiyotaki and Wright's "Model A," type i agents consume only good i and produce only good $i+1$ (mod 3). All agents can store only one unit of good j at a time with a cost c_j, where $c_3 > c_2 > c_1 > 0$. The discount factor is β. Let V_{ij} be the value function of a type i agent holding good j, and p_{ij} be the probability of meeting such an agent, with $i \in \{1,2,3\}$ and $j \in \{\$,1,2,3\}$. A fraction τ of the government agents collect taxes in any period. A fraction m_g of government agents hold fiat money and are buyers. The other government agents are idle as before.

Definition 5. A **monetary equilibrium** is an equilibrium in which all private agents always accept fiat money in trade, they use it to buy their consumption good, and they consume the latter.

I analyze here a fiat money extension of Kiyotaki and Wright's "fundamental equilibrium," in which agents accept commodity moneys only if these have lower storage costs than their own production goods. That is, $(U-C)+V_{12} > V_{10} > V_{12} > V_{13}$, $(U-C)+V_{23} > V_{20} > V_{21} > V_{23}$ and $(U-C)+V_{31} > V_{30} > V_{31} > V_{32}$.

Consider a type 1 agent. Given the others' strategies as described above, type 1's optimal strategy is described by the following equations:

$$V_{12} = -c_2 + \beta\frac{1-\gamma}{3}\left[V_{12} + p_{2\$}\max(V_{12}, V_{1\$}) + p_{21}(U - C + V_{12}) + p_{23}\max(V_{12}, V_{13}) + V_{12}\right] + \qquad (19)$$

$$\beta\gamma\left[m_g\max(V_{12}, V_{1\$}) + \tau(V_{12} - C - P_{12}) + (1 - m_g - \tau)V_{12}\right],$$

$$V_{13} = -c_3 + \beta\frac{1-\gamma}{3}\left[V_{13} + V_{13} + p_{3\$}\max(V_{13}, V_{1\$}) + p_{31}\max(V_{13}, U - C + V_{12})\right] + \tag{20}$$

$$\beta\gamma\left[m_g\max(V_{13}, V_{1\$}) + \tau(V_{12} - C - P_{13}) + (1 - m_g - \tau)V_{13}\right],$$

$$V_{1\$} = -c + \beta\frac{1-\gamma}{3}\left[V_{1\$} + (p_{2\$} + p_{3\$})V_{1\$} + (p_{21} + p_{31})\max(V_{1\$}, U - C + V_{12}) + p_{23}\max(V_{1\$}, V_{13})\right] + \tag{21}$$

$$\beta\gamma\left[m_g V_{1\$} + \tau(V_{12} - C) + (1 - m_g - \tau)V_{1\$}\right].$$

In (18) a holder of good 2 incurs a storage cost and later it is matched with a private agent with probability $(1-\gamma)/3$ for each type. If the holder meets any type 1 agent it never trades. If it meets a type 2 agent that holds fiat money it needs to decide whether to trade. If it meets a type 2 agent that holds good 1 then he trades, consumes, and produces a new unit of good 2. If it meets a type 2 agent that holds good 3 it needs to decide whether to trade. If it meets any type 3 agent (which always rejects good 2) then it cannot trade. With probability γ it is matched with a government agent. If it meets a government buyer it needs to decide whether to trade. If it meets a tax collector, it pays good 2 as a tax, incurs a punishment $P_{12} \geq 0$ and produces a new unit of good 2. The other equations have a similar interpretation.

One necessary condition for the existence of the monetary equilibrium is $V_{10} > V_{12}$. Assuming that type 1 plays the fundamental strategies with fiat money, like the other types, $V_{10} > V_{12}$ iff $P_{12} > \frac{c - c_2 - \beta\frac{1-\gamma}{3}p_{31}(U-C)}{\beta t}$. The condition for $(U - C) + V_{12} > V_{10}$ is $P_{12} < \frac{c - c_2 + \left[1 - \beta\frac{1-\gamma}{3}(2+p_{23})\right](U-C)}{\beta t}$. Given any P_{12}, the condition $V_{12} > V_{13}$ is satisfied iff

$$P_{13} > \frac{\left[\beta t P_{12} - \beta \frac{1-\gamma}{3}(p_{21}-p_{31})(U-C)+c_2-c_3\right]\left[1-\beta\frac{1-\gamma}{3}(2-p_{21})-\beta\gamma(1-m_g-\tau)\right]+\beta\frac{1-\gamma}{3}(p_{30}-p_{20})\left[\beta\frac{1-\gamma}{3}p_{21}(U-C)+c_3-c_0\right]}{\beta t\left[1-\beta\frac{1-\gamma}{3}(1+p_{23}+p_{30})-\beta\gamma(1-m_g-\tau)\right]}.$$

This almost completes the description of the necessary conditions of type 1 for the monetary equilibrium. Those of the other types are similar. In Goldberg (2002) I solve the necessary conditions of the other types and solve for all the p_{ij}'s in a numerical example. I also solve the non-monetary equilibrium and show how the government can rule it out. The implication is that in this environment too the government can guarantee the existence and uniqueness of the monetary equilibrium, as long as the expected number of tax collectors is strictly positive.

9 Monetary History

Casual evidence is clearly favorable to the hypothesis that there is a relation between what is declared legal tender and what circulates as money. Given political and economic stability, indeed the domestic legal tender is typically the general medium of exchange. Countries and currency unions seem to be able to change currency at will, as recently seen in Iraq and the European Union. As Goodhart (1998) notes, money tends to follow political unification and disintegration of federations (e.g., German and Soviet currencies, 1871-1991).

