Economics Bulletin

Volume 30, Issue 1

On the interpretation of the WTP/WTA gap as imprecise utility: an axiomatic analysis

Raphaël Giraud
CRESE-University of Franche-Comté

Abstract

The willingness-to-pay (WTP) and willingness-to-accept (WTA) disparity reported in a rich empirical literature suggests that people have only an imprecise idea of how valuable a good is to them. In this note, we provide axioms that formally relate this imprecision in the evaluation of a good to the imprecision in the utility function, in the sense that x is strictly preferred to y iff the WTP for x is larger than the WTA for y. The preference relation is therefore an interval order (Fishburn (1970)) with ``interval utility' equal to the WTP/WTA interval itself. Applications to preference for liquidity and the strength of the status quo bias are given.

I thank audiences of the FUR 2008 conference in Barcelona and a referee for helpful comments. Funding from ANR project Riskattitude is gratefully acknowledged.
Citation: Raphaël Giraud, (2010) "On the interpretation of the WTP/WTA gap as imprecise utility: an axiomatic analysis", *Economics Bulletin*, Vol. 30 no.1 pp. 692-701.
Submitted: Aug 22 2009. **Published:** March 10, 2010.

1 Introduction

It is common in economics to assume that people know exactly how much a good is worth to them (independently of the fact that they are willing to report this value in a sincere way). However, this assumption is but an idealization: in general, people assess the value of a good with a certain amount of imprecision. Specifically, they tend to submit different willingness-to-pay (WTP) and willingness-to-accept (WTA) values, even in contexts where income effects are negligible (see Schmidt and Traub (2009) for recent evidence, and Plott and Zeiler (2005, 2007) for a dissenting view).

This valuation imprecision may be attributed to a certain amount of imprecision in preferences themselves, which in turn may be interpreted as preference uncertainty, in the standard sense where preferences are random in nature. Such an interpretation, probably because it is amenable to a sound econometric treatment, has been favored in the literature and studied quite extensively, showing that it indeed can explain much, if not all, of the WTP/WTA disparity (see Kingsley (2008) for recent references).

However, preference uncertainty is not the only possible interpretation of preference imprecision. It can also be related to the imperfect ability of the human perception apparatus to discriminate sounds, colors or smells, hence to distinguish similar objects and express a preference between them as a result. The consequences of this limitation in human perception were first studied by Luce in a famous article (Luce 1956) where he introduced the concept of semiorder, generalized by Fishburn with the notion of interval order (Fishburn 1970, p.18). Interval orders have the property that (under suitable conditions in the general case), they can be represented by two functions u and v, with $u \leq v$, in the sense that

$$x \succ y \iff u(x) > v(y).$$

Each object x is therefore mapped to a "utility interval" $[u(x), v(x)]$, i.e. has imprecise utility. The interpretation of the functions u and v is not as transparent as the interpretation of standard utility functions, however. In this note, we provide conditions for an interpretation for these functions as willingness-to-pay and willingness-to-accept, and to the interval $[u(x), v(x)]$ as the set of equivalent monetary valuations for object x, thus bridging the gap between imprecise utility and imprecise valuation in a formal and precise manner. We will introduce axioms that imply that one possible interval utility representation is indeed the interval of values defined by the willingness-to-pay and the willingness-to-accept.

Imprecise utility is indeed related in two natural ways to imprecise monetary valuation. First, it is very likely that an imprecise utility valuation would translate into an imprecise monetary one, as it would be surprising that someone who cannot discriminate clearly between very similar objects would however be able to give them a precise monetary value. Second, money itself is a dimension on which discrimination is not always possible or easy. For instance, one would probably be as willing to buy a given accommodation if its price were 300 000 euros as if it were 299 000 euros, since for an amount of this size a difference of 1000 euros is relatively negligible.

In the next section, we introduce the axioms. The results are presented in section 3 and applications in section 4. Proofs are gathered in the appendix.

2 Axioms

Let A be a nonempty set and let $X = A \times \mathbb{R}$. The intended interpretation of a typical element $x = (a, w)$ of X is that it represents a particular endowment comprising a certain number of non-monetary assets, a, on the one hand, and a certain amount w of money[1]. Notice that this amount can be negative: debts are allowed. For any $x \in X$ and $\lambda \in \mathbb{R}$, we denote $x \oplus \lambda$ the element $(a, w + \lambda)$.

