

# Intertemporal Substitution and Sectoral Comovement in a

## Sticky Price Model\*

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#### Abstract

Strong procyclical fluctuations in the durable production are the most prominent feature of the empirical response to monetary shocks. This paper investigates the role of preferences in matching this feature of the data in a two-sector sticky price model with flexibly priced durables. The reaction of durables depends crucially on whether preferences are separable between labor and aggregate consumption. When preferences are separable, the model exhibits perverse behavior. Flexibly priced durables contract during periods of economic expansion. However, sticky price model with non-separable preferences can replicate the empirically plausible response of durable spending. The key to the model's success hinges upon the fact that the non-separable preferences imply the complementarity between aggregate consumption and labor supply, absent in the separable preference. Finally, we present empirical evidence supporting the non-separable preferences.

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## 1 Introduction

Durable goods feature prominently in discussions of monetary policy. According to the data, the durable goods sector is one of the sectors that seem to respond most procyclically to monetary policy.<sup>1</sup> However, as demonstrated by Barsky et al. (2003, 2007), it is difficult to match this feature of the data by simply incorporating durable goods into sticky price models with separable preferences. In particular, if durable goods have flexible prices, but nondurable goods prices are sticky, then a monetary expansion leads to an *increase* in nondurable goods production but a *decline* in durable goods production, so that aggregate output may not change at all.

This paper shows that the negative sectoral comovement problem in sticky price models is not robust to incorporating non-separable preferences in aggregate consumption and labor. To this end, we consider the King-Plosser-Rebelo utility function, which nests both the non-separable and separable cases. In this class of utility function, the degree of non-separability is governed by the intertemporal elasticity of substitution: the smaller the intertemporal elasticity of substitution, the larger the non-separability. When the intertemporal elasticity of substitution is unity (i.e., the separable case), we replicate the findings of Barsky et al. (2007) that monetary shocks induce a negative output comovement across sectors and have virtually no impact on aggregate output. However, we demonstrate that when the intertemporal elasticity of substitution is less than unity (i.e., the non-separable case), the sticky price model can generate a strong procyclical response of durable goods to a monetary expansion.

We also identify what determines the threshold level of non-separability needed for producing a strong procyclical response of durable goods to a monetary expansion. It turns out that the elasticity of factor supply and the degree of factor mobility across sectors play an important role in generating a strong procyclical response. Variable capital utilization and indivisible labor, implying a more elastic factor supply, and inflexible factor mobility are, either individually or in combination, shown to dramatically decrease the threshold level of non-separability necessary for generating a strong procyclical response of durables.

Why does non-separability between aggregate consumption and labor substantially change

<sup>&</sup>lt;sup>1</sup>Erceg and Levin (2006) document that an exogenous increase in the interest rate, estimated through a structural VAR, reduces consumer durables and residential investment spending nearly three times more than nondurable consumption. Barsky et al. (2003) also report similar results using the *Romer dates* as indicators of exogenous changes in monetary policy. Following the *Romer date*, the production of durables falls far more than that of nondurables.

the behavior of a two-sector sticky price model? This stems from the fact that the non-separable preferences imply that nondurable consumption and labor are complementary<sup>2</sup>. When the intertemporal elasticity of substitution is less than unity, an increase in nondurable consumption reduces the marginal disutility of work in a class of King-Plosser-Rebelo utility function.<sup>3</sup> The strength of the complementarity increases as the intertemporal elasticity of substitution gets lower. The complementarity between nondurables consumption and labor supply in turn affects the reaction of the nominal wage to a monetary expansion. Following such an expansion, the nominal wage tends to rise because nondurable-goods producing firms raise the demand for labor inputs to meet their increased demands due to sticky prices. For producers in the flexibly priced durable sector, the increase in factor price is merely an adverse cost shock. Unless there are forces offsetting the rise in the cost of production, the flexible price sector contracts. However, the complementarity between nondurable consumption and labor supply mitigates the rising pressure on the nominal wage since the increase in nondurable consumption shifts the labor supply curve out. Hence, if the degree of complementarity is large enough, production in the durable sectors could rise.<sup>4</sup>

Basu and Kimball (2002) and Guerron-Quintana (2008) provide compelling evidence showing that preferences displaying additive separability in nondurables and labor are not supported by the data. We elaborate on their evidence against the assumption of additive separability between nondurables and labor. While they consider nondurable goods and labor in estimating the intertemporal elasticity of substitution, we extend the empirical specification of Basu and Kimball (2002) to incorporate durable goods. Even when durable goods are accounted for in the estimation, we find that the data still reject additively separable preferences. Our empirical results show that the intertemporal elasticity of substitution ranges from 0.4 to 0.7, depending on the intratemporal elasticity of substitution between nondurable consumption and the service flow from durable consumption.

<sup>&</sup>lt;sup>2</sup>The non-separability between aggregate consumption and labor also imply that the service flow from durable good and labor are complementary. However, the complementarity between the consumption of durable services and labor has little impact on the behavior of the model. It is because the stock of durable goods changes so slightly following the monetary shock as Barsky et al. (2003, 2007) show.

<sup>&</sup>lt;sup>3</sup>Introducing complementarity between nondurable consumption and labor also has an intuitive appeal. For example, when times are good, workers put in longer hours and enjoy less leisure, but they can make up for this in part by going out to lunch and dinner too.

<sup>&</sup>lt;sup>4</sup>Barsky et al. (2003, 2007) briefly discuss the possibility that the complementarity between nondurables and labor might temper the negative comovement problem.

In addition to the introduction of non-separable preferences, there are several ways to resolve the comovement problem. Barsky et al. (2003, 2007) propose the introduction of a sticky nominal wage as one possible solution to the comovement problem. Carlstrom and Fuerst (2006) explicitly demonstrate that a sticky wage helps generate sectoral comovement. Another way suggested by Barsky et al. (2003, 2007) is to consider the model with a credit constraint. Monacelli (2006) confirms this with numerical simulations. Finally, Bouakez et al. (2008) show that incorporating input-output interactions and limited factor mobility leads to positive comovement across sectors.

The remainder of the paper is organized as follows. Section 2 presents a two-sector sticky price model that includes nondurable and durable goods. Section 3 describes the benchmark values for the key parameters used in the quantitative analysis of the model. In section 4 we demonstrate, both analytically and quantitatively, that sticky price model with non-separable preferences can produce a strong procyclical response of durable spending to a monetary shock. Section 5 investigate some modifications to the model, which dramatically reduce the threshold level of non-separability needed to generate a strong procyclical response of durables. Section 6 presents empirical evidence supporting the non-separable preferences. Section 7 concludes.

## 2 The Model

In this section, we extend the two-sector sticky price model of Barsky et al. (2003, 2007) by incorporating non-separability between aggregate consumption and labor and introducing variable capital utilization.

The economy is populated by a constant number of identical, infinitely-lived households, continua of firms in two sectors that respectively produce differentiated durable and nondurable goods, perfectly competitive final goods firms in two sectors, and a government.

#### 2.1 Households

The representative household receives utility from consumption of the nondurable goods and from consumption of the service flow of durable goods, and incurs disutility from hours worked. Let  $C_t$  and  $S_t$  respectively denote period t consumption of the nondurable goods and consumption of the service flow from the durable consumption, and let  $L_t$  denote labor supply. Households

maximize expected lifetime utility, given by

$$U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, S_t, L_t) \right], \tag{1}$$

where  $\beta \in (0, 1)$  is the subjective discount factor.

We modify the conventional King-Plosser-Rebelo monetary utility function used by Basu and Kimball (2002) and Shimer (2009) to augment the consumption of the service flow from durable goods. The specific form of *U* adopted in this paper is

$$U(C_t, S_t, L_t) = \frac{Z_t^{1 - \frac{1}{\sigma}} \left( 1 + \left(\frac{1}{\sigma} - 1\right) v(L_t) \right)^{\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},$$
(2)

where  $Z_t \equiv g(C_t, S_t) = \left(\psi_c C_t^{1-\frac{1}{p}} + \psi_d S_t^{1-\frac{1}{p}}\right)^{\frac{p}{p-1}}$  and  $v(L_t) = \phi_{\frac{\eta}{1+\eta}} L_t^{\frac{\eta+1}{\eta}}$ .  $Z_t$  is a quantity index that aggregates the consumption of nondurable goods and durable services, and  $v(L_t)$  measures the disutility incurred from hours worked with v' > 0, v'' > 0. Our formulation departs from Barsky et al. (2003, 2007) in that we relax the assumption of additive separability between aggregate consumption and labor. In (2), the degree of non-separability is controlled by a parameter for the intertemporal elasticity of substitution,  $\sigma$ . The lower this parameter is, the larger the non-separability displayed by the utility function. The separable case, for instance, corresponds to  $\sigma = 1$ :

$$\lim_{\sigma \to 1} U(C_t, S_t, L_t) = \log(Z_t) - v(L_t).$$

This separable preference is used in most sticky price models, including Barsky et al. (2003, 2007).