It is easy to find cases of fiat moneys which failed even though they were receivable for taxes. The first task of this section is to show that such cases can be explained *within* the model and the theory outlined above. Some economists are quick to cite many cases of fiat moneys that supposedly circulated without being receivable for taxes. If true, this would mean that the tax-foundation mechanism is not necessary in practice and perhaps contributes nothing. The second task of this section is to show that there is very little substance in such evidence. By and large, both the successes and failures of fiat moneys in reality seem to be related to the tax-foundation mechanism.

9.1 Failures of the Tax-Foundation Mechanism

The model developed above is helpful in explaining why sometimes an object is receivable for taxes and yet does not circulate.

9.1.1 Non-Unique Legal Tender

The Civil War's greenbacks were rejected in the West Coast (Mitchell 1903) even though they were legal tender for taxes. Their failure is consistent with the theory because gold was also legal tender. Proposition 1 shows that if all objects are receivable for tax payments ($T_i = 1\ \forall i$), a fiat money equilibrium always coexists with an equilibrium without fiat money. More recently, the American public has rejected the half-dollar coin, the Susan B. Anthony dollar, and the two-dollar bill. According to Proposition 4 if two types of fiat money have the same storage cost and both are receivable for taxes, then either one of them or both may circulate. Since people do not have to pay taxes in coins and notes of particular denominations, the failure of some denominations does not contradict the theory. In both cases, then, the model features multiple equilibria. Indeed, other evidence shows the exact opposite behavior: during the Revolutionary War gold was hoarded while the paper continental was used in trade (Calomiris 1988); and the recent Sacagawea golden dollar coins were happily received by sellers but were hoarded instead of being spent (Lotz and Rocheteau 2002).

9.1.2 Regime Transition

The observed correlation between a paper money's circulation and the existence of its issuing regime is highly positive but not perfect. Saddam Hussein's money circulated in Iraq half a year after his regime collapsed. On the other hand, Germans abandoned their paper money a few months before the Nazi surrender (Einzig 1966, p. 299). These observations are not inconsistent with the theory. All the propositions and theorems above that show the government's power to promote its favorite currency

are careful to include the condition $t > 0$, where $t \equiv \tau G$ is the expected probability of being taxed in the future by a government that accepts this currency (recall that τ is the probabilistic tax rate and G is the government's survival probability). In such episodes of extreme political instability it is not the current policy of the current government that matters, but the subjective expectations regarding the future government and its policies (Goodhart 1998, King 2004). Iraqis expected the Coalition Provisional Authority to convert Hussein's money into its new legal tender, so there was no reason to reject it. Russians' acceptance of the dead czar's money during the chaotic hyperinflation of the 1920s (Friedman 1992, p. 11-12) can also be explained by a belief that whoever ends up in power would either convert that money or accept it in various payments. The Germans' premature abandonment of their money can reflect expectations that the Allies would treat them harshly and not conduct such a conversion[28].

9.1.3 Ineffective Tax System

The mechanism may fail if the tax system is not functioning (Wray 1998, p. 36), i.e., $\tau = 0$. This may be the case even if the government is expected to remain in power. This is particularly true for new regimes because it takes them time to construct effective tax systems that will detect and punish tax offenders severely enough. This may explain the failure of Japanese money right after Japan occupied Taiwan in 1895 (Li and Wright 1998), and the problems of new fiat moneys in disintegrated federations where tax collection was not important beforehand—the Confederacy during the American Civil War (Lerner 1956) and formerly communist countries. Recalling the similarity to convertibility, it is similar to a situation where the Treasury's gold holdings are lost or expected to be lost. With no immunities to sell to the taxpayers, the fiat money becomes unbacked.

The government may denominate the tax in one unit of account and accept another money as well.

[28] Alternatively, perhaps they expected to be treated the way they treated some occupied peoples. It is well known what monetary theory predicts regarding fiat money in finite horizon economies.

If it accepts the other money according to market value rather than at a pre-determined exchange rate, there is nothing to prevent the collapse of the other money, as shown in Proposition 3. An equilibrium in which both the market and the tax authority see that money as valueless always exists. This actually happened with the continental currency of the American Revolution (Calomiris 1988, p. 59) and the mandat currency of the French Revolution (Nussbaum 1950, p. 50).

9.1.4 Non-exclusive Circulation

In many countries the legal tender currency circulates side by side with foreign currency. It is especially common in periods of high inflation, in which people prefer a foreign, stable currency whenever possible. As shown in Proposition 6, when agents can hold more than $1, sometimes they might choose to conduct transactions in other ways. They may do so as long as they already hold the minimum they need for tax payments in the domestic currency. This is an important constraint on the government's power and the extent to which it can encourage the use of its favorite money in trade. However, it does not completely contradict the theory. At least occasionally agents do demand the domestic currency because they need it for tax payments. It is probably this demand that keeps the price level finite even after people realize that their government is bent on hyperinflation.

9.2 Is Tax Receivability Necessary?

The high positive correlation between the existence of regimes and the success of their currencies raises an intriguing possibility. Perhaps some government intervention is *necessary* to support the value of fiat money. If a certain type of paper money is neither supported by convertibility nor tyrannically forced on all transactions, does it have to be receivable for taxes in order to circulate? Standard monetary models say that this is not the case: pure expectations can sustain the circulation of any fiat money. However, as Prescott and Rios-Rull (2005) show, such monetary equilibria collapse once any agent can issue his own

fiat money. And what prevents each one of us, in reality, from starting our own system of fiat money?