We assume that there exists a family $(\succ_r)_{r \in X}$ of binary relations defined over X. These binary relations model a profile of reference-dependent preferences, RDP for short, that can be interpreted as modeling the *observed* choice behavior of the individual in a context where his or her reference point is an element r of X. We introduce the following axioms for observable preferences:

Axiom 1 (Strict Partial Order). *For all $r \in X$, $\succ_r$ is irreflexive and transitive.*

Axiom 2 (Separability). *For all $x, y, r \in X$, if $x \succ_r y$, then there exists $\lambda > \mu$ such that*

$$x \ominus \lambda \succ_r r \quad \text{and} \quad r \oplus \mu \succ_y y.$$

Axiom 3 (Monotonicity). *For all $r \in X$, for all $\lambda, \mu \in \mathbb{R}$,*

$$\lambda > \mu \quad \Longrightarrow \quad r \oplus \lambda \succ_r r \oplus \mu.$$

Axiom 4 (Strict Buying Price Consistency). *For all $(x, r) \in X^2$, for all $\lambda \in \mathbb{R}$,*

$$x \ominus \lambda \succ_r r \quad \Longrightarrow \quad x \succ_r r \oplus \lambda.$$

Axiom 5 (Strict Monetary Status Quo Bias). *For all $(x, r) \in X^2$, for all $\lambda \in \mathbb{R}$,*

$$x \oplus \lambda \succ_r r \quad \Longrightarrow \quad x \oplus \lambda \succ_x r.$$

In line with the literature on just noticeable differences, we assume that only the strict part of preferences is transitive. That is the essential assumption made in Strict Partial Order. It must be noted, in particular, that we do not make any completeness assumption here. Strong Separability says that if I strictly prefer x to y given endowment r, then there is always a price for which I would be willing to pay for x and a lower price for which I would be willing to sell y. This means that strict preference really means something in terms of price discrimination: if there

[1] All the results in this paper go through if we use as objects of choice any set X endowed with an operation $\oplus : X \times \mathbb{R} \longrightarrow X$ such that:

(i) For all $x \in X$, $x \oplus 0 = x$;

(ii) For all $\lambda, \mu \in \mathbb{R}$, for all $x \in X$,

$$(x \oplus \lambda) \oplus \mu = x \oplus (\lambda + \mu).$$

See Giraud (2009) for details.

is strict preference, then even with imprecision in the evaluation of the objects at hand, their value is sufficiently different for me to be able to choose one above the other. The third axiom simply reflects the fact that more money is better than less. Note however that it does not exclude that for some $x \in X$ different from r, λ, $\mu \in \mathbb{R}$, $\lambda > \mu$ and $x \oplus \mu \succ_r x \oplus \lambda$. Only "pure" money, so to speak, is preferred to less.

The idea of Strict Buying Price Consistency is that, if I am willing to pay λ euros in order to buy x when my current endowment is r, then this means that the subjective value of x relative to r is at least λ, i.e. I strictly prefer x to r even when I receive in addition a windfall amount of λ euros.

The status quo bias is the general tendency to prefer sticking to the current position, only because it is the current position. Samuelson and Zeckhauser (1988) provided convincing evidence for this tendency. Strict Monetary Status Quo Bias conveniently expresses the idea that being the status quo gives an alternative an extra power against other alternatives besides its intrinsic merit: when $\lambda = 0$, it says that if an alternative x beats the status quo r, then a fortiori x must beat r when x is the status quo, since it is thus even more attractive than before; this is exactly the standard status quo bias. When $\lambda > 0$, then the fact that x beats r with a premium of λ may imply that x is only weakly better than r. It does not imply that x itself beats r when r is the status quo. Therefore, even being the status quo it may not beat r. However, it must beat it with the premium λ. In other words, other things being equal moving the status quo from r to x cannot worsen x's attractiveness. In turn, if $\lambda < 0$, then x worsened by the amount λ beats the status quo r. There is thus a strong preference for x, so that when it is the status quo x will still beat r when deprived of the amount λ.