The household enters period t with a stock of private one-period nominal bonds  $(B_{t-1})$ , a stock of nominal money balances  $(M_{t-1})$ , and a fixed stock of capital  $(\overline{K})$ . During the period, the household receives wages, rentals on capital services, dividends paid by firms, a lump-sum transfer  $(T_t)$  from the government, and interest payments on bond holdings. These resources net of the cost of varying capital utilization rate are used to purchase durable and nondurable goods and to acquire assets to be carried over to next period. Then, the household's budget constraint (in nominal term) is

$$P_{c,t}C_t + P_{x,t}X_t + B_t + M_t \le W_t L_t + \sum_{j=c,x} R_{j,t} u_{j,t} \overline{K}_{j,t} + \Pi_t + T_t + (1+i_{t-1})B_{t-1} + M_{t-1} - \sum_{j=c,x} P_{j,t} a(u_{j,t}) \overline{K}_{j,t},$$
(3)

where the subscript *c* and *x* denote variables that are specific to the nondurable and durable sector, respectively.  $P_{x,t}$  and  $P_{c,t}$  are the nominal prices of the durable and nondurable,  $W_t$  is the nominal wage rate,<sup>5</sup>  $\Pi_t$  are profits returned to the consumer through dividends, and  $i_t$  is the nominal interest rate. In addition,  $\overline{K}_j$  is the productive capital stock in sector j = c, x and  $u_{j,t}$  denotes the capital utilization rate in sector j = c, x. Hence,  $K_{j,t} \equiv u_{j,t}\overline{K}_j$  represents the capital services used in each sector and  $R_{j,t}$  is the rental rate of capital services.<sup>6</sup> The increasing and convex function  $a(u_{j,t})\overline{K}_{j,t}$  denotes the cost, in units of the goods in each sector, of setting the capital utilization function,  $a(u_{j,t})$ . First, we require that  $u_{j,t} = 1$  in a steady state. Second, we assume a(1) = 0. Under these assumptions, the steady state of the model is independent of the curvature of the function *a* in steady state,  $\chi \equiv \frac{a''(1)}{a'(1)}$ . The parameter  $\chi$  governs the elasticity of capital utilization. A high value of  $\chi$  corresponds to a small elasticity, implying that varying utilization is highly costly.

The stock of durable goods evolves according to

$$D_t = X_t + (1 - \delta)D_{t-1},$$
(4)

where  $\delta \in (0, 1)$  is the depreciation rate and  $X_t$  denotes newly purchased durables. Following the literature, the service flow from durable goods,  $S_t$ , is assumed to be proportional to the stock of the durable goods,  $D_t$ :

$$S_t = D_t = X_t + (1 - \delta)X_{t-1} + (1 - \delta)^2 X_{t-2} + \cdots$$

<sup>&</sup>lt;sup>5</sup>Note that we assume that labor can flow freely between sectors. Hence, wage rates are identical between sectors. <sup>6</sup>Rental rates in different sectors might not be the same because we consider the case where capital stock is imperfectly mobile between sectors.

The first order conditions associated with the optimal choice of  $C_t$ ,  $L_t$  and  $X_t$  are

$$\frac{\gamma_{c,t}}{P_{c,t}} = \frac{\gamma_{x,t}}{P_{x,t}},\tag{5}$$

$$-U_L(C_t, D_t, L_t) = \gamma_{x,t} \frac{W_t}{P_{x,t}} = \gamma_{c,t} \frac{W_t}{P_{c,t}},$$
(6)

where  $\gamma_{c,t} \equiv U_C(C_t, D_t, L_t)$  denotes the marginal utility of nondurable consumption and  $\gamma_{x,t}$  denotes the shadow value of durable consumption.  $\gamma_{x,t}$  can be written as the present value of future marginal service flow from an additional unit of the durable at time *t*, discounted by the subjective rate of time preference and the depreciation rate:

$$\gamma_{x,t} = MU_t^D + \beta(1-\delta)E_t MU_{t+1}^D + \beta^2(1-\delta)^2 E_t MU_{t+2}^D + \cdots,$$
(7)

where  $MU_t^D \equiv U_D(C_t, D_t, L_t)$  denotes the marginal utility of the service flows from an additional unit of the durable at time *t*.

As in Barsky et al. (2007), money demand is assumed to be proportional to nominal purchases:

$$M_t = P_{c,t}C_t + P_{x,t}X_t. aga{8}$$

#### 2.2 Firms

We assume the existence of a continuum of monopolistically competitive firms, indexed by  $s \in [0, 1]$ , producing differentiated intermediate goods in each sector. A final good in each sector is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods.

#### 2.2.1 Final goods firms

The final good in each sector is aggregated by the CES (constant elasticity of substitution) technology:

$$C_t = \left[\int_0^1 c_t(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad X_t = \left[\int_0^1 x_t(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}},\tag{9}$$

where  $c_t(s)$  and  $x_t(s)$  are the quantity of intermediate goods s used as input in each sector.  $\varepsilon > 1$ is the elasticity of substitution between different intermediate goods. As  $\varepsilon \to \infty$ , intermediate goods become perfect substitutes in the production of final good. Cost minimization by the final good producer in each sector delivers the demand for the intermediate goods

$$c_t(s) = \left(\frac{p_{c,t}(s)}{P_{c,t}}\right)^{-\varepsilon} C_t \quad \text{and} \quad x_t(s) = \left(\frac{p_{c,t}(s)}{P_{x,t}}\right)^{-\varepsilon} X_t, \tag{10}$$

where  $p_{j,t}(s)$  is the price of intermediate good *s* in sector j = c, x and  $P_{j,t}$  is the aggregate price level in sector j = c, x. Finally, the zero-profit condition implies that

$$P_{j,t} = \left[ \int_0^1 p_{j,t}(s)^{1-\varepsilon} ds \right]^{\frac{1}{1-\varepsilon}}, \quad \text{for} \quad j = c, x.$$
(11)

#### 2.2.2 Intermediate goods firms

Intermediate good producers in each sector are monopolistically competitive. Each intermediate goods firm produces its differentiated goods using the following production function:

$$c_t(s) = F(k_{c,t}, l_{c,t}) = k_{c,t}^{\alpha}(s) l_{c,t}^{1-\alpha}(s),$$
(12)

$$x_t(s) = F(k_{x,t}, l_{x,t}) = k_{x,t}^{\alpha}(s) l_{x,t}^{1-\alpha}(s),$$
(13)

where  $k_{j,t}(s)$  and  $l_{j,t}(s)$  are capital services and labor in firms *s* in sector *j* = *c*, *x* at time *t*.

Intermediate goods firms are assumed to set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm in sector j = c, x resets its price with the probability of  $1 - \theta_j$  each period, independently of the time elapsed since the last adjustment. Thus, for each period a measure  $1 - \theta_j$  of firms reset their prices, while a fraction  $\theta_j$  firms keep their prices from the previous period. An intermediate goods firm resetting its price in period *t* in sector j = c, x will seek to maximize the present value of expected future real profits generated while that price remains effective,

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \Theta_j^t \gamma_{j,t} \frac{\Pi_{j,t}}{P_{j,t}} \right], \tag{14}$$

subject to the sequence of demand constraints (10). Here  $\gamma_{j,t}$  is the shadow value of the good produced in sector j and  $\prod_{j,t}/P_{j,t}$  measures the real value of an intermediate goods firm's profit in sector j in period t. It is easy to show that the optimal reset prices in sector j = c, x, denoted as  $p_{j,t}^*$ , are

$$p_{c,t}^* = \frac{\varepsilon E_t \sum_{k=0}^{\infty} \beta^k \theta_c^k \zeta_{c,t+k} (P_{c,t+k})^{\varepsilon - 1} \gamma_{c,t+k} C_{t+k}}{(\varepsilon - 1) E_t \sum_{k=0}^{\infty} \beta^k \theta_c^k (P_{c,t+k})^{\varepsilon - 1} \gamma_{c,t+k} C_{t+k}},$$
(15)

and

$$p_{x,t}^* = \frac{\varepsilon E_t \sum_{k=0}^{\infty} \beta^k \Theta_x^k \zeta_{x,t+k} (P_{x,t+k})^{\varepsilon - 1} \gamma_{x,t+k} X_{t+k}}{(\varepsilon - 1) E_t \sum_{k=0}^{\infty} \beta^k \Theta_x^k (P_{x,t+k})^{\varepsilon - 1} \gamma_{x,t+k} X_{t+k}},$$
(16)

where  $\zeta_{j,t}$  is the nominal marginal cost in sector j. Finally, the equation describing the dynamics for the aggregate price level in sector j = c, x, is given by  $P_{j,t} = \left[ (1 - \theta_j) (p_{j,t}^*)^{1-\varepsilon} + \theta_j P_{j,t-1}^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$ .

### 2.3 Money Supply and Market Clearing

The government finances the transfers to households by printing additional money, so its budget constraint is

$$T_t = M_t - M_{t-1}.$$
 (17)

As in Barsky et al. (2007), it is further assumed that the money supply follows a random walk:

$$M_t = M_{t-1} + \xi_t,$$
(18)

where  $\xi_t$  is an independently and identically distributed (*i.i.d*) disturbance with zero mean.

We construct real GDP  $Y_t$  as  $Y_t \equiv P_cC_t + P_xX_t$ , where  $P_c$  and  $P_x$  are steady state prices for the nondurable and durable good. The GDP deflator is then nominal GDP divided by real GDP.