It is very easy to find references in the literature to currencies that were supposedly intrinsically useless, inconvertible, and not legal tender. A detailed discussion is beyond the scope of this paper (see Goldberg 2007). Here I briefly explain why there is little substance in such popular claims. Bank deposits and checks are not legal tender and are not convertible into goods or services. However, they are convertible by law into some legal tender. As shown in Subsection 4.2 the tax authority may or may not encourage the use of such inside money in trade and tax payments. Similar to checks are modern private banknotes in Scotland and Northern Ireland, which have never been legal tender. They too are legally convertible into some legal tender (Bank of England notes). Private, non-bank local moneys such as the Ithaca HOURS have been either convertible into the issuers' goods and services (in which case they are not fiat money), or convertible into the domestic legal tender. One rare exception is the recent system of *creditos* in Argentina. Hundreds of such private moneys were issued, typically with no promise of convertibility. They circulated briefly before they all collapsed. This fast collapse and the large number of issuers are more consistent with the aforementioned Prescott-Rios-Rull result than with standard monetary models' prediction that fiat money can circulate based on pure expectations.

Among government-issued currencies there are many false examples. The paper moneys of most American colonies, Bank of England notes during the Napoleonic Wars, and notes issued by towns in the U.S. in the nineteenth century, have been cited as not having a legal tender status. The truth is that these currencies were legal tender for taxes but not for contractual debts. That is, their success is clearly explained by the tax-foundation theory. Similarly, Confederate money of the American Civil War, mentioned in the opening quote of this paper, was legal tender for taxes and for debts to banks, but not legal tender for other contractual debts. Federal Reserve notes had been legal tender only for taxes while they were convertible. Recently, Governor of the Bank of England Mervyn King (2004) mentioned that Hussein's older money continued circulating in Kurdish Iraq from 1993 to 2003, even

though its legal tender status had been revoked by Hussein. However, King's claim that this money was no longer supported by *any* government is incorrect. The money was declared legal tender by the Kurdistan Regional Government that actually controlled the area[29].

Another common claim is that traditional societies have used intrinsically useless, inconvertible, non-paper objects as money. The most famous examples are the stone money of the Island of Yap and seashells. As I prove in Goldberg (2005), such claims ignore (unwritten) local laws and religion, and falsely assume that if an object seems intrinsically useless to a modern Western person then it was also considered as such by the natives.

It seems therefore, that if a currency has no other anchor (intrinsic value, convertibility, tyrannical forced usage in all commercial transactions), it can circulate in the long run only if it is receivable for taxes. Of course, a scientific theory has to be refutable. The challenge here for the theory's opponents is to find an example of a currency that circulated for a long time without having *any* such anchor. As far as I know, no such examples exist. The U.S. Congress actually tried something like that. In the antebellum period it authorized the Mint to issue copper coins but refused to grant them any other legal status. Not surprisingly, the public indeed rejected this money (Carothers 1930).

10 Conclusion

In the real world, indeed, there is more to government-issued fiat money than intrinsic uselessness and inconvertibility: its acceptance in tax payments is guaranteed. Generally, no other objects can be forced on the tax authority. The main contribution of this paper is the use of a monetary search model to prove in a dynamic economy that receivability for taxes can make fiat money the general medium of exchange. Other contributions are comparison of this mechanism to convertibility and exploration of its limitations.

[29] Private communication with Mr. Nijyar Shemdin, the U.S. Representative of the Kurdistan Regional Government, 01/12/2004.

There are many ways for a government to promote the circulation of its paper money. The ideal way would assure money-holders that their money could be put to good use, be robust to inflation in the sense that the money will not be completely abandoned, and also be easy and cheap to implement. The tax-foundation mechanism could very well prove ideal. Its assurance that the money can be put to good use was shown to be somewhat equivalent to convertibility. Its robustness to inflation was also shown: circulation can be maintained even while prices increase. As for implementation, this method is also the cheapest. There is no need to obtain and store gold. There is no need to monitor market transactions. There is no need to conduct searches for illegal currencies. Given that the government collects taxes in any case, it can easily promote any money simply by insisting on accepting only that money.

It is easy to find historical cases in which a government supported its money by accepting it for taxes and yet the money failed to circulate. These cases are perfectly consistent with the model presented here. For example, why is it hard to get $1 coins to circulate in the U.S.? After all, they are legal tender for tax payments. The answer is that they are not the *only* legal tender. We can reject these coins and still pay our taxes, so their circulation cannot be guaranteed (Section 4.1). Another potential concern is that near political transitions it often happens that legal tender currencies are abandoned a bit too early or too late. This is also consistent with the model. In all its variants, circulation can be guaranteed only if there is a *subjective* expectation that there will be enough tax collectors in the future who will accept that particular currency. How about the fact that checks and credit cards, which are not legal tender, are used more than cash for shopping and tax payments? This is also consistent with the theory. The tax authority agrees to accept them and so an equilibrium with all the cash in the bank is entirely feasible (Section 4.2). Finally, during hyperinflation people reduce their holdings of the inflated currency to the bare minimum and trade in other ways. The theory explains that as well (Section 5). For a much more detailed analysis of the historical evidence, see Goldberg (2007).