This axiom is a consistency axiom that rules out a very direct form of preference reversal: preferring x to y when y is the endowment and y to x when x is the endowment. Since this kind of preference reversal makes the decision-maker vulnerable to money pumps, ruling them out has been deemed necessary in the literature for a modeling of rational reference-dependent preferences, and therefore similar axioms (with $\lambda = 0$) have been introduced by many authors (Munro and Sugden 2002, Sagi 2006, Masatlioglu and Ok 2005, 2009, Apesteguia and Ballester 2009).

3 Results

Let us first define willingness-to-pay and willingness-to-accept in our framework. The willingness-to-pay for x, given endowment r is given by the function:

$$\begin{aligned} b : X^2 &\longrightarrow \overline{\mathbb{R}} \\ (x, r) &\longmapsto \sup B(x, r) := \{\lambda \in \mathbb{R} \mid x \ominus \lambda \succ_r r\}. \end{aligned}$$

Similarly, the willingness-to-accept for x, given endowment r, relative to $\succ$, is given by the function:

$$\begin{aligned} s : X^2 &\longrightarrow \overline{\mathbb{R}} \\ (x, r) &\longmapsto \inf S(x, r) := \{\lambda \in \mathbb{R} \mid r \oplus \lambda \succ_x x\}. \end{aligned}$$

The main result of this paper is the following theorem:

Theorem. $\{\succ_r\}_{r\in X}$ *satisfies Strict Partial Order, Strong Separability, Monotonicity, Strict Buying Price Consistency and Strict MSQB if and only if*

(i) $x \succ_r y \Longleftrightarrow b(x,r) > s(y,r)$*;*

(ii) $b(x,r) \le s(x,r)$*, for all* $x, r \in X$*;*

(iii) $b(r,r) = s(r,r) = 0$ *for all* $r \in X$*.*

The representation found in the theorem for binary relation $\succ_r$ shows that the axioms imply that it be an interval order, even though none of these axioms (except the first one) are standard axioms for interval orders. As we said above, interval orders are generalizations of weak orders that are suitable to model imprecise (utility) valuation of objects. The theorem allows to relate imprecise utility valuation to imprecise monetary valuation. Indeed, define first the *similarity relation* $\sim_r$ by

$$x \sim_r y \iff \neg(x \succ_r y) \text{ and } \neg(y \succ_r x).$$

This relation is called "indifference relation" by Fishburn (Fishburn 1970, p.12) and he has a very broad interpretation of this concept, since it includes incomparability. As a referee pointed out, however, including incomparability in the concept of indifference might be confusing, and all the more so since Fishburn himself introduces a companion relation that captures more faithfully the concept of indifference, whereby two objects are indifferent if they are perfect substitutes in a preferential judgment. He calls this relation the "equivalence" relation, and his resorting to this very general term shows that there is a terminological problem. The definition of this relation, that we shall call *indifference relation* and denote $\approx_r$, is the following:

$$x \approx_r y \iff (\forall z \in X, x \sim_r z \iff y \sim_r z).$$

Note that $x \approx_r y$ only if $x \sim_r y$, so that indifference implies similarity but not the other way around. Moreover, although transitivity might be considered a defining property of indifference[2], it is not required of similarity in the literature (see e.g. Tversky 1977, Rubinstein 2000).

Then we have the following corollary (the simple proof of which is omitted).

Corollary 1. *Under the conditions of the theorem, for all* $\lambda \in \mathbb{R}$*,*

$$b(x,r) \le \lambda \le s(x,r) \iff x \sim_r r \oplus \lambda.$$

In other words, any monetary value between the WTP and the WTA can be considered as the monetary value of the object assessed, as far as the decision maker is concerned. Notice that since the similarity relation of an interval order is not necessarily transitive, we cannot conclude from this that $b(x,r) = s(x,r)$, thus maintaining the possibility of the WTA/WTP discrepancy.

[2]However Luce (Luce 1956) insisted on the possibility of intransitive indifference, but in this paper indifference is defined as in Fishburn (1970), so the same terminological criticism applies.