Finally, the labor and capital markets equilibriums require

$$L_t = L_{x,t} + L_{c,t}$$
 and  $\overline{K} = \overline{K}_{x,t} + \overline{K}_{c,t}$ , (19)

where  $L_{j,t} = \int l_{j,t}(s)ds$  and  $\overline{K}_{j,t} = \int \overline{k}_{j,t}(s)ds$  is labor and the stock of capital used in sector j = c, x.

## 3 Calibration

This section describes the benchmark values for the key parameters used to compute the response of the model to monetary shocks. Unless otherwise noted, we follow the parameter values used in Barsky et al. (2003, 2007). We set the subjective discount factor  $\beta$  to  $1.02^{-0.25}$ , implying a steady state annualized real interest rate of 2 percent. We choose the parameters  $\psi_c$  and  $\psi_d$  so that a steady-state nondurable share of GDP is 0.75. The parameter  $\rho$  representing the intratemporal elasticity substitution between  $C_t$  and  $S_t$  is set to 1.17, based on the estimate from Ogaki and Reinhart (1998). The parameter  $\eta$ , which corresponds to the Frisch labor supply elasticity when preferences are separable, is set to 1. We normalize the steady state value of hours worked to 1. Durable goods have a quarterly depreciation rate ( $\delta$ ) of 1.25 percent, which implies an annual rate of depreciation on the durable goods equal to 5 percent. Capital's share ( $\alpha$ ) in the production function is 0.33. We choose  $\varepsilon = 11$  to generate a desired markup of 10 percent. We assume that durable goods have perfectly flexible prices ( $\theta_x = 0$ ) and nondurable goods prices are adjusted (on average) every two and half quarters ( $\theta_c = 0.66$ ).

Finally, when the model allows for a variable capital utilization rate, the parameter governing the curvature of capital utilization function,  $\chi$  is set to 0.01, following Christiano et al. (2005). The values chosen for this parameter is somewhat important in the results below. Thus, we will discuss the sensitivity of our results to the choice of this parameter.

## 4 The Role of Non-Separable Preferences in Sticky Price Models

In this section, we show that the behavior of sticky price models depends crucially on how we assume separability between aggregate consumption and labor in preferences. In particular, we find that the larger the non-separability displayed by the utility function, the more likely is the model to generate sectoral comovement. The threshold level of non-separability needed to generate sectoral comovement depends on different assumptions regarding the supply elasticity of the factors of production and the mobility of these factors. To introduce our main results, we simulate the model and present an analytical treatment that provides insight into the underlying mechanism. Note that labor is assumed to flow freely across sectors. We focus on the reaction of

the model to a permanent unanticipated increase in the money supply of one percent.

#### 4.1 Model with Perfect Capital Mobility but Constant Utilization

#### 4.1.1 Simulation results

We begin by examining the case where physical capital is perfectly mobile across sectors but capital utilization rate in each sector remains constant (i.e., inelastic capital services), considered by Barsky et al. (2007). Figure 1 displays the reaction of the model to the monetary shock for various values of  $\sigma$ . It clearly shows that the degree of non-separability significantly affects the response of the model. The lower is the parameter  $\sigma$ , the larger the response of durable spending and aggregate output. In the special case of separable preferences ( $\sigma = 1$ ), our model replicates the results in Barsky et al. (2007). Following the shock, there is a large contraction in the production of durable goods that almost offsets the expansion in the production of nondurable goods, leaving aggregate output virtually unchanged. Thus, money is essentially neutral at the aggregate level in a model with separable preferences, even though the sticky-price nondurable sector accounts for 75 percent of GDP. As the parameter  $\sigma$  takes lower values, however, the contraction of production of durable gets substantially reduced, which in turn results in a positive response of aggregate output.

While the non-separable preferences clearly mitigate the negative comovement problem, extremely low values of  $\sigma$  are needed to produce sectoral comovement in a model with perfect capital mobility and constant capital utilization. For the model to obtain sectoral comovement, the parameter  $\sigma$  needs to be lower than approximately 0.05, even though a positive response of durable production in Figure 1 (d) is somewhat difficult to notice. For this reason, we also report the numerical values for the initial responses of durable production in Table 1. As it confirms, the model can generate a positive response of durable production as long as  $\sigma$  is approximately less than 0.05. Since the threshold level of  $\sigma$  needed to generate sectoral comovement is unrealistically low, however, the model seems to fail to replicate the empirically plausible response of durable spending relative to that of nondurables. Erceg and Levin (2006) estimate that a monetary policy innovation has a peak impact on consumer durable spending that is several times larger than the impact on other expenditure. As Figure 1 and Table 1 show, in contrast, the peak response of

durable goods in the model is far smaller than that of nondurable goods even when  $\sigma = 0.01$ .

Below, we discuss why the non-separability helps to mitigate the comovement problem and what extensions might expand the range of  $\sigma$  consistent with sectoral comovement so that the model is more likely to generate the response of durables observed in the data.

#### 4.1.2 Why does non-separability affect the behavior of the model?

To understand the underlying mechanisms, through which non-separability affects the behavior of model, it is useful to rewrite the labor supply condition (6) in the following manner:

$$-U_L(C_t, D, \underbrace{L_{x,t} + L_{c,t}}_{L_t}) = \gamma_x \frac{W_t}{P_{x,t}} = \frac{\gamma_x}{\mu \zeta_{x,t}} W_t,$$
(20)

where the last equality is implied by the fact that the flexible price of durables is a constant markup ( $\mu$ ) over its marginal cost:  $P_{x,t} = \mu \zeta_{x,t}$ . Note that we drop the time script of  $D_t$  and  $\gamma_{x,t}$  in the equation. Barsky et al. (2003, 2007) show that the stock-flow ratio is high so that even large changes in purchases have only minor effects on the total quantity of the durable good. Small deviations from the steady state of the economy virtually do not alter the stock of durables, and thus their shadow value ( $\gamma_{x,t}$ ) nearly constant at cyclical frequencies.

The nominal marginal cost  $\zeta_{x,t}$  is the cost of hiring an additional unit of a productive input multiplied by the number of inputs required to produce an additional unit of durable goods. Because the production functions in both sectors have constant returns to scale, and because physical capital and labor can flow freely across sectors, all firms have the same marginal cost and choose the same capital-to-labor ratios. Thus,

$$\zeta_{x,t} = \frac{W_t}{F_2(\overline{K}_{x,t}, L_{x,t})} = \frac{W_t}{F_2(\overline{K}, L_t)} = \frac{W_t}{f(L_t)},$$
(21)

where  $f(L_t) = F_2(\overline{K}, L_t) = (K/L_t)^{\alpha}$ . Combining (20) and (21) yields

$$-U_L(C_t, D, \underbrace{L_{x,t} + L_{c,t}}_{L_t}) = \frac{\gamma_x}{\mu} f(\underbrace{L_{x,t} + L_{c,t}}_{L_t}).$$
(22)

This equation shows that the nature of the comovement problem is closely related to the

separability between nondurable consumption and labor. First, suppose that the preference is separable ( $\sigma = 1$ ). In this case, the marginal disutility from labor is only a function of labor, so that (22) becomes

$$-U_{L}(C_{t}, D, L_{t}) = v'(\underbrace{L_{x,t} + L_{c,t}}_{L_{t}}) = \frac{\gamma_{x}}{\mu} f(\underbrace{L_{x,t} + L_{c,t}}_{L_{t}}).$$
(23)

This equation says that if the production of nondurables rises, then employment in the durable sector must fall in a model with separable preferences. The intuition behind this result is straightforward. Following a monetary expansion, nondurable goods firms with sticky prices increase production to meet demand instead of raising their prices. The incipient increase in output in the nondurable goods sector increases the demand for labor, which in turn raises marginal costs (i.e., an increase in disutility of work  $v'(\cdot)$  and a decrease in marginal product of labor  $f(\cdot)$ ). For producers in the flexibly priced durable goods sector, the increase in marginal costs is merely an adverse supply shock. Because there are no forces that can offset a rise in the cost of production, this definitely lowers the labor employed in the durable goods sector and thus the production of durable goods falls.

However, things might be different in a model with non-separable preferences. When  $\sigma < 1$ , the cross-partial derivative,  $-U_{LC}$  is negative in our monetary utility function, (2), which means that the increased level of nondurable consumption would reduce the marginal disutility of work. According to (22), the fact that the cross-partial derivative  $-U_{LC} < 0$  implies that increased nondurable consumption shifts the labor supply curve out. This mitigates the rise in the nominal wage and marginal cost of producing durables and therefore it moderates the contraction in durable production. Since the complementarity between the consumption of nondurables and labor is decreasing with the parameter  $\sigma$  (i.e., $\partial |-U_{LC}|/\partial \sigma < 0$ ), the extent to which production in the durable sector contracts gets smaller as  $\sigma$  takes lower values than unity. When  $\sigma$  takes values below approximately 0.05, the strength of the complementarity is sufficient enough to induce an increase in the production of durable goods following a monetary expansion.