In fact, this theory explains that bare minimum that people continue to hold during hyperinflation.

It is that bare minimum that makes inflationary finance successful at all. As noted by one of the most successful inflation fighters of recent history, Michael Bruno, it is the money needed for tax payments that prevents an immediate abandonment of money as soon as its issuer's intentions become apparent (Bruno 1993, p. 8). Future work with more sophisticated models will need to examine how that minimal money holding (i.e., money demand) affects the rate of inflation.

References

Aiyagari, S. R., and N. Wallace, 1997, Government Transaction Policy, the Medium of Exchange, and Welfare, Journal of Economic Theory 74, 1-18.

Auerbach. A.J., Kotlikoff, L.J. 1998. Macroeconomics: An Integrated Approach. Second edition. Cambridge: MIT Press.

Bank of Canada. 2008. Bank Notes: Law enforcement. Available at

http://www.bankofcanada.ca/en/banknotes/law.html

Bank of England. 2008. Frequently Asked Questions. Available at

http://www.bankofengland.co.uk/banknotes/about/faqs.htm

Barro, R.J. 1993. Macroeconomics. Fourth edition. New York: John Wiley and Sons.

Board of Governors of the Federal Reserve System. 2008. "Frequently Asked Questions." Available at http://www.federalreserve.gov/generalinfo/faq/faqcur.htm#2

Brock, L. V., 1975, The Currency of the American Colonies, 1700-1764 (Arno Press, New York).

Bruno, M., 1993, Crisis, Stabilization, and Economic Reform (Clarendon Press, Oxford).

Calomiris, C. W., 1988, Institutional Failure, Monetary Scarcity, and the Depreciation of the Continental, Journal of Economic History 48, 47-68.

Carothers, N., 1930, Fractional Money (John Wiley & Sons, New York).

Case K.E., Fair, R.C. 2003. Principles of Economics. Upper Saddle River: Prentice Hall.

Cowen, T., and R. Kroszner, 1994, Explorations in the New Monetary Economics (Blackwell, Cambridge).

Einzig, P., 1966, Primitive Money (Pergamon, Oxford).

Ellis, H. S., 1934, German Monetary Theory: 1905-1933 (Harvard University Press, Cambridge).

European Union. 1998. Council Regulation No. 974/98. Available at

http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:31998R0974:EN:HTML

European Union. 2005. Council Regulation No. 2169/2005. Available at

http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2005:346:0001:01:EN:HTML

Forstater, M., 2006, Tax-Driven Money: Additional Evidence from the History of Thought, Economic History, and Economic Policy, in: M. Setterfield, ed., Complexity, Endogenous Money, and Macroeconomic Theory (Edward Elgar, Cheltenham).

Freeman, S. 1996. Clearinghouse Banks and Banknote Overissue. Journal of Monetary Economics 38: 101-115.

Friedman, M., 1992, Money Mischief (Harcourt Brace Jovanovich, New York).

Goldberg, D., 2002, On the Implicit Convertibility of Fiat Money, Chapter 1 in Search and Money, Ph.D. Dissertation, University of Rochester. Available online at

http://www.drorgoldberg.com/dissertation_chapter1.pdf

———, 2005, Famous Myths of "Fiat Money," Journal of Money, Credit, and Banking 37, 957-967.

———, 2006, The Implicit Convertibility of Fiat Money: The Random Case, working paper, Texas A&M University. Available online at http://www.drorgoldberg.com/tax-random.pdf

———, 2007, The Legal Status of Modern Currency, working paper, Texas A&M University. Available online at http://econweb.tamu.edu/dgoldberg/research/legal.pdf

———, 2009, The Massachusetts Paper Money of 1690, Journal of Economic History 69, 1092-1106.

Goodhart, C. A. E., 1998, The Two Concepts of Money: Implications for the Analysis of Optimal

Currency Areas, European Journal of Political Economy 14, 407-432.

Grossman, H. I., 1991, Monetary Economics—A Review Essay, Journal of Monetary Economics 28, 323-345.

King, M., 2004, Ely Lecture: The Institutions of Monetary Policy, American Economic Review 94, 1-13.

Kiyotaki, N, and R. Wright, 1989, On Money as a Medium of Exchange, Journal of Political Economy 97, 927-954.

Klein, B., 1974, The Competitive Supply of Money, Journal of Money, Credit, and Banking 6, 423-453.

Knapp, G. F., 1905, The State Theory of Money. English edition, 1924 (Macmillan, London).

Lerner, A. P., 1947, Money as a Creature of the State, American Economic Review, Papers and Proceedings of the American Economic Association 37, 312-317.

Lerner, E., 1956, Inflation in the Confederacy, 1861-1865, in: M. Friedman, ed., Studies in the Quantity Theory of Money (University of Chicago Press, Chicago).

Li, Y. and R. Wright, 1998, Government Transactions Policy, Media of Exchange and Prices, Journal of Economic Theory 81, 290-313.

Lipsey, R.G., Courant, P.N., Ragan, C.T.S. 1999. Economics. Reading: Addison-Wesley.

Lotz, S. and G. Rocheteau, 2002, On the Launching of a New Currency, Journal of Money, Credit, and Banking 34, 563-588.

Lotz, S. 2004. Introducing a New Currency: Government Policy and Prices. European Economic Review 48: 959-982.

Mankiw, N.G. 2000. Macroeconomics. Fourth edition. New York: Worth Publishers.