For the sake of completeness, we show how a larger willingness-to-pay and a larger willingness-to-accept can be characterized in terms of preference only. Define two new binary relations $\succ_r^b$ and $\succ_r^s$ by:

$$x \succ_r^b y \iff \exists z \in X, x \succ_r z, z \sim_r y,$$

and

$$x \succ_r^s y \iff \exists z \in X, x \sim_r z, z \succ_r y.$$

These binary relations can be seen as approximations of the true preference relation: x is approximately preferred to y in the sense of $\succ_r^b$ if it is preferred to an object similar to y; likewise, x is approximately preferred to y in the sense of $\succ_r^s$ if there exists an object similar to x that is preferred to y. Then:

Corollary 2. *Under the conditions of the theorem,*

$$x \succ_r^b y \iff b(x,r) > b(y,r)$$

and

$$x \succ_r^s y \iff s(x,r) > s(y,r).$$

Moreover,

$$x \approx_r y \iff b(x,r) = b(y,r) \text{ and } s(x,r) = s(y,r).$$

4 Applications

4.1 Preference for Liquidity

Consider the following definition of comparative preference for liquidity:

Definition 1. *Let $(\succ_r^1)_{r\in X}$ and $(\succ_r^2)_{r\in X}$ be the observable preference profiles of two individuals and $r, r' \in X$. Then 1 has a stronger preference for liquidity at r than 2 at r' if, for all $x \in X$ such that $S^1(x,r) = S^2(x,r')$ and $x \succ_r^1 r$ and for all $w \geq 0$:*

$$x \succ_r^1 r \oplus w \quad \Longrightarrow \quad x \succ_{r'}^2 r' \oplus w$$

The intuition is that person 1 with endowment r has a stronger preference for liquidity than person 2 with endowment r' if, given any object x for which they both have the same willingness-to-accept and given any positive amount of money w, whenever person 1 prefers having x to having w, then so does person 2. The requirement that the two decision makers have the same WTA is made to guarantee that the comparison of preference for liquidity is made all things being equal, i.e. controlling for possible divergence in attitudes with respect to the object being evaluated.

Now, it is possible to characterize the preference for liquidity in the context of the theorem, as shown by the next proposition:

Proposition 1. *Assume $(\succ_r^1)_{r\in X}$ and $(\succ_r^2)_{r\in X}$ satisfy Strict Partial Order, Strong Separability, Monotonicity, Strict Buying Price Consistency and Strict MSQB. Then, the following are equivalent:*

(i) 1 has a stronger preference for liquidity at r than 2 at r'.

(ii) For all $x \in X$ such that $s^1(x,r) = s^2(x,r')$ and $x \succ_r^1 r$, $[b^2(x,r), s^2(x,r')] \subseteq [b^1(x,r), s^1(x,r')]$.

In words, this proposition says that if agent 1 has greater preference for liquidity than 2, his or her buying price for a given item will systematically be lower than agent 2's buying price. This is intuitive since, if I buy a good, I lose the advantages of liquidity. The higher these advantages for me, the less will my willingness to abandon them be, and therefore the less my willingness-to-pay for the object since it includes the cost of abandoning them. Moreover, this proposition establishes a connection between preference for liquidity and the size of the WTA/WTP gap: the higher the preference for liquidity, the larger the gap. This seems intuitive if one accepts the idea that WTP incorporates illiquidity costs, and therefore the higher it is, the wider the gap.

4.2 The Strength of the Status Quo Bias

The status quo bias is the fact that the decision maker has a tendency to stick to the status quo. However, there is no reason for this bias to be of equal strength whatever the status quo. To model this fact, we introduced in Giraud (2006) a comparative definition of the strength of this status quo

Definition 2. r is a stronger status quo than r' *(denoted $r \succcurlyeq_{SQB} r'$) if:*

$$\forall x \in X, x \succ_r r \Rightarrow x \succ_{r'} r'.$$

The strength of the status quo bias can be characterized in the context of the theorem:

Proposition 2. *Assume that $(\succ_r)_{r\in X}$ satisfies Strict Partial Order, Strong Separability, Monotonicity, Strict Buying Price Consistency and Strict MSQB. Then,*

$$r \succcurlyeq_{SQB} r' \Longleftrightarrow b(x,r) \leq b(x,r'), \quad \forall x \in X.$$

Hence r is a stronger status quo than r' if the decision maker is never willing to pay more for a given object x when he or she must forego r than when he or she must forego r'.

A Proofs

Proof of the Theorem. **Sufficiency of the axioms** Assume $x \succ_r y$. Then, by Strong Separability, there exists $\lambda > \mu$ such that

$$x \ominus \lambda \succ_r r \quad \text{and} \quad r \oplus \mu \succ_y y.$$

This implies, on the one hand, that $b(x,r) \geq \lambda$ and, on the other hand, that $\mu \geq s(y,r)$. But, since $\lambda > \mu$, this implies that $b(x,r) > s(y,r)$.