#### 4.1.3 What determines the range of the $\sigma$ consistent with sectoral comovement?

It is instructive to log-linearize the equation (22) around a deterministic steady state to understand what factors determine the range of the parameter  $\sigma$  in which the model generates sectoral comovement. Define  $\eta_{LL} \equiv \left(\frac{-U_{LL}L}{-U_L}\right)\Big|_{ss} > 0$  as the own elasticity of marginal disutility from labor and  $\eta_{LC} \equiv \left(\frac{-U_{LC}C}{-U_L}\right)\Big|_{ss}$  as the cross-elasticity of marginal disutility from labor with respect to nondurable consumption, evaluated at the steady state.<sup>7</sup> Log-linearizing (22) around a non-stochastic steady state yields

$$\eta_{LC}\widehat{C}_t + \eta_{LL}(\omega_c\widehat{L}_{c,t} + \omega_x\widehat{L}_{x,t}) = -\alpha(\omega_c\widehat{L}_{c,t} + \omega_x\widehat{L}_{x,t}),$$
(24)

where a circumflex ("hat") over a variable represents proportionate deviations of that variable from its steady state and  $\omega_j = L_j/L$  in sector j = c, x. Using the fact that all firms choose the same capital-to-labor ratio, we can write  $C_t = \overline{K}_{c,t}^{\alpha} L_{c,t}^{1-\alpha} = \left(\frac{\overline{K}}{L_t}\right)^{\alpha} L_{c,t}$  and log-linearize it as follows:

$$\widehat{C}_t = \kappa_c \widehat{L}_{c,t} - \omega_x \widehat{L}_{x,t}.$$
(25)

Here  $\kappa_c \equiv \frac{\partial \widehat{C}_t}{\partial \widehat{L}_{c,t}} = (1 - \alpha \omega_c)$  is the elasticity of nondurable production with respect to labor in the nondurable. Combining equation (25) with equation (24) yields

$$(-\eta_{LC}\alpha + \eta_{LL} + \alpha)\omega_x \widehat{L}_{x,t} = (-\eta_{LC}\kappa_c - (\eta_{LL} + \alpha)\omega_c)\widehat{L}_{c,t}.$$
(26)

This equation confirms a previous discussion that unless labor supply and the consumption of nondurables are complementary (i.e.,  $\eta_{LC} < 0$ ), it is impossible to obtain sectoral comovement. Given that  $\eta_{LC} < 0$ , the condition that generate sectoral comovement is

$$-\eta_{LC}\kappa_c > \nu, \tag{27}$$

where  $\nu = (\eta_{LL} + \alpha)\omega_c$ . This condition has an intuitive interpretation. As discussed, when  $L_{c,t}$  rises to meet higher demand in the nondurable goods following a monetary expansion, it has two offsetting effects on the costs of durable good production. The first term,  $-\eta_{LC}\kappa_c$ , quantifies the extent to which an increase in  $L_{c,t}$  lowers costs of durable good production through the complementarity between labor supply and nondurable consumption. Thus, high values for  $\kappa_c$  strengthen the effects of the complementarity on lowering costs of durable good production.

<sup>&</sup>lt;sup>7</sup>These elasticities are expressed as  $\eta_{LL} = \frac{(1-\sigma)^2}{\sigma} \frac{WL}{\rho_C C} \frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}} + \frac{1}{\eta}$  and  $\eta_{LC} = (1 - \frac{1}{\sigma}) \frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}}$ , respectively.

The second term, v, denotes the extent to which an increase in  $L_{c,t}$  raises costs of durable good production. In this case,  $(\eta_{LL} + \alpha)\omega_c$  corresponds to v since an increase in  $L_{c,t}$  induces a higher disutility of work and a lower marginal product of labor. When the former dominates the latter, a positive response of durable goods production is obtained.

More importantly, the condition above clearly identifies what factors determine the range of the complementarity consistent with sectoral comovement. Higher values of  $\kappa_c$  and lower values of  $\nu$  enable the model to generate sectoral comovement with a smaller degree of the complementarity,  $-\eta_{LC}$ . Below, we show that time-varying capital utilization with imperfect capital mobility increases the value of  $\kappa_c$  and lowers the value of  $\nu$ .

#### 4.2 Model with Imperfect Capital Mobility and Variable Utilization

We modify the assumption of perfect inter-sectoral capital mobility and constant capital utilization. We instead treat productive capital in each sector as a predetermined fixed factor but allow firms to vary their capital utilization rate. These features are consistent with empirical evidence that physical capital is difficult to reallocate across sectors<sup>8</sup> and the capital utilization rate displays pronounced procyclical variability.<sup>9</sup> Figure 2 plots the reaction of the model to the monetary shock when capital is immobile across sectors but capital utilization rate in each sector is allowed to vary. Interestingly, imperfect capital mobility and time-varying capital utilization substantially expand the range of values for the parameter  $\sigma$  that enables the model to generate sectoral comovement. An upper bound for the value of  $\sigma$  to generate a positive response of durable goods is now significantly increased to approximately 0.55. Furthermore, as  $\sigma$  gets lower, the model exhibits the response of durable goods relative to that of nondurable goods quite similar to what observed in the data. For example, when  $\sigma = 0.2$ , durable production rises on impact almost twice more than nondurable production.

To understand why imperfect capital mobility and time-varying capital utilization lead to a wider range of  $\sigma$  that generates sectoral comovement, it is useful to inspect something analogous

<sup>&</sup>lt;sup>8</sup>Ramey and Shapiro (1998) find in their case study of aerospace plant closings that transferring equipment to another sector is costly and that a large discount is required to entice buyers from outside the sector.

<sup>&</sup>lt;sup>9</sup>The fact that equipment and machinery are used more intensively in booms than in recessions is corroborated by the procyclical character of electricity consumption in manufacturing industries (Burnside et al., 1995) and by the fact that expansions are accompanied by the use of two and three shifts in manufacturing sector (Shapiro, 1993).

to (22) in this case, which is given by

$$- U_L(C_t, D, L_{x,t} + L_{c,t}) = \frac{\gamma_x}{\mu} \left( \frac{u_{x,t} \overline{K}_x}{L_{x,t}} \right)^{\alpha}.$$
 (28)

With imperfect capital mobility, labor in the nondurable sector has no impact on the marginal product of labor in the durable sector. Increased labor demand in the nondurable sector raises the cost of production in the durable sector only through a higher disutility of work. Hence, v, which measures the extent to which an increase in  $L_{c,t}$  raises the costs of durable production, decreases from  $(\eta_{LL} + \alpha)\omega_c$  to  $\eta_{LL}\omega_c$ . Thus, a smaller complementarity is required to offset the rise in costs of production.

In addition, time-varying capital utilization also helps to expand the range of the parameter  $\sigma$  consistent with sectoral comovement since it increases the value for  $\kappa_c \equiv \frac{\partial \widehat{C}_t}{\partial \widehat{L}_{c,t}}$ . Variable capital utilization makes the supply of capital services strongly responsive to changes in the labor so that it is sometimes described as leading to short-run production that is nearly linear in labor.<sup>10</sup> As Shapiro (1993) shows, for example, increases in labor are accompanied by increases in the workweek of capital, one measure of capital utilization. Loosely speaking, this observation allows us to write production function as

$$C_t = (u_{c,t}\overline{K}_c)^{\alpha} L_{c,t}^{1-\alpha} = \left(\frac{u_{c,t}\overline{K}_c}{L_{c,t}}\right)^{\alpha} L_{c,t} \simeq BL_{c,t},$$

where  $B = \left(\frac{u_c \overline{K}_c}{L_c}\right)^{\alpha}$ . Thus, variable capital utilization might increase  $\kappa_c$ , the elasticity of nondurable goods production with respect to  $L_{c,t}$ , from  $(1 - \alpha S_c)$  to 1.

Obviously, the extent to which variable capital utilization increases  $\kappa_c$  depends on how costly varying capital utilization is, which is controlled by the parameter  $\chi$ . When varying utilization becomes more costly (i.e., higher  $\chi$ ), an increase in  $\kappa_c$  tends to be smaller, so that a stronger complementarity is required for the model to obtain sectoral comovement. For this reason, we carry out some analysis of the sensitivity of our results to the values of the parameter  $\chi$ . Figure 3 portrays the initial responses of durable and nondurable goods production for different values of  $\sigma$  and  $\chi$ . As  $\chi$  increases, an upper bound on the value for  $\sigma$  that leads to a positive response

<sup>&</sup>lt;sup>10</sup>Previous papers that have used this type production function include Bils and Cho (1994) and Ramey and Shapiro (1998).

of durable goods production gets smaller. For example, when  $\chi = 5$ , the value for  $\sigma$  needs to be less than 0.2 to generate sectoral comovement.

## 5 Modification

We consider additional modifications to the model, which even further decrease the threshold level of complementarity needed for generating sectoral comovement. In particular, we investigate the consequence of incorporating indivisible labor and imperfect labor mobility separately into the model with variable capital utilization and imperfect capital mobility.

#### 5.1 Adding Indivisible Labor

We first examine economies with indivisible labor and lotteries, considered by Rogerson (1988) and Hansen (1985). In this economy, all members in the household are identical but agree on an efficient contract, which allocates some individuals to work a fixed shift of *H* hours in each period while leaving the remaining idle. The typical treatment of the indivisible labor model, as in Rogerson (1988) and Hansen (1985), involves assuming separable preferences. In contrast, we generalize the indivisible labor model with more general preferences, which nest both separable and non-separable preferences. A remarkable feature of this approach is that a stand-in household's preference still retains the original non-separability property and yet  $\eta_{LL}$ , the elasticity of marginal disutility from labor, is substantially reduced.