Mann, F.A. 1982. The Legal Aspect of Money. Fourth edition. Oxford: Clarendon Press.

Mitchell, W. C., 1903, A History of the Greenbacks (University of Chicago Press, Chicago).

Nussbaum, A. 1950. Money in the Law: National and International. Brooklyn: Foundation Press.

Prescott, E. C., and Rios-Rull, J. V., 2005, On the Equilibrium Concept for Overlapping Generations Economies, International Economic Review 46, 1065-1080.

Reserve Bank of Australia. 2008. Legal Tender. Available at

http://www.rba.gov.au/CurrencyNotes/LegalFramework/legal_tender.html

Sargent, T. J., 1982, The Ends of Four Big Inflations, in: R. E. Hall, ed., Inflation: Causes and Effects (University of Chicago Press, Chicago).

Sargent, T.J., Velde, F.R. 2002. The Big Problem of Small Change. Princeton University Press, Princeton.

Selgin, G. A., 2003, Adaptive Learning and the Transition to Fiat Money, Economic Journal 113, 147-165.

Shi, S., 2005, Nominal Bonds and Interest Rates, International Economic Review 46, 579-612.

Shy, O., Tarkka, J. 2002. "The Market for Electronic Cash Cards." Journal of Money, Credit, and Banking 34: 299-314.

Smith, A., 1776, The Wealth of Nations. 1952 edition (William Benton, Chicago).

Smith, B. D., 1985, Some Colonial Evidence on Two Theories of Money: Maryland and the Carolinas, Journal of Political Economy 93, 1178-1211.

Soller-Curtis, E., and C. J. Waller, 2000, A search-theoretic Model of Legal and Illegal Currency, Journal of Monetary Economics 45, 155-184.

Starr, R. M., 1974, The Price of Money in a Pure Exchange Monetary Economy with Taxation, Econometrica 42, 45-54.

———, 2003, Why is There Money? Endogenous Derivation of 'Money' as the Most Liquid Asset: A Class of Examples, Economic Theory 21, 455-474.

Sussman, N., Zeira, J. 2003. "Commodity Money Inflation: Theory and Evidence from France 1350-1436." Journal of Monetary Economics 50: 1769-1793.

United Kingdom, Ministry of Justice. 2008. Currency Act 1983. Available at http://www.statutelaw.gov.uk

United States. 2003. United States Code Annotated. St. Paul: West Publishing.

von Glahn, R., 1996, Fountain of Fortune: Money and Monetary Policy in China, 1000-1700 (University of Carolina Press, Berkeley).

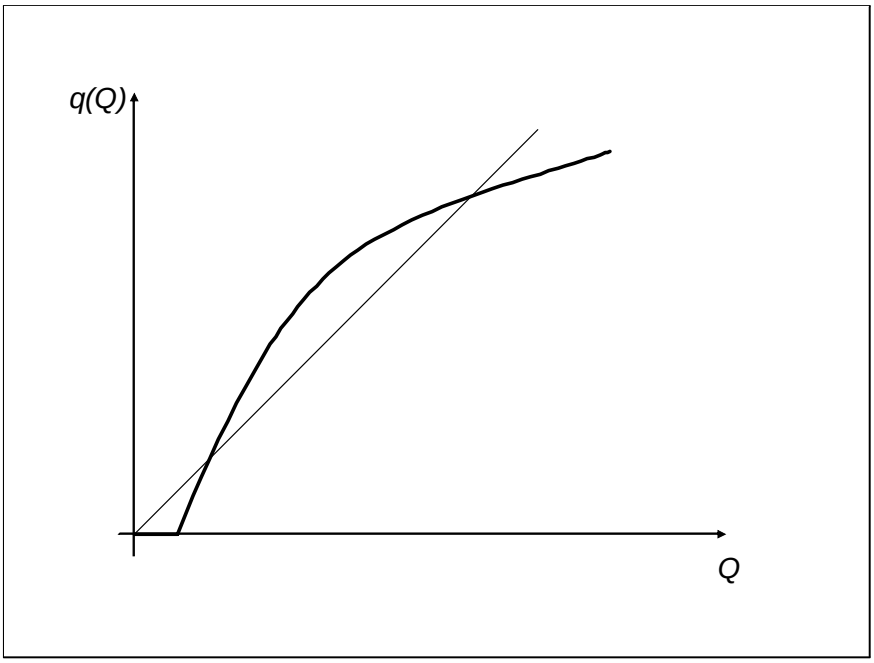
Wallace, N., 1981, A Modigliani-Miller Theorem for Open Market Operations, American Economic Review 71, 267-274.

Williston, S. 2003. A Treatise on the Law of Contracts. Fourth edition, by R.A. Lord. Rochester: Lawyers Cooperative.

Woodward, G. T., 1995, Interest-Bearing Currency: Evidence from the Civil War Experience: Comment. Journal of Money, Credit, and Banking 27, 927-937.

Wray, L. R., 1998, Understanding Modern Money (Edward Elgar, Cheltenham).

Figure 1: Equilibria with Endogenous Prices



Bar-Ilan University
Department of Economics
WORKING PAPERS

1-01 **The Optimal Size for a Minority**

Hillel Rapoport and Avi Weiss, January 2001.

2-01 **An Application of a Switching Regimes Regression to the Study of Urban Structure**

Gershon Alperovich and Joseph Deutsch, January 2001.