Now, assume that $b(x,r) > s(y,r)$. Then, since $b(x,r)$ is the supremum of the set $B(x,r) = \{\lambda \in \mathbb{R} \mid x \ominus \lambda \succ_r r\}$, there exists $\lambda \in B(x,r)$ such that $b(x,r) \geq \lambda > s(y,r)$. By a similar reasoning, there exists $\mu \in \mathbb{R}$ such that $r \oplus \mu \succ_y y$ and $\lambda > \mu \geq s(y,r)$. By Strict Buying Price Consistency, we have that $x \succ_r r \oplus \lambda$ and by Strict MSQB, we have that $r \oplus \mu \succ_r y$. Therefore, since $\lambda > \mu$, Monotonicity and Strict Partial Order imply that $x \succ_r y$.

Irreflexivity of $\succ_r$ for all r implies that for all $x \in X$, not $b(x,r) > s(x,r)$, and therefore $b(x,r) \leq s(x,r)$.

Monotonicity and Irreflexivity imply that

$$B(r,r) = (-\infty, 0),$$

so that its supremum $b(r,r)$ is 0. For the same reason,

$$S(r,r) = (0, +\infty),$$

so that its infimum $s(r,r)$ equals 0.

Necessity of the axioms

Strict Partial Order Irreflexivity follows from (*ii*). For transitivity, take $x, y, z, r \in X$ such that $x \succ_r y$ and $y \succ_r z$. Then, $b(x,r) > s(y,r)$ and $b(y,r) > s(z,r)$. Since $b(y,r) \leq s(y,r)$, we have that $b(x,r) > s(z,r)$.

Strong Separability If $x \succ_r y$, then $b(x,r) > s(y,r)$, so we can repeat the argument given in the sufficiency part.

Monotonicity Let $\lambda > \mu$. Then, for all $r \in X$, $b(r \oplus \lambda, r) - s(r \oplus \mu, r) = b(r \oplus \lambda, r) - s(r \oplus \mu, (r \oplus \mu) \ominus \mu) = \lambda - \mu > 0$ because $b(r,r) = s(r \oplus \mu, r \oplus \mu) = 0$. Therefore $r \oplus \lambda \succ_r r \oplus \mu$.

Strict Buying Price Consistency Let $x, r \in X$ and $\lambda \in \mathbb{R}$ be such that $x \ominus \lambda \succ_r r$. Then, $b(x \ominus \lambda, r) > s(r,r) = 0$. Therefore, $b(x,r) > \lambda = s(r \oplus \lambda, r)$, hence $x \succ_r r \oplus \lambda$.

Strict MSQB Let $x, r \in X$ and $\lambda \in \mathbb{R}$ be such that $x \oplus \lambda \succ_r r$.

Then $b(x,r) + \lambda > 0$, therefore $-b(x,r) - \lambda < 0$, i.e.

$$s(r,x) < \lambda = b(x \oplus \lambda, x), \text{ hence } x \oplus \lambda \succ_x r. \qquad \square$$

Proof of Corollary 2. We prove the first result, the second is proved similarly.

Assume first $x \succ_r^b y$. Then there exists $z \in X$ such that $x \succ_r z$, and therefore $b(x,r) > s(z,r)$, and such that $z \sim_r y$, so that $\neg(y \succ_r z)$, i.e. $b(y,r) \leq s(z,r)$. Combining both yields $b(x,r) > b(y,r)$.

Assume now $b(x,r) > b(y,r)$. Then, there exists $\lambda \in \mathbb{R}$ such that $x \ominus \lambda \succ_r r$ and $b(x,r) \geq \lambda > b(y,r)$. By Strict Buying Price Consistency, this implies $x \succ_r r \oplus \lambda$, and therefore, by Monotonicity and transitivity, $x \succ_r r \oplus b(y,r)$. But, by the preceding corollary, we have $y \sim_r r \oplus b(y,r)$. Therefore, $x \succ_r^b y$.