Let us use the subscript 1 to denote those members who are assigned to work by the random lottery draw and the subscript 2 to refer to the unemployed members. An efficient risk-sharing condition involves maximizing the expected utility of a household prior to the lottery draw subject to feasible allocations of consumptions across the employed and the unemployed:

max 
$$n_t U(C_{1t}, S_{1t}, H) + (1 - n_t)U(C_{2t}, S_{2t}, 0)$$
  
subject to  $n_t C_{1t} + (1 - n_t)C_{2t} = C_t$ , and  $n_t S_{1t} + (1 - n_t)S_{2t} = S_t$ ,

where  $n_t$  is the fraction of the population assigned to work, and *C* and *S* denote consumptions of nondurables and durable services. The efficient risk-sharing condition in this situation of employment lotteries, as in many other contexts, is that the marginal utility of consumption must be equated across types. Using our momentary utility function (2), the condition implies that consumption allocations must satisfy

$$C_{1t} = C_{2t} \left( 1 + \left( \frac{1}{\sigma} - 1 \right) v(H) \right) \quad \text{and} \quad S_{1t} = S_{2t} \left( 1 + \left( \frac{1}{\sigma} - 1 \right) v(H) \right).$$
(29)

According to this specification, if  $\sigma < 1$  there will be more consumption allocated to the employed (group 1) than to the unemployed (group 2). Using these consumption rules along with the expected utility objective,  $n_t U(C_{1t}, S_{1t}, H) + (1 - n_t)U(C_{2t}, S_{2t}, 0)$ , there is a stand-in representative household whose preferences are

$$U(C_t, S_t, L_t) = \frac{Z_t^{1 - \frac{1}{\sigma}} \left( 1 + \left(\frac{1}{\sigma} - 1\right) v^*(L_t) \right)^{\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},$$
(30)

where  $L_t = n_t H$  is the average hours worked and  $v^*(L_t) = L_t v(H)/H$ . There are two points about this expression. First, the stand-in's utility function inherits the original utility function's properties with respect to the effects of changes in nondurable consumption on labor. In particular, when  $\sigma < 1$ , aggregate consumption and labor supply are complementary. Second, a new functional form measuring the disutility incurred from hours worked,  $v^*(L_t)$  is now linear in  $L_t$ , suggesting that  $\eta_{LL}$ , the elasticity of marginal disutility from labor, substantially decreases.

To investigate the extent to which indivisible labor expands the range of  $\sigma$  consistent with sectoral comovement, let us again simulate the model using the new momentary utility function (30). We retain other features of the model such as variable capital utilization and imperfect capital mobility. Figure 4 presents the reaction of the model to the monetary shock. In striking contrasts to the previous cases, the model generates sectoral comovement even though  $\sigma$  is slightly less than unity. Even when  $\sigma = 0.98$ , for example, durable production rises by 0.44% following a monetary expansion. Moreover, even with a relatively modest degree of complementarity, the model produces the response of durable goods consistent with the empirical response in Erceg and Levin (2006). With  $\sigma = 0.93$ , the response of durable production is almost about 3.5 times larger than that of nondurable production.

To better understand the results, it is useful to consider the condition required to generate

sectoral comovement in this case:

$$-\eta_{LC}\kappa_c > \nu = \eta_{LL}S_c,\tag{31}$$

where  $\eta_{LC} = (1 - \frac{1}{\sigma})Z_c$ ,  $\eta_{LL} = \frac{(1-\sigma)^2}{\sigma}\frac{WL}{P_CC}Z_c$  and  $Z_c = \frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho}+\psi_d D^{1-1/\rho}}$ . Notice the change in an expression for  $\eta_{LL}$  due to the introduction of indivisible labor. Compared to the expression for  $\eta_{LL}$  in footnote 7,  $1/\eta$  disappears with indivisible labor because a new functional form measuring the disutility incurred from hours worked is now linear. Hence,  $\nu$ , the extent to which an increase in  $L_{c,t}$  raises costs of durable good production, is significantly mitigated, so that a modest degree of complementarity will be enough to generate sectoral comovement. Substituting expressions for  $\eta_{LC}$  and  $\eta_{LL}$  into (31), one can obtain the exact condition for the model to generate sectoral comovement:

$$\left(\frac{1}{\sigma} - 1\right) Z_c \left(\kappa_c - (1 - \sigma) \frac{WL_c}{P_c C}\right) > 0.$$
(32)

Note that  $\left(\kappa_c - (1 - \sigma)\frac{WL_c}{P_cC}\right) > 0$ . This is because a minimum value for  $\kappa_c \equiv \frac{\partial \widehat{C}_t}{\partial \widehat{L}_{c,t}}$  is  $(1 - \alpha)$ , which corresponds to a case where varying utilization is extremely costly (i.e.,  $\chi = \infty$ ) and  $\frac{WL_c}{P_cC} = (1 - \alpha)\frac{\varepsilon - 1}{\varepsilon}$ .<sup>11</sup> As long as  $\sigma < 1$ , therefore, the condition (32) is satisfied irrespective of the values for  $\chi$ . That is, the introduction of indivisible labor enables the model to generate sectoral comovement with a small degree of complementarity even without variable capital utilization.

This does not imply that variable capital utilization has no impact on the behavior of the model. It turns out that variable capital utilization plays an important role in amplifying the response of durable production. To see this, we examine how our quantitative results change as we vary values for  $\chi$ . Figure 5 portrays the initial responses of durable and nondurable goods production for different values of  $\sigma$  and  $\chi$ . While increasing  $\chi$  does not affect the range of  $\sigma$  consistent with sectoral comovement that much, it substantially dampens the response of durable production. For example, when  $\chi = 5$ , both nondurable and durable production. In contrast, when  $\chi = 0.01$ , the model produces a realistic relative volatility of durable production. Hence, when indivisible labor is introduced, higher elasticity of capital utilization works to amplify the

<sup>&</sup>lt;sup>11</sup>To derive this, we use the following equations. In a steady state,  $P_c = \zeta_c \frac{\varepsilon}{\varepsilon-1}$  and  $\zeta_c = \frac{W}{MPL_c} = \frac{W}{(1-\alpha)C/L_c}$  where  $\zeta_c$  is a marginal cost.

increase in durable production due to the complementarity.

#### 5.2 Adding Imperfect Labor Mobility

We briefly present an alternative way to produce almost the same results as in the model with indivisible labor. It involves relaxing the assumption that labor is perfectly substitutable and mobile across sectors, which insulates durable sector from rising costs in the nondurable sector. While assuming the same momentary utility function as (2), we instead modify the definition of aggregate hours,  $L_t$ , in the following manner:

$$L_t = \left[ L_{c,t}^{\frac{\vartheta+1}{\vartheta}} + L_{x,t}^{\frac{\vartheta+1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta+1}},$$
(33)

where  $\vartheta$  is the elasticity of substitution between hours worked in different sectors. This aggregator represents the idea that labor is imperfectly mobile across sectors and the case of perfect labor mobility corresponds to the case where  $\vartheta \rightarrow \infty$ . We assume that  $\vartheta = 1$  based upon the empirical work by Horvath (2000). He estimates  $\vartheta$  from an Ordinary Least Square regression of the change in relative labor supply on the change in relative labor share using U.S. sectoral data and find  $\vartheta = 0.9996$ . The analog to equation (28) for the case of imperfect labor mobility is

$$- U_L(C_t, D, L_t) \frac{\partial L_t}{\partial L_{x,t}} = \frac{\gamma_x}{\mu} \left( \frac{u_{x,t} \overline{K}_x}{L_{x,t}} \right)^{\alpha}.$$
 (34)

This expression shows that imperfect labor mobility mitigates the rise in costs of durable production due to an expansion in nondurable sector because  $\frac{\partial^2 L_t}{\partial L_{c,t} \partial L_{x,t}} < 0$ . To be more specific, once we log-linearize (34), it is easy to verify that the condition for the model to generate sectoral comovement in this case is given by

$$-\eta_{LC}\kappa_c > \nu = \frac{(1-\sigma)^2}{\sigma} \frac{W_c L_c}{P_C C} Z_c + \frac{1}{\eta} - \frac{1}{\vartheta},$$
(35)

where  $\eta_{LC} = (1 - \frac{1}{\sigma})Z_c$ . The extent to which imperfect labor mobility mitigates the rise in costs of durable production is captured by  $\frac{1}{\vartheta}$ . More importantly, given our parameterization (i.e.,  $\eta = 1, \vartheta = 1$ ),  $\nu$  reduces to the one with indivisible labor. As in the indivisible labor model, therefore, only a very modest degree of complementarity will be necessary to generate sectoral comovement in a model with imperfect labor mobility.

## 6 Empirical Evidence on the Non-Separable Preference

In this section we estimate the intertemporal elasticity of substitution ( $\sigma$ ) to investigate whether non-separable utility in consumption and labor is compatible with the data. While Basu and Kimball (2002) and Guerron-Quintana (2008) present empirical evidence against additively separable preferences, their empirical specifications omit the role of durables in estimating the intertemporal elasticity of substitution. As argued in Ogaki and Reinhart (1998), ignoring durables might lead to misleading results in estimating the degree of the intertemporal substitution. Therefore, we augment Basu and Kimball (2002)'s approach by explicitly considering the role of durables in the estimation.