3-01 **The Kuznets Curve and the Impact of Various Income Sources on the Link Between Inequality and Development**

Joseph Deutsch and Jacques Silber, February 2001.

4-01 **International Asset Allocation: A New Perspective**

Abraham Lioui and Patrice Poncet**,** February 2001.

5-01 **מודל המועדון והקהילה החרדית**

יעקב רוזנברג, פברואר 2001.

6-01 **Multi-Generation Model of Immigrant Earnings: Theory and Application**

Gil S. Epstein and Tikva Lecker, February 2001.

7-01 **Shattered Rails, Ruined Credit: Financial Fragility and Railroad Operations in the Great Depression**

Daniel A. Schiffman, February 2001.

8-01 **Cooperation and Competition in a Duopoly R&D Market**

Damiano Bruno Silipo and Avi Weiss, March 2001.

9-01 **A Theory of Immigration Amnesties**

Gil S. Epstein and Avi Weiss, April 2001.

10-01 **Dynamic Asset Pricing With Non-Redundant Forwards**

Abraham Lioui and Patrice Poncet, May 2001.

11-01 **Macroeconomic and Labor Market Impact of Russian Immigration in Israel**

Sarit Cohen and Chang-Tai Hsieh, May 2001.

Electronic versions of the papers are available at
http://www.biu.ac.il/soc/ec/wp/working_papers.html

12-01 **Network Topology and the Efficiency of Equilibrium**

Igal Milchtaich, June 2001.

13-01 **General Equilibrium Pricing of Trading Strategy Risk**

Abraham Lioui and Patrice Poncet, July 2001.

14-01 **Social Conformity and Child Labor**

Shirit Katav-Herz, July 2001.

15-01 **Determinants of Railroad Capital Structure, 1830–1885**

Daniel A. Schiffman, July 2001.

16-01 **Political-Legal Institutions and the Railroad Financing Mix, 1885–1929**

Daniel A. Schiffman, September 2001.

17-01 **Macroeconomic Instability, Migration, and the Option Value of Education**

Eliakim Katz and Hillel Rapoport, October 2001.

18-01 **Property Rights, Theft, and Efficiency: The Biblical Waiver of Fines in the Case of Confessed Theft**

Eliakim Katz and Jacob Rosenberg, November 2001.

19-01 **Ethnic Discrimination and the Migration of Skilled Labor**

Frédéric Docquier and Hillel Rapoport, December 2001.

1-02 **Can Vocational Education Improve the Wages of Minorities and Disadvantaged Groups? The Case of Israel**

Shoshana Neuman and Adrian Ziderman, February 2002.

2-02 **What Can the Price Gap between Branded and Private Label Products Tell Us about Markups?**

Robert Barsky, Mark Bergen, Shantanu Dutta, and Daniel Levy, March 2002.

3-02 **Holiday Price Rigidity and Cost of Price Adjustment**

Daniel Levy, Georg Müller, Shantanu Dutta, and Mark Bergen, March 2002.

4-02 **Computation of Completely Mixed Equilibrium Payoffs**

Igal Milchtaich, March 2002.

5-02 **Coordination and Critical Mass in a Network Market – An Experimental Evaluation**

Amir Etziony and Avi Weiss, March 2002.

6-02 **Inviting Competition to Achieve Critical Mass**

Amir Etziony and Avi Weiss, April 2002.

7-02 **Credibility, Pre-Production and Inviting Competition in a Network Market**

Amir Etziony and Avi Weiss, April 2002.

8-02 **Brain Drain and LDCs' Growth: Winners and Losers**

Michel Beine, Fréderic Docquier, and Hillel Rapoport, April 2002.

9-02 **Heterogeneity in Price Rigidity: Evidence from a Case Study Using Micro-Level Data**

Daniel Levy, Shantanu Dutta, and Mark Bergen, April 2002.

10-02 **Price Flexibility in Channels of Distribution: Evidence from Scanner Data**

Shantanu Dutta, Mark Bergen, and Daniel Levy, April 2002.

11-02 **Acquired Cooperation in Finite-Horizon Dynamic Games**

Igal Milchtaich and Avi Weiss, April 2002.

12-02 **Cointegration in Frequency Domain**

Daniel Levy, May 2002.

13-02 **Which Voting Rules Elicit Informative Voting?**

Ruth Ben-Yashar and Igal Milchtaich, May 2002.

14-02 **Fertility, Non-Altruism and Economic Growth: Industrialization in the Nineteenth Century**

Elise S. Brezis, October 2002.

15-02 **Changes in the Recruitment and Education of the Power Elitesin Twentieth Century Western Democracies**

Elise S. Brezis and François Crouzet, November 2002.

16-02 **On the Typical Spectral Shape of an Economic Variable**

Daniel Levy and Hashem Dezhbakhsh, December 2002.

17-02 **International Evidence on Output Fluctuation and Shock Persistence**

Daniel Levy and Hashem Dezhbakhsh, December 2002.

1-03 **Topological Conditions for Uniqueness of Equilibrium in Networks**

Igal Milchtaich, March 2003.

2-03 **Is the Feldstein-Horioka Puzzle Really a Puzzle?**

Daniel Levy, June 2003.

3-03 **Growth and Convergence across the US: Evidence from County-Level Data**

Matthew Higgins, Daniel Levy, and Andrew Young, June 2003.

4-03 **Economic Growth and Endogenous Intergenerational Altruism**

Hillel Rapoport and Jean-Pierre Vidal, June 2003.