The final result follows from Fishburn (1970, Theorem 2.6). $\square$

Proof of Proposition 1. (*i*) $\Rightarrow$ (*ii*) Let x such that $S^1(x,r) = S^2(x,r')$ and $x \succ_r^1 r$ and let $\lambda \in [b^2(x,r'), s^2(x,r')]$. Since $x \succ_r^1 r$ and 1 has higher preference for liquidity than 2, it follows that $x \succ_{r'}^2 r'$, and therefore $b^2(x,r') > 0$, hence $\lambda > 0$. Now, $x \sim_{r'}^2 r' \oplus \lambda$, which implies $\neg(x \succ_{r'}^2 r' \oplus \lambda)$. Again, since 1 has higher preference for liquidity than 2, this implies $\neg(x \succ_r^1 r \oplus \lambda)$, hence $b^1(x,r) \leq \lambda \leq s^2(x,r')$. Moreover, since $S^1(x,r) = S^2(x,r')$, they have the same infimum, so that $s^1(x,r) = s^2(x,r')$, hence $b^1(x,r) \leq \lambda \leq s^1(x,r)$.

(*ii*) $\Rightarrow$ (*i*) Straightforward. □

Proof of Proposition 2. Assume first that $r \succcurlyeq_{SQB} r'$ and take $x \in X$ and $\lambda \in \mathbb{R}$ such that $x \ominus \lambda \succ_r r$. Then, by definition of $\succcurlyeq_{SQB}$, we have $x \ominus \lambda \succ_{r'} r'$. Therefore $B(x,r) \subseteq B(x,r')$, and hence $b(x,r) \leq b(x,r')$.

Conversely, assume $b(x,r) \leq b(x,r')$ and assume $x \succ_r r$. Then $b(x,r) > 0$, and therefore $b(x,r') > 0$, implying $x \succ_{r'} r'$. □

References

Apesteguia, José and Miguel A. Ballester (2009) "A theory of reference-dependent behavior," *Economic Theory*, **40**, 427–455.

Fishburn, Peter C. (1970) *Utility Theory for Decision Making*, Wiley and sons: New York.

Giraud, Raphaël (2006) "Reference-Dependent Preferences: An Axiomatic Approach to the Underlying Cognitive Process," in Bernard Walliser and Richard Topol eds. *Cognitive Economics: New trends*, Elsevier: Amstersdam.

Giraud, Raphaël (2009) "Money Matters: An Axiomatic Theory of the Endowment Effect." Mimeo.

Kingsley, David C. (2008) "The Role of Preference Uncertainty in the Willingness to Pay - Willingness to Accept Disparity: An Experimental Test," April. Mimeo, Westfield State College.

Luce, D. (1956) "Semiorders and a Theory of Utility Discrimination," *Econometrica*, **24**, 178–191.

Masatlioglu, Y. and E. Ok (2005) "Rational Choice with a Status Quo Bias," *Journal of Economic Theory*, **121**(1), 1–29. mimeo.

Masatlioglu, Yusufcan and Efe Ok (2009) "A Canonical Model of Choice with Initial Endowments." University of Michigan and N.Y.U.

Munro, A. and R. Sugden (2002) "A Theory of Reference-Dependent Preferences," *Journal of Economic Behavior and Organization*, **50**, 407–428.

Plott, Charles and Kathryn Zeiler (2005) "The Willingness to Pay/Willingness to Accept Gap, the Endowment Effect, Subject Misconceptions and Experimental Procedures for Eliciting Valuations," *American Economic Review*, **95**(3), 530–545.

——— (2007) "Asymmetries in Exchange Behavior Incorrectly Interpreted as Evidence of Endowment Effect Theory and Prospect Theory?,," *American Economic Review*, **97**, p. 1449.

Rubinstein, Ariel (2000) *Modeling Bounded Rationality*, MIT Press: Cambridbge.

Sagi, Jacob S. (2006) "Anchored preference relations," *Journal of Economic Theory*, **127**(1), 283–295, September.

Samuelson, W. and R. Zeckhauser (1988) "Status Quo Bias in Decision Making," *Journal of Risk and Uncertainty*, **1**, 7–59.

Schmidt, Ulrich and Stefan Traub (2009) "An Experimental Investigation of the Disparity Between WTA and WTP for Lotteries," *Theory and Decision*, **66**(3), 229–262, March.

Tversky, Amos (1977) "Features of similarity," *Psychological review*, **84**(4), 327–352.