One difficultly in estimating the intertemporal elasticity of substitution with durable consumption is that the data on service flow from durable goods stock is usually difficult to measure. To get around with this problem, we express our estimation equation in terms of the expenditure share of nondurable goods as in Piazzesi et al. (2007).<sup>12</sup> By doing so, we can avoid using quantity data on durable goods consumption, which is very difficult to obtain and is subject to measurement errors. The expenditure data is much easier to obtain and is more accurate than the quantity data.

#### 6.1 Estimation Strategy

Consider the Euler equation,

$$1 = E_t[M_{t+1}(1+i_t)], (36)$$

where  $M_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{U_C(C_{t+1}, D_{t+1}, L_{t+1})}{U_C(C_t, D_t, L_t)}$ . Since the pricing kernel  $M_{t+1}$  involves quantities of durable consumption, for which available data are not reliable, we will get around the problem by using expenditure share of durable goods consumption, as in Piazzesi et al. (2007).

First, we will start with the optimality condition that the marginal rate of substitution

<sup>&</sup>lt;sup>12</sup>A more straightforward way to resolve this problem would be, as in Ogaki and Reinhart (1998), to use the qualityadjusted data on durable goods constructed by Gordon (1990). However, Gordon's data are available only up to 1983:4.

between non-durable and durable consumption equals the relative price. With the preference (2) and with the assumption of  $D_t = S_t$ , we have

$$\frac{P_{c,t}}{P_{s,t}} = \frac{\psi_c}{\psi_d} \left(\frac{C_t}{S_t}\right)^{-\frac{1}{\rho}},\tag{37}$$

where  $P_{s,t}$  is the user cost of the service flow of the durable and equals  $P_{x,t} - E_t \left[\beta \frac{(1-\delta)\lambda_{t+1}P_{x,t+1}}{\lambda_t}\right]$ . Multiplying both sides by  $C_t/S_t$ , we obtain the expenditure ratio

$$g_t = \frac{P_{c,t}C_t}{P_{s,t}S_t} = \frac{\psi_c}{\psi_d} \left(\frac{C_t}{S_t}\right)^{1-\frac{1}{\rho}}$$
(38)

and thus the expenditure share of nondurable consumption can be written as

$$h_t = \frac{P_{c,t}C_t}{P_{c,t}C_t + P_{s,t}S_t} = \frac{g_t}{1 + g_t}.$$
(39)

Using the nondurable expenditure share  $h_t$ , with some algebra, we can rewrite the pricing kernel as

$$M_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}} \left(\frac{h_{t+1}}{h_t}\right)^{\frac{\rho-\sigma}{\sigma(\rho-1)}} \left(\frac{1 + \left(\frac{1}{\sigma} - 1\right) v(L_{t+1})}{1 + \left(\frac{1}{\sigma} - 1\right) v(L_t)}\right)^{\frac{1}{\sigma}} \left(\frac{P_{c,t}}{P_{c,t+1}}\right).$$
(40)

As in Basu and Kimball (2002), by taking the logarithm, we express the Euler equation

$$0 = E_t \left[ \log(M_{t+1}) + \log(1+i_t) \right] + \text{high order terms}$$

$$= E_t \left[ \left( -\frac{1}{\sigma} \right) \log \left( \frac{C_{t+1}}{C_t} \right) + \left( \frac{\rho - \sigma}{\sigma(\rho - 1)} \right) \log \left( \frac{h_{t+1}}{h_t} \right) + (i_t - \pi_{t+1}^c + \log \beta) + \frac{1}{\sigma} \log \left( \frac{\tilde{v}(L_{t+1})}{\tilde{v}(L_t)} \right) \right]$$

$$+ \text{higher order terms, (42)}$$

where  $\pi_{t+1}^c = \log(P_{c,t+1}/P_{c,t})$  and  $\tilde{v}(L_t) = 1 + (\frac{1}{\sigma} - 1)v(L_t)$ . The higher order terms are non-negative due to the Jensen's inequality.

By rearranging terms and making explicit a rational expectation error term,  $e_t$ , we obtain

$$\Delta \log(C_t) = \sigma(i_t - \pi_t^c + \log \beta) + \left(\frac{\rho - \sigma}{\rho - 1}\right) \Delta \log(h_t) + \Delta \log \tilde{v}(L_t) + e_t + \text{higher order terms.}$$
(43)

The final step is to use a first-order Taylor expansion of  $\log (\tilde{v}(L_t)) = \log \left(1 + \left(\frac{1}{\sigma} - 1\right) v(L_t)\right)$  around

the steady-state level of labor L and to take the first difference. This gives us

$$\Delta \log \tilde{v}(L_t) = \Delta \log \left( 1 + \left(\frac{1}{\sigma} - 1\right) v(L_t) \right) \approx \frac{L\left(\frac{1}{\sigma} - 1\right) v'(L)}{1 + \left(\frac{1}{\sigma} - 1\right) v(L)} \Delta \log(L_t).$$
(44)

We can identify the coefficient  $\frac{L(\frac{1}{\sigma}-1)v'(L)}{1+(\frac{1}{\sigma}-1)v(L)}$  by using the household's optimality condition for labor supply in the steady state,  $-\frac{U_L}{U_c} = \frac{W}{P_c}$ . The coefficient can be uncovered as

$$\frac{L\left(\frac{1}{\sigma}-1\right)v'(L)}{1+\left(\frac{1}{\sigma}-1\right)v(L)} = (1-\sigma)\frac{WL}{P_cC}\frac{\psi_c C^{1-1/\rho}}{z^{1-1/\rho}} = (1-\sigma)\frac{WL}{P_cC}\frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}} = (1-\sigma)\frac{WL}{P_cC}h.$$
 (45)

As a result, we arrive at the following estimation equation:

$$\Delta \log(C_t) - \tau h \Delta \log(L_t) - \left(\frac{\rho}{\rho - 1}\right) \Delta \log(h_t)$$
  
= constant +  $\sigma \left(i_t - \pi_t^c - \left(\frac{1}{\rho - 1}\right) \Delta \log(h_t) - \tau h \Delta \log(L_t)\right) + e_t$ , (46)

where  $\tau = \frac{WL}{P_cC}$  is the steady state ratio of the household's wage income to nondurable consumption expenditure. We calibrate the values of  $\tau$  and h by looking at the long-run averages and impose the value of  $\rho$  based on earlier studies. In this sense, our estimates are conditional on these values.

#### 6.2 Data

Our sample starts from the first quarter of 1950 and ends at the fourth quarter of 2006. Consumption expenditure data and consumption price indices are taken from the NIPA Table 2.3.5 and Table 2.3.4, respectively. We define the universe of consumption goods as the sum of nondurable goods and services. We regard real estate as durable goods. One advantage of treating real estate as durable goods is that NIPA provides a measure of the expenditure on housing services. By using this, we compute the nondurable expenditure share in the total consumption,  $h_t$ .

The growth rate of nondurable consumption  $\Delta \log(C_t)$  and nondurable-goods inflation rate  $\pi_t^c$  are constructed by excluding housing services from our definition of total consumption.<sup>13</sup>

<sup>&</sup>lt;sup>13</sup>This involves aggregation of subcategories in the NIPA table. This procedure is done by calculating the Törnqvist

Data on the interest rate, hours worked, and population are obtained from the FRED of FRB St. Louis. Monthly series are converted to quarterly by using the end-of-quarter observations. The interest rate is measured by the 3-month Treasury yield at a quarterly rate. In order to construct the after-tax interest rate, we mainly rely on the tax data on interest received, which are obtained from the TAXSIM of the NBER.<sup>14</sup> However, the TAXSIM data set only covers from 1966 to present. We use Barro and Sahasakul (1986) for the periods earlier than 1966. Consumption and hours worked are converted to per capita terms by using the Civilian Noninstitutional Population. We use the Non-Farm Business Sector Hours of All Persons for total hours worked.

As mentioned above, we calibrate the values of  $\tau$  and h by looking at the long-run averages. For  $\tau = \frac{WL}{P_cC}$ , it is important to point out that WL is the after-tax labor income earned by the household. Total labor income is obtained from NIPA Table 2.1 as the sum of the compensation of employees and proprietors' income. We use the average marginal income tax rate on wages from the NBER TAXSIM to obtain the after-tax labor income. By taking the average over the periods from 1966 to 2006, we estimate the value of  $\tau$  to be 1.1425. The long-run average value of the expenditure share on nondurable consumption h is 0.8320. Finally, we set  $\rho = 1.17$  based on an estimate reported in Ogaki and Reinhart (1998). As discussed in Piazzesi et al. (2007), although it is difficult to obtain a precise estimate of  $\rho$ , they argue that collective evidence suggest that  $\rho$  is likely to be greater than unity. Since our estimate is conditional on the value of  $\rho$  (and on  $\tau h$ ), we will perform sensitivity analysis on these two calibrated parameters later on.

#### 6.3 Results

We estimate  $\sigma$  in (46) by using two stage least squares with twice lagged values of  $\Delta \log(C_t)$ ,  $\Delta \log(h_t)$ ,  $\Delta \log(L_t)$ ,  $i_t$  and,  $\pi_t^c$  as instruments. To address the time aggregation issue, as in Basu and Kimball (2002), we allow for serial correlation in the error term, so that  $e_t = \varepsilon_t + \theta \varepsilon_{t-1}$ .