5-03 **Remittances and Inequality: A Dynamic Migration Model**

Frédéric Docquier and Hillel Rapoport, June 2003.

6-03 **Sigma Convergence Versus Beta Convergence: Evidence from U.S. County-Level Data**

Andrew T. Young, Matthew J. Higgins, and Daniel Levy, September 2003.

7-03 **Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets**

Mark J. Zbaracki, Mark Ritson, Daniel Levy, Shantanu Dutta, and Mark Bergen, September 2003.

8-03 **First and Second Best Voting Rules in Committees**

Ruth Ben-Yashar and Igal Milchtaich, October 2003.

9-03 **Shattering the Myth of Costless Price Changes: Emerging Perspectives on Dynamic Pricing**

Mark Bergen, Shantanu Dutta, Daniel Levy, Mark Ritson, and Mark J. Zbaracki, November 2003.

1-04 **Heterogeneity in Convergence Rates and Income Determination across U.S. States: Evidence from County-Level Data**

Andrew T. Young, Matthew J. Higgins, and Daniel Levy, January 2004.

2-04 **"The Real Thing:" Nominal Price Rigidity of the Nickel Coke, 1886-1959**

Daniel Levy and Andrew T. Young, February 2004.

3-04 **Network Effects and the Dynamics of Migration and Inequality: Theory and Evidence from Mexico**

David Mckenzie and Hillel Rapoport, March 2004.

4-04 **Migration Selectivity and the Evolution of Spatial Inequality**

Ravi Kanbur and Hillel Rapoport, March 2004.

5-04 **Many Types of Human Capital and Many Roles in U.S. Growth: Evidence from County-Level Educational Attainment Data**

Andrew T. Young, Daniel Levy and Matthew J. Higgins, March 2004.

6-04 **When Little Things Mean a Lot: On the Inefficiency of Item Pricing Laws**

Mark Bergen, Daniel Levy, Sourav Ray, Paul H. Rubin and Benjamin Zeliger, May 2004.

7-04 **Comparative Statics of Altruism and Spite**

Igal Milchtaich, June 2004.

8-04 **Asymmetric Price Adjustment in the Small: An Implication of Rational Inattention**

Daniel Levy, Haipeng (Allan) Chen, Sourav Ray and Mark Bergen, July 2004.

1-05 **Private Label Price Rigidity during Holiday Periods**

Georg Müller, Mark Bergen, Shantanu Dutta and Daniel Levy, March 2005.

2-05 **Asymmetric Wholesale Pricing: Theory and Evidence**

Sourav Ray, Haipeng (Allan) Chen, Mark Bergen and Daniel Levy, March 2005.

3-05 **Beyond the Cost of Price Adjustment: Investments in Pricing Capital**

Mark Zbaracki, Mark Bergen, Shantanu Dutta, Daniel Levy and Mark Ritson, May 2005.

4-05 **Explicit Evidence on an Implicit Contract**

Andrew T. Young and Daniel Levy, June 2005.

5-05 **Popular Perceptions and Political Economy in the Contrived World of Harry Potter**

Avichai Snir and Daniel Levy, September 2005.

6-05 **Growth and Convergence across the US: Evidence from County-Level Data (revised version)**

Matthew J. Higgins, Daniel Levy, and Andrew T. Young , September 2005.

1-06 **Sigma Convergence Versus Beta Convergence: Evidence from U.S. County-Level Data (revised version)**

Andrew T. Young, Matthew J. Higgins, and Daniel Levy, June 2006.

2-06 **Price Rigidity and Flexibility: Recent Theoretical Developments**

Daniel Levy, September 2006.

3-06 **The Anatomy of a Price Cut: Discovering Organizational Sources of the Costs of Price Adjustment**

Mark J. Zbaracki, Mark Bergen, and Daniel Levy, September 2006.

4-06 **Holiday Non-Price Rigidity and Cost of Adjustment**

Georg Müller, Mark Bergen, Shantanu Dutta, and Daniel Levy. September 2006.

2008-01 **Weighted Congestion Games With Separable Preferences**

Igal Milchtaich, October 2008.

2008-02 **Federal, State, and Local Governments: Evaluating their Separate Roles in US Growth**

Andrew T. Young, Daniel Levy, and Matthew J. Higgins, December 2008.

2008-03 **Political Profit and the Invention of Modern Currency**

Dror Goldberg, December 2008.

2008-04 **Static Stability in Games**

Igal Milchtaich, December 2008.

2008-05 **Comparative Statics of Altruism and Spite**

Igal Milchtaich, December 2008.

2008-06 **Abortion and Human Capital Accumulation: A Contribution to the Understanding of the Gender Gap in Education**

Leonid V. Azarnert, December 2008.

2008-07 **Involuntary Integration in Public Education, Fertility and Human Capital**

Leonid V. Azarnert, December 2008.

2009-01 **Inter-Ethnic Redistribution and Human Capital Investments**

Leonid V. Azarnert, January 2009.

2009-02 **Group Specific Public Goods, Orchestration of Interest Groups and Free Riding**

Gil S. Epstein and Yosef Mealem, January 2009.

2009-03 **Holiday Price Rigidity and Cost of Price Adjustment**

Daniel Levy, Haipeng Chen, Georg Müller, Shantanu Dutta, and Mark Bergen, February 2009.

2009-04 **Legal Tender**

Dror Goldberg, April 2009.

2009-05 **The Tax-Foundation Theory of Fiat Money**

Dror Goldberg, April 2009.

2009-06 **The Inventions and Diffusion of Hyperinflatable Currency**

Dror Goldberg, April 2009.

2009-07 **The Rise and Fall of America's First Bank**

Dror Goldberg, April 2009.

2009-08 **Judicial Independence and the Validity of Controverted Elections**

Raphaël Franck, April 2009.

2009-09 **A General Index of Inherent Risk**

Adi Schnytzer and Sara Westreich, April 2009.

2009-10 **Measuring the Extent of Inside Trading in Horse Betting Markets**

Adi Schnytzer, Martien Lamers and Vasiliki Makropoulou, April 2009.

2009-11 **The Impact of Insider Trading on Forecasting in a Bookmakers' Horse Betting Market**

Adi Schnytzer, Martien Lamers and Vasiliki Makropoulou, April 2009.

2009-12 **Foreign Aid, Fertility and Population Growth: Evidence from Africa**

Leonid V. Azarnert, April 2009.

2009-13 **A Reevaluation of the Role of Family in Immigrants' Labor Market Activity: Evidence from a Comparison of Single and Married Immigrants**

Sarit Cohen-Goldner, Chemi Gotlibovski and Nava Kahana, May 2009.

2009-14 **The Efficient and Fair Approval of "Multiple-Cost–Single-Benefit" Projects Under Unilateral Information**

Nava Kahanaa, Yosef Mealem and Shmuel Nitzan, May 2009.

2009-15 **Après nous le Déluge: Fertility and the Intensity of Struggle against Immigration**

Leonid V. Azarnert, June 2009.

2009-16 **Is Specialization Desirable in Committee Decision Making?**

Ruth Ben-Yashar, Winston T.H. Koh and Shmuel Nitzan, June 2009.

2009-17 **Framing-Based Choice: A Model of Decision-Making Under Risk**

Kobi Kriesler and Shmuel Nitzan, June 2009.

2009-18 **Demystifying the 'Metric Approach to Social Compromise with the Unanimity Criterion'**

Shmuel Nitzan, June 2009.

2009-19 **On the Robustness of Brain Gain Estimates**

Michel Beine, Frédéric Docquier and Hillel Rapoport, July 2009.

2009-20 **Wage Mobility in Israel: The Effect of Sectoral Concentration**

Ana Rute Cardoso, Shoshana Neuman and Adrian Ziderman, July 2009.

2009-21 **Intermittent Employment: Work Histories of Israeli Men and Women, 1983–1995**

Shoshana Neuman and Adrian Ziderman, July 2009.

2009-22 **National Aggregates and Individual Disaffiliation: An International Study**

Pablo Brañas-Garza, Teresa García-Muñoz and Shoshana Neuman, July 2009.

2009-23 **The Big Carrot: High-Stakes Incentives Revisited**

Pablo Brañas-Garza, Teresa García-Muñoz and Shoshana Neuman, July 2009.

2009-24 **The Why, When and How of Immigration Amnesties**

Gil S. Epstein and Avi Weiss, September 2009.

2009-25 **Documenting the Brain Drain of «la Crème de la Crème»: Three Case-Studies on International Migration at the Upper Tail of the Education Distribution**

Frédéric Docquier and Hillel Rapoport, October 2009.

2009-26 **Remittances and the Brain Drain Revisited: The Microdata Show That More Educated Migrants Remit More**

Albert Bollard, David McKenzie, Melanie Morten and Hillel Rapoport, October 2009.

2009-27 **Implementability of Correlated and Communication Equilibrium Outcomes in Incomplete Information Games**

Igal Milchtaich, November 2009.

2010-01 **The Ultimatum Game and Expected Utility Maximization – In View of Attachment Theory**

Shaul Almakias and Avi Weiss, January 2010.

2010-02 **A Model of Fault Allocation in Contract Law – Moving From Dividing Liability to Dividing Costs**

Osnat Jacobi and Avi Weiss, January 2010.

2010-03 **Coordination and Critical Mass in a Network Market: An Experimental Investigation**

Bradley J. Ruffle, Avi Weiss and Amir Etziony, February 2010.

2010-04 **Immigration, fertility and human capital: A model of economic decline of the West**

Leonid V. Azarnert, April 2010.

2010-05 **Is Skilled Immigration Always Good for Growth in the Receiving Economy?**

Leonid V. Azarnert, April 2010.

2010-06 **The Effect of Limited Search Ability on the Quality of Competitive Rent-Seeking Clubs**

Shmuel Nitzan and Kobi Kriesler, April 2010.

2010-07 **Condorcet vs. Borda in Light of a Dual Majoritarian Approach**

Eyal Baharad and Shmuel Nitzan, April 2010.

2010-08 **Prize Sharing in Collective Contests**

Shmuel Nitzan and Kaoru Ueda, April 2010.

2010-09 **Network Topology and Equilibrium Existence in Weighted Network Congestion Games**

Igal Milchtaich, May 2010.

2010-10 **The Evolution of Secularization: Cultural Transmission, Religion and Fertility Theory, Simulations and Evidence**

Ronen Bar-El, Teresa García-Muñoz, Shoshana Neuman and Yossef Tobol, June 2010.

2010-11 **The Economics of Collective Brands**

Arthur Fishman, Israel Finkelstein, Avi Simhon and Nira Yacouel, July 2010.