Given  $\tau h = 0.9506$  and  $\rho = 1.17$ , we obtain the following result:

$$\hat{y}_t = \underbrace{0.0146}_{(0.0012)} + \underbrace{0.3845x_t}_{(0.1539)},\tag{47}$$

index. A detailed description is available upon a request.

<sup>&</sup>lt;sup>14</sup>http://www.nber.org/~taxsim/

where  $y_t(\tau h, \rho) = \Delta \log(C_t) - \tau h \Delta \log(L_t) - \left(\frac{\rho}{\rho-1}\right) \Delta \log(h_t)$  and  $x_t(\tau h, \rho) = i_t - \pi_t^c - \left(\frac{1}{\rho-1}\right) \Delta \log(h_t) - \tau h \Delta \log(L_t)$ . The standard errors are reported in the parentheses. The estimated  $\sigma$  of 0.3845 is statistically significant with the *p*-value 0.0132. This result supports for non-separable preferences over the conventional separable preferences with  $\sigma = 1$ .

Although this estimate is not small enough to support the model with perfect capital mobility in Section 4.1, the estimated intertemporal elasticity of substitution is sufficient for generating the sectoral comovement and a realistic volatility of durables in other versions of models presented above. Our point estimate is larger than those reported in Basu and Kimball (2002) (from 0.05 to 0.30),<sup>15</sup> and it is closer to ones estimated by Ogaki and Reinhart (1998) (from 0.32 to 0.45).

As noted earlier, our result is conditional on the calibrated parameters ( $\tau h$  and  $\rho$ ). Figure 6 presents our sensitivity analysis on  $\sigma$ . The top panel plots different point estimates of  $\sigma$  obtained from changing the elasticity of substitution between  $C_t$  and  $S_t \rho$ , together with a two-standarderror band (dashed lines). It appears that  $\hat{\sigma}$  is somewhat sensitive to the value of  $\rho$  used. As  $\rho$  becomes higher,  $\hat{\sigma}$  tends to increase and approach toward unity. Even though the point estimates stay below zero, as  $\rho$  increases, the two-standard-error band becomes wider and it becomes very difficult to tell whether  $\hat{\sigma} = 1$  or not. However, for realistic values of  $\rho$ , the estimation results suggest that the preference is non-separable.

We also examine sensitivity with respect to the steady-state ratio of labor income to nondurable consumption expenditure  $\tau$  and the steady-state share of nondurable consumption expenditure *h*. The bottom panel of Figure 6 depicts  $\hat{\sigma}$  against different values of  $\tau h$ . Unlike the case of  $\rho$ , the results are robust to large changes in  $\tau h$ . In all cases considered, all estimates stay statistically significant.

Overall, the estimation results based on a broad set of calibrated parameters support nonseparable preferences. Except for the model with perfect capital mobility and constant capital utilization, our estimates for  $\sigma$  are well within the range needed for the sticky price model to generate sectoral comovement.

<sup>&</sup>lt;sup>15</sup>See Table 1c in their paper, which reports the estimated intertemporal elasticity of substitution using the sample from 1949:1-1999:2.

## 7 Conclusion

In the data, strong procyclical fluctuations in the production of durable goods are the most prominent feature of the response to monetary shocks. This paper investigates the role of preferences in matching this feature of the data in a sticky price model with flexibly priced durables. The separability between aggregate consumption and labor supply plays an important role in shaping the reaction of durable goods production. When preferences are separable in aggregate consumption and labor, the model exhibits counterfactual behavior. Flexibly priced durable goods production contracts substantially following a monetary expansion if preferences are separable.

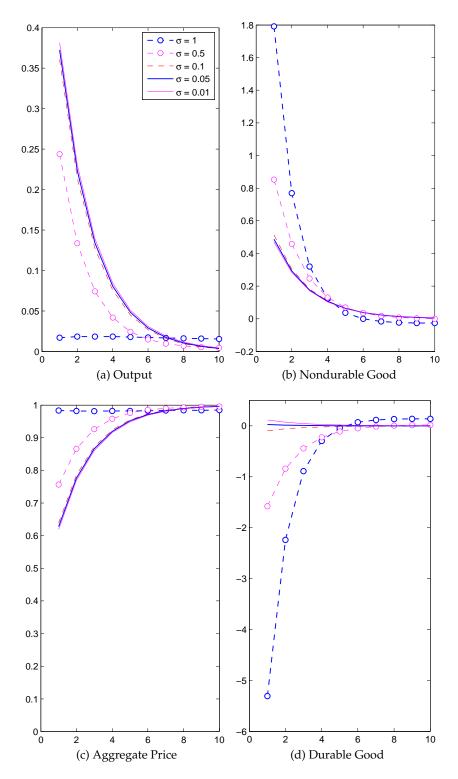
In contrast, the sticky price model with non-separable preferences can replicate the empirically plausible response of durable goods spending to a monetary expansion. The key to the model's success is due to the fact that non-separable preferences imply the complementarity between nondurable consumption and labor supply, which is absent with separable preferences. Variable capital utilization, indivisible labor, and inflexible factor mobility, either separately or in combination, significantly reduce the threshold level of non-separability needed for the model to generate a strong procyclical response of durable spending to a monetary shock.

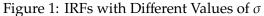
Further, this paper provides empirical evidence against the assumption of additive separability between nondurables and labor, which had previously been shown by Basu and Kimball (2002) and Guerron-Quintana (2008). Our empirical strategy in estimating the intertemporal elasticity of substitution, which governs the degree of non-separability, improves upon the previous estimates by explicitly account for the role of durables in the estimation. We find that even when the durables are explicitly included in the estimation, the data still support non-separable preferences. Our estimates for the intertemporal elasticity of substitution are well below the threshold level needed for the sticky price model to produce a strong procyclical response of durable goods spending to a monetary shock.

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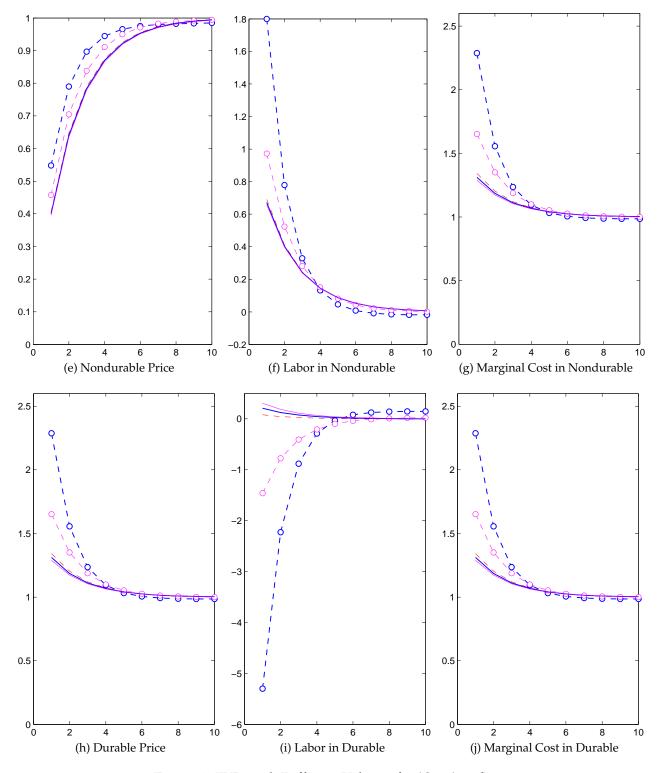
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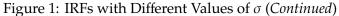
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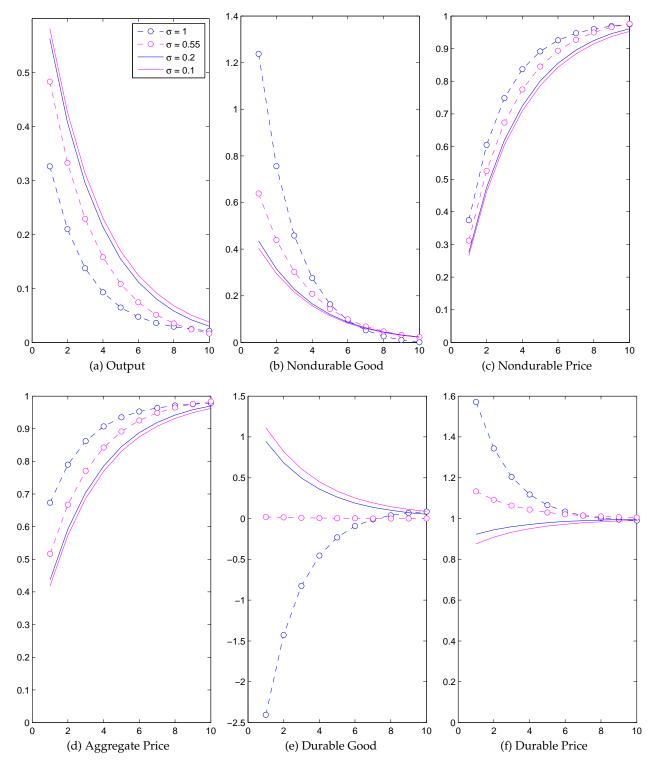


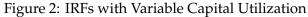
*Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.





*Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.





*Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.

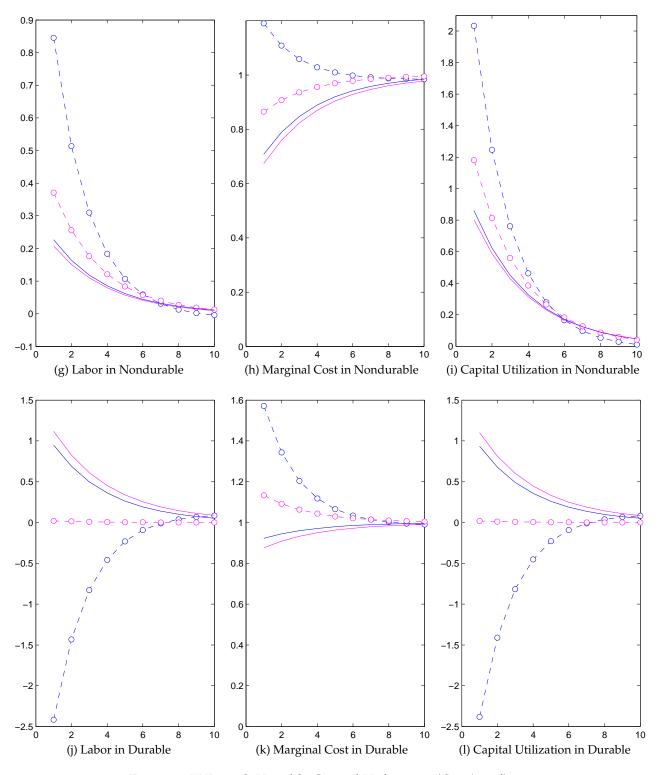
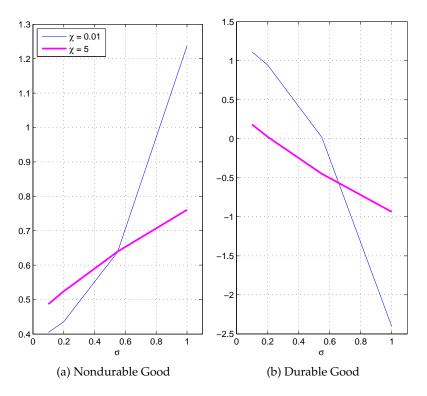
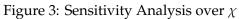


Figure 2: IRFs with Variable Capital Utilization (Continued)

*Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.





*Note:* Vertical axes measure percentage deviations from the steady-state values at the impact period in response to a one percentage increase in money supply, given a value of  $\sigma$  on horizontal axes.

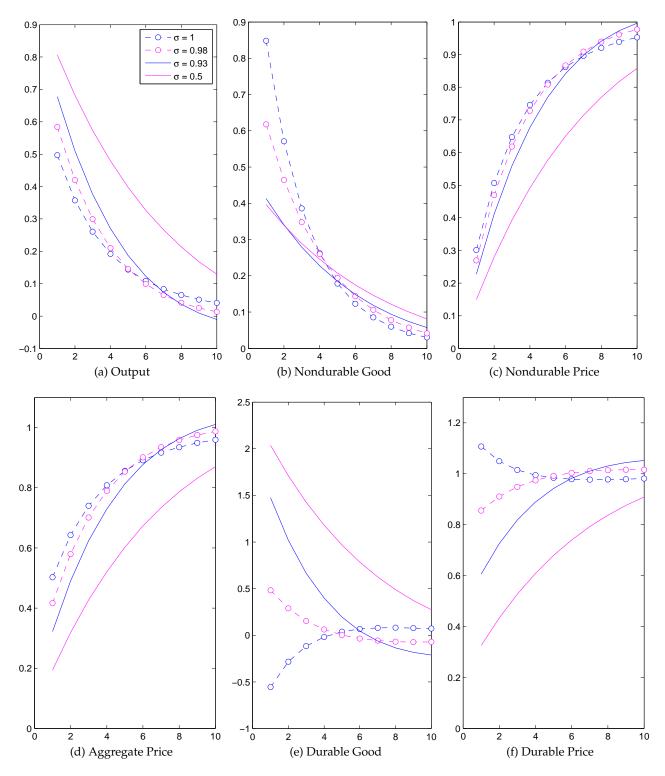


Figure 4: IRFs with Variable Capital Utilization and Indivisible Labor *Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.

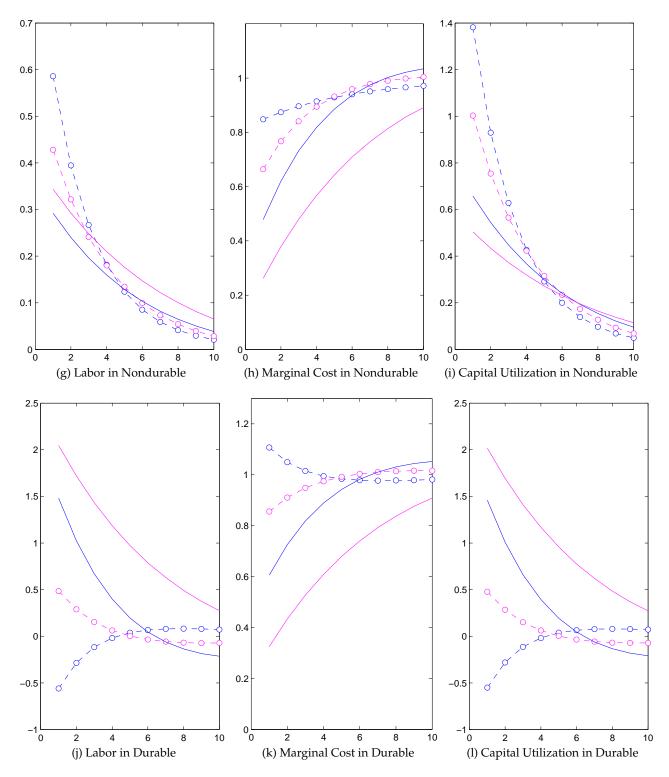


Figure 4: IRFs with Variable Capital Utilization and Indivisible Labor (*Continued*) *Note:* Vertical axes measure percentage deviations from the steady-state values in response to a one percentage increase in money supply. Horizontal axes take period and the shock hits the economy in the first period.

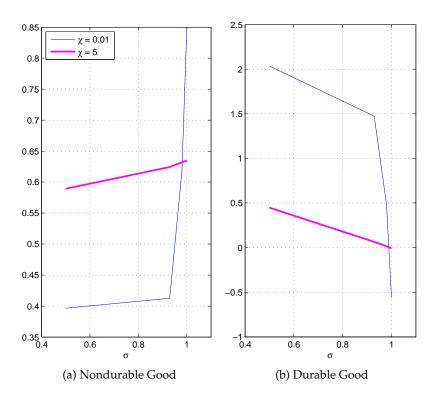


Figure 5: Sensitivity Analysis over  $\chi$  with Indivisible Labor *Note:* Vertical axes measure percentage deviations from the steady-state values at the impact period in response to a one percentage increase in money supply, given a value of  $\sigma$  on horizontal axes.

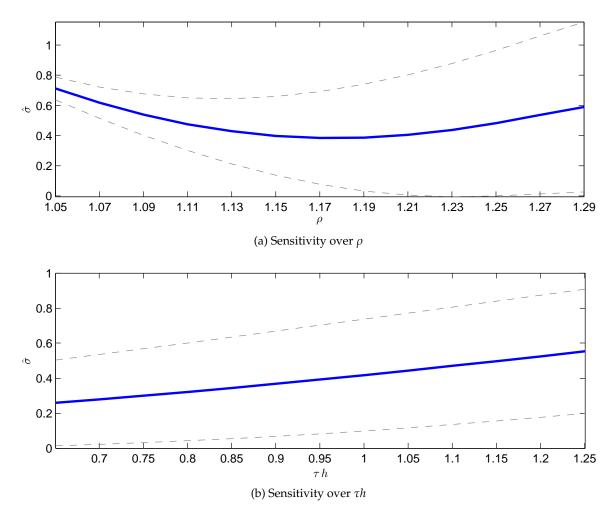


Figure 6: Robustness Check on  $\hat{\sigma}$ 

*Note:* The top panel shows the estimated value of  $\sigma$  with different values of the elasticity of substitution between nondurable and durable consumption,  $\rho$ . The range of  $\rho$  roughly corresponds to  $\pm$  one standard deviation of  $\hat{\rho}$  reported in Ogaki and Reinhart (1998). The bottom panel of this figure presents the sensitivity of the estimated  $\sigma$  with respect to the value of  $\tau h$ , where  $\tau$  is the steady-state ratio of labor income to nondurable consumption expenditure and h is the steady-state share of nondurable consumption expenditure. The dashed lines represent two-standard-error band. In our estimation, we use  $\rho = 1.17$  and  $\tau h = 0.9506$ .

Table 1: Initial Responses of Nondurable and Durable Production

σ	1	0.5	0.1	0.05	0.01	0.001
Nondurable	1.7912	0.8520	0.5128	0.4874	0.4692	0.4653
Durable	-5.3060	-1.5806	-0.0962	0.0271	0.1181	0.1378

Note: Values reported above are percentage deviations from the steady-state values at the impact period in response to a one percentage increase in money supply, given the value of  $\sigma$ .